TUOMAS HANNULA
PARAMETRIC STUDY OF STRESSED SKIN FOR SUPPORTING
INDIVIDUAL MEMBERS AGAINST LATERAL BUCKLING

Master's Thesis

ABSTRACT

TUOMAS HANNULA: Parametric study of stressed skin for supporting individual members against lateral buckling
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In this thesis, support against lateral elastic buckling of steel members is examined using trapezoidal sheeting and metal faced sandwich panels. The main issue is to study, what kind of members can be supported with different stiffening solutions and how various parameters affect the usability of the solutions.

The thesis presents theories for calculating elastic buckling lengths and connector forces. Buckling lengths of members supported by trapezoidal sheeting are calculated with Winkler foundation theory and connector forces are calculated with method from standard SFS-EN 1993-1-1. For members supported by sandwich panels, elastic buckling lengths and connector forces can be calculated with theory presented by Eva Hedman-Pétursson in her doctoral thesis. In this thesis, theory is presented also for different boundary conditions than simply supported beam.

In the parametric study, different supporting solutions and related variables are examined. The results are presented for members with different lengths as a maximum normal force the member can resist before buckling resistance according to standard SFS-EN 1993-1-1 or shear resistance of connector gives utilization ratio one. The main variables affecting the axial force resistance of supported members are the buckling resistance of unsupported member, stiffness of connections and shear resistance of connections.

The parameters in parametric study are for trapezoidal sheeting: cross-section of the member, thickness of the sheeting and distance between connectors. For sandwich panels, the examined parameters are: cross-section of the member, thickness of the inner face of panel and number of screw pairs in panel.

According to the study, examined solutions can be used to prevent buckling in some cases. The benefit of stressed skin is greatest on slender members and the most critical issue in axial force resistance of supported member is bearing resistance of connections due to the thin metal sheets.

TIIVISTELMÄ

TUOMAS HANNULA: Parametrinen tarkasteu levyrakenteille terässauvojen nurjahdustuentana
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Tässä työssä tutkitaan teräksisten sauvojen nurjahdustuentaa muotolevyllä sekä metallipintaisilla sandwich paneeleilla. Työn pääpaino on tutkia parametrisella tarkastelulla, minkä kokoisia sauvoja kyseisillä jäykistysratkaisuilla voidaan tukea ja kuinka eri muuttujat vaikuttavat jäykistysratkaisuiden käytettävyyteen.

Työssä esitetään teoriat nurjahduspituuden ja ruuvivoimien laskentaan. Muotolevyllä tuettujen sauvojen nurjahduspituudet määritetään Winklerin kimmoisesti tuetun palkin teorialla ja ruuvivoimien määrityksessä käytetään standardissa SFS-EN 1993-1-1 esiintyvää menetelmää. Sandwich paneeleilla tuetuille sauvoille nurjahduspituudet ja ruuvivoimat saadaan määritettyä Eva Hedman-Péturssonin väitöskirjassaan esittämän teorian mukaan, joka tässä työssä johdetaan myös muille reunaehdoille, kuin nivelpäisille sauvoille.

Parametrisessa tarkastelussa tarkastellaan eri tuentavaihtoehtoja ja niihin liittyvien parametrien vaikutusta. Tulokset esitetään suurimpana normaalivoimana, jonka eri pituiset sauvat kestävät, jotta standardin SFS-EN 1993-1-1 mukainen nurjahduskestävyys tai liitoksen leikkauskestävyys saavuttaa käyttöasteen yksi. Tuettujen sauvojen normaalivoimakestävyyteen vaikuttavat tukemattoman sauvan nurjahduskestävyys, liitosten jäykkyys sekä liitosten leikkauskestävyys.

Tarkasteltavat muuttujat ovat muotolevylle: sauvan poikkileikkaus, muotolevyn paksuus sekä liitinten välinen etäisyys. Sandwich paneeleille tutkittavat parametrit ovat: sauvan poikkileikkaus, paneelin sisäkuoren paksuus sekä ruuviparien määrä paneelissa.

Tutkimuksen mukaan tutkittuja rakenteita voidaan käyttää nurjahdustuentana joissakin tapauksissa. Levyjäykistyksestä saatava hyöty on suurinta hoikilla sauvoilla ja ohuista levyistä johtuen liitosten leikkausvoima nousee useissa tapauksissa rajoittavaksi tekijäksi tuetun sauvan normaalivoimakestävyydessä.

PREFACE

Tampereella, 12.11.2018

Tuomas Hannula

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LIST OF SYMBOLS AND ABBREVIATIONS

FEM	Finite Element Method, a numerical method for solving problems of engineering
Sandwich panel	Prefabricated element consisting of two thin metal faces with an insulating core

a	proportionality coefficient
A_r	Amplitude of deformation
B	Width of the sandwich panel
c	The distance between screws in trapezoidal sheeting, the distance between screws in a screw pair of a sandwich panel
D	Thickness of the panel at point of fastening
d	Nominal diameter of a screw
d_1	Minor diameter of the threaded part of the fastener
d_S	Diameter of the unthreaded shank of the fastener
E	Modulus of elasticity
e_0	Initial bow imperfection of member
F	Shear force of a connector
F_{max}	Maximum connector force of a member
$f_{u,F2}$	Tensile strength of the internal face
I	Second moment of area
K	Shear stiffness of a connector in trapezoidal sheeting (N/mm)
k	Shear stiffness of connectors in trapezoidal sheeting (N/mm^2)
k_v	Shear stiffness of a connector in a sandwich panel (N/mm^2)
L	Member length
L_{cr}	Buckling length of member
$L_{cr,0}$	Buckling length for Euler's critical load
m	Number of members to be restrained
N	Normal force
N_{Ed}	Design normal force
N_{cr}	Elastic critical buckling load of member
N_{Rd}	Design value of the resistance to axial forces
p	Foundation pressure
q_{Ed}	Equivalent stabilizing force for supporting structure

t	Design thickness of the trapezoidal sheeting $t = t_{nom} - 0.04$ mm
$t_{cor,F2}$	Core thickness of internal face
$t_{cor,sup}$	Core thickness of supporting structure
t_{F2}	Thickness of internal sheeting
t_{nom}	Design thickness of the trapezoidal sheeting
v	Deflection of the member
v_0	Initial deformation of member
V_{Rd}	Design value of shear resistance of a connector
V_{Rk}	Characteristic value of shear resistance of a connector
Δ	Displacement of a connector
δ_q	In-plane deflection of bracing system due to q_{Ed}
γ_{M2}	Material safety factor for connectors
λ	Eigenvalue

1   INTRODUCTION

This Master's Thesis is part of European Union project STABFI: *"Steel cladding systems for stabilization of steel buildings in fire"* funded by the Research Fund for Coal and Steel. The main goal of the project is to examine usefulness of stressed skin for stabilization of structural members and entire frames of single-story buildings in a fire situation. Before the fire situation, it is important to find out the possibilities of stressed skin for stabilization in the ambient temperature. This gives a comparison result for later research.

Many steel buildings have sheet structures on walls and roof. These structures have some capacity to stabilize the buildings but in Finland this capacity is generally not used in structural calculations. Therefore, it is interesting to find out the potential of different stiffening solutions. By stiffening the building with wall and roof structures, material can be use more effectively and some extra parts can be omitted.
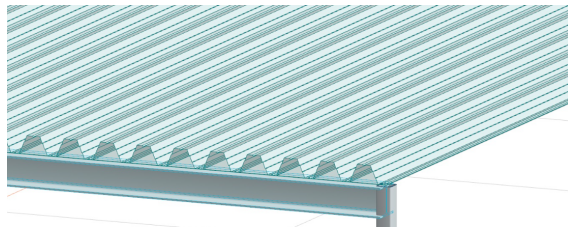
Figure 1. *A trapezoidal sheeting.*
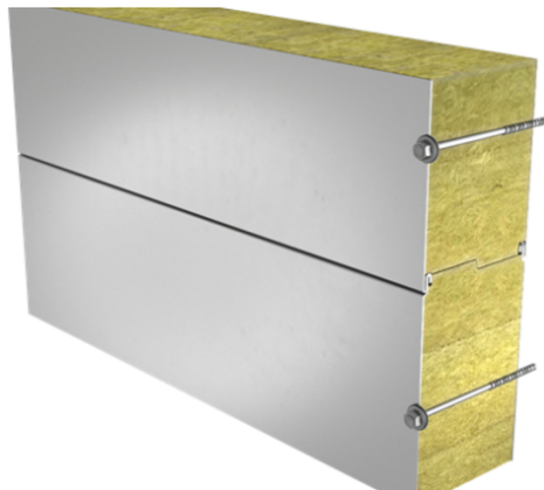
Figure 2. *A metal faced sandwich panel.*

The main goal of this thesis is to compare two different stressed-skin structures, trapezoidal sheeting Figure 1 and metal faced sandwich panels Figure 2, and to

find out if these components can have benefits to the axial force resistance of members.

1.1 Background of stressed skin design

The most commonly used stiffening solution in Finland is the brace stiffening. In this solution, the horizontal loads are passed through the diagonal braces, and the capacity of the sheet structures is not considered at all. Using this solution makes the calculation easy and fast but the solution needs diagonal braces that increase the number of members and thus the steel mass of the building.

Trapezoidal sheeting is commonly used on facades and roofs of industrial halls but usually it is left out from structural calculations. With this solution the building does not necessarily need separate braces. The sheet can be thought to behave like a continuous spring support if it is sufficiently connected to the structure.

The idea of using stressed skin for stabilization is rather old but not much used in Finland. Trapezoidal sheeting has been used for stabilization for a long time and Bryan & Davies have published a manual in the early 1980s on the theoretical background and design guidelines (1982).

The sandwich panels consisting of two thin metal faces with an insulating core are commonly used components on industrial halls because they are easy to use and provide some insulation. Like with trapezoidal sheeting, a building with sandwich panels does not necessarily need separate braces but usually the capability of panels to transfer loads in the longitudinal direction are ignored in the structural calculations. When trapezoidal sheeting can be thought to behave like a continuous spring support, sandwich panels cause force pairs to the member and the stiffening effect is due to their moment.

The theory of using sandwich panels for stabilization is still rather new application. The first comprehensive studies were published in the early 2000s by Hedman-Pétursson (2001) and these studies cover only simple supported beams. The first design rules were not developed until the beginning of the 2010s in European project Ensuring Advancement in Sandwich Construction through Innovation and Exploitation (EASIE 2011), which lead to European Recommendations on the Stabilization of Steel Structures by Sandwich Panels (ECCS 2013).

1.2 Aims and limitations

The main goals in this thesis is to present the theories to use trapezoidal sheeting and sandwich panels as a stiffening structure, expand the theory of using sandwich panels for stabilization to cover other boundary conditions than simply supported members as well and to find out what kind of effect these components can have to the axial force resistance of members and how different parameters affect that. In this thesis, only lateral restraint to individual members is studied, structures are at normal operating temperature, all cross-sections are tubular profiles and members are loaded with axial load only.

1.3 Research methods

Analytical solutions for buckling length and connector forces are derived to members restrained with trapezoidal sheeting from Winkler foundation theory (1867) and to members restrained with sandwich panels from theory presented in Eva Hedman-Pétursson's thesis (2001).

Analytical solutions are compared with the results obtained by linear finite element method (FEM) using Euler-Bernoulli 3D beam elements. Used analysis software is RFem (program version 5.07.11.122642, student version) by Dlubal (Dlubal Software GmbH 2016).

The parametric study is made by calculating axial force resistance with analytical equations with different parameter values. Studied parameter are buckling resistance of unrestrained member, stiffness of connections and shear resistance of connections. The axial force resistance is calculated by increasing axial load until the buckling resistance in the plane of stressed skin is exceeded or until the connector force reaches the bearing resistance of a screw. Buckling resistance is calculated according to SFS-EN 1993-1-1 (2005).

2 LATERAL RESTRAINT FROM STRESSED SKIN

In this chapter, the theoretical framework for stabilization of individual members with trapezoidal sheeting and metal faced sandwich panels is presented.

The stabilization of the entire building by stressed skin is presented in (Dubina et al. 2012). Basic theory, recommendations according to SFS-EN 1993-1-1 (2005), design considerations and design procedures are all presented in (Dubina et al. 2012, Chapter 5) but this thesis focuses on the stabilization of individual structural members. When stabilizing individual members, the main issue is to prevent the buckling of the member laterally, i.e. in the plane plane of the cladding. In contrast, when stabilizing the entire building, the main issue is to prevent the building from overturning and to limit the displacements. Simplified, one idea in both cases is first to make sure that sheeting is strong enough. Then the stiffness of the structure is determined with the sheeting and the sheeting is replaced by a spring system. The spring system should have support reactions where sheeting interact with the structure and spring stiffnesses should match with the sheeting stiffness. After that, structural analysis is performed to make sure that the deflections do not exceed the permitted values.

The main things to stabilize individual members are sufficient support against lateral displacements and sufficient support against rotations. In this thesis only lateral restraint is studied and results are useful mainly for tubular cross-sections because they have good resistance to rotation. Sufficient support means that the strength of the stiffening structure is enough to take all the forces coming from the member, the strength of the connectors is enough to transfer all the forces from the member to the stiffening structure and the stiffness of the stiffening structure is enough to prevent the member from losing its stability. In this study it is assumed that the strength of the stiffening structure is greater than the strength of the connectors, so only the connector forces and stiffness of the stiffeners are taken into account. Sufficient stiffness of the stiffening structure is determined by calculating elastic buckling length of the member because as the stiffness increases, the elastic buckling length decreases and the reduction in the buckling length prevents loss of member's stability.

2.1 Lateral restraint with trapezoidal sheeting

With trapezoidal sheeting, connector forces can be calculated according to Annex BB.2.1 of SFS-EN 1993-1-1 (2005, Clause 5.3.3) and elastic buckling lengths can be calculated with Winkler foundation theory (Winkler 1867) presented by Timoshenko and Gere (Timoshenko & Gere 1961) as it is proposed by Höglund (2002).

2.1.1 Elastic buckling length of member restrained with trapezoidal sheeting

There exist different theoretical models for the beams restrained with sheeting. It is proposed by Höglund (2002) that the trapezoidal sheeting is supposed to be absolutely rigid and all deformations between the supported structure and the sheeting take place at the connectors. Thus, the trapezoidal sheeting is modeled as an elastic foundation for the supported member. In this thesis, one-parameter Winkler model (Timoshenko & Gere 1961) is used to model the elastic foundation, as shown in Figure 3.
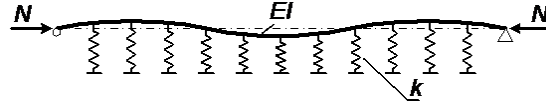
Figure 3. *Beam on elastic foundation.*

In one-parameter Winkler model, the ratio between the foundation pressure $p(x)$ and the deflection of the beam, $v(x)$, is assumed to be a constant *foundation parameter*, k, along the longitudinal x-axis of member.

$$k = \frac{p(x)}{v(x)} \tag{1}$$

The use of this model means that the lateral restraint of the steel member is idealized as a continuous spring support at the centroid of the tubular section and the rotational restraint is neglected. The idealization is acceptable when the torsional stiffness of the section is large and lateral torsional buckling will not be active, which is usually true with tubular cross-sections.

The foundation parameter k depends on the stiffness of the sheeting and its connectors. If the wall thickness of the member is 2.5 times larger than the thickness of the trapezoidal sheet, the stiffness of one screw can be calculated with Equation (2)

from the Swedish code StBK-N5 (1979):

$$K = 1.5d\sqrt{t} \cdot 1 \cdot 10^3 \quad \text{(N/mm)} \tag{2}$$

where

d is diameter of screws;

t is design thickness of the trapezoidal sheet.

Suppose that the spacing of screws is c so the foundation parameter k becomes:

$$k = \frac{K}{c} = \frac{1.5d\sqrt{t} \cdot 1 \cdot 10^3}{c} \quad \text{(N/mm}^2\text{)} \tag{3}$$

Table 1. *Stiffness of the trapezoidal sheeting connectors. Core thickness of the sheeting is $t = t_{nom} - 0.04mm$, where t_{nom} is the nominal thickness of the sheeting, d is the diameter of the connector, K is the shear stiffness obtained from Equation (2) and $k = K/c$ (see Equation (3)) with $c = 500mm$ being the distance between connectors. Sheeting material is S350GD+Z.*

t_{nom} [mm]	d [mm]	K (N/mm)	k (N/mm^2)
0.7	5.5	6702	13
1.0	5.5	8083	16
1.5	5.5	9969	20
0.7	6.3	7677	15
1.0	6.3	9259	19
1.5	6.3	11418	23

In order to calculate the buckling capacity of the member, it is necessary to derive the buckling length of the stabilized member in the plane of the trapezoidal sheeting. After that the member can be designed according to EN 1993-1-1 (2005) using the buckling length obtained here.

The buckling length of the stabilized member can be calculated from the differential equation, Equation (4), of the deflection $v(x)$ for the beam with bending stiffness EI. The beam is loaded by a constant compressive axial load N in the Winkler model with the foundation parameter k and longitudinal coordinate x. (Timoshenko & Gere 1961)

$$EI\frac{d^4v(x)}{dx^4} + N\frac{d^2v(x)}{dx^2} + kv(x) = 0 \tag{4}$$

In this thesis, only simply supported member is considered restraint with trapezoidal

sheeting. The boundary conditions in that case at the ends of the member are:

$$v(0) = 0 \quad \frac{d^2v(0)}{dx^2} = 0 \tag{5}$$

$$v(L) = 0 \quad \frac{d^2v(L)}{dx^2} = 0 \tag{6}$$

The function $v(x)$, which fulfills the boundary conditions of Equations (5) and (6) is:

$$v(x) = C \sin\left(\frac{n\pi x}{L}\right) \tag{7}$$

where C is a real constant and n is integer. Substitution of Equation (4) gives:

$$EI\left(\frac{n\pi}{L}\right)^4 - N\left(\frac{n\pi}{L}\right)^2 + k = 0 \tag{8}$$

$$\Rightarrow N_{cr} = \frac{\pi^2 EI}{L^2}\left(n^2 + \frac{kL^4}{n^2\pi^4 EI}\right) \tag{9}$$

Equation (9) gives the critical load N_{cr} as a function of n, which represents the number of half sine waves in which the member subdivides at buckling. The lowest critical load may occur with $n = 1, 2, \ldots$, depending on the other constants.

When the stiffness of the foundation is very small, the lowest eigenvalue is obtained with $n = 1$. As the stiffness of the foundation increases, the critical buckling mode changes from n to $n + 1$ at certain point, which can be defined from Equation (9):

$$n^2 + \frac{kL^4}{n^2\pi^4 EI} = (n+1)^2 + \frac{kL^4}{(n+1)^2\pi^4 EI} \tag{10}$$

$$\Rightarrow k = \frac{\pi^4 n^2(n+1)^2 EI}{L^4} \tag{11}$$

The critical buckling mode with foundation parameter k can be calculated from Equation (11) by solving n:

$$n = \frac{\sqrt{\dfrac{(2L)^2\sqrt{EIk} + \pi^2 EI}{EI}}}{2\pi} - \frac{1}{2} \tag{12}$$

Equation (12) is derived from a equation that gives stiffness to change the critical buckling mode, so the value should be rounded up to the next integer representing the number of half-waves in the buckling mode.

The elastic buckling load can now be obtained from Equation (9) and the elastic

buckling length can be calculated from buckling load, as:

$$N_{cr} = \frac{\pi^2 EI}{L^2} \left(n^2 + \frac{kL^4}{n^2 \pi^4 EI} \right) = \frac{\pi^2 EI}{L_{cr}^2} \tag{13}$$

$$\Rightarrow L_{cr} = \sqrt{\frac{\pi^2 EI}{N_{cr}}} \tag{14}$$

$$\Rightarrow L_{cr} = \frac{L}{\sqrt{n^2 + \frac{kL^4}{n^2 \pi^4 EI}}} \tag{15}$$

2.1.2 Shear force of connectors with trapezoidal sheeting

There are some different methods for calculating shear force of the connectors from stiffening sheet structure. In this thesis, the method from SFS-EN 1993 (2005, Clause 5.3.3, Eq. (5.12)) is used to calculate shear forces of the connectors when stiffening structure is trapetzoidal sheeting. The method is based on the initial bow imperfection, e_0, that is defined as follows:

$$e_0 = \alpha_m \frac{L}{500} \tag{16}$$

$$\alpha_m = \sqrt{\frac{1}{2}\left(1 + \frac{1}{m}\right)} \tag{17}$$

where

m is the number of members to be restrained;

L is the length of the member.

The total equivalent stabilizing force for supporting structure is given as Eq. (5.13) of EN 1993-1-1 (2005):

$$q_{Ed} = \sum N_{ed} \cdot 8\frac{e_0 + \delta_q}{L^2} \tag{18}$$

where

N_{ed} is the axial force in the stabilized member;

δ_q is the in-plane deflection of the bracing system due to q_{Ed}.
  The exact definition of the δ_q leads to iterative analysis because the in-plane deflection depends both on the deflection of the supporting system and on the deflection of the stabilization system.

The summation is taken over the members to be stabilized when determined the total imperfection force to the stiffening structure. When calculating connector forces to individual members, the summation is not taken into account:

$$F = N_{ed} \cdot 8 \frac{e_0 + \delta_q}{L^2} \cdot c \tag{19}$$

where

c is the distance between connectors.

The shear resistance of the connectors can be calculated based on EN 1993-1-3 (2006, Table 8.1-8.4) when sheet the thickness of the sheeting is less than 3 mm. The shear resistances of self-tapping screws with varying sheet material and thickness are presented in Table 2.

Table 2. *Shear resistance (kN) of self-tapping screws based on (SFS-EN 1993-1-3 2006, Table 8.2). Design thickness is defined as $t_{nom} - 0.04$ mm. Sheet material ultimate strength is calculated according to (SFS-EN 1993-1-3 2006, Table 3.1b).*

Sheet material	t_{nom} (mm) d (mm)	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.5
S220GD+Z	5.5	0.389	0.562	0.755	0.966	1.193	1.436	1.694	3.216	4.047
S250GD+Z	5.5	0.428	0.618	0.830	1.062	1.313	1.580	1.864	3.537	4.452
S280GD+Z	5.5	0.467	0.674	0.906	1.159	1.432	1.724	2.033	3.859	4.820
S320GD+Z	5.5	0.506	0.731	0.981	1.255	1.551	1.867	2.202	4.180	4.820
S350GD+Z	5.5	0.545	0.787	1.057	1.352	1.671	2.011	2.372	4.502	4.820
S220GD+Z	6.3	0.416	0.601	0.808	1.034	1.277	1.537	1.813	3.683	4.636
S250GD+Z	6.3	0.458	0.662	0.889	1.137	1.405	1.691	1.994	4.052	5.099
S280GD+Z	6.3	0.500	0.722	0.969	1.240	1.533	1.845	2.176	4.420	5.563
S320GD+Z	6.3	0.541	0.782	1.050	1.344	1.660	1.999	2.357	4.788	6.027
S350GD+Z	6.3	0.583	0.842	1.131	1.447	1.788	2.152	2.538	5.157	6.490

2.2 Lateral restraint with sandwich panels

With sandwich panels, buckling length of the member can be calculated with the theory presented by Hedman-Pétursson (2001). By giving a member the initial deformation, the connector forces can be calculated from the same theory.

In Hedman-Pétursson's thesis (2001) the theory of stabilization of steel members by sandwich panels has been presented only for simple supported members but a torsional buckling of the member has also been taken into account. The theory is used to calculate analytical expressions in this thesis on for the flexural buckling length and the corresponding screw forces of the steel member supported by sandwich panels. It should be noted, that the same theories may be used for stabilization

of other than steel members, as well.

2.2.1 Elastic buckling length of member restraint with sandwich panels

In this theory, it is assumed that sandwich panels are absolutely rigid in the plane of the faces and all deformations take place at the connectors. Another basic assumption is that the panels can slide without friction and the longitudinal joints of the panels are not fixed. In Hedman-Petursson (2001) is given method for the analysis when it is supposed that the longitudinal joints have stiffness against longitudinal forces and the longitudinal joints. The stiffness may appear due to friction at the joints and/or due to connectors at the joints. This effect has a considerable effect to the stiffness of the stabilizing system, as shown in Hedman-Petursson (2001). Connectors are supposed to behave elastically and due to slip at the joints, the edges of the sandwich panels restrain the deformation of the member by the moment which is caused by force pairs from connectors. The principle is presented in the Figure 4 when there is only one pair of connectors at the edge of each sandwich panels.
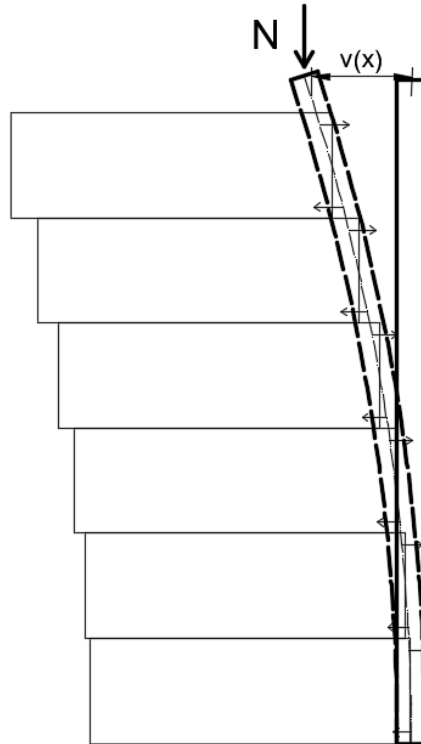
Figure 4. *Sandwich panel restraining the deflection of a cantilever member.*

Width of the sandwich panel is denoted as B, the distance between screw pair is c and the connector displacement is denoted as Δ. The rotation of the member is the

derivative of the deflection of the member, denoted as $v'(x)$ or $\dfrac{dv}{dx}$. Notations are presented in the Figure 5.
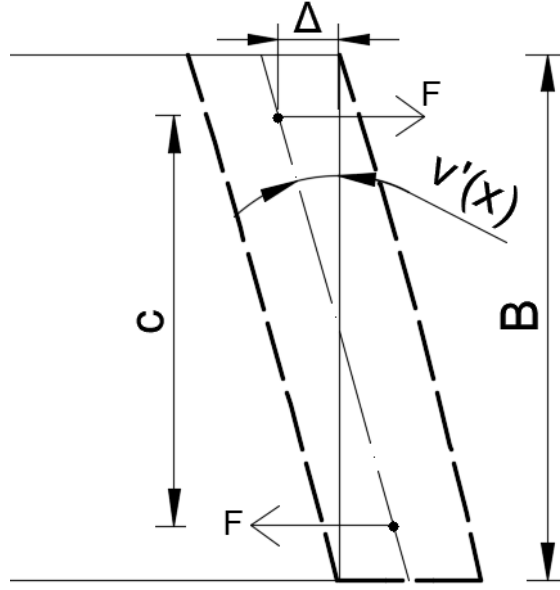
Figure 5. *Deformation of member at the panel joint.*

When deformations are small enough, the connector displacement is:

$$\Delta = \frac{c}{2} \cdot \frac{dv}{dx} \tag{20}$$

Because the connectors are supposed to behave elastically, the connector force F is:

$$F = k_\nu \Delta \tag{21}$$

where k_ν is the shear stiffness of one connector. Shear stiffness of the connector can be calculated according to ECCS Recommendations (2013, Eq. (21)):

$$k_v = \frac{1}{\dfrac{x_F}{k_{F2}} + \dfrac{t_{cor,sup}^2 + 2(1 - x_F)Dt_{cor,sup}}{4C_{sup}} + \dfrac{3(1 - x_F)Dt_{cor,sup}^2 + t_{cor,sup}^3}{24EI}} \tag{22}$$

$$x_F = 1 - \frac{\dfrac{1}{k_{F2}} - \dfrac{DT_{cor,sup}}{2C_{sup}} - \dfrac{Dt_{cor,sup}^2}{8EI}}{\dfrac{1}{k_{F2}} + \dfrac{D^2}{C_{sup}} + \dfrac{D^2(2D + 3t_{cor,sup})}{6EI}} \tag{23}$$

$$EI = 200GPa\frac{\pi d_S^4}{64} \tag{24}$$

$$C_{sup} = 2400MPa\sqrt{t_{cor,sup}d_1^5} \tag{25}$$

$$k_{F2} = \begin{cases} 6.93\dfrac{f_{u,F2}\sqrt{t_{cor,sup}d_1^5}}{0.26mm + 0.8t_{cor,sup}} & \text{if } 0.4 \text{ mm} \leq t_{cor,sup} \leq 0.7 \text{ mm} \\[3mm] 4.2\dfrac{f_{u,F2}\sqrt{t_{cor,sup}d_1^5}}{0.373mm} & \text{if } t_{cor,sup} > 0.7 \text{ mm} \end{cases}$$

In the above,

t_{F2} is the thickness of internal sheeting;

$t_{cor,F2}$ is the core thickness of internal face;

$t_{cor,sup}$ is the core thickness of supporting structure;

d is the nominal diameter of the fastener;

d_1 is the minor diameter of the threaded part of the fastener;

d_S is the diameter of the unthreaded shank;

$f_{u,F2}$ is the tensile strength of the internal face;

D is the thickness of the panel at point of fastening.

Equation (22) is based on component method with five components: Bending stiffness EI of the fastener, clamping of the head of the fastener (rotational spring), clamping of the fastener in the supporting structure (rotational spring with stiffness C_{sup}), hole elongation of the internal face sheet (longitudinal spring with stiffness k_{F2}) and hole elongation of the external face sheet (longitudinal spring with stiffness k_{F1}).

The stiffness of a connector corresponds to the load level at the serviceability limit state. It is assumed that load level not exceed half of the shear resistance of the internal face sheet at serviceability limit state. The shear resistance can be calculated according to (ECCS 2013, Eq. (27)) which is shown in Equation (104) in Section 2.2.2.

Table 3. *Stiffness of the sandwich panel connectors. Modulus of elasticity of the connector $E_{connector} = 200$ GPa and tensile strength of the internal face of the sandwich panel is $f_{y,panel} = 350$ MPa.*

Thickness of internal face of sandwich panel (mm)	Nominal thickness of the connector (mm)	Thickness of the panel at the point of the connector (mm)	Thickness of the supporting structure (mm)	Stiffness of the connectors (N/mm^2)
0.4	5.5	100	8	1933
0.7	5.5	100	8	2938
1.0	5.5	100	8	6036
0.4	6.3	100	8	2145
0.7	6.3	100	8	3296
1.0	6.3	100	8	7029
0.4	5.5	230	8	1929
0.7	5.5	230	8	2905
1.0	5.5	230	8	5784
0.4	6.3	230	8	2151
0.7	6.3	230	8	3285
1.0	6.3	230	8	6829
0.4	5.5	230	10	1854
0.7	5.5	230	10	2740
1.0	5.5	230	10	5172
0.4	6.3	230	10	2090
0.7	6.3	230	10	3148
1.0	6.3	230	10	6272

The moment M_k of a force couple F_k is

$$M_k = F_k c_k \tag{26}$$

If there exist n pairs of identical connectors, with the distance of one pair denoted c_k, the shear force $F_k, (k = 1, 2, \ldots, n)$ of a connector is:

$$F_k = k_\nu \Delta_k = k_\nu \cdot \frac{c_k}{2} \cdot \frac{dv}{dx} \tag{27}$$

The moment of all connectors is:

$$M = \sum_{k=1}^{n} F_k c_k = \frac{k_\nu}{2} \sum_{k=1}^{n} c_k^2 \frac{dv}{dx} \tag{28}$$

This moment is distributed as a uniform moment m to one panel of width B:

$$m = \frac{M_k}{B} = \frac{k_\nu}{2B} \sum_{k=1}^{n} c_k^2 \frac{dv}{dx} \tag{29}$$

Consider a differential element of the member presented in Figure 6.
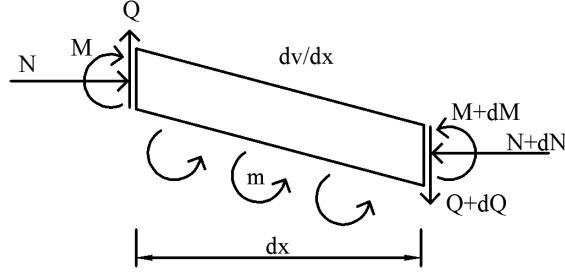
Figure 6. *Differential piece of the member.*

The equilibrium equations of element are:

Horizontal forces: $N - N - \mathrm{d}N = 0 \Rightarrow \mathrm{d}N = 0 \Rightarrow \dfrac{dN}{dx} = 0$ (30)

Vertical forces: $Q - Q - \mathrm{d}Q = 0 \Rightarrow \mathrm{d}Q = 0 \Rightarrow \dfrac{dQ}{dx} = 0$ (31)

Moment: $M - M - \mathrm{d}M + (Q + \mathrm{d}Q)\mathrm{d}x + (N + \mathrm{d}N)\dfrac{dv}{dx}\mathrm{d}x - m\mathrm{d}x = 0$ (32)

$\Rightarrow -\dfrac{dM}{dx} + Q + N\dfrac{dv}{dx} - m = 0$ (33)

$\Rightarrow -\dfrac{d^2M}{dx^2} + \dfrac{dQ}{dx} + N\dfrac{d^2v}{dx^2} - \dfrac{dm}{dx} = 0$ (34)

$\Rightarrow -\dfrac{d^2M}{dx^2} + N\dfrac{d^2v}{dx^2} - \dfrac{dm}{dx} = 0$ (35)

If the effect of shear deformation and shortening of the member are neglected and deformations are small, then:

$M = -EI\dfrac{d^2v}{dx^2}$ (36)

Substituting Equations (36) and (29) to Equation (35) gives:

$EI\dfrac{d^4v}{dx^4} + N\dfrac{d^2v}{dx^2} - \dfrac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\dfrac{d^2v}{dx^2} = 0$ (37)

$\Rightarrow EI\dfrac{d^4v}{dx^4} + \left(N - \dfrac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\right)\dfrac{d^2v}{dx^2} = 0$ (38)

This is the governing linear homogenous differential equation for the member which is restrained by sandwich panels and loaded by the axial compressive force N. If the shear stiffness of the connector, k_ν, is small, the differential equation of axially loaded Euler-Bernoulli member is obtained.

To derive more general solution of Equation (38), re-write the equation as:

$$\frac{d^4v}{dx^4} + \lambda^2 \frac{d^2v}{dx^2} = 0 \tag{39}$$

where

$$\lambda = \sqrt{\frac{N}{EI} - \frac{k_\nu}{2BEI} \sum_{k=1}^{n} c_k^2} \tag{40}$$

Assuming that

$$\frac{N}{EI} - \frac{k_\nu}{2BEI} \sum_{k=1}^{n} c_k^2 \geq 0 \tag{41}$$

yields

$$N \geq \frac{k_\nu}{2B} \sum_{k=1}^{n} c_k^2 \tag{42}$$

If Equation (42) is not satisfied, then denote

$$\bar{\lambda} = \sqrt{\frac{k_\nu}{2BEI} \sum_{k=1}^{n} c_k^2 - \frac{N}{EI}} \tag{43}$$

and the differential equation becomes

$$\frac{d^4v}{dx^4} - \bar{\lambda}^2 \frac{d^2v}{dx^2} = 0 \tag{44}$$

The general solution of Equation (39) is

$$v(x) = C_1 \cos(\lambda x) + C_2 \sin(\lambda x) + C_3 x + C_4 \tag{45}$$

and the general solution of Equation (44) is

$$v(x) = \bar{C}_1 \cosh(\bar{\lambda} x) + \bar{C}_2 \sinh(\bar{\lambda} x) + \bar{C}_3 x + \bar{C}_4 \tag{46}$$

Elastic buckling load N_{cr} can be calculated from Equations (45) and (46) with the boundary conditions of the member. The elastic buckling length L_{cr} in the plane of the panel can be solved from equation:

$$N_{cr} = \frac{\pi^2 EI}{L_{cr}^2} \tag{47}$$

Simply supported member:

Consider first a simply supported member as an example. The solution in this case is shown in Hedman-Pétursson's thesis (2001) also, where the theory has been validated by tests.

The boundary conditions at the ends of the simply supported member are:

$$v(0) = 0 \quad \frac{d^2v(0)}{dx^2} = 0 \tag{48}$$

$$v(L) = 0 \quad \frac{d^2v(L)}{dx^2} = 0 \tag{49}$$

The sine function fulfills the boundary conditions at the supports and the deflection is:

$$v(x) = A_r \sin\left(\frac{r\pi x}{L}\right) \tag{50}$$

Substitution of Equation (50) when $\sin\left(\frac{r\pi x}{L}\right) = 1$ to Equation (38) yields:

$$EI\frac{r^4\pi^4}{L^4} - N\frac{r^2\pi^2}{L^2} + \frac{k_\nu}{2B}\sum_{k=1}^{n} c_k^2 \frac{r^2\pi^2}{L^2} = 0 \tag{51}$$

$$\Rightarrow N_{cr} = EI\frac{r^2\pi^2}{L^2} + \frac{k_\nu}{2B}\sum_{k=1}^{n} c_k^2 \tag{52}$$

It can be seen, that the first eigenmode, $r = 1$, gives always the smallest N_{cr} in this case. Moreover, the first part of N_{cr} is the same as for the member without sandwich panels. The buckling length L_{cr} in the plane of the panel is:

$$N_{cr} = \frac{\pi^2 EI}{L_{cr}^2} = EI\frac{r^2\pi^2}{L^2} + \frac{k_\nu}{2B}\sum_{k=1}^{n} c_k^2 \tag{53}$$

$$\Rightarrow L_{cr} = \frac{L}{\sqrt{1 + \frac{k_\nu}{2B}\sum_{k=1}^{n} c_k^2 \cdot \frac{L^2}{\pi^2 EI}}} \tag{54}$$

Cantilever column:

The boundary conditions for the cantilever member which is fixed at the point $x = 0$ and free at the end $x = L$ with the compressive force N are:

$$v(0) = 0 \quad \frac{dv(0)}{dx} = 0 \tag{55}$$

$$\frac{d^2v(L)}{dx^2} = 0 \quad Q(L) = 0 \tag{56}$$

From Equation (33), the following is obtained:

$$-\frac{dM}{dx} + Q + N\frac{dv}{dx} - m = 0 \tag{57}$$

$$\Rightarrow Q = \frac{dM}{dx} - N\frac{dv}{dx} + \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\frac{dv}{dx} \tag{58}$$

$$\Rightarrow Q = -EI\frac{d^3v}{dx^3} - N\frac{dv}{dx} + \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\frac{dv}{dx} \tag{59}$$

$$\Rightarrow Q = -EI\frac{d^3v}{dx^3} - \left(N - \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\right)\frac{dv}{dx} \tag{60}$$

The cosine function fulfills the boundary conditions at the supports and the deflection is:

$$v(x) = A_r\left(1 - \cos\left(\frac{r\pi x}{2L}\right)\right), \qquad\qquad N \geq \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2 \tag{61}$$

Substitution of Equation (61) to Equation (44) leads to:

$$EI\frac{r^4\pi^4}{(2L)^4} - N\frac{r^2\pi^2}{(2L)^2} + \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\frac{r^2\pi^2}{(2L)^2} = 0 \tag{62}$$

$$\Rightarrow N_{cr} = EI\frac{r^2\pi^2}{(2L)^2} + \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2 \tag{63}$$

In the second case where

$$N \leq \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2 \tag{64}$$

no positive eigenvalues can be obtained.

It can be seen, that the first eigenmode, with $r = 1$, gives always the smallest N_{cr} in this case. Moreover, the first part of N_{cr} is the same as for the member without sandwich panels, again. The buckling length L_{cr} in the plane of the panel can be solved from Equation (47) as:

$$\frac{\pi^2 EI}{L_{cr}^2} = N_{cr} = \frac{\pi^2 EI}{(2L)^2} + \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2 \tag{65}$$

$$\Rightarrow \quad L_{cr} = \frac{2L}{\sqrt{1 + \dfrac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\dfrac{(2L)^2}{\pi^2 EI}}} \tag{66}$$

Cantilever with support at the top:

The boundary conditions at the ends of the cantilever member with support at the top are:

$$v(0) = 0 \qquad \frac{dv(0)}{dx} = 0 \tag{67}$$

$$v(L) = 0 \qquad \frac{d^2v(L)}{dx^2} = 0 \tag{68}$$

With these boundary conditions, the general solution Equation (45) gives equation:

$$(\lambda L)\cot(\lambda L) - 1 = 0 \tag{69}$$

and the smallest eigenvalue can be calculated from that as:

$$\lambda = \frac{4.4934095}{L} \tag{70}$$

Substitution of Equation (70) to Equation (40) leads to:

$$N_{cr} = \lambda^2 EI + \frac{k_\nu}{2B}\sum_{k=1}^{n} c_k^2 \tag{71}$$

and elastic buckling length can be calculated from Equation (47):

$$\Rightarrow L_{cr} = \sqrt{\frac{\pi^2 EI}{N_{cr}}} \tag{72}$$

Fixed supported member:

The boundary conditions for the member which is fixed at the point $x = 0$ and $x = L$ with the compressive force N are:

$$v(0) = 0 \, \frac{dv(0)}{dx} = 0 \tag{73}$$

$$v(L) = 0 \frac{dv(L)}{dx} = 0 \tag{74}$$

The cosine function fulfills the boundary conditions at the supports and the deflection is:

$$v(x) = A_r\left(1 - \cos\left(\frac{2r\pi x}{L}\right)\right), \qquad\qquad N \geq \frac{k_\nu}{2B}\sum_{k=1}^{n} c_k^2 \tag{75}$$

Substitution of Equation (75) to Equation (44) leads to:

$$EI\frac{r^4\pi^4}{(\frac{L}{2})^4} - N\frac{r^2\pi^2}{(\frac{L}{2})^2} + \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\frac{r^2\pi^2}{(\frac{L}{2})^2} = 0 \tag{76}$$

$$\Rightarrow N_{cr} = EI\frac{r^2\pi^2}{(\frac{L}{2})^2} + \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2 \tag{77}$$

In the second case where

$$N \leq \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2 \tag{78}$$

no positive eigenvalues can be obtained.

It can be seen, that the first eigenmode, with $r = 1$, gives always the smallest N_{cr} and the first part of N_{cr} is the same as for the member without sandwich panels, again. The buckling length L_{cr} in the plane of the panel can be solved from Equation (47) as:

$$\frac{\pi^2 EI}{L_{cr}^2} = N_{cr} = \frac{\pi^2 EI}{(\frac{L}{2})^2} + \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2 \tag{79}$$

$$\Rightarrow \quad L_{cr} = \frac{(\frac{L}{2})}{\sqrt{1 + \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\frac{(\frac{L}{2})^2}{\pi^2 EI}}} \tag{80}$$

The critical elastic buckling load N_{cr} can be re-written for simply supported member, cantilever and fixed supported member as:

$$\Rightarrow N_{cr} = EI\frac{r^2\pi^2}{(L_{cr,0})^2} + \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2 \tag{81}$$

and the buckling length L_{cr} in the plane of the panel can be re-written as:

$$\Rightarrow L_{cr} = \frac{(L_{cr,0})}{\sqrt{1 + \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\frac{(L_{cr,0})^2}{\pi^2 EI}}} \tag{82}$$

where $L_{cr,0}$ is the buckling length for Euler's critical load in the corresponding case, which is equal to buckling length for members without support of sandwich panels.

2.2.2 Shear force of connectors with sandwich panels

Connector force is determined in Equation (27). The force depends on the derivative of displacement. Buckling mode can be calculated from differential equation Equation (38) but to solve connector forces the amplitude of deflection must be approximated.

Some initial deformation v_0 is needed in member so that the displacement takes place when member is only axially loaded. In Hedman-Pétursson's thesis (2001), the initial deformation is expressed in terms of the actual deformation as:

$$v_0 = av \tag{83}$$

where a is the proportionality coefficient. The differential equation of buckling in Equation (38) can be expressed with the initial deformation as:

$$EI\frac{d^4v}{dx^4} + N(1+a)\frac{d^2v}{dx^2} - \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\frac{d^2v}{dx^2} = 0 \tag{84}$$

$$\Rightarrow \quad EI\frac{d^4v}{dx^4} + \left(N(1+a) - \frac{k_\nu}{2B}\sum_{k=1}^{n}c_k^2\right)\frac{d^2v}{dx^2} = 0 \tag{85}$$

This can be rewritten as

$$\frac{d^4v}{dx^4} + \lambda^2\frac{d^2v}{dx^2} = 0 \tag{86}$$

where

$$\lambda = \sqrt{\frac{N(1+a)}{EI} - \frac{k_\nu}{2EIB}\sum_{k=1}^{n}c_k^2} \tag{87}$$

The solution of Equation (86) can be expressed as:

$$N = \frac{N_{cr}}{1+a} \tag{88}$$

$$\Rightarrow \quad a = \frac{N_{cr}}{N} - 1 \tag{89}$$

where N_{cr} is defined according to Equation (52) for pinned boundary conditions and Equation (63) for cantilever boundary condition.

The deflection of a simply supported member is as Equation (50)

$$v(x) = A_r \sin\left(\frac{r\pi x}{L}\right) \tag{90}$$

and the deflection of cantilever column is as Equation (61)

$$v(x) = A_r \left(1 - \cos \left(\frac{r\pi x}{2L} \right) \right) \tag{91}$$

The maximum deflection in both cases is:

$$v_{\max} = A_r \tag{92}$$

The maximum initial deformation can now be expressed as:

$$v_{0,\max} = a A_r \tag{93}$$

The amplitude A_r of the subsequent deformation can be expressed as:

$$A_r = \frac{v_0}{a} \tag{94}$$

$$\Rightarrow \quad A_r = \frac{v_0}{\frac{N_{cr}}{N} - 1} \tag{95}$$

Substituting Equation (95) to Equation (50) and Equation (61) yields:

$$v(x) = \frac{v_0}{\frac{N_{cr}}{N} - 1} \sin \left(\frac{r\pi x}{L} \right) \tag{96}$$

$$v(x) = \frac{v_0}{\frac{N_{cr}}{N} - 1} \left(1 - \cos \left(\frac{r\pi x}{2L} \right) \right) \tag{97}$$

Now the connector force can be presented as:

$$F(x) = k_\nu \frac{c_k}{2} \cdot \frac{v_0 \pi}{\left(\frac{N_{cr}}{N_{Ed}} - 1 \right) L} \cos \frac{\pi x}{L} \quad \text{, for simply supported member} \tag{98}$$

$$F(x) = k_\nu \frac{c_k}{2} \cdot \frac{v_0 \pi}{\left(\frac{N_{cr}}{N_{Ed}} - 1 \right) 2L} \sin \frac{\pi x}{2L} \quad \text{, for cantilever member} \tag{99}$$

and maximum connector force in member can be presented as:

$$F_{max} = k_\nu \frac{c_k}{2} \cdot \frac{v_0 \pi}{\left(\frac{N_{cr}}{N_{Ed}} - 1 \right) L} \quad \text{, for simply supported member} \tag{100}$$

$$F_{max} = k_\nu \frac{c_k}{2} \cdot \frac{v_0 \pi}{\left(\frac{N_{cr}}{N_{Ed}} - 1 \right) 2L} \quad \text{, for cantilever member} \tag{101}$$

The initial bow imperfection e_0 is recommended in ECCS Recommendations (2013) to be taken from Eurocode 3 (2005, clause 5.3.3(1)):

$$e_0 = \sqrt{\frac{1}{2}\left(1 + \frac{1}{m}\right)} \cdot \frac{L}{500} \tag{102}$$

where m is the number of members to be restrained. There exist in minimum two members for one panel so when the panel is one span panel from column to column then $m = 2$ and

$$e_0 = \frac{L}{577} \tag{103}$$

The initial bow imperfection e_0 can be used as initial deformation v_0 to calculate displacements and connector forces.

The shear resistance can be calculated according to ECCS Recommendations (2013, Eq. (27)):

$$V_{Rk} = 4.2 \cdot \sqrt{t_{cor,F2}^3 d_1} \cdot f_{u,F2} \tag{104}$$

V_{Rk} is the characteristic value of shear resistance and the design value can be calculated with the material safety factor $\gamma_{M2} = 1.25$ as:

$$V_{Rd} = \frac{V_{Rk}}{\gamma_{M2}} \tag{105}$$

Table 4. *Bearing resistance of the sandwich panel's connectors. Tensile strength of the internal face of the sandwich panel is $f_{y,panel} = 350$ MPa.*

Thickness of internal face of sandwich panel (mm)	Nominal thickness of the connector (mm)	Bearing resistance of the connectors (kN)
0.4	5.5	0.697
0.7	5.5	1.477
1.0	5.5	2.592
0.4	6.3	0.746
0.7	6.3	1.581
1.0	6.3	2.774

The bearing resistances for connectors of sandwich panels with varying thickness of internal face are presented in Table 4.

3   NUMERICAL EXAMPLES

In this chapter, numerical examples is presented to equations derived in Chapter 2 Theory. Same member is studied in all examples.

Used data for member is:

$L = 12$ m   the length of the member;

CFRHS $200x200x10$ cross-section, cold-formed rectangular hollow section;

$f_y = 420$ MPa   yield strength of member;

$E = 210$ GPa   modulus of elasticity of member;

$A = 7257$ mm^2 cross-section area of member;

$I = 4251 \cdot 10^4$ mm^4 second moment of area;

$d = 5.5$ mm   diameter of connectors;

$m = 1$ number of members;

$t = t_{nom} - 0.04$ mm   design thickness of sheeting;

$v_0 = L/577$ initial deformation of member;

3.1 Elastic flexural buckling length of member restrained with trapezoidal sheeting

First example is to calculate the elastic buckling length of a simply supported compressed tubular member restrained with trapezoidal sheeting. The flexural buckling in the plane of the trapezoidal sheeting is considered. The buckling length can be used to calculate the resistance of member according to Eurocode 3 (2005). The member is supported in one direction with trapezoidal sheeting with nominal thickness $t_{nom} = 0.7mm$ and design thickness $t = 0.66mm$. Sheeting is connected to member with screws with diameter $d = 5.5mm$, spacing of the screws is $c = 500mm$ and $m = 1$.

Shear stiffness of one screw is according to Equation (2):

$$K = 1.5d\sqrt{t} \cdot 1 \cdot 10^3 = 1.5 \cdot 5.5\sqrt{0.66} \cdot 1 \cdot 10^3 = 6702 \quad N/mm$$

and foundation parameter is:

$$k = \frac{K}{c} = \frac{6702}{500} = 13.4 \quad N/mm^2$$

The critical buckling mode in the plane of the sheet is according to Equation (12):

$$n = \frac{\sqrt{\frac{(2L)^2\sqrt{EIk} + \pi^2 EI}{EI}}}{2\pi} - \frac{1}{2} = 3.76$$

$\Rightarrow n = 4$ half-waves in the critical buckling mode.

The elastic buckling load from Equation (9) is:

$$N_{cr} = \frac{\pi^2 EI}{L^2}\left(n^2 + \frac{kL^4}{n^2\pi^4 EI}\right) = 22.01 \quad MN$$

and the elastic buckling length is:

$$L_{cr} = \frac{L}{\sqrt{n^2 + \frac{kL^4}{n^2\pi^4 EI}}} = 2.001 \quad m$$

$$\Rightarrow \frac{L_{cr}}{L} = 0.167$$

In the other direction without support of sheeting the elastic buckling load is:

$$N_{cr} = \frac{\pi^2 EI}{L^2} = 0.612 \quad MN$$

and the elastic buckling length is:

$$L_{cr} = \sqrt{\frac{\pi^2 EI}{N_{cr}}} = 12 \quad m$$

$$\Rightarrow \frac{L_{cr}}{L} = 1.0$$

It can be seen that sheeting increases member's buckling resistance significantly against the flexural buckling in the plane of sheeting when only compressive load is consider.

3.2 Shear force of connectors with trapezoidal sheeting

In the second example, the maximum shear force of connectors is calculated for the same case as in the previous Chapter 3.1. Material of the sheeting is S350GD+Z with ultimate strength 420 MPa, nominal thickness of the sheeting $t_{nom} = 0.7mm$ and design thickness is $t = 0.66mm$. Compressive force of the member is $N_{Ed} = 300kN$, in-plane deflection is $\delta_q = 0.5mm$ and $m = 1$. Sheeting is connected to member with screws with diameter $d = 5.5mm$ and spacing of the screws is $c = 500mm$.

Initial bow imperfection e_0 can be calculated using Equations (16) and (17):

$$\alpha_m = \sqrt{\frac{1}{2}\left(1 + \frac{1}{m}\right)} = 1$$

$$e_0 = \alpha_m \frac{L}{500} = 24 \quad mm$$

And after that the shear force of a screw is according to Equation (19):

$$F = N_{Ed} \cdot 8 \frac{e_0 + \delta_q}{L^2} \cdot c = 204 \quad N$$

From Table 2 in Chapter 2.1.2, it can be seen that the shear resistance of the screws is $F_{Rd} = 1.352 kN$, so utilization ratio of screws in this case is 15.1% provided that the utilization ratio 100% means the fully stressed design and thus the screws are sufficient.

3.3 Elastic buckling length of member restraint with sandwich panels

3.3.1 Stiffness of connectors

In this example, the stiffness of connection between member and sandwich panel is as follows.

Used data for a connector is:

$t_{F2} = 0.525$ mm   the thickness of internal sheeting;

$t_{cor,F2} = 0.5$ mm   the core thickness of internal face;

$t_{cor,sup} = 10$ mm   the core thickness of supporting structure;

$d = 5.5$ mm   the nominal diameter of the fastener;

$d_1 = 5$ mm   the minor diameter of the threaded part of the fastener;

$d_S = 5$ mm   the diameter of the unthreaded shank;

$f_{u,F2} = 390$ MPa   the tensile strength of the internal face;

$D = 100$ mm   the thickness of the panel at point of fastening;

The stiffness of connector is then:

$$k_v = \frac{1}{\dfrac{x_F}{k_{F2}} + \dfrac{t_{cor,sup}^2 + 2(1 - x_F)Dt_{cor,sup}}{4C_{sup}} + \dfrac{3(1 - x_F)Dt_{cor,sup}^2 + t_{cor,sup}^3}{24EI}} = 2696 N/mm$$

where

$$x_F = 1 - \frac{\dfrac{1}{k_{F2}} - \dfrac{DT_{cor,sup}}{2C_{sup}} - \dfrac{Dt_{cor,sup}^2}{8EI}}{\dfrac{1}{k_{F2}} + \dfrac{D^2}{C_{sup}} + \dfrac{D^2(2D + 3t_{cor,sup})}{6EI}} = 1.01$$

$$EI = 200GPa\frac{\pi d_S^4}{64} = 6135923 Nmm^2$$

$$C_{sup} = 2400MPa\sqrt{t_{cor,sup}d_1^5} = 424264 Nmm$$

$$k_{F2} = 6.93\frac{f_{u,F2}\sqrt{t_{cor,sup}d_1^5}}{0.26mm + 0.8t_{cor,sup}} = 3142 N/mm$$

3.3.2 Simply supported member

Next example is to calculate elastic buckling length of a simply supported member restrained with sandwich panels. Width of a sandwich panel is $B = 1200mm$ and sandwich panels are connected to the member with a pair of screws. The distance between screws in a panel is $c_k = 1000mm$ and the stiffness of a screw is $k_v = 2696N/mm$.

The elastic buckling load from Equation (52) is:

$$N_{cr} = EI\frac{r^2\pi^2}{L^2} + \frac{k_\nu}{2B}\sum_{k=1}^n c_k^2 = 1.735 \quad MN$$

and thus the elastic buckling length is:

$$L_{cr} = \frac{L}{\sqrt{1 + \dfrac{k_\nu}{2B}\sum_{k=1}^n c_k^2 \cdot \dfrac{L^2}{\pi^2 EI}}} = 7125 \quad mm$$

$$\Rightarrow L_{cr}/L = 0.59$$

Elastic flexural buckling load for the simply supported member without support of panels is:

$$N_{cr} = \frac{\pi^2 EI}{L^2} = 0.612 \quad MN$$

and the elastic buckling length is:

$$L_{cr} = \sqrt{\frac{\pi^2 EI}{N_{cr}}} = 12 \quad m$$

$$\Rightarrow \frac{L_{cr}}{L} = 1.0$$

It can be seen that sandwich panels significantly increase buckling load and thus decrease buckling length.

3.3.3 Cantilever column

This example is to calculate elastic buckling length of a cantilever column restraint with sandwich panels. All input data are the same as in the example above except now the member is cantilever with rigid support at one end and no support in the other. Width of a sandwich panel is $B = 1200mm$ and sandwich panels are connected to the member with a pair of screws. The distance between screws in a panel is $c_k = 1000mm$ and the stiffness of a screw is $k_v = 2696N/mm$.

The elastic buckling load is now according to Equation (63):

$$N_{cr} = EI \frac{r^2 \pi^2}{(2L)^2} + \frac{k_\nu}{2B} \sum_{k=1}^{n} c_k^2 = 1.276 \quad MN$$

and the elastic buckling length is:

$$L_{cr} = \frac{2L}{\sqrt{1 + \frac{k_\nu}{2B} \sum_{k=1}^{n} c_k^2 \frac{(2L)^2}{\pi^2 EI}}} = 8308 \quad mm$$

$$\Rightarrow L_{cr}/L = 0.69$$

Elastic buckling load for cantilever column without support of panels is:

$$N_{cr} = \frac{\pi^2 EI}{L^2} = 0.153 \quad MN$$

and the elastic buckling length is:

$$L_{cr} = \sqrt{\frac{\pi^2 EI}{N_{cr}}} = 24 \quad m$$

$$\Rightarrow \frac{L_{cr}}{L} = 2.0$$

It can be seen that again sandwich panels significantly increase buckling load and decrease buckling length.

3.3.4 Cantilever with support at the top

Third case of boundary conditions is the member with rigid support at one end and a hinge in the other end. Again, width of a sandwich panel is $B = 1200mm$ and sandwich panels are connected to the member with a pair of screws. The distance between screws in a panel is $c_k = 1000mm$ and the stiffness of a screw is $k_v = 2696N/mm$.

The first eigenvalue is according to Equation (70):

$$\lambda = \frac{4.4934095}{L} = 0.374 \quad \frac{1}{m}$$

With that eigenvalue, critical buckling load can be calculated from Equation (71) as:

$$N_{cr} = \lambda^2 EI + \frac{k_\nu}{2B} \sum_{k=1}^{n} c_k^2 = 2.375 \quad MN$$

and elastic buckling length is now:

$$L_{cr} = \frac{\pi^2 EI}{N_{cr}} = 6091 \quad mm$$

$$\Rightarrow L_{cr}/L = 0.51$$

Elastic buckling load for these boundary conditions without support of panels is:

$$N_{cr} = \frac{\pi^2 EI}{L^2} = 1.251 \quad MN$$

and the elastic buckling length is:

$$L_{cr} = \sqrt{\frac{\pi^2 EI}{N_{cr}}} = 8390 \quad mm$$

$$\Rightarrow \frac{L_{cr}}{L} = 0.7$$

It can be seen that even in this case sandwich panels help with buckling.

3.3.5 Fixed supported member

Fourth case of boundary conditions is the member with rigid supports at the both ends. All input data are the same as in the earlier examples.

The elastic buckling load is now according to Equation (77):

$$N_{cr} = EI \frac{r^2 \pi^2}{(\frac{L}{2})^2} + \frac{k_\nu}{2B} \sum_{k=1}^{n} c_k^2 = 3.571 \quad MN$$

and the elastic buckling length is:

$$L_{cr} = \frac{(\frac{L}{2})}{\sqrt{1 + \frac{k_\nu}{2B} \sum_{k=1}^{n} c_k^2 \frac{(\frac{L}{2})^2}{\pi^2 EI}}} = 4967 \quad mm$$

$$\Rightarrow L_{cr}/L = 0.41$$

Elastic buckling load for fixed supported member without panels is:

$$N_{cr} = \frac{\pi^2 EI}{(\frac{L}{2})^2} = 2.447 \quad MN$$

and the elastic buckling length is:

$$L_{cr} = \sqrt{\frac{\pi^2 EI}{N_{cr}}} = 6 \quad m$$

$$\Rightarrow \frac{L_{cr}}{L} = 0.5$$

It can be seen that again sandwich panels increase buckling load and decrease buckling length but the benefit of the panels is reduced when the boundary conditions are stiffer.

3.4 Shear force of connectors with sandwich panels

The last example shows how to calculate the maximum shear force of connectors for members with restraint from sandwich panels. In this example, the same simply supported member and cantilever are examined. The stiffness of a screw is $k_v = 2696N/mm$, the distance between screws in a panel is $c_k = 1000mm$, initial deformation of member is supposed to be according to Equation (103) as $v_0 = \frac{L}{577}$, the axial design force of the member is $N_{Ed} = 300kN$ and length of member is $L = 12m$. The elastic buckling load of simply supported member is $N_{cr} = 1.735MN$ and the elastic buckling load of cantilever is $N_{cr} = 1.276MN$.

Maximum force of a connector for simply supported member can be calculated from Equation (100) as:

$$F_{max} = k_\nu \frac{c_k}{2} \cdot \frac{v_0 \pi}{\left(\dfrac{N_{cr}}{N_{Ed}} - 1\right) L} = 1.18 \quad kN$$

Maximum force of a connector for cantilever can be calculated from Equation (101) as: $F_{max} = k_\nu \dfrac{c_k}{2} \cdot \dfrac{v_0 \pi}{\left(\dfrac{N_{cr}}{N_{Ed}} - 1\right) 2L} = 0.87 \quad kN$

The shear resistance of a connector is according to Equation (104):

$$V_{Rd} = \frac{4.2 \cdot \sqrt{t_{cor,F2}^3 d_1} \cdot f_{u,F2}}{\gamma_{M2}} = 1.09 \quad kN$$

It can be see that utilization ratios of screws are 108% for simply supported member and 80% when the utilization ratio 100% means the fully stressed design. Thus the

connectors are sufficient only in the case of cantilever. It seems that with sandwich panels connector forces limit the member's capacity to withstand axial force more than buckling.

4   VERIFICATION OF CALCULATIONS

In Chapter 2, analytical solutions were obtained for elastic buckling lengths of re-strained members. Analytical solutions are compared with the results obtained by the finite element method for verification purposes. The member is modeled with 120 Euler-Bernoulli beam elements. Software RFem (program version 5.07.11.122642, student version) by Dlubal (Dlubal Software GmbH 2016) was used to solve the finite element model.

4.1 Lateral restraint with trapezoidal sheeting

The examined structure was modeled using beam and spring elements. The stiffness of spring elements is the same than the foundation parameter K from Equation (2). Schematic picture from FEM model is shown in Figure 7.
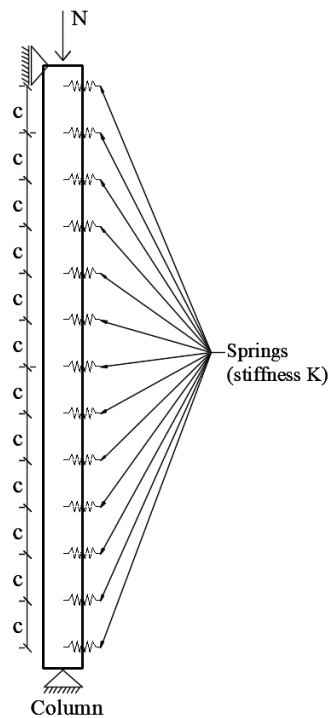

Figure 7. *Schematic picture from FEM model of a column restrained by trapezoidal sheeting. The springs are connected to the mid-points of the column cross-section and the displacements of other end are fixed. The springs can resist only axial forces.*
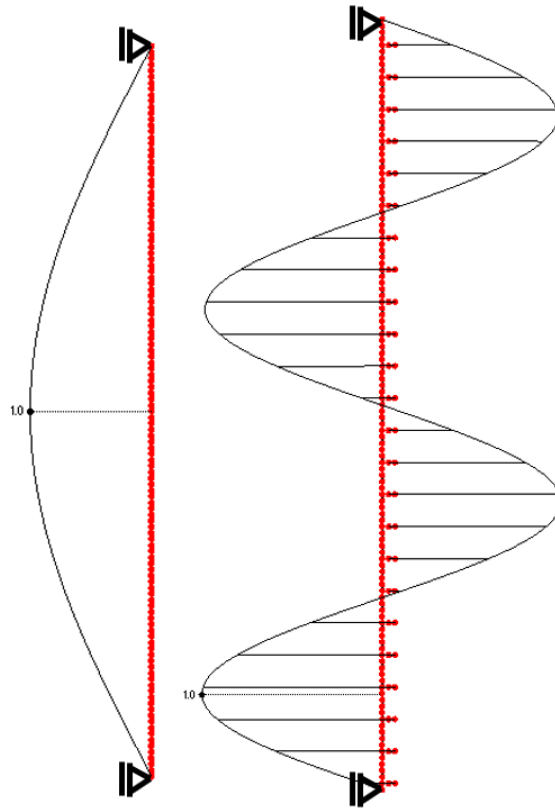
Figure 8. *The smallest eigenmode of simply supported member without and with trapezoidal sheeting. Cross-section CFRHS 200x200x10, length 12 m, thickness of sheeting 0.7 mm and diameter of connectors 5.5 mm.*

Without trapezoidal sheeting, analytical solution for critical elastic buckling load is $N_{cr} = 0.612$ MN as calculated in Section 3.1 and FEM solution is $N_{cr} = 0.610$ MN. Critical elastic buckling loads for simply supported member restrained with trapezoidal sheeting are: analytical solution $N_{cr} = 22.010$ MN and FEM solution $N_{cr} = 21.636$ MN. It can be seen that analytical solutions and FEM solutions are close to each others, analytical solutions are slightly safer than FEM solutions but the difference is very small.

Using the model of Figure 7 the lowest eigenmode for the simply supported and axially loaded column and the distribution of elements is shown Figure 8. The figure shows that number of half-waves in the critical buckling mode of restrained member is $n = 4$ as calculated in Section 3.1.

4.2 Lateral restraint with sandwich panels

FEM model is made to act according to the basic assumptions of the theory: connectors are replaced with linear springs, panels are modeled with rigid plates, only horizontal movement was allowed for panels and the longitudinal joints between panels do not transfer any loads. Schematic picture of the FEM model is shown in Figure 9.
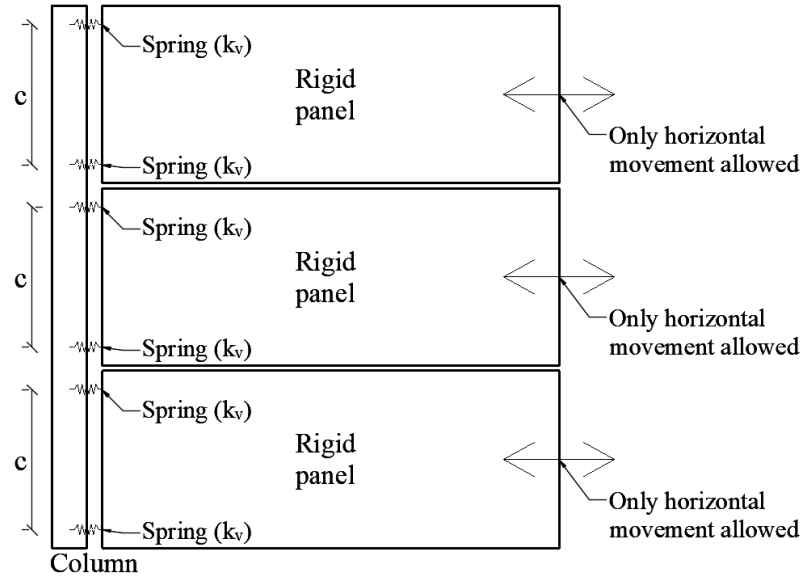

Figure 9. *Schematic picture from FEM model of a column restrained by sandwich panels.*

The comparison between analytical solutions and FEM solutions with different boundary conditions is shown in Table 5. From Table 5, it can be seen that analytical solutions and FEM solutions are close to each others in every case and analytical solutions are slightly safer than FEM solutions. Table 5 shows also how much sandwich panels increase buckling loads and decrease buckling length in every case as calculated in Chapter 3.

Table 5. *Flexural buckling loads for CFRHS 200x200x10 in the plane of the panels,* $L = 12m$, *sandwich panels* $B = 1200mm$, *screws* $k_\nu = 2696N/mm$ *(d = 5.5mm,* $t_f = 0.6mm$, $f_u = 390MPa$), $c_k = 1000mm$, $n = 1$.

	Simply supported	Fixed/hinged	Cantilever	Fixed/fixed
Analytical N_{cr} [kN] without sandwich panels	612	1251	153	2447
FEM N_{cr} [kN] without sandwich panels	610	1245	153	2425
Analytical N_{cr} [kN] with sandwich panels	1735	2375	1276	3571
FEM N_{cr} [kN] with sandwich panels	1728	2363	1272	3520
Analytical L_{cr} [m] without sandwich panels	12	8.390	24	6
Analytical L_{cr} [m] with sandwich panels	7.125	6.091	8.308	4.967
L_{cr} with panels/ L_{cr} without panels	0.59	0.73	0.35	0.83

It is also worth noticing that the difference between unrestrained members and members restrained with sandwich panels is greater with boundary conditions where buckling length of unrestrained member is bigger. In the other words, the greatest benefit from sandwich panels is obtained to cantilever members and members with fixed-fixed support benefit the least from sandwich panels. That is because the support the sandwich panels cause is constant in every case and when buckling load of unsupported member is small the supporting term is relatively bigger.

Figures 10–13 show lowest buckling modes with different boundary conditions without and with sandwich panels. It can be seen that sandwich panels do not change the buckling mode.
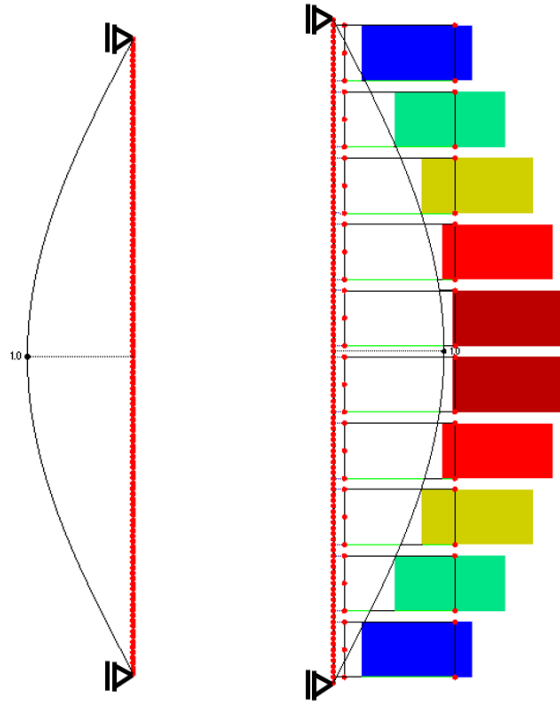
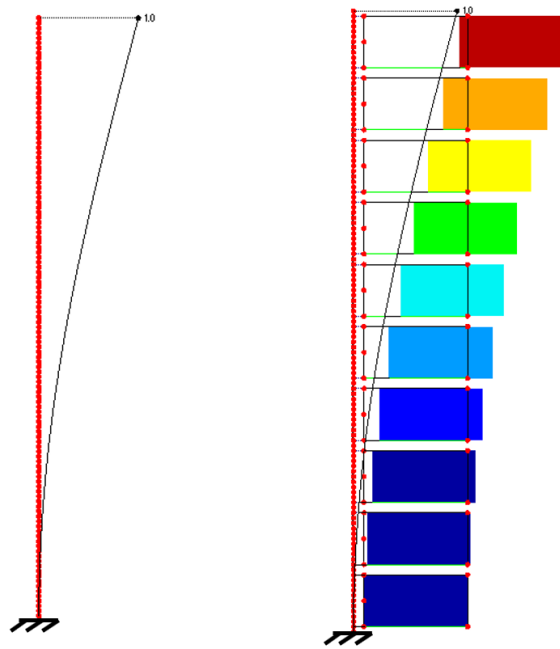**Figure 10.** *The lowest eigenmode of FEM model for simply supported column with and without sandwich panels.*

Figure 11. *The lowest eigenmode of FEM model for the cantilever column with and without sandwich panels.*
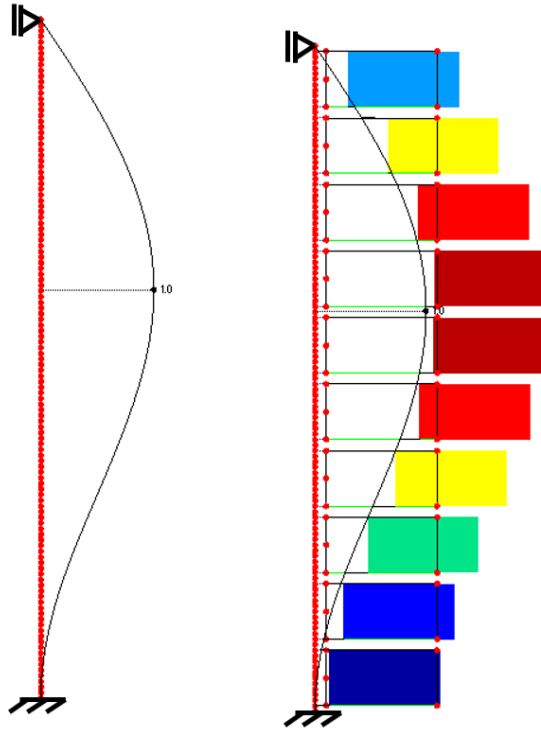
Figure 12. The lowest eigenmode of FEM model for the cantilever column with support at the top with and without sandwich panels.
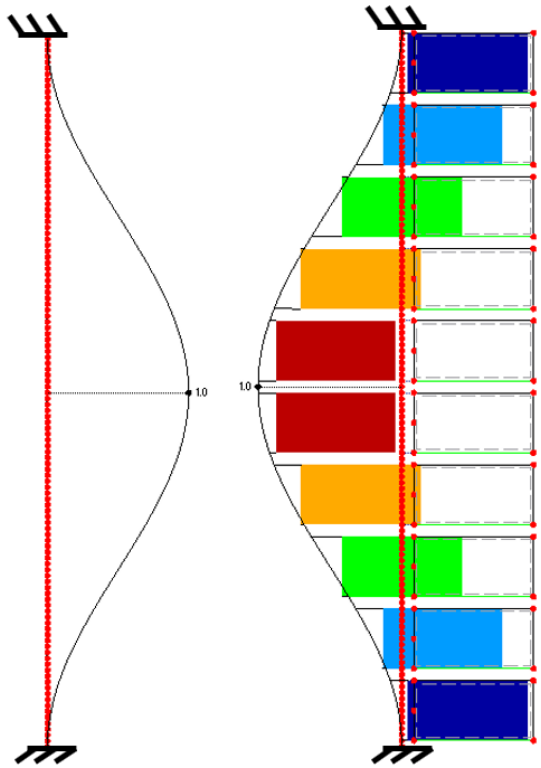

Figure 13. The lowest eigenmode of FEM model for the fixed supported column with and without sandwich panels.

Analytical solutions and solutions from FEM analysis correspond to each other very well with regard of buckling loads and buckling lengths.

4.3 Connector forces with sandwich panels

Figures 14 and 15 show how analytical solutions and FEM solutions correspond to each other with regard of connector forces in the case of axially loaded cantilever member. Analytical results can be calculated from Equation (99). Figure 14 shows real values and Figure 15 shows absolute values of connector forces. It can be seen that in FEM results, connector forces in a sandwich panel are equal like assumed in Section 2.2. Instead, analytical connector forces follow sine function when upper connector takes more force than lower one. It can also be seen that analytical connector forces are bigger than forces from FEM analysis.
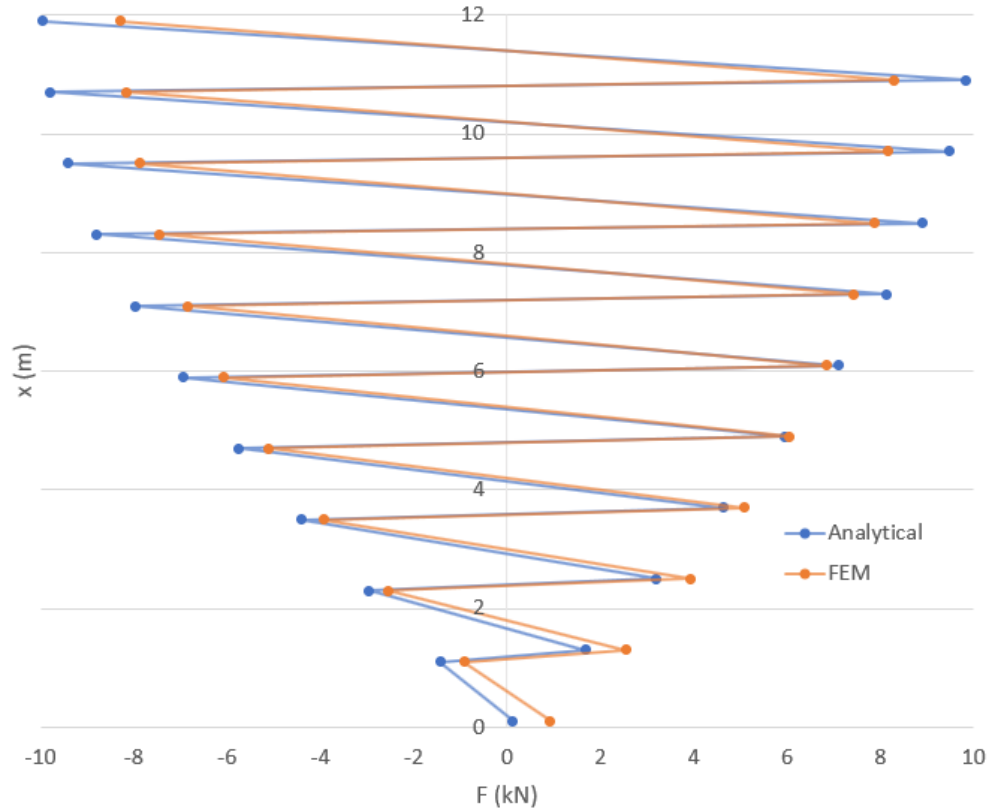

Figure 14. *Connector forces in a 12 m cantilever column with sandwich panels. CFRHS 200x200x10, L = 12 m, k_ν = 2696 N/mm and N_{Ed} = 1000 kN.*
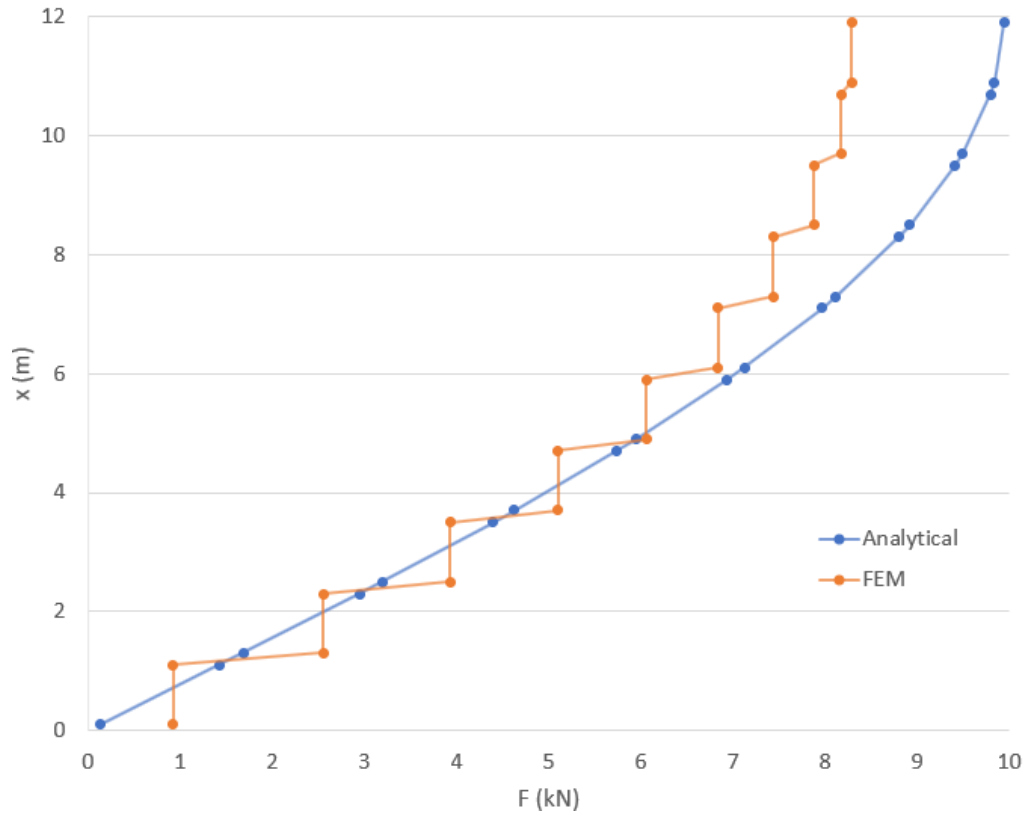
Figure 15. *Absolute values of connector forces in a 12 m cantilever column with sandwich panels. CFRHS 200x200x10, $L = 12$ m, $k_\nu = 2696$ N/mm and $N_{Ed} = 1000$ kN.*

Figures 16 and 17 show how the maximum deflection and maximum connector force of member depend on axial load of member. The maximum deflection can be calculated from Equation (97) and maximum connector force can be calculated from Equation (101). It can be seen that the maximum deflection and maximum connector force behave the same way. When the axial force increases in relation to the buckling load, the deflection and connector force increase at an accelerating speed until the axial load approaches the buckling load and the deflection and connector force approach infinity. Because the connector force approach infinity when utilization ratio of buckling approaches one, connector forces are in all cases the limiting factor instead of buckling when members are restrained with sandwich panels.
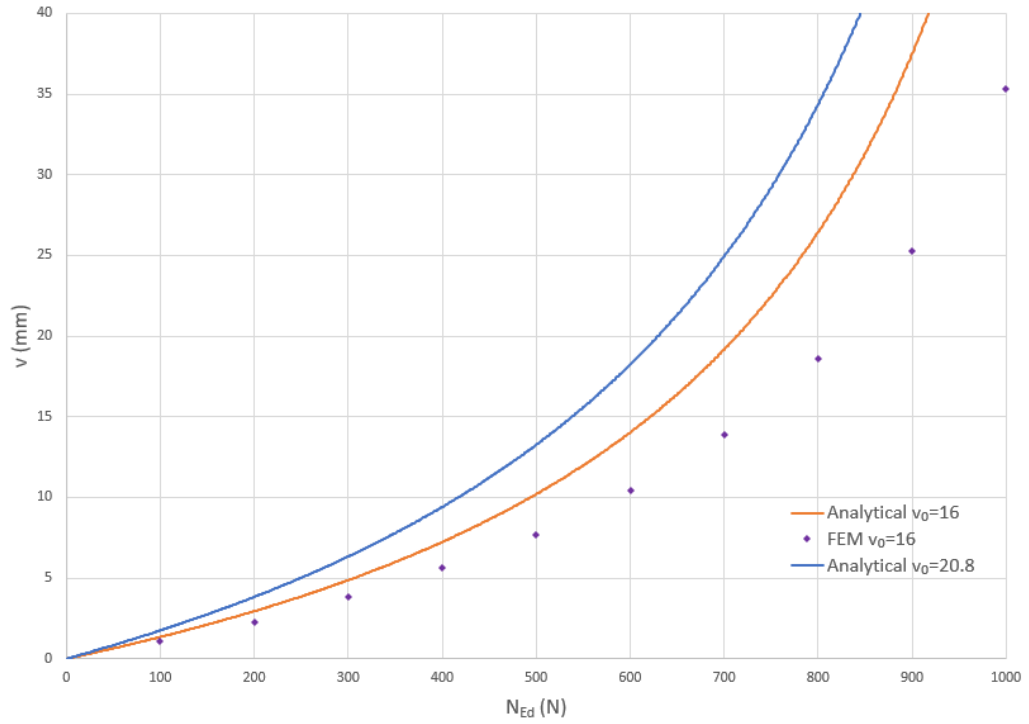
Figure 16. *Maximum deflection of the column (analytical and FEM) versus axial load of the cantilever restrained with sandwich panels. CFRHS 200x200x10, $L = 12$ m and $k_\nu = 2696$ N/mm.*
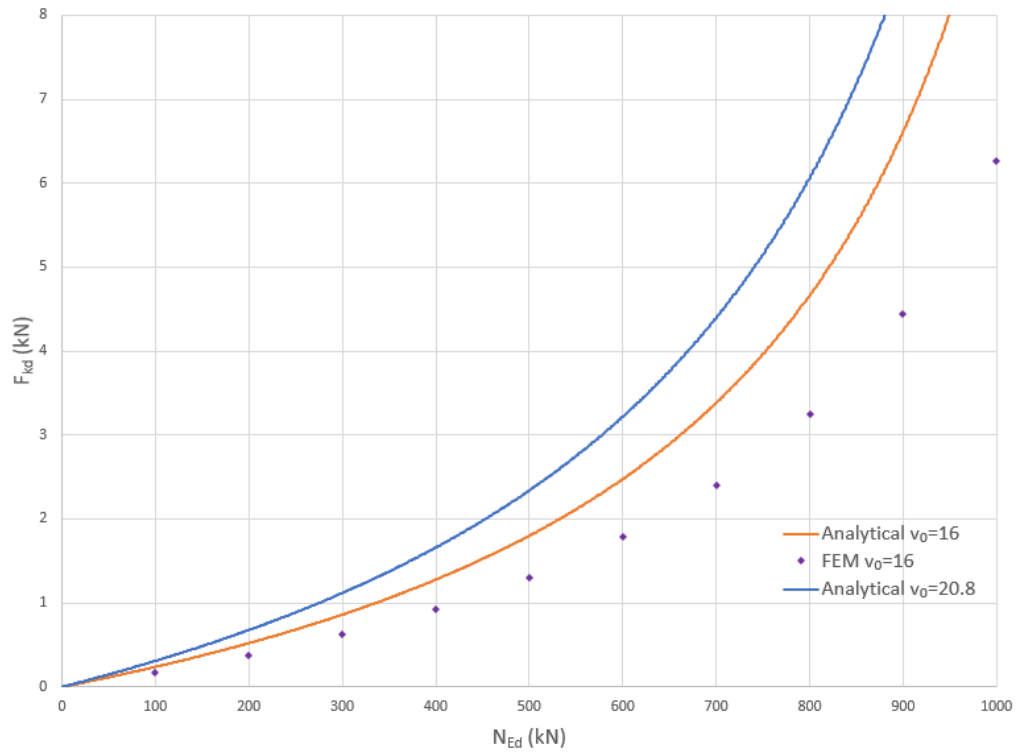

Figure 17. *Maximum force of a connector (analytical and FEM) versus axial load of the cantilever restrained with sandwich panels. CFRHS 200x200x10, $L = 12$ m and $k_\nu = 2696$ N/mm.*

It can also be seen that analytical results are bigger than FEM results and results depends on initial deformation v_0. In Figures 16 and 17 $v_0 = 16$ mm represents initial deformation $L/750$ and $v_0 = 20.8$ mm represents initial deformation $L/577$ calculated in Equation (103) in Section 2.2.2.

5   PARAMETRIC STUDY

In this chapter, a parametric study is made to find out how different parameters affect and when stressed skin can help to get more resistance. The study is made by calculating axial force resistance with analytical equations presented in Chapter 2 and studied parameter are buckling resistance of unrestrained member, stiffness of connections and shear resistance of connections. Only axial load is taken into account and the maximum load is calculated by increasing load until the connector force reaches the bearing resistance of a screw or until buckling resistance in the plane of stressed skin is exceeded. Buckling resistance is calculated according to SFS-EN 1993-1-1 (2005).

For unrestrained members, the maximum load that causes buckling resistance of member to be exceeded, is presented in Figure 18 for simply supported member and Figure 19 for cantilever. These results serve as comparisons to show if different stiffening solutions have benefits.
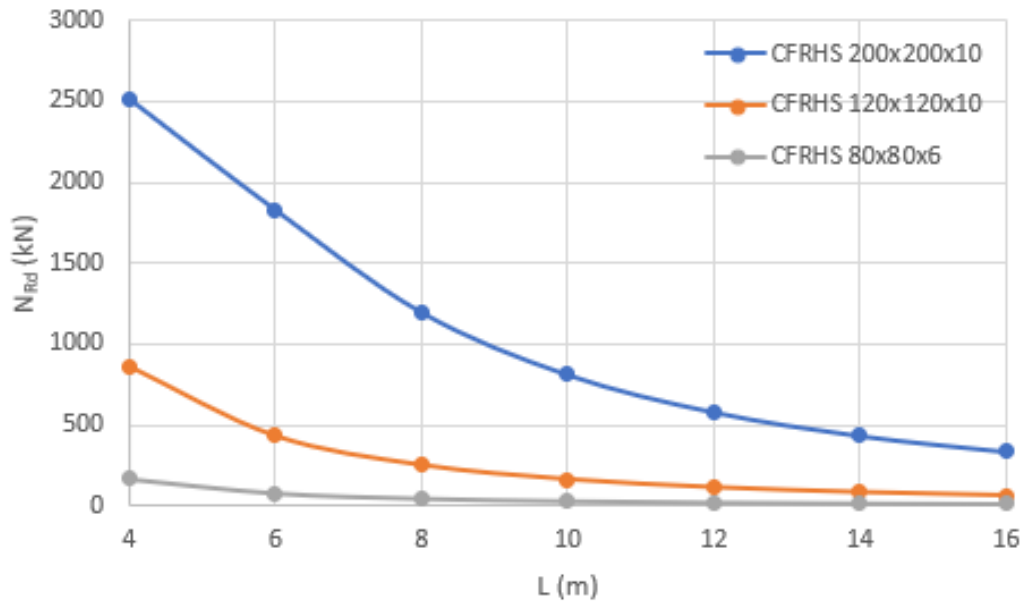

Figure 18. *Maximum axial load for simply supported member without restraint.*
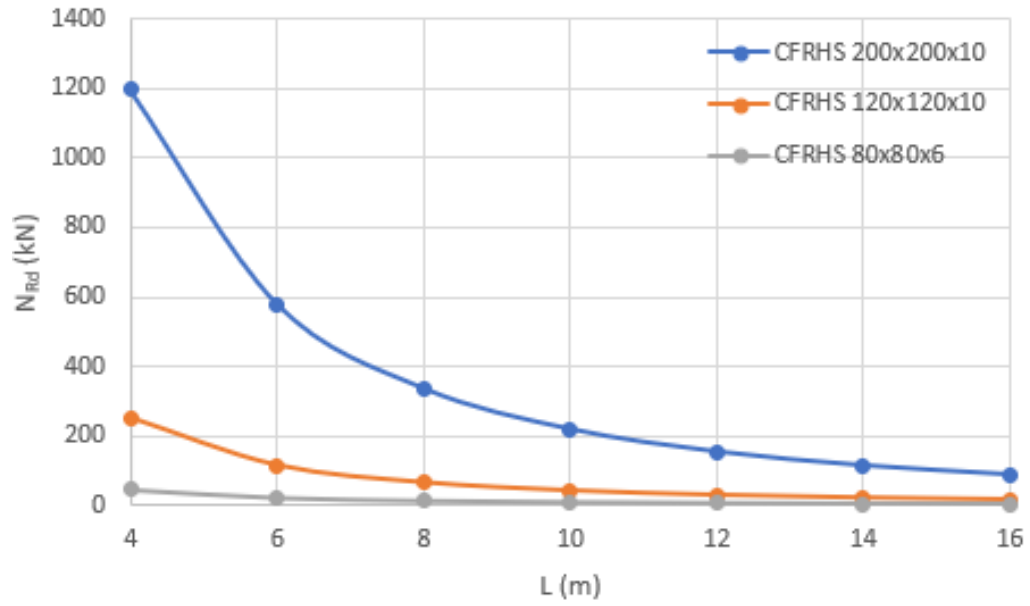
Figure 19. *Maximum axial load for cantilever column without restraint.*

For unrestrained members, the buckling resistance decreases when length of member increases or cross-section is smaller.

5.1 Lateral restraint with trapezoidal sheeting

In this chapter, only simply supported axially loaded member is studied. There are four different parameters that affect the maximum axial load of member with trapezoidal sheeting: length of member, cross-section, stiffness of connections and shear resistance of connections.

In the first case, the effect of member's length and cross-section are studied. Three different cross sections are selected to show the difference and results are presented in Figure 20.
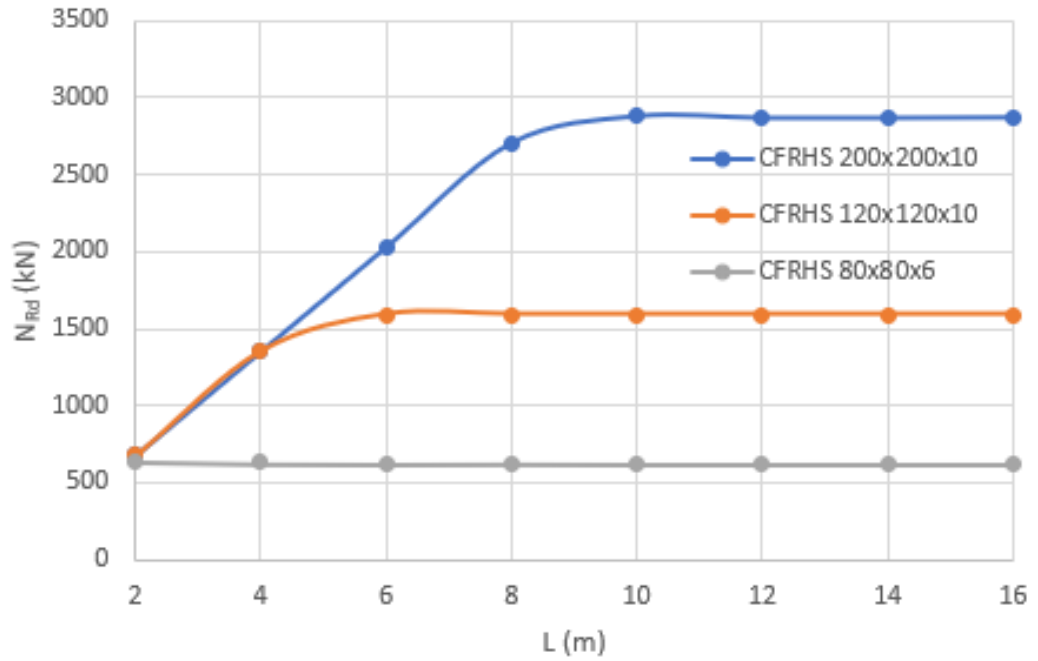
Figure 20. *Maximum axial load versus member length for simply supported member with restraint from trapezoidal sheeting. Thickness of sheeting is $t_{nom} = 0.7$ mm, distance between screws is $c = 500$ mm and diameter of a screw is $d = 5.5$ mm.*

It can be seen that bigger cross-sections have bigger axial force resistance as usual but the shape of graphs is different than without trapezoidal sheeting. When axial force resistance of unsupported member decreases first rapidly and later more evenly approaching zero, the axial force resistance of member restrained with trapezoidal sheeting increases linearly until it reaches constant value. The linear increase is caused by connector forces and linear part is caused by buckling resistance.

Figures 21 and 22 present the comparison between unrestrained member and member restrained with trapezoidal sheeting. It can be seen that with big cross-sections and short members connector forces could limit the axial force resistance of member to be less than resistance of unrestrained member but in most cases the benefit for sheeting is significant.
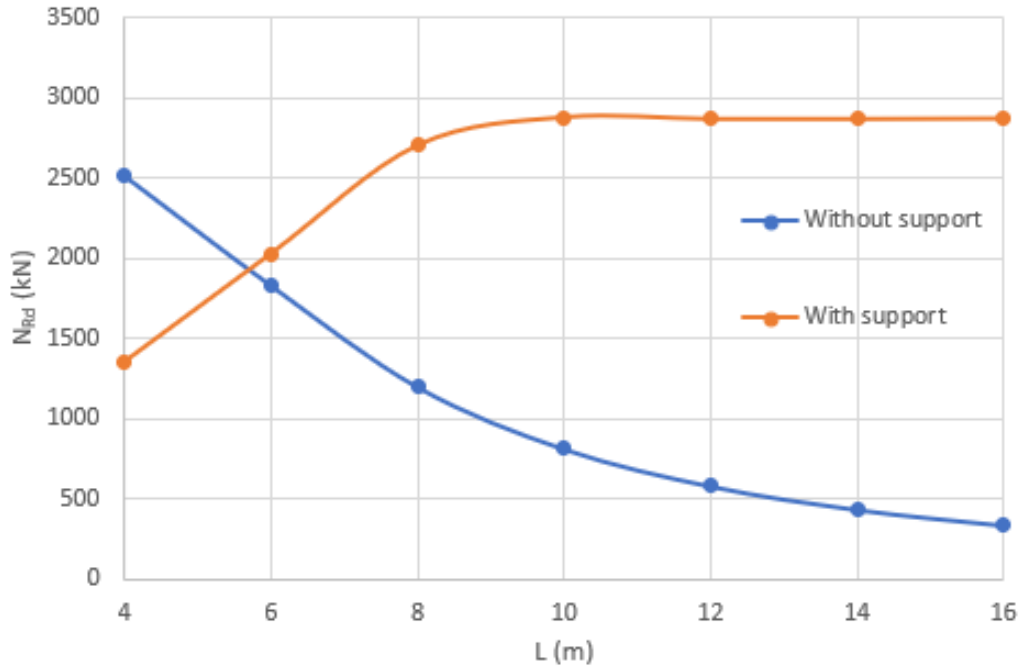
Figure 21. *Maximum axial load versus member length for simply supported member with and without restraint from trapezoidal sheeting. Cross-section is CFRHS 200x200x10, thickness of sheeting is $t_{nom} = 0.7$ mm, distance between screws is $c = 500$ mm and diameter of a screw is $d = 5.5$ mm.*

Figure 22. *Maximum axial load versus member length for simply supported member with and without restraint from trapezoidal sheeting. Cross-section is CFRHS 120x120x10, thickness of sheeting is $t_{nom} = 0.7$ mm, distance between screws is $c = 500$ mm and diameter of a screw is $d = 5.5$ mm.*
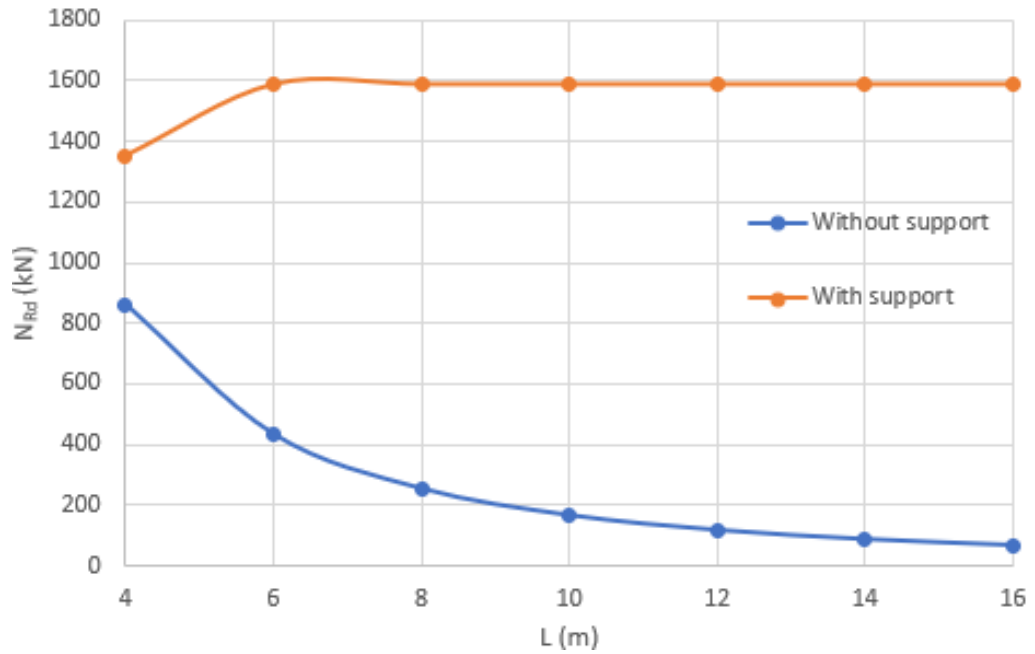
The second case is to study the effect of the thickness of the sheeting. The thickness of the sheet has a significant impact on the shear resistance of connections and it also affects the stiffness of the connections. The diameter of connectors affects shear resistance also but the effect of the thickness is greater. The results are presented in Figure 23.
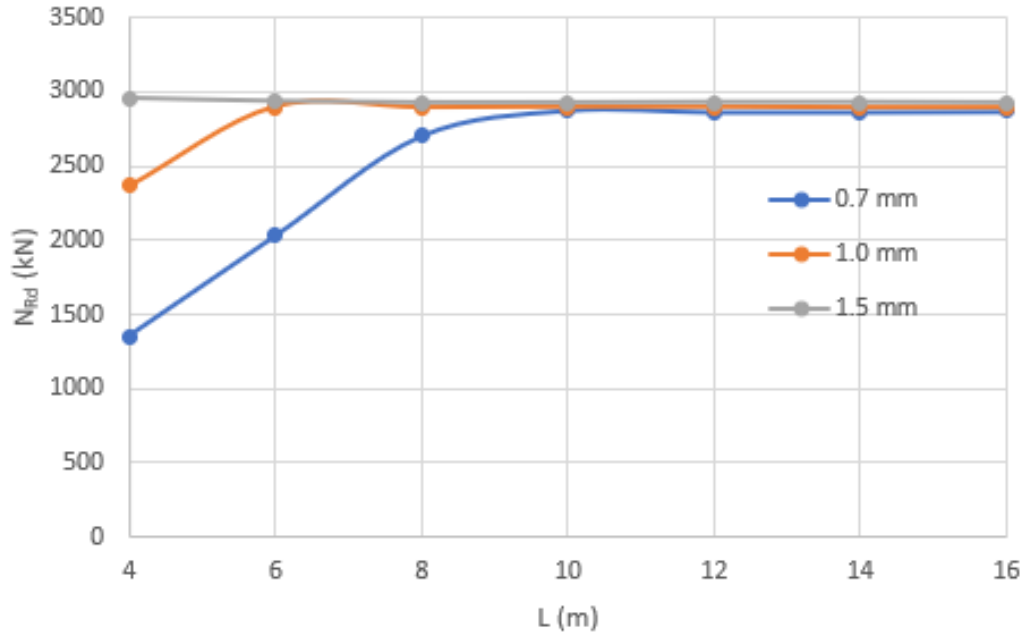

Figure 23. *Maximum axial load versus member length for simply supported member with restraint from trapezoidal sheeting. Cross-section is CFRHS 200x200x10, distance between screws is $c = 500$ mm and diameter of a screw is $d = 5.5$ mm.*

It can be seen that changing the thickness of sheeting does not really affect the maximum constant value the axial force resistance settles but it changes the linearly increasing part caused by connector forces.

The third case is to study the effect of the stiffness of the connections by varying the distance between screws. As shown in Figure 24, the change of stiffness affect mostly to the gradient of the part caused by connector forces.
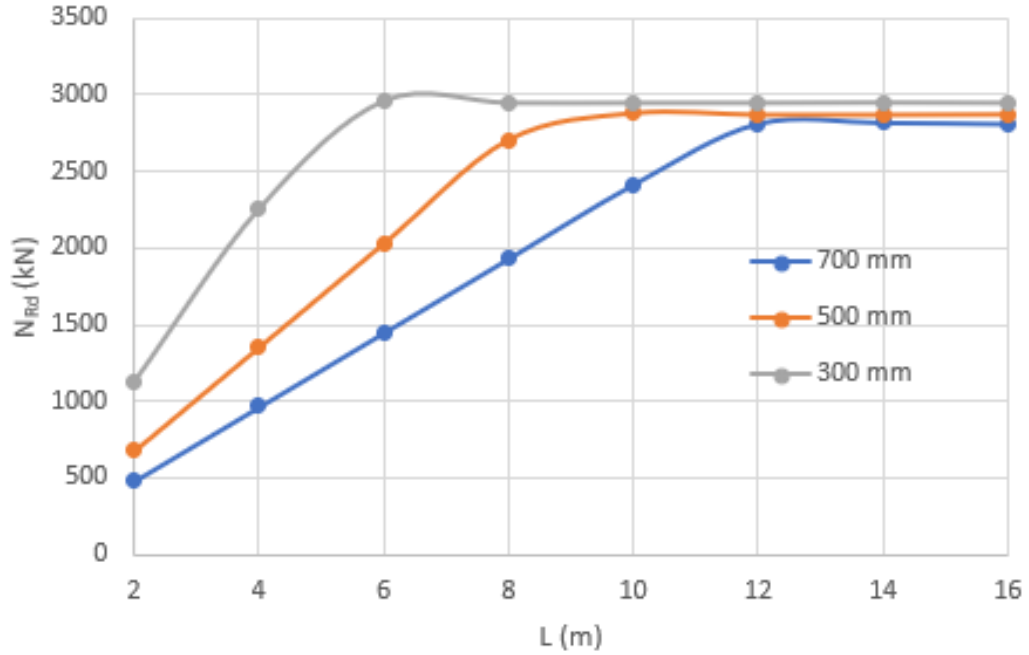
Figure 24. *Maximum axial load versus member length for simply supported member with restraint from trapezoidal sheeting. Cross-section is CFRHS 200x200x10, thickness of sheeting is $t_{nom} = 0.7$ mm and diameter of a screw is $d = 5.5$ mm.*

Trapezoidal sheeting can be used to increase the axial force resistance of the members. The benefit is unquestionable and the difference can be huge, for example CFRHS 200x200x10, thickness of sheeting $t_{nom} = 0.7$ mm, distance between screws $c = 500$ mm, diameter of a screw $d = 5.5$ mm and length $L = 12$ m $\Rightarrow N_{Rd} = 580$ kN without trapezoidal sheeting and $N_{Rd} = 2866$ kN with restraint from trapezoidal sheeting.

5.2 Lateral restraint with sandwich panels

In this chapter, the results are shown with two different boundary conditions, simply supported member and cantilever. The parameters that affect the maximum axial load of member with trapezoidal sheeting are: length of member, cross-section, stiffness of connections and shear resistance of connections.

In the first case, the effect of member's length and cross-section are studied. Three different cross sections are selected to show the difference and results are presented for simply supported member in Figure 25 and for cantilever in Figure 26.
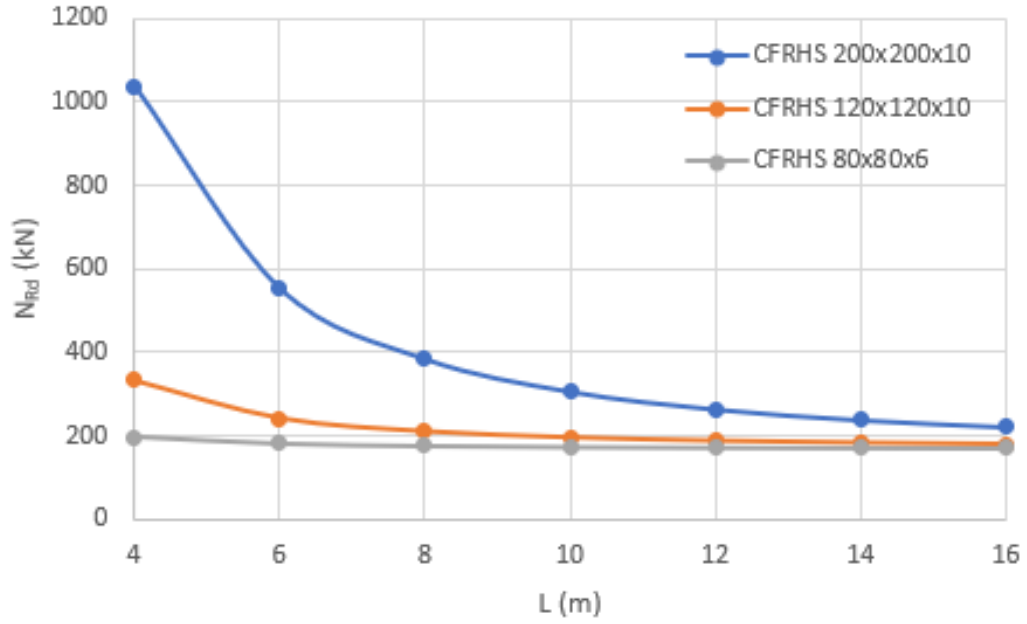
Figure 25. *Maximum axial load for simply supported member with restraint from sandwich panels. The stiffness of a connector is $k_v = 2696$ N/mm, thickness of internal face is $t_{F2} = 0.525$ mm, distance between screws is $c = 1000$ mm and diameter of a screw is $d = 5.5$ mm.*

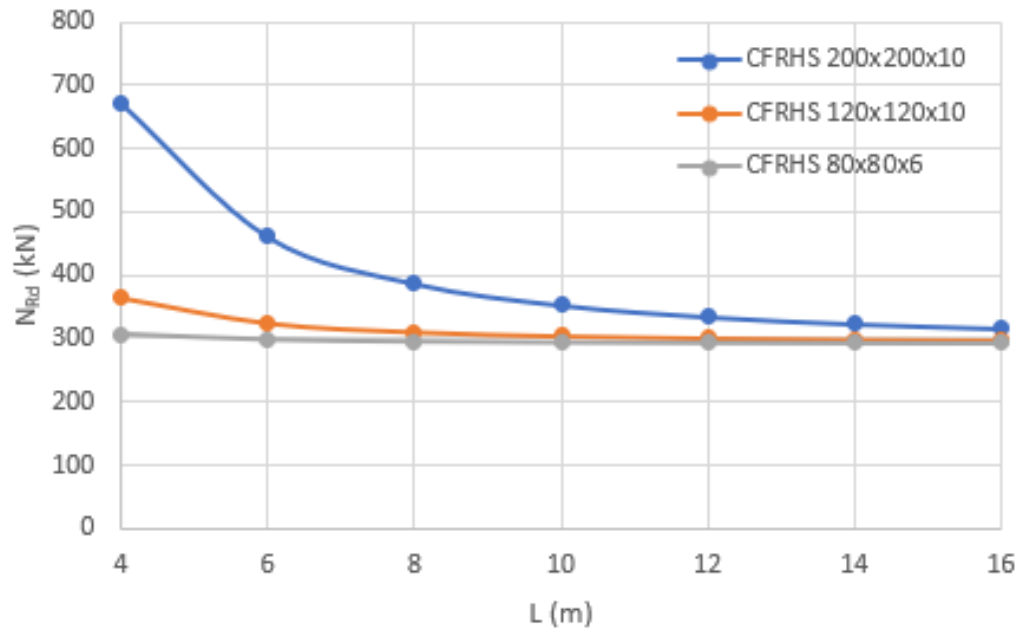**Figure 26.** *Maximum axial load for cantilever column with restraint from sandwich panels. The stiffness of a connector is $k_v = 2696$ N/mm, thickness of internal face is $t_{F2} = 0.525$ mm, distance between screws is $c = 1000$ mm and diameter of a screw is $d = 5.5$ mm.*

It can be seen that the graphs for members restrained with sandwich panels resemble the graphs for unrestrained members. Both sets of graphs have the same

decreasing basic shape but when axial force resistance of unrestrained members approach zero, the axial force resistance of members restrained with sandwich panels approach some constant value. Cross section of member does not seem to affect that constant value significantly. It is worth noticing that the axial force resistance of members restrained with sandwich panels is completely the result of shear resistance of connectors.

Figures 27 and 28 show the differences between unrestrained member and member restrained with sandwich panels in axial force resistance of simply supported member for two different cross-sections. It can be seen that the axial force resistance of CFRHS 200x200x10 is better without sandwich panels at the lengths studied because of the connector forces. For simply supported members, sandwich panels increase the axial force resistance only with slender members, for example CFRHS 120x120x10 longer than 9 meters. Whit these sandwich panels and connections the axial force resistance with sandwich panels stays about 200 kN which is maximum increase this assembly can give.

Figures 29 and 30 show the comparison between unrestrained member and member restrained with sandwich panels for cantilever members. Figures show that the benefit of sandwich panels is easier to see with cantilever members than simply supported members because the buckling resistance of unrestrained cantilever member drops earlier to very small values. It can be seen that sandwich panels increase the axial force resistance of CFRHS 200x200x10 longer than 7 meters and the axial force resistance of CFRHS 120x120x10 is better with sandwich panels in all examined lengths. Whit these sandwich panels and connections the axial force resistance with sandwich panels stays about 300 kN which is maximum increase this assembly can give.
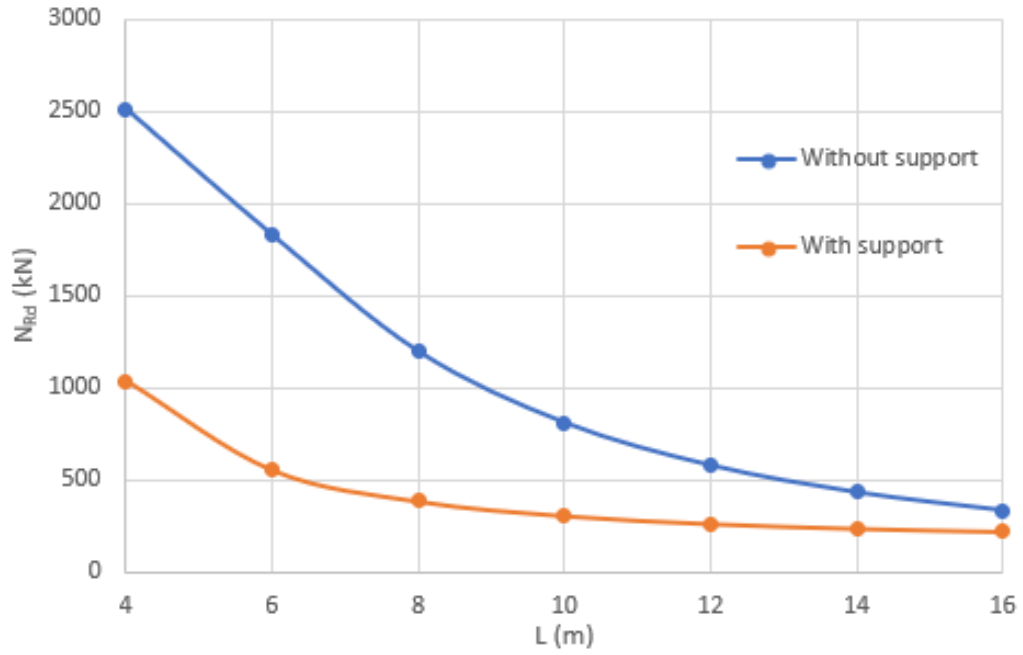
Figure 27. *Maximum axial load versus member length for simply supported member with and without restraint from sandwich panels. Cross-section is CFRHS 200x200x10, stiffness of a connector is $k_v = 2696$ N/mm, thickness of internal face is $t_{F2} = 0.525$ mm, distance between screws is $c = 1000$ mm and diameter of a screw is $d = 5.5$ mm.*
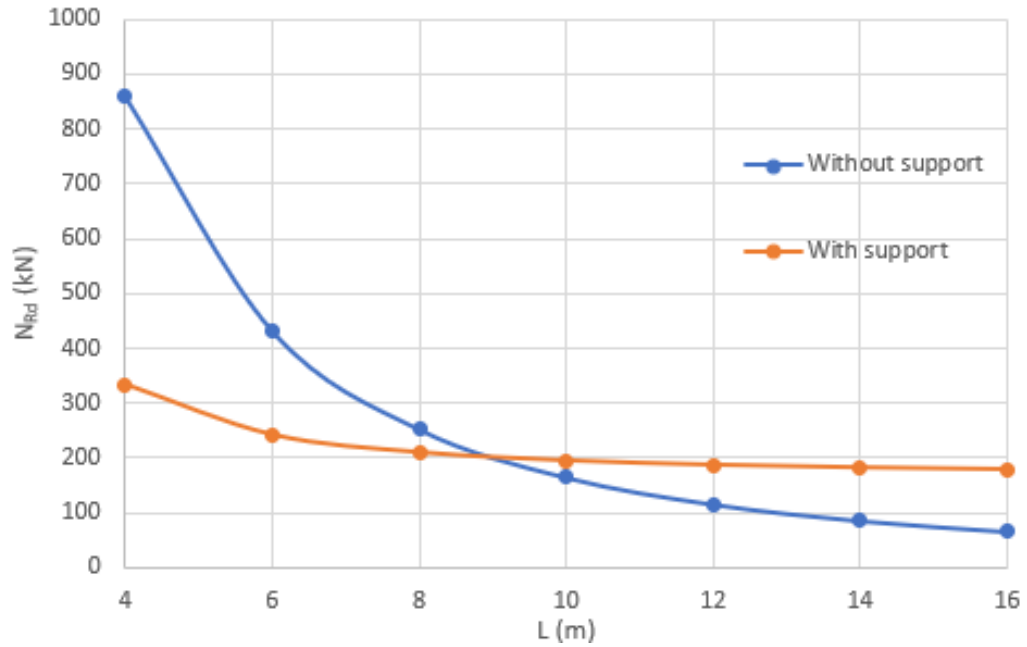

Figure 28. *Maximum axial load versus member length for simply supported member with and without restraint from sandwich panels. Cross-section is CFRHS 120x120x10, stiffness of a connector is $k_v = 2696$ N/mm, thickness of internal face is $t_{F2} = 0.525$ mm, distance between screws is $c = 1000$ mm and diameter of a screw is $d = 5.5$ mm.*

Figure 29. *Maximum axial load versus member length for cantilever member with and without restraint from sandwich panels. Cross-section is CFRHS 200x200x10, stiffness of a connector is $k_v = 2696$ N/mm, thickness of internal face is $t_{F2} = 0.525$ mm, distance between screws is $c = 1000$ mm and diameter of a screw is $d = 5.5$ mm.*
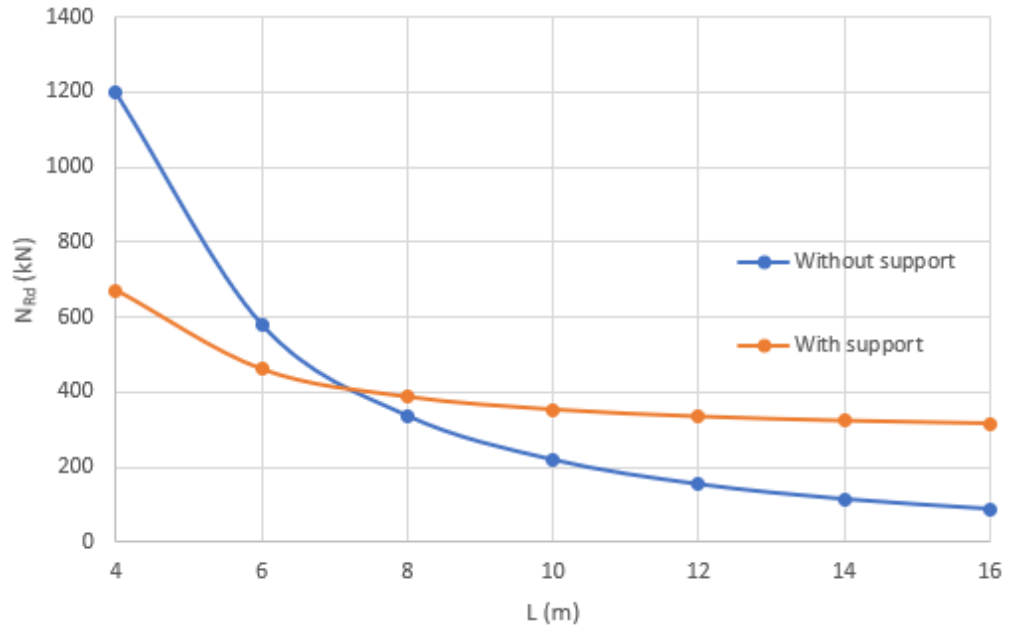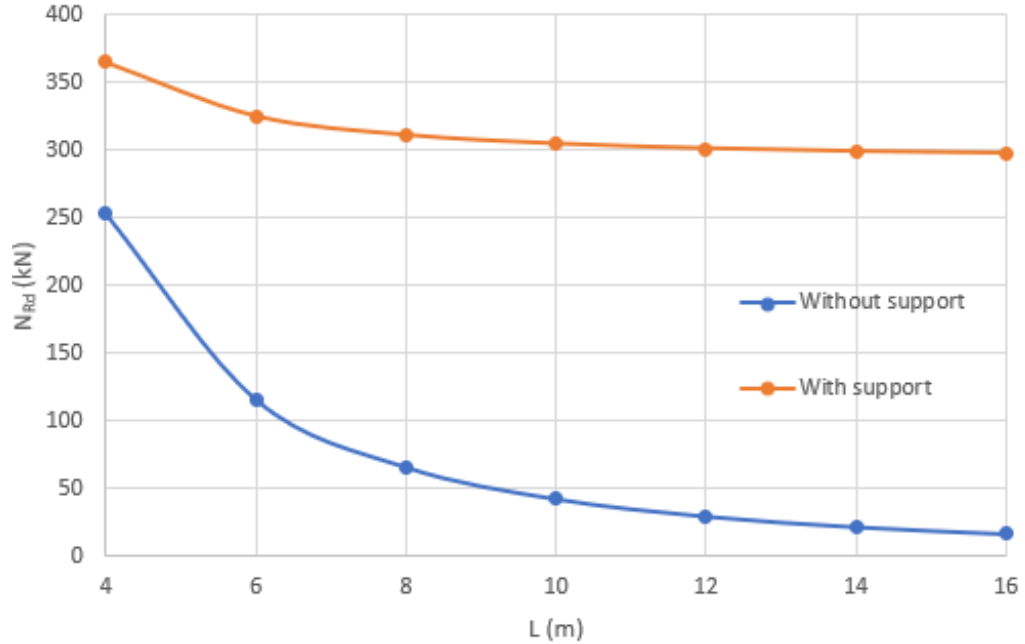

Figure 30. *Maximum axial load versus member length for cantilever member with and without restraint from sandwich panels. Cross-section is CFRHS 120x120x10, stiffness of a connector is $k_v = 2696$ N/mm, thickness of internal face is $t_{F2} = 0.525$ mm, distance between screws is $c = 1000$ mm and diameter of a screw is $d = 5.5$ mm.*

The second case is to study the effect of the shear resistance of connections. Shear resistance is mostly due to the thickness of internal face of sandwich panels and diameter of the screws. Thickness has a greater effect on the shear resistance than diameter, so only the thickness is varied. Results are presented for simply supported member in Figure 31 and for cantilever in Figure 32.

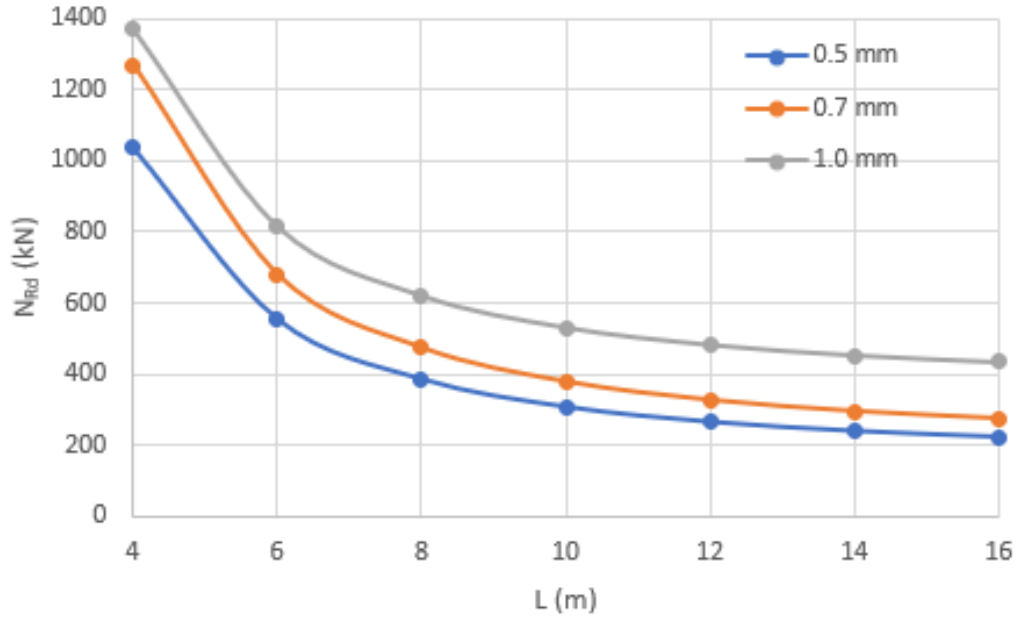**Figure 31.** *Maximum axial load for simply supported member with restraint from sandwich panels. Cross-section is CFRHS 200x200x10, distance between screws is $c = 1000$ mm and diameter of a screw is $d = 5.5$ mm.*

Figure 32. *Maximum axial load for cantilever column with restraint from sandwich panels. Cross-section is CFRHS 200x200x10, distance between screws is $c = 1000$ mm and diameter of a screw is $d = 5.5$ mm.*
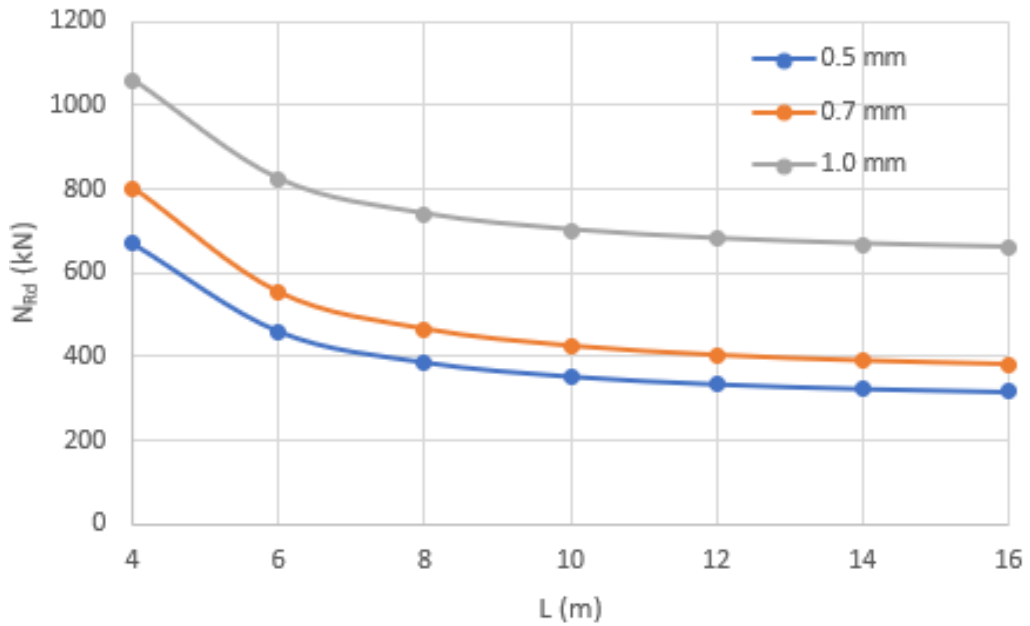
Figures 31 and 32 show that the increase in bearing resistance of connectors increases the axial force resistance of restrained members directly. Doubling the thickness of internal face of sandwich panels can even double the axial force resistance of member so the effect is significant.

The third case is to study the effect of the stiffness of connections. The most effective way to increase effective stiffness of connections is to add more connector pairs. In this study, the effect of effective stiffness is shown with three cases:

- In the first case, each sandwich panel is connected with a screw pair with distance between screws $c_1 = 1000$ mm.

- In the second case, each sandwich panel is connected with two screw pairs with distances between screws $c_1 = 1000$ mm and $c_2 = 800$ mm.

- In the third case, each sandwich panel is connected with three screw pairs with distances between screws $c_1 = 1000$ mm, $c_2 = 800$ mm and $c_3 = 600$ mm.

Distances c_1, c_2 and c_3 are shown in Figure 33. Results are presented for simply supported member in Figure 34 and for cantilever in Figure 35.
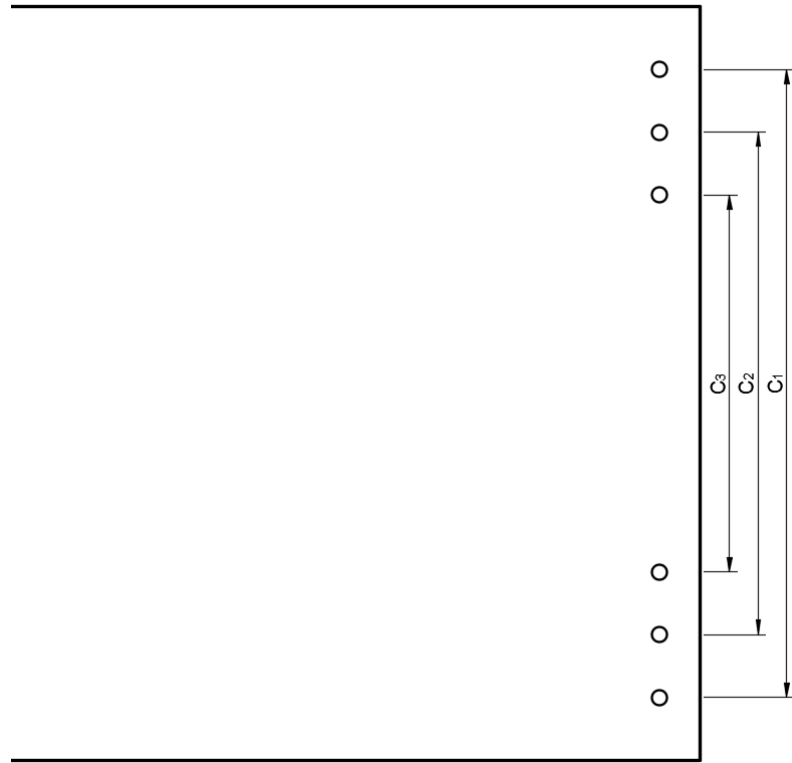

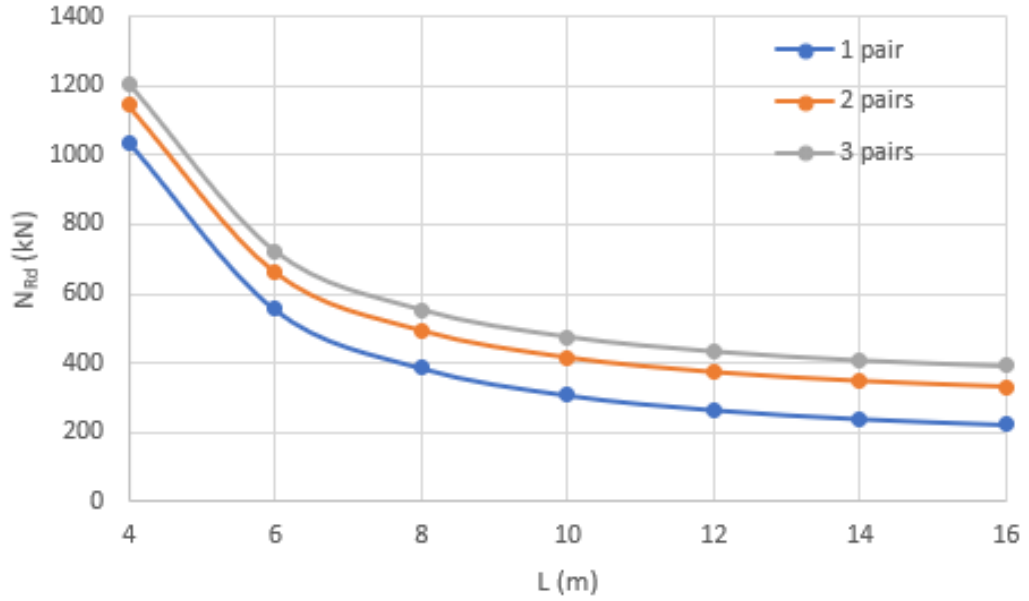*Figure 33.* *Distances between screws.*

Figure 34. *Maximum axial load for simply supported member with restraint from sandwich panels. Cross-section is CFRHS 200x200x10, the stiffness of a connector is $k_v = 2696$ N/mm, thickness of internal face is $t_{F2} = 0.525$ mm and diameter of a screw is $d = 5.5$ mm. Distances between screws are $c_1 = 1000$ mm, $c_2 = 800$ mm and $c_3 = 600$ mm.*
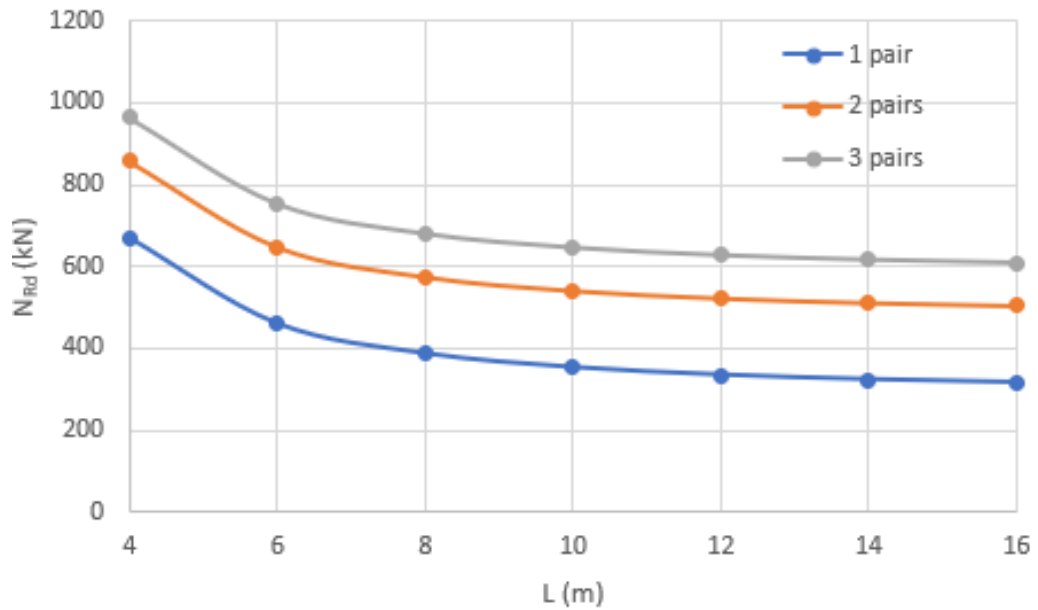
Figure 35. *Maximum axial load for cantilever column with restraint from sandwich panels. Cross-section is CFRHS 200x200x10, the stiffness of a connector is $k_v = 2696$ N/mm, thickness of internal face is $t_{F2} = 0.525$ mm and diameter of a screw is $d = 5.5$ mm. Distances between screws are $c_1 = 1000$ mm, $c_2 = 800$ mm and $c_3 = 600$ mm.*

It can be seen that the axial force resistance of restrained members can also be increased by adding the number of screw pairs. This can be the easiest way to

increase the axial force resistance but the efficiency of that decreases when screw pairs come closer to the center of the panel. The effect of increasing the number of screw pairs from one to three seems to be nearly the same than increasing the thickness of internal face of sandwich panels from 0.5 mm to 1.0 mm.

Sandwich panels can be used to increase the axial force resistance of slender members and the benefit can be significant. For example, 12 meters long CFRHS 200x200x10 with three screw pairs (distances between screws are $c_1 = 1000$ mm, $c_2 = 800$ mm and $c_3 = 600$ mm), thickness of internal face $t_{F2} = 1.0$ mm stiffness of a connector is $k_v = 5172$ N/mm, and diameter of a screw is $d = 5.5$ mm.

For simply supported member $\Rightarrow N_{Rd} = 580$ kN without sandwich panels and $N_{Rd} = 849$ kN with restraint from sandwich panels. The benefit from sandwich panels is 269 kN.

For cantilever $\Rightarrow N_{Rd} = 153$ kN without sandwich panels and $N_{Rd} = 1313$ kN with restraint from sandwich panels. The benefit from sandwich panels is 1160 kN.

6 SUMMARY AND CONCLUSIONS

The main aims of the thesis were to examine if trapezoidal sheeting and metal faced sandwich panels can have benefits to the axial force resistance of members and how different parameters affect that. The thesis showed that both structures, trapezoidal sheeting and metal faced sandwich panels, have a great potential to support slender members against lateral elastic buckling. These structures behave differently and trapezoidal sheeting can be thought to behave like a continuous spring support but sandwich panels cause force pairs to the member and the stiffening effect is due to their moment.

With the trapezoidal sheeting, the axial force resistance is smaller for short members than longer because of the connector forces in all cases. For short members, smaller axial force cause connector force to be over bearing resistance but when the length of member increases, eventually buckling becomes critical and the graph becomes flat. The part of the graphs that depends on connector force is linear. By increasing the bearing resistance or stiffness of connectors, the axial force resistance of short members can be increased even more but these parameters did not seem to affect the resistance of slender members significantly. Trapezoidal sheeting can increase the maximum axial force of members clearly.

With the sandwich panels the axial force resistance drops when slenderness of member increases until it reaches constant value caused by sandwich panels. The buckling resistance of members restrained with sandwich panels increases a lot but the real axial force resistance is limited strongly by bearing resistance of connections and can be increased by increasing the bearing resistance or number of connectors. The effect from sandwich panels are bigger when the boundary conditions are looser because panels add supporting force to the member that only depends on stiffness of connector, width of panel and distances between connectors. That supporting force is relatively bigger when buckling load of unrestrained member is small.

One goal was to find out the possibilities of stressed skin for stabilization in the ambient temperature to give comparison results for later research. Trapezoidal sheeting and sandwich panels can be used to transfer lateral forces so stabilization of the entire building using these structures might be possible but further studies are needed. The theories presented in the thesis can be used to design structures in practice, when studying the stabilization of the entire building and these might also be helpful in fire situation.

The main concerns are how to determine the stiffnesses of the connections with

sufficient reliability, mostly in the case of trapezoidal sheeting or in the fire situation, and how to improve the bearing strength of sandwich panel connections. Also, initial deformation has a remarkable impact to connector forces of sandwich panels so better results can be obtained if that value can be measured more accurately.

Overall, this thesis provides a good base for further research. There is still a lot of work left until these solutions become more common and easy to use.

REFERENCES

Davies, J. M. & Bryan, E. R. (1982). Manual of stressed skin diaphragm design. Granada. 441 pp.

Dlubal Software GmbH (2016). Program RFEM 5. Program description. URL: `https : / / www . dlubal . com / en / downloads - and - information / documents / manuals` (visited on 05/11/2018).

Dubina, D., Ungureanu, V. & Landolfo, R. (2012). Design of cold-formed steel structures. Vol. 1. ECCS - European convention for constructional steelwork. 632 pp.

EASIE (2011). Ensuring Advancement in Sandwich Construction through Innovation and Exploitation. Results avaible (accessed on 05/07/2018): `http : / / www . enveloppe-metallique.fr/wordpress/wp-content/uploads/2015/11/EASIE-research_results.pdf`. URL: `https://cordis.europa.eu/project/rcn/89643_en.htm` (visited on 05/07/2018).

ECCS (2013). European Recommendations on the Stabilisation of Steel Structures by Sandwich Panels. 388 pp.

Hedman-Pétursson, E. (2001). Column buckling with restraint from sandwich wall elements. Doctoral thesis. Luleå, Sweden: Luleå tekniska universitet. 179 pp.

Höglund, T. (2002). Stabilization by stressed skin diaphragm action. Stålbyggnadsinstitutet. 115 pp.

SFS-EN 1993-1-1 (2005). Eurocode 3: Design of steel structures. Part 1-1: General rules and rules for buildings. European Committee for Standardization. 91 pp.

SFS-EN 1993-1-3 (2006). Eurocode 3: Design of steel structures. Part 1-3: General rules. Supplementary rules for cold-formed members and sheeting. European Committee for Standardization. 130 pp.

StBK-N5 (1979). StBK-N5: Tunnplåtsnormen (Swedish Code for Light-Gauge Metal Structures). Stålbyggnadsinstitutet.

Timoshenko, S. & Gere, J. (1961). Theory of Elastic Stability. 2nd Edition. McGraw-Hill. 560 pp.

Winkler, E. (1867). Die Lehre von der Elasticitaet und Festigkeit: mit besonderer Rücksicht auf ihre Anwendung in der Technik für polytechnische Schulen, Bauakademien, Ingenieue, Maschinenbauer, Architecten, etc. Vol. 1. Dominicus. 388 pp.