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THE OPTIMAL PRICING OF COMPUTER SOFTWARE AND OTHER PRODUCTS WITH HIGH SWITCHING COSTS[1]

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Abstract

The paper studies the determinants of the optimum prices of computer programs and their upgrades. It is based on the notion that because of the human capital invested in the use of a computer program by its user, this product has high switching costs, and on the finding that pirates are responsible for generating over 80 per cent of new software sales. A model to maximize the present value of the program to the program house is constructed to determine the optimal prices of initial programs and for those upgrading their programs. It is shown that an upward shift of the demand function of upgrades leads to an increase in the optimum price of upgrades and to a decline in that of initial copies. However, and upward shift of the demand function of initial copies increases their optimal price without affecting that of upgrades. The price of upgrades is higher, the smaller (greater) the absolute value of the derivative of the initial copy demand with respect to the upgrade price (its own price), the smaller the number of pirates purchasing their initial legitimate copies and, normally, the later they do it. The prices of these goods often move in opposite directions despite the fact that the goods can best be characterized as complements. Copy-protection is generally not optimal with standard programs having competing unprotected products, and it is normally optimal to use a drug-pusher's strategy in pricing and in other respects: get the user hooked and cash in on the upgrades.

JEL classification number: D41-42

Key words: pricing of computer software, pricing of goods with high switching costs

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1. Introduction

A computer program is a non-perishable non-rival good, which requires a considerable human capital investment from its user – so considerable that many regard it as greater than the cost of the program. Therefore there has to be a special reason for the user to switch to another program and sink the human capital investment cost involved with the new program. As a non-rival good, a program can be copied, depending on the degree of copy-protection. Indeed, six out of every seven software users in the U.K. have been found to use pirated copies, but pirates were responsible for generating more than 80 per cent of new software buyers (see Givon et al. (1995); see also Goldman (1992)). At least currently, considerable technical progress is characteristic of the evolution of programs over time. This has led program houses to introduce upgrades at frequent intervals, and the programs are purchased not only by new users but also by users of old versions upgrading their programs, and by former pirates. When a user switches to an upgrade, he typically either turns in the old copy or discards it: there is no second-hand market for old programs separate from their platforms, apparently because of obsolescence.

One would expect these special characteristics to have an important effect on the optimal pricing of software. Yet one sees a variety of pricing policies in the marketplace even with standard programs like word processors and spreadsheets, with the price for initial copies typically much higher than for upgrades. Some producers use elaborate copy-protection schemes, while others use none. A common preoccupation of all producers is piracy and the revenues lost as a result.

The general approach to goods with switching costs was pioneered by Klemperer (for a survey, see (1995)). Of earlier writings on computer programs, Conner and Rumelt (1991) analyze the optimal pricing and protection strategies of a single program. An individual buys the program if its value, net of the human capital investment required to use it, exceeds its price. He copies it if the value exceeds the cost of copying, and goes without if it does neither. The value of the program increases, and the cost of copying decreases with the user base. They show that in the presence of this network externality, profits and the optimum price may increase or decrease with copy protection by turning some pirates into buyers, and making others go without. If protection fails to increase legitimate demand, it is not optimal to use it (see also Katz and Shapiro (1986)).

Nascimiento and Vanhonacker (1988) determine the optimum strategic pricing of a single program over time. Purchasers and copiers have a given reservation price, and each of them purchases or copies only one program or none, depending on the reservation price and the cost of the program or a copy. Due to the complicated structure of the model, the authors simulate four different scenarios rather than try to find analytical solutions, and conclude that when the product is not copy-protected, skimming pricing strategies tend to be optimal, and prices will eventually come down to compete against copying. Cumulative profits – while a function of the diffusion rates of copying and buying -- are generally highest when the product is fully protected.

The purpose of this paper is to explore the effects of the characteristics described above on the optimal pricing of software from the point of view of the program house, given the information that can reasonably be expected to be available to the house at the time of the decision. We argue that the market for standard programs like word processors and spreadsheets is characterized by four distinct features, which we will incorporate in the model and which constitute its new features. They are also the respects in which this paper extends the work of Conner and Rumelt (1991), Nascimiento and Vanhonacker (1988) and Givon et al. (1995). The house not only offers a product, but also periodically introduces an upgrade with superior characteristics. The product is sold on two markets, one for initial purchasers and another for users of earlier versions upgrading their programs, and the house can discriminate between them due to registration programs or trading in. Secondly, the services of the program are highly "addictive" due to the differences in the user interfaces and other characteristics of the different programs so that users need a very strong reason to switch to another program and sink the human capital investment cost all over again. Third, pirates constitute a substantial part of program users, which the house does not observe, but pirates are responsible for generating over 80 per cent of new software buyers, as in Givon et al. (1995). They thus constitute the house's quantitatively most important group of potential customers. However, a clear majority of pirates do not even intend to pay for their programs, and therefore it is inappropriate to model their behavior as that of lifetime utility maximizers facing the prices of the initial copy and upgrades.

Finally, the market for computer programs is characterized by rapid technological progress, with new superior products and processes entering at frequent intervals. Uncertainty concerning the characteristics and prices of future programs and hardware is so great that modelling the house and its clients as if they knew the future characteristics and prices a large number of years into the future is of doubtful value. We can therefore model the house’s pricing decision as present-value maximization, given the information available to the house at the time of decision, including static demand functions for the initial program and the upgrades, while allowing for growth. The house makes a new decision when the situation changes.

We will determine the producer's present-value-maximizing prices for the program as well as its upgrades in a model where the public purchases a quantity of initial copies that depends on the initial copy price and the upgrade prices, and subsequently upgrades a price-dependent fraction of the stock of its programs each period. In addition, there is copying depending on the extent of copy protection, which the house does not observe, and, in accordance with Givon et al., after a certain period copiers start purchasing price-dependent quantities of legitimate copies, subsequently upgrading them in the same way as other purchasers.

It will be shown that it is often optimal to use a drug-pusher's strategy in pricing as well as in other respects: the key is to get the user hooked in order to cash in on the upgrades. An upward shift of the demand function of upgrades leads to an increase in the optimum price of upgrades and to a decline in the optimum price of initial copies. However, an upward shift of the demand function of initial copies leads to an increase in their optimal price without affecting the price of upgrades. The optimal price of the upgrade is greater, the smaller (greater) the absolute value of the derivative of initial copy demand with respect to the upgrade price (its own price), the smaller the number of pirates purchasing their initial legitimate copies and, normally, the later they do it. Both prices are higher, the greater the marginal cost of the initial copy and its upgrades. Thus the prices move in opposite directions in many cases despite the fact that the goods are best characterized a complements. Furthermore, its is generally not optimal to copy-protect standard programs with unprotected substitutes such as word processors and spreadsheets, in partial agreement with Conner and Rumelt and contrary to Nascimiento et al..[1]

It has been suggested to us that a highly competitive market and high switching costs are an unlikely combination. However, we submit that the market for the initial copy is highly competitive, whereas that of upgrades is a near-monopoly. The reason for the latter is high sunk costs due to the investment in program-specific human capital in terms of having the routines, possibilities, compatibility with the buyer's other programs, hardware, and network that switching to another program would make necessary.

The program house has a vested interest in keeping up the switching costs in order to maintain its monopoly, and it can do it at a low cost: The buyer is unlikely to pay a premium for low switching costs, because in purchasing a program, he plans to use it or its upgrades for an extended period of time because of the sunk program-specific cost of the human capital investment.

This approach applies to a large number of durable and even non-durable goods. E.g. car manufacturers compete on new car markets and cash in on parts. To do so they have derived various schemes to "enhance customer loyalty".

The paper proceeds as follows. The model is constructed in Section 2 and solved for the optimal prices in Section 3. Finally, Section 4 concludes the paper.

2. The Model

We will analyze the optimal pricing decision of the program house for a standard program with unprotected substitutes. We will determine the prices of the program for initial buyers and for those upgrading their programs, given the information the house can be expected to have in the decision-making situation in this market characterized by considerable technological change: when the situation changes, the house performs another optimization. The house takes the public's behavior function as given. There are also illegitimate copy users, who do not even intend to buy a legitimate program and whose number is a function of the user base. The house observes only the aggregate purchases of initial programs and upgrades. While it knows the result of Givon et al., it does not know which initial purchases are by pirates switching to legitimate copies. The pirates who purchase a legitimate copy do so because of factors like the peace of mind, including fear of viruses, manuals, and technical support.

Specifically, the program house offers its initial program to the public at price P. The basic quantity sold each period (Q) is a function of the price of the original program and its upgrade (N). Each subsequent period the house introduces an upgrade with superior characteristics, exogenous to the model, of which the customers learn on its release. The upgrade sells at price P to initial buyers and at price N to those upgrading their programs. The number of upgrades purchased each period is α times the user base, with α a function of N, and upgraders discard or trade in their old programs.[2] Beginning period (a), a number of illegitimate copy users, which is λ times the legitimate user base, purchase their initial legitimate copies. Having become legitimate users, they subsequently upgrade their programs at the same rate as other users. The parameter λ incorporates the possibility that the pirates' price derivative of demand is different from that of other users, but the ratio of the derivatives with respect to P and N is the same as that of other users.[3] Givon et al., found that the number of pirates turning into legitimate users is $4(Q+S)$ for standard programs, where S is the purchases by upgraders. Since most users of pirate copies do not initially intend to pay the market price in the presence of unprotected substitutes, copy protection has an insignificant effect on Q so that we can ignore this effect. This need not be the case for expensive specialized professional programs with a small potential user base, as suggested by Conner and Rumelt (1991).

The cost function of the programs is linear with a positive intercept, and the cost of each upgrade is $\gamma < 1$ times the cost of the original program. The decision variables for the program house are P and N.

We have:

$$\begin{aligned}\pi &= QP\left(1+\frac{1}{R}+\frac{1}{R^2}+\cdots\right)+\alpha QN\left(\frac{1}{R}+\frac{2}{R^2}+\frac{3}{R^3}+\cdots\right)\\ &+\frac{\lambda QP}{R^a}\left(a+\frac{a+1}{R}+\frac{a+2}{R^2}+\cdots\right)+\frac{\alpha\lambda QN}{R^{a+1}}\left(a+\frac{a+1}{R}+\frac{a+2}{R^2}+\cdots\right)\\ &-C(Q)\left(1+\frac{\gamma}{R}+\frac{\gamma}{R^2}+\frac{\gamma}{R^3}+\cdots\right)-\alpha Q\gamma C_v\left(\frac{1}{R}+\frac{2}{R^2}+\frac{3}{R^3}+\cdots\right)\\ &-\frac{\lambda\alpha Q\gamma C_v}{R^{a+1}}\left(a+\frac{a+1}{R}+\frac{a+2}{R^2}+\cdots\right)-\frac{\lambda Q\gamma C_v}{R^a}\left(a+\frac{a+1}{R}+\frac{a+2}{R^2}+\cdots\right)\end{aligned} \tag{1}$$

$$C(Q)=b+cQ \tag{2}$$

Eq. (1) is the expression for the present value of the program house's profit (π). The first summation is the present value of the sales revenue from the initial purchases of the programs at the rate of (Q) per period, or (QP), where $R \equiv 1+r$, with r the discount rate used by the house. The house has an infinite planning horizon for simplicity and without loss of generality. The summation is followed by the purchases of upgrades costing N at the rate of α of the user base, the stock growing by Q each period: a given version of a program qualifies for upgrading only once, and the programs replaced by the upgrades are discarded, as is usually the case. The third summation is the sales revenue from the initial purchases by users of illegitimate copies, starting in period (a) at the rate of λ times the legitimate user base. These purchasers upgrade their programs at the same rate as other program holders, as indicated in the next summation.

The expressions for the costs of the programs are arranged to correspond to the four kinds of purchases. The first summation is the cost of the initial programs purchased. In the expression, the cost of the first version of the program is C, and that of subsequent versions γC. The cost of the upgraded programs sold is the marginal cost C_v of that upgrade, given that the fixed cost has already been sunk. The same applies to the cost of the initial purchases by users of illegitimate copies (third summation) and their upgrades (fourth summation).

It is worth noting that this formulation nests the case where Q grows at rate g. The growth rate can be incorporated into the discount factor, which would then be $R = 1 + r - g$.

Equation (2) is the cost function of the programs, stating that $C_v = c$.

The basic demand for the initial copy is a declining function of its own price and of the price of the upgrade (Eq. (3)). Thus in choosing a program, the buyer takes into account not only the initial copy price but also the prices of the upgrades. The demand for the upgrade, as a share of the user base α is a function of the upgrade price in Eq. (4):

$$Q = d - fP - nN \tag{3}$$

$$\alpha = k - hN \quad , \tag{4}$$

where d, f, k, h, and n are positive parameters. Of course, these functions are reduced forms of the consumers' optimization problem, nesting such behavior as in Conner and Rumelt (1991) and Nascimiento and Vanhonacker (1988).

Substituting Eq. (2) into Eq. (1) yields, after some manipulation:[4]

$$\pi = \frac{Q}{R-1}\left\{PR - c(R-1+\gamma) + \frac{\alpha R(N-\gamma c)}{R-1} + \frac{\beta}{R-1}\left[PR + \alpha N - \gamma c(R+\alpha)\right]\right\} \tag{5}$$
$$- \frac{b(R-1+\gamma)}{R-1} \quad ,$$

where $\beta \equiv \lambda\left[a(R-1)+1\right]/R^a$.

3. The Optimal Prices

Substituting Eqs. (3) and (4) into Eq. (5) and maximizing the resulting expression with respect to the decision variables P and N yields:

$$N^{*} = \frac{f\rho\kappa - nR(\rho - 1)}{2fh\rho} \tag{6}$$

$$\begin{aligned} P^{*} = \frac{1}{2fR(\rho-1)} \Bigg\{ & \frac{nR(\rho-1) - f\rho\kappa}{4fh\rho}\left[3nR(\rho-1) + f\rho\kappa\right] + dR(\rho-1) \\ & + fc\gamma(k\rho + \beta R + R - 1) + fc(R-1)^{2} \Bigg\} \end{aligned} \tag{7}$$

where $\rho \equiv R + \beta,$ and $\kappa \equiv k + hc\gamma$.

The effects of the different variables and parameters on the optimal prices are displayed in Table 1. The top two rows display the effects in the general model, and the bottom two rows the respective limiting values of the effects as n approaches zero. In interpreting the expressions, we assume throughout that at least some initial programs and upgrades are sold, which requires that the marginal cost of the upgrade is smaller than the price at which no upgrades are sold, or $c\gamma < k/h$.

[Table 1 about here]

It is more intuitive to start with the limiting case of $n \to 0$, i.e. the demand for initial copies is not a function of the price of the upgrade. We then have: $N^{*} = (k/h + c\gamma)/2$: The optimal price of the upgrade is the mean of the price at which no upgrades are sold, and the marginal cost of the upgrade. It is an increasing function of this cost $c\gamma$, and k, or the intercept of the demand function of upgrades. It is a decreasing function of h, or the absolute value of the price derivative of this demand function.

The P^{*} is an increasing function of $d, h, c, \gamma,$ and $\lambda,$ and a decreasing function of k and f. Thus an upward shift in the demand function of upgrades (k) leads to an increase in their optimal price and to a decline in that of initial copies.

However, an upward shift in the demand function of initial copies (d) leads to an increase in their optimal price without affecting that of upgrades. The $P*$ is also greater, the greater the absolute value of the price derivative of the demand for upgrades (h), the greater the marginal cost of the programs (c) and their upgrades (γ), and the greater the number of pirates purchasing their initial legitimate copies (λ). Moreover, it is greater, the smaller the absolute value of the price derivative of the demand for initial copies (f). It is an increasing (decreasing) function of (a) if $(1-a)(R-1)>(<)1$. Normally the latter condition holds, as $(R-1)$ is a few percentage points, and $(1-a)$ is a negative number. Then if pirates purchase their initial legitimate copies later, $P*$ declines in the same way as when d declines.

While the effects of d, f, c, γ, and λ are intuitive, those of k and h call for an explanation, since they affect $N*$ and $P*$ in opposite directions, although the initial copy and the upgrade can best be described as complements. The explanation is based on this very fact: Upgrades can be sold only if initial copies have been sold first. Take $dP*/dk$. An upward shift in the demand curve of upgrades makes it optimal to increase their price, which increases profits and lowers $d\pi/dP$ below zero $(d(d\pi/dP)/dk<0)$. The house restores $d\pi/dP$ to zero by lowering the price of the initial copy. The resulting increase in sales increases the user base, which makes it possible to further increase future profits from upgrades (as $d^2\pi/dP^2<0$ as a condition for maximum). An analogous explanation applies to the effects of (h).

The sign of $\partial P*/\partial R$ is ambiguous. While a decline in R increases the present value of profits, the price at which $d\pi/dP$ is zero may be greater or smaller than the initial price, depending on parameter values. The discount rate does not affect $N*$. The effects of the expected growth rate of sales on the optimal prices are the negatives of those of R, since $\partial R/\partial g<0$. Thus $N*$ is unaffected by g.

When n rises from zero, $N*$ declines, because it now has a depressing effect on initial copy sales. The effect on $P*$ depends on parameter values, as a decline in P increases sales but increases (decreases) the revenues on initial copies if their demand elasticity is greater (smaller) than unity. The user base, however, increases, increasing profits from upgrades later. This relationship is responsible for the fact that some of the effects become ambiguous in the general case. It is unnecessary to further comment on the effects of most parameters, those of d, k, and , conditionally, c and γ remaining qualitatively the same. It is of interest that λ now has a negative, and (a) a conditionally positive, effect on $N*$. If pirates come to the market in larger numbers or earlier, $N*$ declines. This is associated with the fact that these changes cause a conditional increase in $P*$. This increases the user base and thereby the demand for upgrades later, causing a marginal adjustment in $N*$ analogous to the response of $P*$ in response to a change in k above.

The result on the effects of λ has an implication for copy protection. The partial effect of λ on π $(\partial\pi/\partial\lambda)$ is positive. The λ is a declining function of copy protection for standard programs with unprotected substitutes: for a new user without program-specific human capital, the closest substitute for a pirate copy is a pirate copy of another program, all else equal. So copy protection has an insignificant effect on the copies sold initially (Q). However, a copier lost to competition becomes “addicted” to its product’s services and becomes its potential customer. It follows that protecting these programs is generally not optimal. On the contrary, it is optimal to try to attract users to the house’s programs one way or another: [5]

However, protection can be optimal with specialized programs with a small user base and without unprotected substitutes, where it may have a significant effect on Q. Then the present value of profits due to users buying, rather than copying, their first program can be greater than that of pirates switching to legitimate programs later. These results are in agreement with Katz and Shapiro (1986) and Conner and Rumelt (1991), and at variance with Nascimiento et al., who find the opposite about cumulative profits. One reason for the latter discrepancy is that the appropriate maximand is not cumulative profits but the present value of profits (which the authors duly maximize in their simulations). Moreover, the major source of sales (and profits) in our model - four times standard sales according to Givon et al. - comes from copiers switching to legitimate copies, which was discovered only in 1995, and from upgrades, whereas their individuals acquire only one program, and the producer has to compete against copiers, which generally lowers their optimal price later on.

4. Concluding Comments

We have determined the optimal pricing of computer software and other products with high switching costs in a model where the public purchases initial copies of the program and subsequently upgrades them, also taking into account the purchases by users of pirate copies.

It was shown that an upward shift of the demand function of upgrades leads to an increase in the optimum price of upgrades and to a decline in that of initial copies. However, an upward shift of the demand function of initial copies leads to an increase in their price without affecting that of upgrades. The price of upgrades is greater, the smaller (greater) the absolute value of the derivative of the initial copy demand with respect to the upgrade price (its own price), the smaller the number of pirates purchasing their initial legitimate copies and, normally, the later they do it. Both prices are higher, the greater the marginal cost of the initial copy and its upgrades. The prices move in opposite directions in many cases despite the fact that the goods can best be characterized as complements.

Experiments with plausible numerical parameter values suggest that it is often optimal to use a drug pusher's strategy in pricing standard programs with unprotected substitutes, and pay the victim to take the first shot to get him hooked, and cash in on the upgrades. In these cases, it is not optimal to copy-protect standard programs, since protection would only lead potential customers to competing products. [6]

The above policy is different from what one sees in the marketplace: Upgrading typically costs a fraction of the price of the initial copy. In terms of this model, that can be rationalized by the fact that in an upgrade, the user purchases only the marginal product of the new version over the old one. The user already having a close substitute, his k is lower and his h higher. However, the same is also broadly true of the house's most important customer (according to Givon et al.), the user of pirate copies buying his initial legitimate copy, which lowers d and increases f.

However, other components of the drug pusher's strategy are widely used, making it possible for the house to collect both on the initial copy and on the upgrades. Many houses fail to copy-protect their products, introducing upgrades frequently, apparently counting on the λ factor of the individual illegitimate user. Some houses use product differentiation. They offer inexpensive and often unprotected student versions with limited capabilities in addition to the standard version. This policy can also be expected to make the (a) of the full version smaller, thereby further increasing both the present value of the program to the house and often also its $P*$ and $N*$. However, if the standard version is copy-protected without substantial academic discounts, it turns away faculty users, and with them also their students: The diffusion process never gets started. You have to get the teacher hooked to hook his disciples.

These results lead us to conclude that while it is optimal for software houses as an industry to intimidate and scare users of pirate copies, as individual competing businesses they would do well to court them, concentrating on the sales to be gained rather than on those lost: In pricing, they would do well to give less emphasis to the rational user maximizing lifetime utility, given the expected present value of the program's marginal product over its cost over his lifetime, -- which no one could possibly know -- and more emphasis to the initial user of possibly illegitimate or subsidized programs who does not even intend to pay the full fare, to get him hooked. Thereafter they should make the transition to legal copies as easy and as attractive as possible. Thus site licenses and generous offers to colleges and schools can often be in the long term interest of the house. Houses could also further exploit the “addictive” property of programs by using the same user interfaces in different programs. This would enchance the strategic significance of getting the user hooked on the house's first program -- points that appear to be well understood by Microsoft and the Department of Justice.[7]

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Notes

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1. However, protection can be optimal with specialized programs with a small user base and without close unprotected substitutes, when leakage to substitutes is not a serious danger. Then protection can affect not only addicted copiers' purchases of legal copies later, but also current sales due to buyer substitution between legal and pirate copies of the same program. See page 10 below.

2. The α function is a reduced form of the user's optimization, given his expectations, where the user weighs the perceived marginal product of the new version over the user's version against the price of, and learning cost associated with, the new version.

3. We think this formulation of copy protection is reasonably close to the observed behavior of individual users, since there are always unprotected substitutes available for standard programs. To be considered, the protected program has to dominate the unprotected competitor by wide enough a margin, to compensate for the fact that it cannot be tried out on the user's own computer as easily as the unprotected substitute. Adjusted for this, the amoral buyer then weighs both programs' marginal utilities against their marginal costs in the conventional way.

Extending the model to include copy protection as a decision variable is straightforward, but it is beyond the scope of this paper, which studies standard programs. It would also complicate the expressions significantly, since Q and λ are both functions of protection.

4. Substituting Eq. (2) into Eq. (1) yields Eq. (5) as follows. The series $a+(a+1)/R+(a+2)/R^2+\cdots$ is the sum of the series $a+a/R+a/R^2+\cdots$ and the series $1/R+2/R^2+3/R^3+\cdots$. While the value of the former series is straightforward, the latter series equals

$R(d/dR)\left[-R^{-1}-R^{-2}-R^{-3}-\cdots\right]=R(d/dR)\left[-1/(R-1)\right]$, which in turn equals $R/(R-1)^2$.

5. Outside this model, in increasing the total user base (including pirates) they also increase the program's value to others via network externality effects, as shown by Conner and Rumelt, and make them potential customers.

6. These results differ e.g. from Klemperer (1995), because our program house can discriminate between new purchasers and repeat purchasers. Furthermore, the pirates' behavior prior to considering a legitimate copy is not affected by the prices P and N, given Q, since when they become users, they do not even intend to buy a legal copy of the program.

If the initial copy price is much lower than that of upgrades, individual users would have an incentive to arbitrage between the two markets. They would upgrade their programs by purchasing the new versions as initial programs in the names of their family members, friends, or Mickey Mouse. This arbitrage possibility is easily incorporated in the model, but it complicates it unnecessarily. While the arbitrage possibility can be reduced by various means, it is likely to considerably limit the amount by which the price of the upgrade can exceed that of the initial copy.

7. Providers on other markets have increased customer switching costs with discount coupons like frequent flier programs. Warranties are made conditional on having the product serviced and repaired at high-cost authorized dealers. Use of non-standard parts is another device, but it can backfire. Indeed, several manufacturers of producer durables have refocused their activities and now expect to generate most of their profits from the service and maintenance of their products. The sale of the product has then turned from an end into a means towards the service revenues.

Table 1. The Effects of Different Parameters and Variables on the Optimal Prices

		$/df$	$/dn$	$/dh$	$/dd$	$/dk$	$/dR$	$/dc$	$/d\gamma$	$/d\lambda$	$/da$
	$dN^*/$	+	–	?	0	+	(+)$^{4)}$	+	+	–	(+)$^{5)}$
	$dP^*/$	(-)$^{7)}$	(±)$^{2)}$	(±)$^{6)}$	+	-	?	(+)$^{1)}$	(+)$^{1)}$	(+)$^{8)}$	(-)$^{9)}$
$n \to 0$	$dN^*/$	0		–	0	+	0	+	+	0	0
	$dP^*/$	–		+	+	–	?	+	+	+	(±)$^{3)}$

7) if $nR(\rho-1)$ is dominated in the expression

2) ± if $3nR(\rho-1) \gtrless f\rho\kappa$

3) ± iff $(1-a)(R-1) \gtrless 1$

4) sufficient condition $\rho[1+a(R-1)(a-1)]>1$

5) if $(1-a)(R-1)<1$

6) ± iff $f\rho(\kappa-hc\gamma)-nR(\rho-1) \gtrless 0$

7) sufficient condition for (-): $3nR(\rho-1)> f\rho\kappa$

8) if $fc\left[(\gamma-1)(R-1)^2-\gamma k\right]$ is dominated in the expression

9) if 5) and 8) hold.