

# **Optimal Capital Structure: An Option Theory Approach**

Mikko Nieminen  
Master's Thesis  
Department of Economics  
University of Tampere  
June 2005

University of Tampere

Department of Economics

**NIEMINEN, MIKKO: Optimal Capital Structure: An Option Theory Approach**

Master's Thesis, 70 pages.

Economics, finance, financial economics

June 2005

---

The objective of this work is to introduce a model which is able to produce an optimal capital structure (OCS) for maximizing total firm value. The model is calibrated to reflect actual firm characteristics and uses contingent-claim methods to value tax shields and costs of financial distress associated with debt financing. It also maintains a long-run target debt to capital ratio by refinancing maturing debt; this provides a dynamic setting for capital structure choice. The model is derived from known financial theories and is theoretically consistent with other theories describing capital structure choice.

Several papers have emerged focusing on OCS with a variety of contingent-claim models. They all present similar characteristics concerning model calibration and the assumption of a tradeoff between tax shields and costs associated with leverage. However, the model presented in this in this work differs considerably from previous studies. The single most decisive factor is the valuation method of required returns on claims issued by a firm. Our model is entirely based on option theory and their pricing. The reason is that we are able to simultaneously incorporate several different factors which affect firm value within the capital structure choice. Another reason which makes option theory appealing is the fact that we are also able to extract market values and expected returns from within the model as they continuously alter accordingly to changes in leverage.

The model is derived from the earlier works of Black and Scholes (1973), Merton (1973) and Vassalou and Xing (2004). These papers as well as an introduction to option pricing theory and an over-all presentation of contemporary capital structure theories are revisited. We will produce some comparative static results which at first seem contradictory to earlier results and stipulations. However, while considering other capital structure theories, these findings provide new insight for maximizing firm value with optimal leverage. The empirical work focuses on the implications of our model and the results derived from Finnish companies during 1994–2003 period.

## TABLE OF CONTENTS

<b>1.</b>	<b>INTRODUCTION</b>	<b>3</b>
<b>2.</b>	<b>CAPITAL STRUCTURE THEORIES</b>	<b>5</b>
<b>2.1</b>	<b>The agency theory</b>	<b>6</b>
2.1.1	Conflicts between equityholders and management	7
2.1.2	Conflicts between equityholders and debtholders	9
<b>2.2</b>	<b>The asymmetric information theory</b>	<b>10</b>
2.2.1	Investment decision and capital structure	11
2.2.2	Signaling with debt	12
2.2.3	Managerial risk aversion and signaling	14
<b>2.3</b>	<b>Corporate control contests</b>	<b>14</b>
<b>2.4</b>	<b>The tradeoff theory</b>	<b>18</b>
<b>3.</b>	<b>MODEL OF OPTIMAL CAPITAL STRUCTURE</b>	<b>22</b>
<b>3.1</b>	<b>Basics of option theory</b>	<b>23</b>
<b>3.2</b>	<b>Option valuation</b>	<b>25</b>
<b>3.3</b>	<b>Option theory and firm's balance sheet</b>	<b>28</b>
<b>3.4</b>	<b>Market values of equity and debt</b>	<b>30</b>
<b>3.5</b>	<b>Leverage and cost of debt</b>	<b>34</b>
<b>3.6</b>	<b>Leverage and cost of equity</b>	<b>41</b>
<b>3.7</b>	<b>Total firm value</b>	<b>46</b>
<b>4.</b>	<b>COMPARATIVE STATICS OF MODEL</b>	<b>48</b>
<b>4.1</b>	<b>Asset value volatility</b>	<b>49</b>
<b>4.2</b>	<b>Debt maturity</b>	<b>51</b>
<b>4.3</b>	<b>Dividend policy</b>	<b>53</b>
<b>4.4</b>	<b>Interest rate level</b>	<b>55</b>
<b>5.</b>	<b>EMPIRICAL EVIDENCE ON LEVERAGE</b>	<b>57</b>
<b>6.</b>	<b>CONCLUSIONS</b>	<b>63</b>
	<b>REFERENCES</b>	<b>65</b>

## 1. Introduction

The pursue of optimal capital structure has inspired relentless production of studies and new theories which year after year captivate the attention of scholars and capital market professionals. Capital structure choice has been declared a much complex problem than the optimal dividend policy and much has been left unsaid regardless of the wide variety of theories and results where corporate finance policy makers have been able to lean on.

However, empirical methods in corporate finance have lagged behind those in capital markets for several reasons. First, models of capital structure decisions are less precise than asset pricing models. The major theories focus on the ways that capital structure choices are likely to affect firm value. But rather than being formal as a precise mathematical formula, like capital asset pricing model or option pricing model, the existing theories of capital structure provide at best qualitative or directional predictions. They identify major factors like taxes, bankruptcy costs and informational incentives associated with leverage and firm value, but they do not actually quantify how much the effect is.

Second, most of the competing theories of optimal capital structure are not mutually exclusive. Evidence consistent with one theory generally does not allow us to conclude that another factor is unimportant. It seems that taxes, agency and information costs as well as costs of financial distress all play some role in determining a firm's optimal capital structure. Third, many of the variables that are believed to affect optimal capital structure are difficult or impossible to measure. Signaling theory suggests that private information about the firm's future prospects play an important role in their financing decisions, but there is no obvious way of identifying and quantifying such information asymmetries.

Fourth, different companies face different business environments regarding the volatility of revenues in comparison with overall economic situation or maturity of their debts. Even companies, which seem similar or are in the same branch of industry, may in fact have very differently proportioned fundamental factors, which affect capital

structure choice and subsequently firm value. These factors need to be directly observed or easily calculated in order to identify how much firm value is affected. For all these reasons, it is advocated, that the art of corporate financing policy is less developed than asset pricing. It is therefore the focus of this work to bridge the gap between these two fields of study and produce a suggestion which combines both capital structure theory and asset pricing. We will achieve this by applying a completely new approach of option theory application in corporate finance.

We continue on the path started by Brennan and Schwartz (1978), (1984); Leland (1994), (1998); Leland and Toft (1996), Fischer et al. (1989) and Bradley et al. (1984). They have all presented continuous-time contingent claim models, which study firm value and debt policy. The development of our model is based on work by Black and Scholes (1973), Merton (1973), and Vassalou and Xing (2004). We adopt an option pricing framework which is used to relate the value of a levered firm to the value of an unlevered firm, the amount of debt, debt maturity, interest rate level, dividend policy, and volatility of cash flows or business risk.

We will introduce a model, which endogenously generates relative market values for both equity and debt along with expected returns for both claims. We assume a basic tradeoff theory of capital structure, where major forces affecting firms' optimal financing decisions are corporate tax shields and costs of financial distress. Some of the results are surprising and give support to firm specific characteristics and various capital structure theories which play an important role for maximizing total firm value with optimal leverage. Therefore optimal capital structure is not based on industry means. This finding is similar to the one reported by MacKay and Phillips (2002).

The remainder of the paper is organized as follows. Section 2 reviews the major capital structure theories and their implications. Section 3 describes our optimal capital structure model and introduces the methodology how the model is derived from option pricing theory. Section 4 presents comparative statics on the model's predictions of unique optimal capital structures. Section 5 reports empirical findings on capital structures among Finnish firms, whereas section 6 concludes.

## 2. Capital Structure Theories

Capital structure research and the introduction of new theories have been abundant since Modigliani and Miller (1958) first published their controversial propositions of capital structure irrelevance on corporate value. This caused huge debate on their views and they were more or less overwhelmed by criticism led by traditional views of corporate borrowing which advocated and rationalized moderate borrowing. However, they also produced the so called correction paper (1963), which stated benefits of debt in a form of tax shield that increases firm value.

In the special case of corporate debt being permanent and the firm having taxable income, the value of the tax shield and increase in firm value is equal to the amount of debt multiplied by the corporate tax rate. The firm value can be written as:

Market value of levered firm = market value of equity + present value of tax shields

$$V = E + D \cdot \tau_c$$

However, corporate debt was also assumed to be risk-free and produce an expected return equaling government bonds regardless of the current debt / equity ratio. Therefore issuing new debt had no effect on market values of outstanding debt and the relative values of debt and equity.<sup>1</sup>

The general academic view by the mid-1970s, although not a consensus, was that the optimal capital structure involves balancing the tax advantages of debt against the present value of bankruptcy costs. This is known as the tradeoff theory. However, a central issue in corporate finance research has been the question of why, despite the potentially large tax advantages enjoyed by debt, firms seemed to have fairly low leverage ratios.

---

<sup>1</sup> In the real world, increased leverage causes bond downgrades. This will result in a reduction of market values. Subsequently MMI-proposition does not hold.

Trading benefits and costs was considered an incomplete description of corporate borrowing. This question motivated much of the early research on agency theory and information asymmetries, which appeared in the later part of 1970s and during 1980s. Large numbers of takeover activities in 1980s have also generated theories combining capital structure choices and corporate control considerations.

Currently these different strands of theories can broadly be categorized as:

1. Conflicts of interest between different groups holding claims to the firm's assets (the agency theory)
2. The conveyance of private information to capital markets or mitigation of adverse selection effects (the asymmetric information theory)
3. Corporate control contests and their outcomes
4. The tradeoff theory of debt financing, benefits versus costs

There are also other theories concerning capital structure choice, but we will exclude them from this paper. Harris and Raviv (1991) survey paper provides more information and insight on this matter.

## 2.1 The agency theory

Conflicts of interest between different groups accumulate as agency costs and research in this area was first introduced by Jensen and Meckling (1976). They identified two types of conflicts, which arise from different portions of gains on corporate performance. The first is conflicts between equityholders and management; the second is conflicts between equityholders and debtholders. Both are comprised of the same basic thought that one party tries to benefit from others by decreasing their wealth. This can be done by transferring firm's resources to their own as perks ("corporate life") or by having an incentive to invest suboptimally in projects that are considered risky compared to the firm's normal line of business ("asset substitution").

### 2.1.1 Conflicts between equityholders and management

Conflicts between equityholders and firm management arise because managers hold less than 100 % of the residual claim. As a result, they do not capture the entire gain from good company performance owed to profit enhancement activities, but they do however bear the entire cost of these activities. This can lead the managers to invest less of their effort to manage the firm's resources and consequently consume these resources for their own gain. Comfy corporate life includes corporate jets, golf tournaments, empire building and lavish offices all of which deplete equityholders' wealth. If managers refrain from spending, they bear the entire cost of forfeiting the easy life, but in turn they capture only a fraction of the gain. Managers may well become overindulged in pursuing corporate life instead of concentrating maximizing firm value and equityholders' wealth.

In order to reduce or eradicate agency costs, the firm should reduce free cash flow which enables extravagant spending by managers.<sup>2</sup> This can be achieved by increasing managers' fraction of the firm's equity or their residual claim. If the managers' absolute level of investment in the firm is kept constant, replacing equity with debt also increases their fraction of the residual claim. Since debt commitments increase the firm's obligations to pay out cash as coupon payments, it also reduces the amount of free cash flow available for managers' spending. This constitutes the benefits of debt financing and the optimal capital structure can be obtained by trading off the agency cost of debt against these benefits (Jensen and Meckling 1976).

Harris and Raviv (1990) begin by assuming operating disagreements between managers and investors. Managers are assumed to always favor the continuation of the firm's current operations. On the other hand, investors prefer liquidation of the firm. This stark contrast between these two parties results in increasing the amount of debt in the firm and increasing the likelihood of default. By doing so, debt mitigates the conflict by giving investors the option to force liquidation if cash flows are poor and below their expectations.

---

<sup>2</sup> The theory predicts that mature industries with slow growth prospects should be more levered. Large cash inflows without NPV > 0 projects create the opportunity to "empire building".

However, while investors gain control over the firm through bankruptcy, the liquidation proceedings entail costs related to information gathering concerning the firm's prospects. In other words, here the optimal capital structure trades off improved liquidation decisions against higher information costs.

The model proposed by Harris and Raviv predicts that firms with large amounts of tangible assets as well as firms with low information costs should also have higher liquidation values. This supports the view, that these firms should also pursue higher levels of debt in order for investors to increase the probability of default and subsequent liquidation. They also argue that as liquidation value increases, the probability of restructuring the firm decreases while the probability is independent of information costs. In addition, while assuming a constant-returns-to-scale, they prove that debt level relative to expected firm income, default probability, bond yield and the probability of restructuring are independent of the firm size. Combining these arguments yield, that higher leverage is associated with higher firm value, higher debt level relative to expected income and lower probability of restructuring after default.

Stulz (1991) begins with the same assumption as Jensen (1986). Firms receive large, positive cash flows and are lacking of good investment projects. Managers are assumed to always want to invest all available funds even if investors prefer the firm paying out the excess cash as dividends. This spending would lead to an over-investment problem with the firm throwing away valuable resources in non-performing or bad investments, which are  $NPV < 0$ . Therefore the model entails a disagreement over operating decisions between managers and investors, like Harris and Raviv (1990) have proposed.

Capital structure is once again determined by trading off benefits and costs of debt. The over-investment problem is mitigated by increasing debt because it reduces free cash flows, but on the other hand, Stulz argues that debt repayments may more than exhaust free cash flow. This might reduce available funds even for profitable investment projects and the firm would end up with another problem, namely underinvestment.

The conflicts between equityholders and management can be summarized as:

**Table 2.1 Agency theories on manager-investor conflicts**

Model	Conflict	Benefit of debt	Cost of Debt
Jensen & Meckling (1976)	Managerial perquisites	Increase managerial ownership	Asset substitution
Jensen (1986)	Overinvestment	Reduce free cash flow	Unspecified
Harris & Raviv (1990)	Failure to liquidate	Allows investors option to liquidate	Information costs
Stultz (1991)	Overinvestment	Reduce free cash flow	Underinvestment

Source: Harris & Raviv 1991

### 2.1.2 Conflicts between equityholders and debtholders

Diamond (1989) has shown that managers have an incentive to pursue relatively safe projects out of reputational considerations when using debt financing. Since lenders can only observe the firm's default history, firms can build a reputation for assured repayments by first investing in safe projects. However, due to asset substitution, the firm could engage in a myopic maximization of equity value and would choose a risky  $NPV < 0$  project with equal probabilities of either success (full repayment) or failure (default). In other words, managers' behavior increases equity value while decreasing the debt values. This results in agency costs of debt.

Since investors cannot distinguish the firm's preferences beforehand, the initial lending rate reflects their beliefs about the projects chosen by firms on average.<sup>3</sup> The firm can then try to convince lenders, that it only has a safe project. If successful, the firm would therefore enjoy a lower lending rate. However, investors are not easily fooled (most of the time). Restrictions such as loan covenants play an important role in these situations. The provision of immediate loan repayment is frequently used as a management control device.

---

<sup>3</sup> The same problem of asymmetric information problems is dealt with in insurance business as contracts are priced between different quality level customers.

Diamond's theory implies that older, more established firms find it optimal to choose the safe project, since asset substitution could lead to a loss of valuable reputation. Younger firms with little reputation may well choose the risky project, since there is always the possibility, that the risky project could be successful. After avoiding default, they will eventually choose safe projects in the future. Therefore, firms with long, solid track records will have lower default rates and as a result, a lower cost of debt.<sup>4</sup>

Hirschleifer and Thakor (1992) approach the same issue from the managers' own reputational reasons. Here managers maximize the probability of success while equityholders prefer higher expected returns. Once again, there are two different projects (risky and safe) with two outcomes: success or failure. If the result is a failure, the outcome is the same regardless of project risk. From the equityholders' perspective, the risky project is better, because it yields higher expected returns now and higher realized returns if successful. However, the risky project also implies a higher probability of default and if the project is a failure, managers will suffer due to weaker reputation in the labor markets and subsequent career opportunities. Therefore managers will choose a safe project; this reduces the agency cost of debt.

## 2.2 The asymmetric information theory

Financial markets are especially characterized by informational differences between buyers and sellers. Borrowers typically know their own characteristics, such as their collateral and moral integrity, better than lenders who would benefit from knowing the true nature of borrowers. The verification of these true characteristics by outsiders may be costly or simply impossible. Since information transfer is incomplete, the markets may also perform poorly and valuation of projects would merely reflect average project quality like Diamond has proposed. If informational asymmetry is substantial, the supply of poor projects would be large relative to the supply of good projects.<sup>5</sup>

---

<sup>4</sup> Many start-up companies and other young enterprises may be completely unable to receive debt financing. Instead, they need equity to finance growth.

<sup>5</sup> Compare to market of used cars referred as "lemons" by Akerlof (1970).

Private information and modeling it to financial economics has created another perspective on capital structure choice. These theories proclaim that insiders, such as management, are assumed to possess superior, private information about the firm's characteristics concerning income stream as well as investment opportunities. One approach relies on capital structure choice signaling this information to outside investors. Another approach is based on assuming, that capital structure choice is designed to mitigate inefficiencies in investment decisions caused by information asymmetries between insiders and outsiders.

### 2.2.1 Investment decision and capital structure

Information asymmetry between management and outside investors concerning the value of the firm's assets can lead to mispricing the market value of equity. If the firm is required to finance new projects by issuing new equity, this mispricing may in fact lead to a situation, where new equityholders would capture more than the NPV of the new project. In other words, old equityholders would experience a loss in their share of the firm's future profits. Since management should always maximize the current equityholders' value, even  $NPV > 0$  projects might be rejected and this would result in underinvestment problem.

Since all new equity issues are or need to be underpriced, management should pursue to issue other securities, which are not so severely undervalued by the market. Normally internal funds such as retained earnings and issuance of corporate debt will be preferred over a new equity issue. Myers (1984) refers to this as the pecking order theory, which states that capital structure choice is influenced by the firm's desire to finance new projects. However, the pecking order does not actually stipulate an optimal capital structure. It merely concludes that observable capital structures are simply a by-product of earlier investment decisions.

The pecking order theory also implies that an announcement of an equity issue would lower the market value of the firm's existing equity.<sup>6</sup> However, financing a project with retained earnings or by riskless debt would not convey any superior information to investors and therefore it would not result in a stock price reaction.

However, Narayanan (1988) states that since project acceptance is associated with issuing new debt, debt issues are good news and would therefore lead to an increased stock price. Only good firms are assumed to issue debt and the market perceives that the new project is in fact  $NPV > 0$ . In addition, firms with fairly little tangible assets relative to firm value are more subject to information asymmetries and are therefore expected to become more levered over time. Instead of Myers and Majluf's (1984) model, which entails informational asymmetry both in the project quality and value of assets-in-place, in Narayanan's model there is asymmetric information only about the quality of the new project.

### 2.2.2 Signaling with debt

In contrast to the underinvestment problem, we now consider that investments are fixed and capital structure serves as a signaling device of private information to outsiders. The original idea of signaling is based on works of Akerlof (1970) and Spence (1973), although neither focuses on financial markets. Ross (1977) refined their thoughts into the context of financial economics and stated, that higher debt levels indicate better firm quality. In his model the firm management knows the actual distribution of firm returns, but investors do not. Since managers benefit from higher stock prices through a known incentive scheme, such as stock options or cash bonuses, and are penalized if the firm goes bankrupt, they are forced to choose a debt level, which maximizes their compensation or more precisely, the firm value.

---

<sup>6</sup> Korajczyk et al. (1991) argue that underinvestment is not a problem when equity issues are announced together with or close to earnings reports.

The firm has to establish a signaling equilibrium so, that debt financing conveys the insider information to outside investors. High quality firms need to signal their superior ability so they issue more debt. Investors will realize this and the stock price rises due to private information conveyed to investors.<sup>7</sup> Since lower quality firms have higher marginal expected probability of bankruptcy for any debt level, managers of low quality firms will not imitate higher quality firms by issuing more debt. In other words, firm value and profitability is positively related to debt / equity ratio.

Heinkel (1982) has introduced a model, where insiders know both the true value of their firm, and the value of any given debt repayment promise made by the firm, while outside investors know neither. As a result, insiders can potentially profit by selling overpriced securities. A firm which represents itself as being of high value benefits by realizing a high price for its shares, on the other hand, it can sell debt only at a low price. Higher quality firms' return distributions are higher overall, but their debt values are lower (bonds trading under par). This causes their equity value to be higher.

This results in a zero-cost separation of firm types as insiders maximize the value of their residual claim. The reason is that firms attempt to convince investors that they are a different type than they actually are. If successful, they will gain on overvaluation of one security but at the same time it will lose from undervaluation of another. Therefore high value firms issue more debt. If a low value firm wants to imitate a high value firm, it must issue more underpriced debt and reduce the amount of overpriced equity. Conversely, to imitate a low value firm, a higher value firm needs to issue less overpriced debt and more underpriced equity. Since high quality firms have higher value, they issue more debt.

---

<sup>7</sup> The financing decision is made at period 0 and the benefits are received at the next period. If the managers do not care about future time periods, they will have no incentive to signal correctly.

### 2.2.3 Managerial risk aversion and signaling

Leland and Pyle (1977) present a model, where increases in leverage allow managers to retain a larger fraction of the equity; this reduces managerial welfare due to risk aversion. However, the decrease is smaller for managers of higher quality projects and therefore high quality firms can signal this by having more debt. It can be shown that equilibrium ownership increases with firm quality. Increases in owners' expected return results in an increase in management ownership, but increased ownership produces two opposing effects on leverage.

As ownership is increased, more funds would need to be required by raising debt. However, firm value is larger for higher ownership fraction since equityholders may have to pay more for the smaller fraction of the firm. As a result, debt level does not have to increase to order to finance the increased ownership fraction. Leland and Pyle derive results, that debt increases with ownership fraction and as insiders own more of the firm; this signals higher quality of the firm. This willingness to invest their own wealth is in fact considered a positive signal by lending markets of the true quality of the project, firm and its prospects. The result implies that information transfer through signaling possesses a key efficiency property. An interesting conclusion is that projects, which are more distinct, or they have higher project specific risk, have lower signaling costs in equilibrium.

### 2.3 Corporate control contests

Takeover activities were especially high in the 1980s and financial economics began to examine the relationship between corporate acquisitions and capital structure. Basic notion is that common stock carries voting rights whereas debt does not. New theories developed from waves of corporate acquisitions all focus on how capital structure influences and effectively changes outcomes of takeover contests. The role of the takeover target's management and their ownership of the firm play an important part in these contests for corporate control.

In Harris and Raviv (1988), the managers maximize their expected payoff, which consists of the value of their equity stake and the value of other benefits. These benefits can be thought of as private control benefits or as the value of cash flows, which they can expropriate from the firm if they remain in control.<sup>8</sup> The optimal ownership share of the management is determined by the incumbent managers who trade off capital gains on their stake against a loss of any personal benefits derived from not being in control. The rest of the firm is assumed to be owned by passive investors.

When a rival appears, the incumbent managers try to manipulate the method and probability of a successful takeover by increasing their fraction of ownership. They can increase their equity stake by having the firm repurchase equity from the passive investors, financing the buy-backs by issuing debt. Now the maximization of the managers' payoffs is actually accomplished by choosing a debt level that determines the optimal fraction. Since control benefits decrease as debt level rises, it is optimal to choose the lowest possible level. As managers increase their fraction of ownership, the probability of a successful takeover decreases and the incumbent remains in control along with possession of private benefits. However, if ownership and leverage is increased too much, the value of the firm and the managers' equity stake are reduced.

Depending on the choices of equity ownership by the incumbent and rival, the takeover contest can have three possible outcomes. First, the incumbent's stake is so small that, the rival succeeds in taking over the firm. This is referred as a successful tender offer. Second, the incumbent's stake may be so large, that the takeover will fail. This is referred as an unsuccessful tender offer. Third, the incumbent will win if and only if its abilities are higher than the rival's in the perspective of passive investors. This is called a proxy fight, where the outcome is determined by passive investors through voting.

It follows from the above arguments that in the case of a successful tender offer being optimal, the firm will have no debt. It is also shown that generally proxy fights require some debt, and guaranteeing that the tender offer is unsuccessful requires even more debt.

---

<sup>8</sup> Corporate life and personal perquisites.

Thus, takeover targets will increase their debt levels on average and targets of unsuccessful tender offers will issue more debt on average than targets of successful tender offers or proxy fights. Also, firms that increase leverage tend to have either unsuccessful tender offers or proxy fights.

As rivals bid for shares held by passive investors, the firm value increases. Thus, on average, debt issues are accompanied by stock price increases. However, if the incumbent is perceived to possess higher ability than the rival, less debt is required to affect a proxy fight. Since winning a proxy fight is positively related to the probability of being more able, the likelihood of the incumbent winning is also associated with less debt. Therefore, in a sample of firms experiencing proxy fights, one would expect to observe less leverage among firms in which the incumbent remains in control.

Stulz (1988) has derived a model, which similarly focuses on the management's manipulation of ownership as it tries to remain in control and consume private benefits. In order for the rival to acquire control in the firm (more than 50 % of the shares), it has to purchase shares from passive investors. This reflects the assumption that passive investors vote for the incumbent in any takeover contest. In addition, they have heterogeneous reservation prices for selling their shares. Intuitively, the larger the incumbent's stake in the firm, the larger the fraction of the passive investors' shares that must be acquired by the rival, hence the more it must pay. The rival will bid a premium if and only if its expected benefit exceeds it.

Again, increasing the firm's leverage can increase management's ownership fraction. As a result, takeover targets have an optimal debt level, which maximizes the value of outside investors' shares. Targets of hostile takeovers will have more debt than firms that are not targets. Since becoming a takeover target is good news, one would expect share buy-backs (debt for equity switch) that accompany such an event to be associated with stock price increases as the rival purchases remaining shares. Accordingly, the probability of a takeover is negatively related to the target's debt / equity ratio, and the takeover premium is positively related to this ratio.

A similar approach has been taken by Israel (1991). Here too increases in debt levels increase the gain to target shareholders if takeover occurs. However, the mechanism is different from Stulz. The reason is that debt commands a fixed share of any gains from a takeover whereas the target shareholders along with acquirer shareholders bargain only over the portion, which is not already committed to debtholders. The more debt the target has, the less cash flow is left for the target and the acquirer. In addition, target shareholders can capture the gains accruing to the debtholders when the debt is issued, at the same time they capture all of the gain not going to the acquirer. As debt levels are increased, the expected payoffs to target shareholders increase if the takeover occurs.

The optimal debt level is determined by trading off the increased gain in payoffs against the reduced probability of takeover as debt levels increase. Israel reports three different comparative statics results. First, an increase in the costs of a takeover contest results in a decrease in leverage but an increase in the appreciation of target equity if a takeover occurs. Second, if the distribution of potential takeover gains shifts to the right (incumbent's ability decreases), debt level increases. Third, the optimal debt level increases, and the probability of takeover and the gain to target equity in a takeover decrease, the better the rival's bargaining power.

Although capital structure theories derived from corporate control contests are widely studied, these theories focus only on short-term changes in capital structure taken in response to imminent takeover threats. As a result, these theories have nothing to say about the long run capital structure of firms. On a general level, takeover targets will on average increase their debt levels and this will be accompanied by a positive stock price reaction.<sup>9</sup>

---

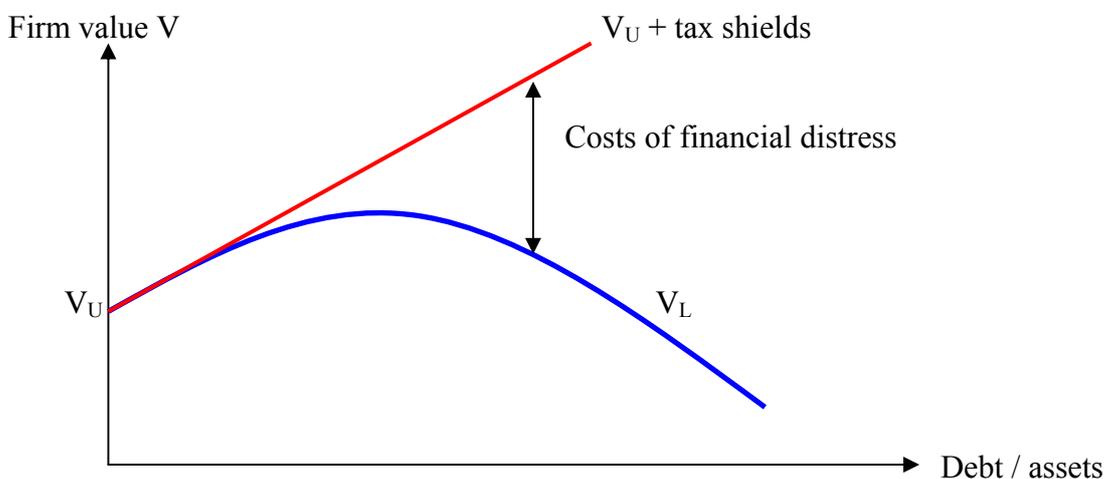
<sup>9</sup> Acquirers always have to pay a premium, especially when the target is publicly owned.

## 2.4 The tradeoff theory

A typical firm's optimal debt ratio can be viewed as a tradeoff between costs and benefits of debt financing, while holding the firm's investment decisions and assets constant. This balancing is portrayed as substituting debt for equity or equity for debt. In other words, firms are thought to have a target debt to assets ratios, which they try to pursue through gradual movements. This is very similar to dividend policies, which follow a target path. As a result, management is bargaining between interest tax shields and accumulating costs of financial distress. The theoretical optimum is reached when these benefits and costs offset each other and firm value is unaffected by changes in leverage. The financing structure and firm value are combined of:

$$\left( \text{Value of levered firm } V_L \right) = \left( \text{value if all - equity financed } V_U \right) + \left( \text{present value of tax shields} \right) - \left( \text{costs of financial distress} \right)$$

This can also be illustrated as:



**Figure 2.1** The tradeoff theory of capital structure

When a non-levered firm decides to finance its operations with debt, it creates a shield which increases firm value since interest payments on debt are tax-deductible. In the special case of debt being permanent and the firm having a steady target debt ratio, the present value of tax shields are equal to the amount of debt multiplied by the corporate

tax rate. This amount adds firm value above its unlevered counterpart. However, the existence of tax shields depends on the firm's ability to produce taxable income. Otherwise, tax shields are useless and do not increase value. Therefore, the greater the marginal tax rate and the greater the amount of debt, the greater the gain in firm value. Modigliani and Miller (1963) presented this result as a correction paper to their 1958 seminal paper. However, they concluded that the firm is always able to issue risk-free debt and that new issues do not affect ratings or the risk class of previous debt issues. Therefore they did not take costs of financial distress into consideration and how they affect expected returns on the firm's claims.

Costs of financial distress include the legal and administrative costs of bankruptcy, as well as the subtler agency, moral hazard, monitoring and contracting costs, which erode firm value even if formal default is avoided. Investors acknowledge these costs by raising their expected return on claims issued by the firm and thereby resulting in a decline in their market value. This constitutes for direct costs of bankruptcy, but there are also indirect costs such as loss of revenues, client relationships and skilled employees as the firm's reputation is damaged by dire straits.

Altman (1984) was the first to also incorporate indirect costs in estimating changes in firm values prior to filing bankruptcy. He reported results which clearly indicate the significance of materialized bankruptcy costs as firms become financial distressed under high leverage. In many cases costs exceeded 20 % of the value of the firm measured just prior to bankruptcy and even some cases this measured several years prior. On average, bankruptcy costs ranged from 11 % to 17 % of firm value up to three years prior to filing bankruptcy.

Altman also concludes that the present value of expected bankruptcy costs for many of the bankrupt firms exceed the present value of tax benefits accrued from leverage. This implies that firms were overleveraged and that a potentially important ingredient in the discussion of optimal capital structure is indeed the bankruptcy-cost factor (Altman 1984, 1088). This comparison of benefits and costs strongly indicate an optimal leverage ratio, which can be achieved by using moderate levels of debt financing.

Further evidence on costs of financial distress has been presented by Andrade and Kaplan (1998). They studied 136 highly leveraged transactions (HLT) that took place in the end of the 1980s and found that high leverage is the primary cause of financial distress but not economic distress. Poor firm and industry performance played much smaller roles compared to debt servicing obligations. Andrade and Kaplan estimated the costs of financial distress as ten to twenty percent of firm value and even some values of nearly 25 percent. As a result, costs of financial distress are identified as significant sources of firm value decline. However, they also note that if their sample is exposed to selection bias, their estimates may understate the costs for the typical firm.

The existence of debt financing benefits is not considered as straightforward as Modigliani and Miller (1963) intended. There has always been controversy how valuable tax shields are and how individual firms can exploit them. Researchers face several problems when they investigate how the tax incentives affect corporate financial policy and firm value. Among these problems is the difficulty of calculating corporate tax rates due to data problems and the complexity of tax codes in certain countries. Another problem is quantifying the effect of interest taxation at a personal level in countries where tax levels differ between dividends, interest income and capital gains. However, for simplicity, this work assumes equal taxation levels for dividends, capital gain, interest income and corporate tax rate.

Graham (2000) has found evidence, that typical firm interest tax shields are 9,7 % of total firm market value, and some LBOs have even produced increases of up to twenty percent. Also, should debt conservative firms decide to maximize their values, the result would increase firm values by 15,7 %. To his surprise, he also discovered that firms, which use debt conservatively, are large, profitable, liquid, in stable industries, and face low ex ante costs of distress. However, these firms also have growth options and relatively few tangible assets. In addition, he found that debt conservatism is persistent, positively related to excess cash holdings, and weakly related to future acquisitions.

Clear and robust evidence of substantial tax effects has also been reported by MacKie-Mason (1990). He found that the relationship between tax shields and the marginal tax rate is important and that the marginal tax rate does effect financing decisions. In

addition, firms with high tax loss carryforwards are much less likely to use debt, since these firms are unlikely to be able to use interest deductions. This is because loss carryforwards in one year are highly correlated with zero-tax status in following years and they are likely to lower the effective marginal tax rate while crowding out interest deductions. Additional evidence consistent with tax-based theories is reported by Masulis (1983).<sup>10</sup>

Empirical studies have also reported debt ratios to differ significantly from one firm to another. Any cross-sectional test of financing behavior should specify whether firms' debt ratios differ because they have different optimal ratios or because their actual ratios diverge from optimal ones.<sup>11</sup> Adjustment costs may also affect observed debt levels as they produce lags in capital structure changes. Especially large and even low adjustment costs could possibly explain the observed wide variation in actual debt levels, since firms would be forced into long excursions away from their optimal ratios. Some scholars, like Myers (1984), have therefore proposed the incorporation of recapitalization costs when capital structure choice is examined. This implies a dynamic aspect in financial policy and shareholder value maximization as Brennan and Schwartz (1984) have similarly advocated.

Cross-sectional dispersion of actual debt ratios may also imply that firms do in fact have different optimal capital structures,<sup>12</sup> even within an industry, and that there must be several factors influencing financing decisions. Firms with safe, tangible assets and plenty of taxable income to shield should have high target ratios. Unprofitable firms with risky, intangible assets should rely primarily on equity financing. High operating cash flow volatility implies a higher volatility in asset value, which increases default risk and increases expected returns. This in turn lowers the debt / equity optimum. In order to identify these different firm specific factors, we present an alternative approach to optimal capital structure modeling in the following chapter.

---

<sup>10</sup> Exchange offers which increase leverage transfer wealth from debtholders and preferred stockholders to common stockholders. The opposite applies for decreasing leverage.

<sup>11</sup> Which was first, the chicken or the egg?

<sup>12</sup> Fama and French (1998) and Shyam-Sunder and Myers (1994) have not found evidence of the tradeoff theory in firm financing decisions.

### 3. Model of optimal capital structure

The objective of this paper is to introduce a closed-form model, which is able to produce an optimal capital structure that maximizes total firm value. The model is calibrated to reflect actual firm characteristics and uses contingent-claim methods to value tax shields and costs of financial distress associated with leverage. It also maintains a long-run target debt to assets ratio by refinancing maturing debt; this provides a dynamic setting for capital structure choice.

Several papers have emerged dealing this subject with similar contingent-claim models.<sup>13</sup> They all present similar characteristics concerning model calibration and the assumption of a tradeoff between tax shields and costs associated with leverage. However, the model presented in this paper differs from previous studies. The single most decisive factor is the valuation method of required returns on claims issued by a firm. Our model is entirely based on option theory and therefore completely new. We are now able to simultaneously tackle many different factors that affect capital structure choice and combine capital structure theories. Another reason, which makes the application of option theory appealing, is that we are also able to extract market values and expected returns from within the model as firm leverage is changed.

Firms operating in different industries face very different business environments. In cyclical industries the annual volatility of cash flows is higher and firms in capital-intensive industries have usually preferred debt with longer maturity. Dividend policies also differ from one firm to another as well as their expected future growth.<sup>14</sup> It is therefore crucial for capital structure models to take these different variables into account when studying different firms. Our option theory approach enables the model flexibility to adjust quickly when examining different industries or even firms within a certain industry.

---

<sup>13</sup> As mentioned in the introduction.

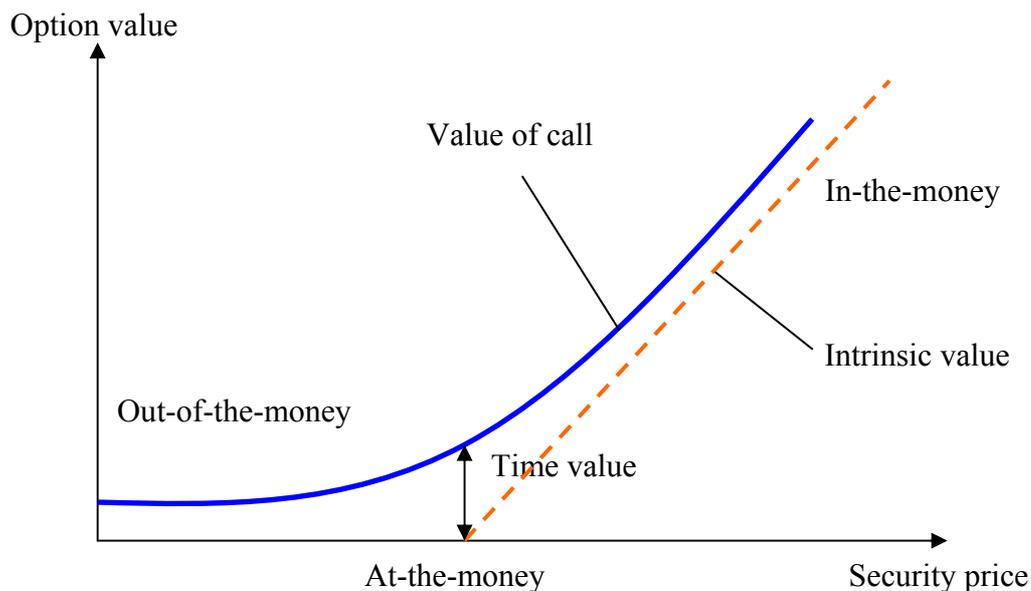
<sup>14</sup> Firms in growing industries should retain a bigger fraction of their profits in order to invest in  $NPV > 0$  projects. On the other hand, agency theory suggests firms in slowly growing industries should distribute a considerable fraction of profits as dividends, according to Jensen (1986).

### 3.1 Basics of option theory

An option is a security giving the right to buy or sell an asset, subject to certain conditions, within a specified period of time. This right is given by the issuer or writer of the option. A call option gives the right to buy the security, while a put option gives the right to sell the security. In order to give effect to the right to buy or sell, the option has to be exercised. This can be done either only at the date of expiry if the option is a “European”, whereas an “American” option can be exercised at any time before the date of expiry. In return for the insurance offered by the option contract, the buyer has to pay the issuer an option premium, which consists of two components:

$$\text{Option premium} = \text{Intrinsic value} + \text{Time value} \quad (3.1)$$

Intrinsic value exists if the current stock price is higher than the exercise price (in-the-money). Therefore an investor would be able to make a certain profit by buying options. However, if the stock price is lower than the exercise price (out-of-the-money), the option has no intrinsic value, but this does not mean that the option is worthless. Depending on the possible future performance of the stock, the option does have time value. We can illustrate this as:



**Figure 3.1** Call option value before expiration

There are five factors, which influence the option premium. Since we are focusing on European options,<sup>15</sup> we will only consider factors influencing them. The premium for a call depends on:

$$C = F(S, X, T, r, \sigma) \quad (3.2)$$

where

- $C$  = premium on a European call
- $S$  = spot price of the underlying asset
- $X$  = exercise price
- $T$  = time to expiry
- $r$  = riskless rate of return
- $\sigma$  = volatility of the return on the underlying asset

It is obvious, that the higher the security price, the more valuable the call option. Therefore  $C$  is positively related to  $S$ . The opposite applies to the exercise price, so  $X$  and  $C$  are negatively related. In turn, the maturity  $T$  and  $C$  are positively related. An increase in the riskless rate of interest  $r$  increases the option value because the money saved by purchasing the option rather than the security itself can be invested at the riskless rate until the option expires. Therefore, an increase in  $r$  increases the attractiveness of holding the option relative to the underlying security and therefore raises the option premium.

An increase in the volatility of the asset value also increases the option premium. This is due to the increased probability that the security price will lie in the tails of the distribution of the security price when the option expires. Since the distribution above the exercise price is relevant, the potential gains on the option are unlimited. If we compare two different underlying securities with options written on them at the same conditions, same expiry and exercise price, the security with a higher volatility will produce a higher premium on the option. This is called stochastic dominance and implies that if one option payoff pattern is stochastically dominates that of another, the option will have a higher value (Blake 2000, 304).

---

<sup>15</sup> This paper will concentrate on European options, because they cannot be exercised beforehand. The same intuition applies for corporate debt; it does not expire before the actual payback date.

Dividend payments will also affect the option premium if they occur before the expiration. When the security becomes ex-dividend, the security's price will be lower even though the option holders are not entitled to receive any dividends unless they also own the actual security. When the stock becomes ex-dividend, the security price goes down and this will lower the value of the option.<sup>16</sup>

Changes to a company's capital structure can also affect the security price. Examples are capitalization issues, rights issues, demergers, takeovers and consolidations. In such cases, the exchange will alter the exercise price or the number of securities underlying the option but will leave both optionholders and writers unaffected by the changes made.

### 3.2 Option valuation

Before we introduce an exact formula for pricing options, we need to consider conditions, which affect option premiums. In order to value the premium, we must introduce a few boundary conditions, which restrict the possible premium values. First, the option must have a positive value throughout its life because there is some possibility that the option will expire in the money. Therefore it must be that:

$$C \geq 0 \quad (3.3)$$

Second, a call option cannot be worth more than the underlying security, that is, no one will pay more for the option on a security than the security itself. Therefore it must be that:

$$C \leq S \quad (3.4)$$

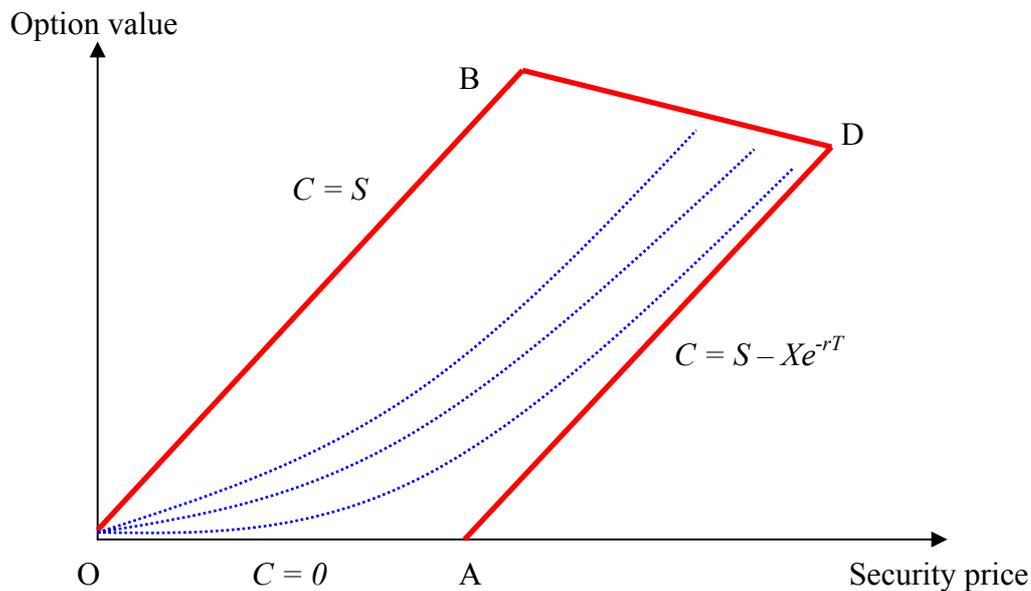
---

<sup>16</sup> Dividend payments will increase the value of a put option written on the same security. This will in turn affect the required returns on the company's liabilities.

Third, the value of a call option will always be greater than the value of the security price minus the discounted value of the exercise price:

$$C \geq S - Xe^{-rT} \quad (3.5)$$

These conditions are combined in figure 3.2, which shows the boundary conditions that apply for a European call option. The option premium will lie in the area OADB restricted by the above conditions.



**Figure 3.2** Boundary conditions for a European call Source: Blake (2000)

Once the value of a European call option is known,<sup>17</sup> it can be used to calculate the value of a European put option written on the same underlying security with the same exercise price and expiration date. This relationship is called put-call parity and was first introduced by Stoll (1969).

<sup>17</sup> The premium can be calculated either by using a discrete-time binomial model by Cox, Ross and Rubenstein (1979) or by using a continuous-time diffusion process model by Black and Scholes (1973). American calls have to be calculated with the binomial method.

While assuming perfect markets, it is possible to create a riskless hedge portfolio by combining long positions in the security and the put option with a short position in the call option (same exercise price  $X$  and expiry period  $T$  as put option). At expiration, if the price of the security equals or exceeds the exercise price, then the value of the portfolio is:

	Value of security	$S$
+	Value of put option	$0$
-	Value of call option	$-(S - X)$
<hr/>		
=		$X$

Then there is the alternative case. If the share price is less than the exercise price when the call expires, the value of the portfolio is:

	Value of security	$S$
+	Value of put option	$X - S$
-	Value of call option	$0$
<hr/>		
=		$X$

In either case, the value of the portfolio at expiration is  $X$ . The portfolio is completely riskless and will earn a risk-free return. Therefore, the value of the portfolio at the beginning of the period is  $Xe^{-rT}$ , where the exercise price is discounted with the riskless rate of return until expiration. For this reason the following relationship (put-call parity) must hold for European, non-dividend paying options:<sup>18</sup>

$$P = C - S + Xe^{-rT} \quad (3.6)$$

From this expression we can deduce, that buying a put is equal to buying a call, short-selling the share and investing the proceeds in a safe asset. This relationship will prove essential for estimating the cost of debt financing and the effects of conflicts between debt and equityholders, as we will see in later sections.

---

<sup>18</sup> If the stock makes dividend payments before the final exercise date, the investor who buys the calls misses out on this dividend. In this case the relationship is:  $C + PV(X) = P + S - PV(\text{dividends})$

### 3.3 Option theory and firm's balance sheet

A firm's balance sheet is comprised of real and financial assets, which produce cash flows in the future. These assets are financed by either equity, debt or both depending on the company. On the left hand side are the real assets and on the right hand side the sources of finance. This can be illustrated with a market value balance sheet:

<b>Current assets</b>	<b>Current liabilities</b>
Cash	Accounts payable
Inventory	Short-term debt
Accounts receivable	Long-term debt
Plants and buildings	Preferred stock
Equipment	Equity
<b>Value of firm</b>	<b>Value of firm</b>
<b><math>V = A</math></b>	<b><math>D + E = A</math></b>

**Figure 3.3** Market value balance sheet

It is generally realized, that corporate liabilities other than warrants may be viewed as options. Consider a company, which decides to lever its assets and it therefore issues a bond, which is sold to investors. When the debt matures the equityholders face the choice of either paying back the loan or to default in which case the equity would become worthless and the firm would fall into the hands of the debtholders. It is clear, that the equityholders have the equivalent of an option on their company's assets. In effect, the debtholders own these assets, but they have given options to the equityholders to buy them back.

When the debt matures, equity value will be the value of the company's assets minus the face value of the debt, or zero, whichever is greater. If the firm's asset value is under the current expiring debt value, default is imminent. On the other hand, if the assets are worth more than the debt, it is naturally in the best interest of the equityholders to pay off the maturing debt and in essence exercising their option on the firm's assets. Now the ownership is transferred from the debtholders back to the equityholders.

Within the option theory, all the variables will be redefined. The call option becomes equity, stock price becomes firm's assets and the option's exercise price becomes the promised payment to debtholders, that is, the nominal value of the debt in the balance sheet (Brealey & Myers 1998, 594). Applying the put-call parity this relationship can be illustrated as:

$$\begin{aligned}
 \text{Value of firm} &= \text{Value of firm's assets} \\
 \text{Market value of debt} &= \text{Present value of promised payment to} \\
 &\quad \text{debtholders} - \text{value of put} \\
 \text{Market value of equity} &= \text{Value of call} \\
 &= \text{Value of firm's assets} - \text{present value of promised} \\
 &\quad \text{payment to debtholders} + \text{value of put}
 \end{aligned}$$

When a firm is a risky borrower, it is also subjected to some probability of default. The firm's risky debt is equal to a safe debt less the value of the equityholders' option to default, which is equal to the put option on the firm's assets. In other words, the put value can also be defined as the expected loss to debtholders.

The growth in the put's value causes an asset substitution between equity and debtholders. The value of the put increases the equity value. Although the firm value as a whole does not increase, it does alter the relative values of equity and debt. This effect is realized as a capital loss for debtholders and a capital gain for equityholders. Needless to say, the asset substitution poses a threat to lenders as section two indicated. Therefore different covenants and indenture clauses are typically associated with debt issues.

Leland (1994) has presented a study concerning bond covenant effects on yield spreads, bond valuation and capital structure optimum. He finds that short-term debt financing seems to correspond to protected debt, whereas long-term debt rarely has a positive net-worth covenant and therefore implies a status of unprotected debt. He also concludes that protected debt values and unprotected investment grade debt values behave similarly (leverage increases expected return). However, unprotected junk bonds exhibit quite different behavior as leverage increased. Here, an increase in firm risk would actually increase debt value as will a decrease in the coupon.

### 3.4 Market values of equity and debt

This paper utilizes a continuous-time framework for analyzing relative market values for debt and equity and their expected returns. Because of this framework and also for simplicity, the binomial option pricing model is excluded. In order to apply the Black–Scholes option pricing formula for valuation, we will assume “ideal conditions”<sup>19</sup> in the market:

- 1) The short-term interest rate  $r$  is known and is constant through time.
- 2) The asset price follows a random walk in continuous time with a variance rate proportional to the square of the asset price. Thus the distribution of possible asset prices at the end of any finite interval is log-normal. The variance rate of the return on the assets is constant.
- 3) The stock pays no dividends or other distributions.<sup>20</sup>
- 4) The option is European, that is, it can only be exercised at maturity.
- 5) There are no transaction costs in buying or selling the stock or the option.
- 6) It is possible to borrow any fraction of the price of the security to buy it or to hold it, at the short-term interest rate.
- 7) There are no penalties to short selling.

Under these assumptions, the value of the option (equity) will depend only on the price of the stock (assets), time and on variables, which are taken to be known constants. It is therefore possible to create a hedged position, consisting of a long position in the stock (assets) and a short position in the option. Writing  $w(A,T)$  for the value of the option as a function of the asset price  $A$  and time  $T$ , the number of the options that must be sold short against on share of asset long is:

$$\frac{1}{w_1(A,T)} \tag{3.7}$$

---

<sup>19</sup> These ideal conditions are not very often observed in the capital markets since return distributions are not actually normally distributed. However, within a theoretical model we can accept these deviations.

<sup>20</sup> Dividends can be taken into account by subtracting them from the asset value before calculating the option premium.

In this expression, the subscript refers to the partial derivative of  $w(A,T)$  with respect to its first argument. Since the hedged position contains one share of stock long and  $1/w_1$  options short, the value of the equity in the position is:

$$A - \frac{w}{w_1} \quad (3.8)$$

The change in the value of the equity position in a short interval  $\Delta T$  is therefore:

$$\Delta A - \frac{\Delta w}{w_1} \quad (3.9)$$

By using stochastic calculus,  $\Delta w$  which is  $w(A + \Delta X, T + \Delta T) - w(A,T)$ , can be expanded as follows:

$$\Delta w = w_1 \Delta A + \frac{1}{2} w_{11} \sigma^2 A^2 \Delta T + w_2 \Delta T \quad (3.10)$$

In equation (3.10), the subscripts on  $w$  refer to the partial derivatives and  $\sigma^2$  is the variance of the return on the assets or the variance of their value. Substituting from equation (3.10) into expression (3.9), the change in the value of the equity in the hedged position can be written as:

$$-\left( \frac{1}{2} w_{11} \sigma^2 A^2 + w_2 \right) \cdot \frac{\Delta T}{w_1} \quad (3.11)$$

Since the return on equity in the hedged position is certain, the return must be equal to  $r\Delta T$ . If this were not true, speculators would try to profit by borrowing large amounts to create such hedged positions and in the process force the returns down to the short-term interest rate.

Due to arbitrage by market participants, abnormal profits disappear and the change in the equity value (3.11) must equal the value of the equity (3.8) times  $r\Delta T$ :

$$-\left(\frac{1}{2}w_{11}\sigma^2 A^2 + w_2\right) \cdot \frac{\Delta t}{w_1} = \left(A - \frac{w}{w_1}\right) \cdot r\Delta t \quad (3.12)$$

Eliminating  $\Delta T$  from both sides and rearranging, the differential equation for the value of the option can be written as:

$$w_2 = rw - rAw_1 - \frac{1}{2}\sigma^2 A^2 w_{11} \quad (3.13)$$

Writing  $t^*$  for the maturity date of the option and  $X$  for the exercise price, boundary conditions for possible option premiums can be written as:

$$\begin{aligned} w(A, t^*) &= A - X, & A &\geq X \\ &= 0, & A &< X \end{aligned} \quad (3.14)$$

There is only formula  $w(A, T)$ , that satisfies the differential equation (3.13) subject to the boundary conditions above and therefore this formula is the option valuation formula. In order to solve this differential equation, the following substitution has to be made. By writing  $w(A, T)$  as:

$$\begin{aligned} w(A, T) &= e^{r(T-t^*)} y \left[ \left( \frac{2}{\sigma^2} \right) \cdot \left( r - \frac{1}{2}\sigma^2 \right) \right. \\ &\quad \left. \left[ \ln \frac{A}{X} - \left( r - \frac{1}{2}\sigma^2 \right) \cdot (T - t^*) \right], \right. \\ &\quad \left. - \left( \frac{2}{\sigma^2} \right) \cdot \left( r - \frac{1}{2}\sigma^2 \right)^2 \cdot (T - t^*) \right] \end{aligned} \quad (3.15)$$

With this substitution, the differential equation (3.13) becomes simply  $y_2 = y_{11}$ .

The boundary condition (3.14) will also be rewritten as:

$$\begin{aligned}
 y(u,0) &= 0, & u < 0 \\
 &= X \left[ e^{\frac{u(\frac{1}{2}\sigma^2)}{(r-\frac{1}{2}\sigma^2)} - 1} \right], & u \geq 0.
 \end{aligned} \tag{3.16}$$

The differential equation  $y_2 = y_{11}$  is equal to the heat-transfer equation of physics and its solution in this notation is:

$$y(u,s) = \frac{1}{\sqrt{2\pi}} \int_{-\frac{u}{\sqrt{2s}}}^{\infty} c \left[ e^{\frac{(u+q\sqrt{2s})(\frac{1}{2}\sigma^2)}{(r-\frac{1}{2}\sigma^2)} - 1} \right] \cdot e^{-\frac{q^2}{2}} dg \tag{3.17}$$

Substituting from equation (3.17) into equation (3.15) and simplifying, we find the familiar form of option pricing equation:<sup>21</sup>

$$C = S \cdot N(d_1) - X \cdot e^{-rT} \cdot N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{Xe^{-rT}}\right)}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2} \tag{3.18}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where

- $C$  = value of call - option
- $S$  = value of firm's stock
- $X$  = strike price of option
- $r$  = risk - free rate of return
- $T$  = maturity of option
- $\sigma$  = volatility of stock price
- $N(d_i)$  = cumulative normal distribution

(Black & Scholes, 1973)

---

<sup>21</sup> This presentation is a scaled down version of the original Black and Scholes (1973) paper.

This is known as the Black–Scholes equation, which was introduced in 1973. It is widely used by option dealers every day throughout the world as they execute deals worth billions of dollars. The attractiveness of this option valuation method is its simplicity and flexible use for the purpose of studying optimal capital structure. This enables us to model the effects on pricing claims issued by the firm when capital structure is changed. The equation’s continuous-time assumption is also ideal for our model, which assumes a dynamic refinancing scheme when debt matures.

In order to apply the Black–Scholes formula for equity and debt valuation, the equation variables have to be redefined. The exercise price  $X$  becomes the debt liabilities (outstanding debt), option’s time to maturity  $T$  becomes the debt maturity, risk-free rate of return  $r$  is the return on government bonds with the corresponding maturity  $T$ , stock price  $S$  becomes assets in the balance sheet (market value of firm) and volatility  $\sigma$  is the business cycle smoothed volatility of free cash flow. All of these variables can easily be observed or calculated, compared to signaling or asymmetric information factors. Next, we will extract expected returns for both claims.

### 3.5 Leverage and cost of debt

MMI –proposition holds that corporate debt faces the same miniscule default risk as government bonds, or risk-free bonds. However, this is not true in the real world. Investors expect higher return on debt, which is associated with any probability of default. The value and the expected return of a particular issue of corporate debt depend essentially on three items:

- 1) The required rate of return on risk-free debt (government bonds or very high grade corporate bonds)
- 2) The various provisions and restrictions contained in the indenture (maturity, coupon rate, call terms, seniority, etc.)
- 3) The probability that the firm will be unable to satisfy some or all of the indenture requirements (that is, the probability of default)

Suppose there exists a security whose market value  $Y$  at any point in time can be written as a function of the value of the firm and time,  $Y = F(V,T)$ . The dynamics of this security's value can then formally be written in a stochastic differential equation form as:

$$dY = [\alpha_y Y - C_y] \cdot dT + \sigma_y Y \cdot dz_y \quad (3.19)$$

where  $\alpha_y$  is the instantaneous expected rate of return per unit time on the security,  $C_y$  is the euro payout per unit of time of the security,  $\sigma_y^2$  is the instantaneous variance of the return per unit of time and  $dz_y$  is a standard Gauss-Wiener process. There is an explicit functional relationship between the  $\alpha_y$ ,  $\sigma_y^2$  and  $dz_y$  in (3.19) and the corresponding variables  $\alpha$ ,  $\sigma$  and  $dz$ . Applying Ito's Lemma, the dynamics for the security's value  $Y$  can be rewritten as:

$$dY = F_V dV + \frac{1}{2} F_{VV} (dV)^2 + F_T \left[ \frac{1}{2} \sigma^2 V^2 F_{VV} + (\alpha V - C) \cdot F_V + F_T \right] \cdot dT + \sigma V F_V \cdot dz \quad (3.20)$$

where subscripts denote partial derivatives. Comparing terms in (3.20) and (3.19), we have that:

$$\alpha_y Y = \alpha_y F \equiv \frac{1}{2} \sigma^2 V^2 F_{VV} + (\alpha V - C) \cdot F_V + F_T + C_y$$

$$\sigma_y Y = \sigma_y F \equiv \sigma V F_V \quad (3.21)$$

$$dz_y \equiv dz$$

Now consider a composition of a three-security portfolio containing the firm, the particular security and a risk-free debt such, that the aggregate investment in the portfolio is zero. This can be achieved by using the proceeds of short-sales and borrowings to finance the long positions in different securities.

If  $dx$  is the instantaneous euro return to the portfolio, then:

$$\begin{aligned}
 dx &= W_1 \cdot \frac{(dV + C \cdot dT)}{V} + W_2 \cdot \frac{(dY + C_Y \cdot dT)}{Y} + W_3 r \cdot dT \\
 &= [W_1 \cdot (\alpha - r) + W_2 \cdot (\alpha_Y - r)] \cdot dT + W_1 \sigma \cdot dz + W_2 \sigma_Y \cdot dz_Y \\
 &= [W_1 \cdot (\alpha - r) + W_2 \cdot (\alpha_Y - r)] \cdot dT + [W_1 \sigma + W_2 \sigma_Y] \cdot dz
 \end{aligned} \tag{3.22}$$

where  $W_1$  is the number of euros of the portfolio invested in the firm,  $W_2$  is the number of euros invested in the security and  $W_3$  ( $= -[W_1 + W_2]$ ) is the number of euros invested in risk-free (government) debt.

A portfolio strategy ( $W_j = W_j^*$ ) is chosen so, that the coefficient of  $dz$  is always zero and then the euro return on the portfolio would be non-stochastic. Since the portfolio requires zero net investment, it must be so in order to avoid arbitrage profits, the expected return on the portfolio with the chosen strategy is zero. Therefore:

$$W_1^* \sigma + W_2^* \sigma_Y = 0 \quad (\text{no risk}) \tag{3.23}$$

$$W_1^* \cdot (\alpha - r) + W_2^* \cdot (\alpha_Y - r) = 0 \quad (\text{no arbitrage})$$

A nontrivial solution to (3.23) exists if and only if:

$$\left( \frac{\alpha - r}{\sigma} \right) = \left( \frac{\alpha_Y - r}{\sigma_Y} \right) \tag{3.24}$$

We can substitute from (3.21) for  $\alpha_Y$  and  $\sigma_Y$  and rewrite (3.24) as:

$$\frac{\alpha - r}{\sigma} = \left( \frac{1}{2} \sigma^2 V^2 F_{VV} + (\alpha V - C) \cdot F_V + F_T + C_Y - rF \right) / \sigma V F_V \tag{3.25}$$

By rearranging terms and simplifying, (3.25) can be rewritten as:

$$0 = \frac{1}{2} \sigma^2 V^2 F_{VV} + (rV - C) \cdot F_V - rF + F_T + C_Y \quad (3.28)$$

This equation is a parabolic partial differential equation for  $F$ , which must be satisfied by any security whose value can be written as a function of the value of the firm and time. A complete description of the equation needs two boundary conditions and an initial condition. These boundary condition specifications distinguish one security type from another, that is, debt from equity.

It can be easily noticed which variables and parameters appear in the equation and therefore identify the reasons for changes in a security's value. In addition to the value of the firm and time,  $F$  depends on the interest rate, the volatility of the firm's asset value (or its business risk), the payout policy of the firm and the promised payout to the holders of the security.

However,  $F$  does not depend on the expected rate of return on the firm nor on the risk-preferences of investors nor on the characteristics of other assets available to investors beyond the three mentioned earlier. Therefore, two investors who agree on the volatility of the firm's value will for a given interest rate and current firm value, agree on the value of the particular security. This is assumed by the implication of perfect markets; investors have homogenous expectations of future firm prospects.

In the context of this paper we will assume the simplest case of corporate debt pricing, a risky discount bond.<sup>22</sup> Suppose the firm has two classes of claims:

- 1) A single, homogeneous class of debt
- 2) The residual claim, equity

---

<sup>22</sup> There is also a modification for a coupon paying debt. See Merton (1973).

We will further suppose that the indenture of the bond issue contains the following provisions and restrictions:

- 1) The firm promises to pay a total of B euros to the bondholders on the specified date
- 2) In the event this payment is not met, the bondholders immediately take over the company rendering the equity worthless
- 3) The firm cannot issue any new senior claims on the firm nor can it pay cash dividends (salvage assets) or do share repurchases prior to the maturity of the debt

If  $F$  is the value of the debt issue, the parabolic partial differential equation (3.26) can be rewritten as:

$$\frac{1}{2}\sigma^2V^2F_{VV} + rVF_V - rF - F_T = 0 \quad (3.27)$$

where  $C_V = 0$  because there are no coupon payments (the debt is considered a bullet-debt),  $C = 0$  from restriction (3.21) and  $T$  is length of time until the debt matures.

To solve for the value of the debt, two boundary conditions and an initial condition must be specified as we mentioned earlier. These conditions are derived from the provisions of the indenture and the limited liability of claims. Since we have the value of the company  $V = F(V,T) + E(V,T)$ , where  $E$  is the value of the equity, and both debt and equity can only take non-negative values, we have that:

$$F(0,T) = f(0,T) = 0 \quad (3.28)$$

$$\frac{F(V,T)}{V} \leq 1$$

On the maturity date  $T$ , the firm must either pay the promised payment, which is the face value of  $B$  to the bondholders or else the current equity becomes worthless due to default. If at maturity  $V(T) > B$ , the firm should pay the bondholders because the value of equity will be  $V(T) - B > 0$ . If the firm is unable to pay the bondholders, that is if  $V(T) < B$ , then the firm will default on its loans unless equityholders invest additional

funds to save the company from bankruptcy. Therefore, the initial condition for the debt at  $T = 0$  is:

$$F(V,0) = \min[V, B] \quad (3.29)$$

We now have the boundary conditions and the initial condition. To determine the value of the equity  $E(V,T)$ , we note that  $E(V,T) = V - F(V,T)$  and substitute for  $F$  in (3.27) and (3.28) in order to deduce the partial differential equation for equity:

$$\frac{1}{2}\sigma^2V^2f_{VV} + rVf_V - rf - f_T = 0 \quad (3.30)$$

Subject to:

$$f(V,0) = \max[0, V - B] \quad (3.31)$$

and boundary conditions (3.28); we can see that (3.29) and (3.30) are identical to the equations for a European call option on a non-dividend-paying common stock, where firm value in (3.29) and (3.30) corresponds to stock price and  $B$  corresponds to the exercise price. This relationship between levered equity and the call option allows us to write down the solution to (3.29) and (3.30) directly. From the Black–Scholes equation (3.18) we have that:

$$C = S \cdot N(d_1) - Xe^{-rT} \cdot N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{Xe^{-rT}}\right)}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

From (3.19) and  $F = V - E$ , we can simplify and write the value of the debt issue as:

$$F(T) = B \cdot e^{-rT} \left[ \left( \frac{1}{d} \right) \cdot N(h_1) + N(h_2) \right]$$

$$h_1 = \frac{-\left[ \frac{1}{2} \sigma^2 T - \ln(d) \right]}{\sigma \sqrt{T}} \quad (3.32)$$

$$h_2 = \frac{-\left[ \frac{1}{2} \sigma^2 T + \ln(d) \right]}{\sigma \sqrt{T}}$$

where

- $B$  = face value of bond
- $T$  = maturity
- $d = B \cdot e^{-rT} / A$
- $\sigma^2$  = variance of firm's asset value
- $r$  = risk - free rate of return
- $A$  = value of assets
- $N(h_i)$  = cumulative normal distribution (Merton, 1973)

Since this paper also focuses on the required returns on different securities, we can rewrite (3.32) in terms of spread between the risk-free bond and the risky corporate bond as:

$$r_D - r = -\frac{1}{T} \ln \left[ N(h_2) + \left( \frac{1}{d} \right) \cdot N(h_1) \right] \quad (3.33)$$

As we can see from (3.33), for a given maturity, the risk premium is a function of only two variables on which all investors homogeneously agree upon:

- 1) The volatility  $\sigma$  of the firm's asset value or cash flows
- 2) The ratio  $d$  of the present value (at the risk-free rate) of the promised payment to debtholders and the current value of the firm

Within our option theory approach and with the help of put–call parity, the required spread (3.33) can also be written in a much simpler form as:

$$\begin{aligned}
 r_D - r &= -\frac{1}{T} \ln \left[ 1 - \frac{P}{PV(X)} \right] \\
 &= -\frac{1}{T} \ln \left[ 1 - \frac{P}{Xe^{-rT}} \right]
 \end{aligned}
 \tag{3.34}$$

The theoretical put option value we derived earlier in section three is in fact the expected loss in euros to the bondholders. As  $P$  becomes higher in comparison to the promised payback to debtholders, therefore implying a higher risk of default, the required premium also increases. As a result, the firm is able to refinance itself with worse rates and the market value of the debt declines.

### 3.6 Leverage and cost of equity

Mehra and Prescott (1985) have been the first to observe an equity premium puzzle in the capital markets. Especially after the 1950s, the realized return on equity has consistently been above the expected rate of return derived from dividend and earnings growth models. Fama and French (2001) have reported similar findings. The average real return for 1872–2000 on the S&P index is 8,81 % per year. The average real return on six-month commercial paper (a proxy for the risk-free interest rate) is 3,24 % indicating an equity premium of 5,57 %. The estimate of the expected real equity premium for 1872–2000 from the dividend growth model is 3,54 % per year. The estimate from the average stock return, 5,57 %, is almost 60 % higher. The difference in estimated and realized returns between the two is largely due to the last fifty years.

The equity premium for 1872–1950 from dividend growth models, 4,17 % per year, is close to the estimate from the realized average return, 4,40 %. In contrast, the equity premium for 1951–2000 produced by the average return, 7,43 % per year, is almost three times the dividend estimate, 2,55 %. The estimate of the expected real equity premium for 1951–2000 from the earnings growth model, 4,32 % per year, is larger

than the estimate from the dividend growth model. But the earnings growth estimate is still less than 60 % of the estimate from the average return. (Fama & French 2001)

The competition between the dividend growth model and the average stock return is more interesting for 1951–2000 period. The dividend growth estimate of the expected return, 4,74 %, is less than half the realized average return, 9,62 %. The dividend growth estimate of the equity premium 2,55 % is 34 % of the estimate from returns, 7,43 %. The 1951–2000 estimates of the expected stock return and the equity premium from the earnings growth model, 6,51 % and 4,32 %, are higher than for the dividend growth model. But they are well below the estimates from the realized average returns, 9,62 % and 7,43 %. (Fama & French 2001)

It has been argued that the realized equity premiums are due to higher risk. In asset pricing theory, the Sharpe ratio is related to aggregate risk aversion. Fama and French found that the ratios for the 1872–1950 and 1951–2000 equity premiums from the dividend growth model and the earnings growth model suggest that aggregate risk aversion is roughly similar in the two periods. Therefore, higher returns are not explained by investments in securities in higher risk class.

They also suggest that much of the high return for 1951–2000 is unexpected capital gain. Most of the decline in the price ratios seems to be due to the unexpected decline of expected returns and for the values to end far below the mean. Their evidence suggests that rational forecasts of long-term dividend and earnings growth rates are not unusually high in 2000 and they therefore conclude that the large spread of 1951–2000 capital gains over dividend and earnings growth is largely due to a decline in the expected stock return. Since the majority of the sample firms were using debt financing, this would seem to imply that expected return on equity might not in fact follow a linear relationship with firm leverage as MMII–proposition indicates.

Our method of estimating the equity premium in a levered company is based on the same idea as in equation (3.33). Instead of using a theoretical value of a put option, we adopt a figure, which will estimate the expected loss to equityholders in case of default. For this we need a theoretical probability of default in order to deduce the expected loss.

Our approach to calculating the default risk is based on the work of Vassalou and Xing (2004). We can calculate the estimated probability of default at any given leverage level and extract the expected loss due to default. Our method is again based on the same methodology used in previous sections. The default probability is the probability, that the firm's assets will less than the book value of the firm's liabilities. In other words, the default probability at time t is:

$$P_{def,t} = \Pr ob(V_{A,t+T} \leq X_t | V_{A,t}) = \Pr ob[\ln(V_{A,t+T}) \leq \ln(X_t) | V_{A,t}] \quad (3.35)$$

By assuming, that the asset value of the company again follows a geometric Brownian motion with constant drift and volatility, the value of the firm's assets at any time t is given by:

$$\ln(V_{A,t+T}) = \ln(V_{A,t}) + \left( \mu - \frac{\sigma_A^2}{2} \right) \cdot T + \sigma_A \sqrt{T} \cdot \varepsilon_{t+T}$$

$$\varepsilon_{t+T} = \frac{W(t+T) - W(t)}{\sqrt{T}} \quad (3.36)$$

$$\varepsilon \sim N(0,1)$$

Therefore, we can write the default probability as:

$$P_{def,t} = \Pr ob \left[ \ln(V_{A,t}) - \ln(X_t) + \left( \mu - \frac{\sigma_A^2}{2} \right) \cdot T + \sigma_A \sqrt{T} \cdot \varepsilon_{t+T} \leq 0 \right] \quad (3.37)$$

$$= \Pr ob \left[ - \frac{\ln \left( \frac{V_{A,t}}{X_t} \right) + \left( \mu - \frac{\sigma_A^2}{2} \right) \cdot T}{\sigma_A \sqrt{T}} \geq \varepsilon_{t+T} \right]$$

From this we can then define the distance to default as:

$$DD_t = \frac{\ln\left(\frac{V_{A,t}}{X_t}\right) + \left(\mu - \frac{1}{2}\sigma_A^2\right) \cdot T}{\sigma_A \sqrt{T}} \quad (3.38)$$

Default occurs when the ratio of the asset value to debt liabilities is less than one, or its logarithm is negative. The distance to default tells us by how many standard deviations the log of this ratio needs to deviate from its mean in order for default to occur. An important notice is that DD depends on the drift of the assets. In other words, DD depends on the future value of the assets (Vassalou and Xing 2004). Since our model is based on constant growth measured in long run, the drift is the same as the required return on unlevered equity, which is the real drift the firm's assets are able to produce.

Since we are assuming the same theoretical distribution used in Black–Scholes option valuation and Merton's estimation of risky bond pricing, the theoretical probability of default will be given by:

$$P_{def} = N(-DD) = N\left[-\frac{\ln\left(\frac{V_{A,t}}{X_t}\right) + \left(\mu - \frac{1}{2}\sigma_A^2\right) \cdot T}{\sigma_A \sqrt{T}}\right] \quad (3.39)$$

Vassalou and Xing call this measure default likelihood indicator (DLI) instead of default probability since it does not correspond to the true probability of default in large samples. However, we will use this measure of default since it is consistent with the same assumptions utilized in other parts of this work. It is important to note that the difference between our measure of default risk and those produced by Standard & Poor's, Moody's or KMV. Their estimates are calculated using the empirical distribution of defaults. Our theoretical default probability of a firm is a positive nonlinear function of its default probability, that is, the required equity premium does not follow the linear function proposed by Modigliani and Miller (MMII).

As leverage increases and therefore the probability of default grows, the expected equity return increases slowly at first, but then starts to climb rapidly as default probability is increased. As a result, expected equity returns are at a lower level during reasonable leverage ratios, much closer to the ones reported by Fama and French (2001) and Mehra and Prescott (1985). The estimates lie well below the predictions of MMII–proposition. Furthermore, discarding older methods of estimating expected returns derived from the original MMI–proposition and applying a market-based model seems sensible and is in line with other assumptions behind option theory.

We can now estimate the required premium charged on the levered equity. If default occurs, the equity will be worthless. If the firm is able to pay off the maturing debt and refinance itself while keeping a target debt ratio, the equity’s market value will not change. By applying equation (3.34) with the same intuition as in valuing debt premiums, we can rewrite the equation in terms of equity premium:

$$r_L^E - r_U^E = -\frac{1}{T} \ln \left[ 1 - \frac{\text{expected loss to equity holders in levered firm}}{\text{present value of promised payment to unlevered equity}} \right] \quad (3.40)$$

$$= -\frac{1}{T} \ln \left[ 1 - \frac{E_L - (E_L \cdot (1 - P_{def}))}{E_U \cdot e^{-r_U^E T}} \right]$$

where

- $r_L^E$  = required return on levered equity
- $r_U^E$  = required return on unlevered equity
- $E_L$  = market value of levered equity
- $E_U$  = market value of unlevered equity
- $P_{def}$  = theoretical probability of default

The expected loss to equityholders is merely a probability weighted estimate (if firm defaults, equityholders receive nothing) of loss in euros whereas the promised payment to unlevered equity is the basis for the equity return in a default-free situation, that is, a firm which is not faced by costs of financial distress.

### 3.7 Total firm value

Now that we have been able to produce the relative market values for the firm's liabilities (equations 3.6 and 3.18) and the required returns for them (equations 3.34 and 3.40), we can calculate the cost of capital charged on the firm's assets. The objective is to maximize the tax shields while at the same time minimizing the costs of financial distress. At the optimum point, total firm value is maximized and the growth in tax shields is completely offset by increasing costs of financial distress.

By holding the firm's future cash flows constant and assuming that capital structure does not affect the firm's ability to produce cash flows, we can isolate the effect of capital structure change on the firm's value. Applying the after-tax weighted average cost of capital (WACC), we can estimate the correct discount rate for calculating the present value of the cash flows given changes in leverage.

$$WACC = \frac{E}{E + D} \cdot r_E + \frac{D}{E + D} \cdot r_D \cdot (1 - \tau_C) \quad (3.41)$$

where

- $E$  = market value of equity
- $D$  = market value of debt
- $r_E$  = return on equity
- $r_D$  = return on debt
- $\tau_C$  = corporate tax rate

The tax advantages of debt financing are implicitly taken into account in a lower discount rate.<sup>23</sup> The required returns and the relative values of claims issued by the firm change in a continuous fashion as leverage increases. When we hold all other parameters constant and change only the leverage, we are able to extract a concave curve, which presents firm value to investors while simultaneously taking into account the tax shields and costs of financial distress. In this paper, the WACC is merely a numerical manifestation of the tradeoff theory of capital structure.

---

<sup>23</sup> Some countries apply a progressive corporate tax rate, which changes the effective tax rate considerably. Our model is based on a constant tax rate unaffected by the profits.

The WACC is then applied to calculate the present value of the firm's free cash flow, that is, the cash that is left over to serve debt repayments and payoffs to equityholders. The free cash flow is defined as:

$$\begin{array}{r}
 \text{EBITDA} \\
 - \text{taxes} \\
 - \text{capital expenses} \\
 \underline{- \text{increase in working capital}} \\
 \text{FREE CASH FLOW}
 \end{array}$$

EBITDA stands for earnings before interest, taxes, depreciation and amortization.<sup>24</sup> Capital expenses are combined of investments in assets (mostly tangible) and working capital is the difference between the firm's short-term assets (inventories, accounts receivable) and liabilities (accounts payable).

It should be emphasized, that this work focuses on maximizing the whole firm value, instead of maximizing only the equity value. Since the objective is to find an optimal leverage ratio for the whole firm and provide support for a tradeoff theory, we will leave equity maximizing approaches to others. If the only objective is to increase equityholders wealth, it can be achieved by implementing operating decisions, which produce asset substitution. We on the other hand would like to focus on the determinants, which increases the firm's appeal to all investors. Titman and Tsyplakov (2004) present a model, where an equity value maximizing firm has a suboptimal recapitalization strategy when debt matures. This causes agency costs to be quite significant, which implies that static capital structure may considerably underestimate the effect and the value of the agency costs due to debtholder – equityholder conflicts.

---

<sup>24</sup> IFRS 3 has changed the way amortization affects EBITDA. Now goodwill is not written-off on a yearly basis, but it has to be tested annually in order to verify its contribution to the firm's future cash flows. Therefore, goodwill is "eternal", if there is no need for impairment charges.

#### 4. Comparative statics of model

In this section we will show the comparative statics of our optimal capital structure model. The total value of the firm is normalized to 100 when the assets are unlevered. Calibration is also needed, so the model requires estimates of:

1. Maturity of corporate debt
2. The risk-free rate of return on the corresponding maturity
3. The effective tax rate
4. The volatility of the cash flows or asset value
5. The level of dividends paid by the firm
6. The drift parameter for the total value of the firm

The volatility of cash flow and debt maturity are similar to some Finnish paper companies. The volatility is the business cycle smoothed annual rate of percentage volatility. The risk-free interest rate equals a Finnish government 7-year bond (April 2005) and the drift component is equal to the growth rate of assets in the long run, which is assumed to equal the expected return on unlevered equity. Dividend level is the long run dividend per asset ratio. The corporate tax rate is taken exogenously.

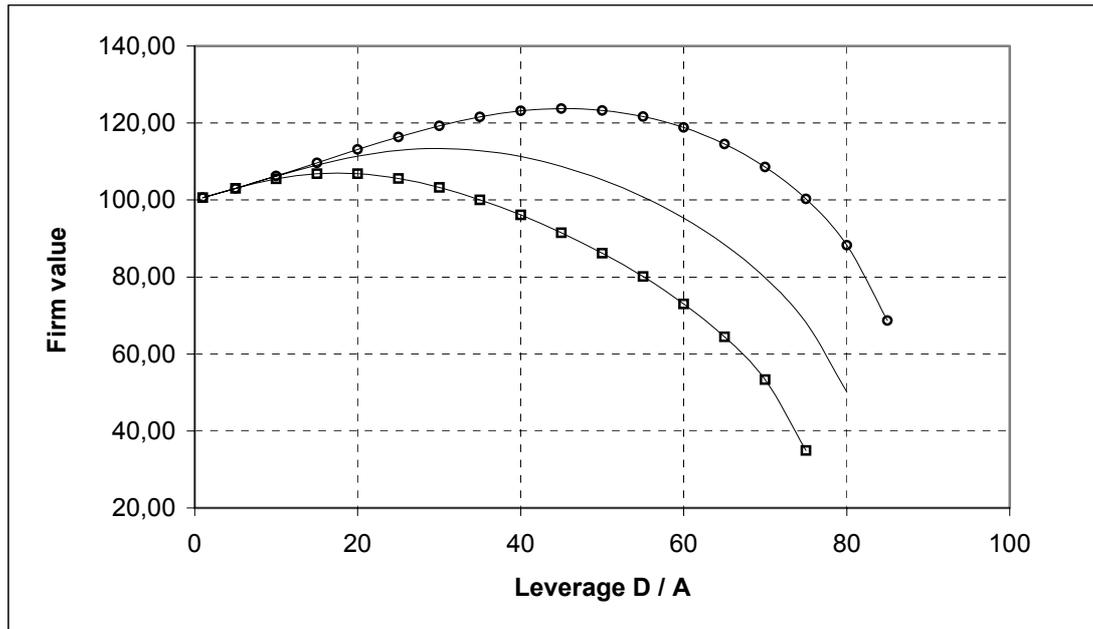
The applied base case parameters are:

Corporate tax rate	30,00 %
Debt maturity	7 years
Volatility	0,30
Risk-free return	3,50 %
Drift	10,00 %
Dividend level	2,00 %

Some studies have incorporated transaction costs, bankruptcy boundaries and recovery rates in contingent-claim models. We have however excluded them from this analysis. They will be left for future examination of this option theory framework.

#### 4.1 Asset value volatility

Figure 4.1 illustrates the effect of cash flow / asset value volatility on the value of the levered firm and the optimal leverage ratio. As would be expected, the riskier the firm, the less the advantage of debt and the lower the optimal leverage ratio. A riskier firm suffers from higher debt payments and expected equity return, because of higher default probability. Therefore, we observe lower optimal leverage ratios for riskier firms.



**Figure 4.1 Firm value and cash flow / asset value volatility.**

The lines plot firm value  $V$  for three levels of cash flow or asset value volatility  $\sigma$ : 25 percent (open circle), 30 percent (solid line) and 35 percent (square). Other parameters are same as base case.

With an annual asset value volatility of 25 percent, firm value increases to 123,73 when leverage ratio  $D / A$  is 0,45. For 30 percent, firm value can be increased to 113,38 at leverage ratio of 0,30. Finally, 35 percent volatility requires a leverage ratio of 0,18 to maximize firm value at 107,03.

Similar evidence of asset value volatility on firm value has been reported by several studies. Brennan and Schwartz's (1978) model produce a value graph, which presents same characteristics of volatility and value. Bradley et al. (1984) conclude that the volatility of firm earnings is an important, inverse determinant of firm leverage. They further argue that strong intra-industry similarities in firm leverage ratios and of persistent inter-industry differences, together with the highly significant inverse relation

between firm leverage and earnings, tend to support the modern balancing theory of optimal capital structure. However, Titman and Wessels (1988) fail to produce support for volatility influencing observed leverage ratios. They do admit, that their model may not capture the relevant aspects of the attributes effecting capital structure choice. This may be due to variables, which may not adequately reflect the nature of those attributes.

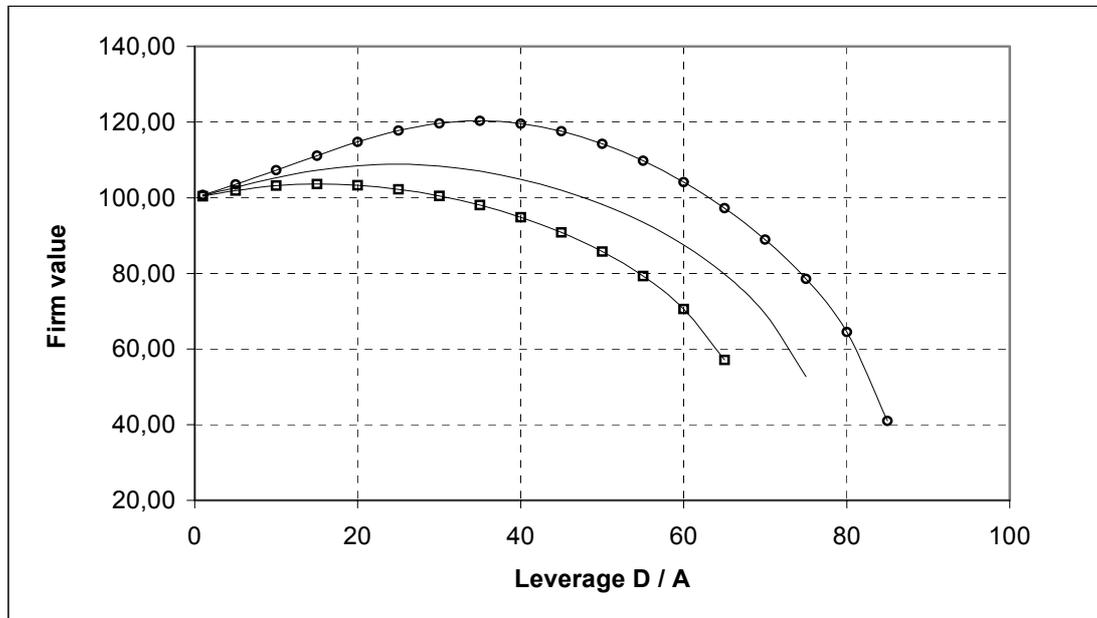
Leland (1994) and Leland and Toft (1996) derive findings, which are similar to ours and advocate, that the risk of asset substitution increases debtholders' expected returns. As a result, this implies increasing agency costs as volatility becomes higher. In return, optimal leverage ratio will decrease along with firm value. However, if the corporate bonds are rated as junk, volatility increase and higher interest rate may actually produce opposite results. In other words, debt values might increase. This is due to the very different way how junk bond duration changes. Instead of having a "normal" convex present value profile, junk bonds may well have concave profiles that produce such results.

According to Leland (1998), very different results may also appear when volatility changes. For realistic parameters, the agency costs of debt related to asset substitution are far less than the tax advantages of debt. Relative to an otherwise-similar firm that can precontract risk levels before debt is issued, the firm will choose a strategy with higher average risk. Leverage will be lower and debt maturity will be shorter. Yield spreads rise as the potential asset substitution increases. But relative to an otherwise-similar firm that has no potential for asset substitution, optimal leverage may actually rise. This contradicts the presumption that optimal leverage will fall when asset substitution is possible. (Leland 1998, 1237)

Even though volatility versus optimal leverage and firm value are regarded to have an inverse and mostly strong negative relation, opposing results have also been reported. Since accruing costs might force firms to deviate substantially from their optimal leverage, a leverage range is advocated to be more suitable for examining observed leverage ratios. If adjustment costs are realized and an optimal leverage range is applied, such as Fischer et al. (1989) have proposed, increased volatility widens the range and could in fact produce an optimal leverage which is higher.

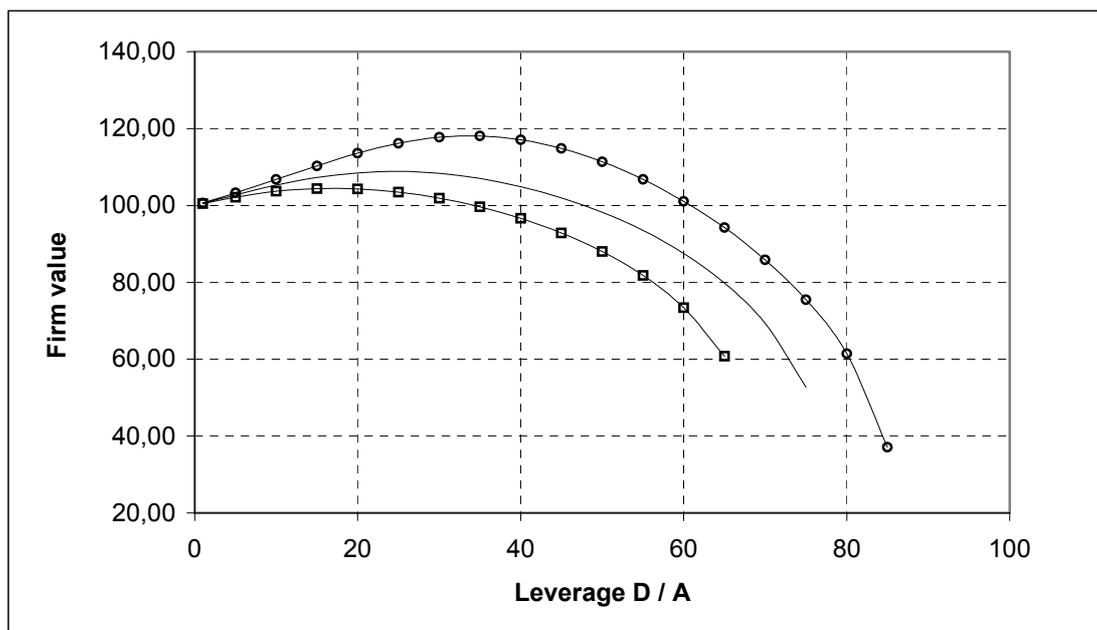
## 4.2 Debt maturity

Figure 4.2 illustrates the effect of debt maturity on the value of the levered firm and the optimal leverage ratio. The risk-free interest rates correspond to actual observed levels of Finnish government bonds in April 2005. Figure 4.3 entails a constant interest rate.



**Figure 4.2** Firm value and debt maturity with actual risk-free interest rates.

The lines plot firm value  $V$  for three levels of debt maturity  $T$ : 5 years and 3,0 percent (open circle), 10 years and 3,5 percent (solid line), 15 years and 4,0 percent (square).



**Figure 4.3** Firm value and debt maturity with constant risk-free interest rate of 3,5 %.

Maturity  $T$ : 5 years (open circle), 10 years (solid line) and 15 years (square).

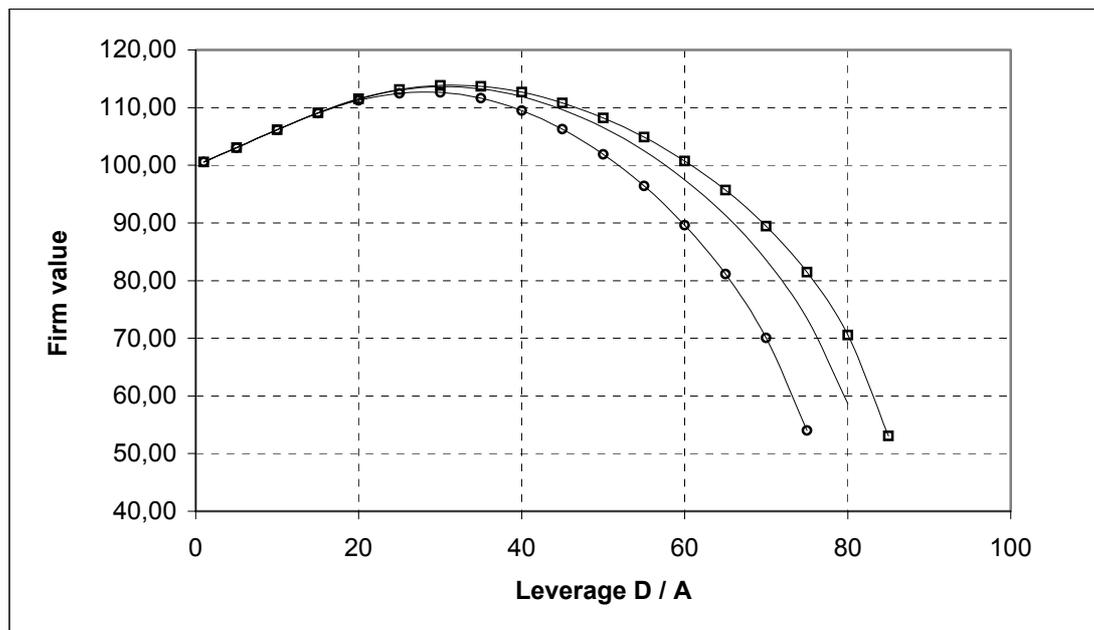
In the case of actual interest rates, firm value is maximized at 120,30 when leverage is 0,35 and maturity is five years. Firm value is 108,82 at leverage ratio 0,25 and 103,64 at leverage 0,15 with maturity of 10 and 15 years respectively. There is a marginal difference comparing actual rates and a constant one. In April 2005, the term structure was not steep nor was it completely flat. More importantly, the shape of the interest rate term structure at a given economic state influences maturity choice. The end result is surprising, shorter maturities imply a higher optimal leverage and higher firm value. At first, this seems counterintuitive. Brennan and Schwartz (1978) produce a result that seems to verify our finding. Their model also implies a negative relation between debt maturity and optimal firm leverage, but in contrast they argue that firm value is higher at an optimum leverage the longer the maturity.

Leland and Toft (1996) have drawn conclusions that a longer maturity pushes the optimal leverage higher and firm value is also positively related to maturity. Empirically this has been reported by Barclay and Smith (1995). However, Leland and Toft also indicate that longer maturities increase credit spreads. This is exactly what our model indicates. At any given leverage ratio, theoretical default probabilities are higher when maturity is longer. Therefore debt with shorter maturity reduces the incentives for equityholders to increase firm risk. As a result, agency costs of asset substitution will be substantially lower when short-term debt is used. Naturally, this increases firm value. Guedes and Opler (1996) found little support for signaling and tax-based theories of debt maturity choice, but they did however find a negative relation between maturity and term premium for new corporate debt issues.

Another major influence, which differentiates Leland and Toft's (1996) and our results is that their model as well as Leland (1994) assume bankruptcy at any given moment before debt repayment. This is achieved by incorporating bond covenants, which limit possible debt values by triggering bankruptcy if firm asset values decline. These are called positive net-worth covenants. We, on the other hand, assume default only at maturity. Debtholders cannot force the firm into bankruptcy as they can in the above-mentioned cases. Therefore, longer maturity enables firm management to pursue riskier ventures and the risk of this materializing gives debtholders as well as equityholders reason to increase their expected return on the firm's financial claims.

### 4.3 Dividend policy

Figure 4.4 illustrates the relationship between different dividend policies and the value of the levered firm and the optimal leverage ratio. Larger dividend ratios imply higher firm value and a positive relation with optimum leverage even though debt issues become more costly.



**Figure 4.4 Firm value and dividend level.**

The lines plot firm value  $V$  for three dividend / assets level: zero percent (circle), 3 percent (solid line) and 5 percent (square). Other parameters are same as base case.

Firm value reaches a maximum point 112,72 at 0,28 leverage ratio with a zero percent dividend ratio. For three percent dividends, maximum firm value is 112,84 at 0,31 leverage ratio and five percent dividend ratio increases firm value to 113,92 at 0,32 leverage ratio. We assume that bond covenants do not prohibit dividend payments.

Instead of waiting for a pay-off until the debt is retired, equityholders are able to receive a dividend flow during the debt financing period. Larger dividends increase firm value because equityholders experience a lower default probability. They can expropriate funds from the company and reduce the total amount lost if default occurs when the debt matures. Even though debtholders will increase their expected return as cash outflows to owners grow larger (theoretical put value increases), the lower expected return for equityholders more than offsets this effect. Therefore WACC is actually

reduced and optimal leverage ratio increases. The effect becomes much more apparent as leverage ratios increase and firms become overlevered. Another interesting implication is that dividend policy has a barely noticeable affect on firm value when firms are underlevered. This raises arguments for and against dividend policy irrelevancy.

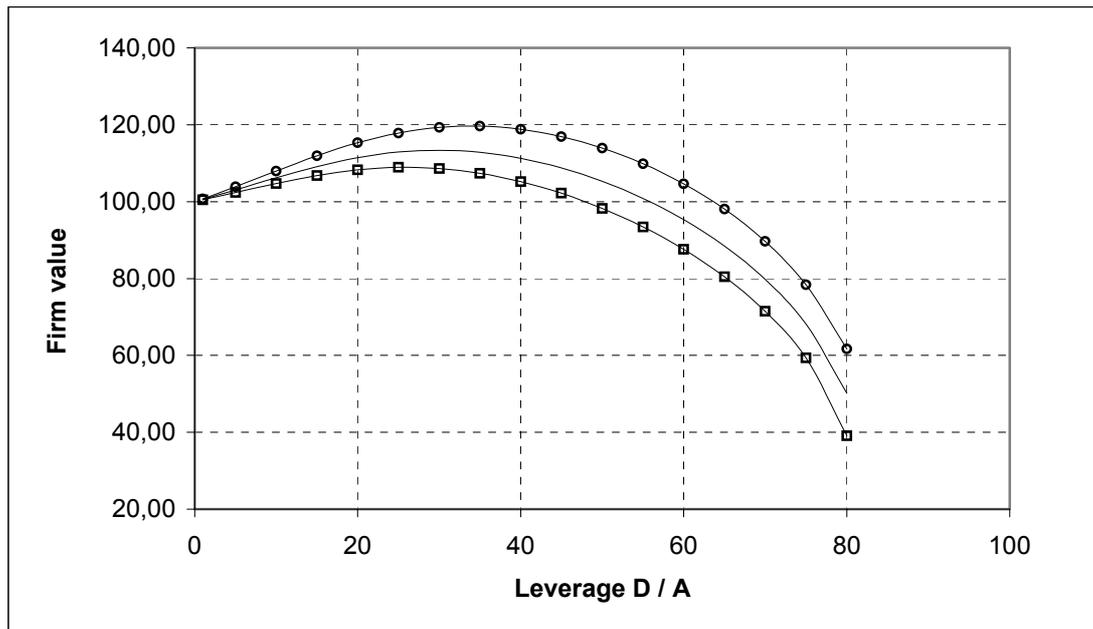
It is interesting to observe these results, but then again they might provide some evidence for well-known dividend policy theorems. Gordon-Lintner's "bird-in-the-hand" –theory states that equityholders prefer dividend payments instead of capital gains, which are perceived uncertain. This is clearly evident when firms are overlevered and the expropriation of assets benefits equityholders. The other observation, however, provides support for the dividend policy irrelevance advocated by Miller and Modigliani who stipulate that the level of dividend out flow does not affect firm value.

However, it is obvious that changes in expected losses due to bankruptcy affects valuation of the firm's securities. We cannot dismiss the MM–argument at low levels of debt financing, but increased leverage together with debt issues without very constricting covenants seem to imply that dividend policy does affect firm value. Normally investment policy is an important determinant of firm value and its optimal leverage. Increasing dividend flows may reduce investment opportunities if outside financing is harder to obtain, this in turn reduces the amount of net investment and leads to a lower firm value and optimal leverage ratio in the long run.

According to Leland and Toft (1996), firms with lower payout rates should have a lower optimal amount of long term debt but higher amount of optimal short term debt. When using long term debt instead of short term debt, the incremental firm value is significantly reduced when payout ratio falls. They also advocate that the incremental agency costs associated with asset substitution are lower for firms with lower payout ratios. If the firms have greater growth prospects, it advocates lower payout ratios or cash outflows, greater risk and higher bankruptcy costs. These firms are expected to have lower optimal leverage ratios. Leland (1994) derives a result where a higher dividend ratio causes optimal leverage to reduce and yield spreads on debt issues increase. Accordingly, bankruptcy is more likely and firm value falls.

#### 4.4 Interest rate level

Figure 4.5 illustrates the relation between risk-free interest rate level, firm value and optimal leverage. It does not come as a surprise, that lower interest rates increase firm value. The negative relation between optimal leverage and interest rates is also apparent. Lower debt servicing costs benefit the firm as it can increase its leverage without increasing the probability of default. Tax shields become larger and firm value is higher.



**Figure 4.5 Firm value and risk-free interest rate level.**

The lines plot firm value  $V$  for three levels risk-free interest rate  $r$ : 2,0 percent (open circle), 3,5 percent (solid line) and 5,0 percent (square). Other parameters are same as base case.

Optimum leverage is reached at 0,33 leverage ratio and firm value peaks at 119,02 when interest rates are at 2,0 percent. As interest rates increase to 3,5 percent, optimal leverage falls to 0,28 and firm value to 112,77. Finally, at 5,0 percent level optimal leverage is at 0,24 and firm value reaches 108,47. It seems that the prevailing interest rates affect the point of optimal leverage and firm value much more than dividend levels executed by the firm.

Leland (1994) reports opposing results in relation between interest rates and optimal leverage. He found curious aspects that a rise in the interest rates leads to greater optimal leverage. Normally one might suggest that higher interest rates would reduce the amount of optimal leverage but he suggests it does not. Higher rates generate greater

tax benefits, which in turn advocate more debt capacity despite its higher cost. The added tax shield more than offsets the increased cost of debt financing when rates are high. Similarly the optimal debt for firms with higher bankruptcy costs may carry a lower interest rate than for firms with lower bankruptcy costs. This is because firms will choose significantly lower optimal leverage when bankruptcy costs are substantial, making debt less risky. (Leland 1994, 1248).

More in line with our findings, a negative relation between interest rates and optimal leverage is found by Leland and Toft (1996). However, when asset value is kept constant and interest rates fall, the level of optimal leverage is reduced since lower required coupons will reduce the tax deduction of debt. Accordingly for any given maturity, previously-issued debt values fall when asset value volatility or interest rates increase if leverage is reasonably low. For newly-issued debt, rising interest rates increase credit spreads of shorter term debt but decreases spreads for long term debt. Lower bankruptcy costs lead to the use of particularly greater leverage for intermediate term debt, thereby increasing yield spreads for optimal debt with this maturity (Leland and Toft 1996, 1003).

Not only does higher interest rates change the level of optimal leverage and firm value, Fischer et al. (1989) report additional interesting implications. Since large adjustment costs and even small recapitalization costs lead to wide swings in firms' debt ratios over time, their study incorporated a leverage ratio range within the optimal leverage. They conclude that increasing interest rates reduce the optimal debt ratio and produces a higher initial debt ratio because it similarly increases the tax advantage of debt. In addition, an increase in rates will decrease the optimal leverage ratio range, which is derived from their model. Therefore firms will focus more on their leverage when interest rates are increasing. The same deduction also applies for increases in corporate tax rate and bankruptcy costs.

## 5. Empirical evidence on leverage

We examine 175 companies which are among the largest in Finland measured by their annual turnover. The data, covering a ten year period between 1994 and 2003, includes both publicly traded and privately owned companies and is compiled by ETLA. It originally covers 600 companies. Unfortunately, the data is incomplete and for several years data is unavailable. We have therefore selected only those firms for which there exists enough information for our purposes and firms with incomplete data strands are excluded. Due to their distinctive business sectors and composition of their balance sheets, banks and insurance companies are also excluded from the estimations.

We run four different regressions with two alternative estimation methods and two different leverage ratios. Dependent variables are the leverage ratios of interest bearing debt to total assets ( $IBD / A$ ) and total debt to total assets ( $TOTD / A$ ). The independent variables are chosen to reflect the four main categories which affect optimal capital structure under option theory framework. Business risk or the volatility of firm asset value is measured by free cash flow volatility scaled by the value of the firm's total assets ( $FCFVOL / A$ ). These figures are easily calculated from the data set. As Bradley et al. (1984) point out, this kind of volatility measure has been used by others and it does not suffer from the statistical problems associated with alternative measures of firm volatility. Dividend levels are also readily available from the data, which provides a dividend yield level of dividends per asset value ( $DIV / A$ ). Other factors are much more difficult to measure and this will undoubtedly subject results to some level of lower validity.

The first problem is the interest rate. The basic lending rate of the Bank of Finland, calculated as a year average, will serve as a proxy for the risk-free interest rate ( $INT$ ). There is reasonable questioning whether or not this is the appropriate measure. However, there are certain difficulties in obtaining other suitable interest rate data such as helibor / euribor quotations for a wide range of maturities. Some of this information is either not readily available and would become relatively expensive to obtain. We presume that the basic lending rate is adequate for our needs and gives enough

directional indication of relationship between observed capital structures and interest rate levels. The other problematic situation concerns the actual nature of the firms' liabilities. Debt maturity is another complicated variable since the utilized data does not contain a single figure which would indicate the remaining life of debt liabilities in the corporate balance sheets. Again we must introduce a proxy variable which would capture the composition of the maturity. We have chosen a ratio of long term debt to short term debt (LT / ST) which indicates a higher figure as longer interest bearing maturities increase. Given the limitations of our data, no other type of proxy could serve as representative figure of debt maturity.

**Table 5.1 Summary statistics for regression variables**

	Mean	Median	Mode	Max	Min	Std
IBD / A	0,2685	0,2599	0,0000	0,9228	0,0000	0,1872
TOTD / A	0,5672	0,5734	0,6644	1,2088	0,0845	0,1801
DIV / A	0,0249	0,0096	0,0000	1,1745	0,0000	0,0556
INT	4,1771	4,0000	4,0000	5,2708	3,1250	0,7070
FCFVOL / A	0,1986	0,1139	0,0720	5,2007	0,0199	0,3128
LT / ST	0,7782	0,4905	0,0000	16,9855	0,0000	1,3325

**Table 5.2 Correlation matrix with total debt to assets**

	TOTD / A	DIV / A	INT	FCFVOL / A	LT / ST
TOTD / A	1,0000	-0,1993	0,1441	0,1832	0,2669
DIV / A	-0,1993	1,0000	-0,1154	0,0556	-0,1271
INT	0,1441	-0,1154	1,0000	0,0846	0,0880
FCFVOL / A	0,1832	0,0556	0,0846	1,0000	-0,0786
LT / ST	0,2669	-0,1271	0,0880	-0,0786	1,0000

**Table 5.3 Correlation matrix with interest bearing debt to assets**

	IBD / A	DIV / A	INT	FCFVOL / A	LT / ST
IBD / A	1,0000	-0,2404	0,1553	-0,1239	0,5134
DIV / A	-0,2404	1,0000	-0,1154	0,0556	-0,1271
INT	0,1553	-0,1154	1,0000	0,0846	0,0880
FCFVOL / A	-0,1239	0,0556	0,0846	1,0000	-0,0786
LT / ST	0,5134	-0,1271	0,0880	-0,0786	1,0000

Table 5.1 contains summary statistics for the regression variables. Considerable variability between extreme ends can easily be observed from the produced figures, especially for both leverage ratios. However, all included factors present a fairly stable set of observations throughout the data set. Tables 5.2 and 5.3 are in turn the correlation matrixes for the variables.

The four different regression results are reported in table 5.4 which is divided into two major sections. The left hand side shows the OLS regressions whereas the right hand side shows ML regressions.<sup>25</sup> The reported data is also divided into two different categories according to the explained leverage ratio.

**Table 5.4 OLS and ML regression results of firm leverage**

	OLS	OLS	ML	ML
	IBD / A	TOTD / A	IBD / A	TOTD / A
CONSTANT	0,1277 (5,90)	0,4403 (18,10)	0,1143 (6,98)	0,4267 (24,80)
DIV / A	-0,5492 (-3,31)	-0,5456 (-3,41)	-0,3128 (-5,63)	-0,3054 (-5,48)
INT	0,0269 (5,91)	0,0216 (4,53)	0,0309 (9,93)	0,0232 (7,44)
FCFVOL / A	-0,0514 (-2,80)	0,1182 (5,32)	-0,0697 (-5,50)	0,0373 (2,91)
LT / ST	0,0670 (4,34)	0,0343 (4,93)	0,0599 (17,40)	0,0566 (16,20)
R square	0,3105	0,1513	0,2497	0,2103
F-statistics	196,48	77,77	145,15	116,16

All reported variables and their coefficients as well as the models' F-values are statistically significant at one percent risk level. If leverage ratio is indicated by interest bearing liabilities instead of taking all liabilities into consideration, the explanatory powers or  $R^2$  -values are higher.<sup>26</sup> In other words, the observed levels of interest bearing debt are better explained by the chosen independent variables and therefore in estimating firm value, more emphasis should be on IBD. This is exactly what is done by analysts in securities firms and investment banks. Non-interest bearing liabilities, such as accounts payable, are given less consideration in firm valuation although long-term trends in the development of their level are scrutinized closely.

<sup>25</sup> ML is identical to OLS when examining large samples. On a theoretical basis, OLS is considered a better estimator. ML regressions above are presented only as a verification that we used a large sample.

<sup>26</sup> Due to data problems stated earlier, these results should be considered more as qualitative indications than actual estimations of financing behavior among Finnish companies.

Another noteworthy difference between IBD and TOTD is that the signs of the volatility coefficients are opposite from each other. Higher free cash flow volatility seems to lower the amount of interest bearing debt but on the other hand, total leverage is increased! There are two natural explanations for this kind of financing behavior. Chapter three indicated that the pricing of risky corporate debt essential depends on two separate factors, the firm's current leverage ratio and the volatility of asset value which is based on operating income volatility. Therefore, when volatility level or business risk is higher, firms are able to receive debt financing at worse interest rates and this makes it a less appealing alternative.

On the other hand, business operations have to be financed somehow and if operating income and retained earnings are not sufficient, the other alternative is to issue new common stock. However, considering the costs associated with new equity issues and the possible dilution of current stockholders' wealth, it would seem more likely that firms will instead turn to other methods of debt related financing such as accounts payable.

According to our findings, Finnish companies seem to rely on good customer-vendor relations and utilize this by inflating their balance sheets with accounts payable since higher volatility increases total debt levels. There are also other ways to increase short term liabilities. During the past ten years Finnish capital and money markets have grown extensively and this has enabled firms to acquire reasonably priced short term debt on the open market. Many firms have signed commercial paper agreements with the largest Finnish banks which act as dealers operating between investors and the firms. This has turned the traditional bank oriented lending markets into better performing capital markets. Increased liquidity and borrowing possibilities have enabled both medium and large firms to acquire relatively inexpensive debt financing with short notice and at the same time, banks' own operating risks have decreased as lending does not strain their balance sheets.

Observed dividend policies and leverage ratios have unexpected effects on each other. Our theoretical model predicted a marginal increase in optimal leverage as dividends increase, but empirical evidence does not support this argument. Whether IBD or TOTD

is examined, larger dividend levels have a negative impact on observed leverage ratios. Generous cash outflows to owners seem to imply a profitable business where less debt is required to finance operations. Similarly, debt covenants may prohibit the generous dispersion of dividends. Therefore, companies, which are willing to reward their owners, probably need to lower their leverage levels in order to avoid debt contract violations.

However, we have not taken any view on whether or not observed leverage ratios are over or under the firm-specific theoretical optimum and if the observed companies were pursuing maximization of their total value. As we pointed out, firm value is hardly affected by dividend level changes when firms are underlevered. The opposite applies when leverage has increased beyond the optimum and if resources can be expropriated without restrictive debt covenants. It would seem unlikely that highly levered firms would be able to pay out excessive amounts of cash dividends due to economic distress. In this respect, pecking order theory would seem a plausible answer to financing behavior among Finnish companies. Profitable firms use mainly internal sources of funds and then turn to debt financing.

Another striking change in financing behavior is the fact that new equity issues have more or less disappeared. Before the severe depression in the beginning of the 1990s, equity issues were almost an every-week event for the largest companies even though bank lending was heavily exploited. The majority of firms invested heavily on capital expenditures and corporate acquisitions. This transformed the largest companies into conglomerates operating in a wide selection of business areas, which had nothing to do with each other. This trend of overinvestment has disappeared and companies have begun to sell off different business units. Similarly, the need for funding these activities has decreased.

Observed relationship between interest rate level and firm leverage also differs from the model's predictions, which indicated an inverse relation. Even though we cannot observe changes in firm value, higher debt costs should produce a lower optimal leverage. The empirical findings support contradictory results if we assume that firms have pursued an optimal capital structure. The produced coefficients imply that increases in interest rates drive leverage ratios higher. Understandably, this kind of

behavior can also be assumed logical since higher debt servicing costs lowers taxable income. However, we feel that the reasons have to be searched somewhere else.

The majority of Finnish companies were extremely highly levered before the depression. When the economic scene was hit by severe problems, many companies ran into severe difficulties in debt servicing when interest soared to nearly twenty percent. The end result for many firms was the inevitable bankruptcy and subsequent liquidation or merger with a healthier company. However, due to a phenomenal economic recovery and a “collapse” in interest rates, the economy as well as companies prospered.

During these years, the survived Finnish companies’ financing behavior changed radically. Instead of relying on heavy amounts of debt and frequent equity issues, the focus was set on internal performance enhancement and productivity. The excessive amounts of debt were repaid very quickly and balance sheets transformed into more conservative state. Accordingly, the results have been very good for the economic state of firms. Nowadays companies are in a very solid financial state and wealthy up to a point where international investors and banks are unable to find profitable investment opportunities in corporate debt markets.

The final factor influencing leverage and firm value is debt maturity. Our model produced a curious, inverse relation between maturity and optimal leverage ratio. In contrast, this is completely different from traditional beliefs which advocate longer maturities for maximizing optimal leverage. However, our model considers agency costs and the risk of asset substitution. Combining these two topics with debt maturity produced our results. Nevertheless, leverage ratios and debt maturity also seem to follow a contradictory result in comparison to the model’s predictions.

If leverage is increased, longer maturities are preferred by Finnish finance starved companies. When debt is issued, companies prefer longer maturities than one year therefore implying that money market instruments and accounts payable are utilized conservatively. We must also consider the fact, that our proxy for maturity may not be an adequate measure. However, future research with better data will undoubtedly present other results which may lend support to our theoretical suggestion.

## 6. Conclusions

This paper developed a synthesis of asset pricing and capital structure theory in order to produce a balancing effect or tradeoff between tax shields and costs of financial distress. We introduced an option theory framework which illustrated the effects of leverage changes on firm value. We also identified several major factors which need to be taken into consideration when financing policy is set. These firm specific factors are debt maturity, business risk, risk-free interest rate and dividend policy. Some variables and their comparative statics generated expected changes in optimal leverage ratios and firm values, but some surprising results were also discovered.

The absence of default triggering bankruptcy boundaries enables firm value maximization with higher leverage ratios by using shorter term debt. This is due to agency costs or asset substitution risk being lower for any given level of debt. As maturity is increased, equity value maximization becomes easier for firm management and debtholders will accordingly increase their expected return. Another implication of longer maturities is the increased price risk due to changes in interest rate levels. When durations increase, the risk of adverse price reactions become more apparent. Therefore, higher duration increases investors' risk and expected returns are adjusted accordingly.

Our model predicts an inverse relation between optimal leverage and business risk which is also confirmed by regression results when interest bearing liabilities are examined. Otherwise observed leverage ratios and volatility produced a positive correlation, which can also be explained by good customer-supplier relationships. Lower risk-free interest rate drives optimal leverage higher and firm value increases at the same time since a lower rate enables a lower bankruptcy probability at higher leverage ratios. Higher leverage ratios in turn lead to greater tax shields.

Dividend policy and firm value together with the existence of an optimal capital structure produces two confronting arguments on dividend policy irrelevance. We observed that firm value is hardly affected by dividend policy when underleverage is the prevailing state. In essence, this supports the view of dividend policy irrelevance

especially when default probability is considered to decrease as optimal leverage is reached. However, opposite results emerged when optimal leverage point was exceeded. As a result, higher dividend levels increased total firm value although debt pricing rose due to higher probability of bankruptcy. Without very constricting debt covenants, stockholders may expropriate firm resources before the debt matures and increasing the company's probability of default. This supports the opposite view that dividend policy does affect firm value since dividends are perceived a more secure flow of income than capital gains.

The empirical evidence recovered from Finnish companies produced disappointing results for our model. However, we feel that data problems which we experienced during our research caused these perverse findings. Another complicated fact which produces certain biases in our results can be accounted for the extraordinary events that occurred in Finland just before and during 1994–2003 period. Ideally our theoretical model and its implications should be empirically tested with better data such as COMPUSTAT files on the US markets.

We have shown that new methods of examining optimal capital structure have not disappeared even though this field has been extensively studied for the past five decades. Future examination and study of option theory applications to capital structure choice are strongly recommended. The flexibility and simultaneous nature of this method is beneficial for focusing on critical factors affecting firm value and combining implications of agency and asymmetric information theories. We studied only non-coupon paying debt and ignored the possibility of convertible bonds and default boundaries which would enable debtholders to have a stronger hold on the financing and operative decisions of the company as well as to force liquidation. These model specifications should be scrutinized more carefully and adopted to an option pricing framework.

## REFERENCES

Akerlof, George A. (1970). The Market for “Lemons”: Quality Uncertainty and the Market Mechanism. *The Quarterly Journal of Economics*, Vol. 84, No. 3, 488–500.

Altman, Edward I. (1984). A Further Empirical Investigation of the Bankruptcy Cost Question. *Journal of Finance*, Vol. 39, No. 4, 1067–1089.

Barclay, M. & Smith, C. (1995). The Maturity Structure of Corporate Debt. *Journal of Finance*, Vol. 50, 609–631.

Black, Fischer & Scholes, Myron (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy*, Vol. 81, No. 3, 637–654.

Blake, David (2000). *Financial Market Analysis*. (2<sup>nd</sup> ed.). West Sussex, England: John Wiley & Sons Ltd.

Bradley, Michael, Jarrell, Gregg A. & Kim, E. Han (1984). On the Existence of an Optimal Capital Structure: Theory and Evidence. *Journal of Finance*, Vol. 39, No. 3, 857–878.

Brealey, Richard A. & Myers, Stewart C. (2000). *Principles of Corporate Finance*. (6<sup>th</sup> ed.). New York, USA: The McGraw–Hill Companies Inc.

Brennan, Michael J. & Schwartz, Eduardo S. (1978). Corporate Income Taxes, Valuation, and the Problem of Optimal Capital Structure. *The Journal of Business*, Vol. 51, No. 1, 103–114.

Brennan, Michael J. & Schwartz, Eduardo S. (1984). Optimal Financial Policy and Firm Valuation. *Journal of Finance*, Vol. 39, No. 3, 593–607.

Cox, J.C., Ross S.A. & Rubinstein, M. (1979). Option Pricing: A Simplified Approach. *Journal of Financial Economics*, Vol. 7, 229–264.

Diamond, Douglas W. (1989). Reputation Acquisition in Debt Markets. *The Journal of Political Economy*, Vol. 97, No. 1, 828–862.

Fama, Eugene F. & French, Kenneth R. (1998). Taxes, Financing, and Firm Value. *Journal of Finance*, Vol. 53, No. 3, 819–843.

Fama, Eugene F. & French, Kenneth R. (2001). The Equity Premium. *The Center for Research in Security Prices, University of Chicago Graduate School of Business*, 522.

Fischer, Edwin O., Heinkel, Robert & Zechner, Josef (1989). Dynamic Capital Structure Choice: Theory and Tests. *Journal of Finance*, Vol. 44, No. 1, 19–40.

Graham, John R. (2000). How Big are the Tax Benefits of Debt? *Journal of Finance*, Vol. 55, No. 5, 1901–1941.

Guedes, Jose & Opler, Tim (1996). The Determinants of the Maturity of Corporate Debt Issues. *Journal of Finance*, Vol. 51, No. 5, 1809–1833.

Harris, Milton & Raviv, Artur (1988). Corporate Control Contests and Capital Structure. *Journal of Financial Economics*, Vol. 20, 55–86.

Harris, Milton & Raviv, Artur (1990). Capital Structure and the Informational Role of Debt. *Journal of Finance*, Vol. 45, 321–349.

Harris, Milton & Raviv, Artur (1991). The Theory of Capital Structure. *Journal of Finance*, Vol. 46, No. 1, 297–355.

Heinkel, Robert (1982). A Theory of Capital Structure Relevance Under Imperfect Information. *Journal of Finance*, Vol. 37, No. 5, 1141–1150.

Hirschleifer, David & Thakor, Anjan V. (1992). Managerial Conservatism, Project Choice and Debt. *The Review of Financial Studies*, Vol. 5, No. 3, 437–470.

Israel, Ronen (1991). Capital Structure and the Market for Corporate Control: The Defensive Role of Debt Financing. *Journal of Finance*, Vol. 46, No. 4, 1391–1409.

Jensen, Michael C. & Meckling, William (1976). Theory of Firm: Managerial Behavior, Agency Costs and Capital Structure. *Journal of Financial Economics*, Vol. 3, 305–360.

Jensen, Michael C. (1986). Agency Costs of Free Cash Flow, Corporate Finance and Takeovers. *American Economic Review*, Vol. 76, No. 2, 323–339.

Korajczyk, Robert, Lucas, Deborah & McDonald, Robert (1991). The Effects of Information Releases on the Pricing and Timing of Equity Issues. *The Review of Financial Studies*, Vol. 4, 685–708.

Leland, Hayne E. & Pyle, David (1977). Information Asymmetries, Financial Structure and Financial Intermediation. *Journal of Finance*, Vol. 32, No. 2, 371–387.

Leland, Hayne E. (1994). Corporate Debt Value, Bond Covenants and Optimal Capital Structure. *Journal of Finance*, Vol. 49, No. 4, 1213–1252.

Leland, Hayne E. & Toft Klaus Bjerre (1996). Optimal Capital Structure, Endogenous Bankruptcy and the Term Structure of Credit Spreads. *Journal of Finance*, Vol. 51, No. 3, 987–1019.

Leland, Hayne E. (1998). Agency Costs, Risk Management and Capital Structure. *Journal of Finance*, Vol. 53, No. 4, 1213–1243.

MacKay, Peter & Phillips, Gordon M. (2002). Is There an Optimal Industry Financial Structure? *NBER working paper series*, 9032.

MacKie-Mason, Jeffrey K. (1990). Do Taxes Affect Corporate Financing Decisions? *Journal of Finance*, Vol. 45, No. 5, 1471–1493.

Masulis, Ronald W. (1983). The Impact of Capital Structure Change on Firm Value: Some Estimates. *Journal of Finance*, Vol. 38, No. 1, 107–126.

Mehra, Rajnish & Prescott, Edward (1985). The Equity Premium: A Puzzle. *Journal of Monetary Economics*, Vol. 15, 145–162.

Merton, Robert C. (1973). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance*, Vol. 29, No. 2, 449–470.

Modigliani, Franco & Miller, Merton H. (1958). The Cost of Capital, Corporation Finance and the Theory of Investment. *American Economic Review*, Vol. 48, No. 3, 261–297.

Modigliani, Franco & Miller, Merton H. (1963). Corporation Income Taxes and the Cost of Capital: A Correction. *American Economic Review*, Vol. 53, 433-443.

Myers, Stewart C. (1984). The Capital Structure Puzzle. *Journal of Finance*, Vol. 39, No. 3, 575–592.

Myers, Stewart C. & Majluf, Nicholas S. (1984). Corporate Financing and Investment Decisions When Firms Have Information That Investors Do Not Have. *Journal of Financial Economics*, Vol. 13, No. 2, 187–221.

Narayanan, M.P. (1988). Debt Versus Equity under Asymmetric Information. *The Journal of Financial and Quantitative Analysis*, Vol. 23, No. 1, 39–51.

Ross, Stephen (1977). The Determination of Financial Structure: The Incentive Signaling Approach. *Bell Journal of Economics*, Vol. 8, No. 1, 23–40.

Shyam-Sunder, Lakshmi & Myers, Stewart C. (1994). Testing Static Trade-off Against Pecking Order Models of Capital Structure. *NBER working paper series*, 4722.

Spence, Michael (1973). Job Market Signaling. *Quarterly Journal of Economics*, Vol. 87, 355–374.

Stoll, Hans R. (1969). The Relationship Between Put and Call Option Prices. *Journal of Finance*, Vol. 24, No. 5, 801–824.

Stulz, René (1988). Managerial Control of Voting Rights: Financing Policies and the Market for Corporate Control. *Journal of Financial Economics*, Vol. 20, 25–54.

Stulz, René (1991). Managerial Discretion and Optimal Financial Policies. *Journal of Financial Economics*, Vol. 26, 3–27.

Titman, Sheridan & Wessels, Roberto (1988). The Determinants of Capital Structure Choice. *Journal of Finance*, Vol. 43, No. 1, 1–19.

Titman, Sheridan & Tsyplakov, Sergey (2004). A Dynamic Model of Optimal Capital Structure. Unpublished manuscript, University of Texas at Austin and University of South Carolina.

Vassalou, Maria & Xing, Yuhang (2004). Default Risk in Equity Returns. *Journal of Finance*, Vol. 59, 831–868.