



TAMPERE ECONOMIC WORKING PAPERS
NET SERIES

THE NEW THEORY OF COMMERCIAL BANKING
AND BANK LENDING BEHAVIOR

Pekka Ahtiala

Working Paper 43
June 2005

<http://tampub.uta.fi/econet/wp43-2005.pdf>

DEPARTMENT OF ECONOMICS AND ACCOUNTING
FI-33014 UNIVERSITY OF TAMPERE, FINLAND

ISSN 1458-1191
ISBN 951-44-6368-4

June 10, 2005

THE NEW THEORY OF COMMERCIAL BANKING AND BANK LENDING BEHAVIOR*

Pekka Ahtiala**

Abstract

The paper studies the bank's lending decision, based on three observed phenomena: Banks earn substantial profits from off-balance sheet activities and services, which they take into account in their lending decisions. Secondly, the critical point in the customer relation is the loan decision: the probability of the customer staying with the bank is a function of the loan extended each time one is applied for. Third, what is at stake in the loan decision is the expected value of the entire customer relation, which is the probability times the present value of expected future profits. The bank is a maximizer of this expected present value, while making decisions on individual loan applications. It is shown that the bank is in a corner solution with respect to its good customers, and other customers often have an incentive to get to a corner. Therefore corner solutions may be the rule rather than the exception in the bank's customer relations, and there is no mechanism making the bank indifferent, at the margin, between lending to different customers. It can be optimal to extend loans to (present and expected future) good customers at an interest rate loss. A rationed customer with concave enough a probability function can get a bigger loan by asking for less. Loyalty increases the customer's value to the bank but improves its loan terms only if the customer makes it conditional on the loan extended.

JEL classification numbers G21, E51

Key words: bank-customer relationship, bank lending, credit rationing.

* Scottish Journal of Political Economy, forthcoming.

** Professor Pekka Ahtiala, Department of Economics, 33014 UNIVERSITY OF TAMPERE, FINLAND, phone: +358-3-364 5122, fax: +358-3-364 5122, e-mail: pekka.ahtiala@uta.fi.

1. Introduction

Rapid technological change and deregulation have caused banks to dramatically refocus their activities. With competition depressing margins in lending, the share of non-interest income of commercial banks has more than tripled from less than 10 per cent of total income in 1980 to over 25 per cent in 1994, (see Rajan (1996)), and bank executives are on record as stating that they expect the share of this non-traditional business to rise to 50 per cent by the turn of the millennium (Round Table (1996) p. 29).

Following the refocusing, also a new theory of commercial banking is emerging. Its key features, from the point of view of this paper, are the following (for excellent surveys, see James and Houston (1996) and Rajan (1996); see also Kashyap *et al.* (2002)). The bank is primarily not in the separate businesses of accepting deposits from, and extending loans to its customers, but these products constitute the joint business of liquidity provision for the customers: providing liquidity on demand by meeting the customer's liquidity shortfalls by extending loans, and by standing ready to pay interest on its excess liquidity. In connection with this and the payment processing function the bank obtains plenty of information on the customer, which is a non-rival good: the bank can use it over and over again not only for its information-intensive lending decisions, but also for other information-intensive products like bank guarantees, standby letters of credit, and sale of credit information (see Chan *et al.* (1986)). Information is thus a joint input, which gives the bank a comparative advantage in information-intensive activities, especially if the customer demands several kinds of services.

Both the bank and the customer can utilize scope economies with a relationship involving both liquidity provision and services¹. Possession of information makes the incumbent bank a low-cost producer for established customers demanding several kinds of services. It also makes the bank unable to cost the services separately. In addition, the customer saves search and negotiation costs with one-stop banking.

The customer relation is the bank's main asset, its "crown jewel". (Round Table (1996) p. 35 and Rajan (1996) p. 125). The bank has an implicit contract with its customer, whereby it undertakes

to provide the customer liquidity at a “reasonable” price also in adversity. The customer, in turn, gives the bank the first right of refusal on all banking business, and as long as the bank’s offer is “reasonably competitive”, it will not shop further (Round Table (1996) p. 35). Thus it will not even verify the competitiveness of the offer. The bank and the customer view the relationship as a whole, involving loans, deposits, and services, though banks have only recently become able to monitor individual customer profitability (Round Table (1996) p. 31, 37). It can then be rational for the bank to extend loans even on a loss-leader basis in order to get the profits from non-credit business (Rajan (1996), p. 117; Round Table (1996), p. 32).² Correspondingly, banks have been found to be reluctant to lend cheaply even to big companies, unless they make a profit out of them from other activities (The Economist (1994), p. 83; See also (2004)).

If the bank and the customer view the relationship as a whole, the traditional marginal analysis of the loan decision, based on additively separable production and revenue functions, is likely to be inappropriate. According to that analysis, the bank equates the marginal cost of funds with the marginal revenues, and is indifferent, at the margin, about lending to different customers. Not only do bank executives take all the present and future revenues and costs of a relationship into account, but they also claim to have profitable and less profitable customers all the time (see Round Table (1996)). This being the case, one has a reason to question all the propositions based on the traditional analysis.

In an early approach integrating banking services with lending along basically traditional lines, Cukierman (1978) postulates that other banking services are priced above their marginal cost because of the non-competitive nature of the banking industry, and the bank equates marginal cost with marginal revenue, including excess profits from services, in its lending. He specifies the demand for services as a function of the loan stock outstanding. In the presence of these dependencies, he gets a variant of the conventional marginal conditions and indifference. Greenbaum et al. (1989) and Sharpe (1990) propose that high-quality customers are informationally captured by their old bank, which makes it possible for the old bank to earn rents on the customer, whereas competition forces the bank to lend to new unknown customers at an expected loss. However, as pointed out by Rajan (1992), the bank has an incentive to "behave"

with its old customers to get in on subsequent projects. Evidence provided by James (1992) on investment banks is consistent with this view: the underwriter spread was significantly lower in the initial public offerings in which the issuing firm made a subsequent equity offer, and the customer was the more likely to switch the poorer the investment banker's prior pricing performance (see also Boyd and Prescott (1986), Diamond (1984, 1991), Diamond and Rajan (2001), Fama (1985), Flannery (1983), Lummer and McConnell (1989), and Osborne and Zaher (1992)).

The purpose of this paper is to examine the implications of the “new bank theory” for the loan decision, given the information the bank can reasonably be expected to have or at least to form a perception of, by focusing on the interplay of three notions, all well supported by empirical evidence. First, as suggested above, off-balance sheet activities and banking services like bank guarantees, acceptances, foreign exchange operations and trust operations have become quite profitable for banks, but they are part of a customer relation and thereby tightly connected to their lending function.

Secondly, since the bank's main asset is the customer relationship, it is appropriate to model the optimization with respect to all the expected profits from the customer over the bank's entire planning horizon rather than that of the loan in question over its life.

The third notion is that the entire customer relationship - the capital - is potentially on the line each time the customer's loan application is decided on. This was first argued on a theoretical level by Kane and Malkiel (1965). On the empirical side, Haines *et al.* (1991) surveyed small Canadian businesses. As summarized in Table 1, they found that 60 % of the firms that got less than 50 % of the loan they asked for shopped, while the figure for firms with 50-99 % granted was 55 %, and that of firms with 100 % granted 35 %. Of those who shopped 47 % actually switched. So if the two sets of percentages are independent, 28 % of the first, and 16 % of the last group switched. It is worth noting that the other regressors that came out significant in the shopping equation were measures of the quality of the bank's service, and those of the activity from the part of other banks in trying to attract the firms' business, but not e.g. commissions, spreads, or even deposit or loan rates. Consequently, unlike the pricing of other banking services,

each lending decision appears to be potentially critical for the continuation of the relationship, the probability of the customer staying being a function of the loan extended, given that applied for.

[Table 1 about here]

This paper incorporates all the profits from the customer relationship in the loan decision, highlighting the fact that the whole relationship is at stake each time a loan decision is made. Therefore the critical variable is the entire capital of the customer relationship, of which the expected service profits can be treated as given for an individual loan decision, since they are mainly a function of the nature of the customer's business. This "customer-specific capital" is the expected present value of the future profits from the customer. This expected value is the product of the capital and the bank's subjective probability of keeping the customer and the capital, and the bank is a maximizer of this expected present value. As shown by Kane and Malkiel (1965) and Haines *et al.*(1991), the probability is a function of the loan extended each time one is applied for. Thus the bank is concerned with the entire specific capital but makes decisions on individual loan applications. This modelling solution is the novel feature of our model, and we feel that it is a more appropriate way to model the loan decision. Of course, this approach is not limited to banking but can be applied to firms operating on most customer markets.

The subjective probability distribution and the "face value" of customer-specific capital appear to provide an explanation for several of the paradoxes that one encounters in the light of the old theory. As will be shown, the bank is in a corner solution with respect to its profitable customers, and other customers often have an incentive to get to a corner. Therefore corner solutions may be the rule rather than the exception in the bank's customer relations. As a result, the bank is characteristically not indifferent, at the margin, between lending to different customers, and there is no mechanism bringing about indifference. Thus the best customers can get all the loans on preferential terms in all conceivable situations, as is well known to practical bankers. Specific capital causes resources to be reallocated towards the bank's profitable customers' projects (as already proposed by Cukierman), though customer profitability is often inversely related to the

rate of return on its investments or depends on its production function. It can be optimal to extend loans to good customers at an interest rate loss. A rationed customer with concave enough a probability function can get a bigger loan by asking for less. It can also be shown that if the bank cannot change the loan rate, the customer cannot reach its maximum obtainable loan stock in one try, except if the expected revenue function on its loans starts at the origin and is convex enough. The intermediate-to-large firm's mobility - through access to other banks and the money market - affects its loan terms, unambiguously improving them if the bank cannot change the loan rate. The really large firm's ability to extract a greater share of the rents at the source worsens its loan terms. Loyalty, while increasing the customer's value to the bank, improves the customer's loan terms only to the extent that the customer makes it conditional on the loan extended. The conventional optimum results in the special case where the non-interest profits on each customer are zero and the bank is sure of keeping the customers' business - or it does not care e.g. because of perfectly competitive markets.

Our model nests the theories of Sharpe (1990) and Rajan (1992) as building blocks, and there it is possible that it is the established good customer who gets all the rents, in contrast to Sharpe.

In the following, the loan decision and other implications for bank lending behavior are studied in Part 2, and Part 3 is the conclusion. Appendix 2 studies the case under an interest rate floor (or ceiling) constraint e.g. because the interest rate cannot be negative, or a prime rate convention is binding.

2. The Capital of a Customer Relationship and Bank Lending Behavior

a. The Model and the Optimum

To focus on the problem on hand, we will assume that the specific capital and the interest earnings on loans are independent over customers. We will also treat implicitly the process of information collection and its cost (by treating the cost of the bank's information stock (x) as predetermined) and other issues off our focus. This enables us to specify the bank's profit as the

sum of the profits from its customers, enabling the bank to examine each customer relationship separately.

The basic setting is the following. The bank maximizes its expected net present value by setting the loan rate \hat{r}_L and the loan size ΔL in response to customer i 's given loan request ΔL_i^D , where $\Delta L_i \leq \Delta L_i^D$. Thus if the bank decides to increase the loan rate or extend a smaller loan than the customer asked for, it must expect to increase its profit at the expense of the customer. However, it also increases the probability that the customer leaves. Such an event would result in the loss of all of specific capital. We have:

$$\pi = \sum_{i=1}^n \{P_i M_i - \Delta x_i\} \quad (1)$$

$$M_i \equiv Z_i + (r_{Li} - r)L_i \quad (2)$$

$$L_i = L_{0i} + a_i \Delta L_i \quad (3)$$

$$P_i = P_i(\Delta L_i, \hat{r}_{Li}, r) \quad (\partial P / \partial \Delta L)_i \begin{cases} = 0; & \kappa_i < P_i = \beta_i \leq 1 \\ & \text{if } \Delta L_i = \Delta L_i^D \geq 0 \\ \geq 0; & P_i = \kappa_i \geq 0 \\ & \text{if } 0 = \Delta L_i < \Delta L_i^D \\ \geq 0; & \kappa_i \leq P_i \leq \beta_i \\ & \text{if } 0 < \Delta L_i < \Delta L_i^D \end{cases} \quad (4)$$

$$r_{Li} = \hat{r}_{Li} - \tilde{r}(L_i, x_i) \quad (5)$$

$$\Delta L_i^D - \Delta L_i \geq 0 \quad (6)$$

Equation (1) is the expression for the present value of the bank's expected profit. It is the sum of the expected profits from the bank's customers, where (P_i) is the bank's subjective probability of customer (i) staying with the bank, and M_i the "face value" of the bank's specific capital on the customer. The Δx_i is the exogenous marginal cost of the additional information on the customer that the bank acquires during the period. Equation (2) spells out the face value. The Z_i is the capitalized expected non-interest profits generated by the customer, and r_{Li} is the expected

average risk-adjusted interest earnings on its expected average loan stock (L_j) over the bank's planning horizon. The r is the expected average marginal opportunity cost of the loan stock to the bank in the future. (Naturally, Z , r_L , and \hat{r}_L are unobservable to the outsider, while the customer observes \hat{r}_L .) The expression is spelled out in footnote 3 below. To keep the expressions simple, we will treat r as exogenous to the loan decision, and Z , x , Δx , and L_0 as predetermined - without loss of generality: what matters for the loan decision is the value of specific capital, not its source.³

In Equation (3) the customer's expected average loan stock is its initial expected average loan stock L_{0j} , plus the share (a_j) of the loan extended (ΔL_i) that the bank believes adds to the long-run average.

In Equation (4), the bank's subjective probability of customer (i) staying with the bank is specified, for simplicity, as an increasing function of the difference between the loan extended and the loan applied for, and a declining function of the loan rate (\hat{r}_L) in excess of the free market rate (r), given the customer's idiosyncratic variables, in accordance with the evidence presented in the Introduction (See Haines et al. (1991) and Kane and Malkiel (1965)). Accordingly, if the entire request ΔL_i^D is extended (or equals zero), the bank expects to keep the customer's business with the maximum probability of $\beta_i \leq 1$: If the bank lends the customer all the customer asks for, it cannot make the customer more satisfied so that the probability cannot rise above this value. Hence $\partial P / \partial \Delta L = 0$. Yet β can be smaller than unity, since the bank may not expect to keep the customer with certainty. If the application is completely rejected, the probability reaches its minimum value of $\kappa_i \geq 0$: the κ need not be zero, as the bank need not expect the customer to leave for sure. Finally, if part of the application is extended and part rationed, the probability is between these two values. Thus in the last two cases $\partial P / \partial \Delta L$ is zero or positive. The bank observes ΔL_i^D , but it does not know the critical value of ΔL at which the customer moves all or part of its business to another bank. Therefore it bases its optimization on a subjective probability distribution. The value of P_j is thus determined by the bank's evaluation of the customer's propensity to shop and if so, of the offers that the customer is likely to get from other banks, as elaborated in Appendix 1, where the P function is derived from an optimizing

model. Of course, while bank investment in new specific capital is a straightforward extension of the present analysis, it is beyond the scope of this paper (see Klemperer (1995) and Petersen and Rajan (1995)).^{4,5}

In Equation (5), the bank's risk-adjusted interest earnings r_L equal the loan rate \hat{r}_L minus the risk premium and administrative costs. The premium is an increasing function of L mainly because, all else equal, credit risk is an increasing function of the customer's leverage. It is a declining function of the bank's stock of information on the customer (x_i), because more information makes the bank's subjective probability distribution of the customer's earnings more compact. The \hat{r}_L includes non-price loan terms such as compensating balances.

The P and \tilde{r} are assumed to be twice-differentiable monotonic functions of ΔL (the P function naturally only in the range between zero and ΔL^D). This differentiability also holds for P as a function of \hat{r}_L . The second derivatives of the P function have signs opposite to the signs of the first derivatives, while $\partial^2 \tilde{r} / \partial \Delta L^2$ is positive.

Our formulation accounts for the fact that information is a reusable joint input: the bank needs to invest in it when a new customer applies for a loan, but once the information has been acquired, it can be used, at a negligible cost, also for subsequent loans, bank guarantees, and other off-balance sheet activities.²

Finally, Equation (6) states that the loan extended ΔL_i is no greater than the request ΔL_i^D . Of course, the parameter values of Equations (2) through (5) are likely to vary over customers.

The Kuhn-Tucker conditions for the maximum of the bank's expected present value are, dropping the subscripts (i):

$$\frac{d\pi}{d\Delta L} = \frac{\partial P}{\partial \Delta L} [Z + (r_L - r)(L_0 + a\Delta L)] + aP \left[r_L - r - \frac{\partial \tilde{r}}{\partial L} (L_0 + a\Delta L) \right] - \lambda \leq 0;$$

$$\Delta L \geq 0; \quad \Delta L (\partial \pi / \partial \Delta L) = 0 \tag{7a}$$

$$\frac{d\pi}{d\hat{r}_L} = \frac{\partial P}{\partial \hat{r}_L} [Z + (r_L - r)(L_0 + a\Delta L)] + P(L_0 + a\Delta L) \leq 0; \quad \hat{r}_L \geq 0; \quad \hat{r}_L \frac{d\pi}{d\hat{r}_L} = 0 \tag{7b}$$

$$\frac{d\pi}{d\lambda} = \Delta L^D - \Delta L \geq 0; \quad \lambda \geq 0; \quad \lambda(\Delta L^D - \Delta L) = 0, \quad (7c)$$

where λ is the Lagrangean pertaining to Eq. (6).

Equations (7a), (7b), and (7c) yield:

$$D(L_0 + a\Delta L)^2 + aP(L_0 + a\Delta L) + \frac{\partial P}{\partial \hat{r}_L}(aZ + \lambda) \leq 0, \quad (8)$$

$$\text{where } D \equiv \frac{\partial P}{\partial \Delta L} + a \frac{\partial \tilde{r}}{\partial L} \frac{\partial P}{\partial \hat{r}_L}.$$

Thus if inequality holds in Eq. (7c) and rationing is optimal, $\lambda = 0$ and equality holds in Eq. (9) below, which expresses the optimal loan, given ΔL^D and L_0 . Correspondingly, if equality holds in Eq. (7c), λ is strictly positive, inequality holds in Eq. (9) and we have a corner solution. We shall return to Eqs. (8)-(10) shortly.

The critical nature of the loan decision is reflected in an asymmetry in the optimum condition of Eq. (7a). It is due to the fact, suggested in the Introduction, that the bank is concerned with the entire specific capital but makes decisions on individual loans. In the loan decision, the customer's whole business is on the line: The bank weighs the marginal gain in all of specific capital, (first term), which greatly depends on the increment to the loan stock, ΔL , against the marginal net interest cost of its expected average loan stock (second term), which does not include Z and is zero if $a = 0$. Therefore, the former returns are not the rent flow on specific capital over the life of the loan, but the entire capital. Thus its significance, relative to the loan's net interest earnings, is greater, the smaller the loan and the shorter its life relative to the bank's planning horizon, as can be seen from footnote 3. Correspondingly, the average interest earnings from the customer r_L gain importance relative to the marginal interest earnings, as will be elaborated in Proposition 1 below. The reason is that the marginal cost and marginal revenue of the loan extended ΔL are relevant only to the extent to which they affect the average cost and the average revenue of the customer's expected average loan portfolio during the bank's planning horizon, i.e. if $a > 0$ and $\partial \tilde{r} / \partial L > 0$. In addition, the terms of the loan, but only of the loan in

question, affect also the probability of getting both the interest and non-interest earnings during the bank's planning horizon.

It is also seen that the conventional equilibrium results in the special case of $P = 1$, $Z = 0$, i.e., the non-interest profits on each customer are zero and the bank expects to keep the customer's business with certainty, or it does not care e.g. because of perfectly competitive markets.

The loan decision is illustrated in Figure 1, with $L_0 + a\Delta L$ on the horizontal axis, and R and C , to be explained below, on the vertical axis. The two curves in the Figure depict Eq. (8). The r_L curve below the horizontal axis is obtained from Eq. (7b). Initially, the customer's expected loan stock outstanding is L_0 , the interest earnings r_{L_0} , and the "face value" of specific capital is $Z + (r_{L_0} - r)L_0$.

[Figure 1 about here]

When the customer applies for ΔL^D , the expected future rents become contingent on the bank's loan decision. The R curve represents the first term of Eq. (8). Its multiplier (D) is the partial effect of the net marginal gain, in terms of P , of the loan extended in response to a customer's request: A marginal loan increases P directly (first term of the expression for D). In addition, it increases the risk premium (Eq. (5)). To cover this cost, the loan rate has to increase, which reduces P (second term of the expression for D).⁶ The D is thus positive, and a marginal loan increases P , if the direct effect on P dominates the effect via the loan rate, and vice versa. Haines et al. find that $\partial P / \partial \hat{r}_L$ was not significant. Partly because of this we will examine the case of a positive D to limit the number of cases, while observing that the probability of a negative D increases with highly indebted customers with a high $\partial \tilde{r} / \partial L$. (Eq. (5)). (Analysis of this case is available from the author.)

As $a\Delta L$ increases, the value of R increases roughly to the square of $(L_0 + a\Delta L)$ up to the quantity applied for (ΔL^D) , observing that D is also a function of ΔL . At ΔL^D the customer is fully satisfied and cannot be made more so. The bank now expects to keep the customer's business with the maximum probability of β , which implies $\partial P / \partial \Delta L = 0$ (Eq. (4)). The curve then drops vertically, observing that $d\Delta L$ is zero at ΔL^D by Eq. (7c).

The C curve represents the negative of the remaining terms in the Equation. It intercepts the vertical axis at $-a[(\partial P / \partial \hat{r}_L)(Z + \lambda/a) + PL_0]$ and declines linearly down to ΔL^D . It coincides with the R curve thereafter because of Eq. (6c), as explained above. The optimal loan is at the intercept of the curves at $a\Delta L^* < a\Delta L^D$, and we have an interior solution with rationing (the other intercept in Figure 1 obviously not applying). The optimal interest earnings r_L^* is at the projection of $a\Delta L^*$ on the r_L curve.

The reason for an interior optimum is the following: Returning to Eq. (1), the value of specific capital is the probability of the customer staying (P) times the “face value” of this capital $M \equiv [Z + (r_L - r)L]$. In the loan decision, the non-interest profits (Z) constitute a fixed “revenue” for the bank in the sense of being predetermined and thereby not a function of the loan extended. All else equal, the bank can increase its expected present value by increasing P in either of two ways. If it increases ΔL , it eventually also reduces the net marginal interest earnings of the loan stock because r_L is a declining function of ΔL due to credit risk (Eq. (5)). This reduces the marginal face value. Alternatively, it can cut \hat{r}_L . This reduces the net marginal interest earnings on the loan, which also reduces the marginal face value. Rationing is optimal if the product of the probability and the face value hits a maximum before ΔL^D , the two effects offsetting each other at the margin.

Another illustration is provided in Figure 2, which determines a sufficient condition for the interior optimum. There, $1/\hat{r}_L$ is depicted on the vertical axis, and ΔL on the horizontal axis. In Equation (1), the interior optimum condition is $PdM + MdP = 0$, requiring $dM = dP = 0$. The “iso-face value” curve M is a locus of points where M is constant. It is declining in a rationing equilibrium, as stated, since $r_L - r - a(\partial \tilde{r} / \partial L)L$ is negative as $dM / d\Delta L$, or the marginal revenue of loans: As just stated, the interior optimum calls for the marginal face value of specific capital being offset by marginal gain in P , or $\partial P / \partial \Delta L$. The iso- P curves are the loci of the points where P is constant. Again, we have to take into account the fact that an increase in the size of the loan, in addition to increasing P directly, also increases credit risk, which is a cost. So an increase in the loan rate to cover the risk as in Eq. (5), does not increase r_L but reduces P . Thus the effect of ΔL on P is the net effect of the direct effect and the effect through the loan

rate, or D . We limited ourselves to examining the case of a positive D . In the Figure, the P curves are descending at a diminishing rate, and we have substituted D in the equation for their slope, restricting of course D to be constant. We have drawn these curves as lines to keep the Figure clearer. The initial optimum is at the tangency of the M and P curves at Ω_0 , where the marginal rates of substitution between $1/\hat{r}_L$ and ΔL are equal in both functions.

b. The Interior Optimum

The expressions for ΔL^* and r_L^* read:

$$\Delta L^* \leq \frac{1}{a} \left\{ \frac{N}{2D} - L_0 \right\}, \quad (9)$$

where $N \equiv -aP \pm A^{\frac{1}{2}}$

$$A \equiv a^2 P^2 - 4a \frac{\partial P}{\partial \hat{r}_L} ZD.$$

$$r_L^* = -2ZD / N + r - P / (\partial P / \partial \hat{r}_L) \quad (10)$$

In an interior optimum equality holds in Eq. (9) and λ is zero. With $D > 0$, the second term in the expression for A is positive in the range $0 \leq \Delta L \leq \Delta L^D$, observing its sign, which implies that $A^{\frac{1}{2}}$ is greater than aP in the expression for N . Since we examine only cases where L is positive, the plus sign applies to $A^{\frac{1}{2}}$ in this expression. This extremum is a maximum if the term $(\partial P / \partial \Delta L)[2a(r_L - r) - (1 + a)(\partial \tilde{r} / \partial L)L]$ is either negative, or if positive, dominated in the expression for $d^2 \pi / d\Delta L^2$. Moreover, the term involving $\partial^2 P / \partial \hat{r}_L^2$ has to be dominated in the expression for $d^2 \pi / d\hat{r}_L^2$.

Proposition 1. It can be optimal for the bank to extend loans to a valuable customer at an interest rate loss. As can be seen from Eq. (10), $r_L^* - r = -Z / L^* - P / (\partial P / \partial \hat{r}_L)$ is negative if the customer's Z/L is high enough and the bank is not certain of keeping the customer's business. (Note that $L^* = N / 2D$ in Eq. (9).)⁷ The r_L can be even smaller than the minimum indicated by the Equation, as long as the face value of specific capital is positive (implying $r_L - r > -Z / L$; see Eq. (1)), but then the bank is in a corner solution: If the bank thinks that total or partial rejection of the application leads to the loss of the

customer with high enough a probability. As seen, the critical variable is now the average, rather than marginal, rate of return on the customer relationship. The reason is again that the bank is concerned with the entire specific capital but makes decisions on individual loans. It can thus be optimal to make a loan at an expected loss, provided the value of specific capital is positive, and rejection of the application would lead to a loss of the capital with high enough a probability.

It is easy to see that if $Z = 0$, the expression is positive: a customer not generating non-interest income has to pay a loan rate making (the expected future loan stock's) r_L greater than r ; If the customer has no loans outstanding or in prospect initially, the marginal loan earnings $R_{ML} \equiv r_L - a(\partial\hat{r}/\partial L)L$ have to exceed r (see Eq. (7a)).

Of course, the above observations also apply to loans that do not add to the expected long-run loan stock like seasonal loans, as can be seen from Eq. (9) by setting $a = 0$.

The phenomenon that the bank is concerned with the entire specific capital but makes decisions on individual loan applications has another interesting implication, known to practical bankers. A potential or actual problem debtor can force the bank to choose between making one more risky loan (ΔL) with even a negative expected return in the hope of saving the entire specific capital $[Z + (r_L - r)L]$, whereas by refusing the request the bank takes a high probability of losing much of the loan capital and all of the rest of specific capital.

Our explanation for the bank's equilibrium is closest to that of Cukierman (1978), who determines the optimal loan stock with $P = 1$; $\partial P/\partial \Delta L = 0$ in our notation. His service profits (the counterpart of dZ) are a continuous function of the loan stock. His optimum is an interior solution where, in our notation, $0 = [\partial Z/\partial L + R_{ML} - r]dL$. Therefore his marginal interest earnings plus the marginal service profits equal the marginal opportunity cost of funds. The source of the inequality between marginal interest earnings and the marginal opportunity cost of funds is then the marginal service profits, which have to be positive.

The ΔL^* and r_L^* depend on $\partial P/\partial \Delta L$, $\partial P/\partial \hat{r}_L$, and Z as follows. In Figure 1, an increase in $\partial P/\partial \Delta L$ causes the R curve to shift up and become steeper for any given value of $L_0 + a\Delta L$, which leads to a decline in ΔL^* and r_L^* . In Figure 2, the partial of the expression for the slope of the P curve with respect to $\partial P/\partial \Delta L$ is positive. Thus an increase in $\partial P/\partial \Delta L$ makes the P curve flatter at P_1' , and the P_1' curve intersects the M curve at Ω_0 . The bank can now move to a higher P curve while keeping M constant by trading ΔL for $1/\hat{r}_L$ along the M curve, until it reaches a tangency of the curves at Ω_1 . The reason for this somewhat surprising result is that first, the P curve becomes flatter because of the interdependence between $\partial P/\partial \Delta L$ and $\partial P/\partial \hat{r}_L$ through D , as explained: An increase in ΔL increases P directly. It lowers it because the increase it causes in credit risk causes an increase in the loan rate, and we assumed the direct effect to dominate. Then the effect of $\partial P/\partial \Delta L$ through $\partial P/\partial \hat{r}_L$ cancels out its direct effect in the expression of the slope, only the effect of D remaining.⁸ It follows that if $D=0$, the slope of the P curve remains unchanged.

In other words, an increase in $\partial P/\partial \Delta L$ makes it optimal for the bank to increase P to increase the value of specific capital. In the new situation it can do it by cutting the loan rate and the loan quantity. Namely, with a positive D , $(\partial P/\partial \hat{r}_L)Z$ has to dominate PL^* in Eq. (9), which makes a cut in the loan rate more attractive, credit risk reducing the attractiveness of an increase in the loan size.

Of course, if the bank is unable to change the loan rate, as is often the case with the best customers, an increase in $\partial P/\partial \Delta L$ leads to an increase in ΔL , since ΔL is now the only variable affecting P . See Appendix 2.

An increase in $|\partial P/\partial \hat{r}_L|$ lowers the R curve and makes it flatter for any given value of ΔL in Figure 1. It also shifts up the C curve. This leads to an increase in ΔL^* . The r_L^* curve shifts “down” towards the horizontal axis, reducing the value of r_L^* for any given value of $L_0 + a\Delta L^*$. So r_L^* declines if the movement of the curve dominates the movement along the curve, and vice versa. In Figure 2, the derivative of the expression for the slope of the P curve with respect to $\partial P/\partial \hat{r}_L$ is negative. So the P curve becomes steeper at P_2' , intersecting the M curve. Like above, it is now optimal to move to a tangency at Ω_2 , with a larger ΔL and \hat{r}_L according to the

Figure, leading to a greater P without sacrificing M . As we suggested, however, the change in the interest rate is actually ambiguous, because \hat{r}_L is not independent but a function of ΔL , which Figure 2 does not fully take into account: the increase in ΔL causes the loan rate to rise (Eq. (5)), while the other effects cause it to decline, in accordance with Figure 1. The explanation is a mirror image of the above.

An increase in non-interest profits (Z) leads to an upward shift of the C curve, causing the intercept of the curves to move out in Figure 1, which leads to an increase in the optimal loan. It makes the r_L curve shift “down” towards the horizontal axis so that the change in the loan rate depends on whether the movement of the r_L curve dominates the movement along it. The algebra shows, however, that the loan rate declines.⁹ In Figure 2, the M curve shifts out (not shown) while the slope of the P curve remains unchanged. The increase in the face value of specific capital makes it optimal to improve the customer’s loan terms. Therefore it is optimal to use both variables -- provided of course that the customer made its loyalty conditional on the loan ($\partial P / \partial \hat{r}_L < 0$, $\partial P / \partial \Delta L > 0$).

If the bank is unable to change the loan rate, an increase in Z leads to an increase in ΔL , since ΔL is now the only variable affecting P , as can be seen in Appendix 2.

An increase in P shifts the C curve down and makes it steeper, which leads to a decrease in ΔL^* with D positive. The r_L curve shifts “up” away from the horizontal axis so that r_L^* increases if the movement of the curve dominates the movement along it, and vice versa.

Proposition 2. A rationed customer can induce the bank to extend a larger loan by asking for less if its P function is concave enough with respect to ΔL . We obtain from Eq. (9), where equality holds, observing that $\partial P / \partial \Delta L^D = -\partial P / \partial \Delta L$ (Eq. (3)):

$$\frac{d\Delta L^*}{d\Delta L^D} = \frac{1}{2DA^{\frac{1}{2}}} \left[\frac{\partial P}{\partial \Delta L} \left(\mp aP + A^{\frac{1}{2}} \right) - 2 \frac{\partial^2 P}{\partial \Delta L^2} \left(\frac{P \left(\mp aP + A^{\frac{1}{2}} \right)}{2D} \pm \frac{\partial P}{\partial \hat{r}_L} Z \right) \right] \quad (11)$$

$$\frac{dr_L^*}{d\Delta L^D} = \frac{(\partial P / \partial \Delta L)[PN / 2D + (\partial P / \partial \hat{r}_L)Z]}{(\partial P / \partial \hat{r}_L)[PN / 2D + 2(\partial P / \partial \hat{r}_L)Z]} \quad (12)$$

With D positive, the multiplicand of the first term in the brackets of Eq. (11) is positive, as shown in connection with Eq. (9), making that term positive. In the multiplicand of the second term, the minus sign applies to aP and the plus sign to the second term if $D > 0$. The first term of this expression equals $PN / 2D$ in Eq. (9), or PL^* . The multiplicand as a whole thus equals the last two terms of Eq. (8), and with $D > 0$ it has to be negative for the equation to have positive real roots. The derivative is positive and the conventional result obtains if the first term in the brackets dominates, or the second term is positive, observing its sign: if $\partial^2 P / \partial \Delta L^2$ is positive. However, if $\partial^2 P / \partial \Delta L^2$ is negative and the second term dominates, the rationed customer - which by definition would like to have a bigger loan - can obtain a bigger loan by asking for less.

If such a customer reduced its request, the vertical part of its R curve in Figure 1 would move in. The curve would decline for any given value of $L_0 + a\Delta L$ smaller than $L_0 + a\Delta L^D$, as $dR / d\Delta L^D [= -(\partial^2 P / \partial \Delta L^2)L^2]$ is positive when $\partial^2 P / \partial \Delta L^2$ is negative. The C curve would also decline, as $dC / d\Delta L^D [= a(\partial P / \partial \Delta L)L]$ is positive. When the second term in Eq. (11) dominates, the R curve declines by more so that ΔL^* increases.

The $dr_L^* / d\Delta L^D$ is negative because the multiplicand in the numerator of Eq. (12) is negative as the multiplicand of the second term in Eq. (11). Then the multiplicand in the denominator is also negative, making the denominator positive and the whole expression negative. So the r_L curve shifts “up” away from the horizontal axis, causing r_L^* to increase. Thus ΔL^* and r_L^* both increase. The reason for this result is that by reducing the loan request the customer with a concave enough a P function reduces $\partial P / \partial \Delta L$ for any given value of ΔL , and a decline in $\partial P / \partial \Delta L$ makes it optimal for the bank to satisfy more of the request at a higher loan rate, as shown above.¹⁰

As the customer continues to reduce ΔL^D , these developments continue until a maximum ΔL^* is reached. If this maximum is a corner solution at $L_0 + \Delta L^{D*}$, the loan granted starts declining by the decline in the loan request thereafter.

If the bank is unable to change the loan rate, the Proposition holds for a customer with convex enough a P function, as shown in Appendix 2.

Proposition 3. The intermediate-to-large firm's mobility - through access to other banks and the open market - affects its optimal loan terms, adjusted for scale, unambiguously improving them if the bank cannot adjust the loan rate. However, the really large firm can also extract a greater share of the rents at the source. This worsens its loan terms. Let the firm expand in scale. Then also its optimal loan stock increases proportionately. However, the intermediate-to-large firm typically has lower switching costs than smaller firms, having established relations with several banks, and access to the open market, which gives it a higher $\partial P / \partial \Delta L$ and $|\partial P / \partial \hat{r}_L|$. As shown above, if $D > 0$, the increase in $\partial P / \partial \Delta L$ lowers ΔL^* and r_L^* , and the increase in $|\partial P / \partial \hat{r}_L|$ increases ΔL^* , while its effect on r_L depends on whether the movement along the r_L curve dominates the movement of the curve.

However, the really large firm's transactions and their commissions are often so large that it pays to negotiate them individually, which makes it possible for these firms to extract more of the rents already at the source. This reduces Z, which lowers the optimal loan quantity and increases the loan rate, strengthening the effects of $\partial P / \partial \Delta L$, and weakening the effects of $\partial P / \partial \hat{r}_L$, on ΔL^* .

If the bank is unable to change the loan rate, the increase in $\partial P / \partial \Delta L$ increases ΔL , while a decline in Z reduces it. Thus banks' unwillingness to lend to big firms at favorable rates, unless they make profits on them on services, is thus consistent with rationality in this model. ¹¹

These effects contrast with Blackwell and Santomero (1982), who find that the veteran or prime customer's loans are the first to be rationed, because its loan demand function is more (interest) elastic and it is thus charged a lower loan rate. They assume that competition keeps Z at zero. Of

our key customer characteristics, $\partial P / \partial \hat{r}_L$ is closest to theirs, and we obtain an increase in ΔL in response to an increase in its absolute value, r_L declining for any given ΔL . An increase in $\partial P / \partial \Delta L$ leads to a decline in ΔL^* and r_L^* . The reason for the discrepancy is that our customer has a nonnegative Z and responds to unfavorable loan terms not by reducing its borrowing while staying on as a customer, but by considering taking all its business, both L and Z , elsewhere.

Greenbaum et al. (1989) use a search-theoretic model and also find that the veteran customer's loan terms are worse, because the customer's profit function is convex and the variance of the cash flows declines with tenure. Our counterpart is the "loyal customer", whose Z is greater and which gets better loan terms provided it makes its loyalty conditional on the loan.

c. The Global Optimum

We are now ready to examine the loan decision from a broader perspective. As proposed, in Figure 1, an increase in Z causes the C curve to shift up and the optimal loan to increase, while the r_L curve shifts "down", causing the optimal loan rate to decline. As Z increases, the optimal loan increases up to the point where the interior solution calls for the whole request being granted: the intercept reaches the point where the R curve becomes vertical, the r_L curve shifting "down" towards the horizontal axis. The bank now ends up in a corner solution: it would be prepared to extend more loans if only the customer applied for more. It has an incentive to invite the customer to apply for more (i.e. to increase ΔL^D) only if the marginal revenue on loans, net of the risk premium, exceeds r . If the marginal revenue is smaller than r , the bank will not invite the customer to borrow more, since $\partial \pi / \partial \Delta L < 0$ for $P = \beta$; $\partial P / \partial \Delta L = 0$; (Eq. (7a)), equality holding in Eq. (7c): The bank cannot increase the probability of the customer staying, and thus its specific capital, after the point where the customer got all it asked for. This property differs from that of Cukierman (1978): his service profits are a monotone function of the loan stock, which produces an interior solution. However, the optimal loan rate keeps declining according to Eq. (10) as Z increases.

There is a limit, however, to how far the loan rate can decline, because the interest rate cannot be negative. The prime rate convention may raise the floor still higher. When the floor is binding, the bank cannot cut the loan rate. It is then in the regime of Appendix 2.

Correspondingly, a decline in Z leads to a downward shift of the C curve and an “upward” shift of the r_L curve. The optimal loan rate begins to increase after \hat{r}_L^* has exceeded the floor rate \hat{r}_L . The optimal loan is unaffected by a change in Z or by changes in other variables affecting ΔL^* as long as the customer is in a corner solution. After an interior solution has been reached, ΔL^* begins to decline, doing so until the intercept of the C curve with the vertical axis coincides with the corresponding intercept of the R curve. From this point on, the entire application is rejected. Naturally, as proposed in connection with Proposition 1, a customer with a zero Z can get a loan only by paying a risk-adjusted interest rate (now or in the future) greater than r : it is optimal to make a loan only to a customer with a positive expected net present value to the bank.

The fact that the loan terms that the bank is willing to offer to the customer improve with Z and specific capital in general causes resources to be reallocated in favor of profitable customers' projects, by lowering both their probability of being rationed and often also their loan rate. Hence these customers face a lower opportunity cost of investment than less profitable customers, a point made by Cukierman (1978). This is problematic from the resource allocation point of view, since the size of a rational customer's deposits (whose profits to the bank are part of Z) is a declining function of their opportunity cost, i.e. the marginal rate of return on its investments. The commission revenues, in turn, are a function of the customer's production function. A customer demanding plenty of inputs sold profitably by the bank is a profitable customer, whose investments are financed over a less profitable customer's more profitable investments: customer profitability to the bank becomes a factor in resource allocation at the expense of the expected rate of return of the investment.¹²

Proposition 4. The bank is not indifferent, at the margin, between lending to different customers, except by chance, and there is no mechanism bringing about indifference. Eqs. (7a)-(7c) imply for customers i and j : $(\partial\pi/\partial\Delta L)_i \leq (\partial\pi/\partial\Delta L)_j \leq 0$. Inequality holds if either or both

customers are in corner solutions, except by chance. Thus equality holds only if both customers are in interior solutions during the loan decision, i.e. $(\partial\pi/\partial\Delta L)_{i,j} = 0$, except by chance. After the loan decision, some customers may leave, others staying. For those who stay in the bank's set of customers, P_i rises to the customer-specific value of β_i . Thus the marginal profits are characteristically different even for customers who were in an interior solution during the loan decision, except by chance, and there is no mechanism bringing about indifference. Thus the fact that banks have profitable and less profitable customers all the time, as observed by practical bankers, is consistent with optimality in our model. Moreover, since the valuable customers are on average those paying the lowest interest rates (Equation (10)), it is understandable why bank management would get upset about losing them rather than the customers paying the highest rates.

Why will competition not depress the loan rate of good customers until the bank becomes indifferent, at the margin, about lending to each customer? First, loan rate floors may be binding, and if so, they typically apply to good customers' loans. Naturally, outside banks are normally subject to the same floors. Competitive pressure from outside institutions is limited by the fact that much of the profits is rent on accumulated information capital, part of which is not transferable, being private. Of course, the incumbent bank has an incentive not to transfer information on a customer that is contemplating switching. At the very least, information makes the bank's subjective probability distribution of the return on the customer's investments more compact. This increases the expected loan earnings by reducing the probability of default, given that a prudent bank keeps the loan payments below the expected returns on the borrower's investments. The incumbent bank (and the customer) knows therefore that it is the low cost provider of loans, mainly thanks to the information capital accumulated in the past. So as long as its offer is "reasonably competitive", the customer, economizing on transaction costs, will not shop further, as suggested by practical bankers in the Introduction. Therefore, the customer normally does not even verify the competitiveness of the offer. It is also not optimal for the bank to invite the customer to borrow more, except if the marginal risk-adjusted revenue of loans exceeds r . Note that in the expressions for the optimal loan quantity and loan rate, all the arguments are customer-specific except for the market rate. Optimal bank behavior vis a vis

customers with respect to which the loan rate floor is a binding constraint is analyzed in Appendix 2.¹³

A related observation is worth noting. It is optimal for the bank to extend loans to the customer as long as the customer's specific capital for it is positive. The customer can increase its share of specific capital at the expense of the bank by making credible that its $\partial P / \partial \Delta L$, $|\partial P / \partial \hat{r}_L|$ are high, that is, the alternative to accommodation is likely to be the loss of the customer. In this situation, it is optimal for the bank to accommodate the customer as long as the bank has a marginal share of specific capital to itself. In this model, such a customer may thus get practically all the rents from the capital, because it, too, has bargaining power, in contrast to Sharpe (1990). The reluctance of banks to lend to large customers at favorable rates, unless they get service revenue from them, as mentioned in the Introduction, suggests that a point of indifference has been reached with these customers.

Proposition 5. Assume that the bank cannot change the loan rate e.g. because of an interest rate floor like the prime rate convention. Then the customer cannot reach its maximum obtainable loan stock in one try, except if it has convex enough a P function starting at the origin. Proof: Available from the author.

3. Concluding Comments

We have developed a model of bank lending behavior in the presence of customer-specific capital and the critical nature of the loan decision, which are mainly due to joint production in information and transactions.² Therefore, if the profits are independent across customers, the bank maximizes the expected present value of its profits by maximizing the product of the probability of the customer staying with the bank (which is a function of the loan extended each time one is applied for) and the face value of the specific capital of each customer. Thus the bank is concerned with then entire specific capital but makes decisions on individual loan applications.

The bank's loan decision was shown to be asymmetrical. On the one hand there is the marginal gain in the probability of the customer staying times all of specific capital (i.e. a function of the increment), and on the other the expected net marginal interest cost of the average loan stock, which does not include non-interest profits (Z) and is zero if the loan is not expected to increase the long-run stock. The Z improves the customer's loan terms, reallocating resources to profitable customers' projects. Customer profitability for the bank thus becomes a factor in the allocation of resources. Adjusted for customer size, it is often negatively correlated with the profitability of the customer's investments or depends on the customer's production function. Its relative significance increases, the smaller the loan and the shorter its maturity. The conventional optimum results in the special case where the non-interest profits on each customer are zero and the bank is sure of keeping their business - or it does not care e.g. because of perfectly competitive markets.

It can be optimal to extend loans to a valuable customer at an interest rate loss. Cutting the loan rate is not always possible, however, partly because of the nonnegativity of the interest rate and the prime rate convention, which accentuates the role of specific capital.

A rationed customer with concave enough a P function can get a bigger loan by asking for less. The following can also be shown. Suppose the bank cannot change the loan rate because of the above interest rate floors or ceilings. Then the customer cannot reach its maximum obtainable loan stock in one try except if it has convex enough a P function starting at the origin.

The intermediate-to-large firm's mobility affects its loan terms, unambiguously improving them if the bank cannot adjust the loan rate. However, the really large firm can extract a greater share of the rents at the source, which worsens the terms. Loyalty increases a customer's value to the bank but improves the customer's loan terms only to the extent that the customer makes it conditional on the loan extended.

There is no mechanism bringing about an equilibrium where the bank is indifferent, at the margin, between lending to different customers. The bank is in a corner solution with respect to its good customers, and other customers often have an incentive to try to get to a corner.

Therefore corner solutions may be the rule rather than the exception in bank-customer relations. The best customers are immune to monetary policy if they are in a corner solution and the interest rate floor is binding. They can thus get all the loans on preferential terms in all conceivable situations, as is well known to practical bankers.¹⁴

On the whole, these findings suggest that the contribution of the banking system to the efficiency of resource allocation is likely to be smaller than has commonly been perceived. The allocative effect of information capital through the interest earnings on loans can be justified on efficiency grounds to the extent that it contributes to the appropriate pricing and rationing of loans. The same applies to banking services priced on the basis of risk. However, the allocative effects of non-interest earnings on loan terms are likely to be a source of serious inefficiency, which can be expected to persist into the future. The economies of joint production are substantial, and recent product innovations in banking services have enhanced the significance of specific capital on resource allocation, whereas the switch to cost-based pricing has weakened it.

Another implication of specific capital is that there are efficiency gains to be made by allowing banks to further expand the scope of their operations to e.g. the investment banking and information business - although the profits of this business may have effects on loan terms similar to those of Z, which works in the opposite direction.

Finally, the harsh judgment on banks' lending to present problem debtors does not appear to be entirely justified when viewed in the light of the present approach. In offering loans to prospective customers on concessionary terms banks invest in specific capital. If non-interest earnings on the loans are taken into account in addition to net interest earnings and credit losses, the picture changes substantially: For example, Brazil was not a big net drain of funds but Citybank's second most profitable source of business in 1988, surpassed only by the United States (The Economist 1989, p. 69). Moreover, a potential or actual problem debtor can take advantage of the decision asymmetry when asking for a loan: it can force the bank to choose between making one more risky loan even with an expected loss in the hope of saving the entire specific capital, whereas by refusing the request the bank takes a high probability of losing much of the loan capital and all of the rest of specific capital.

The present approach is not limited to banking but can be applied to firms operating on most customer markets.

The research agenda for the future points in several directions. Further empirical research is called for to quantify the relationships. Secondly, the bank-customer relationship is a repeated game with asymmetric information, where each party reveals to the other parts of its behavior function each round. Thus a game-theoretic study of the bank-customer relationship based on the present approach could provide interesting insights on the working of the monetary system. Finally, this approach may open interesting avenues in the theory of loan pricing and credit rationing in general.

References

- Adar, Zvi, Tamir Agmon, and Yair E. Orgler. "Output Mix and Jointness of Production in the Banking Firm." Journal of Money, Credit, and Banking 7 (May 1975), 235-243.
- Blackwell, Norman R., and Anthony Santomero, " Bank Credit Rationing and Customer Relation", Journal of Monetary Economics 9, 1982, 121-129.
- Boyd, John, and Edward Prescott. "Financial Intermediary Coalitions." Journal of Economic Theory 38(2) (April 1986), 211-232.
- Chan, Yuk-Shee, Stuart Greenbaum, and Anjan Thakor. "Information Reusability, Competition, and Bank Asset Quality." Journal of Banking and Finance (June 1986), 243-253.
- Cukierman, Alex. "The Horizontal Integration of the Banking Firm, Credit Rationing, and Monetary Policy." Review of Economic Studies (February 1978), 165-178.
- Diamond, Douglas W. "Financial Intermediation and Delegated Monitoring." Review of Economic Studies (July 1984), 393-414.
- , "Monitoring and Reputation: The Choice Between Bank Loans and Directly Placed Debt." Journal of Political Economy 99, 1991, 689-721.
- , and Raghuram G. Rajan, "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking", Journal of Political Economy 109, no 2, 2001, 287-327.
- Fama, Eugene F. "What's Different About Banks?" Journal of Monetary Economics 15, 1985, 29-39.
- Flannery, Mark J. "Enforcing Implicit Contracts: An Application to the Theory of Banking." Working Paper, The Wharton School, University of Pennsylvania, 1983.
- Greenbaum, Stuart I., George Kanatas, and Itzhak Venezia, "Equilibrium Loan Pricing under the Bank-Client Relationship", Journal of Banking and Finance 13, 1989, 221-235.
- Haines, George A., Allan Riding, and Roland Thomas. "Small Business Bank Shopping in Canada." Journal of Banking and Finance 15, 1991, 1045-1056.
- James, Christopher. "Relationship-Specific Assets and the Pricing of Underwriter Services". Journal of Finance XLVII, No 5 (December 1992), 1865-1885.
- James, Christopher, and Joel Houston. "Evolution or Extinction: Where Are Banks Headed?" Journal of Applied Corporate Finance Vol. 9 no 2, Summer 1996, 8-23.
- Kane, Edward J., and Burton G. Malkiel. "Bank Portfolio Allocation, Deposit Variability, and the Availability Doctrine." Quarterly Journal of Economics 79 (February 1965), 113-134.

Kashyap, Anil, Raghuram Rajan, and Jeremy C. Stein. "Banks as Liquidity Providers: An Explanation for the Coexistence of Lending and Deposit-Taking." Journal of Finance 57,1, February 2002, 33-74.

Klemperer, Paul. "Competition when Consumers Have Switching Costs: An Overview with Applications to Industrial Organization, Macroeconomics, and International Trade." Review of Economic Studies 62, 1995, 515-539.

Lummer, Scott. L, and John J. McConnell. "Further Evidence on the Bank Lending Process and the Capital Market Response to Bank Loan Agreements." Journal of Financial Economics 25, 1989, 99-122.

Osborne, Dale K., and Tarek S. Zaher. "Reserve Requirements, Bank Share Prices, and the Uniqueness of Bank Loans." Journal of Banking and Finance 16, 1992, 799-812.

Petersen, Mitchell A., and Raghuram G. Rajan. "The Effect of Credit Market Competition on Lending Relationships." Quarterly Journal of Economics 110, 1995, 407-443.

Rajan, Raghuram G. "Insiders and Outsiders: The Choice Between Informed and Arms-Length Debt". Journal of Finance, XLVII, No 4 (September 1992), 1367-1400.

------. "Why Banks Have a Future: Toward a New Theory of Commercial Banking." Journal of Applied Corporate Finance Vol. 9 no 2, Summer 1996, 114-128.

Sharpe, Steven A. "Asymmetric Information, Bank Lending, and Implicit Contracts: A Stylized Model of Customer Relationships". Journal of Finance, XLV, No 4 (September 1990), 1069-1087.

Stiglitz, Joseph, and Andrew Weiss. "Incentive Effects of Termination: Applications to the Credit and Labor Markets." American Economic Review 73(5), (December 1983), 912-927.

"Current Issues in Commercial Banking: Strategic Planning, Performance Measurement, and Incentive Compensation." Round Table Discussion, Journal of Applied Corporate Finance Vol. 9 no 2, Summer 1996, 24-51.

The Economist, March 21, 1987.

The Economist, January 7, 1989.

The Economist, March 12, 1994.

The Economist, April 17, 2004, A Survey of International Banking.

Notes

The paper was written while the author was a Visitor at Northwestern University. He wishes to thank Ernst Baltensperger, Charles Calomiris, Thomas Gittings, Donald Hodgman, Glenn Hubbard, Edward Kane, Bernt Stigum, anonymous referees, as well as the participants of the economics workshops of the Federal Reserve Bank of Chicago, University of Haifa, Northwestern University, Ohio State University, Tel Aviv University, University of Illinois, Champaign-Urbana, and University of Oslo for useful comments. I owe special thanks to Stuart Greenbaum and Yair Orgler. Financial support from the Academy of Finland, the Association of Banks in Israel, and the Yrjö Jahnsson Foundation is gratefully acknowledged.

1. I am not aware of studies on the joint production of information and banking services. That of deposits and loans has been studied on a general level by Adar, Agmon, and Orgler (1975).

2. A reason for the important role of loans is economizing on transactions costs. The pricing decisions of the bank are made by costly executives, and it would be very costly to negotiate individually the prices of high-volume transactions involving small unit sums like deposit rates and commissions: they are typically posted prices. The terms on larger loans are typically negotiated and tailored individually in any case. It is thus efficient to make also other individual adjustments in loan terms, the bank having an incentive to share the rents on the relationship with the customer here, to keep the customer from abandoning its non-shopping attitude.

3. The face value of the bank's specific capital is composed as follows:

$$M_i \equiv Z_i + (r_{Li} - r)L_i \equiv E \left\{ \sum_{t=0}^T \frac{1}{(1+i)^t} [z_{it} + (r_{Lit} - r_t)L_{it}] \right\},$$

where $z_{it} \equiv \sum_{q=0}^n S_{qit} (R_{qt} - C_{qt})$, and Z_i is the net present value of the expected rents from services, i the bank's discount rate, T its planning horizon, t time, S_{qit} the number of transactions of type q made by customer i in period t , and R_{qt} and C_{qt} their marginal revenue and marginal cost to the bank, respectively. It is seen that Z_i is a function of the nature of the customer's business, given R_{qt} , C_{qt} , and i . Since specific capital is the expectation of the returns, it is optimal to give a loan to a new customer only if the relationship offers a positive expected net present value.

4. Note that $P = pb$, where the customer is expected to keep the share b of its business with the bank with probability p . Thus the specific capital is the same if the customer keeps all its current business ($b = 1$) with the bank with probability $p/2$ as it is if it keeps a half of it ($b = 1/2$) with

probability p , all else equal. Then of course M is the face value of the customer's total specific capital, which the bank does not know.

5. Note, however, that r_L and \hat{r}_L are the average interest rates on the entire loan stock in Eqs. (1) and (4). In the P function, the loan rate is the rate on the loan in question ΔL , of which \hat{r}_L is a function. Specifying P directly as a function of \hat{r}_L means that the absolute value of $\partial P / \partial \hat{r}_L$ is greater than the partial with respect to the rate on ΔL .

6. Eq. (4) and the Equation for r_L in the Figure yield:

$$d\hat{r}_L = \left[a(\partial\tilde{r}/\partial L) + aZ/L^2 - (\partial P/\partial\Delta L)/(\partial P/\partial\hat{r}_L) \right] d\Delta L + dr,$$

where the expression in the brackets is positive. Thus an increase in the loan size leads to an increase in the loan rate.

7. The optimal marginal revenue on loans R_{ML}^* can also be smaller than r , since it is smaller than r_L^* : $R_{ML}^* = r_L^* - a(\partial\tilde{r}/\partial L)L^*$.

8. The optimality condition reads:

$$d\Delta L / d(\partial P / \partial \Delta L) = \left(1 / DA^{\frac{1}{2}} \right) \left[P \left(-aP + A^{\frac{1}{2}} \right) / 2D + (\partial P / \partial \hat{r}_L)Z \right] < 0.$$

9. We have:

$$d\hat{r}_L^* / dZ = \left(4aD^2 / N^2 A^{\frac{1}{2}} \right) \left\{ \left[P \left(\mp aP + A^{\frac{1}{2}} \right) / 2D \right] \pm (\partial P / \partial \hat{r}_L)Z \right\}$$

The minus sign applies for aP and the plus sign for $(\partial P / \partial \hat{r}_L)Z$. With a positive D , Equation (8) has positive real roots if its last two terms add up to a negative number ($C > 0$ in Figure 1). This implies $|\partial P / \partial \hat{r}_L|Z > PL^*$. Therefore the multiplicand is negative, making the whole expression negative.

10. Note that the R curve is still likely to be convex due to L^2 . Moreover, P in Eq. (4) is a function of the part of the application not granted $\Delta L - \Delta L^D$. For example, $P = \beta$ both at the new lower value of ΔL^D and at the old value, and it equals κ at $\Delta L = 0$ in both cases. Likewise, the $\partial P / \partial \Delta L$ are equal at the new and the old values of $\Delta L^D - \varepsilon$, where ε is small.

11. An implication is that multiple banking relationships are rational for the customer if their effects on $\partial P / \partial \Delta L$ dominate those of Z , net of the customer's share of investment costs. Another

implication is that loyalty increases the customer's value to the bank, since the bank can expect to get the revenues longer into the future, which increases M (note 4). However, loyalty improves the customer's loan prospects only to the extent that the customer makes it conditional on the loan extended ($d\Delta L^* / \partial Z = dr_L^* / dZ = 0$ when $P = 1$; $\partial P / \partial \Delta L = \partial P / \partial \hat{r}_L = 0$, as can be inferred from the discussion above).

12. Of course, the profitability of the customer's projects can affect the bank's loan decision indirectly through r_L if it affects the customer's assessed credit risk or its willingness to pay interest, and possibly through Z if it affects its demand for bank services, but these are second-order effects.

13. The reader can see that monetary tightening increases r , which reduces $\hat{r}_L - r$ in the P function (Eq. (4)), and the net interest earnings on loans $r_L - r$ in Eq. (1). The customer in a corner is unaffected by monetary conditions if also the loan rate ceiling or floor is binding.

Appendix 1. The Bank's Subjective Probability Function

Let the customer's profit be: $\pi = r_K(E + L) - \hat{r}_L L$, where r_K is the average rate of return on the customer's capital consisting of its loan stock plus its equity E . Maximizing profit yields: $\lambda = r_K' - \hat{r}_L$, where r_K' is the marginal rate of return on the customer's capital and λ the Lagrangean of the loan stock constraint. A necessary condition for the customer to switch to another bank is the condition for the customer to engage in search, i.e. that the present value of its expected future profits increases by more than the sum of the search costs (s) and its risk premium γ : in case of search, s is a sunk cost and has to be borne even if the search leads to the conclusion that switching is not optimal. We have:

$$\left[r_K' - E(\hat{r}_L^A) \right] E(L^A) - \left[r_K' - E(\hat{r}_L) \right] E(L) > s + \gamma, \quad (A1)$$

where $E(\hat{r}_L^A)$ and $E(L^A)$ are the expected loan rate and loan stock with the other bank, respectively. The customer's $E(L)$ is based on the information it obtains from the bank: the bank's responses to its given loan requests (ΔL^D). If the customer is not rationed and $\Delta L = \Delta L^D$, $E(L)$ can be expected to remain the same or increase. If he is rationed and $\Delta L < \Delta L^D$, the customer may revise $E(L)$ downward, which increases the left hand side of Eq. (A1), given $E(L^A)$ and $E(\hat{r}_L^A)$. The customer starts searching if the new value of that side exceeds $s + \gamma$. The customer then forms a better informed opinion of $E(L^A)$ and $E(\hat{r}_L^A)$ on the basis of the values of ΔL^A and \hat{r}_L^A that the best alternative bank offered in response to ΔL^D , and switches if the left-hand side of Eq. (A1) is positive. Given this, it is rational for the old bank to expect the probability of the customer staying with it to be a rising function of $\Delta L - \Delta L^D$ and a declining function of \hat{r}_L , given $E(L^A)$, $E(\hat{r}_L^A)$, s , and γ , where we proxy $E(\hat{r}_L^A)$ by the free market rate (plus a margin). This yields Equation (3) in the text.

Appendix 2. The Optimum Loan When the Bank Cannot Change the Loan Rate

When the nonnegativity or prime rate (or loan rate ceiling) constraint is binding, the loan rate is the floor or ceiling rate \hat{r}_L , which translates into the interest earnings of \bar{r}_L via Eq. (5). In this regime, the bank can increase P only by increasing ΔL so that an incentive to invest in P always translates into an increase in ΔL , except in a corner solution. Eq. (7a) yields:

$$\Delta L^* \leq \frac{1}{a} \left[\frac{(\partial P / \partial \Delta L)Z + aP(\bar{r}_L - r)}{(\partial P / \partial \Delta L)(r - \bar{r}_L) + aP(\partial \tilde{r} / \partial L)} - L_0 \right] \quad (\text{B1})$$

If inequality holds in Eq. (7c) and rationing is optimal, equality holds in Eq. (B1). Correspondingly, if equality holds in Eq. (7c), inequality holds in Eq. (B1) and we have a corner solution.

The propositions are affected as follows. Proposition 1 holds on conditions analogous to those of the general case.

Proposition 2 holds for a customer with a convex enough P function:

$$\frac{d\Delta L^*}{d\Delta L^D} = \left[\left(\frac{\partial P}{\partial \Delta L} \right)^2 - P \frac{\partial^2 P}{\partial \Delta L^2} \right] \left[Z \frac{\partial \tilde{r}}{\partial L} + (\bar{r}_L - r)^2 \right] / G^2, \quad (\text{B2})$$

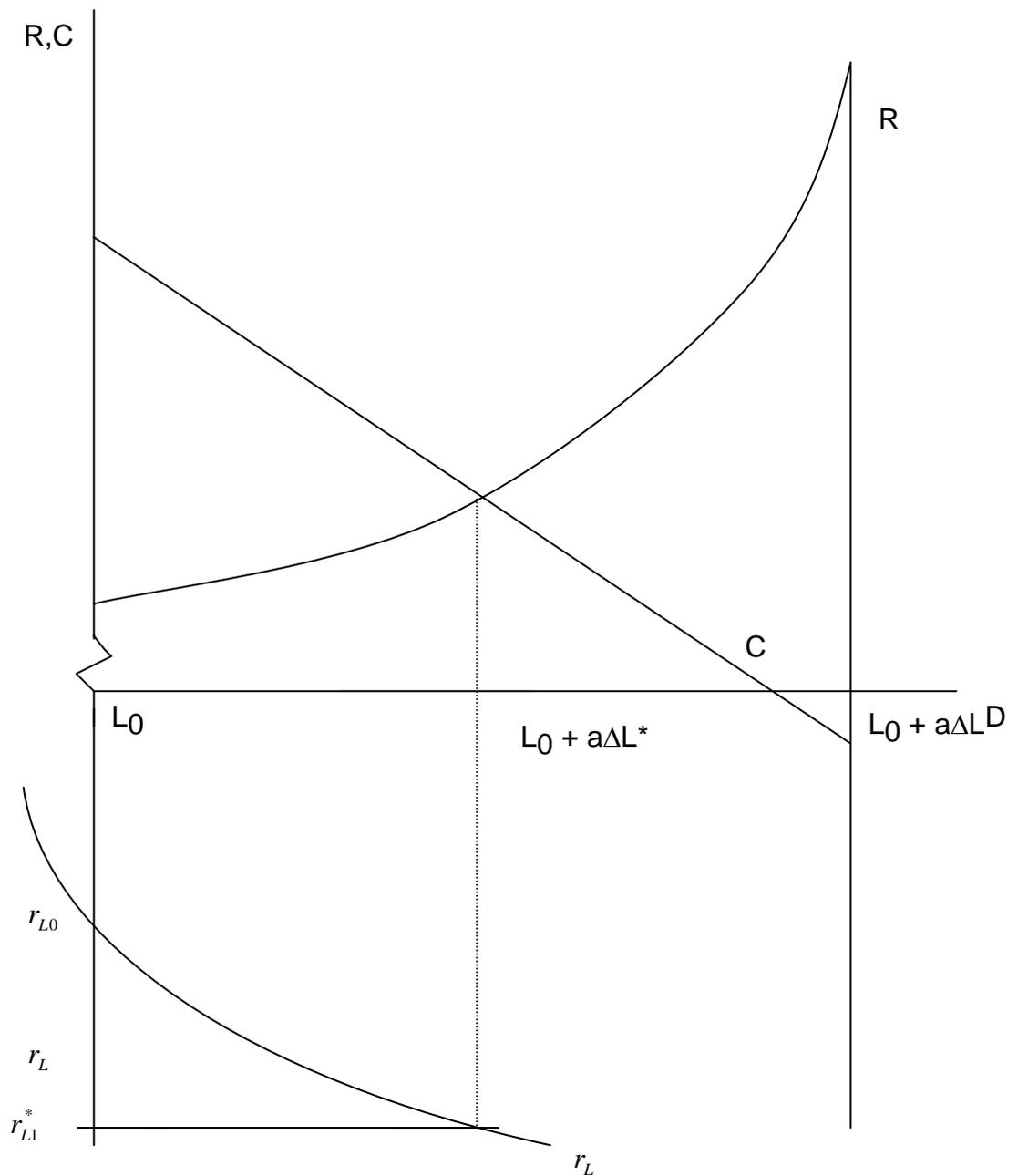
where G is the denominator of Eq. (B1). The denominator of Eq. (B2) is of course positive. In the numerator, the second factor is positive since its both terms are positive. The first factor and the derivative are positive if the P function is concave ($\partial^2 P / \partial \Delta L^2 \leq 0$), or if convex, if the first term dominates. Then the conventional result is obtained: the loan extended increases with the loan request. However, if the P function is convex ($\partial^2 P / \partial \Delta L^2 > 0$) (rather than concave, as in the general case) and the second term dominates in the first factor, the loan extended increases when the loan request is reduced. As the customer reduces the loan request, $\partial P / \partial \Delta L$ increases for any given value of $\Delta L \leq \Delta L^D$, and ΔL^* increases. It is optimal for the customer to reduce the request until either Eq. (B2) changes sign, or the customer reaches a corner solution. In the latter case, it is optimal for the customer to try to avoid an interior solution and try to stay in a corner solution.

Proposition 5: Proof available from the author.

Table 1. Small Business Bank Shopping in Canada

	Percentage of Application Extended		
	< 50	50-99	100
Percentage of applicants shopping	60	55	35
If 47 % of shoppers switch, percentage of applicants switching	28	26	16

Source: Hines, George, A., Allan Riding, and Roland Thomas. "Small Business Bank Shopping in Canada", Journal of Banking and Finance, 15, 1991, 1045-1056.

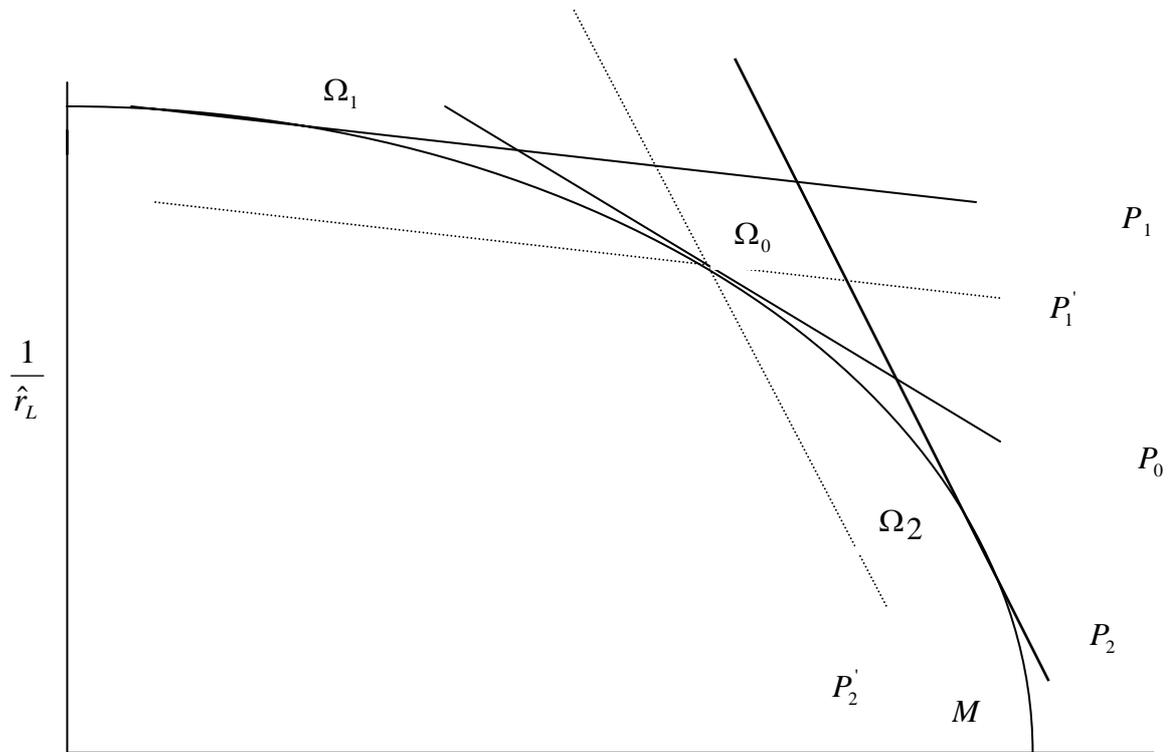


$$R \equiv \left(\frac{\partial P}{\partial \Delta L} + a \frac{\partial \tilde{r}}{\partial L} \frac{\partial P}{\partial \hat{r}_L} \right) (L_0 + a\Delta L)^2$$

$$C \equiv -a \left[\frac{\partial P}{\partial \hat{r}_L} \left(Z + \frac{\lambda}{a} \right) + P(L_0 + a\Delta L) \right]$$

$$r_L = -\frac{Z}{L_0 + a\Delta L} + r - \frac{P}{\partial P / \partial \hat{r}_L}$$

Figure 1. Determination of the Optimal Loan and Loan Rate When $D > 0$.

Figure 2. The Bank's Interior Equilibrium ΔL

$$M = Z + (r_L - r)L$$

$$P = P(\Delta L - \Delta L^D, \hat{r}_L - r)$$

$$\frac{\partial P}{\partial \Delta L} + a \frac{\partial \tilde{r}}{\partial L} \frac{\partial P}{\partial \hat{r}_L} - D = 0$$

$$\frac{d(1/\hat{r}_L)}{d\Delta L_{(M)}} = \frac{1}{\hat{r}_L^2} \left(\frac{r_L - r}{L} - a \frac{\partial \tilde{r}}{\partial L} \right) (< 0)$$

$$\frac{d(1/\hat{r}_L)}{d\Delta L_{(P)}} = - \frac{a(\partial P / \partial \Delta L)(\partial \tilde{r} / \partial L)}{\hat{r}_L^2 (\partial P / \partial \Delta L - D)} < 0$$