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THE U.S. DOLLAR REAL EXCHANGE RATE

A REAL OPTION APPROACH

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Abstract

The aim of this paper is to discuss the determinants of the U.S. dollar real exchange rate fluctuation. We focus our analysis on a nominal exchange rate effect on tradable prices. We explicitly consider the effects of profit maximizing foreign firms' entry decisions on the domestic tradable prices through the supply changes after a large appreciation. If firms face sunk entry costs when breaking into foreign markets, the extent of pass-through will depend on the expected changes of nominal exchange rate. Typically, exchange rate uncertainty is determined as a volatility of continuous time series process. We enlarge the discussion to consider also possible jumps in the expected exchange rate time path. Finally, an interesting perspective is provided by a real option approach that emphasize dynamic supply effects through sunk costs and uncertainty.

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1. INTRODUCTION

During some subperiods of floating rates traditional macroeconomic models, such as monetary or Harrod-Balassa-Samuelson model, explain monthly or quarterly exchange rate movements reasonably well and during other subperiods their explanatory power completely disappear.¹ If there were no relative structural shocks between two currency areas, a real exchange rate should be a stationary variable and it should follow the purchasing power parity hypothesis (hereafter PPP). However, empirical evidence for large and persistent deviations from PPP is overwhelming. Recent evidence also suggests that the failures of the law of one price are not only significant, but they also play a dominant role in a behavior of real exchange rate.²

A prominent question in this research is why the U.S. real exchange rate exhibit large and persistent deviations from PPP. We discuss the determinants of the U.S. dollar real exchange rate fluctuation since the collapse of the Bretton Woods system of fixed exchange rates in the early 1970's. The real exchange rate is measured using CPI deflators.

Number of studies have given quite strong evidence that deviations from PPP derive in large part from differences in relative traded goods prices across countries.³ Recent studies have especially increased interest in the effect of fluctuations of U.S. dollar on U.S. tradables prices. Engel's (1999) results concerning the U.S. dollar suggest that consumers prices for tradables goods behave very much in the same way as non-tradables consumer prices. These results are quite puzzling in a view of Harrod-Balassa-Samuelson hypothesis which holds that the relative tradables prices show little long-term variation across countries compared with the variation in relative non-tradables prices. Canzoneri, Cumby and Diba (1999) also argue that the problems with the Harrod-Balassa-Samuelson hypothesis lie in the failure of PPP to explain the U.S dollar traded goods prices. Above evidence seems to suggest that the dominant component of real exchange rate behavior is a nominal exchange rate even in a long run through the incomplete pass-through. Thus, we focus our analysis on a nominal exchange rate effect on tradable prices.

The incomplete pass-through of international price setting has been addressed in various ways in the literature. The most common analytical tool to examine incomplete pass-through has probably been

¹ See the discussion in Goldberg (2001).

² As a survey see Rogoff (1996).

³ See, for example, Asea and Mendoza(1994) and De Gregerio and Wolf (1994).

pricing-to-market approach which presuppose short term rigidities and the market power of importing companies. These market imperfections allow foreign suppliers to set the markup of prices over the marginal cost. The assumptions of pricing- to- market approach are, however, in many ways under the much debate and not necessarily even sufficient for long-term deviations in aggregate price index analyzed in this study. Aggregate price index, such as a consumer price index, definitely includes goods produced by industrial sectors which are best characterized by the incomplete international competition but also by industrial sectors which are almost competitive. Thus, we have to extend our tool kit a bit more.

Among all other economic explanations of incomplete pass-through we have limited this paper to the (international) real option investment theory inspired by MacDonald and Siegel (1986), Pindyck (1988), Dixit (1989a and b) and Dixit (1993). Using a real option theory we examine foreign firms' entry and exit decisions in competitive domestic importing markets.

Since foreign firms are focused the exchange rate uncertainty is especially considered. At first, a nominal exchange rate is determined by net capital flows. The value of nominal exchange rate is assumed to be revealed before the firms make their decision whether or not to be in the market. We explicitly consider the effects of profit maximizing foreign firms' entry and exit decisions on the domestic tradable prices through the supply changes.

Dixit (1989a and b) assumes that the exchange rate follows a geometric Brownian motion and the inaction band (no entry or exit) around the base value is either stable or determined by the market share of foreign firms if the expected uncertainty is assumed to be constant. Although values given for entry and exit trigger points in Dixit (1989b) are realistic especially in the median firm case, we argue that the dynamic structure of the pass-through process is at least partly problematic. Deviations from the parity level are too volatile and persistent to be explained just by the changes in number of firms or changes in expected uncertainty if uncertainty is generated by the continuous time series process. To avoid these problems we enlarge the discussion considering also possible jumps in the expected exchange rate time path.

The estimates in Sollis, Leybourne and Newbold (2002) show that there is a weaker mean reversion when the U.S. dollar real exchange rate is overvalued relative to historical averages. The adjustment to PPP level is not only nonlinear (inaction band around mean) but also asymmetric. Thus, we especially consider entry decisions after a large appreciation. Furthermore, we assume that the

exchange rate appreciation is expected to be temporary and the adjustment is expected to be relatively rapid. This gives us an opportunity to analyze the possibility that the U.S exchange rate follows the mixed Brownian motion-Poisson jump process after a large appreciation. Together with the real option investment theory this seems to offer an interesting explanation for the time path of the U.S. dollar real exchange rate. After a large positive shock in nominal exchange rate foreign suppliers do not completely adjust their supply since there is a substantial probability for a large negative shock. We are now able to explain large and persistent deviations from the parity level which are not constant in magnitude without assuming market imperfections or a systematic link between trade flows and the deviations of the real exchange rate from trend.

2 TRADABLE GOODS PRICES AND U.S DOLLAR REAL EXCHANGE RATE

Traditionally tradable prices are assumed to follow the rule of the one price. In this chapter we will discuss the collapse of PPP for tradable goods. We examine more closely the time path of the U.S. dollar real exchange rate and its deviations from the parity condition. Some special characters of the U.S tradable prices are also considered.

2.1. INCOMPLETE PASS-THROUGH

As discussed in Obstfeld and Taylor (1997), even the studies most favorable to long-run PPP suggest an extremely slow decay rate for international price differentials. Estimated half-lives for PPP deviations for most countries and time periods are found to be on the order of four to five years.⁴ These estimates appear to imply more sluggishness than one can attribute entirely to nominal rigidities alone.⁵

Persistent deviations from PPP indicate incomplete pass-through. One explanation commonly offered for incomplete pass-through puzzle is an international price discrimination. Krugman (1987) labeled the phenomenon of exchange rate induced price discrimination in international markets as “pricing-to-market”, hereafter PTM. According to the PTM approach international markets for

⁴ See, for example, Wei and Parsley (1995) or Frankel and Rose (1996). Panel estimations, such as Papell (1998) and Oh (1996), generally find somewhat more rapid reversion with half lives on the order of 2 to 2,5 year.

⁵ Cheung and Lai (2000b) reports hump-shaped adjustment paths of the U.S. real exchange rates, which are also incompatible with standard sticky price models.

manufacturing goods are sufficiently segmented that producers or retailers can, at least over some horizon, tailor the prices they charge to the specific local demand conditions prevailing in different national markets. Thus, firms set different prices for its goods across segmented national markets to compete with firms in those markets. According to Dornbusch (1987) the degree of pass-through depends on i) substitution between domestic and foreign goods ii) market integration iii) market organization.

Since we analyze aggregate price index (CPI), it is partly problematic to take the ability to price discriminate through the substitution effect as being absolute. As emphasized by Rogoff (1996) a segmentation might be the case for some goods, such as automobiles, where differences in national regulatory standards combined with need for warranty service allow firms great leeway to price discriminate across countries. There is, however, a substantial amount of tradable goods which are homogenous in different countries. Also findings of Knetter (1993), which show that pricing to market seems to characterize even the most mundane goods, are not in line with this substitutability assumption.

An other possible explanation for persistency in deviations is that traded goods market are not completely integrated. It is possible that trade frictions, such as transportation costs, allow tradables prices to differ within some range without inducing profitable arbitrage. Estimates of average transport costs across all tradable goods range between 6 and 10 per cent.⁶ In addition, tariffs and non-tariff barriers can be important frictions. Thus, transaction costs should provide some scope for deviations from PPP. Based on this assumption there should be one rapid convergence band when price differences exceed transaction costs, and one slow or non-convergence band when price differentials are relatively small. Michale, Nobay and Peel (1997), and Obstfeld and Taylor (1997), among others, have found evidence that large PPP deviations die out more rapidly than small ones.⁷

The market organization which provides market power for firms is, albeit essential for PTM, under the much research and debate.⁸ In Cournot oligopolistic market formulation with homogeneous goods, Dornbusch (1987) summarized the elasticity of the equilibrium price with respect to the exchange rate as

⁶ See the discussion in Rogoff (1987).

⁷ Michale, Nobay and Peel (1997) use long time series spanning 1791-1992 and postwar time series for bilateral exchange U.S. real exchange rates. Obstfeld and Taylor (1997) use data measured relatively to the US after 1980.

⁸ See the discussion in Cheung, Chinn and Fujii (1999).

$$\varphi = (n^* / N)(xw^* / p)$$

The elasticity formula has two determinants: the relative number of foreign firms and the ratio of marginal cost of foreign firms (w^*) in home currency (x) to price of foreign suppliers in domestic currency (p)⁹ The first determinant of elasticity formula simply tells us that the higher share of imports increases the elasticity of tradables price level for exchange rate changes. The last determinant of elasticity formula is a price discrimination determinant. It is essential for the pricing-to-market literature which has focused on the issue of markup adjustment as a possible explanation for very slow response of tradable goods prices to exchange rate movements.

Recent literature supplements the PTM assumption with an extra assumption on local-currency pricing, i.e. prices are assumed to be sticky in the local currency of the buyer.¹⁰ It is again important to notice that we concentrate on consumer price indices, i.e. trade at the consumer level. Something important must be happening between the consumer level, where the medium-term effect of exchange rates on prices is virtually zero for many goods, and the wholesale level, where price effects tend to be less than proportional but also significantly greater than zero.¹¹ According to Obstfeld and Rogoff (2000a), local currency pricing is pervasive at the retail level, which should explain these findings.¹² Price contracts are not, however, generally thought to be very long-lived.¹³ Bergin and Feenstra (2001) found that price contracts combined with PTM are able generate endogenous persistence beyond the exogenously imposed rigidity. Nevertheless, they are not able to reproduce the degree of persistency observed in the data.¹⁴

Finally, we again would like to stress the importance of trade cost for incomplete pass-through. Obstfeld and Rogoff (2000b), emphasize the importance of trade cost in output market as an explanation for resolving PPP puzzle since they prevent resales of goods. As pointed out by Obstfeld and Taylor (1997), advances in the theory of investment under uncertainty imply, however, that a band of no-arbitrage should be interpreted as a resulting not only from concrete

⁹ As shown in Dornbusch(1987), qualitative results do not depend strictly on the chosen model. Dornbusch (1987) examines Cournot oligopolistic model and a more keynesian type of Dixit-Stiglitz model.

¹⁰ See, for example, Devereux (1997).

¹¹ See the results in McCarthy (1999).

¹² Obstfeld and Rogoff (2000a) argue that if a local-currency pricing was common practice in manufacturing then country's manufacturing terms of trade improve if its currency depreciate. This is inconsistent with the data.

¹³ Typically at most one to two years.

¹⁴ Chari, Kehoe, and McGrattan(1998) find, using only the exogenous persistence of price contracts, that sticky prices can help replicate persistence, but only if at least 3 years price contracts are assumed.

shipping costs and trade barriers, but also from sunk costs of international arbitrage.¹⁵ Thus, one interesting perspective, which partly combines above arguments, is provided models that emphasize dynamic supply effects. Dixit (1989a and b) show that when firms face sunk entry costs when breaking into foreign markets, the extent of pass-through will depend on the expected changes of nominal exchange rate. Different pricing behavior on different markets now depends on entry and exit decisions of competitive firms. Prerequisites for entry into foreign market are, for example, investment in marketing and distribution network, which are especially important at the consumer level.

2.2. U.S. TRADABLE CONSUMER PRICES

There are some special issues related to U.S. tradable prices. Canzoneri, Cumby and Diba (1999) using panel data argue that failure of PPP to explain traded goods prices is especially important for the U.S. dollar. It is certainly true that a non-tradable component is important in determination of tradables prices. Engell (1999) results makes clear that more than just this must be going on since prices for tradable goods do not seem to respond any faster to exchange rate movements than do the prices of non-traded goods. Engell (1999) has also pointed out that tradables prices appear to account for a large part of the movement of U.S. real exchange rates independently on the chosen real exchange rate between the U.S. and high income countries.¹⁶

It is also interesting to view the relation between the nominal exchange rate and the price differential. We use the U.S. dollar/German mark variables as an example. The large deviations of nominal exchange rate from the long-run price trend are shown in Figure 1, where X is the U.S. dollar/German mark nominal exchange rate and dif is the difference of consumer price indices in the U.S.A. and Germany.

¹⁵ O'Connell and Wei (1997), provide a continuous time model on goods arbitrage that highlights the relative importance of proportional and fixed cost of transactions. Their evidence indicates that the fixed component is dominant.

¹⁶ Engel (1999) uses aggregate price indices, but there are also similar findings even for highly disaggregate data. See Giovannini (1988) and Engel and Rogers (1996).

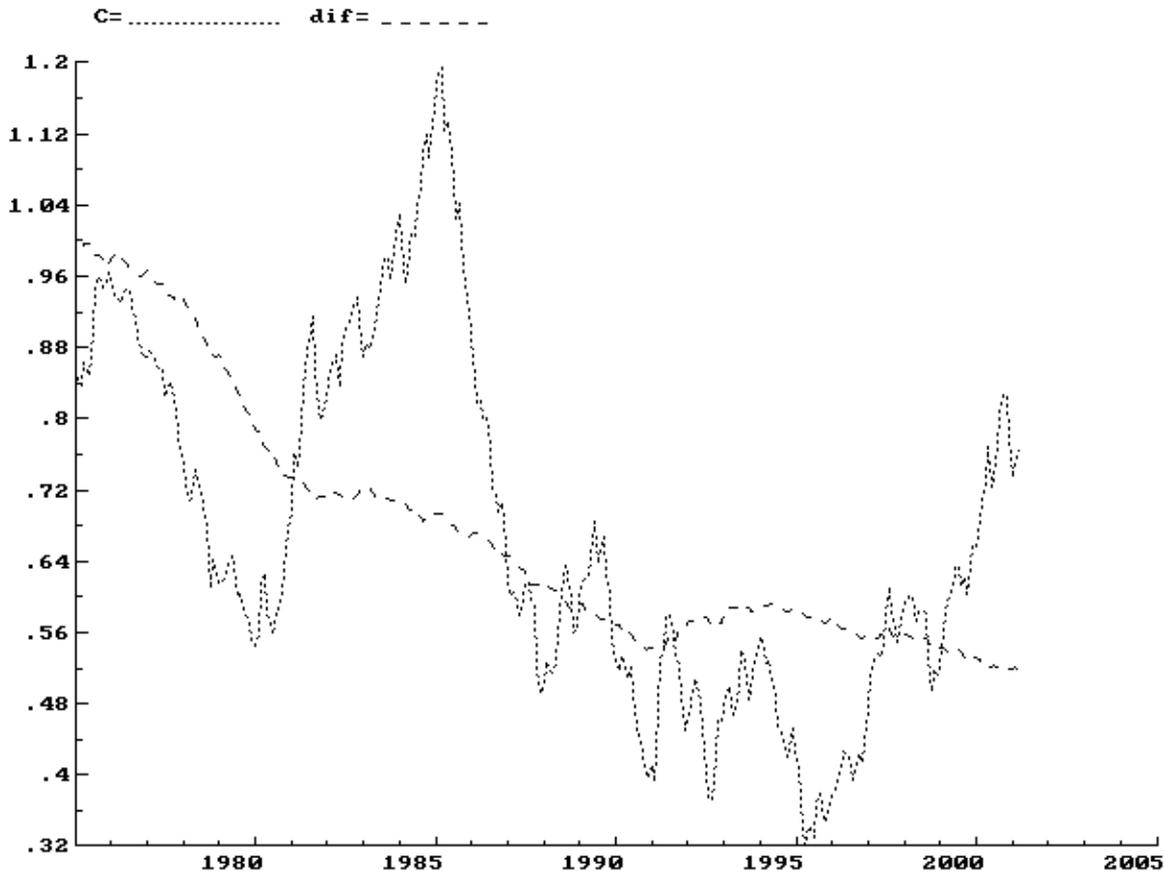


Figure 1. Nominal exchange rate and price differential.

Deviations from the parity level do not seem to be an order of magnitude constant but there are large changes in magnitude. They are too volatility to be explained by the constant converge band especially in the middle of the eighties and probably also in the beginning of new millennium. There are also large jumps in the time path of the nominal exchange rate. Note that most of the large deviations from the parity level seem to emerge simultaneously with large swings in the nominal exchange rate.

Pass-through from the dollar exchange rate to U.S tradable prices fall in the 1980s.¹⁷ The estimates in Sollis, Leybourne and Newbold (2002) show that there is weaker mean reversion when the U.S. dollar real exchange rate is overvalued relative to historical averages. This puzzling fall during the time period of appreciating dollar has been pointed out various ways in the literature. Marston (1990) points out that nominal exchange rates surprises lead only to temporary changes in pass-through due to preset prices. Permanent changes are also possible if there are fundamental changes

¹⁷ See, for example, Dornbusch (1987), Froot and Klemperer (1989).

in PTM behavior. PTM behavior is, in turn, determined in Marston (1990) by the differences in price elasticities of demand curves in each importing countries. Dixit (1989b) assumes that the sunk cost of entry decreases pass-through if exchange rate appreciation is not large enough. However, sufficiently large appreciation can raise pass-through permanently even if appreciation itself is only temporary.¹⁸ Froot and Klemperer (1989) show that a model with consumer switching costs will lead exporters to respond differently to temporary and permanent changes in exchange rate. They examine the effects of temporary appreciation of the dollar focusing on dynamic demand side effects in an oligopolistic market. In their model temporary appreciation increases the value of current, relative to future, dollar profits expressed in foreign currency. When the dollar is temporary high, foreign firms will find investments in market share less attractive, and will prefer instead to let their current profits to increase.

2.3. U.S. DOLLAR EXCHANGE RATE

As discussed in Goldberg and Knetter (1997), the assumption of temporary exchange rate change in Froot and Klemperer (1989) is partly problematic since the literature of exchange rate determination shows only a very weak evidence in favor of reversion to PPP, meaning that most changes might be viewed as permanent. Thus, we take a different approach by assuming that exchange rate follows a mixed random walk process. Especially, after a large appreciation there is a certain possibility for unexpected jumps in the process.

Obviously, we should first find some evidence for the mixed random walk process. In particular, the time period between 1980 and 1985 is notable. The dollar appreciation was started by the effects on interest rate of the increasing US fiscal deficit. As Froot and Klemperer (1989) discuss, nominal interest rate differential, a common measure of expected depreciation, shows that the dollar was expected to depreciate most rapidly in the early 1980s, just when the rate of appreciation was also the greatest. However, as pointed out by Juselius and MacDonald, (2000) the prolonged nature of the appreciation would seem to be unwarranted solely in terms of interest rate differential or “safe haven” effect. They also argue that it might be a speculative bubble which was intensified by the role of dollar as a reserve currency in the internationally monetary system.

¹⁸ Permanent effect can be cancelled only if the appreciation period is followed by the large depreciation period. In Chapter 5 we will discuss the dynamic structure of model more precisely.

Another interesting evidence is provided by literature which discuss the relationship between oil price and the U.S. dollar appreciation. In articles such as Amano and van Norden (1998), and Lahtinen (2000), the U.S real exchange rate is shown to be a positive function of the real oil price, i.e. higher oil price will appreciate the U.S real exchange rate.¹⁹ The net effect of oil price shock on nominal exchange rate depends on capital account changes, i.e. whether investment in dollar currency is more or less than America's share of the industrial world's current account deficit. The unstable and unpredictable nature of the oil price process makes it a text book example of the non-discrete jump process. We should also notice that oil price is a special kind of source of capital inflow, since it is not determined in competitive markets. Furthermore, Krugman (2000), offers a multiply equilibrium explanation for the oil price. This makes it even more vulnerable for unexpected jumps.

Above discussion gives us an opportunity to study the order of pass-through after an exchange rate appreciation. The appreciation is expected to be temporary and the adjustment to the parity level is expected to be relatively rapid. Regardless of the source of exchange rate shock the effects are similar, if investors perceived even a small probability of a large and rapid depreciation. Next we will examine the effect of mixed process assumption on pass-through using a supply side model developed by Dixit (1989).

3. A MODEL FOR INCOMPLETE PASS-THROUGH

To examine exchange rate pass-through we introduce a simple market structure following Dixit (1989b). In this model demand is assumed to be stable, i.e. importing goods prices are supply side determined. Supply side of economy is competitive, i.e. importing firms act as price-takers. Let the demand for import goods be determined by the inverse demand function $p = P(n)$. For all n there is a function such that $p = P(n) = U'(n)$, i.e. the n th firm's marginal contribution to utility is equal to the market price $P(n)$. We can write

$$P(n) = U(n) - U(n-1) \quad \text{and} \quad U(n) = \sum_{j=1}^n p_j \quad (3.1)$$

¹⁹ Amano and Van Norden (1998) examine the real effective exchange rate and Lahtinen (2000) use German mark/U.S dollar real exchange rate.

Only trade is considered, not the location of production. Similarly only supply of importing firms with profits measured in foreign currency is considered, not the supply changes of domestic firms. We also assume that each foreign firm sells one unit of output per unit of time, i.e. supply changes depend strictly on changes in number of firms. Thus, price level is completely determined by the number of foreign active firms.

Foreign firms are characterized as follows. There are same sunk costs for every foreign firm to enter home market.²⁰ They are not able to recoup these sunk costs if the firms decide to quit at a later date. We let, however, firms to differ in their variable cost.²¹ The variable cost of the first n firms are defined as

$$w_n = W(n) - W(n-1) \quad \text{and} \quad W(n) = \sum_{j=1}^n w_j \quad (3.2)$$

The firms are labeled so that w_n is increasing. Since w_n is increasing, $W(n)$ is increasing and convex. Variable cost of foreign firms are dominated in foreign currency. All foreign firms also have same relative technological progress, i.e. they differ only in their variable costs.

Foreign firms are risk neutral and have rational expectations, i.e. they maximize the expected present value of profits in foreign currency. The maximand is

$$E \left\{ \int_0^{\infty} \{X_t U(n_t) - W(n_t)\} e^{-\rho t} dt - \sum_i X_i \{k[\Delta n_i]\} e^{-\rho i} \right\} \quad (3.3)$$

where X is nominal exchange rate and ρ is a real interest rate, which is used as the discount rate by the foreign firms. At instant $t=i$, when the numbers of foreign firms change, there is a sunk cost k for every foreign firm to enter the market.

Nominal exchange rate X_t determines the price level through the investment decision of foreign importing firms. At the beginning of the period the size of net capital inflows is revealed. In order to

²⁰ There is no specific cost to quit home market.

²¹ Sarantis (1999) has argued, that the transition between the convergence regimes based on transaction costs is smooth. This is probably due to the heterogeneous investors.

assess the simple feedback from net capital inflows to the nominal exchange rate, we assume that nominal exchange rate is completely determined by net capitals inflows in a medium run. Thus, there is no feedback from firms' investment decisions to the nominal exchange rate process, i.e. exchange rate process is exogenous in respect of investment decisions of foreign importing firms.

After the size of net capital inflows has been revealed firms decide whether to enter or exit. Depreciation of foreign currency increases the demand for foreign goods. There are also necessary sunk costs for entry in home markets. Thus, investment decisions (importing decisions) are based on maximand given above. In an interval of time when no entry or exit takes place, supply is fixed and prices are proportional to the nominal exchange rate shock. Maximization of the expected present value of investments determine, in turn, a range of current values of exchange rate that will lead no firms to either enter or exit. Thus, prices get determined due to the competitive risk neutral foreign firms' entry and exit decisions.²²

4. INTERNATIONAL INVESTMENTS AND REAL OPTION INVESTMENT THEORY

In this Chapter we restrict our analysis on foreign firms investment decisions at the industry level. Foreign firms' entry and exit decisions are based on the expected discounted value of future profits of foreign investment. To determine the expected discounted value of future profits of foreign investment we use a real option investment theory, which has been originally developed by MacDonald and Siegel (1986), Pindyck (1988); and further developed by Dixit (1989a,b) and Dixit (1993). Real option investment theory is a general solution method which uses Itos's lemma for the analysis of a stochastic investment income process.

4.1. REAL OPTION THEORY

For the entry into the market, the crucial importance in the real option theory is the moment in time when firms decide to invest in a single project. Firms own an option to enter the market at any moment in time. This option has an exercise price which is also a sunk cost of entering the market. The value from exercising the option is the expected present discounted value of future profits from serving that market. Since the value of investment is unknown, there is an opportunity cost to invest

²² We underestimate the pass-through since we do not consider expansion of firms already established in the market.

today. In terms of option theory, the investment rule can be stated as follows: invest when the value of the project exceeds its costs at least by an amount equal to the option value of waiting to invest.²³ Similarly, if firms consider to left the market, investments are typically at least partly irreversible, incurring again sunk cost to the firms.

In a real option literature it has been generally assumed that the higher level of uncertainty increase the option value and this leads more distant critical value for option exercise, i.e. there is a inverse relationship between uncertainty and investment since greater uncertainty increases the value of option to wait.²⁴ Thus, fewer events of entry will observe. This argument implicitly includes the assumption of concavity of the investment value since we know from the general theory of choice under uncertainty that greater uncertainty will increase the expected value of an action if the payoff is convex in random variable, and decrease if it the payoff is concave.²⁵

We examine the effect of uncertainty on foreign investment in competitive markets. The increasing volatility of a geometric Brownian motion means, however, that higher price levels may be achieved. Cabellero (1993) has noticed, that under perfect competition between firms where the elasticity of demand is almost infinitive, the increase of achieved prices rises the value of the investment both directly through the price change and also indirectly through the increase in optimal output. The latter effect creates the convexity of the profit function. This would induce greater amount of investments, i.e. leads to the positive relationship between uncertainty and investments. However, Dixit (1993) have argued that even in the case of perfect competition the level of price about which to make the demand more elastic is itself endogenous and, moreover, it acts as a ceiling or reflecting barrier. More precisely, competitive firm's investment decisions are restricted by the price ceiling at certain level \bar{P} imposed by firm rational expectations of other firms' entry decisions. Thus, ceiling barrier will make expected profits of investment concave in price process and increases the option value to wait if uncertainty is increased even when a perfect competition is assumed.

²³ As discussed in Dixit and Pindyck (1994), this decision rule gives completely different results than a traditional net present value decision rule.

²⁴ See, for example Pindyck (1988).

²⁵ Although this definition builds on Jensen inequality and risk averse investor, it is possible to show that, under certain conditions, this is also true even for a risk neutral investor. See Dixit (1993).

Above uncertainty is determined as an increasing volatility of time series process. There is, however, also other possible source of uncertainty. In the following analysis we discuss unexpected jumps as a source of uncertainty.

4.2. MIXED POISSON JUMP AND GEOMETRIC BROWNIAN MOTION PROCESS

The time path of the net capital inflow follows random walk but we also allow a possibility that, at some random point in time, the time path will take a Poisson jump. In a following analysis we use the appropriate version of Ito's Lemma which combines random walk and Poisson jump effect. We discuss the expected time series process after a large appreciation.

Random walk and Poisson jump effect is, more accurately in a continuous time representation, a mixed geometric Brownian motion and Poisson jump process, where the former goes on all time and the latter occurs infrequently. We denote the process as

$$dX = \alpha X dt + \sigma X dz + X dq$$

where X is a non-stationary exchange rate variable at time t , σ is standard deviation of the process and dX is the infinitesimal change in X over the infinitesimal interval of time dt . Without any jumps in the process the change in the variable over this interval of time is dz . The random variable dz presents an accumulation of random influences over the interval dt . The expected value of dz is zero. dq is the increment of a Poisson process. dq and dz are independent so that $E(dz \cdot dq) = 0$. The expected value of dX is equal to the sum of deterministic component, αdt and Poisson component $X dq$. We assume that if an event occurs, q changes by some fixed percentage θ . The increment of a Poisson process takes a form such that:

$$dq = \begin{cases} 0 & \text{with probability: } (1-\lambda)dt \\ \theta & \text{with probability: } \pi\lambda dt \\ -\theta & \text{with probability: } (1-\pi)\lambda dt \end{cases}$$

where the probability of Poisson jump effect is λ and π is a probability of positive jump.

The decision to invest is equivalent to decide when to exercise an option. Thus, we can analyze firm's investment behavior by analyzing the option value of investment $H_n(X)$. Then, Ito's Lemma gives us the following differential equation

$$\begin{aligned} & \frac{1}{2}\sigma^2 X^2 H_n''(X) - rH_n(X) + (r - \pi\lambda\theta + (1-\pi)\lambda\theta - \delta)XH_n'(X) + XU(n) - W(n) \\ & + \pi\lambda H_n(X) - (1-\pi)\lambda H_n(X) + \pi\lambda H_n[(1-\theta)X] + (1-\pi)\lambda H_n[(1-\theta)X] = 0 \end{aligned} \quad (4.1)$$

where $U(n)-W(n)$ is the flow of future dividends, r is a real interest rate, and δ is the difference between α and r . This has a general solution

$$H_n(X) = A(n)X^{-\alpha} + B(n)X^\beta + XU(n)/(r-\alpha) - W(n)/r \quad (4.2)$$

where $A(n)$ and $B(n)$ are constant to be determined and $-\alpha$ and β are roots of the quadratic equation in ξ

$$\begin{aligned} f(\xi) = & \frac{1}{2}\sigma^2 \xi(\xi-1) + (r - \pi\lambda\theta + (1-\pi)\lambda\theta - \delta)\xi + (\lambda\pi - r - \lambda(1-\pi)) \\ & - \pi\lambda(1-\theta)\xi + (1-\pi)\lambda(1-\theta)\xi = 0 \end{aligned} \quad (4.3)$$

The last two terms on the right hand side of (4.2) give the expected present discount value of maintaining exactly n firms forever, starting with the exchange rate X . Then, the other terms must be the values of the options to change the number of firms. We provide a full derivation of the problem in Appendix 1. It closely follows the cited literature. However, two important modifications have been done compared to standard analysis. First, the time path of the expected value of the investment now follows a mixed process, since nominal exchange rates might be affected by random Poisson jump shocks. Secondly, the market determined expected rate of return is assumed to remain constant, i.e. higher probability of sudden fall of the investment income is accompanied by an increase in real interest rate, which here equals to the profits of the competitive firm.

For sensible parameter values there are two roots to this equation, one of which is greater than unity (β) and another which is negative ($-\alpha$). These roots together with constants A and B give the exchange rate values for entry and exit of foreign firms. Obviously, there is a range between the

exchange rate values which does not create an incentive to enter or an incentive to exit. This creates hysteresis in the real exchange rate, i.e. an effect that persists after the cause that brought it about has been removed.²⁶

We are, however, especially interested in the entry decision after a large shock in exchange rate value. Thus, we only consider entry decision of foreign firms and exclude exit decisions.²⁷ Assuming firms never shutting down, the solution takes the form

$$H_n(X) = B(n)X^\beta + XU(n)/(\rho - \mu) - W(n)/\rho \quad (4.4)$$

As shown in Dixit (1989b) if I_n is the optimal exchange rate for entry then the relationship between the investment value at which it is for n th firm optimal to invest and sunk cost of the investment is given by Value matching condition

$$H_{n-1}(I_n) = H_n(I_n) - kI_n, \quad (4.5)$$

where k is an entry cost. This endogenizes the value of investment and the number of firms. Smooth Pasting condition is

$$H'_{n-1}(I_n) = H'_n(I_n) - k \quad (4.6)$$

Inserting these two boundary conditions in (x.x.), we get

$$-b_n I_n^\beta + I_n p_n / (\rho - \mu) - w / \rho - k I_n = 0 \quad (4.7)$$

and

$$-\beta b_n I_n^{\beta-1} + p_n / (\rho - \mu) - k = 0 \quad (4.8)$$

²⁶ As discussed in Baldwin and Krugman (1989), after a large appreciation hysteresis effect will also change the structure of goods market and this may also lead to more persistent current account deficit.

²⁷ Excluding the possibility of exit we increase the entry values. This problem will be discussed in Chapter 6.

where $b_n = B(n-1) - B(n)$ for $n=1,2,\dots,N$ firms. There are now two equations for two unknowns; I_n and b_n .

5. ECONOMIC INTERPRETATION OF REAL OPTION THEORY

Above we have argued, using a real option investment theory, that sunk costs and uncertainty affect on investment decisions. In this Chapter we first discuss micro data evidence for this relationship and after that we examine the importance of nature of risk for investment behavior.

5.1. FOREIGN IMPORTING FIRMS AND EXCHANGE RATE RISK

Empirical research considering the importance of uncertainty and sunk-cost hysteresis in real exchange determination has typically focused on asymmetries in the response of import and export prices to exchange rate changes without explicitly focusing on the entry and exit decision into the market. As an example Baldwin (1988) examines the behavior of the U.S. import prices to exchange rate changes. He finds a structural shift in the relationship between U.S. aggregate import prices and the dollar appreciation during the early 1980s. Giovanni (1988) also finds evidence for incomplete pass-through in the U.S. using highly disaggregate import price data but, on the other hand, Parsley and Wei (1993) cannot find exchange rate volatility to be significant predictor of trade flows between the U.S. and Canada.

Indeed, large theoretical work in firms entry and exit decisions contrasts with the very few empirical evidence for the behavior of foreign firms. Since explicit entry decisions especially in the U.S.A are important for our analysis, the interesting study is provided by Campa (1993). He focus on the effect of exchange rate changes on entry decision into the U.S import market.²⁸ The empirical estimations confirm an inverse relation between the exchange rate uncertainty and entry decisions. However, Campa (1993) is not able to discover a relationship between exchange rate trend (α) and the number of firms that decide to enter the market. This could be due to the fact that firms cannot predict even a trend behavior of the exchange rate.

²⁸ Using other currencies Roberts and Taylor (1997) analyze exporting experience on the decision of Columbian manufacturing plants to participate in foreign market and finds support for the hysteresis behavior. Campa (2000) looks at the responsive of a country's export supply to exchange rate changes using Spanish data. This paper supports the importance of sunk cost hysteresis in entry behavior but finds exchange rate uncertainty unrelated to export supply.

5.2. RISK

As discussed in Chapter 2, deviations of the exchange rate from its parity level do not seem to be constant in magnitude. In Dixit (1989b) model, which is based on an assumption about geometric Brownian motion, this can be explained by either large and persistent changes in expected risk or large and persistent changes in a relative number of foreign firms. The time path of the expected risk is illustrated by Figure 2, which plots quarterly series of the differences of the trade weighted U.S. real exchange rate. The importance of changes in the relative number of foreign firms is, in turn, shown in the Figure 3.

In general, the differenced real exchange rate is a stable variable for most of the time.²⁹ Although the value 0,04 is a relatively rare event, there are few large peaks which are an order of magnitude almost 0,06. Thus, the time path of exchange rate is not best characterized by the high and low level regimes of uncertainty but by the few large peaks in time series. Although there is no unique way how firms form expectations on risk, it might be plausible to assume that the overall expected risk combines the effects of the continuous motion parameter σ and the jump parameter dq .³⁰

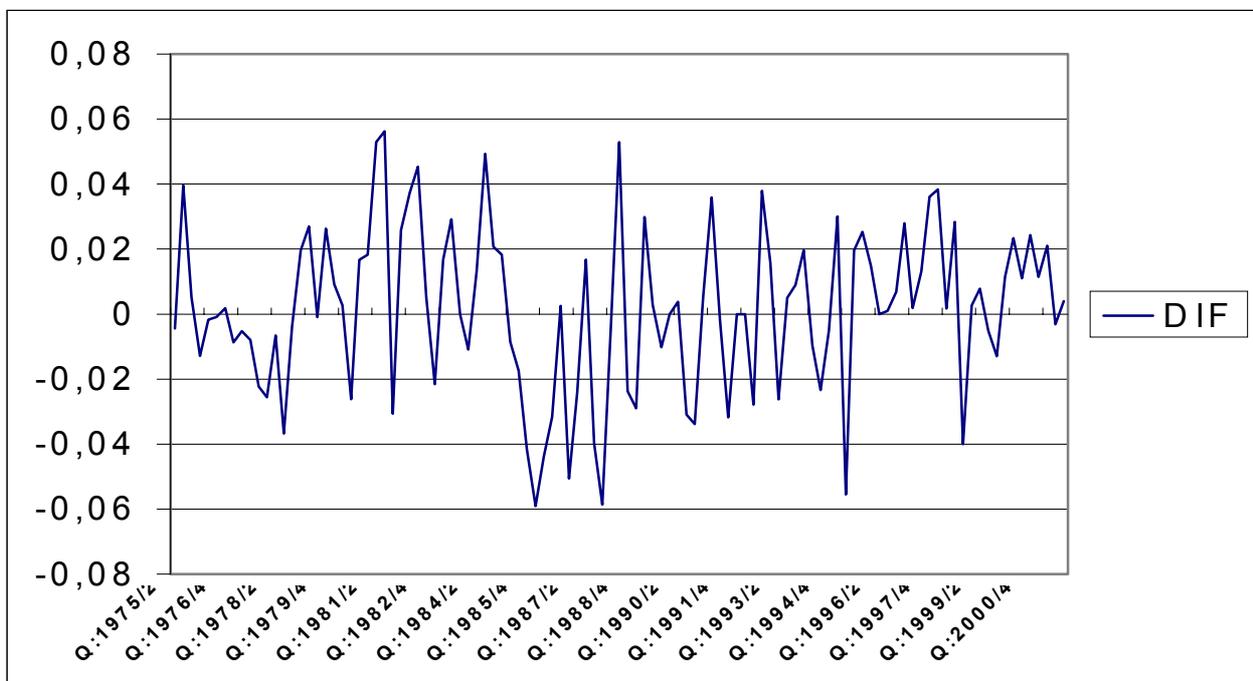


Figure 2. Differenced real exchange rate.

²⁹ GARCH(1,1) model, for example, is not statistically acceptable.

³⁰ Common assumptions are perfect foresight or static expectations based on historical data..

The effect of large and persistent changes in a relative number of foreign firms can be seen in Figure 4 below. As an expression for this effect we have used the ratio of imported goods to private consumption of goods. The import share has been increased quite steadily during the last two decades. This is mainly due to the structural changes on global goods market and the complement relationship between trade and increased foreign direct investment.³¹ The increased share of imports should increase the pass-through as discussed in Chapter 2. However, changes in imports are not able to explain changes in magnitudes of deviations from PPP level. During some subperiods with appreciating dollar, the import share of private consumption goods increases more than just by trend but during the other subperiods the effect of appreciation for imports shares is relatively small.³²

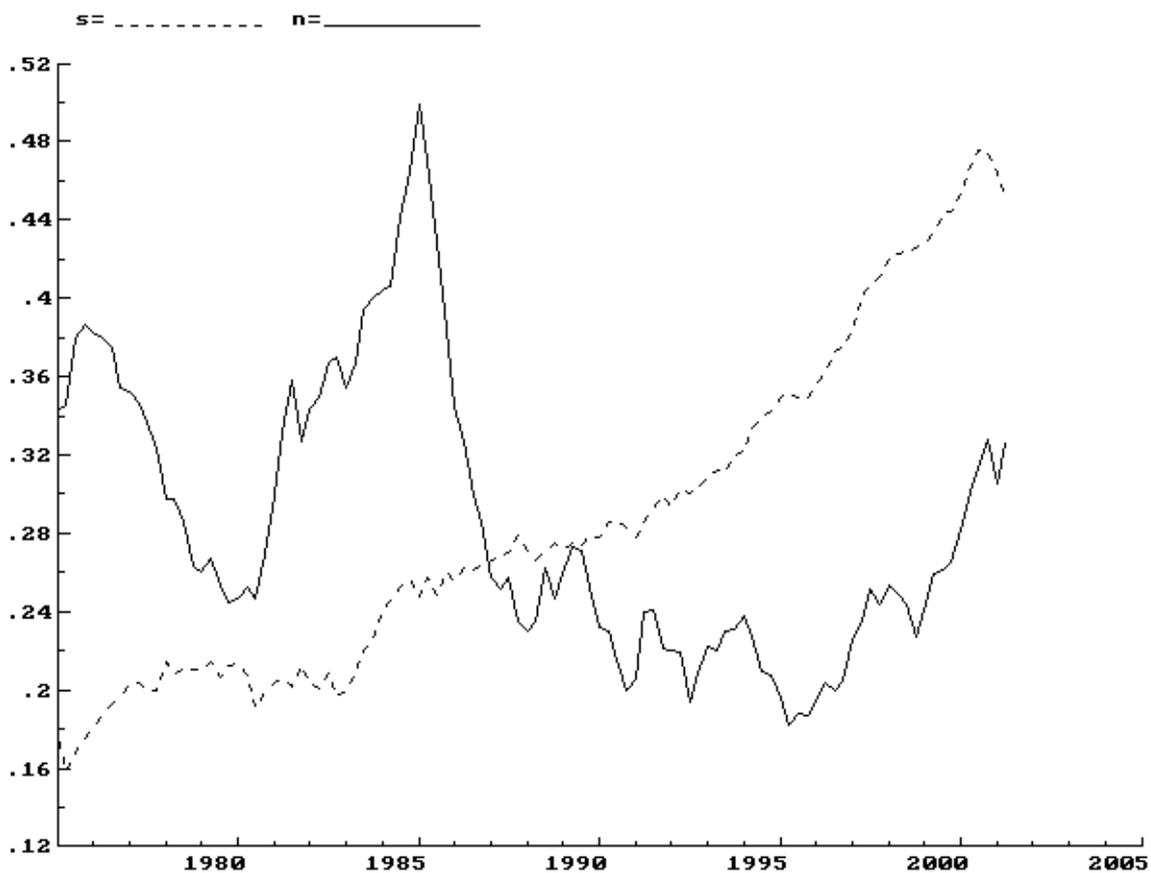


Figure 3. Import share

³¹ See the discussion in Baldwin and Krugman (1989)

³² Campa and Wolf (1997) also find (using G7 data) that the deviations of import share from trend do not systematically co-vary with the deviations of real exchange rates from trend.

In the following analysis we will examine numerically the effects of the mixed process on the time path of the real exchange rate. This might help us to understand deviations from the parity level which are not an order of magnitude constant.

5.3. CALCULATIONS

Interesting economic interpretation of the above equations is given by examining the sensitivity of results for parameters changes. We have calculated dependence of I_n on θ and λ . To compare our results with Dixit (1989b) we make similar assumptions on firms and use similar domestic demand function for imports. (For details see Appendix 2). At first we have assumed that the annual standard deviation is 0,10, which is approximately a standard deviation of the U.S dollar real exchange rate. The probability of negative jump is assumed to be very high ($1-\pi=0,8$) because we would like to examine an investment behavior after a large shock. Interest rate is fixed at the two and half per cent level, the sunk cost of investment is 2 and the maximum number of foreign firms is $n = 100$. The expected time until next jump is $1/\lambda$.

Table 5.1. Size and time. Stdv 0,10

	time	0,125	0,25	0,5
size	0,1	1,58	1,6	1,66
	0,2	1,65	1,74	1,91
	0,3	1,76	1,91	2,33

Table 5.1 displays the results. As the value of the Poisson jump parameter increases, the value of waiting also increases. If only a geometric Brownian motion is assumed, the critical value of investment need to be 1,55, i.e. 55 per cent above the stationary value 1.³³

The magnitude of the shock also depends on the expected time until next jump. The critical value is higher the shorter the expected time to the next shock. If Poisson jump occurs frequently, it is rational to assume that foreign importers do wait it out. However, we get relatively large deviations from the base level even if the parameter λ is assumed to be at the minimum.

³³ Net present value (NPV) decision rule gives values very near to one. See the discussion in Dixit (1989b)

Above we have assumed that heterogeneous investors face sunk entry cost $k=2$ when breaking into the foreign markets. The annualized sunk cost is then five percent of the full cost for median firm ($n=50$). Since the amount of sunk costs for entry into the foreign market at the consumer level may vary significantly, we have also made calculations based on the assumption $k=1$, i.e. the annualized sunk cost is two and half percent of the full cost for median firm. This assumption decreases I_n from 1,74 to 1,70 if the size of expected shock is 0,2 and time 4 years. A geometric Brownian motion assumption now gives the value 1,50 for entry.

The sensitivity of results can be discussed also in the light of the assumptions made about risk. In following analysis we have decreased standard deviation from 0,10 to 0,05. Results are reported in Table 5.2 below.

Table 5.2. Size and time. Stdv 0,05

	time	0,125	0,25	0,5
size	0,1	1,28	1,32	1,38
	0,2	1,37	1,49	1,69
	0,3	1,50	1,73	2,12

Also for a lower level of standard deviation, the critical values increases as parameters σ and $1/\lambda$ increase. The combined effect of size and time is large, i.e. when the size of the shock increases the time effect is more pronounced. If only a geometric Brownian motion is assumed, the entry value is 1,25.

As a standard deviation of the real exchange rate is 0,1 with the geometric Brownian motion assumption, we get the value 1,55, which is approximately the same value as we get by assuming the standard deviation 0,05 and thirty per cent expected drop during next eight years or twenty per cent drop during next four year as displayed in Table 6.2. Thus, we get a similar effect by assuming higher expectations for standard deviations as assuming mixed process.

5.4. EXCHANGE RATE PASS-THROUGH

First, we examine an exchange rate pass-through assuming the exchange rate lies on stationary value 1 with the standard deviation 0,05 and the median case, i.e. $n=50$ (market share 25%), $p = 1$

and $w = 0,95$, size of the shock is 0,2, and the expected time four years. As long as the number of foreign firms is constant at n , there is no pass-through at all. This is 49% above the stationary value, if we assume a mixed process and 25% if a geometric Brownian motion is assumed. When exchange rate appreciate even more 10 per cent from 1,49 to 1,64 prices changes six per cent and the pass through is 0,6. We also get almost similar results if the number of foreign firms is 70, i.e. a market share is 32%. Then the exchange pass-through is 0,55.

6. CONCLUSIONS

We did not allow exit. This increases the critical value needed for entry. For a geometric Brownian motion with the standard deviation 0,1 the overvaluation is 24%, if there is no cost for exit. Assuming entry and exit cost equals 2, the overvaluation is 18%. Using these values for the mixed process, we get the limits 1,4 and 1,47 for the median firm. This is approximately the real overvaluation of the dollar in the early 1980s.

Our quantitative results are, however, far from perfect. We have assumed that foreign firms sell one unit of output per unit of time. In reality, however, firms are not tied to sell just a unit output as we have assumed. More flexible operational space would decrease the exchange rate level which is necessary for the positive expected income of investment. By assuming unit output we also underestimate an expansion of the previously established firms. As discussed in Dixit (1989b), this is an important phenomenon especially if exchange rate takes the values, which are high enough for entry. Thus, the exchange rate pass-through is probably close to one in the phase with entry.

The most important contribution is not, however, precise numerical result but the general dynamics of the adjustment process to the parity level. We can conclude that the necessary level of exchange rate for profitable investment depends on expected Poisson shock and this, in turn, allow long run deviations from the purchasing power parity level, if the source of the expected Poisson shock is persistence. Obviously, for example, oil price fulfill these conditions.

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APPENDIX 1

The outcome of the maximization problem (3.3) can be written as $H_n(X)$. Since we assume that the firm is risk neutral, its discount rate is equal to interest rate, $\rho=r$. Then the Bellman equation for $H_n(X)$, the value of the investment opportunity, is

$$rH_n(X)dt = E[dH_n]$$

As discussed in Chapter X, we assume that the expected value of the foreign investment follows a mixed process after a large appreciation. The stochastic process of mixed geometric Brownian motion and Poisson process is described by the equation

$$dX = \alpha X dt + \sigma X dz + X dq \quad (1)$$

where Poisson process (dq) takes a form given in Chapter 4. As long as the number of foreign firms remains unchanged, the evolution of $H_n(X)$ is given by Ito's Lemma, which combines these two effects. This expansion by Ito's lemma is

$$dH_n(X) = \left[\alpha X H_n'(X) + \frac{1}{2} \sigma^2 X^2 H_n''(X) \right] dt + \sigma X H_n'(X) dz + \left\{ \pi \lambda [H_n(X) - H_n\{(1-\theta)X\}] - (1-\pi)\lambda [H_n(X) - H_n\{(1-\theta)X\}] \right\} dt \quad (2)$$

Since the variance term of the Brownian motion (dz) is defined as $dz = \epsilon \sqrt{dt}$, where ϵ is a normally distributed random variable with a mean of zero, the expected value of dz is also zero. Note that over of an interval of exchange rate values where the number of firms is constant, the capital gain is given by above Ito's Lemma. Thus, taking expectations we can write

$$E[dH_n(X)] = \alpha X H_n'(X) dt + \frac{1}{2} \sigma^2 X^2 H_n''(X) dt + \left\{ \pi \lambda [H_n(X) - H_n\{(1-\theta)X\}] - (1-\pi)\lambda [H_n(X) - H_n\{(1-\theta)X\}] \right\} dt \quad (3)$$

As discussed in Chapter 4, the flow dividend term for foreign importing firms is $[XU(n) - W(n)]$. Including this term in above equation and equating the sum of flow dividend term and the capital gain to normal return of competitive firm ($rH_n(X)$), the equation goes to

$$rH_n(X)dt = \alpha XH_n'(X)dt + \frac{1}{2}\sigma^2 X^2 H_n''(X)dt + \{\pi\lambda[H_n(X) - H_n\{(1-\theta)X\}] - (1-\pi)\lambda[H_n(X) - H_n\{(1-\theta)X\}]\}dt + [RU(n) - W(n)]dt \quad (4)$$

α should be less than the discount rate of risk neutral firm, r , otherwise it is always optimal rather to wait than to invest. Let δ denote the difference $r-\alpha$ and assume $\delta>0$. Replacing α with $r-\delta$ and rearranging terms, then above equation can be rewritten as

$$\frac{1}{2}\sigma^2 X^2 H_n''(X) - rH_n(X) + (r-\delta)XH_n'(X) + XU(n) - W(n) + \pi\lambda H_n(X) - (1-\pi)\lambda H_n(X) + \pi\lambda H_n[(1-\theta)X] + (1-\pi)\lambda H_n[(1-\theta)X] = 0 \quad (5)$$

We may assume that the market determined expected rate of return should remain constant (relative to the Brownian motion case) although there is now a certain possibility for negative shock. As pointed out by Dixit and Pindyck (1994) otherwise no investor would choose to hold the project. Then $\alpha-\lambda$ should remain constant because higher probability of sudden fall of the investment income is now accompanied by an increase in α . We replace $r-\delta$ in equation (5) with $r-\pi\lambda\theta + (1-\pi)\lambda\theta - \delta$. In this case the increased probability of negative shock (positive shock) would be equivalent to an increase (decrease) in the risk free interest rate, which determines the expected rate of return in a competitive market. Taking account this we can write

$$\frac{1}{2}\sigma^2 X^2 H_n''(X) - rH_n(X) + (r-\pi\lambda\theta + (1-\pi)\lambda\theta - \delta)XH_n'(X) + XU(n) - W(n) + \pi\lambda H_n(X) - (1-\pi)\lambda H_n(X) + \pi\lambda H_n[(1-\theta)X] + (1-\pi)\lambda H_n[(1-\theta)X] = 0$$

Over an interval of time when the number of foreign firms remains constant the expected value of investment is equal discounted value of dividends $(XU(n)/r-\alpha-W(n)/r)$. Solution for the values of options to change the numbers of firm takes the form $B(n)X^\xi$. Derivatives are

$$H_n'(X) = \xi B(n) X^{\xi-1} \quad \text{and} \quad H_n''(X) = \xi(\xi-1)B(n)X^{\xi-2}$$

Inserting above derivatives into the equation (x.x), we get a quadratic equation:

$$\begin{aligned} \frac{1}{2}\sigma^2 \xi(\xi-1) + (r - \pi\lambda\theta + (1-\pi)\lambda\theta - \delta)\xi + (\lambda\pi - r - (1-\pi)\lambda) \\ - \pi\lambda(1-\theta)^\xi + (1-\pi)\lambda(1-\theta)^\xi = 0 \end{aligned} \quad (6)$$

APPENDIX 2

The maximum number of foreign firms is $N=100$. The net import demand function is

$$Q = 250 - 200p$$

which gives the price equation

$$p_n = 1,25 - n/200$$

and market share

$$s_n = n/(160 + 0,8n)$$

The profile of variable cost is increasing and takes the form

$$w_n = 0,85 + n/500$$

For the median firm

$$w + \rho k = 1 = p$$