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GENERAL EQUILIBRIUM ELABORATION OF THE CLASSIC MODEL OF MIGRATION

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Abstract

The classic type of analysis of inter-regional migration is derived from the traditional trade doctrine, expressed in terms of labour markets. The approach is here given a general equilibrium elaboration by including descriptions of local production functions, IS-LM frameworks and aggregate supply and demand schedules. The comparative static properties of the model are illustrated concerning the mechanism of migration. The effects of capital movements are also studied. A key finding is that interior solutions of spatial resource allocation are not granted in general.

Key words: real wage equalisation, capital movements, spatial resource allocation
JEL classification: 931

1 Introduction

The classic theory on inter-regional migration is based on a labour market application of the traditional trade doctrine. In the classic model, migration is described by free mobility of labour motivated by inter-regional differences in real wages (Isard, 1975, p. 172-175; McCann, 2001, p. 191-197). Under the assumption of full employment and perfect wage flexibility, the real wages adjust so that the inter-regional labour market clears. The classic labour market model of migration is a simple but powerful device of comparative static analysis on the rather complex set of factors and effects of migration.

In its simplicity, the labour market model is a reasonable framework for migration analysis for several reasons. First, employment possibilities, seek for better jobs, cultivation of special skills, building up careers and other such work related issues constitute a major part of the determinants of average people's location decisions in practice. Second, the connection between work and residence is quite unquestionable and, at least from the economic point of view, the concepts of labour market area and residential area match accurately enough. To generalise even further, the concept of a local market area can be applied. And third, the setting is convenient because the real wage, or the purchasing power of wages, is a common approximation of individual welfare. This is to say that the migrants' utility functions can be omitted and only the budget constraints that determine consumption utility can be considered. The model not only facilitates wide interpretation from strict orientation on work to a broader vision over differences in perceived welfare, but also treats the issue in such terms that can be empirically observed and measured.

The main message of the classic theory of labour migration is very intuitive: existing real wage (or welfare) differences motor up rational migration, which eventually equalises the differences between localities. The adjustment ends up to a stable and efficient market equilibrium, where

nobody can benefit from relocation, and where social welfare is maximised. Migration thus acts as a real world reflection of the abstract idea of factor reallocation. The simplest versions of the model ignore closer description of the real wage equalisation mechanism, and local evolution of nominal wages and prices remains implicit in the analysis. Nevertheless, the model suggests that, in the free migration equilibrium, both nominal wages and prices may well differ between localities although the real wage is equalised. This is reasonable since nominal wages and prices, such as the price of housing in particular, are both sensitive to migration, but have also local determinants.

This paper provides closer scrutiny on the adjustment of real wages between and within local market areas, which are henceforth called localities. The locality can be treated as a country, a region, a city or a municipality although the interpretations of the results may be of different relevance depending on the treatment. The paper constructs a general equilibrium elaboration of the classic model of inter-locality migration, and examines the adjustment of real wages more explicitly than in the conventional textbook versions. Examination of the comparative static properties of the model yields further intuition on local development under the circumstances of factor mobility.

The paper proceeds as follows. Chapter 2 constructs the basic model to illustrate the economic basis of local welfare. The model is composed of standard neoclassical components so that any findings in the model should fit into the neoclassical market paradigm. In chapter 3, inter-locality migration is introduced and the migration equilibrium is examined from the viewpoints of small and bigger localities. Chapter 4 studies the implications of capital movements and examines the existence of an interior market solution of spatial factor allocation. Chapter 5 concludes the findings.

2 The basic model

The model is a local economy elaboration of a basic general equilibrium model (for conventional presentations see e.g. Burda and Wyplosz, 1997; Heijdra and van der Ploeg, 2002; and also Brown & Jackson, 1978). In the model, production, denoted by q , is given by the production function

$$(1) \quad q = f(L, \underline{K}),$$

where L denotes the labour input and K denotes the capital input. The local capital stock \underline{K} is taken to be constant in the short term. Therefore, production depends on the amount of labour, measured in labour time units. The standard assumptions concerning the production function are made, namely constant returns to scale and $f_1, f_2 > 0, f_{11}, f_{22} < 0, f_{12} = f_{21} > 0$, where the subscripts refer to first and second partial derivatives of the function in order of appearance of its arguments.

Define the short-run profits of the local firm sector as $\pi = pq - wL$. Recalling function (1), competitive profit maximisation with respect to labour use yields

$$(2) \quad w = pf_1$$

for the demand for labour in the locality. Function (2) is written in terms of nominal wages, and it says that the local price level together with the marginal product of the labour input determine local labour demand - the demand for labour equals the value of the marginal product of labour (Krugman & Wells, 2005, p. 284-289). Following from the assumption of diminishing marginal product of labour, the demand curve is downwards sloping in L - w space.

Normalising total available time of the households to unity, and assuming that work is the only source of income, the households maximise their utility $U(q, 1-L)$ subject to the budget constraint $wL = pq$. Under the assumption of perfect foresight, local supply of labour, derived from the optimal time use decisions of the households then reads

$$(3) \quad w = pg(L),$$

where $g(L)$ describes the households' valuation of their time taking into account the dual use of it. It is assumed that the opportunity cost of labour time is positive, $g' > 0$, which is to say that the substitution effects dominate the income effects in supplying labour. The labour supply function (3) is therefore upwards sloping in L - w space.

Given that the capital stock is constant, the local labour market conditions together with the production function determine the equilibrium output, or the aggregate supply, in the local goods market. Under perfect foresight, the derivation of the aggregate supply function in q - p space is straightforward because examination of the labour market equilibrium by using functions (2) and (3) reveals that, since the price parameters cancel out from the equations, aggregate supply is invariant to the price level. This classical result is to say that the aggregate supply (AS) curve is vertical in q - p space, and its horizontal position is given by the labour market solution and the production function.

The aggregate demand conditions of the local economy are determined from the IS-LM model. The equilibrium of the real side of the economy, the IS curve, is given by

$$(4) \quad q = c(q) + i(r),$$

where c denotes consumption, i denotes investment and r denotes the market interest rate. By assumption, $c' > 0$ and $i' < 0$. Totally differentiating (4) and rearranging gives

$$(5) \quad \frac{dr}{dq} = \frac{1-c'}{i'} < 0,$$

which says that the IS curve is declining in q - r space. The respective monetary equilibrium, the LM curve, of the local economy is given by the money market equation

$$(6) \quad m/p = l(q, r),$$

where the supply side is written in terms of real money, the amount of nominal money m deflated by the price level. On the right-hand side of (6), function l depicts the demand for money, which is reasoned by two purposes: transaction use that depends on real production or factor incomes q , and inter-temporal use that depends on market rate of return r , which is the opportunity cost of holding liquid money. By assumption, $l_1 > 0$ and $l_2 < 0$. The LM curve thus is upwards sloping in q - r space.

The properties of the IS-LM setting can be seen by totally differentiating equations (4) and (6), and substituting, which yields

$$(7) \quad dq = \frac{1}{p c' l_1} dm - \frac{m}{p^2 c' l_1} dp - \frac{1}{c'} \left(\frac{l_2}{l_1} + i' \right) dr.$$

The first two terms on the right-hand side of expression (7) describe the aggregate demand (AD) curve in the goods market. The first term says that $\partial q / \partial m > 0$ and the second term says that $\partial q / \partial p < 0$. Both terms point to the effects of changes in the supply of real money: an increase in nominal

money and/or a decrease in the price level increases real money m/p and vice versa. The third term on the right-hand side of expression (7) describes the equilibrium conditions in the IS-LM framework itself. Keeping real money constant ($\partial m = \partial p = 0$) yields $\partial r / \partial q > 0$, which says that an increase (decrease) in production implies a rise (fall) in the equilibrium interest rate.

Figure 1 illustrates the properties of the model. The basic figure consists of four quadrants, the labour market, specified in terms of nominal wages in the northeast quadrant, the production function in the southeast quadrant, the aggregate goods market in the southwest quadrant and the real wage in the northwest quadrant. An additional fifth quadrant is supplemented in the far southwest end of the figure to incorporate the IS-LM setting of the local economy.

(Figure 1 about here)

In Figure 1, the equilibrium e_0 in the labour market is brought up by the adjustment of nominal market wages and prices to w_0 and p_0 , which give $w_0/p_0 = \omega_0$ for the equilibrium real wage. The labour market equilibrium determines labour use at L_0 , which in turn yields production of q_0 along the production function. The equilibrium combination p_0, q_0 is the aggregate supply in the goods market presented in the southwest quadrant of the figure. Since aggregate supply is invariant to the price level around the equilibrium point, it can be presented in full shape by the vertical graph AS_0 .

The p_0, q_0 combination is sustainable given that the goods market is in equilibrium. That is, aggregate supply, derived above from the labour market and production conditions must equal aggregate demand, which is derived from the IS-LM setting. The far southwest quadrant of Figure 1 presents the respective IS-LM equilibrium at the intersection point of the downwards-sloping IS_0 curve and the upwards-sloping LM_0 curve, which gives r_0 for the equilibrium interest rate. The full

shape of the aggregate demand curve AD_0 can be derived by letting the price level vary, which shifts the LM-curve outwards as prices fall and inwards as prices rise, and reading the consequent changes in aggregate demand from the intersection points along the IS_0 curve. The experiment follows the message of the second term on the right-hand-side of (7), which says that the aggregate demand curve of the local economy is declining in q - p space.

Furthermore, recall that $\partial r/\partial q > 0$ by expression (7). In terms of Figure 1, this relationship can be read in the IS-LM setting along the LM_0 curve with respect to shifts of the IS curve. If there occurs a rise in production/income while the supply of real money remains unchanged, the IS curve shifts outwards due to the increase in the consumption component on the right hand side of equation (4). The money market equilibrium requires that the interest rate rises to release money from inter-temporal use to the increased transaction purposes, which in turn decrease the investment component of equation (4).

3 Migration

3.1 Mobility of labour

The standard way to analyse the factors and effects of migration is to construct a comparative-static setting, where the autarky equilibrium of one considered locality is set into comparison with that of the outer economy. To simplify the analysis further, concentrate on migration alone and assume that capital remains fixed and thus immobile, and that there is no trade in goods and services between localities. This is to say that the labour input is the only mobile element in the economy. Full employment, wage flexibility, competitiveness and perfect foresight throughout the economy are also assumed.

Immobility of capital and non-tradability of goods mean that the domestic interest rates and prices are determined purely locally. This implies that the presented IS-LM framework is relevant without further restrictions, which actually means that the equilibrium of the goods market could simply be assumed to hold. Of course, the assumption limits the practical interpretations of the model – the assumption fits best to very big localities with autonomous goods and money markets and full interest rate flexibility. Especially the concept of local money supply becomes crucial in this respect.

The localities are assumed to constitute perfect (labour) market areas so that working in a locality necessitates residing in it. Therefore, taking a job in another locality necessitates moving into it. Migration is triggered by inter-locality differences in real wages. Perfectly foresighted households are capable to perfectly monitor these differentials. Assume also that migration, after it becomes free in the comparative-static setting, is costless so that there are no migration thresholds of any kind.

3.2 Migration equilibrium in a small locality

Examine migration first from the point of view of residents in a locality of atomistic size in the economy. This is to say that the considered locality must fully adjust to the market conditions of the outer economy - migration flows in either direction do not change circumstances in the rest of the economy. Figure 2 illustrates the comparative statics of shifting from autarky to free migration from the viewpoint of one such locality.

(Figure 2 about here)

In Figure 2 the initial autarky equilibrium in the considered locality is given by the solution in the competitive labour market. The market prices and nominal wages are settled so that the real wage is $\omega = w_0/p_0$. The equilibrium occurs at point e in the labour market, yielding the consequent outcomes in labour use, L_0 and production, q_0 . The respective equilibrium interest rate is r_0 .

Consider the effects of opening up migration. Suppose that the real wage is higher outside the locality, denoted by ω^* in the northwest quadrant of Figure 2. Under the conditions of perfect mobility of labour workers are employed in the economy-wide labour market, and the local market conditions in the considered small locality must adjust to the situation in the rest of the economy. The domestic real wages adjust upwards to ω^* so that migration ends up to a new stable equilibrium, where nobody can benefit from migration. In order to get a deep insight into the real wage adjustment process, analyse the adjustment separately from the points of view of pure nominal wage adjustment and pure price adjustment.

First, explore pure nominal wage adjustment. Fix the price level to p_0 and examine the required adjustment in terms of nominal wages. Read the adjustment vertically along the line that goes through p_0 and find that the adjustment from ω to ω^* necessitates a rise in the nominal wage from w_0 to w_1 . For that nominal wage, labour demand and labour supply are determined along the original D_0 and S_0 schedules so that demand is L_1 and supply is L_2 . The excess supply of labour –the amount of emigration that is necessary to restore domestic labour market equilibrium at the constant price level – amounts to $L_2 - L_1$. Since the domestic labour use falls from L_0 to L_1 , domestic production falls from q_0 to q_1 and the aggregate supply curve shifts from AS_0 to AS_1 . The IS curve of the economy shifts inwards from IS_0 to IS_1 , with an adjacent inwards shift of the aggregate demand curve from AD_0 to AD_1 . The equilibrium interest rate falls from r_0 to r_1 along the unchanged LM_0

curve. The fall of the interest rate restores the equilibrium in the domestic money market – the amount of money released from transaction demand shifts into inter-temporal purposes.

Second, consider pure price adjustment. Fix nominal wages to w_0 and study the required adjustment in terms of prices alone. Reading horizontally at w_0 , the adjustment from ω to ω^* necessitates that the price level falls from p_0 to p_1 . By function (2), the fall in local market prices makes the labour demand curve to shift inwards and turn flatter. The demand curve shifts from D_0 to D_1 in the northeast quadrant of Figure 2. By function (3), the labour supply curve shifts, too – the fall in prices makes the supply curve to shift outwards and turn flatter. Under perfect foresight the shift is from S_0 to S_1 so that the new demand and supply curves intersect vertically below point e reflecting the property that aggregate supply is invariant to prices. At the nominal wage w_0 , domestic labour demand is L_1 and labour supply is L_2 . Therefore, there exists excess supply of labour amounting to $L_2 - L_1$, which is the emigration necessary to restore equilibrium in the local labour market at the constant nominal wage. As the domestic labour use falls to L_1 , domestic production falls to q_1 , and the vertical aggregate supply curve shifts again inwards from AS_0 to AS_1 . The aggregate demand curve shifts inwards from AD_0 to AD_2 intersecting the AS_1 curve at the new price p_1 . Note that the shift in aggregate demand is now larger than in the above case of pure nominal wage adjustment. This is because the fall in prices increases real money in the local economy, manifesting in an outwards shift from LM_0 to LM_1 . Unless nominal money is withdrawn from circulation, the equilibrium of real and monetary sides of the economy at q_1 necessitates that the IS curve shifts from IS_0 to IS_2 , and the interest rate falls from r_0 to r_2 .

And third, simultaneous adjustment of both prices and nominal wages is also possible. In practice it is most reasonable that emigration reduces both local labour supply, which makes the nominal wages to rise, and consumption demand, which makes the prices to fall. The reduction in

consumption demand is evident because the wage sum is reduced in spite of that the remaining consumers are better off than before. Simultaneous adjustment means that prices and nominal wages both adjust partially as compared to the above experiments: the prices fall less than $p_0 - p_1$ and the wages rise less than $w_1 - w_0$. This kind of simultaneous adjustment is presented in Figure 2 by the price-wage combination p_2, w_2 , which results in the dashed labour demand and labour supply schedules D_2 and S_2 in the labour market. As to migration and production, the solution is equal to the above cases of unilateral adjustment. The aggregate supply curve shifts from AS_0 to AS_1 . But, since the price level adjusts from p_0 to p_2 , the LM curve shifts from LM_0 to LM_2 , the IS curve shifts from IS_0 to IS_3 and the aggregate demand curve shifts from AD_0 to AD_3 . The interest rate in the locality falls from r_0 to r_3 , which is higher than r_2 but lower than r_1 .

The above experiments show that higher real wages outside the considered locality motor up emigration from the locality, and induce adjustment in the local labour, goods and money market. The real wages of those who stay put in the locality rise and the market interest rate falls. The real effects are the same not depending on if nominal wages, prices or both adjust. However, the effect of emigration on the local interest rate depends on the mode of adjustment: the interest rate falls the more the larger is the adjustment in prices as compared to that in nominal wages. In the setting of Figure 2 the initial autarky circumstances are unfavourable to the considered locality. The reverse effects occur in a locality that confronts lower real wages in the outer economy. Illustration of this case can be omitted since it is merely a mirror image conversion of the presented one.

3.3 Equilibrium between two big localities

Assume now that the economy consists of two symmetrically big localities, A and B. Both localities are initially in their competitive autarky equilibriums. Suppose that there is a difference in real

wages between the localities, and examine the effects of opening up free migration between these two localities.

Figure 3 below presents the initial autarky situation in the two localities, the market mechanism of simultaneous adjustment, and the final allocation of the perfectly mobile labour resources. In the figure, panel (i) is a four-quadrant presentation of the general equilibrium in locality A, panel (iii) is the respective presentation of locality B, and panel (ii) presents the inter-locality labour market, drawn in terms of real wages. In the figure, the IS-LM configurations are ignored, but the respective aggregate demand schedules are still drawn.

(Figure 3 here)

The initial autarky solution in locality A in panel (i) of Figure 3 is given by the labour market equilibrium at point a , corresponding to real wage ω^A . The respective autarky solution in locality B in panel (iii) is at b given by the real wage ω^B . Free migration evokes adjustment in both localities such that matches to the equilibrium solution in the inter-locality labour market. In the following, assume that both prices and nominal wages adjust simultaneously in the two localities.

Consider first locality A. Since the real wage is elsewhere lower, the workers in A are happy with their location because they cannot improve their welfare by moving. Thus, motives for emigration do not exist. On the other hand, the firms in locality A would be willing to employ people for lower wages. Therefore, there exists over-demand for workers from B at lower real wages than ω^A . There is a pulling force for immigration, which amounts to the horizontal difference between the domestic labour demand and supply curves below w_0^A in panel (i). This excess demand is redrawn in panel

(ii), where the downwards-sloping demand curve $D_L = D_L^A - S_L^A$ illustrates the labour demand in the inter-locality labour market.

In locality B the difference in real wages is anticipated from the opposite angle. The firm sector hires all the workers it can for the real wage ω^B , but there are workers, who would be willing to accept work for higher real wages. Therefore there exists a pushing force for emigration in locality B: the over-supply of labour above the real wage ω^B constitutes the upwards-sloping labour supply curve $S_L = S_L^B - D_L^B$ in the inter-locality labour market.

At the moment when mobility of labour becomes free, labour demand and labour supply meet in the inter-locality labour market in panel (ii) of Figure 3. The real wage adjusts to ω^* generating the inter-locality labour market equilibrium at point e . In the equilibrium, inter-locality migration is L_e .

In locality A the adjustment means that the nominal wage falls from w^A_0 to w^A_1 and the price level rises from p^A_0 to p^A_1 in panel (i) of Figure 3. The rise in prices shifts the labour demand curve outwards to $D^{A'}$ and the labour supply curve inwards to $S^{A'}$. Reading horizontally at w^A_1 along the $S^{A'}$ and $D^{A'}$ curves, domestic labour supply falls from L^A_0 to L^A_1 , but labour use rises to L^A_2 . The difference $L^A_2 - L^A_1 = L_e$ is filled up by immigration. Therefore, production increases from q^A_0 to q^A_1 , the aggregate supply curve shifts from AS^A_0 to AS^A_1 and the aggregate demand curve shifts from AD^A_0 to AD^A_1 .

In locality B the adjustment goes the other way round: the nominal wage rises from w^B_0 to w^B_1 and the price level falls from p^B_0 to p^B_1 in panel (iii) of Figure 3. The fall in prices induces the labour demand curve to shift inwards from D^B to $D^{B'}$ and the labour supply curve to shift outwards from S^B_0 to S^B_1 . Reading horizontally at w^B_1 , domestic labour use falls from L^B_0 to L^B_1 . The difference

between labour supply and labour demand $L^B_2 - L^B_1 = L_e$ flows out in the form of emigration. As a result, domestic production reduces from q^B_0 to q^B_1 . The aggregate supply schedule shifts inwards from AS^B_0 to AS^B_1 , and the aggregate demand schedule shifts inwards from AD^B_0 to AD^B_1 .

By the above analysis it is most plausible that, with free mobility of people, there exists an interior solution of residence (or spatial allocation of labour) in the multi-locality economy. The analysis also points out that, while free migration equalises the real wages in the two localities, nominal wages and price levels are not necessarily equalised. For example the prices of housing, which constitute a substantial component in households' living costs, usually differ considerably between in- and out-migration localities. Furthermore, the IS-LM analysis in the above chapter concerning Figure 2 suffices to confirm that, in the present case of Figure 3, the local interest rates adjust in opposite directions: the interest rate in locality A rises and that in locality B falls. If capital is immobile and there is no trade in goods and services, big differences in local prices, nominal wages and interest rates may thus exist. This property raises the question, what may be the implications in a more general situation, where other elements than the labour input are also mobile.

4 Capital accumulation

4.1 The effects of capital inflow in autarky

In the above analysis, capital is assumed non-mobile. There is, however, a good reason to believe that capital movements change the picture of migration flows somewhat. The basic intuition of the standard factor endowment model of international trade is that full tradability of consumption goods is a substitute for mobility of labour and capital – the factors simply move in the form of traded goods (Krugman & Obstfeld, 2003, p. 160-175). The intuition holds also the other way round: with no trade the mobility of the factors does the trick. Since the trade model rests on the initial differences in the K/L ratios of the integrating countries, full factor mobility results in equalisation

of the K/L ratios. Therefore, if both labour and capital are mobile, their movements should occur in opposite directions.

To check the above intuition in the present model, examine the effect of capital mobility on migration. Stick to the simple comparative static analysis, and do not allow perfect mobility of capital but consider only the effects of exogenous changes in the capital stock.

To examine the effect of capital inflow to aggregate supply in the locality, use functions (2) and (3), totally differentiate the labour market equilibrium condition, and have

$$(8) f_{11}dL + f_{12}dK = g'dL,$$

where $f_{11} < 0$ and $f_{12} > 0$ by assumption. Totally differentiating the production function (1), solving for dL from (8) and substituting reads, after manipulation

$$(9) \frac{dq}{dK} = \frac{f_2(f_{11}-g')-f_1f_{12}}{f_{11}-g'} > 0.$$

Expression (9) states the fact that production increases as the capital stock in the economy is increased. To put the message in terms of graphical presentation, an increase in the capital stock shifts the vertical aggregate supply curve horizontally outwards.

Expression (8) already tells that changes in the capital stock have implications also to the labour market. In particular, $f_{12} > 0$ says that higher capital stock implies higher productivity of labour at the marginal, which must be reflected in the labour market equilibrium. To see the exact effect in the

labour market, solve functions (2) and (3) for the real wage $w/p = \omega$, totally differentiate both equations, substitute for dL from the latter to the first one, manipulate and have

$$(10) \quad \frac{d\omega}{dK} = \frac{f_{12}g'}{g'-f_{11}} > 0.$$

By expression (10), real wages in the economy rise if the capital stock increases and fall if the capital stock decreases.

Figure 4 below illustrates the effects of an exogenous capital inflow in one particular locality. The figure includes again the IS-LM supplement. The initial equilibrium is given by point e_0 in the labour market. The equilibrium nominal wage is w_0 , price level is p_0 , and the real wage is ω_0 . Employment is L_0 , production is q_0 , and the aggregate supply curve is AS_0 . The IS_0-LM_0 equilibrium gives AD_0 as the respective aggregate demand curve. The market interest rate is r_0 .

(Figure 4 about here)

An exogenous increase in the capital stock shifts the production function upwards from $f(L, K_0)$ to $f(L, K_1)$ in the southeast quadrant of Figure 4. Expression (10) says that the real wage must also rise, which is presented by the clockwise shift from ω_0 to ω_1 in the northwest quadrant. Furthermore, marginal productivity of labour is increase, which is presented by the parallel shift of the labour demand curve outwards from D_0 to D_1 in the northeast quadrant. The effects imply adjustment in the labour market, goods market and money market. The adjustment mechanism is again analysed step by step from the perspectives of pure nominal wage adjustment and pure price adjustment.

To consider pure nominal wage adjustment, fix the price level to p_0 and read the adjustment from ω_0 to ω_1 vertically in terms of nominal wages. Because the increase in the marginal productivity of labour has shifted the labour demand curve to D_1 , the nominal wage adjustment necessary to reproduce the labour market equilibrium along the unchanged S_0 at point e_1 is from w_0 to w_1 . Labour use increases from L_0 to L_1 , production increases, according to expression (9), from q_0 to q_1 and the aggregate supply curve shifts outwards from AS_0 to AS_1 . The respective aggregate demand curve shifts outwards from AD_0 to AD_1 , which is given by the adjacent shift of the IS curve from IS_0 to IS_1 along the LM_0 curve. The market interest rate rises from r_0 to r_1 .

Analyse pure price adjustment by fixing the nominal wage to w_0 , and reading the adjustment horizontally from ω_0 to ω_1 . The price level falls from p_0 to p_1 , which makes labour demand to fall from D_1 , which is the demand after the increase in productivity, to D_2 . Under perfect foresight, labour supply shifts outwards from S_0 to S_1 . Note that both labour demand and labour supply curves have become flatter because of the fall in prices. The labour market equilibrium is at point e_2 , and employment and production are L_1 and q_1 , respectively. The aggregate supply schedule therefore shifts to AS_1 just like in the above experiment of pure nominal wage adjustment. The aggregate demand schedule, however, shifts outwards from AD_0 only to AD_2 . This is because the fall in prices increases real money, which shifts the LM curve outwards from LM_0 to LM_1 . Therefore, the IS curve shifts from IS_0 only to IS_2 , and the interest rate rises from r_0 only to r_2 .

Under simultaneous adjustment of nominal wages and prices, the price level is determined between p_0 and p_1 , say, to p_2 , and the nominal wage is determined between w_0 and w_1 , say, to w_2 in Figure 4. The labour market equilibrium, that is the intersection of the D_3 and S_2 curves, occurs along the vertical line between e_1 and e_2 at e_3 . This kind of a labour market equilibrium is accompanied by the

AD_3-AS_1 equilibrium in the goods market, and by the IS_3-LM_2 equilibrium in the far southwest quadrant. The market interest rate rises from r_0 to r_3 .

The conclusion from the above experiments is that the positive effects of capital accumulation on employment and production are the same irrespective if the adjustment is carried out by changes in nominal wages or prices or both. In practice, pure price adjustment may not be very relevant, but an important finding is that price adjustment dampens the effects of capital accumulation on the market interest rate. The more prices adjust, the smaller is the opposite change in the market rate of interest

4.2 Effects of capital inflow on migration

The above analysis on the effects of exogenous capital inflow shows that the domestic autarky equilibrium is considerably changed. Now, examine the effects of capital inflow to migration. Carry out the comparative static procedure in two steps: first, introduce capital inflow into the locality and second, allow for free migration. The result of the analysis actually is the effect of exogenous capital inflow to the migration potential of the locality. Figure 5 presents the analysis of a small locality in the basic four-quadrant illustration.

(Figure 5 here)

In Figure 5 the initial autarky equilibrium is at point e_0 in the labour market at the intersection of the D_0 and S_0 curves. In the autarky equilibrium, the nominal wage is w_0 , the price level is p_0 , and the real wage is ω_0 . Employment is L_0 , production is q_0 , and the aggregate supply curve is AS_0 . The aggregate demand curve is AD_0 by the assumption of prevailing equilibrium in the omitted IS-LM setting. The pre-growth migration benchmark can be simply derived by considering pure nominal wage adjustment. Measuring at p_0 , adaptation from ω_0 to the market real wage ω^* of the outer

economy necessitates nominal wage adjustment from w_1 to w_1 . Reading along the original D_0 and S_0 curves, domestic employment reduces from L_0 to L_1 , and domestic production reduces from q_0 to q_1 . Furthermore, as the total labour supply increases to L_2 , the difference $L_2 - L_1$ measures the pre-growth emigration potential in the locality.

To introduce the exogenous capital inflow, let the capital stock in the locality to increase from K_0 to K_1 . The increase in capital shifts the production function outwards from $f(L, K_0)$ to $f(L, K_1)$ in the southeast quadrant, rotates the real wage schedule clockwise from ω_0 to ω_1 in the northwest quadrant, and shifts out the labour demand curve from D_0 to D_1 in the northeast quadrant of Figure 5. Considering pure nominal wage adjustment and reading at p_0 , the new autarky equilibrium is at point e_1 in the labour market, produced by the adjustment of nominal wages from w_0 to w_2 . This results in a rise of employment from L_0 to L_3 , an increase in production from q_0 to q_2 .

Finally, open up migration under the circumstances of experienced growth. Allowing free migration for the constant market real wage ω^* implies, keeping prices fixed to p_0 , further adjustment of the domestic nominal wage from w_2 to w_1 . Reading at w_1 along D_1 , domestic employment now amounts to L_4 instead of the pre-growth L_1 . Domestic production is q_3 , which gives AS_1 for the relevant aggregate supply curve. It is evident that $q_2 > q_3 > q_1$, but it depend on the slopes of labour demand and supply curves if q_3 is higher or lower than q_0 . Nevertheless, potential emigration is the horizontal difference of labour supply S_0 and labour demand D_1 , which amounts to $L_2 - L_4$. Comparison of the post-growth migration potential to the pre-growth benchmark reveals that, since $L_4 > L_1$, then also $L_2 - L_4 < L_2 - L_1$. The result says that the potential for emigration is dampened by the increase in the domestic capital stock. This is because capital inflow has increased the productivity of labour in domestic use. The result is quite intuitive.

The same result can be derived also by examining pure price adjustment and fixing the nominal wage at w_0 . The price adjustment can also be split into two parts, the growth effect that makes the price level to fall from p_0 to p_1 , and the migration effect that makes the price to adjust further to p_2 . As a result, the labour demand curve shifts first parallelly outwards from D_0 to D_1 because of capital accumulation, and then inwards from D_1 to D_2 because of the fall in domestic prices. The labour supply curve shifts accordingly outwards from S_0 to S_1 . As prices fall, the labour demand and labour supply curves become flatter. Measured at w_0 , emigration is $L_2 - L_4$, which is the same as above. The result remains unchanged in terms of migration also if both prices and nominal wages are allowed to adjust. One such solution is presented by the p_3, w_3 –pair of prices and nominal wages, which implies shifts of the demand and supply curves to D_3 and S_2 , respectively.

The above result supports the intuition that an inflow of capital is a substitute for emigration. The larger the capital inflow is, the smaller is emigration. As a matter of fact, the inflow of capital can fully offset the need for emigration, if the impact of capital accumulation is strong enough to make the domestic real wage the same as the market wage in the outer economy, or $\omega_l = \omega^*$ in terms of Figure 5. Moreover, the impact may be even high enough to make the domestic real wage line steeper than that of the outer economy, $\omega_l > \omega^*$, which is to say that the initial migration pattern is reversed and emigration turns into immigration. The result rises up the question about the nature of the market equilibrium in the case of full factor mobility.

4.3 Adjustment in the factor market

The present framework is static in nature and proper analysis of the factor market dynamics remain out of its scope, but at least some comparative static indications can be derived about the equilibrium properties of the model. As a matter of fact, Figure 6 below manages to yield a strong result concerning the nature of the equilibrium. The figure presents the case of an economy

consisting of two symmetrically big localities, between which both labour and capital as well as consumption goods can all be regarded as mobile. Yet, in order to get any results, the focus is on the movements of capital and labour.

Figure 6 consists of two panels, panel (i) for locality A and panel (ii) for locality B. As compared to the earlier Figure 3, the presentation of the inter-locality labour market is omitted, but the IS-LM frameworks for the two localities are added. The tradability of goods is treated by the assumption that the price level is fixed to p^* in both localities A and B. This can be regarded to reflect the free trade equilibrium of consumption goods in the economy.

(Figure 6 here)

In Figure 6, the initial situation in locality A is given by the labour market equilibrium a_0 at the real wage ω^A in panel (i) of the figure. At capital stock K^A_0 and employment L^A_0 , production is q^A_0 . The initial equilibrium interest rate is r^A_0 . The respective equilibrium in locality B in panel (ii) of the figure is described by point b_0 at real wage ω^B in the labour market. The capital stock K^B_0 and employment L^B_0 give q^B_0 for production. The interest rate is initially r^B_0 .

First, consider the motives for migration. In the initial situation there is a difference in real wages in favour to locality A, $\omega^A_0 > \omega^B_0$. This implies migration from locality B to locality A. The migration equilibrium is found by measuring the required adjustment in nominal wages vertically at the fixed price level p^* in both localities. The presentation of the inter-locality labour market is omitted, but nevertheless it is evident that the migration equilibrium is reached to by adjustment of the local nominal wages so that a common equilibrium real wage ω^* is settled. This is to say that, prices

being fixed to p^* in both localities, the nominal wages must be equalised, too. It is also evident that production and the interest rate rise in locality A and fall in locality B.

Second, introduce simultaneous capital movements. In the initial situation in Figure 6, the market interest rate in locality B is higher than that in locality A, $r^B_0 > r^A_0$. The difference in the market interest rates attracts capital from locality A to locality B. Recalling the message of Figure 5, these kinds of capital movements offset the migration flows described above as the production function and the labour demand curve shift inwards in locality A and outwards in locality B. These movements also cause a negative pressure on the interest rate in locality A and a positive pressure on the interest rate in locality B

The simultaneous equilibrium in labour and capital markets is given by the set p^*, ω^*, r^* . Reading the result in terms of labour, the migration equilibrium is $L^A_1 - L^A_2 = L^B_3 - L^B_1$ (that is, immigration to A equals emigration from B) at the equilibrium real wage ω^* . As capital increases from K^B_0 to K^B_1 in locality B and decreases from K^A_0 to K^A_1 in locality A, the demand for labour in locality A shifts inwards from D^A_0 to D^A_1 , and the demand for labour in locality B shifts outwards from D^B_0 to D^B_1 . The result is that $L^A_3 - L^A_1$ is the amount of immigration that the capital outflow offsets in locality A, and $L^B_1 - L^B_2$ is the amount of emigration that the capital inflow offsets in locality B. In the general equilibrium, production in locality A increases from q^A_0 to q^A_1 , and production decreases in locality B from q^B_0 to q^B_1 .

The analysis in Figure 6 suggests that there exists an interior market solution of spatial factor allocation between the two localities. The key finding from the analysis is, however, that interior solutions are possible only in the special case, where the motives for factor movements draw the factors to opposite directions. In the context of the figure this precludes that $\omega^A_0 > \omega^B_0$ and $r^A_0 < r^B_0$,

or $\omega^A_0 < \omega^B_0$ and $r^A_0 > r^B_0$. It is easy to verify in the figure that all other kinds of real wage and interest rate relations imply parallel factor movements inwards and outwards, which leads to a corner solution.

The conclusion is that, in the present model without further restrictions and policy considerations, the market mechanism is incapable to secure a stable and efficient interior solution of employment, production and residence in an economy consisting of a multiple of localities. Corner solutions, where some localities flourish and some others end up to be deserted, may well exist. This property of the model can be traced back to the assumption of positive second cross partials of the production function ($f_{12} = f_{21} > 0$), and it can be read also from the standard textbooks presentations of the factor market effects of economic growth (see for example Burda & Wyplosz, 1996, p. 110-113). Growth models with migration and capital mobility usually apply special assumptions to avoid corner solutions (Andersson & Kuenne, 1986; Barro & Sala-i-Martin, 2004, p. 383-407).

5 Conclusions

The paper constructs a general equilibrium elaboration of the classic model of migration. The basic properties of the model are illustrated and its usefulness in analysing the rather complex set of factors and effects connected to migration is demonstrated. The virtue of the model is in its simplicity: the analysis is manageable in terms of graphics. The model gives a sound basis for policy considerations and other further experiments.

The model yields intuition to the adjustment mechanism of real wages under free migration. In the migration equilibrium, real wages are equalised while both nominal wages and prices may well differ between locations. Local adjustment can happen through the evolution of nominal wages, prices or both. The outcome is the same in all versions in terms of migration, domestic employment

and production, but the effect on local interest rates is different: the interest rates are affected the more the larger share of the adjustment is carried out by local prices. The proportional share of price adjustment can be considerable because of the important role of housing costs in the determinants of local price levels. In the housing market the supply side is usually rather inelastic in the short run, and changes in demand manifest rapidly in rents, real estate values and other housing prices.

The effects of capital accumulation are also studied. An exogenous increase in the capital stock fosters productivity of labour and real wages. The real wage can improve through a rise in nominal wages, a fall in prices or through a simultaneous change in both. The real effects are the same in all cases, but the effects on the rate of interest again differ: the interest rate rises the more the less the prices fall. In practice the fall in prices may be quite moderate. The analysis also shows that a capital inflow acts as a substitute for emigration. This is to say that capital accumulation reduces potential emigration and vice versa.

A major finding is that in an economy consisting of a multiple of localities with fixed capital endowments and no trade, free migration quite plausibly ends to an interior spatial solution of residence. The question becomes more problematic, when capital is mobile, too. The result of a comparative static analysis is that an interior solution is possible but not granted. Corner solutions, in which some localities become deserted may well exist. The result seems to be in odds with the famous convergence hypothesis of economic growth, but, since the present model is not a growth model, and it cannot challenge the hypothesis. Still, being derived in a most elementary set-up with standard neoclassical elements, the finding reserves attention. In the present model, the neoclassical paradigm seems dismal to less developed areas, which, instead of economic growth and convergence, often find them in a vicious circle of economic decline caused by parallel outflow of labour and capital.

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Figure 1: The basic model

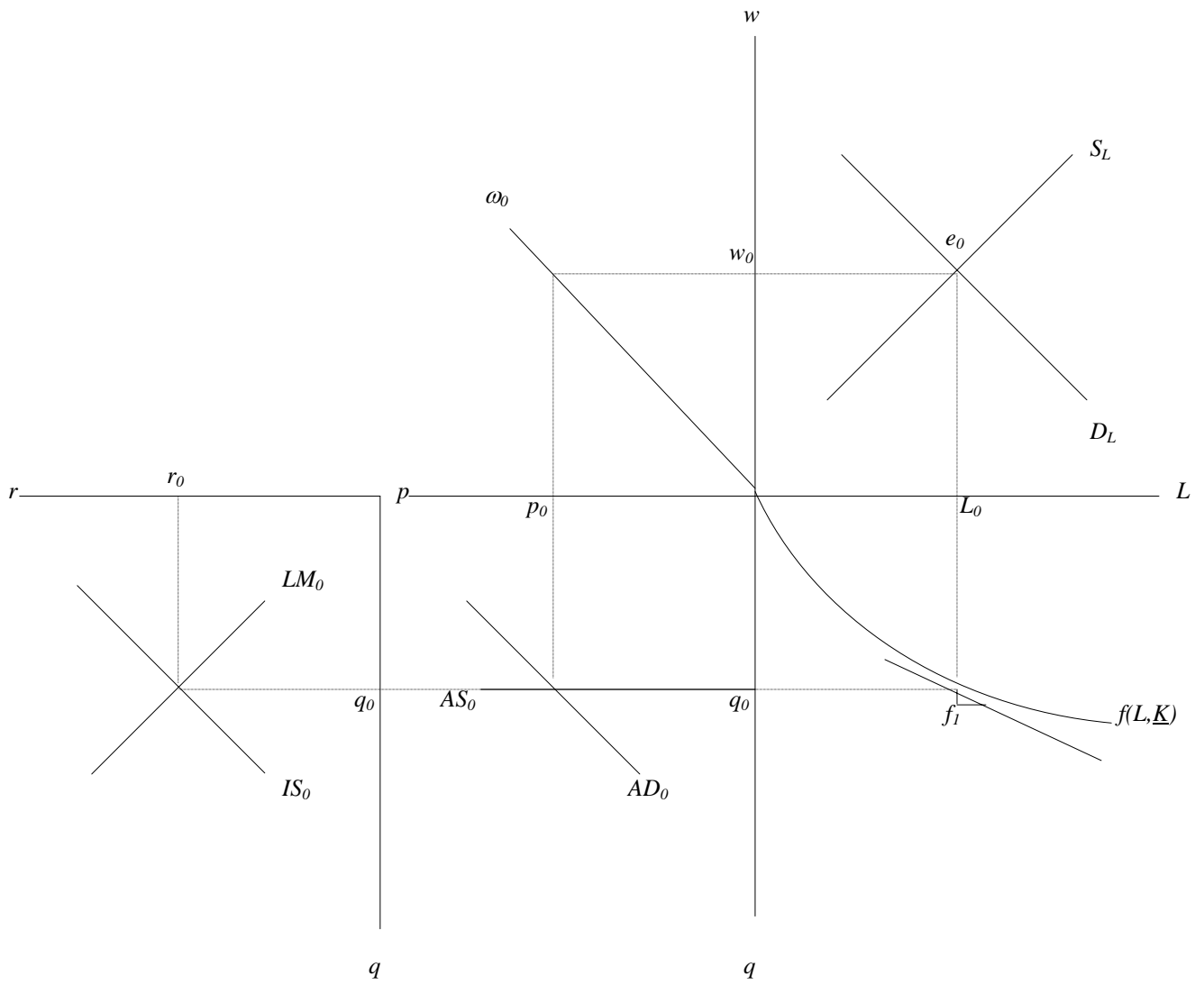


Figure 2: Emigration as a response to real wage differentials

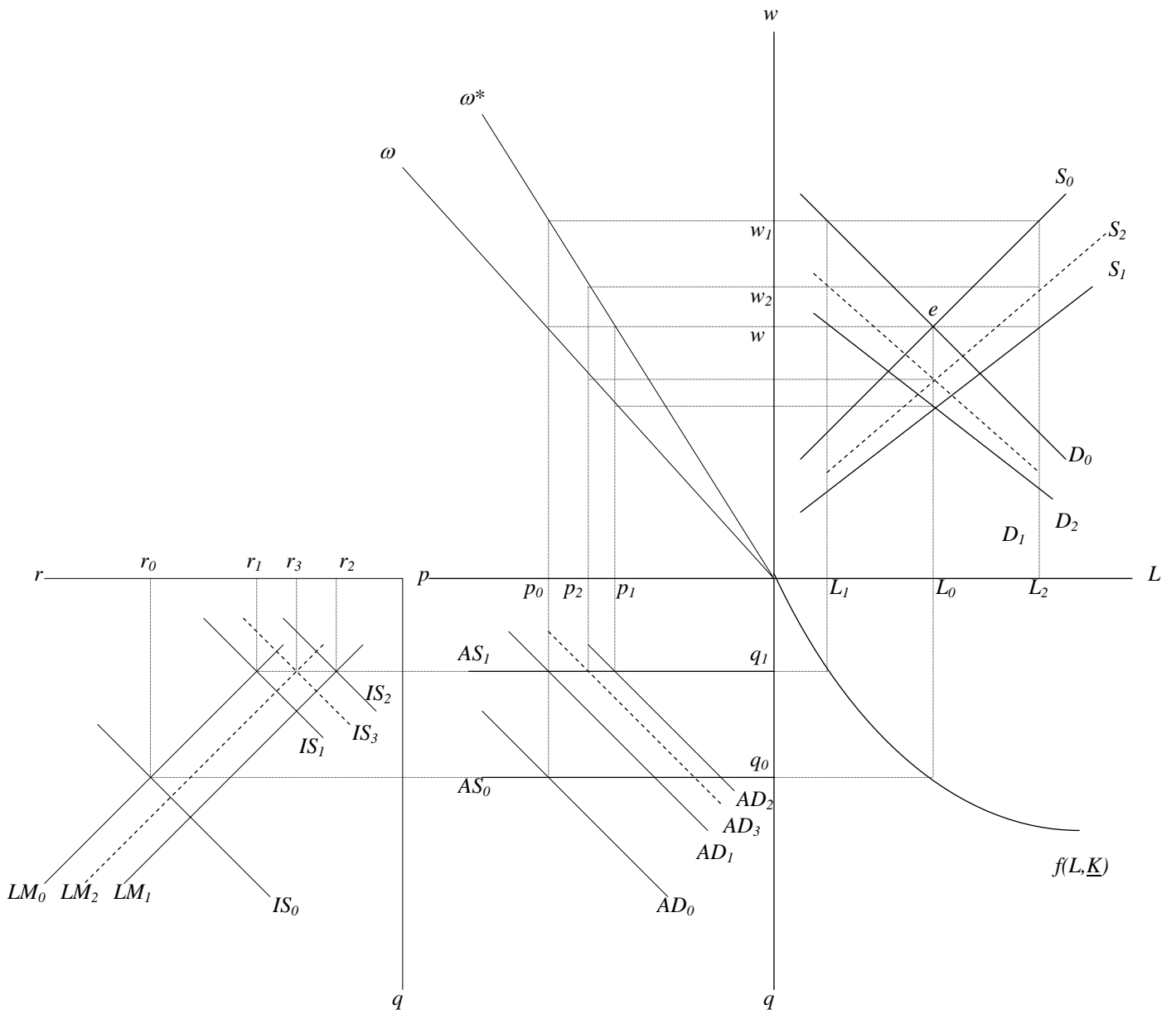
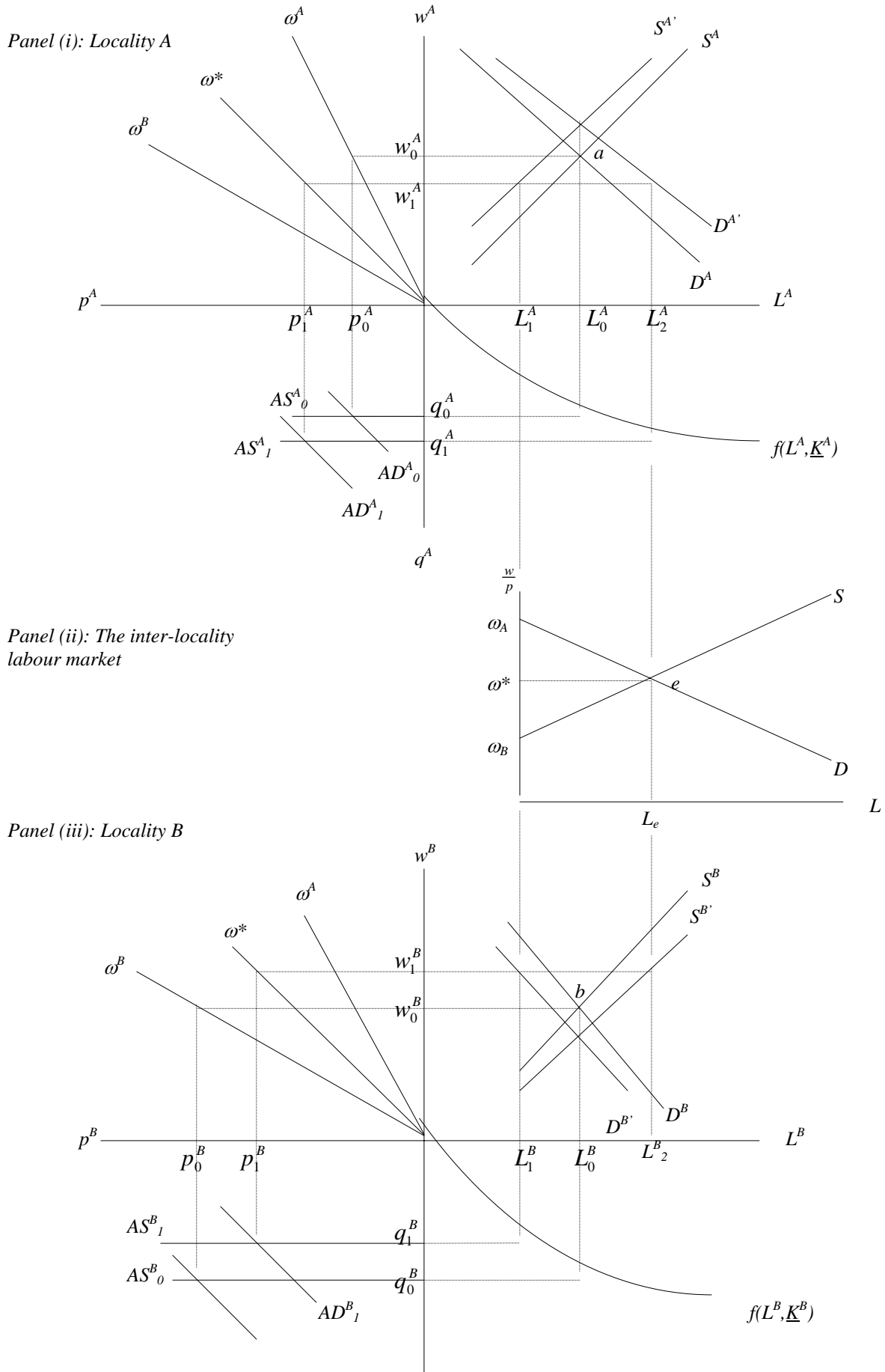


Figure 3: Migration equilibrium between two big localities



AD^B_0 q^B

Figure 4: The effects of capital inflow

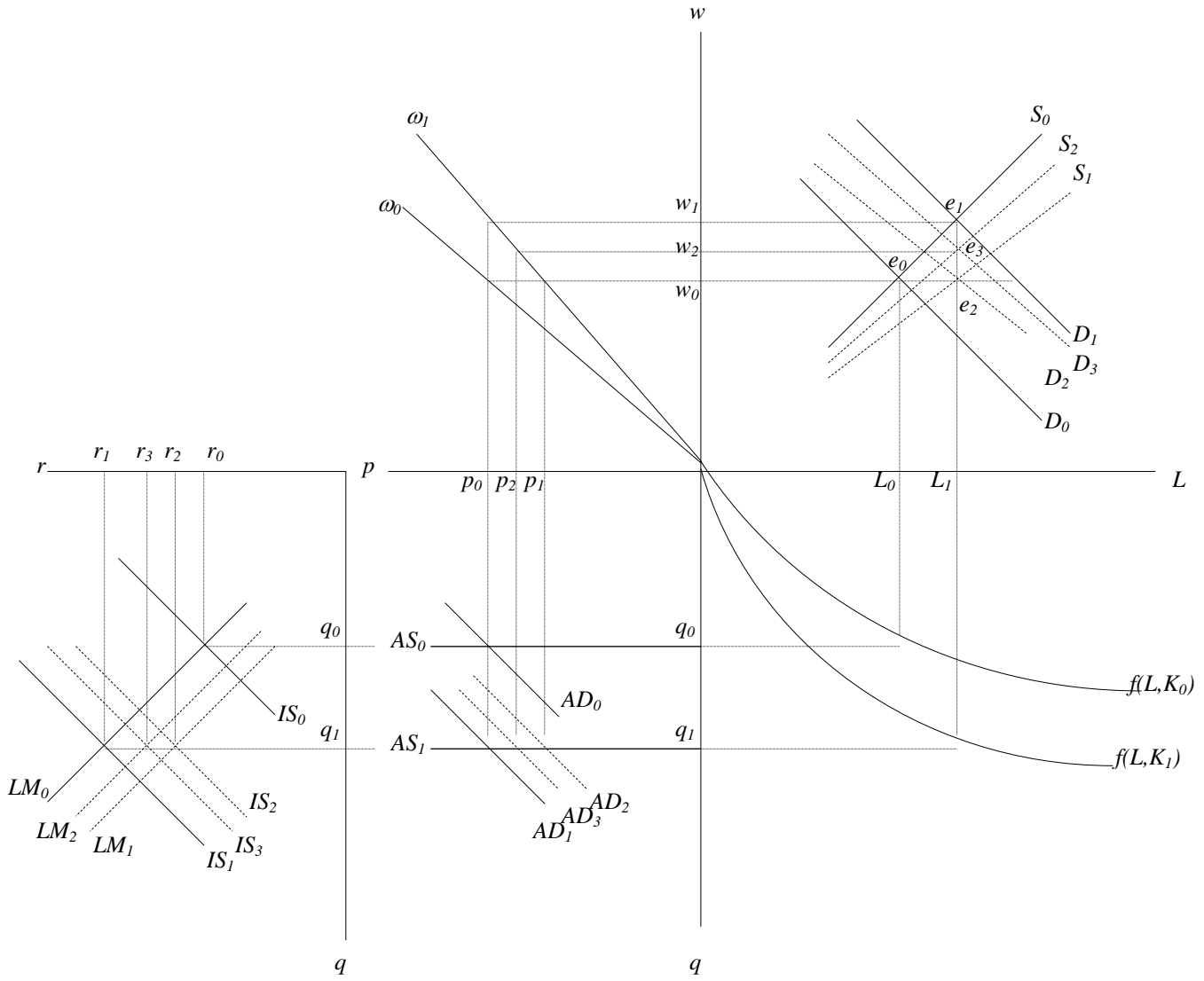


Figure 5: The effect of capital inflow to emigration

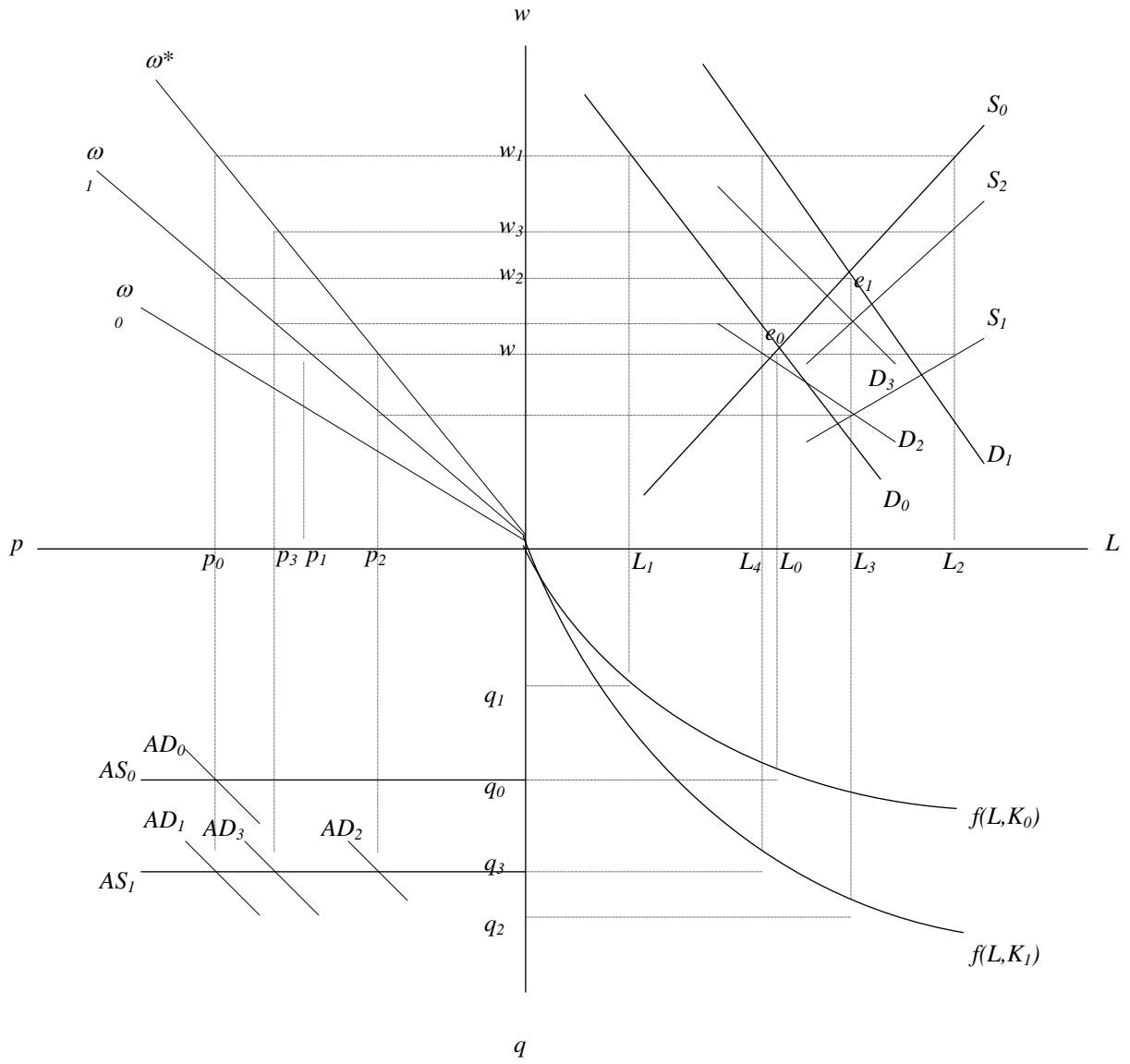


Figure 6: Capital movements and migration between two localities

