TAMPERE UNIVERSITY OF TECHNOLOGY

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JUMP DETECTION IN STANDARD & POOR'S 500 -INDEX USING MODEL-FREE IMPLIED VOLATILITY

Master of Science Thesis

Examiner: Juho Kanniainen
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ABSTRACT

TAMPERE UNIVERSITY OF TECHNOLOGY
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Jumps are large and fast price movements in asset prices, which cannot be explained by traditional Brownian motion in models for stock price dynamics. In equity prices, jumps are often caused for example by significant macroeconomic or company-specific announcements. Recent financial literature has immensely studied jumps and methodologies to detect them, especially in high-frequency data. An important aspect in determining whether the price has jumped or not is the market spot volatility at the moment of the large price movement. Since spot volatility is not directly observable, multiple ways in estimating it has been suggested in literature. Existing jump detection methodologies often use historical realized variation as a proxy for spot volatility.

This thesis studies jumps in S&P 500 index using minute-by-minute high-frequency data. Using this data, VIX index and its corridor implied equivalent, CX index are computed from observable option prices. Jump detection test on S&P 500 price data is then run using both realized bipower variance and both implied volatility measures as a spot volatility estimators. Detailed analysis is made on the detected jumps yielded from both methodologies. The object is to identify, how the characteristics of detected jumps differ when using different volatility measures in jump detection.

The results suggest that implied spot volatility measure is often lower than realized bipower variance, which results in total number of detected jumps being significantly higher using implied spot volatility measures. However, the implied spot volatility is more robust spot volatility measure especially when there are jumps or large shifts present in the volatility time series. Realized bipower variance often results in false detections of price jumps when upward volatility jumps occur. Similar behaviour is not visible when using implied spot volatility estimators. Implied volatility appears more robust especially during first minutes of the trading day.

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Hypyt ovat suuria ja nopeita liikkeitä omaisuuserien hinnoissa, joita ei voida selittää Brownin liikkeeseen perustuvilla hinnoittelumenetelmillä. Osakkeiden hinnoissa hyppyjä aiheutuu usein esimerkiksi merkittävien makrotaloudellisten tai yrityskohtaisten uutisten seurauksena. Viimeaikaisessa kirjallisuudessa on tutkittu paljon hyppyjä ja menetelmiä niiden tunnistamiseksi, erityisesti suuritaajuisessa datassa. Tärkeä muuttuja hyppyjen tunnistuksessa on markkinan spot-volatiliteetti suuren hintaliikkeen aikana. Koska spot-volatiliteettia ei voida suoraan havainnoida markkinoilta, on kirjallisuudessa esitetty useita menetelmiä sen estimoimiseksi. Nykyiset hyppyjentunnistusmenetelmät käyttävät usein historiallista realisoitunutta volatiliteettia estimoimaan spot-volatiliteettia.

Tämä diplomityö tutkii hyppyjä S&P 500-indeksin suuritaajuisesta datasta. Datan optiohintojen perusteella lasketaan VIX-indeksi ja vastaava käytävävolatiliteetti-indeksi (CX-indeksi). S&P 500 hintadatasta tunnistetaan hyppyjä sekä historialliseen volatiliteettiin perustuvalla menetelmällä että molempiin implisiittisiin volatiliteetteihin perustuvilla menetelmillä. Näillä kolmella eri menetelmällä tunnistettuja hyppyjä analysoidaan yksityiskohtaisesti. Työn tarkoituksena on tunnistaa, miten eri menetelmillä tunnistettujen hyppyjen ominaisuudet eroavat toisistaan.

Tuloksista havaittiin, että implisiittinen spot-volatiliteetti on usein matalampi kuin realisoitunut volatiliteetti, mistä syystä implisiittistä volatiliteettia käyttäen tunnistetaan kokonaisuudessaan huomattavasti enemmän hyppyjä. Implisiittinen volatiliteetti on vakaampi menetelmä hyppyjen tunnistuksessa etenkin kun volatiliteetissa havaitaan hyppyjä tai suuria liikkeitä. Tutkimuksessa havaittiin myös, että realisoituneen volatiliteetin avulla tunnistetaan virheellisesti hyppyjä ylöspäin suuntautuvien volatiliteettihyppyjen yhteydessä. Vastaavaa ei havaita hyppytunnistuksessa implisiittisen volatiliteetin avulla. Implisiittinen volatiliteetti vaikuttaa käyttökelpoisemmalta menetelmältä etenkin kaupankäynnin ensimmäisten minuuttien aikana.

PREFACE

Preparing this thesis has been at the same time the most challenging, rewarding, and educational part of my studies. Choosing this subject was never going to be the easiest route, but in hindsight the lessons learnt were definitely worth the effort. I would like to express sincerest gratitude to my supervisor Juho Kanniainen for providing valuable advise and feedback during the process. Without the help from Juho this thesis would most likely be missing most of the insights.

Finishing this thesis was the final stretch in my university studies. Thanks to all my friends for all the great memories and experiences during the years. Very special thank you goes to my parents for constant support during the studies. Last but not least, I want to thank Lenita for the support in pushing me to graduate (and keeping the home clean).

Mikko Virtanen

Helsinki, March 5, 2018

CONTENTS

LIST OF SYMBOLS AND TERMS

$MFIV$	Model-free implied volatility
$CBOE$	Chicago Board Options Exchange
VIX	Volatility index published by the CBOE
CX	Corridor-implied volatility index
RV	Realized volatility
BPV	Realized bipower variance
$S\&P500$	Standard & Poor's 500-index

1. INTRODUCTION

Asset prices have traditionally been modelled with stochastic processes. The most common stochastic process used in asset price modelling is geometric Brownian motion. The asset-specific variables needed for modelling an assets price development with geometric Brownian motion are the expected rate of return and volatility. However, this simplified approach in asset price modelling requires multiple assumptions, which have nowadays been proven false. Among others, the model assumes constant expected rate of return and volatility. Most importantly in this thesis point of view however, the model assumes that asset returns should follow lognormal distribution.

Indeed, empirical research has shown that often stock returns do not follow lognormal distribution as assumed. More precisely, compared to normal distribution, the return distribution has heavy tails and is skewed to the left (Kou 2002). These heavy tails can be observed in the markets as abnormally large movements in asset prices, commonly referred to as jumps. Usually these movements are associated with unexpected information flow to the market. Depending on the asset class, the jump can be caused by different information flow. For example, an announcement of merger, surprising earnings announcement or other firm-specific information could be causing jumps in an individual equity. On the other hand, stock index jumps are more commonly associated with macroeconomic news such as FOMC meetings or natural disasters. (Lahaye et al. 2011) The presence of jumps in asset price paths is the foundation for this thesis.

Jumps can be formally defined as unexpected, abnormally large price movements in asset price over a very short period of time. As such, asset price process with jumps cannot be modelled with geometric Brownian motion. If large jumps exist, the ordinary diffusion model cannot explain the price path. An alternative approach was first suggested by (1976). Merton notes, that Black-Scholes model does not hold when the behavior of the underlying asset cannot be represented by a stochastic process with a continuous sample path. Hence, Merton proposes an alternative approach to model stock price evolution to accurately price options, so called Jump-diffusion process. The suggested method is a combination of the classical geometric Brownian motion and a Poisson-process modelling the jump component. After Merton, many different approaches have been suggested to model an asset price process including discontinuous jump part. For example Bates (1996) studies deutsche mark options and proposes a stochastic volatility model with jump component to model FX-process.

Jumps have been a major point of interest in financial research and they have empirically been proven a fact. Naturally, it is also interesting to analytically model jump occurrence. Observable volatility is the most important parameter when investigating whether a jump has occurred or not. When volatility is high, more radical price movements can be explained by the diffusion process and vice versa. The fundamental idea in jump detection statistic introduced by Lee and Mykland (2008) is whether or not the resulted asset price can be a product of the log-normal distribution. The price movement is considered as a jump if it is too large to credibly be a result of the diffusion process. The estimator for spot volatility in jump detection methods described in the literature are invariably based on historical realized volatility. This can be problematic in a multitude of ways. For example, previously occurred jumps can considerably increase historical volatility. Additionally, jumps in the volatility process are not captured in the historical volatility instantly. Both of these problems have large impact on existing jump detection methods. The problems with historical volatility can potentially be resolved using instead implied volatility in the jump detection tests. Implied volatility should capture market's expectations of future volatility and should, in theory, be superior to historical volatility in jump detection.

This thesis focuses on jump detection in S&P 500 index. On way of measuring S&P 500 volatility is VIX index, which is calculated via observable option prices. VIX is a measure of model-free implied volatility, which refers to the fact that the volatility measure is not dependent on any particular option pricing model. Thus, model-free implied volatility is deemed as a robust way of measuring implied volatility. Compared to realized volatility, implied volatility is a forward looking measure and often deemed as the markets expectation of future realized volatility. Now the interesting question becomes whether or not it is relevant to use implied volatility measure to conduct jump detection. Intuitively, jump or quick large increase in market volatility might lead to false detection of jump using historical volatility based methods. This is due to the fact that larger market volatility allows larger price movements not to be interpreted as jumps. Historical volatility based estimator does not contain the new information about quick market volatility shifts, and could therefore lead to false detections. However, implied volatility based spot volatility estimator should contain all new market information.

This thesis studies jumps in S&P 500 -index over the period of 2006-2010. High-frequency, minute-by-minute data is used to detect jumps using similar method introduced by Lee and Mykland (2008). The jump detection is then repeated using computed data on two different model-free implied volatility measures. The detected jumps using realized volatility is compared to detected jumps using implied volatility. This thesis aims to address, whether or not there are major differences in jump intensity, timely occurence or other characteristic factors of the two jump distributions. Main research problem is defined as: *How does the characteristics of historical volatility detected jumps differ from model-free implied volatility detected jumps?* This research problem is addressed by comparing the results of jump detection using both volatility measures described earlier. The characteristics of detected jumps are thoroughly analyzed for each different jump detection method. Most importantly, the behaviour of different spot volatility estimators around detected jumps is studied. The target of this thesis is to analyze, whether implied volatility based jump detection methods could be more robust and valid in different market conditions compared to historical volatility based methods. The differences between these methods is analyzed based on dividing into jumps detected by one method but not the other.

The analysis is based on S&P 500 -index data from 2006-2010. The data includes prices for market quoted option surface, both puts and calls and different maturities. The data is sufficient for computing VIX-index and corresponding corridor implied volatility index for each minute of the trading day. Total of six trading days spanning across 2006-2010 are missing from the data due to interest rates being unavailable and for a small range of minutes computing VIX is not possible due to too few option quotes being available in the data for an accurate computation.

The structure of the thesis is as follows: Chapter 2 discusses different approaches to modelling asset price process, jumps and jump detection. Chapter 3 examines the concept of volatility and more precisely implied volatility. Chapter 4 describes VIX index and the methodology behind calculating it. Chapter 5 describes the empirical data and methodologies used in analyzing it. Chapter 6 discusses the results of the study and chapter 7 concludes analyzing the implications of the results and possible areas of further research.

2. VOLATILITY

One of the key variables in providing a price for a derivative is the volatility of the underlying asset (see for example formulae 2.1 and 2.2). Ever since the introduction of Black-Scholes model, implied volatility has been a major topic in financial literature. Volatility can be measured in different ways, and thus it is important to point out the fundamental difference between realized volatility and implied volatility. This chapter examines fundamental definitions and studies most important literature regarding volatility.

2.1 Continuous asset price process

In order to understand the importance of volatility to asset prices, random processes simulating asset prices are first described. An important tool in modelling financial variables is stochastic calculus, which is essential to understand in modelling random processes. Wiener process is one type of stochastic process, where the mean of change is zero and variance 1 per year. Wiener process for variable W can be formally defined following Hull (2006) with following two properties:

1. The change ΔW_t during a small time Δt is $\Delta W_t = \epsilon_t \sqrt{\Delta t}$, where ϵ is a standardized normal variable.
2. The values of ΔW_t for any two different time intervals Δt are independent

From the first property, it is obvious that ΔW_t has a normal distribution with $E(\Delta W_t) = 0$ and $Var(\Delta W_t) = \Delta t$. If the change in the value of W_t is taken over a longer time period T, Δt can be denoted by $W_t - W_0$. If time interval T is divided into N smaller time intervals of length Δt, we can denote:

$$W_T - W_0 = \sum_{i=1}^{N} \epsilon_i \sqrt{\Delta t}.$$

The second property of Wiener process states that the values of ϵ_i are independent. From the properties of normal distribution (Durrett 2010, p. 44) we recall that the sum of independent normally distributed random variables is also normally distributed with mean being the sum of means and variance being the sum of variances. Hence, $W_t - W_0$ is normally distributed with mean 0 and variance T, which is important property when considering simulating asset price paths. Wiener process alone is not enough to model asset price process since the mean change per time unit, *drift rate*, is constant zero and variance per time unit, *variance rate*, is constant 1. Extension of the basic Wiener process is the generalized Wiener process, which can be defined for variable z as (Hull 2006)

$$dz_t = adt + bdW_t,$$

where a is the drift rate, b^2 is the variance rate, and dW_t is the Wiener process. It is worth noting, that in generalized Wiener process drift rate and variance rate are still constant. In stock markets, this is a false assumption, because normally the return investors require is a percentage of the stock price. Applying Itô's lemma allows a and b to be functions of z and t, which solves the problem with constant drift rate. This from is referred to geometric Brownian motion (Hull 2006):

$$\frac{dS_t}{S_t} = \mu dt + \sigma_t dW_t, \tag{2.1}$$

where S_t is the price of the underlying asset at time t, μ is the expected rate of return, σ_t is the volatility of the asset price at time t, and dW_t is the Wiener process. As shown, the price process can be divided into two separate parts. The diffusion part is deterministic, and is only dependent on the expected price appreciation. It is often referred to as the drift term. The second term on the other hand is stochastic, a random process. The degree of randomness is decided by the volatility, which is assumed constant in the classic models. Equation 2.1 is still commonly used to model stock price diffusion in basic level, but it fails to consider a couple of empirical facts. Commonly volatility is assumed to be constant in time, which has multiple times been proven inaccurate assumption empirically. In equation 2.1 this is taken into account by denoting volatility as a process in time. More importantly in this thesis' point of view, the model does not include possible discontinuities in the diffusion model, so called *jumps*.

Cox and Ross (1976) were first to suggest alternative stochastic processes to model asset prices. They argue, that a good alternative could be continuous time jump process. In contrast to diffusion process, a jump process follows discontinuous sample paths. In short, a jump process assumes that there are two possible future states for the asset price: the same as the previous state or jump a certain percentage. In contrast to the diffusion process, the jump process is modelled with a continuous time Poisson process. The Poisson process specifies the intensity of the jumps. Cox and Ross (1976) provide multiple alternative processes with different assumptions and approaches. However, their approaches are also lacking the possibility of large, rare discontinuities.

Merton (1976) combines these two approaches to a hybrid model. Merton points out, that even though many of the original Black-Scholes assumptions can be relaxed and the model still holds, the assumption of continuous price path invalidates the solution. As a more accurate solution, Merton suggests a jump-diffusion process. The underlying assumption is, that stock price is composed of two different types of changes. First, the continuous part from log-normal diffusion modelled by Brownian motion and second, the discontinuous jump part modelled by a Poisson-driven process. Defining the price process like this allows for the proven discontinuities observable in the market. Merton writes the process for asset price S as

$$\frac{dS_t}{S_t} = (\mu - \lambda k)dt + \sigma_t dW_t + dq_t, \tag{2.2}$$

where σ_t is the volatility at time t conditional on no Poisson event occurring. Merton's (1976) model originally assumed volatility to be constant in time, which can be relaxed using a stochastic process for volatility. This is pointed out in the formula by denoting volatility σ_t as a process in time. Additionally, dW is Wiener process, μ is the instantaneous expected return and $q(t)$ is the Poisson process describing jump intensity. λ is the average number of jumps per time unit. In addition, $k \equiv \epsilon(Y-1)$, where $(Y-1)$ is the percentage change in S in case of a Poisson event and ϵ the expectation operator over Y. Merton (1976) notes, that equation 2.2 is in line with the observed discrepancies in option market prices, for example deep-in-the-money or deep-out-the-money options quotes being seemingly too high. Another explanation to the same observation would later take the form of volatility smile.

As noted, the assumption of constant volatility is empirically proven false a multitude of times. Heston's (1993) stochastic volatility model is an extension to modelling asset prices with geometric Brownian motion and constant volatility. In Heston's model, volatility is modelled as a mean-reverting stochastic process called Ornstein-Uhlenbeck process. Heston's model can be written as:

$$\frac{dS_t}{S_t} = \mu dt + \sigma_t dW_{1t} \tag{2.3}$$

$$d\sigma_t^2 = \kappa(\theta - \sigma_t^2)dt + \gamma\sigma_t dW_{2t}, \tag{2.4}$$

where μ is the drift term, σ_t is volatility at time t, κ is the mean reversion speed of variance and θ is the long-term mean of volatility. W_1 and W_2 are two Brownian motions correlated with coefficient ρ. Empirical study proves, that most commonly the correlation ρ is highly negative when studying stock prices. Stochastic volatility models are seen more realistic than constant volatility models. However, Heston's model also lacks the jump component and cannot explain many of the price movements visible in empirical data.

Bates (1996) proposed an extension to Heston's model incorporating also the possibility of jumps in the asset price process. Bates' model can be written as

$$\frac{dS_t}{S_t} = \mu dt + \sigma_t dW_{1t} + Y_t dJ_t, \tag{2.5}$$

where J_t is a standard Poisson counting process with intensity λ and Y_t is the Poisson jump amplitude conditional on a jump occurring. Bates also defines the distribution of jumps as follows:

$$ln(1 + Y_t) \sim N(ln(1 + \overline{Y_t}) - \frac{1}{2}\alpha^2, \alpha^2).$$

The volatility process is definded similarly as in 2.4. Simply put, Bates model assumes jumps independent from the Wiener processes in the asset price process. Multiple similar models incorporating discontinuous elements and/or stochastic volatility into asset price paths have subsequently been developed. For example Hull and White (1987) find patterns causing mispricing in Black-Scholes model and propose a stochastic volatility model for option pricing. In addition to stochastic volatility models trying to include the smile features in the pricing, there has been substantial research on how to incorporate leptokurtosis in the models (see e.g. (Barndorff-Nielsen and Shephard 2001)). Leptokurtosis refers to the observed asymmetry of the return distribution, i.e. fat tails and high peaks. Different asset pricing models and their extensions would yield material for whole another thesis, but discussing them further will fall out of scope in regard to this study.

2.2 Realized volatility

Volatility refers to the standard deviation of a financial asset. It can be intepreted as the uncertainty or risk associated with the stock returns. Volatility reflects how much the asset price is expected to vary in certain timeframe. (Joshi 2003) It is commonly reported on annualized levels, but depending on the purpose even minute-level volatilities are sometimes significant. Additionally, it is commonly assumed that the mean of returns is zero (Hull 2006). Lahaye et al. (2011) define quadratic variation in continuous time jump-diffusion process as

$$QV_t = \int_0^t \sigma_t^2 + \sum_{0<s\leq t} \kappa_t^2, \tag{2.6}$$

where σ denotes volatility of the asset and κ denotes the size of a jump. In other words, the quadratic variation is the sum of integrated variance plus the sum of squared jumps.

Obviously, in real-world only discrete observations are available. The empirical way to approximate quadratic variation is realized variance, which is calculated using a historical time series of log-prices of the underlying asset. Literature often refers to realized variance as realized volatility. In order to avoid misconseptions in the future, in this thesis realized volatility is referred to as the square root of realized variance. Denoting a series of log-returns as r_i, the realized variance based on n daily returns is given by (Barndorff-Nielsen and Shephard 2004)

$$RV = \sum_{i=1}^{n} r_i^2.$$

By definition, realized volatility is square root of realized variance. An important remark in terms of this thesis is that realized quadratic variation is inconsistent in the presence of jumps in the asset price process (Lee and Mykland 2008). Barndorff-Nielsen and Shephard (2004) introduce realized bipower variation, which is a more robust volatility measure in precense of rare jumps. Realized bipower variation consistently estimates the integrated variance as stated in the first term of equation 2.6. Realized bipower variation is defined as

$$BPV = \sum_{i=2}^{n} r_{i-1} r_i. \tag{2.7}$$

Realized bipower variation is important consept in this thesis due to its consistency no matter how large jumps the asset price path includes. Jumps will be discussed more in chapter 4.

2.3 Implied volatility

The consept of implied volatility is closely interlinked with Black-Scholes model. Implied volatility refers to the markets expectations of the future volatility, and is commonly seen as the volatility to apply in Black-Scholes formula to acquire the option price observable in the market. The Black-Scholes model can be presented by the following partial differential equation following Hull (2006):

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC, \tag{2.8}$$

where C denotes the price of an European call option, t denotes time, r denotes risk-free rate, S denotes asset price, and σ denotes the volatility of the asset. Analytical solution to Black-Scholes option pricing model is

$$C = SN(d_1) - Ke^{-r\tau}N(d_2), \tag{2.9}$$

where

$$d_1 = \frac{ln\frac{S}{K} + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}},$$

$$d_2 = \frac{ln\frac{S}{K} + (r - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} = d_1 - \sigma\sqrt{\tau},$$

where τ denotes time to maturity and $N(d)$ is the cumulative probability distribution function for a standardized normal distribution[1].

Implied volatility can be obtained from option prices quoted in the market. By applying Black-Scholes formula, one can find the implied volatility of the option in question. Cont and da Fonseca (2002) define Black-Scholes implied volatility σ_t^{BS}

[1] Complex detailed derivation is omitted since it is not relevant for the purpose of this thesis. More information on BS-model and formula can be found from multiple books and articles, for example Black and Scholes (1973) and Hull (2006).

as the volatility parameter, which equates to market price of the option in question:

$$C_{BS}(S_t, K, \tau, \sigma_t^{BS}(K,T)) = C_t^*(K,T), \tag{2.10}$$

where S_t denotes the spot price of the asset at time t, K denotes the strike price, τ denotes the time to maturity and T denotes the maturity date. $C_{BS}(S_t, K, \tau, \sigma_t^{BS}(K,T)$ is the Black-Scholes price for the option and $C_t^*(K,T)$ is the market observed price for the option.

Black-Scholes model is based on assumption of constant volatility over time (Black and Scholes 1973). Various empirical studies have proven this assumption false. For example, Rubinstein (1994) found that after the stock market crash in 1987, S&P 500 index option implied volatilities started depending more heavily on strike price. The way implied volatility behaves as a function of the strike price is referred to as volatility smile. The shape of the smile has later been a concern of many academic studies. For example, Canina and Figlewski (1993) find that implied volatility often forms a parabolic curve where the minimum is found near- or at-the-money. As equation 2.10 shows, the implied volatility σ_t^{BS} is dependent on the strike price K, but also time to maturity T. Cont and da Fonseca (2002) study the dynamics of implied volatility surfaces, which is a three-dimensional plot of implied volatility in respect to K and T. They find a downward sloping term structure in S&P 500 index, meaning that the closer the time-to-maturity is, the higher the implied volatility usually is. The shape of the volatility surface seems to indicate traders pricing jump risk in the option prices.

The fact that volatility is not constant with respect to strike price raises additional questions. If observable option prices indicate different implied volatilities for similar options (apart from strike price), what is the "correct" way of measuring market expectation of the future volatility? Research on the historical behaviour of implied volatilities in comparison to realized volatilities provides incoherent results to this question. Christensen and Prabhala (1998) find evidence from the S&P 100 index supporting the predictive power of ATM implied volatility. They use non overlapping low-frequency data, and find that implied volatility forecasts the future more efficiently than past volatility. Jorion (1995) conducted a similar study on FX-markets. Using data on deutsche mark, japanese yen, and swiss franc, he finds that option implied forecasts outperform historical data. Howerer, he also notes that the option implied volatilities tend to be upward biased compared to actual realized volatilities. On the other hand, Canina and Figlewski (1993) study the same index and find that implied volatility has little or no correlation with the future realized volatility. They conclude, that the informational content of implied volatilities include also other factors, such as liquidity and investor desire for specific payoff patterns. The net effect of these factors undermines the predictive power of implied volatilities. More recently the predictive power of different stochastic volatility models have been studied (see e.g. (Andersen and Bollerslev 1998)), but are only noted as they fall out of the scope of this thesis.

This far we have referred to implied volatility only the measure obtained from Black-Scholes formula. However, it is obvious that the implied volatility obtained from option prices is dependent on the pricing model being used to deduce it. Therefore it is obvious that the implied volatility measure aquired via an option pricing model is not unique. This has been a motivation to develop an alternative method to estimate implied volatility. Britten-Jones and Neuberger (2000) introduced their model-free implied volatility measure, which is not based on any specific option pricing model. The advantage of their suggested method is that it does not suffer from inconsistencies in contrary to model-based approaches. The model requires complete strike range of option prices for the given time horizon. Assuming we forecast volatility from current time 0 to a future time t, Britten-Jones and Neuberger (2000) define the following equation to estimate the return variance with a continuum of options with strike prices K expiring on t:

$$E_0^F\left[\int_0^t \left(\frac{dF_t}{F_t}\right)^2\right] = 2\int_0^\infty \frac{C^F(t,K) - max(F_0 - K, 0)}{K^2} dK. \tag{2.11}$$

Where $\int_0^t (\frac{dF_t}{F_t})^2$ is the quadratic variation between time 0 and time t, F_t is the forward asset price and C^F denotes the forward option price. Forward asset price is defined as:

$$F_0 = S_0 e^{rT},$$

where S_0 denotes the spot price of the asset, r denotes the risk-free rate and T denotes the time to delivery.

Square root of the right-hand side of equation 2.11 is by definition the model-free implied volatility. Jiang and Tian (2005) further studied the concept of MFIV and its informational content. They claim that MFIV is superior to Black-Scholes measure in multiple ways. First, independence of any particular option pricing model causes it to be consistent and unbiased estimate. Second, the information in MFIV is extracted from a continuum of option prices instead of a single option. This should intuitively make its informational content superior to B-S implied volatility. Third, conducting tests with MFIV do not include the potential misspecifications included in a specific pricing model. Thus, the tests can be deemed as direct tests of market efficiency. Jiang and Tian (2005) also conduct empirical tests, which support the informational superiority of MFIV. Their results confirm, that MFIV includes all the information contained in Black-Scholes implied volatility and is also more efficient in estimating future realized volatility.

Having the scope of this thesis in mind, it is worth noting that Britten-Jones and Neuberger (2000) built their method on the assumption of a continuous price process (i.e. no jumps) and underlying asset prices are viewed as forward prices. Jiang and Tian (2005) showed that equation 2.11 also holds when the price process contains jumps, but also identified issues in implementing the method to real world. One source of error is truncation. It refers to the availability of strike prices from 0 to ∞ as suggested in equation 2.11. In real world, far out-the-money options might not be quoted at all. Hence, the tails of the distrubution will be ignored when approximating MFIV. Another source of error is discretization. Since option prices are discrete, numerical integration needs to be used in calculating MFIV. The smaller the ΔK is, the smaller the discretization error. Jiang and Tian (2005) argue, that discretization has little impact on the calculation of MFIV. Third problem might arise from applications that require the use of spot prices. This problem can easily be addressed by converting the forward prices from equation 2.11 to spot prices by simple variable change. Finally, there exists only finite number of observable strike prices between the limited range. This causes additional discretization error with the numerical integration. Table 1 summarizes the error sources and suggested corrections.

Table 1: Sources of error in model-free implied volatility calculation (Jiang and Tian 2005)

Source of error	**Cause of error**	**Correction method**
Truncation	Strike range does not include tails	Extrapolation beyond available range
Discretization and limited availability of strikes	Only finite number of strikes are quoted	Volatility interpolation by curve fitting
Spot prices	Equation is expressed in forward terms	Variable change

The truncation in the tails and covered strike range varies differs considerably across trading days depending on the market quoted options. These issues along with the fact that far out-of-the-money options tend to be less liquid and contain more pricing errors induce random noise to MFIV measure. Andersen and Bondarenko (2007) provide exposition on the concept of corridor implied volatility (CIV), which aims to apply more coherent and reliable volatility measure in contrast to pure MFIV. The idea behind CIV is to focus only over a certaing region of the risk-neutral density (i.e. strike range), applying a consistent truncation to the tails of the distribution. Naturally, this technique does not represent the full scale of MFIV and CIV should be seen as a downscaled variant of MFIV (Andersen and Bondarenko 2007). They construct the CIV with following equation:

$$CIV_0(B_1, B_2) = \sqrt{2 \int_{B_1}^{B_2} \frac{M_0(K)}{K^2} dK}, \tag{2.12}$$

where

$$M_0(K) = min(P_0(K), C_0(K)),$$

the minimum of put and call at observation time, B_1 is the lower barrier and B_2 upper barrier. The strike range between these consistently defined barriers are deemed as the corridor. Andersen and Bondarenko (2007) were the first to discuss empirical CIV in the literature. CIV can roughly be interpreted to be somewhere between Black-Scholes implied volatility and MFIV. Important difference to MFIV is that the illiquid and potentially erroneously priced far OTM options are disregarded in the calculation. Thus, broad corridor CIV is a good proxy for MFIV, but can be modelled with a better precision. The market pricing of the variance risk is also strongly present in MFIV and broad corridor CIV measures. Andersen and Bondarenko (2007) note, that narrow corridor CIV seems to perform best in forecasting future volatility. All in all, CIV measure is a theoretically coherent manner of measuring implied volatility, and can arguably be seen as the best method of predicting future realized volatility.

The information content of implied volatility has been researched substantially. Especially its prediction power to future movements is of interest in this thesis. It is deemed to be the markets expectation of the volatility of the underlying asset, and thus it should provide good estimates of the future movements. However, the literature on the subject is controversial. Canina and Figlewski (1993) argue that Black-Scholes implied volatility predicts future movements poorly. On the other hand, for example Christensen and Prabhala (1998) use nonoverlapping data and longer time span than previous studies and find that Black-Scholes implied volatility has been a better predictor of future volatility especially after the 1987 crash. More recent studies show, that MFIV is a very efficient forecast for future volatility. Jiang and Tian (2005) find that MFIV contains all information in Black-Scholes implied volatility and realized volatility, and is thus superior forecast to future volatility. Andersen and Bondarenko (2007) point out that in addition to forecasting volatility, MFIV also includes pricing of the risk associated with volatility, being a biased estimate. They also argue, that best method of extracting volatility from markets might be corridor implied volatility. Baruník and Hlínková (2016) used wavelet regression methods and corridor implied volatility to estimate the prediction power. Their empirical results point out, that corridor implied volatility efficiently forecasts future realized volatility in the long term.

3. VIX AND MODEL-FREE IMPLIED VOLATILITY

3.1 Origins of VIX

The Chicago Board of Options Exchange (CBOE) introduced the CBOE Volatility Index VIX in 1993. The index was originally designed to measure the markets expectations of 30-day implied volatility of S&P 100 Index. At that time, options on the S&P 100 index were the most actively traded index options in the U.S. Originally, VIX represented the implied volatility of eight 30-day near-the-money index options on S&P 100 index. In 2003, CBOE revised the underlying index from S&P 100 to S&P 500. At the same time, the old VIX was renamed as the VXO, and CBOE continued to provide quotes on the old model as well. One motivation behind the index change was that options on S&P 500 had become the most actively traded index options in the U.S. This is a very relevant factor, since the liquidity of the options is also reflected in the prices. The more liquid options in S&P 500 provides also more accurate quotes, including less noise and mispricing. CBOE also updated the options used in VIX calculation to include only out-of-the-money options. This was motivated firstly due to the changes in trading activity. Secondly, out-of-the-money option prices contain information on the whole volatility surface making VIX more robust measure of overall implied volatility. Including more options also makes the index less sensitive to single option prices. As VIX should be presentation of the expected stock market volatility, using only ATM options would be theoretically dubious. As discussed previously, the volatility smile indicates that OTM options provide additional information on the market expectation of the volatility. (Whaley 2009)

During the years, VIX became a market standard in measuring the U.S. stock market volatility. It is often referred to as the fear gauge due to its strong negative correlation with realized S&P 500 returns. One of the reasons behind this is the strong demand from investors to insure their portfolio against potential drops in the stock market. The VIX is used as an insurance against potential drops in portfolio values. Whaley (2009) proves the negative correlation using a regression model. He finds, that on average VIX will fall by 2.99% for every 1% S&P 500 rises and vice versa, VIX will rise by 4.493% for every 1% the S&P 500 falls. The larger slope in market downturn also suggests that the demand for VIX could be higher especially during times of increased downside risk. After all, the demand for insurance is one of the drivers of the value of VIX. Figure 1 plots S&P 500 index values and computed VIX values, highlighting the negative relationship.

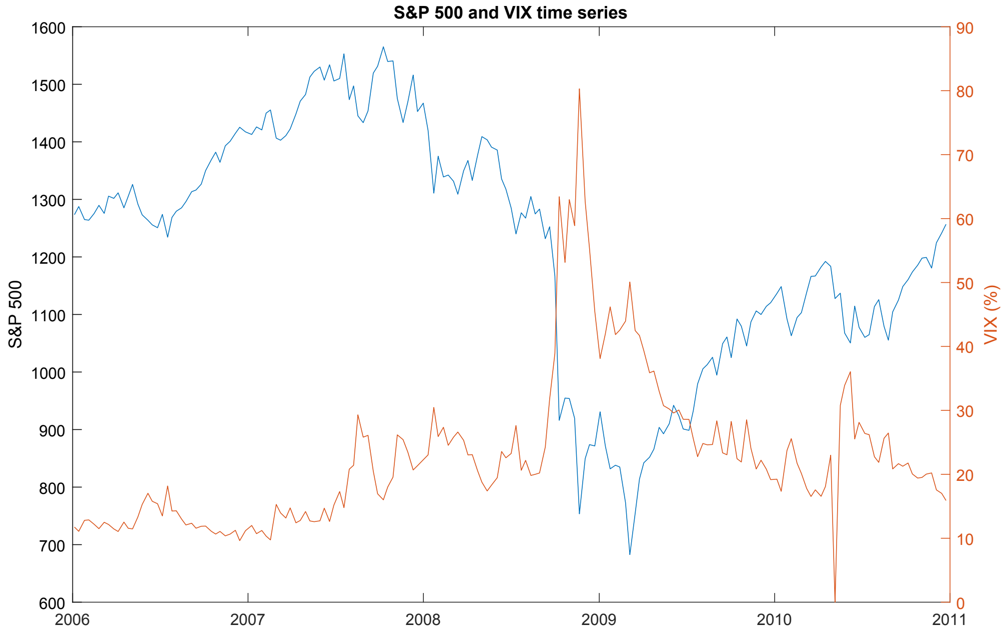

Figure 1: Time series of S&P 500 index and computed VIX-index. The negative correlation between index value and VIX is particularly visible in the late 2008.

As only eight near-the-money options are included in the calculation of the old VXO, it is more or less a measure of ATM implied volatility. As such, VXO is often deemed to represent the market expectation of the future realized volatility. Carr and Wu (2006) study the behaviour of the two indices and the motivation to the switch. They point out, that the VXO is essentially a good approximation of the volatility swap rate, but the economic relevance is otherwise unclear. Moreover, volatility swaps are difficult to replicate and therefore hedge. On the other hand, the new VIX index represents the price of a large portfolio of options. In contrast to the old VXO, the price of this portfolio is a good approximation of the variance swap rate. Variance swaps can easily be hedged with a portfolio of options and futures. Therefore, the derivative market on VIX is much more active and nowadays even exchange-traded products on VIX exist.

3.2 The VIX methodology

After the VIX index was revised in 2003, the calculation became more complicated. The revised calculation rules consider the prices of a portfolio of out-of-the-money calls and puts weighted inversely proportional to the squared strike price. The modern VIX index has therefore information on option prices over the whole volatility surface instead of only at-the-money options. The formula used by CBOE in the VIX calculation is:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2, \tag{3.1}$$

where σ is $\frac{VIX}{100}$, T is time to expiration, F is the implied forward index level, K_0 is the first strike below F, K_i is the strike price of $\mathrm{i^{th}}$ out-of-the-money option (call or put), ΔK_i the interval between strikes, R the risk-free rate and $Q(K_i)$ the mid-price for each option with strike K_i. The second term $\frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$ is included to adjust the discrepancy between K_0 and actual implied forward price F. (CBOE 2009, Andersen et al. 2015)

Essentially, VIX is measure of model-free implied volatility. The calculation rules follow roughly the MFIV presented in equation 2.11 with a few notable differences. Since a continuum of option prices can not be observed in the real world, a numerical integration needs to be applied. Therefore, equation 3.1 is presented as a sum rather than integration. Most of the times, there is not observable option prices with strike at implied forward F. This is problematic, because in theory the calculation should only include out-of-the-money options. CBOE addresses this issue by replacing the mid price $Q(K_i)$ with the average of call and put prices at K_0. Another issue is the availability of far OTM strike prices. To avoid using illiquid options with potential pricing errors, CBOE has decided to use a special cutoff rule. Whenever two consecutive strikes with zero bid-price are found, no options with lower strikes are included in the calculation, regardless of having non-zero bids. The implications of these decisions is discussed further in the following section. (CBOE 2009)

Since VIX is defined for a maturity of 30 calendar days and only a few option maturities are quoted at a given time, CBOE uses linear interpolation between two maturities to compute the VIX. The options considered eligible for the official calculation have more than 23 days and less than 37 days to expiration. The calculation includes so called near-term and next-term options, near term having less than or equal to 30 days to expiry and next term having more than 30 days to expiry. The interpolation is calculated as follows:

$$VIX = 100 * \sqrt{\left\{T_1\sigma_1\left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}}\right] + T_2\sigma_2\left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right]\right\} * \frac{N_{365}}{N_{30}}},$$

where

- N_{T_1} is number of minutes to settlement of the near-term options
- N_{T_2} is number of minutes to settlement of the next-term options
- N_{30} is number of minutes in 30 days
- N_{365} is number of minutes in 365-day year (CBOE 2009).

3.3 Problems and suggested improvements in VIX calculation

VIX is arguably the world's most famous volatility index. It is widely used measure of the US volatility and overall market sentiment. However, multiple problems are recognized surrounding VIX. According to academics, the calculation rules are especially problematic in some cases. Jiang and Tian (2007) even claim that CBOE's implementation is significantly flawed. The sources of these major errors arising from calculation rules are same as described earlier in section 2.3 with a couple of specific additions. Namely, the expansion error caused by the Taylor series expansion used in CBOE's correction term and the interpolation error arising from interpolation between two maturities are error sources specific to VIX. Nevertheless, truncation error and discretization error are recognized as the most meaningful error sources. As CBOE uses a rule on ignoring the tails of the option price distribution, the truncation error underestimates the volatility. On the other hand, the numerical integration method applied by CBOE overestimates the volatility. This is especially significant at strike price K_0, since the average of call and put price can be considerably different from fair value of an option with at-the-forward strike. Jiang and Tian (2007) conclude, that the net error is usually negative, leading to underestimation of the MFIV.

As a solution to the recognized issues in VIX calculation, Jiang and Tian (2007) suggest a smoothing method based on an interpolation-extrapolation scheme. The idea of the method is to build a smooth function that fits all the observed Black-Scholes implied volatilities at a given moment. The tails of the distribution, for which option prices are not available, are then extrapolated using observable implied volatilities. The implied volatility smile obtained using this function can thereafter be translated to option price at any required strike using Black-Scholes model. Jiang and Tian (2007) argue, that the smoothing method provides robust values and is significantly less prone to error than the CBOE method. They note, that the CBOE method underestimates VIX over 90% of the time. Due to the discussed bias, it is relevant to question whether or not current VIX should even be seen as a measure of MFIV. The applied truncation method suggests, that VIX is closer to corridor-implied volatility than a true measure of MFIV (Andersen and Bondarenko 2007).

Far OTM options tend to be unliquid and have high bid-ask spread. Therefore the mid-prices can include considerable pricing errors and be poor indicators of underlying implied volatility. This justifies the truncation rule used by CBOE, but also brings notable randomness and noise in VIX. Andersen's et. al (2015) approach aims to eliminate these artificial jumps by taking VIX towards CIV rather than MFIV. They provide detailed analysis on how this cutoff rule introduces unjustified jumps and discontinuities in VIX. They define the *effective range* for the purpose of analyzing how jumps in VIX are associated with widening (narrowing) of the strike range used. The conclusion is, that the discrepancies in strike range are significantly contributing to artificial jumps present in the measure. This is particularly present during times of market distress, VIX is more vulnerable to these artificial shifts. Being highly sensitive to option market liquidity, the index can be seriously downward biased during times of crisis when the market liquidity is low. Additionally, when the confidence and thus liquidity returns to the market after, VIX can become upward biased due to sudden widening of the effective range. Andersen et. al (2015) conclude, that during times of high volatility VIX is most likely to be largely biased. Unfortunately, those are also the times it is most needed.

As a fix to the problems mentioned above, Andersen et. al (2015) suggest a corridor fix, which takes VIX one step further from MFIV measure, towards corridor-implied volatility. This fix provides solution to the major problems arising from randomness in the effective range. The idea is to provide transparent and robust method to calculate effective range in order to mitigate the occurrence of artificial jumps compared to the official VIX methodology. In other words, they define a method which captures economically invariant portion of the strike range used in the index calculation. The definition of this range should include four features. As VIX is calculated real-time, the corridor should be defined with observable option prices. Secondly, as near ATM options have highest weight in index, the range should naturally center around the forward level. Third, the range should adjust as option prices and thus volatility in the market fluctuates. Finally, the calculation of this measure should be model independent. (Andersen et al. 2015) The trivial calculation rule for ratio statistic $R(K)$ is defined

$$R(K) = \frac{P(K)}{P(K) + C(K)},$$

where $P(K)$ is the mid-price of put with strike K and $C(K)$ the mid-price of call with strike K. Defining the ratio statistics as above provides some convenient implications. Because at-the-forward call and put prices are in theory identical, the forward price is easily extracted as the 50^{th} percentile of $R(K)$:

$$K_{0.50} = R^{-1}(0.50) = F.$$

In addition, $R(K)$ is Andersen et. al (2015) suggest, that 3% truncation for the tails is enough to capture a broad enough corridor, but at the same time ensuring the included option quotes are reliable and available over high-frequency intervals. Figure 2 presents how the strike range is selected using this method on an illustrative moment.

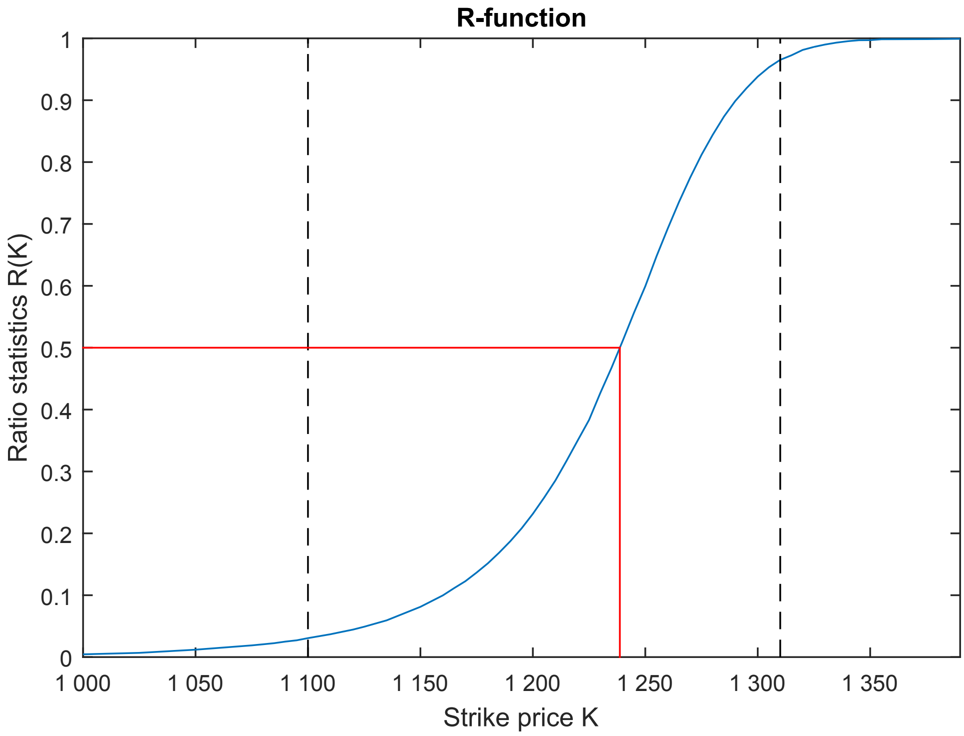

Figure 2: $R(K)$-function and truncation level on 13 June 2006 09:31:00. Dashed lines present the smallest and largest strike price included in the corridor (1100 and 1310, respectively). Red line presents the implied forward price: $R^{-1}(0.50) = 1238$

3.4 The dependence of VIX and realized volatility

The uncertainty about the return is often deemed as the measure of risk in investment decision. Return variance is the variable describing the uncertainty regarding returns. As stated before, it is widely accepted fact that return variance itself is also stochastic, and is therefore a major factor affecting the uncertainty in returns. Carr and Wu (2008) study, how the uncertainty regarding return variance is priced in the market and how investors manage the risk induced by it. Using a broad data sample on five indices and 35 individual stocks, they find that the variance risk premiums are clearly negative in S&P 500, S&P 100 and Dow Jones Industrial Average indices. In other words, investors are willing to pay to hedge against upward movements in market volatility. Their findings also include, that neither the capital asset pricing model nor other recognized risk factors can explain the size of the premium. Most of this premium must thus be generated by a separate variance risk factor. Todorov (2009) finds, that this risk premium is by are by a large extent explained by jump activity. Especially after price jumps, the variance risk premius are higher. This indicates that investors fear negative price jumps, and the risk premium is strongly associated with it. Carr and Wu (2008) define this variance risk premium as the difference between realized variance and variance swap rate,

$$RP_{t,T} = RV_{t,T} - SW_{t,T}.$$

As said previously, VIX can be interpreted as the variance swap rate. In the precense of negative variance risk premiums, it is therefore likely that VIX overestimates the future realized volatility. In addition to being a biased estimate of MFIV, VIX is also a biased forecast of future volatility. Carr and Wu (2008) run the following linear regression to analyze the relationship between variance swap rate and realized variance:

$$RV_{t,T} = a + bSW_{t,T} + e.$$

In case of zero variance risk premiums, the regression should yield values 0 and 1 for parameters a and b, respectively. As expected, the computed b is significantly lower than 1, indicating that the observed negative risk premiums are dependent on variance swap rate. When the regression is run on log-returns, the risk premium is very close to being constant. The relevant implication of their result in regard to this thesis is, that VIX cannot directly be used as a proxy for expectation of future realized volatility. Nevertheless, a regression model provides reasonable estimates.

In their earlier work, Carr and Wu (2006) ran similar regressions between realized volatility and VIX. According to their research, the average ratio $RV/VIX^2 = 0.6027$. This can be interpreted as one way of measuring the average risk premium. Additionally, they note that there is a strong negative correlation between market returns and return variance. This explains the negative market risk premiums in part, but it cannot fully explain the magnitude of the negative risk premiums, neither in VIX or the earlier VXO-index. Therefore, on average, shorting variance swaps on S&P 500 will yield positive returns.

4. JUMP DETECTION

Derivative pricing is essentially based on replicating portfolio. Replicating portfolio is a portfolio of assets, which provides the same payoff as a derivative. The fundamental idea is, that the cost for a derivative and its replicating portfolio should be same. Black and Scholes (1973) groundbreaking option pricing model is effectively based on replicating the payoff of a stock option with the underlying stock. In order to do this, the price process of the underlying stock needs to be modelled. Black and Scholes assumed that stock prices follow log-normal diffusion process.

The main point of interest in this thesis is to detect jumps in high-frequency data. In order to understand jumps, we need to understand how to model asset price behavior. In this chapter, the basic concepts and theories regarding asset price processes are discussed. We start from basic continuous stochastic processes and continue to discuss processes with jumps included. The chapter ends with studying the literature on detecting jumps in asset prices.

4.1 Jumps in financial assets

As discussed previously, financial literature has long agreed on the fact that asset price paths are not continuous, i.e. they contain discontinuities called jumps. The precense of jumps has important impact for example on asset pricing, portfolio management and derivative hedging. Therefore, financial literature has extensively studied these discontinuities in asset price paths ever since the work of Merton (1976). Bates (1996) finds that stochastic volatility models cannot explain the volatility smile observed from deutsche mark options, and argues that jump-diffusion model can explain an 8 percent appreciation in USD/deutsche mark exchange rate. However, Bates acknowledges that the timeseries with infrequent jumps lacked explanatory power, and notes that hypothesis of no jumps is also as plausible. Duffie et. al (2000) introduce pricing models allowing jumps for a scale of derivatives. Their results show, that allowing jumps in both asset returns and volatility have significant impacts on the observed volatility smiles and thus derivative prices.

Johannes (2004) analyzes the role of jumps in interest rate derivatives and bonds.

He finds, that multi-factor diffusion models cannot explain the observed changes in interest rates, and suggests a jump-diffusion model to model the behaviour of the yield curve. According to his research, jumps in interest rates are often induced by three separate factors: first, official announcments regarding the current state of economy, second, official announcements regarding changes in Fed policy and third, other political events in macroeconomically important countries. Lahaye et. al (2011) agree on the fact that jumps occur more frequently on days with macroeconomical announcements. More specifically, often jumps occur during the beginning of the trading day. This is consistent with the hypothesis that announcements are one of the main reasons causing jumps, because they are often released at 08:30 EST.

The fact that jumps are likely to occur shortly after macroeconomical releases is recognized also by Lee (2011). She studies, whether it is possible to predict the arrival of jumps and which predictors are the most significant ones, finding that different macroeconomical information arrivals have different effect on individual stock jumps. Similarly, stock specific news might be predicting jumps in single equities. Some type of information is more likely to cause systematic jumps, whereas other tends to cause idiosyncratic jumps to particular stocks. Additionally, the cluster effect is recognized, meaning that sometimes previously occurred jumps increase the likelihood of jump occurring also in the future. Table 2 summarizes the findings.

The role of jumps is especially important in derivative pricing, but also have some implications in regard to other financial modelling problems, including bond pricing (Johannes 2004). The effect of jumps in pricing of derivatives calls for actions also in hedging the claims. After all, pricing a derivative is essentially finding the price for a replicating portfolio. Hedging in the precense of jumps is therefore also an immensely studied subject. Cont et. al (2007) present a hedging strategy, which aims to minimize the hedging error when the asset price paths contain jumps. They argue, that the hedging with a combination of the underlying asset and available options can significantly reduce the risk in comparison to classic delta hedging. In more recent work, Bandi and Ren (2016) raise the question on price and volatility co-jumps. They find that price jumps are often associated with simultaneous jumps in volatility with opposite sign, leading to large negative correlation. They find strong evidence supporting this from high-frequency data on VIX and S&P 500. The negatively correlated co-jumps are also important to recognize in pricing, risk management, and hedging. For example, an investor recognizing the co-jumps would value near-the-money put close to expiry much higher than an investor not informed about this phenomenon, because of the jump risk associated with this investment.

Table 2: Significant jump predictors (Adapted from (Lee 2011))

Information factor	**Scope**	**Finding**
Time of the day	Systematic	More jumps tend to occur during opening hours of the trading day (09:30-11:00 a.m.)
Market jump	Systematic	More jumps occur shortly after S&P 500 jumps
FOMC news	Systematic	More jumps occur shortly after FOMC announcements
Payroll report	Systematic	More jumps occur shortly after payroll reports are released
Jobless claims	Systematic	More jumps occur shortly after jobless claims are released
Earnings announcement	Idiosyncratic	Jumps tend to occur within one day before the earnings announcement
Analyst recommendations	Idiosyncratic	Jumps tend to occur shortly after analyst recommendations
Past jumps	Idiosyncratic	Jumps tend to occur within three hours after previous jumps
Dividend release	Idiosyncratic	Jumps tend to occur in the morning hours of ex-dividend dates

4.1.1 Jump detection using realized bipower variation

The implications and causes of the jumps discussed in the previous section has been an incentive to develop tests in identifying jumps from asset price data. In order to understand the jump diffusion process, it is naturally interesting to understand how jumps behave in the real world. Even though jumps are extensively studied and the implications they have on different financial variables are widely known, they are hard to identify from discrete time data. The models are often based on continuous-time models, and many academics identify this problematic (see e.g. (Lee 2011)). Using discrete data in continuous-time models always leads to some kind of approximation errors. Nevertheless, multiple efficient methods for identifying jumps have been developed in the recent years. A few different jump detection tests are introduced next.

Even though the methodology and used test statistics among the developed tests differ much, the fundamental idea of the jump detection tests is similar. They study whether or not the resulted historical price path can feasibly be a result of a continuous stochastic process. If the test statistics suggests this is not the case, the null hypothesis of no jumps is rejected. Barndorff-Nielsen and Shephard (2006) were one of the earliest to conduct empirical jump tests on high-frequency data. They base their test on different characteristics of realized bipower variation (BPV) and realized quadratic variation (QV), splitting the realized QV into the jump component and continuous component. Noting that realized QV and realized BPV behave similarly when there are no jumps in the process, they use realized BPV as a tool for estimating the two different components realized QV consists of. A very simplified explanation of the test statistics is, that it measures the differences between realized QV and a jump-robust variance measure, which is based on realized BPV. If realized quadratic variation is significantly higher than this variance measure, the null hypothesis of no jumps is rejected.

An alternative approach has been suggested by Jiang and Oomen (2008). Their test is based on the result, that in the absence of jumps, a log contract and a continuously rebalanced long position in the underlying asset can be used to perfectly hedge a variance swap. On the other hand, if there are jumps in the asset prices, the hedging error can be fully explained by the realized jumps. This leads to the insight that jumps can be detected by comparing the difference between so called "Swap Variance" (the accumulated difference between simple returns and log returns) and the realized variance. Under the null hypothesis of no jumps, this difference is not significantly different from zero. On the other hand, when a jump occurs, the difference measures the magnitude of the jump. According to the empirical and simulated results by Jiang and Oomen (2008), their test often outperforms the one suggested by Barndorf-Nielsen and Shephard (2006). The difference can be especially significant when jump sizes are relatively small.

An addition to these two tests is another nonparametric test developed by Lee and Mykland (2008), which is applicable to all financial time series where high-frequency data is available. Their test is used to detect jumps in the empirical part of this thesis and is therefore examined much closer than the two previous ones. Similarly to other tests, the basis for their test is to assume that log-asset prices follow the diffusion model stated in equation 2.1 when there are no jumps in the market. If a jump occurs, the asset price S_t is given by 2.5.

The question at this stage is, if the asset return can reasonably be a result of equation 2.1. Naturally, if the spot volatility is high, significantly larger price movements can be a result of the diffusion model in equation 2.1. Therefore it is important to measure the spot volatility in a standardized manner, which filters out the effect of possible previous jumps. Lee and Mykland (2008) also utilize realized bipower variation (equation 2.7) in measuring this *instantaneous volatility* σ_{t_i},

$$\widehat{\sigma_{t_i}}^2 = \frac{1}{K-2} \sum_{j=i-K+2}^{i-1} \left| log S_{t_j}/S_{t_{j-1}} \right| \left| log S_{t_{j-1}}/S_{t_{j-2}} \right|, \tag{4.1}$$

where K is the window size used in measuring the instantaneous volatility and S_t is the stock price at time t. This window size is chosen in a way that the effect of possible previously occurred jumps disappears and is dependent on the sampling frequency of the jump detection. Denoting number of observations per day as *nobs*, this is achieved when $\sqrt{252 * nobs} \leq K \leq 252 * nobs$ (Lee and Mykland 2008). In other words, instantaneous volatility is the realized bipower variation of $K-1$ observations before the testing time t_i. Lee and Mykland argue, that selecting the lowest possible integer within the above range is reasonable, since larger K offers only minor contribution, but increases the computational load.

Lee and Mykland (2008) define the test statistics $\mathcal{L}_i$ testing for a jump from t_{i-1} to t_i as

$$\mathcal{L}_i \equiv \frac{log S_{t_i}/S_{t_{i-1}}}{\widehat{\sigma_{t_i}}}. \tag{4.2}$$

The test statistics is approximately normally distributed with a mean of 0 and a variance of $\frac{1}{c^2}$, where $c = \sqrt{\frac{2}{\pi}}$ if there is no jump at the testing time. If there is a jump, the statistics is very large and the null hypothesis of no jump is rejected. When the value of the instantaneous volatility in the denominator is high, the price movement needs to be much more radical to be considered as a jump. Lee and Mykland (2008, p. 9) provide analytical solution on how large the test statistic can become without being evidence of a jump. The idea is to compare the acquired test statistic value to that of the normally distributed asymptotic distribution of the test statistic. If the observed test statistics is unlikely to be result of the diffusion model, the test indicates a jump at the testing time. Under the null hypothesis of no jumps and as $\Delta t \to 0$ we can denote[2]:

$$\frac{|\mathcal{L}_i| - C_n}{S_n} \to \xi \tag{4.3}$$

[2] $C_n = \frac{(2\log n)^{1/2}}{c} - \frac{\log \pi + \log(\log n)}{2c(2\log n)^{1/2}}$ and $S_n = \frac{1}{c(2\log n)^{1/2}}$

where ξ has cumulative distribution function $P(\xi \leq x) = exp(-e^{-x})$. In order to select a rejection region for the null hypothesis, we can then set a desirable significance level for ξ. Figure 3 shows a sample of the data, where a jump has been detected using the method of Lee and Mykland. The detected jump is highlighted in the data with a red color.

As visible from figure 3, the detected jump occurs from 15:16 to 15:21. The magnitude of the movement is well described by the middle panel: the log return at that time is almost 10 times the instantaneous volatility. Assuming the returns follow normal distribution, and volatility describes the standard deviation, a price move of this magnitude is extremely unlikely. The bottom panel shows the behaviour of the jump statistics defined in equation 4.3. The red line indicates the barrier, above which a price movement is regarded as a jump. As can be seen, this detected jump is clearly above it. An expected notice of the market behaviour can also be made. After the jump, the volatility clearly rises, indicating some magnitude of uncertainty or panic effect following the jump.

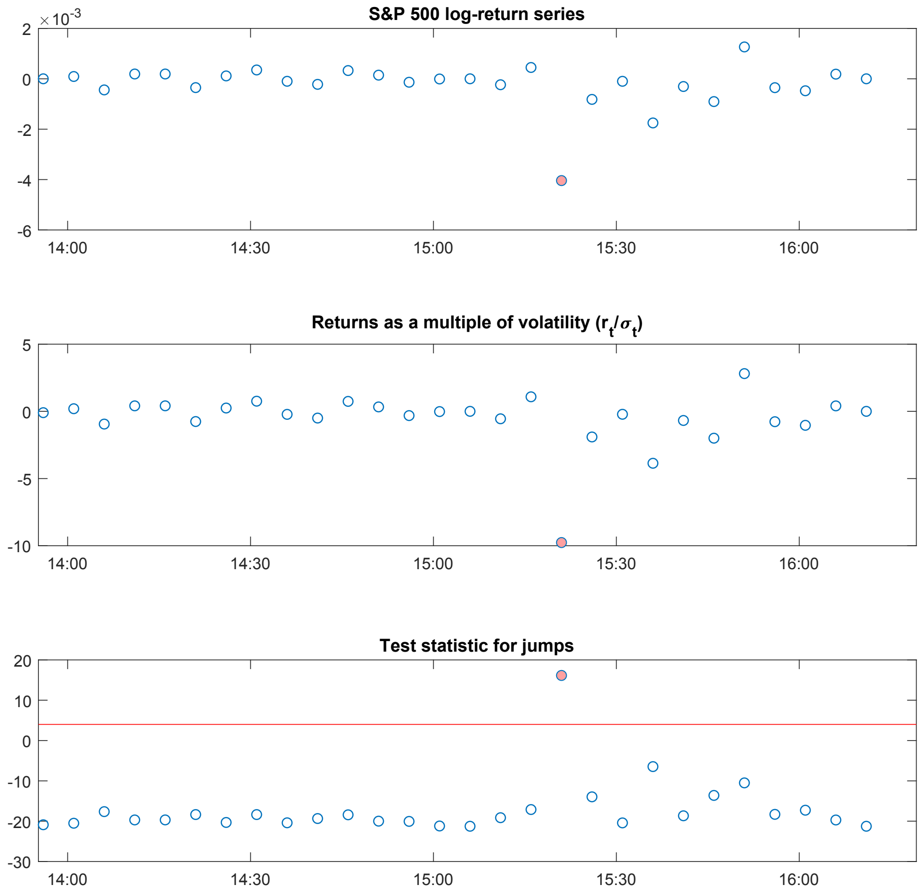

Figure 3: Jump detection data at the end of trading day, 1 May 2006. The occurred jump from 15:16 to 15:21 has been highlighted with a red color. The red horizontal line refers to the threshold above which a price move is categorized as a jump.

4.1.2 Jump detection using model-free implied volatility

The spot volatility estimator used in jump detection should describe the current volatility environment as accurately as possible. As discussed, there are often large differences between realized volatility and corresponding implied volatility. Therefore, using realized bipower variation as a spot volatility estimator in jump detection tests can be misleading. As a solution to this problem, a spot volatility estimator based on model-free implied volatility is suggested. Building this estimator is based on an assumption of the following linear relationship between VIX-index (or CX-index) and instantaneous volatility as defined in equation 4.1:

$$\sigma_i = \alpha + \beta \overline{VIX_i} + \epsilon, \tag{4.4}$$

where σ_i is the instantaneous volatility estimator on day i using the selected sampling interval, $\overline{VIX_i}$ is the average value of VIX (or corresponding CX-index) on day i using selected sampling interval. Such linear relationship in VIX suggested for example by Yang (2015). Strong R-squared of the results in empirical data (see chapter 5) additionally suggest that the assumed regression is applicable. If the regression is run on sampled data but more frequent observations are available, the robustness of σ_i is increased by following method:

$$\sigma_i = \frac{1}{n} \sum_{k=1}^{n} \sigma_{i_k}$$

where n is the sampling interval and σ_{i_k} is instantenous volatility on day i starting the calculation from minute k. In other words, a total of n instantaneous volatility estimators are calculated for each day, starting point being each minute from $1...n$. For each day, σ_i used in the regression will then be the average of n computed daily σ_i:s. This way every observation can be included in calculations and robustness of the daily σ_i is increased.

The obtained values for α and β are then used to convert sampled VIX or CX to spot volatility σ_{VIX} (or σ_{CX}) to be used in jump detection test defined in equation 4.2 as follows:

$$\sigma_{VIX_t} = \alpha + \beta VIX_t$$

The expected advantages of using this methodology in jump detection arise especially in situations where there are jumps in the volatility process but not in the price process. When volatility jumps, VIX is expected to react almost immediately, whereas realized bipower variation and thus instantaneous volatility as defined in equation 4.1 will follow with a lag depending on the sampling interval. Therefore, it is possible that jump detection test suggested by Lee and Mykland (2008) will detect false jumps in asset prices when volatility jumps. The new method in estimating spot volatility is suggested in order to avoid such false detections and provide more robust estimator of spot volatility.

4.2 Jump detection in simulated data

When stock price data with no jumps present is simulated, jump detection tests should detect no jumps besides from margin of error. This assumption is tested using the jump detection test by Lee and Mykland (2008) and stock price data simulated using Heston model (Heston 1993) without jump component (see equations 2.3 and 2.4). More accurate description of methodologies used in the jump detection is described in chapters 5 and 6. In this chapter, only a short summary of the results using simulated data is presented. Stock price data is simulated using five minute sampling interval for one year, assuming 252 trading days and 7.5 hours per trading day yielding a total of 22 680 observations. Table 3 shows the parameters used in the simulation and figure 4 plots the results using daily sampling interval.

Table 3: Parameters used in Heston model simulation (Yang and Kanniainen 2017)

Parameter	**Definition**	**Value**
S_0	Stock price at time 0	100
r and μ	Risk-free rate and drift term	0.02
σ_0	Volatility at time 0	0.2007
θ	Long term mean of volatility	0.2007
κ	Mean reversion speed of volatility	2.9688
γ	Volatility of volatility	0.3181
ρ	Correlation between Brownian motions simulating stock price and volatility	-0.7435

Heston put and call prices for full strike range are simulated for each moment in order to calculate corresponding VIX- and CX-indices. A linear regression (see equation 4.4) is run in order to transfer both VIX and CX indices into spot volatility

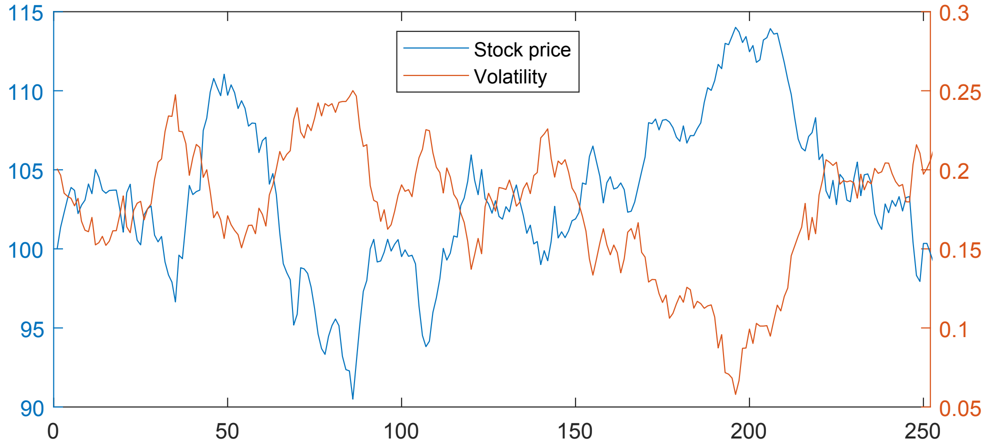

Figure 4: Time series of simulated stock price and volatility data using Heston model

measures. The result of the linear regression for VIX-index is presented in figure 5. For VIX-index, α and β resulted in -0.045 and 0.812, respectively. The regression for CX-index shows very similar behaviour, with α and β resulting in -0.042 and 0.863, respectively. R-squared for both regressions equals 0.921, indicating very strong coefficient of determination.

When detecting jumps with simulated data with no jump component, the jump test by Lee and Mykland (2008) expectedly yields no jumps whatsoever regardless of whether the spot volatility estimator is implied or historical volatility based. Interesting question becomes how would the jump detection statistics behave if there are jumps present in the volatility process but not in the price process itself. This is tested by manually inducing ”jumps” in the volatility process, otherwise using same random variables and parameters as in simulation without jumps. The stock price at each observation of the process is simulated modelling the behaviour of volatility as follows:

$$\sigma_{tj} = \sigma_t + 10\alpha\sqrt{dt},$$

where σ_{tj} is the volatility at time t under an assumption of a jump, σ_t is the actual Heston-model simulated volatility, α is the volatility of volatility and dt is the time step in years. In other words, an increase of 10 standard deviations in the volatility is assumed to represent a jump. Thereafter, each observation in the price process is re-simulated using the new volatility parameter. Next, the jump detection test is repeated using returns from the process including volatility jumps, but calculating realized bipower variation from the original process without volatility jumps. By this methodology we can test each moment independently, assuming each manually induced volatility jump would be individual in the process and no prior jumps would be present.

When volatility jumps are present, a total of 506 false price jumps are detected using historical bipower variation as spot volatility estimator. For each of these moments, VIX and CX indices are recalculated under the assumption of σ_{tj} representing the volatility to be used in Heston model option pricing. The jump detection test is then repeated using σ_{VIX} and σ_{CX} as spot volatility estimators. Using σ_{VIX} the test detects 120 jumps and using σ_{CX} 114 jumps. The significantly lower amount is a result of implied volatility indices capturing the effect of increased volatility, and therefore more accurately describing the minute-to-minute spot volatility. On the other hand, realized bipower variance is based on history and therefore underestimates the spot volatility under the precense of volatility jumps. This observation naturally raises the question whether similar behaviour is present in empirical data. Table 4 shows the jump detection results using simulated data.

Table 4: Detected jumps with each method using simulated data

	Detected jumps using simulated data	
	Without volatility jumps	**With volatility jumps**
$\boldsymbol{\sigma_{BPV}}$	0	506
$\boldsymbol{\sigma_{VIX}}$	0	120
$\boldsymbol{\sigma_{CX}}$	0	114

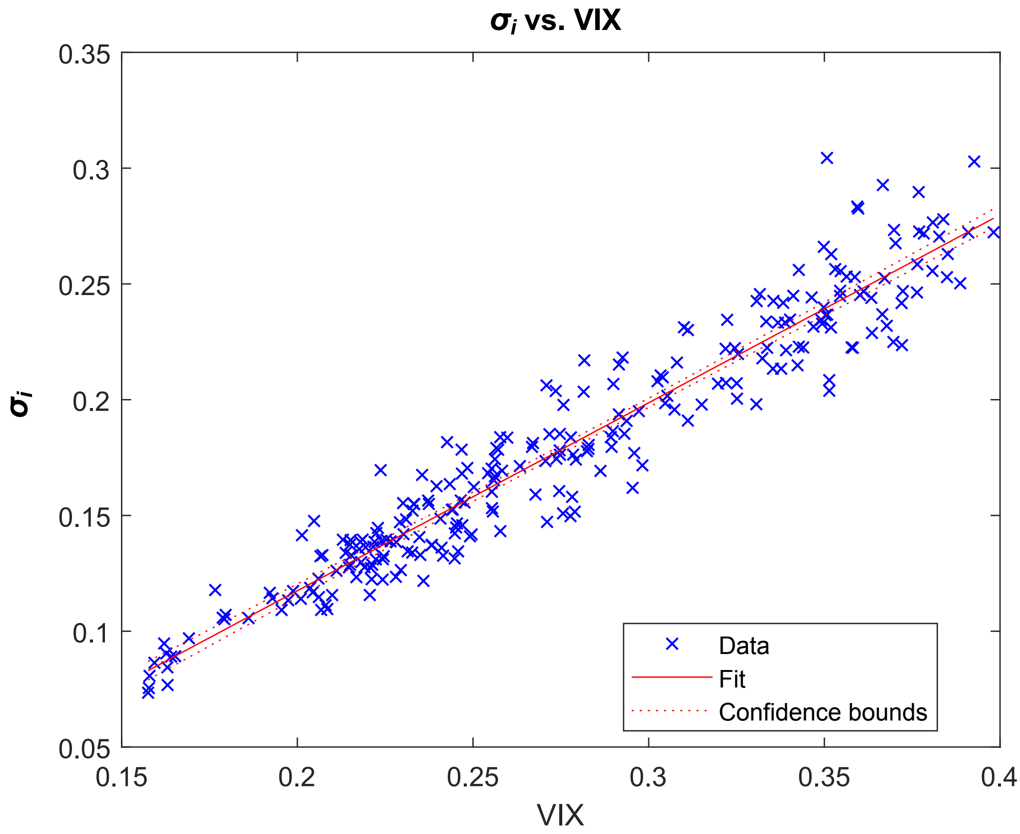

Figure 5: Results of linear regression transforming VIX to spot volatility estimator

5. DATA

This thesis analyzes high-frequency data on S&P 500 index options. The data is used to simulate model-free implied volatility for each second of the observation period. The data consists of minute-by-minute observations on a five-year period starting from January 2006 and ending December 2010. During the timespan there are total of 1259 trading days. For October 9 2006, October 8 2007, November 12 2007, October 13 2008, November 11 2008, and October 12 2009 there were no risk-free interest rates available in the data. Hence, the trading days in question are ignored in the simulations. There are ten observed variables in total for each option, each minute. The variables included for each observation among are listed in table 5 below.

Table 5: Description of the variables included in the data

Variable	**Variable description**
Root	A coded number reporting the option category
Call/Put	Binary tag reporting whether the option is call or put
Time	Matlab serial number reporting the observation time
Time to maturity	Time to maturity in calendar days
Strike	The strike price of the option
Bid price	The bid price of the option
Ask price	The ask price of the option
Mid price	The average of bid price and ask price
Underlying	S&P 500 index value on observation moment
Rate	The risk-free rate on observation moment

There were 15[3] option categories included in VIX calculation over this thesis' sample period (Andersen et al. 2015). Observations belonging to these categories were identified, and the original data was filtered accordingly. Additionally, the observation was filtered out if:

- Both bid and ask price were zero

[3] The option categories are identified with a three-letter code. Simulations included options in categories SPB, SPQ, SPT, SPV, SPX, SPZ, SVP, SXB, SXM, SXY, SXZ, SYG, SYU, SYV, and SZP.

- The interest rate for observation moment and corresponding maturity was not available
- The maturity was more than 730 days

After the filtering, the data included on average 847 observations per minute. This filtered data was then used to simulate VIX replication index. Since official high-frequency VIX values were unavailable, this replication index is simply denoted as VIX. This replication index was calculated percisely following the CBOE method (CBOE 2009). Additionally, a corridor implied volatility index (CX) was computed minute-by-minute. CX was calculated according to the method suggested by Andersen et al. (2015). During the first minute or first few minutes of some observation dates the data was missing adequate amount of observations to calculate value for MFIV. This is the case for a total of 56 trading days during the observation period. Furthermore, in some rare mid-day minutes MFIV could not be calculated. CBOE provides official open- and close-values for VIX on their website[4], which were then compared to the computed values to test for robustness. Figure 6 illustrates a scatter plot of computed close values compared to official close values. The correlation between successfully computed and official values is .9997. There is some deviation visible especially when the measure is high, which can be resulting for example from different dataset or different risk-free rate used by CBOE.

As expected, the values of computed VIX are clearly higher than those of computed CX-index. In addition, both of these values are higher than 30-day realized volatility. Figure 7 shows the results of computed VIX and CX indices compared to subsequently realized 30-day volatility on annualized levels. Both figures are plotted with a weekly sampling interval to reduce noise. As expected, in both cases the implied volatility measure is on average higher than the realized volatility, but CX index is usually closer to realized volatility. This is indicating the discussed variance risk premiums. When large spikes in realized volatility occur, both implied volatility measures follow with a lag often ending up higher than earlier realized volatility. This on the other hand supports referencing VIX as the fear gauge, as the demand for hedging portfolios rises during turbulent times. After large spikes, the investor nervousness causes volatility risk to be reflected in implied volatility measures.

The objective of computing VIX and thereafter CX is to calculate an implied spot volatility estimator of the model-free implied volatility measures to be used as a parameter in jump detection as defined in equation 4.2. As discussed, VIX-index and corresponding CX-index measure 30-day implied volatility, but in jump detection

[4] http://www.cboe.com/publish/scheduledtask/mktdata/datahouse/vixcurrent.csv

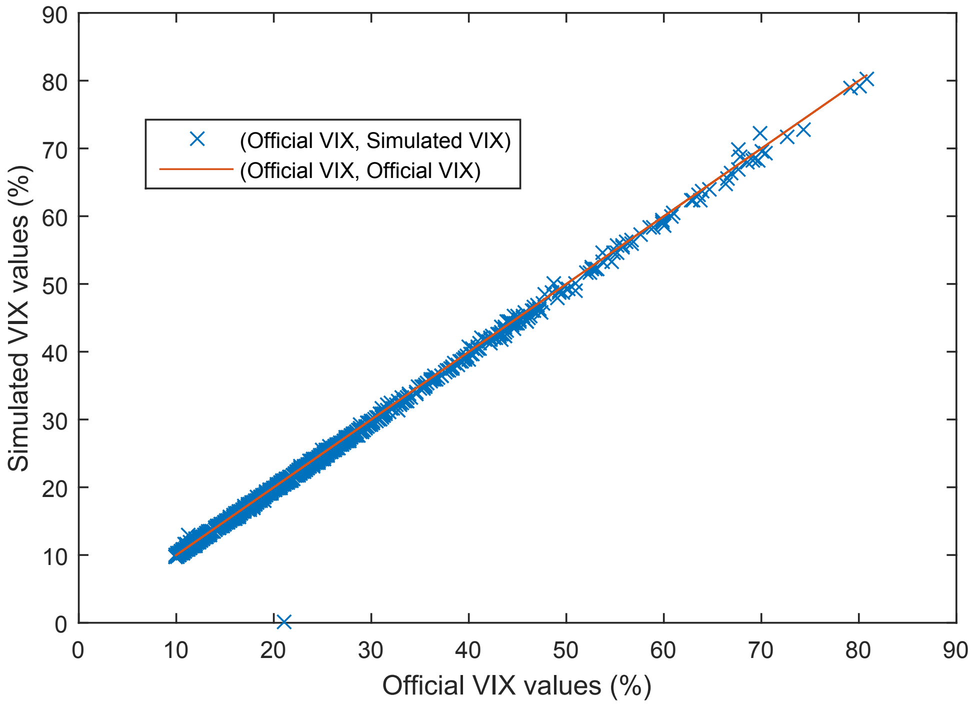

Figure 6: Scatter plot of simulated and official daily close values of VIX-index during the observation period.

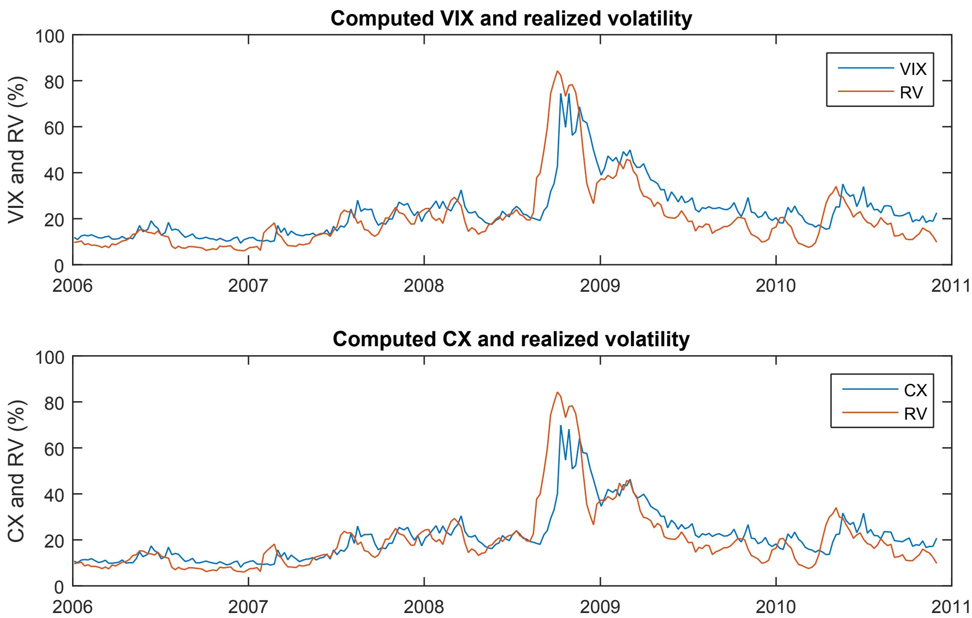

Figure 7: Computed VIX and CX index compared to realized volatility in annualized terms. Realized volatility is computed using daily closing values for 30 calendar day period starting from measurement time of the corresponding implied volatility measure.

spot volatility needs to be used. The volatility measure obtained by computing VIX and corresponding CX must be converted to intra-day spot volatility in order for it to be comparable to realized bipower variance used by Lee and Mykland (2008) and applicable in jump detection test. Therefore the linear regression decribed in equation 4.4 is applied to computed VIX and CX to obtain the spot volatility. Figures 8 and 9 show the results of the regression for VIX and CX indices, correspondingly. The high R-squared implies, that the assumption of linear relationship is reasonable.

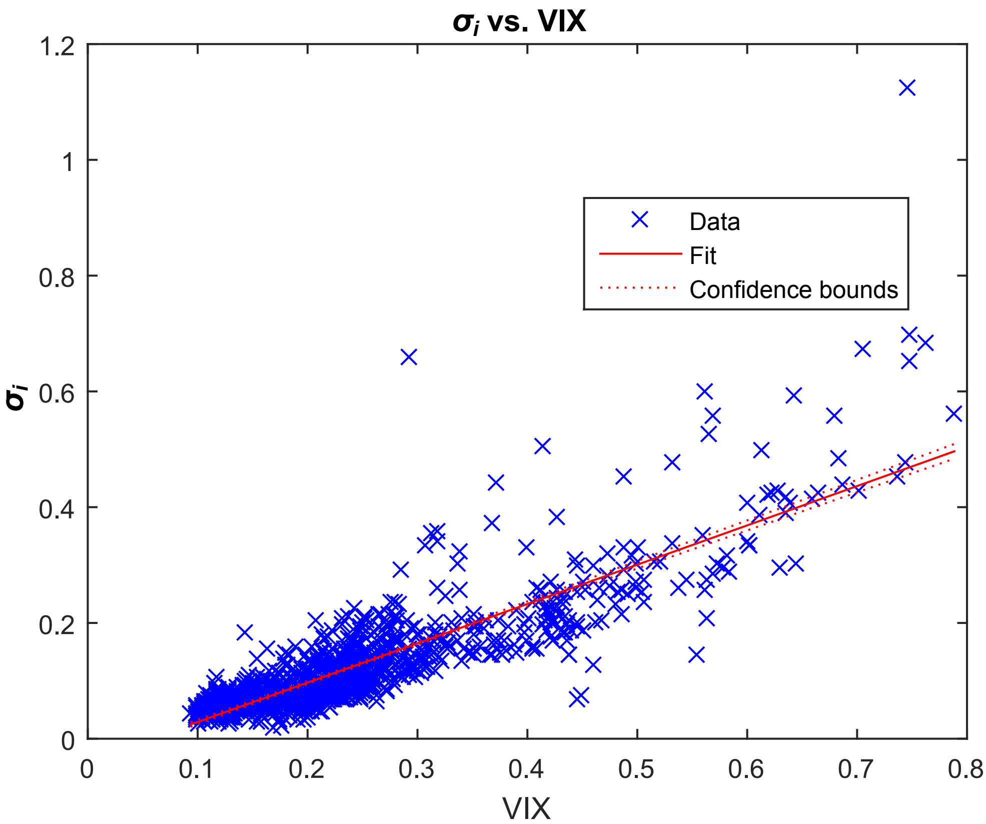

Figure 8: The results of the linear regression for VIX as stated in equation 4.4. The results are stated in annualized volatility. The value of α and β resulted in -0.039 and 0.679, correspondingly. R-Squared of the regression resulted in 0.74

The regressions are run on 5- and 15-minute sampling interval, after which the values of VIX and CX -indices are converted to their spot equivalents using the regression results. These measures of spot volatility are then used in jump detection in S&P 500 index. In this thesis, the jump detection methdology discussed by Lee and Mykland (2008) is used. One of the limitations of the test is that high-frequency data should be used, which is fulfilled with the used dataset. The results of the jump detection and comparison to more traditionally used bipower variation method is discussed in chapter 6.

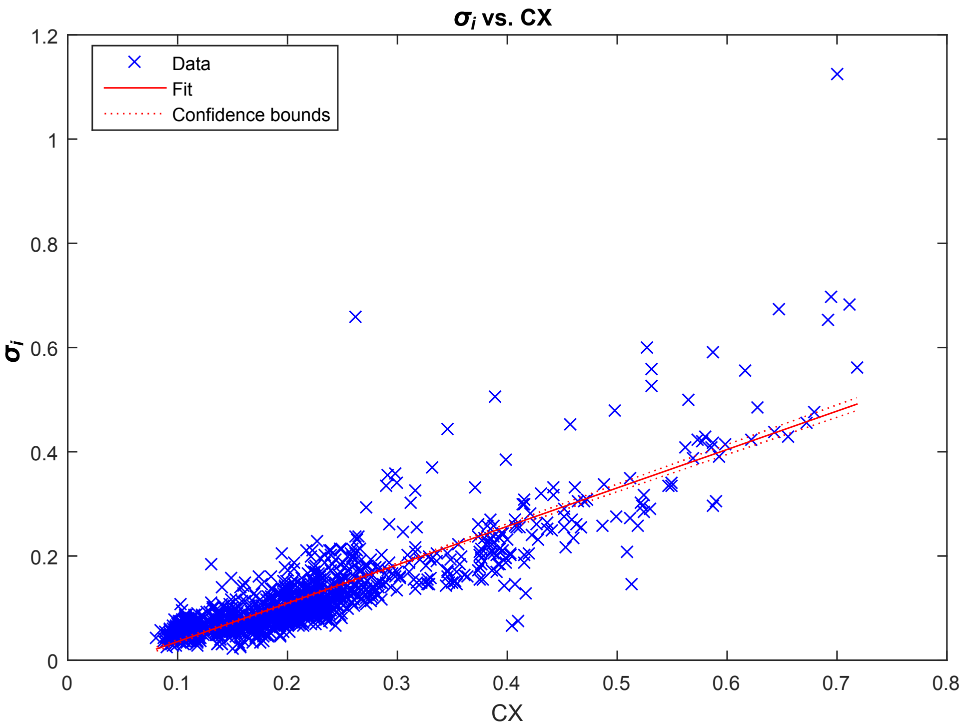

Figure 9: The results of the linear regression for CX as stated in equation 4.4. The results are stated in annualized volatility. The value of α and β resulted in -0.038 and 0.737, correspondingly. R-Squared of the regression resulted in 0.748.

6. EMPIRICAL RESULTS

This chapter analyzes how the behaviour of the detected jumps differs when using realized volatility method and model-free implied volatility method. First, the used spot volatility measures are analyzed. Then, the distribution and behaviour of the detected jumps is analyzed further. The spot volatility estimators obtained by linear regression 4.4 are denoted by σ_{VIX} and σ_{CX}. For the sake of consistency, instantaneous volatility as defined in equation 4.1 is denoted by σ_{BPV}.

6.1 Results of jump detection using different spot volatility estimators

Table 6 reports the characteristics of σ_{BPV}, σ_{VIX}, and σ_{CX} used in the jump detection test. Both the mean and standard deviation of σ_{BPV} are significantly higher than in either of the implied volatility measures. The results behave similarly regardless of sample interval. It is worth noting, that the mean and standard deviation are rather similar to both implied volatility measures even though the truncation of tails in the calculation of corridor implied volatility (discussed in chapter 3) assures that its value is fundamentally lower than VIX value. The result is attributed to the used linear regression, since the regression coefficient β from equation 4.4 is smaller for CX index. Therefore, the higher average level of VIX index in comparison to CX index diminishes when regressing to the spot volatility estimates σ_{VIX} and σ_{CX}. The kurtosis and skewness are also remarkably higher for σ_{BPV} than for the implied volatility measures. This indicates that market shocks cause significantly more fluctuation in S&P 500 prices than in the underlying options. This result is in line with the comparison of realized volatilities and implied volatilities reported in figure 7.

Figures 10 and 11 present the distributions of each different spot volatility estimator. As evident also from table 6, the values of σ_{BPV} include fat tails, which is not visible in implied volatility measures. Additionally, the mean value of σ_{BPV} is slightly higher than for the implied volatility measures, stemming from the amount of outliers.

Table 6: Statistical variables for different spot volatility estimators

	σ_{BPV}	σ_{VIX}	σ_{CX}	σ_{BPV}	σ_{VIX}	σ_{CX}
	5 minute sampling			**15 minute sampling**		
Mean	9.46E-04	8.44E-04	8.44E-04	1.66E-03	1.42E-03	1.42E-03
SD	7.42E-04	5.71E-04	5.75E-04	1.23E-03	9.45E-04	9.50E-04
Skewness	3.43	1.80	1.77	2.69	1.80	1.77
Kurtosis	22.12	6.89	6.85	12.58	6.86	6.83

Table 7 reports the intra-day distribution of the detected jumps with different volatility estimates. The columns specify the starting point of each time interval. Distinctly, most jumps take place in the first hour of the trading day. The following trading hours from 10:31 to 13:30 introduce only few jumps, after which from 13:31 to 15:30 the jump intensity increases. During the final period from 15:31 to 16:00 the intensity decreases again. Using 15 minute sample interval yields notably less jumps than 5 minute sample interval. Longer sample interval allows larger price movements to plausibly be resulted from diffusion process, and therefore the result is expected. Otherwise, the intra-day distribution of the detected jumps behaves similarly regardless of sample interval. Overall, the amount of detected jumps is significantly higher using the implied volatility measures in comparison to .σ_{BPV}. This is expected, since as discussed earlier the average spot volatility is higher for σ_{BPV} than in σ_{VIX} or σ_{CX}. The higher volatility allows for price movements of much larger magnitude to be interpreted as a part of the continuous component of the price process.

Detecting two concurrent jumps with opposite directions immediately after each other could indicate noise in the observed data or recording errors in the underlying prices. Therefore, the amount of such jumps is studied. On 5-minute sampling interval, 3 such jumps are detected using σ_{VIX} or σ_{CX} and only 1 using σ_{BPV}. On 15 minute sampling interval, none such jumps are detected. Therefore it can be concluded the detected jumps are not induced by recording errors or noisy data.

Focusing on the very first minute of the trading day provides interesting observations. Detecting jumps with σ_{BPV} during the first minutes of a trading day relies on the realized bipower variance of the previous trading day as a spot volatility estimator. It is therefore justified to ask whether using σ_{BPV} as an estimator is particularly biased during this period. The results show that especially the first minute of the trading day is commonly associated with a jump. The percentage of detected jumps occurring in the first minute of the trading day is 66% (38%), 51% (21%), and 49% (22%) for σ_{BPV}, σ_{VIX}, and σ_{CX}, respectively (15 minute sample

Table 7: The intra-day distribution of the detected jumps

Method	09:31	10:31	11:31	12:31	13:31	14:31	15:31	Sum
	5 minute sampling							
σ_{BPV}	310	3	4	1	20	18	4	360
σ_{VIX}	381	24	10	6	34	34	12	501
σ_{CX}	385	24	11	7	39	34	12	512
	15 minute sampling							
σ_{BPV}	66	2	1	0	4	12	1	86
σ_{VIX}	91	11	4	1	7	19	3	136
σ_{CX}	94	12	4	1	7	20	3	141

interval in parenthesis). A clearly lower percentage of the detected jumps occur immediately after the break in the trading when using implied volatility methods. However, considering that there are 1253 trading days in the data sample, it is clear that a break in the trading does not necessarily induce jumps.

Table 8 reports the amount of mutual and non-mutual jumps for each method. There is a significant amount of jumps detected with σ_{BPV} which are not detected with implied volatility methods. The first trading minute discussed earlier is a vital reason for the difference in the detected jumps. The amount of the jumps detected with σ_{BPV} and not detected with σ_{VIX} or σ_{CX} occurring on first or second observation of the trading day is 55 out of 94 in both cases. The difference in detected jumps using the two implied volatility measures is not significant. The reason for σ_{CX} detecting more jumps than σ_{VIX} is natural, since the absolute level for σ_{CX} is by definition lower.

Table 8: Detected mutual and nonmutual jumps for each different method. The rows indicate the method against which the results are compared to. For example, the first row states how many jumps identified by BPV method were not identified by other methods.

	Mutual jumps			**Non-mutual jumps**		
	5 minute sampling					
	σ_{BPV}	σ_{VIX}	σ_{CX}	σ_{BPV}	σ_{VIX}	σ_{CX}
σ_{BPV}	360	266	266	-	94	94
σ_{VIX}	266	501	488	235	-	13
σ_{CX}	266	488	512	246	24	-
	15 minute sampling					
σ_{BPV}	86	57	59	-	29	27
σ_{VIX}	57	136	132	79	-	4
σ_{CX}	59	132	141	82	9	-

The Jaccard similarity index can be used to measure similarity between two data sets. The Jaccard index is calculated as follows:

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|}, \tag{6.1}$$

where A and B are the compared sets. Table 9 reports the Jaccard index for jump distributions using different volatility estimators. The similarity for implied volatility methods is very large, whereas the similarity between σ_{BPV} and implied volatility methods is rather low. However, this result is expected as table 8 shows significant difference between jumps detected by different methods.

Table 9: Jaccard similarity index measures the similarity of detected jumps between different volatility measures

	Jaccard similarity index	
	5 minute sampling	**15 minute sampling**
$\boldsymbol{\sigma_{BPV}/\sigma_{VIX}}$	0.4471	0.3455
$\boldsymbol{\sigma_{BPV}/\sigma_{CX}}$	0.4389	0.3512
$\boldsymbol{\sigma_{VIX}/\sigma_{CX}}$	0.9295	0.9103

The correlation of each different spot volatility estimator is reported in table 10. The correlation is high, but as scatter plots 12 and 13 show, there are also outliers in the data, especially moments when σ_{BPV} is significantly higher than corresponding implied volatility measure. These moments are associated with the 2008 financial crisis. As evident from figure 7, in the end of 2008 realized volatility and thus σ_{BPV} was significantly higher than implied volatility.

Table 10: The correlation coefficients of each spot volatility measure.

	$\boldsymbol{\sigma_{BPV}}$	$\boldsymbol{\sigma_{VIX}}$	$\boldsymbol{\sigma_{CX}}$	$\boldsymbol{\sigma_{BPV}}$	$\boldsymbol{\sigma_{VIX}}$	$\boldsymbol{\sigma_{CX}}$
	5 minute sampling			**15 minute sampling**		
σ_{BPV}	1.0000	0.8697	0.8729	1.0000	0.9016	0.9042
σ_{VIX}	0.8697	1.0000	0.9992	0.9016	1.0000	0.9991
σ_{CX}	0.8729	0.9992	1.0000	0.9042	0.9991	1.0000

As the robustness of volatility measure is important in jump detection, next the effect of jumps in volatility is studied. Namely, we analyze what the difference in spot volatility estimator on next observation after a detected jump is in comparison to spot volatility estimator at the moment of detected jump. Table 11 reports the change in percentage terms. The standard deviation and average is clearly higher in BPV-measure. In both implied volatility measures the spot volatility behaves much more stable. This indicates that implied volatility is clearly more robust in the precense of jumps.

Table 11: The mean change and standard deviation of the mean change in different volatility estimators after a detected jump.

	σ_{BPV}	σ_{VIX}	σ_{CX}	σ_{BPV}	σ_{VIX}	σ_{CX}
	5 min sampling			**15 min sampling**		
Mean	7,24 %	0,54 %	0,49 %	7,84 %	0,02 %	0,24 %
SD	0,12	0,05	0,04	0,11	0,04	0,05

6.2 Behaviour of spot volatility in the precense of detected jumps

In order to understand what causes the differences in detected jumps under different volatility measures, the following two situations are analyzed:

1. σ_{CX} detects a jump but σ_{BPV} does not detect a jump.
2. σ_{BPV} detects a jump but σ_{CX} does not detect a jump.

The amount of such jumps is reported in table 8. Only σ_{CX} is separately analyzed, since it behaves very similarly with σ_{VIX}, making separate analysis not necessary.

Using 5 minute sampling interval and studying situation 1, there are a total of 246 jumps to be analyzed. The distribution of jump timing is similar to total jumps detected using σ_{CX}, 65% of the jumps occurring in the first trading hour and a small peak in jumps being visible around 14pm (11% of total situation 1 jumps). σ_{CX} does not significantly change directly after a detected jump, but increases by 2.08% on average in 10 observations after a detected jump. On the other hand, σ_{BPV} increases on average 2.6% directly after a detected jump and 8.8% 10 observations after a detected jump. Studying behaviour of the volatility 10 observations prior to the detected jump shows that σ_{CX} remains rather stable, actually decreasing on average by 0.7%. Similar analysis shows that σ_{BPV} increases on average by 3.3%. Median increase however is only 1.7%, as there are a few outliers in the data.

Using 15 minute sampling interval and studying situation 1, there are a total of 82 jumps to be analyzed. The behaviour is very similar to when analyzing 5 minute sampling interval and situation 1. On average σ_{CX} slightly adjusts upwards directly after a detected jump, by 0.9%. However, ten observations after a detected jump there is no trend visible. However, σ_{BPV} increases on average 4.0% directly after a detected jump and 9.8% ten observations after a detected jump. The behaviour of the volatility prior to the detected jump is also similar. On average, σ_{CX} decreases by 2.8% prior to situation 1 jumps, whereas σ_{BPV} does not significantly change.

These results seem rather normal, indicating that σ_{CX} usually increases as a reaction to price jumps. Additionally, situation 1 jumps are often connected with a slight decrease in σ_{CX} prior to the jump, allowing for smaller price movements to be categorized as jumps. Additionally, even though σ_{BPV} did not detect jumps, it adjusts upwards as a result of relatively large price variance almost categorized as a jump. The behaviour of σ_{BPV} and σ_{CX} prior to and after the situation 1 jumps suggests that the detected jumps are actual price jumps instead of false detections caused by jumps in volatility.

Using 5 minute sampling interval and studying situation 2, there are a total of 94 jumps to be analyzed. The distribution of jump timing is similar to jumps detected by σ_{BPV} overall, 90% landing in the first trading hour and 6% landing around 14 pm. Studying the behaviour of spot volatility indicates that σ_{BPV} behaves similarly as described in table 11, increasing on average by 7.89% directly after the detected jump. An interval of 10 observations after a detected jump, σ_{BPV} increases on average as much as 20.76%. On the other hand, σ_{CX} does not change significantly after a situation 2 jump, decreasing on average by 0.2% in both situations (1 or 10 observations after a detected jump). When studying how the spot volatility developed prior to the detected jump, an interval of 10 observations (50 minutes) prior to the detected jump is analysed. σ_{CX} increased on average 3.37% during that time interval, whereas σ_{BPV} remained stable with no apparent increases or decreases visible.

Using 15 minute sampling interval and studying situation 2, there are 27 jumps to be analyzed. The mean change in σ_{BPV} directly after a detected jump is 9.4%, whereas σ_{CX} does not change significantly. In 10 observations after a detected jump, σ_{BPV} increases on average by 28%, whereas σ_{CX} still does not change significantly. 10 observations prior to the detected jump however, σ_{BPV} decreases on average by 3.4%. However, σ_{CX} increases on average by 5.6%.

The results demonstrate that most situation 2 jumps are associated with previous increase in implied spot volatility and how σ_{BPV} does not immediately react to the switch in market implied spot volatility. The extremely large average increase in 1 and 10 observations after a detected jump indicates that σ_{BPV} does not capture the current market conditions when implied spot volatility has previously increased. Therefore, the outdated spot volatility estimator might lead to false detections. This conclusion is reinforced by the fact that σ_{CX} does not significantly alter after situation 2 jump detection, indicating that market participants do not notice abnormal behaviour in S&P 500 prices.

Figure 14 plots the behaviour of implied spot volatility and historical spot volatility around an illustrative situation 2 jump. As visible, implied volatility moves clearly upwards during the pause in trading between two trading days. However, σ_{BPV} does not contain the new market information and thus detects a price jump, catching up to σ_{CX} with a delay. Option prices and thus σ_{CX} have reacted to market sentiment immediately, therefore not detecting a price jump.

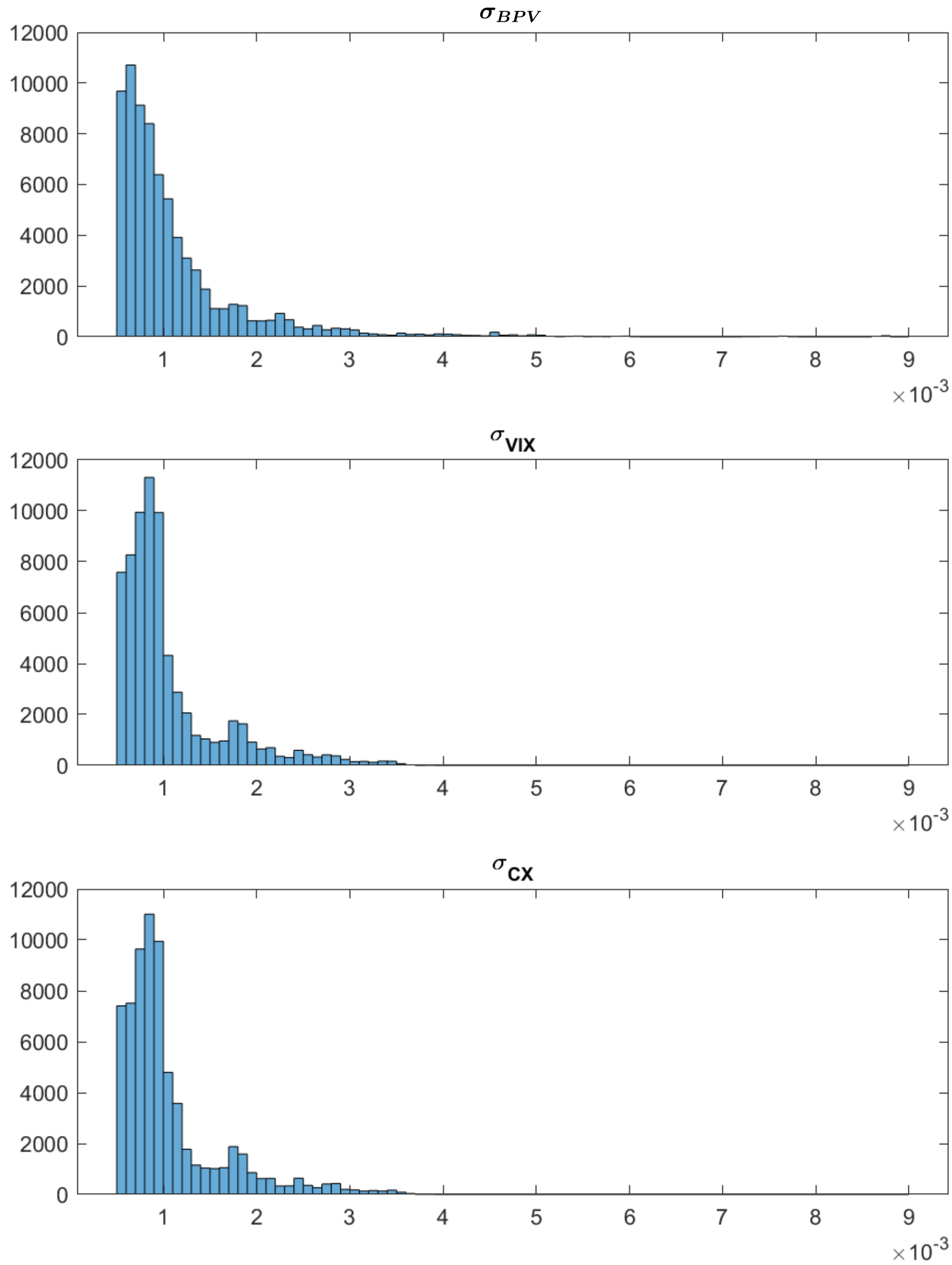

Figure 10: Distribution of each spot volatility estimator, 5 minute sampling frequency.

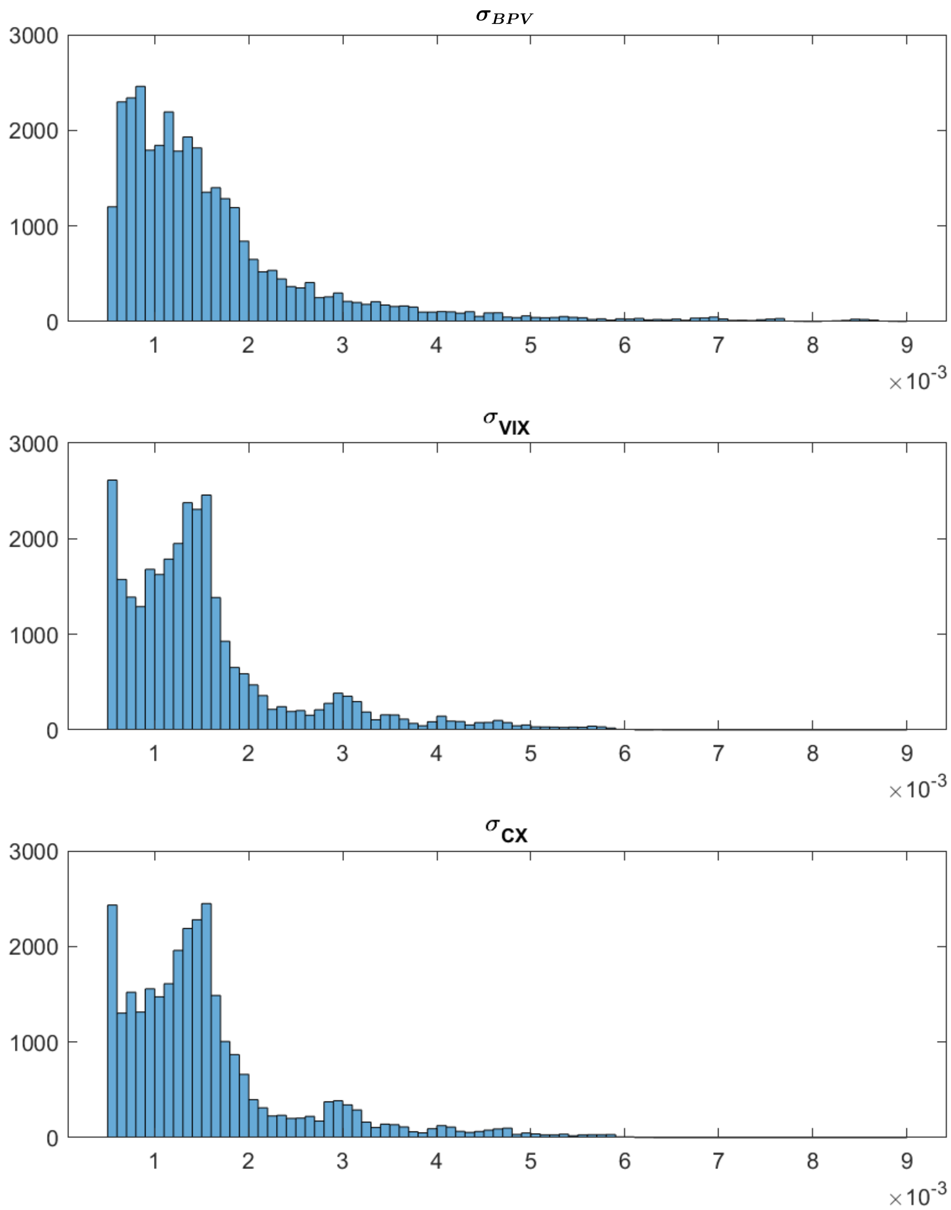

Figure 11: Distribution of each spot volatility estimator, 15 minute sampling frequency.

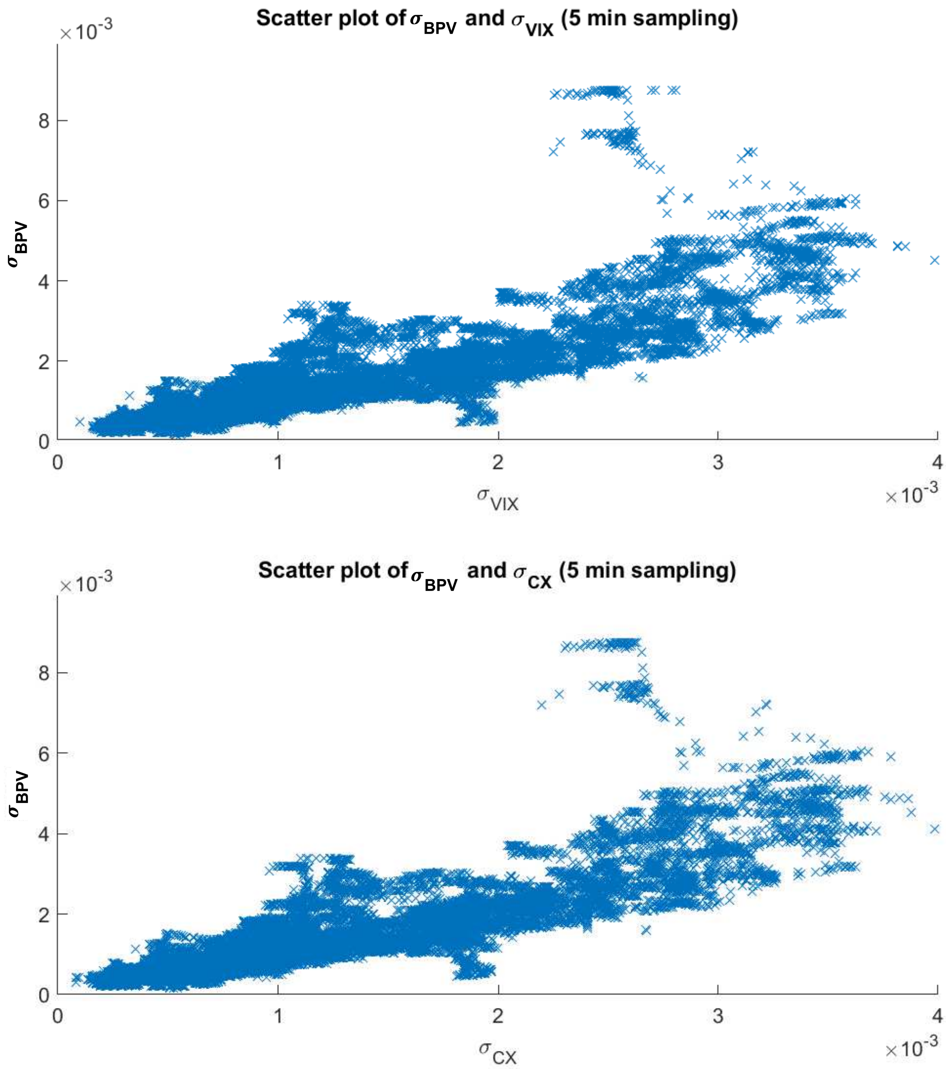

Figure 12: Scatter plot for spot volatility estimators, 5 minute sampling frequency.

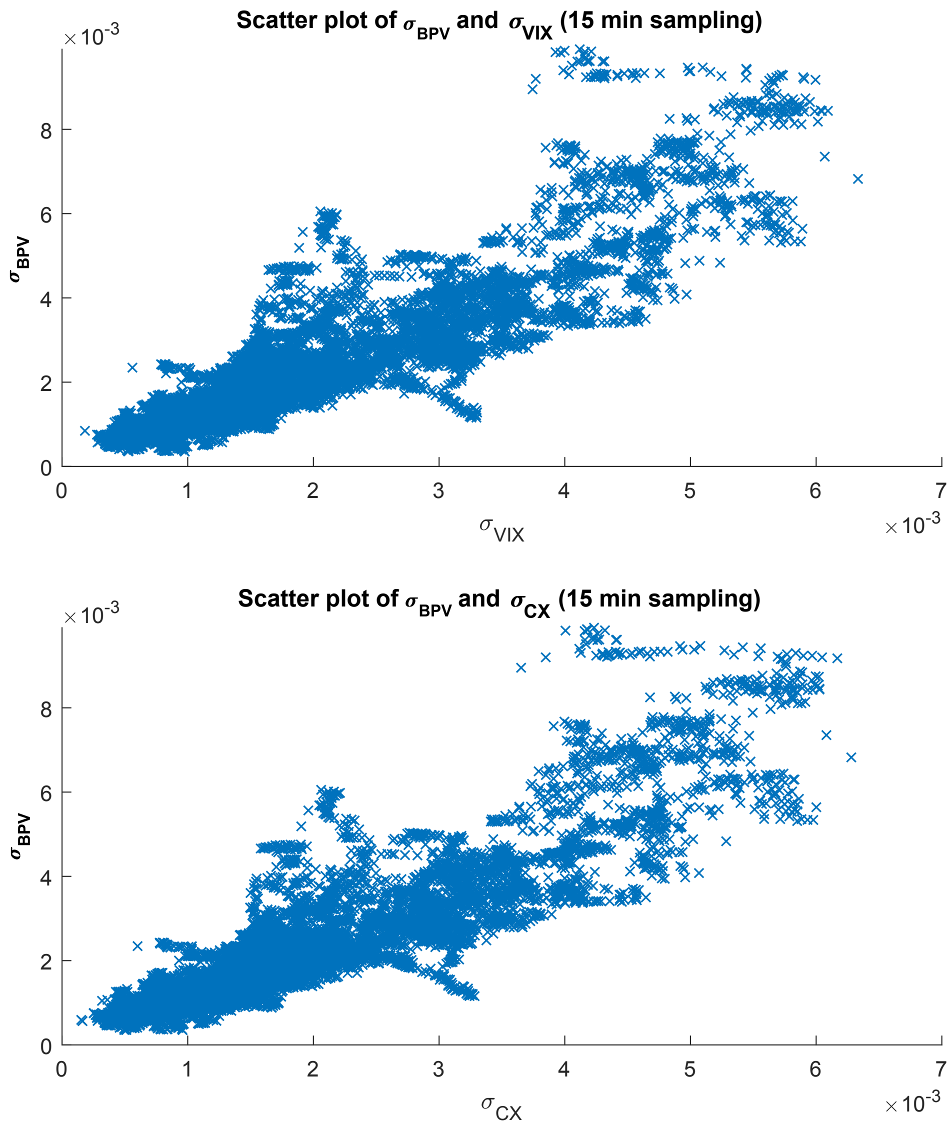

Figure 13: Scatter plot for spot volatility estimators, 15 minute sampling frequency.

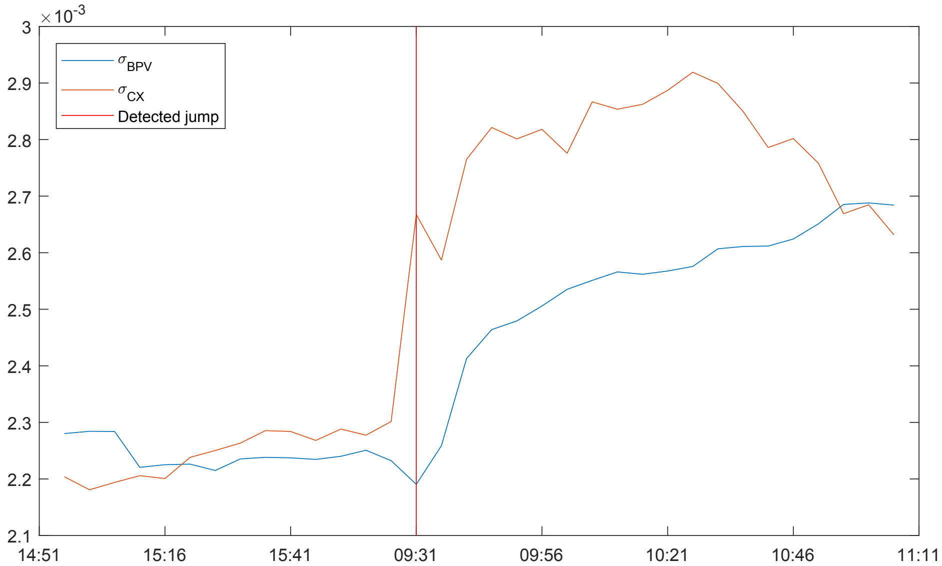

Figure 14: Illustrative behaviour of spot volatility estimators with 5 minute sampling interval around a detected jump at 09:31 on 22 October 2008 demonstrating the effect of σ_{BPV} reacting to changes in market environment with a significant delay. σ_{VIX} moves clearly upwards after the change in trading day, whereas σ_{BPV} reacts to the change in implied volatility environment with a delay.

7. CONCLUSIONS

The chosen volatility measure has decisive impact on how jumps are detected in a financial time series. In recent scientific literature, jumps and jump detection has been immensely studied. Different methodologies have been suggested, but the spot volatility estimate used in these methodologies is without exception based on historical measures. Since implied volatility is commonly accepted to interpret the market expectation of future volatility, it is relevant to question whether or not implied volatility measure could be used also in jump detection. That idea was the starting point to this thesis. Jump detection has important implications in, among others, hedging and portfolio rebalancing after a detected jump. Therefore the results of this thesis also have practical implications and the presented methodology could also be considered among practicioners.

According to the results, implied spot volatility measures are in general of lower magnitude that historical bipower variation. This obviously leads to significantly higher amount of jumps being detected using implied spot volatility in jump detection. However, the correlation between these spot volatility estimators is high, allowing the conclusion that each method is applicable in the used jump detection test. The intra-day distribution of detected jumps is fairly similar regardless of the method used. However, there is one important difference visible. In the early minutes of a trading day, especially the very first minute, implied spot volatility measures detect proportionally less jumps than realized bipower variation. Regardless of the spot volatility estimator, a vast majority of detected jumps occur during the first trading hour.

In general, the results suggest that jumps or rapid upward shifts in spot volatility might lead to false detections of jumps using realized bipower variation. The volatility jumps are not captured quickly enough in the historical spot volatility, whereas option prices (and therefore implied volatility) adjust almost instantly. In these cases, implied volatility measures are far more robust in jump detection. This is especially visible during the first few minutes of the trading day, when realized bipower variation estimates are outdated. The results indicate that using realized bipower variance often detects jumps in the opening minutes of the trading day, which are not detected using implied volatility measures. The long overnight pause in the trading leads to realized bipower variance being outdated in the early trading minutes. Therefore, implied volatility, which reflects the market's expectations of future volatility at each time in question could potentially be much more accurate estimate of spot volatility before realized bipower variance adjusts to the market conditions.

In the presence of jumps, the implied spot volatility estimators behave much more stable than historical volatility based BPV estimator. The results show that the effect of jumps on model-free implied volatility are minor, whereas jumps induce clear changes in bipower variation. The increase in bipower variation after jumps might result in some jumps leaving undetected as the spot volatility estimator shifts upwards influenced by jumps. Similar behaviour is not visible in model-free implied volatility. This indicates that market pricing of the options takes the possibility of jumps into account, and therefore the effect of large moves in the underlying index is only minor in option prices.

Further research on the topic would be required to validate the displayed results. Additionally, the generalization of the results could be tested for example by testing the methodology using a diversity of underlying indices distributed geographically. The time series used in this thesis centers around the financial crisis, when market conditions were highly abnormal. Using more recent data from more stable market conditions would perhaps allow for higher coefficient of determination in regressing VIX and CX to spot volatility. Further research could focus on more recent time series and investigate whether similar behaviour is visible aroung more stable market conditions.

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