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**EVALUATION OF BEAMFORMING
ALGORITHMS FOR MASSIVE MIMO**
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ABSTRACT

Muhammad Nasir Khan: Evaluation of Beamforming Algorithms for Massive MIMO
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Massive MIMO relay system is an expansion of the Multiple-Input-Multiple-Output (MIMO) which enabled multiple users and antennas to communicate with each other for data sharing. A relay system with multiple antenna system has an advantage over simple MIMO system as it interconnects base station and users with each other for sharing of information and both BS and users are independent of many antennas. High data rate applications such as Machine-to-Machine communication and wireless sensor networks are experiencing transmit power loss, channel capacity and mismanagement of data. The demand for the Massive MIMO relay system is opening a door for ultra-high latency wireless network applications in case of saving transmit power and transmission of accurate information over the wireless networks.

Due to the loss in transmit power and mismanagement of information over wireless networks, it is difficult to get better performance. Different approaches were made to optimize the overall transmit power of communication systems. One of the approaches was explained in this thesis work. The focus of the thesis is the use of beamforming algorithms named as Maximum Ratio Combining (MRC) and Zero-Forcing (ZF) to maximize the overall capacity of the MIMO system. These algorithms were evaluated on different scenarios to handle the performance and behavior with different network conditions. Various use cases were used for analyzing the beamforming algorithms. The performance of both algorithms was observed by considering the scenarios such as varying the transmit and receive antenna's size and modulation schemes. Singular Value Decomposition (SVD) Method was used at the main MIMO channel to optimize the channel capacity. SVD divides the MIMO channel into different subchannels and optimizes the channel capacity of individual channels.

The summary of results showed that MRC and ZF in CP-OFDM environment when number of RX antennas increased then they gave better BER performance as compared to the single antenna system. On the other hand, with higher modulation schemes efficiency was not good but with lower modulation scheme performance was satisfactory.

Keywords: Massive MIMO, MRC, ZF, SNR, BER, Beamforming, SVD

PREFACE

This master thesis, "Evaluation of Beamforming Algorithms for Massive MIMO" was done in partial fulfilment of the requirement for the Master of Science degree in Communication Systems and Networks major, in the Electrical Engineering Unit. I would specially like to thank my thesis supervisor, Prof. Jari Nurmi who has given me this topic as my master's thesis and supported me throughout the completion of this work.

I would like to thank Almighty Allah who has given me this opportunity to pursue my master's degree here in Tampere University, Finland.

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LIST OF SYMBOLS AND ABBREVIATIONS

AoA	Angle-of-Arrival
ADC	Analog-to-Digital Converter
BS	Base Station
CSI	Channel State Information
CSR	Cooperative Spatial Reuse
CoMP	Cooperation/Cooperated Multipoint Transmission
DAC	Digital-to-Analog Converter
DF	Decode-and-Forward
DL	Downlink
FBCP	Fixed Beamforming Channel Precoding
FDD	Frequency Division Duplexing
I.I. D	Independent and Identical Distribution
IoT	Internet of Things
ITU	International Telecommunication Unit
LAN	Local Area Network
LCFS	Last Come First Serve
LS	Least Square
MIMO	Multiple-Input-Multiple-Output
MRC	Maximum Ratio Combining
ML	Maximum Likelihood
MISO	Multiple-Input-Single-Output
MS	Mobile Station
MMSE	Minimum-Mean-Square-Error
MS	Minimum Square Error
MAC	Media Access Control
NGMN	Next Generation Mobile Networks
QoE	Quality of Experience
<i>R_X</i>	Receiver
SHF	Super High Frequency
SNR	Signal-to-Noise Ratio
SE	Spectral Efficiency
SISO	Single-Input-Single-Output
SIMO	Single-Input-Multiple-Output
<i>T_X</i>	Transmitter

SVD	Singular Value Decomposition
TDD	Time Division Duplexing
TDMA	Time Division Multiple Access
UL	Uplink
WAN	Wide Area Network
ZF	Zero-Forcing
3GPP	3rd Generation Partnership Project
5G	5th Generation
A	Channel Detection Matrix
B^H	Hermitian of detection Matrix B
b_k	k th Column of Detection Matrix B
$C^{M \times K}$	Channel Matrix for the Combination of BS Antennas and Users
H	Channel Matrix
K	Number of Users
$M \times K$	Combination of BS Antennas and Users
$M \times 1$	k th Column of Channel Matrix H
$M \gg K$	M is Much Larger Than k
n_k	Noise for k th User
P_T	Transmit Power
S	BS Transmitted Signals to Users k
S_K	k th Signals for Users
U	Signals from Users
V	Transmitted Signals from BS
γ	Precoding Matrix

1 INTRODUCTION

Multiple-input-multiple output (MIMO) is a modern wireless technology in which a multiple antenna relay system Base Station (BS) is used for establishing and maintaining connection to hundreds of users present on the receiver side. Basically, Multiple antenna system consists of the Base Station, users and communication channel. The BS and users both have large number of antennas and channel provides paths for data exchange between antenna systems. The processing in MIMO is done in two perspectives uplink and downlink directions i.e. Base Station is a main participant in Massive MIMO which helps the users to obtain channel state information in the uplink side during channel estimation as users lack enough data base for it [1]. Applications of massive MIMO relay systems include Large-Scale MIMO radar commonly used for object tracking and mapping. The accurate localization of user in indoor and outdoor environment over multipath channel is obtained by detecting Angle of Arrival (AoA) for user on all base station antennas and measuring the source location by the triangulation method [2]. Some relay networks have critical role in hybrid networks e.g., power control and data exchange in heterogeneous cellular networks and wireless sensor networks respectively [1].

Modern wireless networks are facing huge loss in the form of insufficient throughput, energy efficiency and communication reliability. The value of Signal to Noise Ratio is the main indicator defining all above factors. To make the SNR value high we need to increase the number of antennas in the existing MIMO system. However, the main issue is that with the large number of users in multiantenna system increases the complexity of mobile network. Different challenges are needed to take care of reducing the system's complexity. The first one is pilot contamination and second one propagation channel improvement [3].

Massive MIMO beamforming is the solution where there are a massive number of antennas at the relay network BS and lesser on the user sides are used and linear detection algorithms are applied on them to perform the tasks such as precoding, channel estimation and data detection. Beamforming is a technique used to steer the beam towards the desired user. The weights of antenna array elements are adjusted to focus on the incoming signal and directing them towards the intended user and ignoring user interference

coming from adjacent users. In this thesis, two beamforming algorithms are used: Maximum ratio combining and Zero-forcing. From the performance point of view, zero-forcing gives more signal-to-noise ratio but ignores noise and its implementation is very complex due to matrix inversion. On the other hand, with maximum ratio combining the SNR is better and it is very simple to implement but it ignores the signal-to-inter-user-interference (SINR) from different users. Signal to noise ratio is an important factor. It determines the Performance of a large antenna system in the form of high Spectral Efficiency (SE) and Bit Error Rate (BER) [3].

In this thesis, focus is to develop a MATLAB model for two different linear processing schemes (MRC, ZF). MATLAB simulation will be performed on preferred algorithms and conclusions made for different use cases.

1.1 Motivation and Purpose

The evolution of massive MIMO has opened a door for current 4G, 4.5G and future 5G mobile networks. These mobile networks can increase the cell coverage, reduce the interference and enhance overall system capacity by introducing many antennas at the relay systems. It has a common feature with Internet of Things (IoT) with respect to network size. Different small networks in Massive MIMO relays and Internet of Things (IoT) networks unite to form huge networks by providing efficient performance in one side but increase in the network size can also create network troubles between devices on the opposite direction. A few issues which are common with huge networks include channel interference, data traffic and power allocation. It is shown in Figure 1 that network technologies are improved, and new features are added in mobile wireless networks with the passage of time [4].

This thesis deals with couple of issues such as inter symbol interference (ISI) and Signal-to-Inter-User-Interference. Beamforming algorithms within OFDM scheme are implemented and different use cases are made on different Signal-to-Noise-Ratio (SNR) and Bit-Error-Rate (BER) values to minimize these issues. It is possible to make signal strong and reduce the transmit power by using linear processing schemes. One of the techniques is to make the signal beam more powerful i.e. concentrate the signal beam to intended user equipment while unwanted beams must be cancelled using the beamformers. In one side focused beam directed towards the desired users increases the Signal to noise ratio and on the other side it will save the transmit power by cutting down the unused beams through beamformers [5]. Focus of this thesis is to evaluate the Mas-

sive MIMO beamforming algorithms. Different use cases are implemented for beamforming algorithms in MATAB and summarized with respect to better results in signal-to-noise ratio and BER.

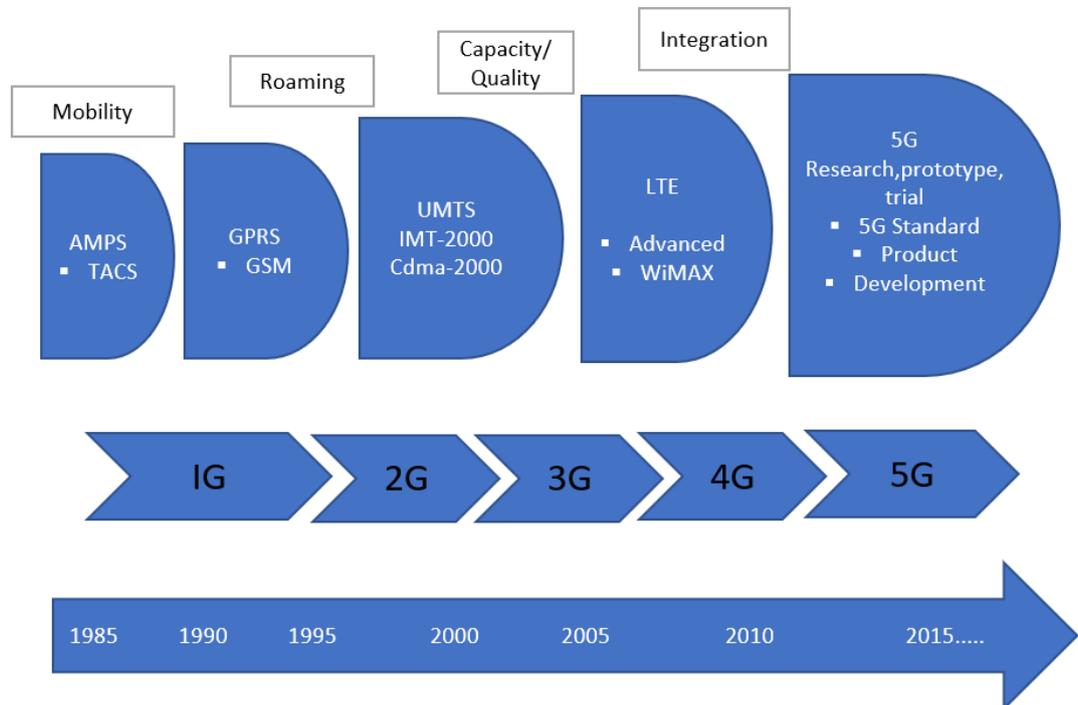


Figure 1. Evolution of Wireless Networks [4]

This Master thesis is pursued in this way: chapter 2 includes the massive MIMO fundamentals where working principle about massive MIMO and several diversity techniques are briefly discussed. In Chapter 3 the emphasis is on Massive MIMO system model and beamforming algorithms for uplink and downlink transmission. In chapter 4 various theories and laws are discussed which are involved in the mathematical computation of Maximum ratio processing and zero-forcing. Chapter 5 deals with the review of the state of the art. Chapter 6 explains the Modulation techniques. Simulation and measurements are described in chapter 7. Chapter 8 contains the Applications of Massive MIMO. Chapter 9 contains conclusion and future improvement in the massive MIMO beamforming.

2 MASSIVE MIMO FUNDAMENTALS

2.1 Background and Literature

Massive MIMO exploits beamforming and antenna diversity techniques to transmit and receive multiple beams over same radio channel to get full spectral efficiency. In the introductory section, the MIMO evolution with different antenna configurations is described.

2.1.1 SISO (Single Input Single Output)

A radio network having one transmitter and one intended receiver are used to exchange information over a channel, as shown in Figure 2. In the language of MIMO, a Tx antenna sends a single input and Rx receives a single output.

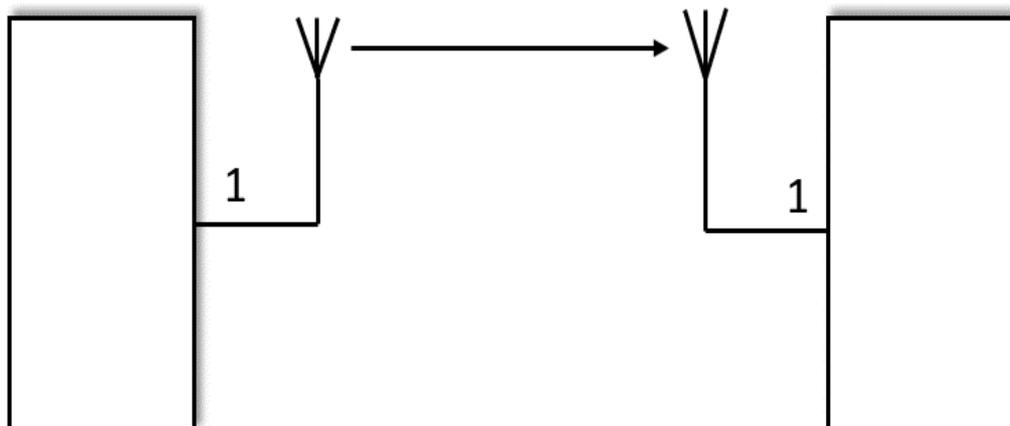


Figure 2. Single Input Single Output Antenna Configuration [6]

2.1.2 SIMO (Single Input Multiple Output)

A radio network having one antenna at transmitter and multiple antennas at receiver are used to exchange information over a channel as shown in Figure 3. In the language of MIMO, a Tx antenna sends input and Rx antenna receives multiple output.

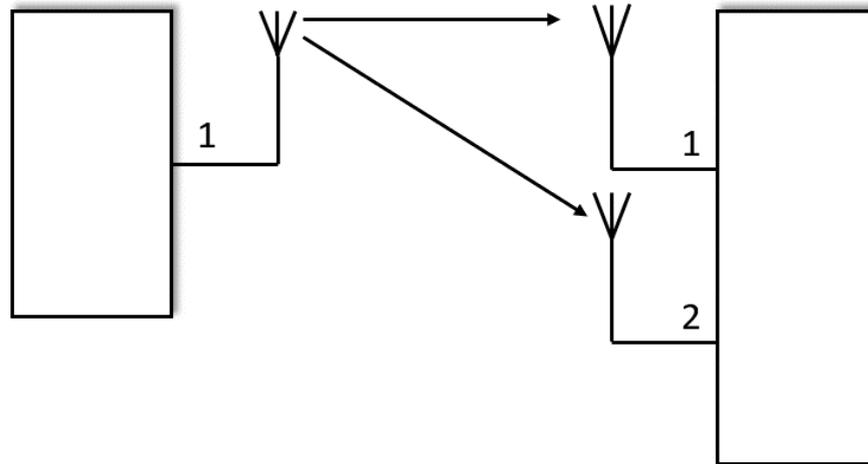


Figure 3. SIMO Antenna Configuration [6]

2.1.3 MISO (Multiple Input Single Output)

A radio network having multiple antennas at transmitter and a single antenna at receiver are used to exchange information over a channel as shown in Figure 4. In the language of MIMO, multiple Tx antennas send multiple inputs and Rx receives output.

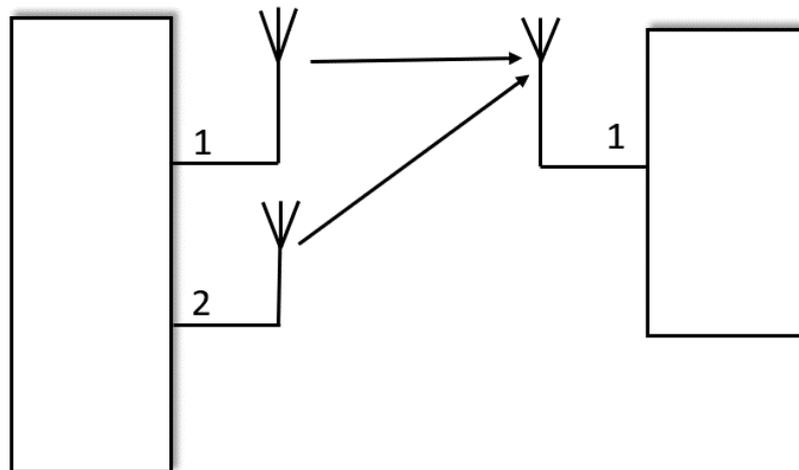


Figure 4. MISO Antenna Configuration [6]

2.1.4 MIMO (Multiple Input Multiple Output)

A radio network having multiple antennas at transmitter and multiple antennas at receiver are used to exchange information over a channel as shown in Figure 5. In the language of MIMO, Multiple Tx antennas send multiple inputs and Rx antennas receives multiple outputs [6].

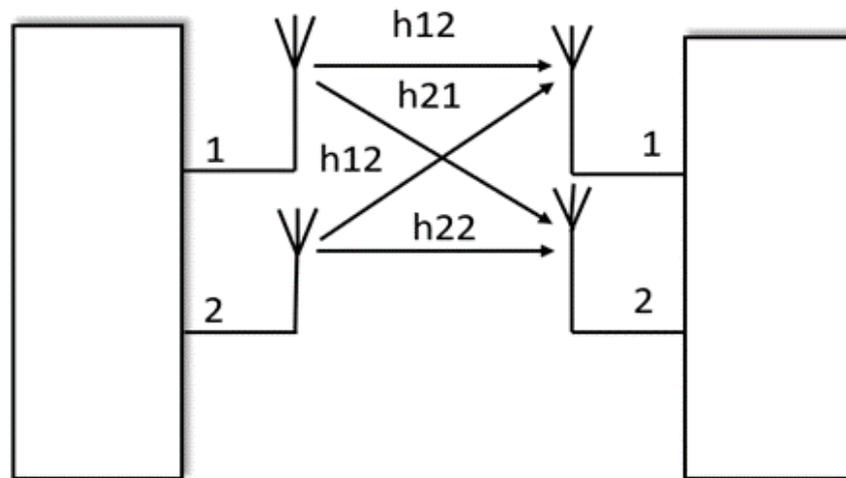


Figure 5. MIMO Antenna Configuration [6]

Table 1. Performance Comparisons SISO, SIMO, MISO and MIMO

Parameter Name	SISO	SIMO	MISO	MIMO
Quality of signal at receiver	Poor/Weak	Multiple antennas so, best reception is selected	Improved quality due to multiple transmission	Best quality
BER	Maximum	Medium	Medium	Minimum
Throughput	Very less	Better than SISO	Slightly better than SIMO	Best throughput
Complexity of design	Simplest	Moderate design	Moderate design	Complex

It is shown in Table 1 that MIMO is best in term of capacity system from other configurations but due to large number of antenna system at transmitter and receiver it is costly as well as more complex [7].

2.2 Massive MIMO

Massive MIMO is a type of multiuser relay network where many BS antennas simultaneously facilitate hundreds of users using the same time-frequency resource. The communication in relay network takes place in the form of uplink and downlink direction. In the uplink phase, BS is used for channel estimation of users.

2.2.1 TDD Link Protocol

Time division duplexing is a two-way protocol in MIMO channel. It is used to make reciprocity of channel i.e. to perform sending and receiving operations using same communication channel. It is shown in Figure 6 that Frequency Division Duplexing (FDD) is suitable when number of BS antenna are lower in numbers. As the number of antennas on the BS side in massive MIMO increases, pilot overhead also increases whereas time Division Duplexing (TDD) protocol is more suitable. Because it has much more to capacity to carry pilot overhead compared to frequency division duplexing (FDD) [8].

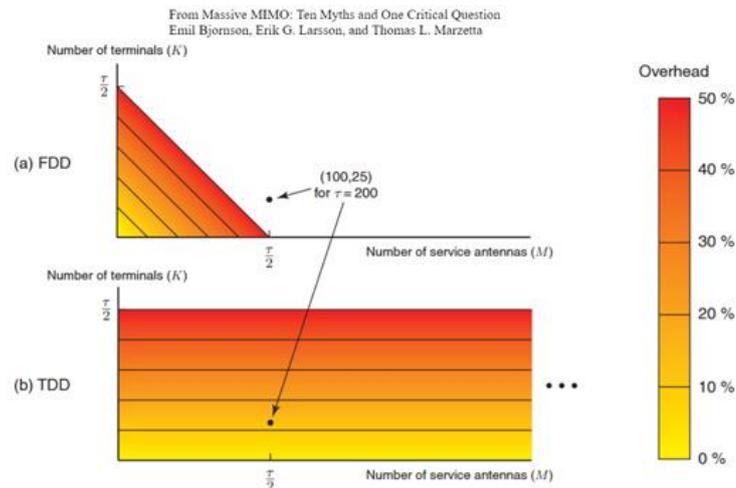


Fig. 6: Illustration of the typical overhead signaling in Massive MIMO based on (a) FDD and (b) TDD operation. The main difference is that FDD limits the number of antennas, while TDD can have any number of antennas. For a coherence block with $\tau = 200$ even a modest Massive MIMO setup with $M = 100$ and $K = 25$ is only supported in TDD operation.

Figure 6. Illustration of Typical Overhead Signaling in Massive MIMO [8]

It is shown in Figure 7 that the transmission starts when users send orthogonal pilot sequence and data to BS, shown in the diagram as uplink pilot and data phase. First, BS uses orthogonal pilot sequence coming from users and gets channel state information in the uplink pilot and data phase have given below. After that BS pre-codes users' pilot and extracts the channel matrix for further channel estimation. BS sends the data to users in downlink data phase. Users detects the pre-coded signal from effective channel

gain (a scalar constant) which has already obtained by BS through linear precoding techniques [9].

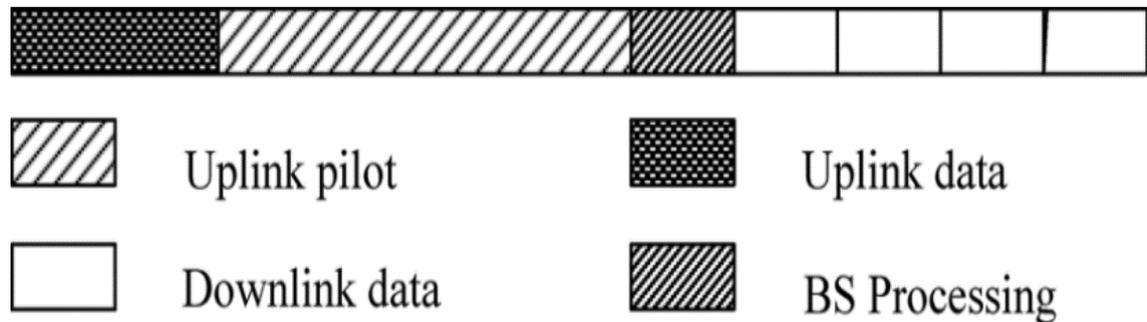


Figure 7. Time Duplexing Operations [10]

2.3 Working Principle in Massive MIMO

2.3.1 Channel Estimation

A reliable communication in multi antenna system is possible when channel information is known. There are two basic steps that are used to find the channel state information. Firstly, BS initiates the process and sends the training data to the desired users. Secondly, users receive the training sequence and perform channel estimation to find the CSI. Now the estimated CSI is returned to BS. The training data is also known as orthogonal pilot sequence. In uplink, users send the sequence to the BS that pre-codes and estimates them. TDD is suitable because it creates channel reciprocity i.e. each user uses a coherence time interval for orthogonal sequence in uplink stage, whereas in FDD different frequency bands are used in uplink and downlink phase for data transmission. Main difference between these protocols is that in TDD the estimation is dependent on the BS antennas while users are responsible in FDD to estimate the CSI.

2.3.2 Uplink data transmission

It is shown in Figure 8 that users on the receiver side use coherence time interval for data transmission in uplink using the same time/frequency resource. BS detects the pre-coded data after using linear processing techniques over incoming data from users.

2.3.3 Downlink data transmission

Figure 2.7 shows that BS detects the data in the uplink and sends it to all desired users after precoding in downlink phase. It also takes help from user incoming data and estimates it for precoding of data. After linear processing the BS combines the symbols within channel estimation to create the pre-coded signals and sends these signals to all intended users [11].

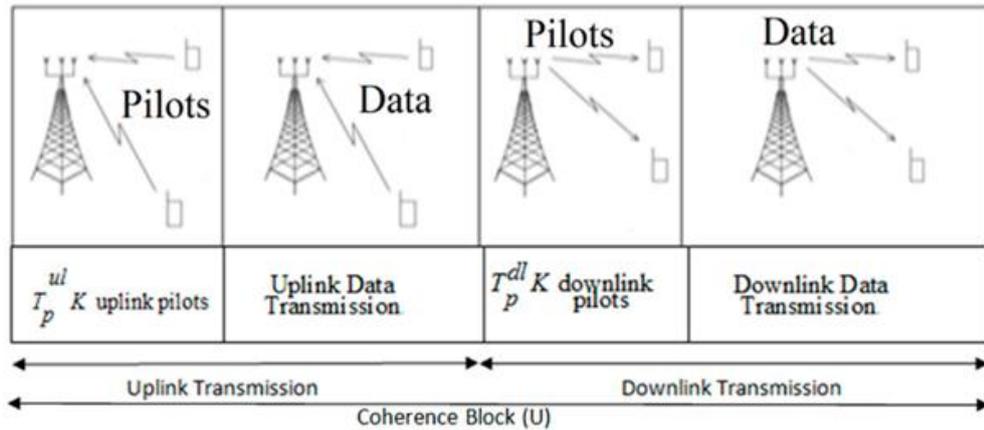


Figure 8. Steps involved in Uplink and Downlink Transmission [12]

2.4 Benefits of Next Generation Massive MIMO

Massive MIMO is becoming a source for creating new 5G broadband data networks. These networks will be different from previous technologies in providing energy efficiency, system robustness, security and bandwidth efficiency. The use of TDD makes the bandwidth efficient while using the same frequency time resources. Similarly, use of right beam of signals for right user also enhances the capacity of network. The next sub-section is describing potential benefits achieving through linear processing of signals in combination with multiple antenna arrays. The linear processing of signals in combination with multiple antenna arrays gives potential benefits the one described in the next sub-sections.

2.4.1 Energy Efficiency

The existence of antenna arrays and spatial multiplexing can enhance the system capacity more than 100 times. The fundamental principles in massive MIMO are constructive and destructive interference. The beams which are needed for specific user terminals are shaped with beamformer such as MRC and transmitted out from antenna after

being combined constructively. The beams that are not needed for the intended users are suppressed destructively, for instance, Zero-forcing beamformer cuts the unused beams and as a result reduces the transmitted power and increases the energy efficiency. Following formula describing the relationship between Energy Efficiency, throughput and total transmit power:

$$EE = \frac{R}{P} \quad (2.1)$$

where, R is the throughput and P is the total power spent in achieving the R [13].

2.4.2 Power Control

Massive MIMO is a game changer technology for 5G because it makes use of low power and inexpensive hardware components in the design. For example, using the milliwatt amplifier instead of 50 W amplifiers. The use of massive antenna array eliminates bulky coaxial cables and concise the hardware circuits. The presence of large number of antennas make use of less transmission power and give information with great accuracy and range to the hardware devices. The network keeps itself stable in the circumstances where failure of few antenna components happens. As large array of antenna just facilitates few terminals, so degree of freedom is large. The steering of signal beams through beamformer make use of less power because unused beams are suppressed with the help of linear signal processing. A couple of issues are originated with multiple antenna system which are discussed in the next sub-sections.

2.5 Challenges in the Next Generation Massive MIMO

2.5.1 Pilot Contamination

A single cell environment is contamination free where each user uses orthogonal pilot sequence and shares the same frequency resources. For multicell setup, each user in each cell is not provided with pilot sequence during small coherence time so different users reuse the same frequency resources. It is shown in Figure 9 that when two different users send the signals simultaneously to the base station then BS gets the superposition of signals. In the uplink, BS fails to separate the signals during estimation hence pilot sequence is contaminated. This phenomenon is called pilot contamination. As the number of antennas grows signals are contaminated more and more because signal and noise power are proportional to the number of antennas. Different techniques are discussed to eliminate it known as clever channel estimation algorithms or even blind techniques [14].

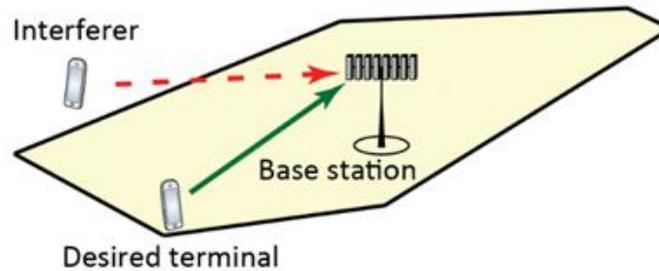


Figure 9. *Pilot Contamination in Massive MIMO [15]*

2.5.2 Channel Reciprocity

Channel reciprocity can obtain full spectral efficiency by exploiting the multiple antennas in massive MIMO. TDD makes the channel reciprocal. It helps the BS to obtain downlink CSI during uplink channel estimation. The propagation channel and hardware for DL and UL in transceiver must be symmetric to each other. However, in real scenario it is difficult that hardware symmetry in transmitter and receiver side will be identical. There are two main problems attached with channel reciprocity as known as hardware mismatch and reciprocity calibration. Firstly, reciprocity calibration is a big challenge due to limitation of transmit pilots because every user is not given with a separate pilot and many users in multiple cells are using same pilots, so channel estimation is impossible so TDD is the best solution. In case of hardware mismatch several methods are proposed. One method is to make one antenna as reference with other antennas during transmission of beams. The ratio of weights between forward and reverse radio channel is compared with the reference antenna [16].

3 SYSTEM MODEL

We consider a massive antenna array system which consists of relay (BS) and many users. The relay system is equipped with M antennas and receiver consist of K single-antenna users as shown in Figure 10. In general, a single user may have more than one antenna. Here, we suppose that each user terminal has a single antenna and uses the same time frequency resources. To make the process simple, we assume that BS and user have perfect knowledge of channel state information. The communication around channel can be done by the means of protocols such as TDD or FDD. Frequency division duplexing is difficult to achieve because uplink and downlink use different frequency spectrum. Hence, Channel reciprocity is well handled with TDD because it gives duplex modes for both UL and DL. As multiple antenna array system feeds information to hundreds of users that is why M antennas on the transmitter as compared to K users ($M > K$) so that each user during CSI detect $K - 1$ transmitted symbols [17].

The channel between relay and users is denoted by $H^{M \times K}$ where M are the BS antennas and K is the users. The entries of the channel include h_k , where $M \times 1$ is the channel vector and k th column of channel $H^{M \times K}$. The entries of channel are Independent and Identical Distribution (I.I.D) gaussian distributed with zero mean and unit variance.

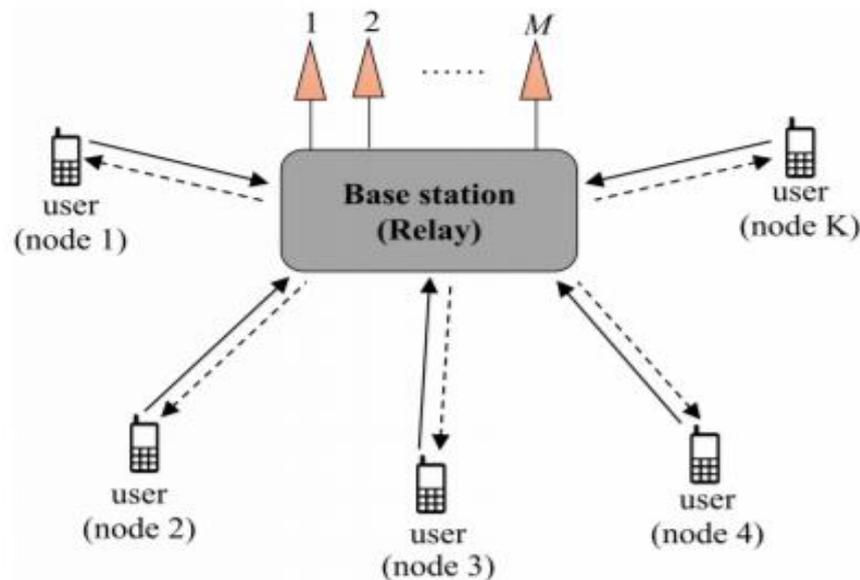


Figure 10. Block Diagram of Massive MIMO Relay Network [1]

3.1 Uplink Transmission

To start a transmission in a channel, BS as well as users must have knowledge about CSI. However, the users are lack of channel knowledge and an appropriate signal processing system. In uplink, BS processes the signals when users transmit towards it for channel estimation. If all the users send signals to BS then k th signals are denoted as $E\{|U_k|^2\} = 1$, where combination of signals take position in $K \times M$ vector in channel matrix H

$$y_{UL} = \sqrt{p_T} \sum_1^K h_k U_k + n \quad (3.1)$$

$$y_{UL} = \sqrt{p_T} H U_k + n \quad (3.2)$$

Where P_T is the transmit power by each user, $U \triangleq [U_1, U_2, \dots, U_K]^T$ are the signals for BS coming from users, n is the additive noise vector consisting of Independent and Identical Distribution (I.I.D) variables with zero mean and unit variance and h_k is the k th column of channel matrix H [17].

3.2 Downlink Transmission

After channel estimation BS starts to transmit the signals to K users simultaneously. Suppose, $s_k \in C^{M \times 1}$, where $E\{|s_k|^2\} = 1$ is a vector which BS transmitted to users. The received vector for users K is given below

$$y_{DL} = \sqrt{p_T} \sum_1^K h_k^T s_k + n_k \quad (3.3)$$

$$y_{DL} = \sqrt{p_T} \sum_1^K H^T s_k + n_k \quad (3.4)$$

Where P_T is the transmit power of each user, $s \triangleq [s_1, s_2, \dots, s_K]^T$ are the signals for users coming from BS, n_k is the additive noise, $n \triangleq [n_1, n_2, \dots, n_K]^T$ is the noise vector consists of Independent and Identical Distribution (I.I.D) variables with zero mean and unit variance and h_k is the k th column of channel matrix H [17].

3.3 Linear Processing

To obtain the optimal gain, we must need complex signal processing techniques because they deal well with the complex signals and when number of antennas are large. In this thesis however, we consider linear processing schemes such as MRC and ZF which are used as linear receiver in the uplink and linear precoder in the downlink [17].

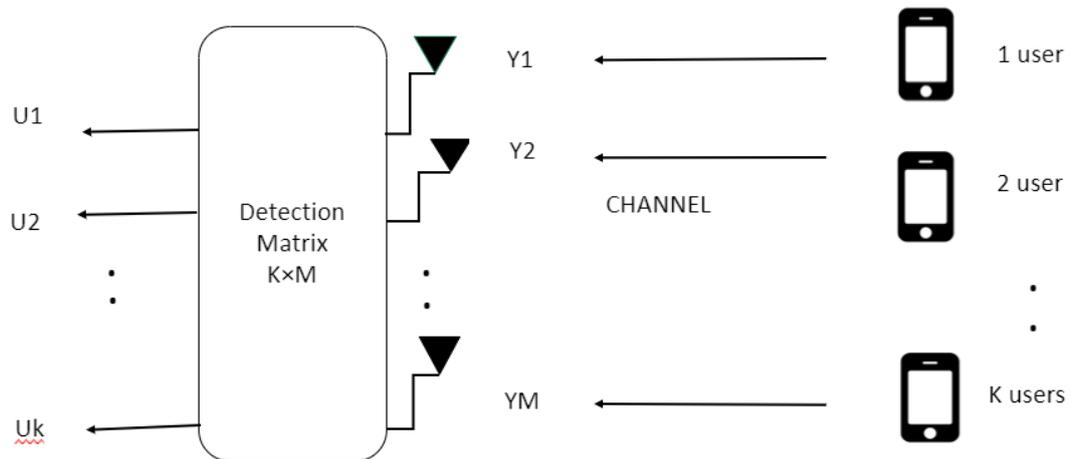


Figure 11. Block Diagram of Linear Detection at B [17]

3.4 Linear Detection in Uplink

The received signals are separated with the help of linear detection schemes at BS, simply, received signals are multiplied with the $K \times M$ detection matrix B as shown in Figure 11.

$$\tilde{U} = B^H y_{UL} \quad (3.5)$$

Here, \tilde{U} are the received signals that are separated at BS when $K \times M$ detection matrix B is multiplied with y_{UL} , B^H indicates Hermitian of linear detection matrix B and y_{UL} is the received signal, containing the input signal, channel and noise.

$$\tilde{U} = B^H \left(\sqrt{p_T} \sum_1^K h_k U_k + n \right) \quad (3.6)$$

$$\tilde{U} = B^H \left(\sqrt{P_T} \sum_1^K b_k h_k U_k + n \right) \quad (3.7)$$

Here, h_k is the k th column of channel matrix H , b_k is the k th column of detection matrix B and U_k are the k th signals of users and P_T is the total transmit power.

Here, collection of all symbols is decoded independently at the complexity of kU_k . Thus, the k th element is decoded as

$$\tilde{U} = \left(\sum_1^K \sqrt{P_T} b_k^H h_k U_k \right) + \left(\sum_1^K \sqrt{P_T} b_k^H h_k U_k \right) + (b_k^H n) \quad (3.8)$$

Here signal to interference plus noise is considered as effective noise and b_k is the k th element of detection matrix B .

$$SINR_K = \frac{P_T |b_k^H h_k|^2}{P_T \sum_{k'}^K |b_k^H h_{k'}|^2 + P b_k P^2} \quad (3.9)$$

We first discuss about the Maximum ratio combining. It has a very big advantage over ZF because it maximizes the Signal to Noise ratio and does not consider the $SINR$. The signal to noise ratio can be calculated as

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}}$$

The MRC at k th column of detection matrix B is

$$SNR_{mrc} = \frac{P_T |b_k^H h_k|^2}{P b_k P^2} \quad (3.10)$$

$$b(mrc) = \text{constant} \cdot h_k$$

$$\frac{P_T |b_k^H h_k|^2}{P b_k P^2} \leq \frac{P_T P b_k^H P^2 P h_k P^2}{P b_k P^2} \quad (3.11)$$

$$SINR_K = \frac{P_T |b_k^H h_k|^4}{P_T \sum_{k'}^K |b_k^H h_{k'}|^2 + P b_k P^2} \quad (3.12)$$

$$\frac{P_T |b_k^H h_k|^4}{\sum_{k'}^K |b_k^H h_{k'}|^2} \quad (3.13)$$

Advantages of MRC:

1. The computation is very simple with MRC e.g., it multiplies each coming stream with the conjugate complex of the detection matrix B which are then separated and decoded.
2. At low SNR, it has almost same the gain as singular user system.

Disadvantages of MRC:

1. It ignores the signal to inter user interference plus noise.
2. It does not perform well where value of power is large.

Zero-Forcing

With respect to MRC, ZF considers the inter user interference plus noise and ignores the effect of noise. The streams are projected at the orthogonal complement of each user to

null out the multiuser interference. The detection matrix B which satisfies the conditions of eq. (3.15) is called pseudo inverse of the channel matrix H .

$$\begin{cases} b^H_{zf,k} h_k \neq 0 \\ b^H_{zf,k} h_{k'} = 0, \forall k' \neq k \end{cases} \quad (3.14)$$

$$y_{zf,ul} = (H^H H)^{-1} H \cdot y_{UL} \quad (3.15)$$

$$= \sqrt{P_T} U + (H^H H)^{-1} H \cdot n \quad (3.16)$$

$$SINR_{zf,k} = \frac{P_T}{\langle (H^H H)^{-1} \rangle_{kk}} \quad (3.17)$$

Advantages of ZF:

The linear processing is easy and considers the signal to inter-user interference plus noise.

The $SINR$ can be increased by increasing the transmit power.

Disadvantages of ZF:

- 1) It does not consider the noise and performs poorly.
- 2) Zero forcing is much more complex than MRC because of the pseudo inverse computation [17].

3.5 Linear Precoding in Downlink

The collection of symbols is transmitted from M antennas to the intended users at receiver end after applying precoding techniques. Suppose v are precoded symbols for intended users K , then the precoder vector for transmitted symbols is S_K such that $E\{\|S_k\|^2\} = 1$, as shown in Figure 12.

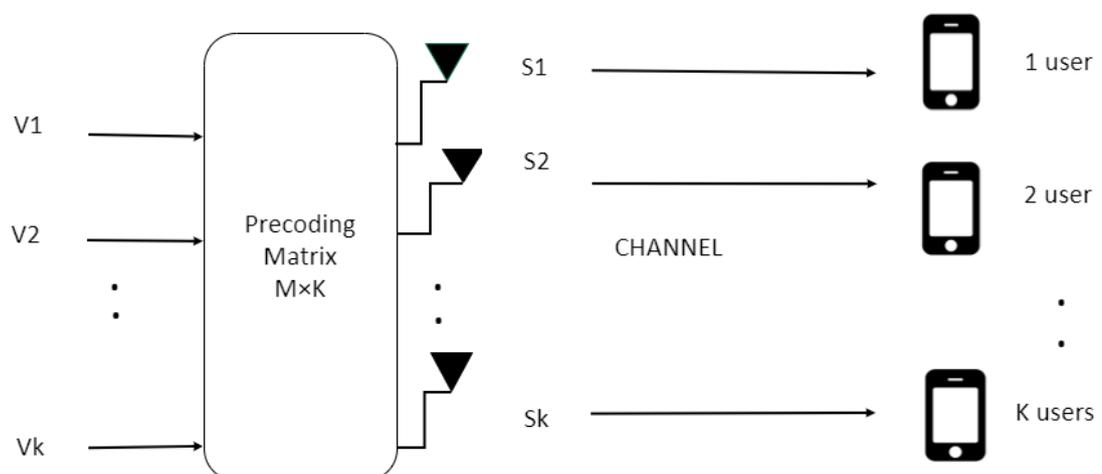


Figure 12. Block Diagram of Linear Precoder at BS [17]

$$s = \sqrt{\beta} \gamma v \quad (3.18)$$

Where $v \triangleq [v_1, v_2, \dots, v_K]^T$ a vector of all symbols for users, $\gamma \in \mathbb{C}^{M \times K}$ is the precoding matrix and we use β for the normalization of power which is given:

$$\beta = \frac{1}{E\{\text{tr}(\gamma \gamma^H)\}} \quad (3.19)$$

Putting equation (3.18) into equation (3.3), we get

$$y_{d,k} = \sqrt{\beta p_T} h_k^T \gamma v + n_k \quad (3.20)$$

$$y_{d,k} = \sqrt{\beta p_T} \gamma_k v_k + \sqrt{\beta p_T} \sum_{k' \neq k}^K h_k^T \gamma_{k'} v_{k'} + n_k \quad (3.21)$$

To get $SINR$ for from BS kth users is obtained by simplifying the equation (3.20)

$$SINR_k = \frac{\beta p_T |h_k^T \gamma_k|^2}{\sqrt{\beta p_T} \sum_{k' \neq k}^K |h_k^T \gamma_{k'}|^2 + 1} \quad (3.22)$$

The conventional precoder or conjugate beamformer for linear processing such as MRT and ZF receivers are simplified and given the brief formula in equation [17][18][19][20].

4 LAWS AND THEORIES

4.1 Random Matrix Theory

Let $S_1, S_2, S_3, \dots, S_n$ be the independent and identical distribution variables with mean ($\mu = 1$) and variance ($0 < \sigma^2 < \infty$), then

$$\text{the sample mean } (\overline{S}_n) = \frac{1}{n} \sum_{i=1}^n S_i \text{ or } \mu = E(S_i) \quad \forall_i \quad (4.1)$$

4.1.1 Law of Large Numbers

From equation (4.1), Law of large number is defined as

the probability where sample mean (\overline{S}_n) approaches to average mean ($\mu = 1$) and variance ($0 < \sigma^2 < \infty$) [20].

4.1.2 Central Limit Theorem

Equation (4.1) can be multiplied by a factor $\frac{n}{\sigma^2}$, we get

$\sqrt{\frac{n}{\sigma^2}} \left(\frac{1}{n} \sum_{i=1}^n S_i - \mu \right) \rightarrow$ approaches to normal distribution $N(0,1)$ where variance ($\sigma^2 = 1$) and average mean ($\mu = 1$) as shown in Figure 13,

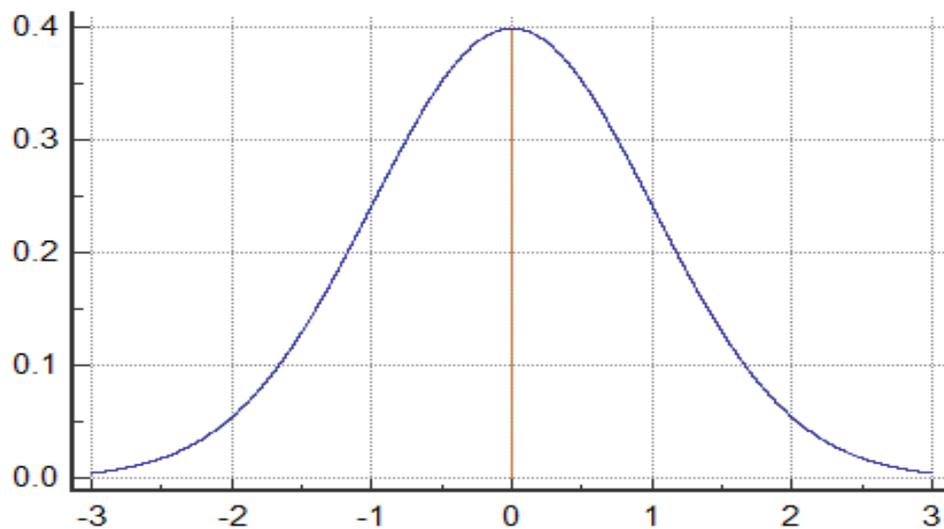


Figure 13. Normal Distribution with $N(0,1)$ with $\mu = 1$ and σ^2 [21]

4.2 Cauchy-Schwarz Inequality

For any random variables S, V ,

$$E[SV]^2 \leq ES^2EV^2$$

Proof. For $p, q \in R$ Let $C = pS - qV$

$$\text{Then } 0 \leq E[C^2] = E[(pS - qV)^2] = pE[S^2] - 2pqE[SV] + q^2E[V^2] \quad (4.2)$$

quadratic in a with at most one real root and therefore has $det \leq 0$ [22].

4.3 Transpose of a Matrix

Let A be the given $m \times n$ matrix, then transpose of A is given as

$$A = A^T \quad (4.3)$$

where rows of matrix A^T are becoming columns of matrix A [23].

4.4 Inverse of a Matrix

Suppose $A = a_{ij}$ is the given $n \times n$ matrix. Inverse of matrix exists when $det \neq 0$ as describe [24].

$$A^{-1} = \text{transpose of } \left(\frac{(-1)^{i+j} \text{Det}(A_{ij})}{\text{Det}(A)} \right) \quad (4.4)$$

5 REVIEW OF THE STATE-OF-THE-ART

This thesis has two main parts. The first part is to describe the basics of the massive MIMO relay networks in the performance perspectives, e.g., Bit-Error-Rate. It also includes the process about information transmission and protocols used in the thesis with channel effect such as pilot contamination and the solution for it. Second part is to design models for maximum ratio processing and zero-forcing and to analyse them based on different factors such as BER, Spectral Efficiency, power and number of antennas in array.

In [1] by Chung Duc Ho et al., “a Multi-Way Massive MIMO relay networks with maximum ratio processing” a relay system is proposed where Base Station (BS) multiple antenna system. It transmits the information to many users when channel state information is already known. In case of improper CSI, users send the orthogonal pilot sequence to the BS for channel estimation. The precoding scheme, MRC, is used on the relay system. The information is then broadcasted to each intended user. Two approaches have been analysed in this research paper. First, saving the power and second, the energy efficiency regime. The transmit power on both relay and user side can be reduced by the ratio of M number of antennas. Linear processing schemes are used on relay system and analysed that ZF is more efficient in the multi-way antenna system because it cancels the effect of inter-user interference but difficult to compute due to matrix inversion. Zero-forcing is not performed well in antenna systems with large number of antennas due to increase in computation complexity. On the other hand, MR is very simple to implement in large antenna systems and can be used in a distributed manner. Due to massive array system it can be helpful in wireless conferencing, sensor networks and power control in heterogeneous cellular networks.

In[25] by Chung Duc Ho et al., “On the Performance of Zero-forcing Processing Way Massive MIMO relay Network” a multi-way relay network is proposed having many antennas with imperfect CSI is considered. The uplink pilot sequence is used for obtaining the channel estimation. Time division duplexing is introduced which is a network protocol. It gives the same time frequency resources to BS antennas and users for sending and receiving data. The data sending to intended users in a linear processing technique, ZF, is used at relay. The main function of it is to beamform the signals to the intended user and cut the unused beams for wastage of coverage area and transmit power. A couple of things are analysed from the closed expression of spectral efficiency. The spectral efficiency (SE) can be achieved when the number of antennas is inversely proportional

to inter-user interference and noise. The numerical results show that at high SNR values, spectral efficiency can be higher when using ZF processing at relay instead of MRC. The research for imperfect CSI and linear processing techniques is very less. The combination of relay network for different field such as wireless sensor networks and data fusion centres made massive relay networks so important that it is obvious to get more knowledge of Imperfect CSI. The solution of channel estimation will make the MIMO relay system an important next generation technology.

In [9] by Shi Jin et al., “On Massive MIMO Zero-Forcing Transceiver Using Time-Shifted Pilots is proposed the characteristics of a modern wireless networks”. A quality of experience (QoE) is needed in the form of efficient frequency spectrum reuse and channel quality. Massive MIMO seems to be the solution for all the problems. It works well when the number of user terminals is limited. In a wireless cell many users within a cell can use few pilots and interference is started when more than one user from different cells try to use the same pilots. Pilot contamination is the major issue in the new wireless systems. Several linear precoding methods are proposed such as MMSE and messages reuse but do not become practical due to unavailability of CSI in large number of users. One of the best techniques named as conjugate beamforming is derived. It uses time shifted pilot sequences for different cells as shown in Figure 14

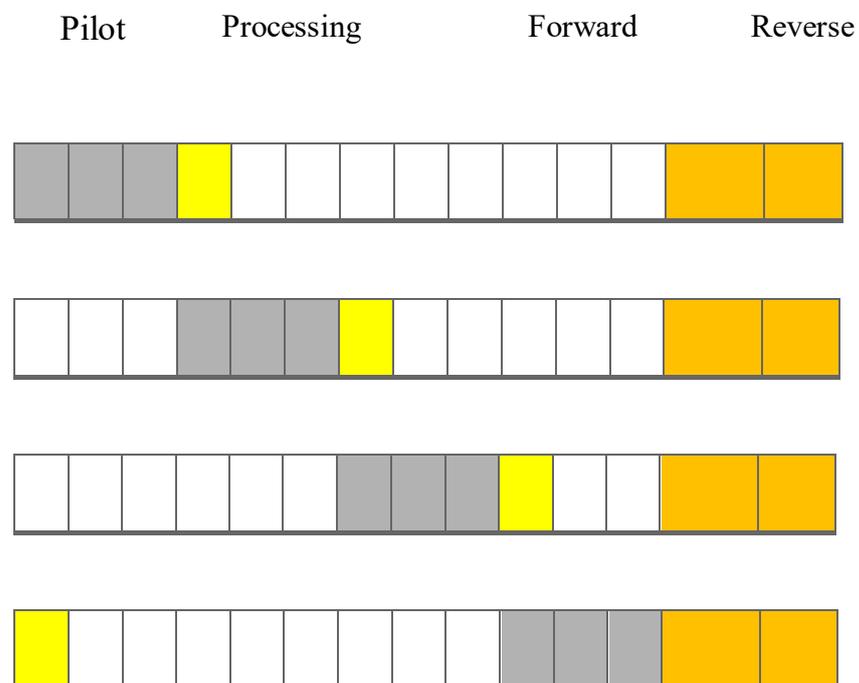


Figure 14. Time Shifted Pilot Processing [9]

The pilot contamination is mitigated when using the same pilots used in the forward link and rearranging them for reverse link. Different parameters such as cell radius, transmit power, number of antennas are set in time shifted pilot to check the performance. It is analysed that without CSI, throughput can be maximized by applying conjugate beamforming. The comparison shows that conjugate beamform precoder is more efficient than ZF precoder. The performance difference of conjugate beamforming and zero-forcing precoder in the form of enhancing cell throughput devised a large-scale-channel-fading-based user scheduling (LCFS) algorithm, used to mitigate the pilot contamination.

In [26] by Claire Masterson et al., “Massive MIMO and Beamforming: The Signal Processing Behind The 5G Buzzwords”, it is proposed that increase in the electronic devices in the urban environment especially creates high data traffic which have saturated available frequency spectrum. The efficient utilization of the current frequency spectrum can be possible with the inclusion of Massive MIMO beamforming cellular system. Multiple antenna system makes use of the radio multipath propagation phenomenon to get more powerful signals. Beamforming steers the beam towards intended users constructively and cuts the undesired beams destructively to avoid energy wastage. Channel state information (CSI) is an important factor to manage the system capacity and energy regime. It is a combination of all spatial data transfer between antennas and users in the form of a channel matrix. In Massive MIMO, BS has a large antenna array and users also. A mobile wireless network is said to be performance driven when base station has the complete knowledge of channel state information. Two techniques can be used for channel estimation. Time division Duplexing (TDD) uses the same path for uplink and downlink to utilize frequency resources while frequency division duplexing (FDD) uses different paths for the utilization of frequency spectrum. Precoding/detection is performed with linear processing schemes such as MRC, ZF or MMSE. The integrated transceiver product of the analogue Devices Radio Verse™ family enhances the available system capacity and can be Massive MIMO friendly.

In [27] by Juan Wang et al., “Optimal Power Allocation of MIMO Relay System Under the Background of 5G”, it is proposed that 5G mobile technologies aim to provide the modern world with high speed internet, increased coverage, efficient use of frequency spectrum and improved channel capacity. The evolution of 4G to 5G mobile technology has a lot of challenges before being implemented in the form of technical maturity, standardization and commercialization. The organization named as Cisco Visual Networking Index reported that with the passage of time mobile technology grows explosively [28]. On the other hand, new technologies enhance the terminal capacity, recognize the source and destination terminals intelligently and virtually but on the other hand create high data

traffic. To develop standards and modern 5G telecom industry, several organizations made combined effort. With the struggle of International telecommunication unit (ITU), 3rd generation project partners (3GPP) and Next generation mobile networks (NGMN) started formulating the standards for new mobile technologies. The capacity of network can be increased with the help of Shannon capacity formula which consists of coverage, channel, bandwidth and SNR factors. It is evident that previous technology cannot be changed totally to achieve the system performance, but it can be modified and extended by combining the relay systems. This mixed technology is called as hybrid technology. The best example is the massive MIMO relay system where a relay is used before and after the terminal to provide the data processing and convey the data where the coverage of base station is impossible. The MIMO in this hybrid system uses the antenna diversity to enhance the coverage. A mathematical model was designed from Shannon capacity formula, which optimized the power and channel capacity both for uplink and downlink in 5G network.

The paper “Power Control for Cellular Networks with Large Antenna Arrays and Ubiquitous Relaying” [29] by Raphael T.L. Rolny et al., focuses on a cellular network which evolves from complex network to simpler network with main goals as higher data rates, coverage, reliability and interference mitigation. The first scenario shows a network consisting of BSs and massive relay systems. The relay system works as relay carpet and exchanges information processed by the BSs to the users, separated on a huge geographical area. This network is suitable for less users and with a higher number of users the relays are overcrowded, resulting in less performance. For higher user densities, one use case is to increase the number of BSs and reduce the cell sizes. However, in practice, it is difficult to implement because the network becomes complex and costly. Another use case can be suitable if the existing network is extended when BSs relate to massive antennas. The benefit is that massive antennas suitable for many users instead of many BS. A virtual form of network where base station (BS) are jointly working with mobile station (MS) is called cooperation/coordinated multipoint transmission (CoMP) gives better

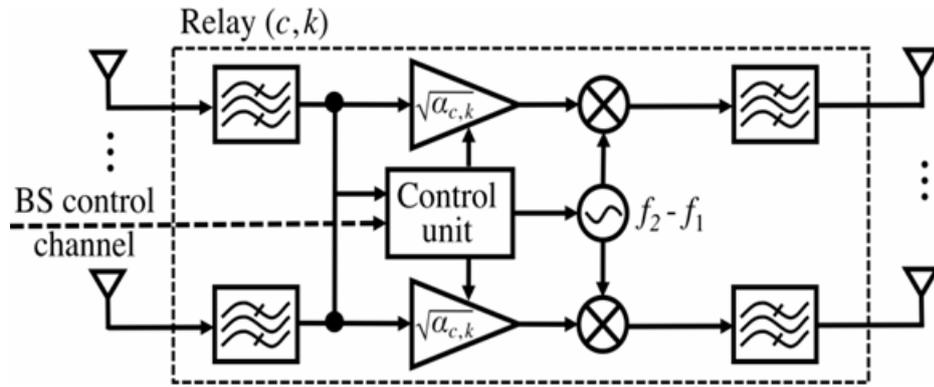


Figure 15. Conceptual Schematic of FDD AF Relays [29]

coverage to users than a physical channel formed from just mobile stations or base stations. A base station (BS) connects more relays in the relay system networks and gives coverage to more users than mobile station (MS) that connects few users and relays. The user separation is achieved with the help of beamforming. The issue for this network is that more pilots are attached with the signals for channel state information (CSI) and data exchange among all nodes, hence increasing the overhead. The crowded traffic of nodes decreases the performance of the whole system. The issue of pilots can be solved with two-hop network. Figure 15 shows that a base station (BS) must be attached with fixed-position relays and for transmit channel state information (CSI) the channel is considered as constant. Beamforming and channel estimation become simple. A mobile station (MS) on the other hand deals with few antenna and relays on favourable channel, saving the channel from deep fade. The distributed network makes signal processing easy, minimizing the power and mitigating the interference.

There are two ways to create a two-hop network. One is to use time division duplexing (TDD) while the other uses frequency division duplexing (FDD). From this paper it is learned that TDD introduces delays for signal processing while FDD decreases delays. The performance and power are balanced with the use of frequency division duplexing (FDD) in jointly beamforming network.

In [30] by Shohei Yoshioka et al., “5G Massive MIMO with Digital Beamforming and Two-Stage Channel Estimation for Low SHF Band”, it is proposed that the abundance of electronic devices has made the communication system saturated. Long-term evolution (LTE) and advanced long-term evolution (LTE-A) will not be able to cope with the increase of bit rate and channel bandwidth in near future. 5G and internet of things (IoT) communication systems will take care of the future wireless networks. One of the fundamental properties of 5G is that it uses super high frequencies such as Low-SHF, high-SHF and EHF and facilitates the networks with throughput and large bandwidth. One of

the technologies known as massive MIMO beamforming can reduce the path-loss by using the spatial diversity. Here, two types of methods are considered for multiple input and multiple output system. Firstly, MIMO can be used as pure digital fixed beamforming named as digital fixed beamforming channel precoding (FBCP). The practical implementation for this network has some drawbacks. 5G uses super high frequency so power consumption will be high and ADC to DAC converters are very costly. Secondly, Hybrid FBCP method can be used in the form of analog beamforming and digital precoding. The extension of fully digital FBCP can be solved by applying the lower frequencies instead of higher frequencies, which results in low power consumption and easy to implement. Again, there are few complexities to make it practical, precoding matrix computation is very complex and channel estimation at DL and UL must be solved. Four important steps are introduced to fix all these problems. Initially a limited number of beams is chosen as (No of beams \leq No of transmit antennas), which means that less pilots are used for channel estimation and less will be the overhead. After that, power of the pilot signals coming from users for channel estimation used to select the weight w and BS estimates in channel matrix. The BS pre-codes the weights in the channel matrix and lastly sends them to the users according to the power of channel matrix. Result from the link level computer simulation shows that throughput obtained from digital-FBCP is much higher than conventional digital precoding without fixed Beamforming.

In [31] by Ashwani Kumar Pandey et al., "Performance Analysis of MIMO Multi Relay System in Cooperative Relay Network", it is proposed that wireless networks often use single antenna at source and destination to exchange the information between the channel. From a modern networks point of view, it is considered that as information and data traffic have been increasing so networks must be capable to carry more data and more diverse. A new concept has been introduced often named as Cooperative network. A network where a transmitter attaches large number of relays as partners to decode and forward the data towards destination. Relays in MIMO network decode and forward the signals coming from transmitter to destination. The DF strategy works well in limited number of relays, but it fails, and relays become overcrowded and causing performance degradation. Several DF schemes are purposed for high and low complexity environment such as Maximum Likelihood Equalizer and Decision Feedback Equalizer respectively, but both degrade the performance. Finally, opportunistic scheme is purposed which suits the DF processing. The mechanism is very easy. A scenario where a centralized BS is designed which globally knows the CSI of all the partners relays. During processing between source and destination, it uses the best relay to deal with the specific task but

becomes overhead and fails to be practical. In this paper, opportunistic scheme is improved by providing a few important steps. Especially a relay node selection algorithm is proposed which decides the best relay and task for specific relay. Also, a power scaling matrix is designed which evaluates the cooperative diversity for transmitted symbols in probability error closed form expression. Simulations show that at for practical SNR regime low value of power with high normalization factor DF MIMO relays performs efficiently.

In [32] by Chenguang Lu et al., “Cooperative Spatial Reuse with Transmit Beamforming is proposed that beamforming” is the technique in modern wireless network which use spatial diversity and multiplexing to increase the system capacity and energy efficiency. Wireless Local area networks are increasing enormously over a large geographical area. They lack centralized node for dealing and deciding among the users for using channel resources. The channel spectrum is wasted due to contention between users which becomes a reason for creation of contention region. An important scheme known as TDMA-based MAC is proposed in which an 802.11 MAC protocol to coordinate with different nodes. It makes use of spatial diversity. It allows one link to operate in contention region while others need to be silent. Current MAC scheme deals well with small size of contention region, but ad-hoc networks are very large, so they need more links to transmit the data instead of one link with multiple users. The modern idea came into being in the form of cooperative spatial reuse. CSR enables the current TDMA-based MAC scheme to expand the wireless network on a large area. Here, more slots are given to the links to transmit the data over channel. The channel capacity is increased when each node contributes its own data. The cooperative links only participate in the network when they get benefits otherwise leave the network. To check the channel capacity and energy efficiency over a channel, a comparison between ZF-CSR and MRC-TDMA is made. It is shown that ZF-CSR has more potential to save power and energy efficiency compared to MRC-TDMA in 2×1 MISO wireless networks.

In [33] by M. Hasbullah Mazlan et al., “Investigation of pilot training effect in massive-MIMO TDD system”, it is proposed that Massive MIMO is the vital technology for modern 5G cellular networks in upcoming years. The main reason is that it uses multiple antennas at transceiver to serve less users efficiently. Spectral Efficiency and Energy efficiency can be improved when BS maintains the rank of channel matrix and mitigates the interference. Massive MIMO provides different paths to the users in TDD environment to simultaneously exchange data stream back and forth. The steering of beams towards desired users saves the transmits energy. Beamforming through linear processing helps

to separate the users and mitigates the inter-user interference. The inexpensive infrastructure and focused beams increase the EE and SE instead of using expensive amplifiers. The large antenna system creates technical challenge for BS in the form of channel estimation. Pilot contamination is caused due to large antenna and limited training sequence, which hinders the overall system performance. So, the BS is unable to achieve full CSI in uplink and downlink with a smaller number of pilots training available for users in different cells. The paper investigates the pilot and noise contamination with different performance metrics such as average rate per user, bit energy, average achievable rate and SE. TDD transmission environment is used to evaluate the effects of pilot training sequence on the performance of Least Square (LS) and Minimum Mean-Square Error (MMSE) estimator. The increase in pilot sequence duration and number of antennas in both uplink and downlink increase the channel estimation performance and have greater impact in terms of Minimum Square Error (MSE).

6 MODULATION TECHNIQUE

6.1 OFDM Modulation and Demodulation:

The technique dealing with high data rate in advanced communication systems is the orthogonal frequency division multiplexing (OFDM) [34]. Different other communication techniques such as CDMA (Code Division Multiple Access) and OFDMA (Orthogonal Frequency Division Multiple Access) are single carrier and deal with low data rates.

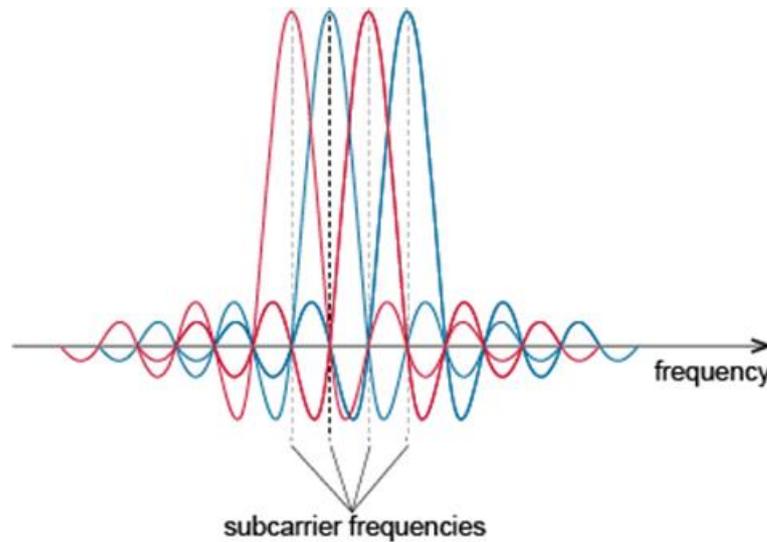


Figure 16. *The Subcarrier Spacing in OFDM communication [35]*

They also use carriers for signal transmission and remove interference between cells but lack a system which deals with the complex algorithms to minimize the complexity during channel equalization process [36]. On the other hand, OFDM utilizes communication bandwidth efficiently. It divides the bandwidth into a number of subcarriers as given below:

$$N = \frac{W}{\Delta_f} \quad (6.1)$$

Where N is the number of sub-channels, W is the total bandwidth and Δ_f is the subcarrier spacing [37].

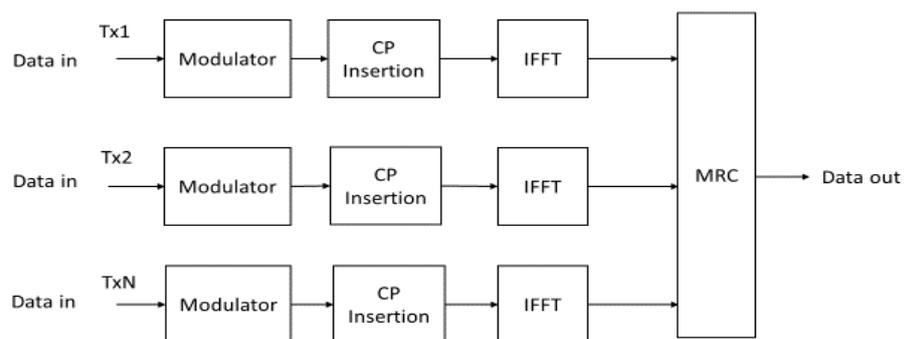
It is shown in Figure 16 that subcarriers are orthogonal to each other during the allocation of communication bandwidth. It is shown in Table 2 that OFDM modulation has much more bandwidth than other schemes [38].

Table 2. A Comparison of Different Modulation Schemes

Modulation scheme	Bandwidth Efficiency	Complexity
BPSK	Medium	Low
FSK	Medium	Low
SFSK	Low	Medium
OFDM	High	High

6.2 Steps involved in OFDM Modulation

At transmitter, modulator converts the random data to complex data. Input data is modulated according to the sub-carrier's amplitude and phase for the required modulation scheme. Inverse Fast Fourier Transform (IFFT) converts the data into time domain. Maximum Ratio Combining combines the streams as shown in Figure 17. To make the data symbols periodic Cyclic Prefix (CP) is attached with them to save them from Inter-Symbol-Interference (ISI) and Inter-Cell-Interference (ICI). All time domain symbols are then passing through the channel.

**Figure 17.** Block Diagram of OFDM Transmitter [39][40]

6.3 Steps involved in OFDM Demodulation

It is shown in Figure 18 that received symbols on receiver side are converted into frequency domain with the Fast Fourier Transform (FFT) and the cyclic prefix is removed. The channel is inverted for equalization using zero-forcing equalizer and data streams are combined with maximum ratio combining and the Bit-Error Rate (BER) measured according to SNR of each data stream.

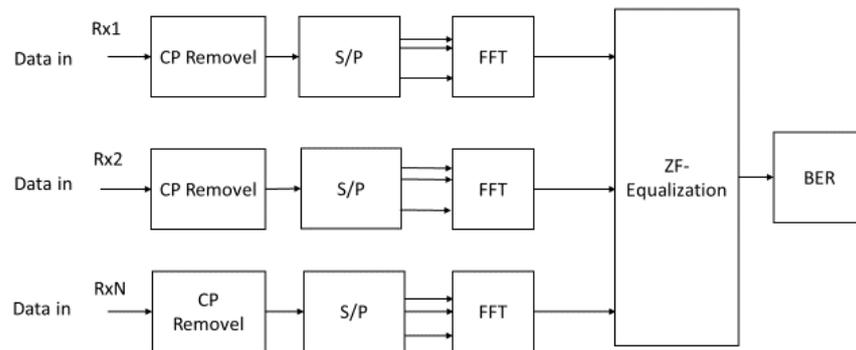


Figure 18. Block Diagram of OFDM Receiver [39][40]

Lastly, the data is converted from serial to parallel data stream at the receiver.

6.4 Singular Value Decomposition Method for Massive MIMO

Singular Value Decomposition method is used to compute the overall capacity and channel gain when data travels from Transmitter to receiver over channel. It is a very important mathematical tool that can be utilized for the complex matrix computation. In this thesis, a MIMO channel is supposed where a number of antennas for transmitter and receiver side are present.

Consider a MIMO channel setup defined as

$$Y = Hx + n$$

Here, if H contains the non-real values then we have an option to convert these values into real. So, H is configured with antennas as $N=2$, $M=2$ then it is modified with SVD in MATLAB as given below

$$H = (\text{ones}(N, M(c)) + 1i * \text{ones}(N, M(c))) / \text{sqrt}(2);$$

$$[U, S, V] = \text{svd}(H);$$

$$U =$$

$$\begin{bmatrix} -0.5000 - 0.5000i & 0.7060 - 0.0394i \\ -0.5000 - 0.5000i & -0.7060 + 0.0394i \end{bmatrix}$$

$$S =$$

$$\begin{bmatrix} 2.0000 & 0 \\ 0 & 0.0000 \end{bmatrix}$$

$$V =$$

$$\begin{bmatrix} -0.7071 + 0.0000i & -0.7071 + 0.0000i \\ -0.7071 - 0.0000i & 0.7071 + 0.0000i \end{bmatrix}$$

Where S is a diagonal matrix with singular values, U and V are orthogonal matrices. The output of SVD is multiplied with the input values x and noise is added before sending towards the receiver [41].

6.5 Importance of Cyclic Prefix in OFDM

OFDM modulation is very important when it is compared with other modulation schemes because it provides high speed data transmission, high bandwidth. Due to high data rate it also includes Inter-Symbol-Interference (ISI) among the data. In this modulation, Guard interval is introduced at the beginning of the OFDM symbols by appending Cyclic Prefix (CP). From the IFFT data, last samples are duplicated and appended at the start of the data symbols. The process of inserting Cyclic Prefix is shown in the Figure 19.

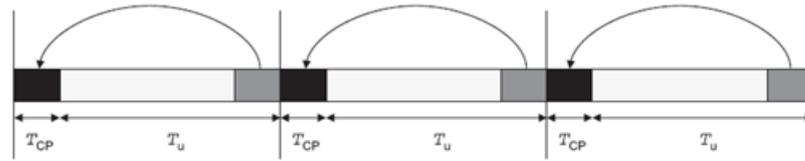


Figure 19. Cyclic Prefix Insertion in OFDM Data [42]

Cyclic Prefix transforms the linear (aperiodic) convolution to circular (periodic) convolution. ISI can be eliminated completely only when length of CP is longer than the impulse response of the channel. CP on the receiver is removed in reverse process as happens in the transmitter. CP-OFDM is very efficient because it converts the frequency selective channel to flat fading channel. The equalization process becomes very easy when parallel streams of data passes to flat fading channel [42].

6.6 Flow Chart of Zero-Forcing Equalizer

Figure 20 shows all the important steps involved in the theoretical model of zero-forcing equalizer. ZF equalizes the streams by inverting the channel response.

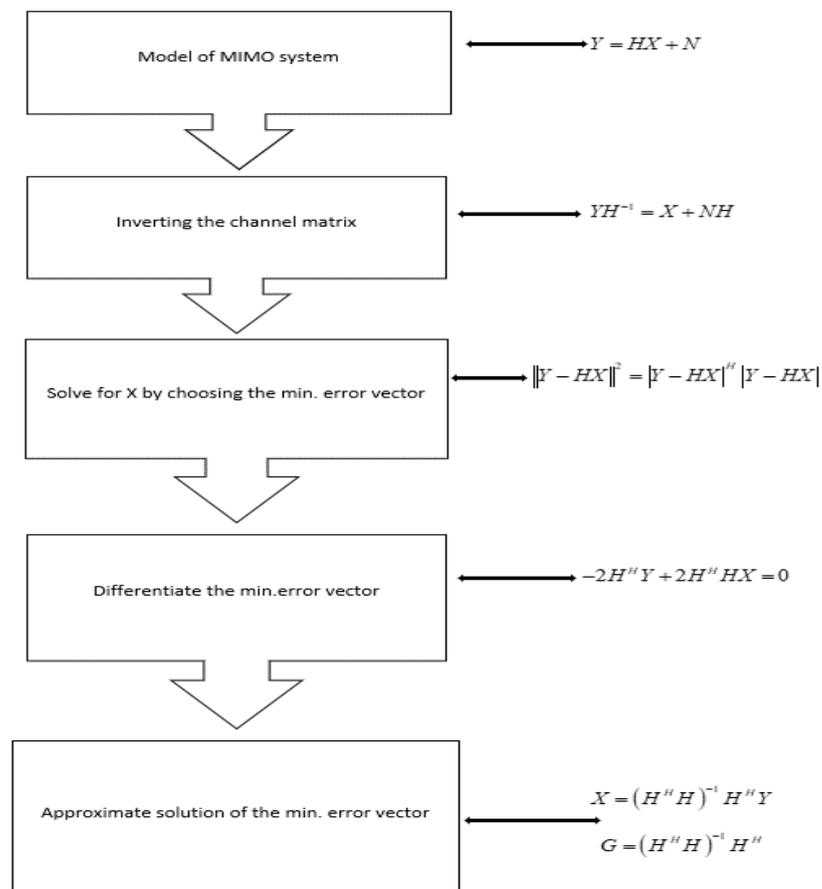


Figure 20. Flow Chart of Zero-Forcing Equalizer [43]

6.7 Flow Chart of Maximum Ratio Combining

Figure 21 shows all important steps performed by maximum ratio combining algorithm. Streams are combined at transmitter side with respect to SNR values. Phases are adjusted when streams are multiplied with weights.

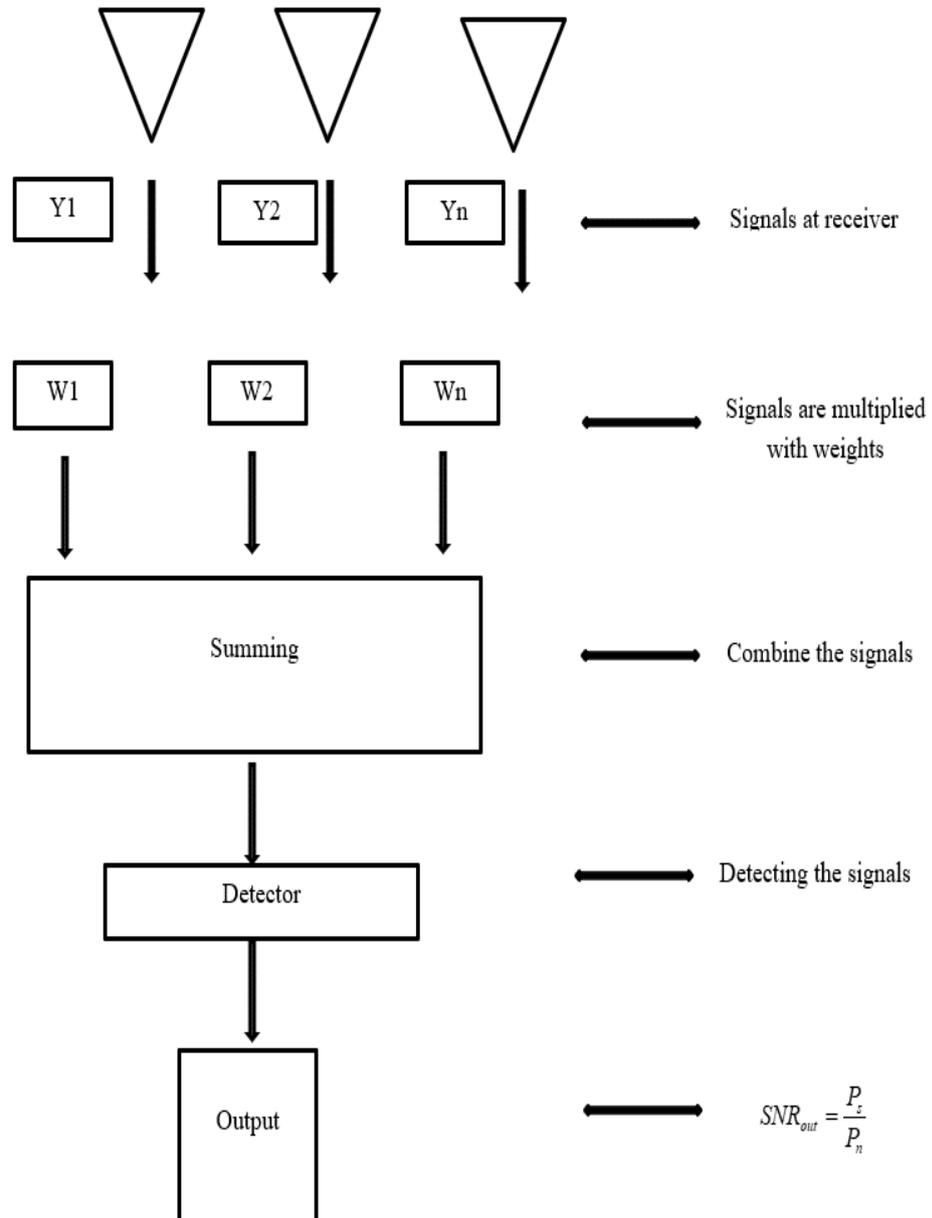


Figure 21. Flow Chart of Maximum Ratio Combining [44]

7 MEASUREMENT AND RESULTS

The following section demonstrates the simulation results obtained from the MATLAB simulation Model. This section describes the evaluation of beamforming algorithms in the cyclic prefix-Orthogonal Frequency Division Multiplexing (CP-OFDM) environment. This section also considers different use cases for evaluating the performance, the antenna configuration and modulation schemes.

7.1 Algorithms Processing

The simulation parameters for the processing of algorithms are given in the table below. The data that travels from transmitter to receiver follows the flow chart given below. The Random data was generated by setting the number of subcarriers, modulation scheme and OFDM symbols. The modulation schemes according to Table 3 were used and CP of length 8 was also used to remove the ISI from data. Training bits were concatenated with IFFT data of size 2048 to perform equalization at the receiver end.

Table 3. *Simulation Parameters*

Parameters	Values
IFFT Size	2048
Number of Subcarriers	1200
Modulation Schemes	QPSK, 16-QAM, 32-QAM, 64-QAM
Signal to Noise Ratio	-30:2:20 dB
Number of Transmit Antennas	64
Number of Receive Antennas	1,2,4,6
Cyclic-Prefix	8
OFDM Symbols	15

A single stream of data is generated and transmitted from each antenna. Singular Value Decomposition (SVD) method is used to optimize the MIMO channel capacity.

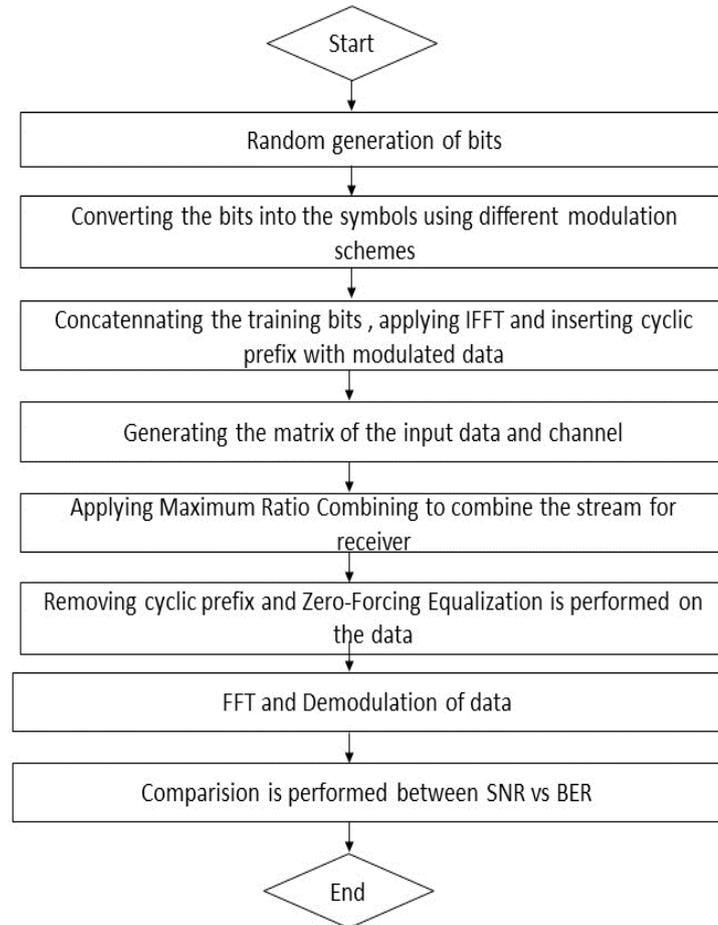


Figure 22. Working of Beamforming Algorithms in CP-OFDM Process [39][40]

The Transmitted signal passed through the MIMO channel where noise was added. Maximum Ratio Combining (MRC) combined the individual streams for each receiver antenna one by one according to individual SNR. On the receiver side, CP was removed and FFT of the data performed. The received data was equalized with Zero-Forcing equalization by inverting the frequency response of the channel. The bit stream before the modulation and the after demodulation are compared to check the BER as shown in the Figure 22. The simulation parameters as shown in Table 3 were varied in different use cases to evaluate the BER.

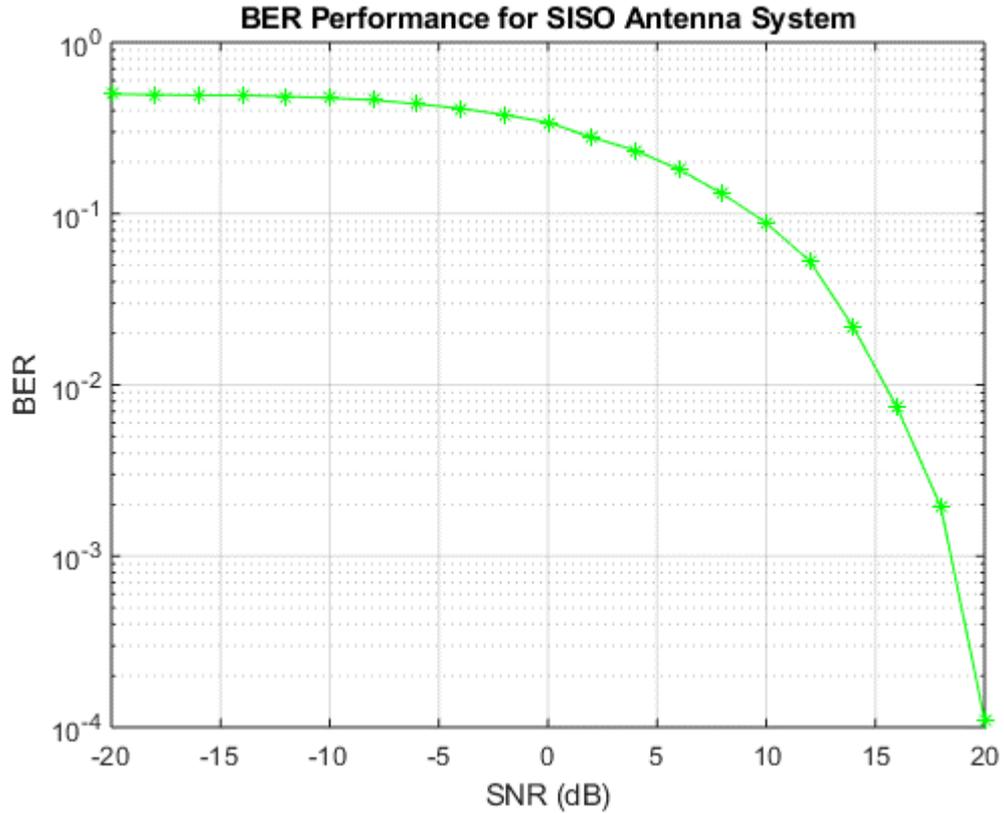


Figure 23. The Figure shows the BER vs SNR plot for SISO Channel with Modulation scheme QAM=16, Subcarriers=1200, CP=8 and OFDM Symbols=15, numtx=1

The beamforming algorithms are used in the SISO channel to measure the BER performance with respect to SNR. From Figure 23 when the single antenna system is configured in the CP-OFDM system with MRC and ZF algorithms on the transmitter and receiver side respectively, the BER value is becoming better and better. As SNR values are increased in the signal, the BER curve continuously decreasing. For instance, at SNR=-20 dB the value of BER is, BER=0.49 and at SNR=0 dB, BER=0.3385 and similarly, it decreases in the same fashion as SNR increases, so at SNR=28dB, it is BER=0.0015. So, before simulation and after, it is clear that the performance with a single antenna configuration is improving as SNR value is increasing.

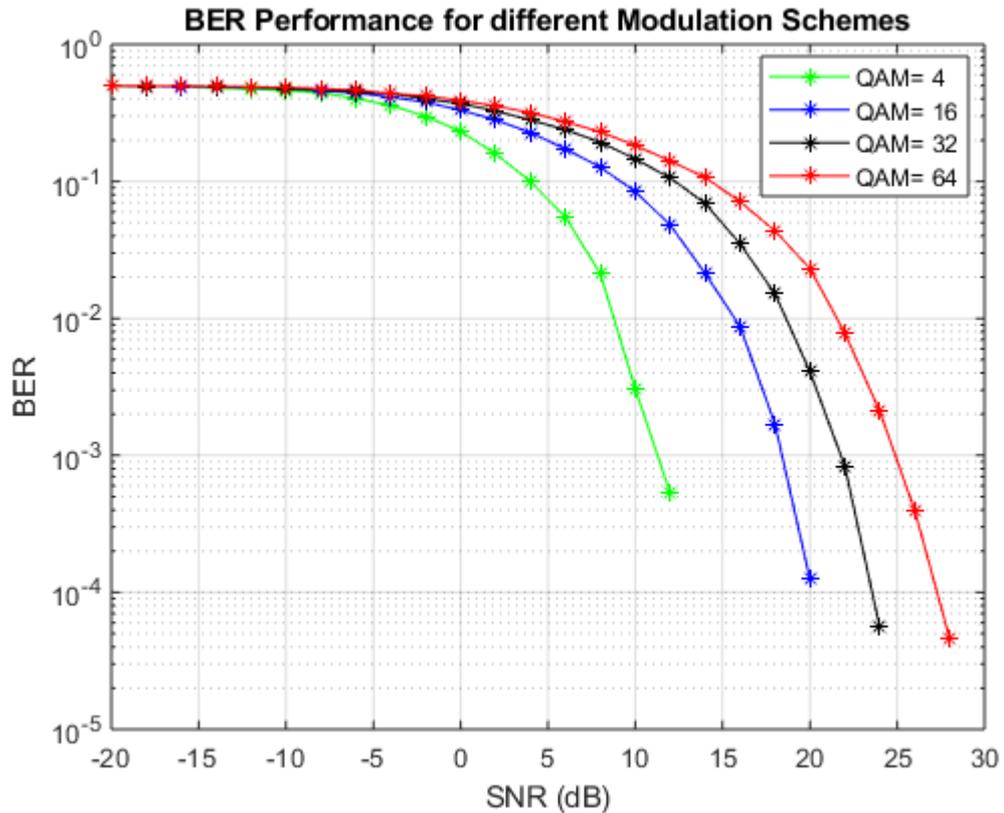


Figure 24. The Figure shows the BER vs SNR plot for MISO channel with different Modulation schemes QAM=4, QAM=16, QAM=32, QAM=64, Subcarriers=1200, cp=8, OFDM Symbols=15, numtx=64 and numrx=1

Figure 24 illustrates the BER performance comparison of different modulation schemes keeping simulation parameters such as antennas, OFDM symbols, subcarriers and CP length fixed. Different modulation schemes were implemented to evaluate the beamforming algorithms. It is shown in the above figure that as modulation schemes are going higher from QAM-4 to QAM-64, performance start to decrease. The QAM=64 gave very high BER values when comparing with other modulation schemes. At SNR=-20 dB, BER=0.49, all modulation schemes gave the same performance. But as SNR values increased, BER value decreased for all schemes. At SNR=0dB, BER=0.2249,0.3278, 0.3690,0.3917 for QAM=4,16,32 and QAM=64 respectively. When we further increased the SNR values then the performance difference between all schemes became huge. At SNR=16dB, BER=0,0.0078,0.0369,0.0723 for QAM=4,16,32 and 64 respectively. After a comparison of all modulation schemes, it is concluded that QAM=64 at high noise gave worst performance. On the other hand, QAM-4 showed good performance.

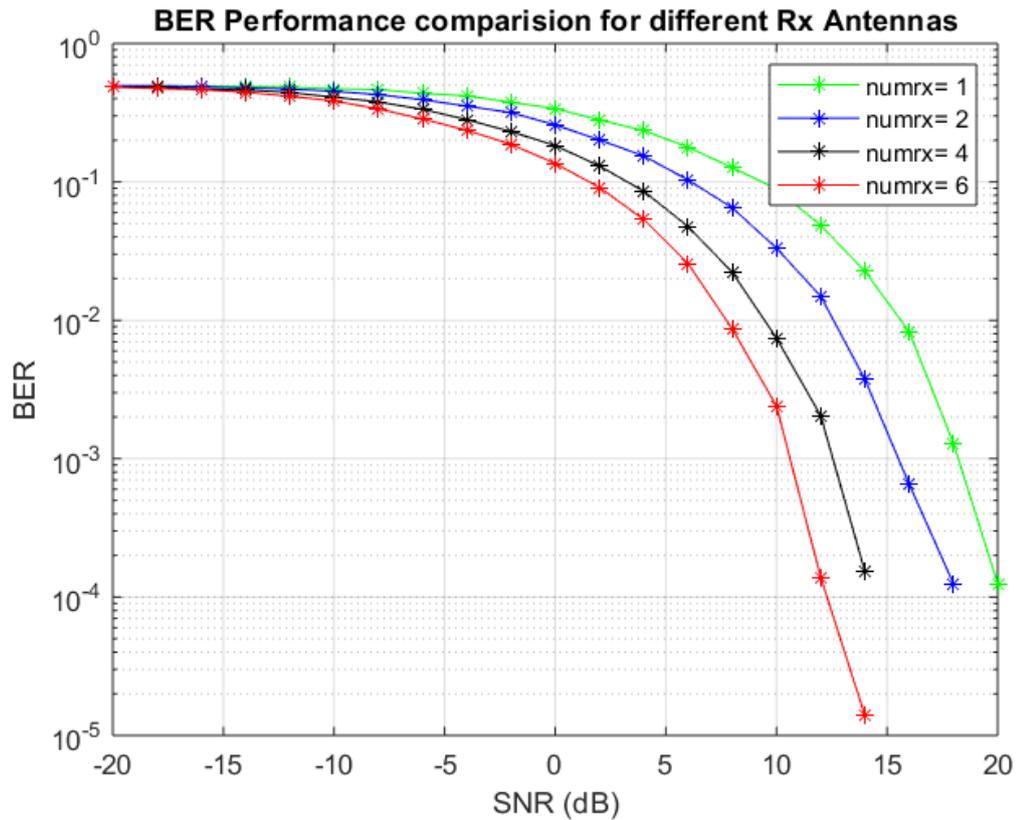


Figure 25. The Figure shows the BER vs SNR plot for MIMO channel with different Receive antennas, numrx=1,2,4,6, QAM=16, Subcarriers=1200, cp=8, OFDM Symbols=15 and numtx=64

Figure 25 illustrates the BER performance comparison of different receive antennas considering fixed simulation parameters such as modulation scheme, transmit antennas, OFDM symbols, subcarriers, and CP length. A use case was made where different receive antennas were implemented to evaluate the beamforming algorithms in the CP-OFDM system. At SNR=-20 dB, BER=0.4977, all receive antennas got the same power and BER was also the same. As SNR values increased BER value decreased for all receive antennas. At SNR=0dB, BER=0.3415,0.2633, 0.1803,0.1326 for numrx=1,2,4 and numrx=6 respectively. When we further increased the SNR values then performance difference between all receive antennas became large. At SNR=20 dB, BER= 2.7e-05,0,0,0 for numrx=1,2,4 and 6 respectively. After a comparison of all receive schemes, it is concluded that from all receive antennas, numrx=6 gave much better performance in CP-OFDM system as compared to other antennas.

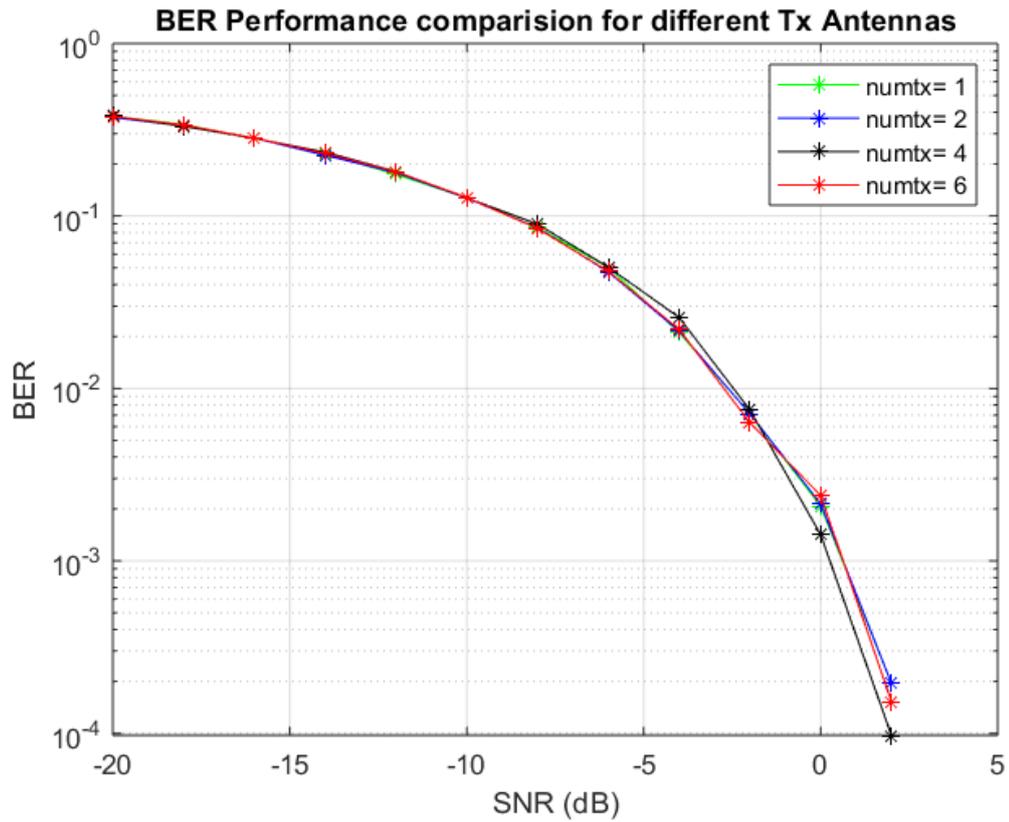


Figure 26. The Figure shows the BER vs SNR plot for MIMO channel with different Transmit antennas, $numtx=1,2,4,6$, QAM=16, Subcarriers=1200, $cp=8$, OFDM Symbols=15 and $numrx=64$

Figure 26 illustrates the BER performance comparison of a different transmit antennas considering fixed simulation parameters such as modulation scheme, receive antennas, OFDM symbols, subcarriers, and CP length. Different transmit antennas were implemented to evaluate the beamforming algorithms. It is seen from Figure 26 that all transmit antennas at the same SNR values gave the same BER values. So, we can say that by increasing the SNR values and providing equal transmit power, all transmit antennas had same BER in the CP-OFDM environment.

8 APPLICATIONS

Massive MIMO technology evolved from mobile communications and achieved significant milestones over the time in terms of throughput, BER, cost, energy, spectral efficiency and latency. The transition of mobile networks has taken place when SISO at mm-Wave frequencies expanded to the conventional MIMO, named as Massive MIMO mm-Wave. Massive MIMO can be used also in the sub-6 GHz frequencies, in fact it is easier to deploy there as illustrated in Figure 27

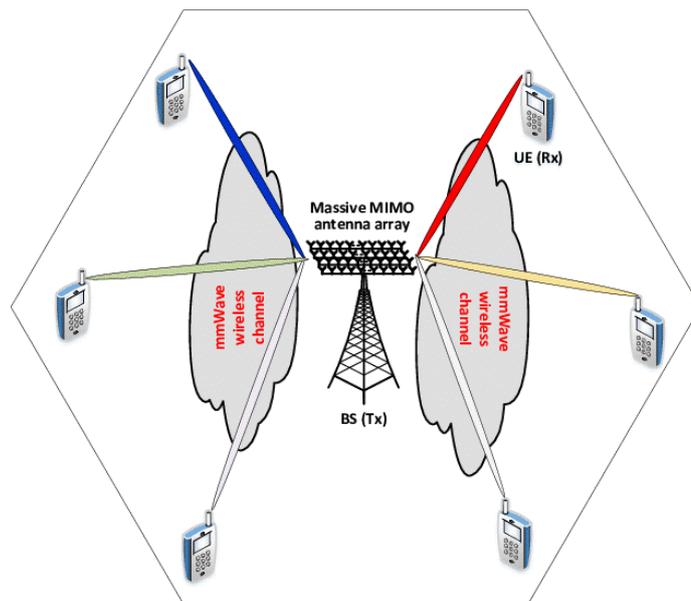


Figure 27. *Mm-Wave Massive MIMO System Model [45]*

A MmWave massive MIMO system model consists of Base station with antenna arrays, Channel and User Equipment. One of the key characteristics of the above model is the access of Channel State Information (CSI). Throughput can be increased through the precoding where transmitter sends the intended signal along the best position of the users. Beamforming is another characteristic used in the Massive MIMO system model to steer the beam to the accurate user and cut down the beam power which is unused to save the transmit power as explained in Figure 28 [45].

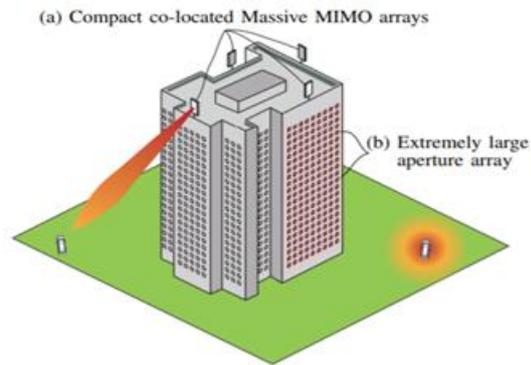


Figure 28. *Massive MIMO Compact Array for Users [46]*

Another application of Massive MIMO is the configuration of cluster antennas for those users which are in the near field of the array and the separate antennas using LoS (Line of Sight) path to send the user residing in the far field of the antenna arrays.

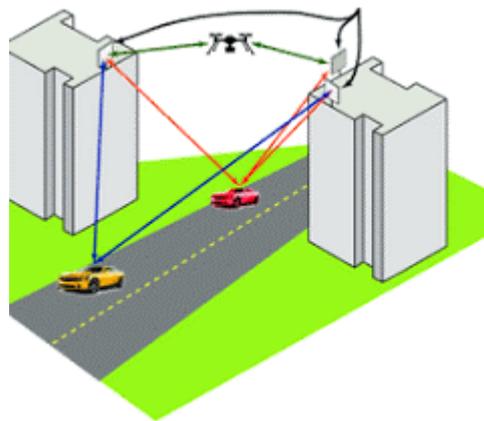


Figure 29. *Objects Tracking through Large-Scale MIMO Radars [46]*

Large-Scale MIMO radar makes use of Electromagnetic waves to detect the location and speed of the object. A signal processing technique is used where radar transmit signals towards the object is shown in Figure 29. The signal echoes back. Now-a-days radars are considered to be intelligent machines because they can send and receive different waveform, process and decode them. Different algorithms are used to process the received signal and optimize the parameters. MIMO radars are mainly of two types. The radar consists of distributed antennas and co-planner antennas.

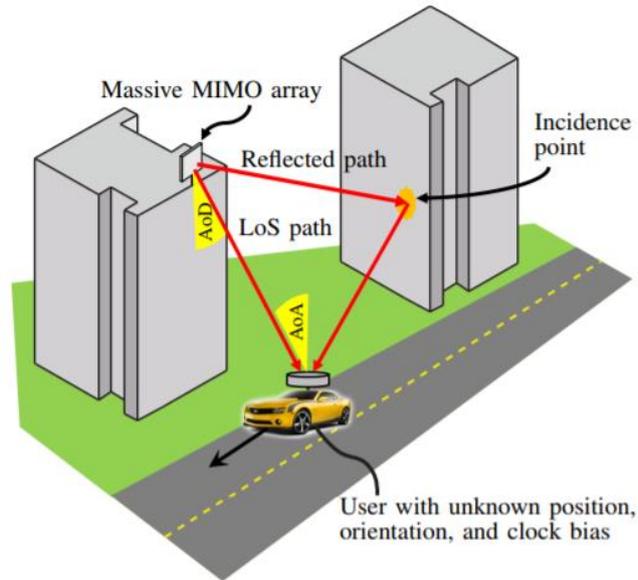


Figure 30. *Objects Tracking through Massive MIMO Compact Antenna Arrays [46]*

Radar often used in military application for security and in measurement of weather. Wireless communication system uses the Global Navigation Satellite System (GNSS) to locate the user. BS has the data base which can add new physical quantities by CSI, named as Angle of Arrival (AoA) in uplink and Angle-of-Departure (AoD) in downlink. Figure 30 is showing mapping and positioning in real time environment and to improve the accurate positioning and mapping of objects, still algorithms are needed [46].

9 CONCLUSIONS AND FUTURE WORK

Focus of the thesis was evaluating the Linear Beamforming Algorithms for multiple antenna systems with different use cases to measure the Bit-Error-Rate as function of SNR. Algorithms are combined in the OFDM system to process the data on the transmitter and Receiver side in Massive MIMO configuration. One of the algorithms named as Maximum Ratio Combining used at the Transmitter side to combine the individual data stream according to their SNR ratios. After passing through channel, noise was added in data streams. The channel was formed with random data containing non-real values, so Singular Value Decomposition method was utilized to optimize the channel and obtained the channel capacity. On the receiver side, Zero-Forcing was performed to inverse the frequency response of the received signal to get the streams one by one at the receiver antennas.

To measure the performance of the MIMO system model, different use cases were applied. Firstly, by varying the antennas on the receiver side, BER also varied in the same fashion as receiver antennas. RX large antenna arrays provided satisfactory BER performance. Secondly, using different modulation acted inverse from first use case. Increasing the modulation schemes from 4-QAM to 64-QAM reduced the system performance. In contrast, MRC and ZF performed very bad on SISO system. With small antenna configurations, system performance decline.

The summary of the results showed that combination of MRC and ZF performed worse when modulation schemes were increasing otherwise enhanced in the system performance. Another important factor was observed that both linear algorithms with CP-OFDM reduced the ISI from the information. Aim of utilizing linear processing schemes in Massive MIMO is to open the door for massive MIMO application which can use lower power and will increase high system capacity.

This research dealt with the Transmit diversity where a single stream beamforming is applied, and all transmit antennas sent the same signals to the receive antennas. In future, precoding can be implemented where multiple different streams sent to the receivers.

There are lot of benefits which can be performed with precoding. Channel estimation is one of the key performance indicators. It will cut down the power lobs when transmitter knows the users. In addition, MMSE and MRC can be used in the next future to maximize the received SINR because they will include more antennas and acted well for noisy

environment but ZF was not perform well. In case of saving the power with linear process algorithms over the wireless networks will open the door for many wireless applications which will provide accurate data with higher bandwidth.

Pilot contamination is another research area which reduces the power of wireless systems. The user fights each other to get the system's resources result the Inter user Interference.

Two ways to reduce the interference between the users, when cell size will reduce, and frequency reuse factor is set according to the cell requirement.

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