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A GENERAL EQUILIBRIUM ANALYSIS

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**Publicly provided private goods and redistribution:  
A general equilibrium analysis\***

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**Abstract:**

This paper focuses on two sets of impacts of public provision of private goods that have been neglected in the self-selection framework of optimal taxation, the by-now standard approach in examining public provision. We first show, using a general formulation whereby production depends on labour supply of different households and the level of public provision, that there can be a role for public provision because of productivity and wage-structure effects, even if preferences are weakly separable between goods and leisure. Second, we deal with a specific example of public provision of education that provides an intuitively appealing case for the production side impacts. Finally, we address the role of public provision in a dynamic, overlapping generations, economy, whereby public provision may affect efficiency and social costs of redistribution of future generations as well, opening up a way to combine inter- and intra-generational impacts of public provision.

**Keywords:** Optimal taxation, public provision, education, overlapping generations.

## 1. Introduction

One of the key interest areas in the recent economic research following the asymmetric information approach to optimal redistribution has been the attempt to explain the role of public provision of private goods (such as education, health care, day care and care of the elderly) as a part of redistributive set of instruments (see e.g. Blomquist and Christiansen (1995), (1998a) and (1998b), Boadway and Marchand (1995) and Boadway, Marchand and Sato (1998))<sup>1</sup>. All these studies model the production side in a simple way with exogenous wage rates for different types of households in a static setting.

The aim of this paper is to extend the literature above in two main ways. First, we allow for a richer description of the production technology that enables the before-tax wage rate to depend on the labour supply of different households and, in various ways, the level of publicly provided private goods. We find this to be a quite important extension because many of the publicly provided goods (e.g. education) can have sizeable influence on the productivity of different households. It seems to us that in practice the government involvement in public provision is indeed often motivated by the attempt to reduce wage inequality. Allowing for endogenously determined wages is also interesting because it gives rise to new forms of distorting taxation and production inefficiency, along the lines of the important recent contribution by Naito (1999).

Second, building on our earlier work (Pirttilä and Tuomala (2001)), we consider public provision of private goods in a genuinely dynamic, overlapping generations economy. Many of the publicly provided goods have arguably interesting dynamic and cross-generational effects, because of financing arising from a different generation (pensions) or potential externality effects between generations (education). The government may be interested in the provision of such goods precisely because they have a special characteristic to enhance human capital. Part of the effects of the publicly provided good can thus be modelled as a stock that preserves from period to another and provides a potentially intriguing link between the

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<sup>1</sup> For an excellent overview of this literature, see Balestrino (1999).

taxation of different households. In such a case, the decision rule for the publicly provided private good involves efficiency and distributional considerations both within and between generations.

We apply the self-selection approach to optimal non-linear taxation, along the lines of e.g. Stiglitz (1982), with two types of households that differ in their income earning abilities. The government cannot observe productivity and must design its redistributive policy subject to the self-selection constraint that the skilled households do not want to mimic the choice of the unskilled households. In addition to the non-linear income tax, the government can provide part of a private good to the households.<sup>2</sup> The households may supplement this good from private markets. The publicly provided good is modelled in a versatile way, affecting potentially both household utility, their own income earning abilities and, through an externality, the productivity of the entire economy. Thus, we address in principle a large set of publicly provided private goods. Throughout the analysis we discuss, however, the findings in the education context.

In the first part of the analysis (section 2), we examine a simple static set up but with the endogenous wage structure discussed above. We show that in contrast to results in earlier literature, there can be reasons for public provision of private goods, even if we assume weakly separable preferences between goods and leisure. The reason is that public provision can affect productivities of different kinds of labour, thus affecting wage dispersion, and the possibilities for redistribution. This result bears resemblance to that of Naito (1999) who shows that under similar circumstances, differentiated commodity taxation can be welfare improving. In section 3 we consider an example whereby the public provision itself influences productivity – i.e. education that raises income-earning abilities – and discuss the characteristics of optimal public provision in those circumstances.

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<sup>2</sup> An alternative formulation, based on the idea that government may support the consumption of a good through price subsidies instead of public provision, would be possible as well. However, the recent papers by Blomquist and Christiansen (1998b) and Boadway, Marchand and Sato (1998) have shown that, even when optimal commodity taxation is allowed for, the use of public provision can still be welfare-improving.

In section 4 we extend the analysis into a dynamic, infinite horizon, OLG framework, where the individuals live two periods, supplying labour at the first and retiring at the second. The purpose of this section is to demonstrate how the extension of the production side of the economy to include a stock of publicly provided private good that preserves from period to another can affect the public provision rule. In particular, the publicly provided private good may be motivated both because of intragenerational and intergenerational reasons. Conclusions are given in section 5.

## 2. Public provision with productivity and relative wage effects: A general case

In this section we consider a simple static formulation of public provision, inspired by e.g. Boadway and Marchand (1995). There are two types of households in the economy: Households of type 1 are less skilled and earn income  $w_1$ . The more skilled household, type 2, earns a wage  $w_2 (> w_1)$ . The number of households of each type is 1. Households supply labour, denoted by  $l$ , and consume two types of goods: a normal private good,  $c$ , and a quasi-private<sup>3</sup> good  $z$ . The overall level of the latter good for the household is a sum of the public provision of the good, depicted by  $g$ , and the private, topping up, part,  $z$ . We apply the terminology introduced by Boadway and Marchand (1995) and call the situation where an increase in  $g$  leads to a reduction in the private purchases  $z$  as one where the household's private purchases of the quasi-private good are crowded out. The government supplies the same amount of  $g$  to all households. In addition we assume that  $g$  cannot be resold.

It is useful to break the household optimisation into two parts. In the first, the households make the optimal consumption decision for a given after-tax income ( $x$ ) and public provision. Denote household utility function by  $u(c_i, l_i, z_i + g)$ , where the subscript  $i \subseteq (1, 2)$  refers to household type and is dropped below. Then, using the household budget constraint, the

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<sup>3</sup> The term 'quasi private' only refers to the idea that this good is both provided by the government and purchased by the household itself.

household maximisation of  $u(x - z, l, z + g)$  with respect to  $z$  yields the following Kuhn-Tucker conditions:

$$-u_c + u_z \leq 0; \quad z(-u_c + u_z) = 0, \quad (1)$$

where subscripts refer to partial derivatives.<sup>4</sup> Conditions in (1) give the conditional indirect utility function  $v(x, l, g) = u[x - z(x, l, g), l, z(x, l, g) + g]$  and conditional demand for  $z$ ,  $z(x, l, g)$ . Using the envelope theorem, the following properties hold:

$$v_x = u_c, \quad v_l = u_l, \quad v_g = u_z. \quad (2)$$

In the second step of consumer optimisation, the household maximises  $v(x, l, g)$  with respect to his labour supply, subject to a given tax schedule,  $T(y)$ , and the budget constraint  $x = y - T(y)$ , where  $y = wl$  denotes households gross income. The optimisation enables the marginal income tax to be expressed in terms of the utility function:

$$T'(y) = MTR(y) = \frac{1}{w} \frac{v_l}{v_c} + 1. \quad (3)$$

The production side of the economy is modelled using an aggregate, constant returns to scale, production function  $F(l_1, l_2, e)$ , where  $e$  denotes the aggregate level of the quasi-private good in the economy, i.e. the sum of private and public parts for the two consumers. Note that the same technology is used to produce both goods. They have thus similar producer prices as well. For simplicity, the prices for both goods are normalised to unity. This specification captures two important features of the model. First, the wage rates are endogenous in a similar way as in Stern (1982) or Stiglitz (1982). In the following,  $\Omega = \frac{w_1}{w_2}$  depicts the relative wage

of the low-skilled type. Assuming competitive labour market,  $\Omega$  is a function of  $l_1/l_2$  and  $e$ ,

$\frac{w_1}{w_2} = \frac{F_1(l_1, l_2, e)}{F_2(l_1, l_2, e)}$ . It captures the idea that the relative wage rate of type 1, determined at the

market, is a decreasing function of  $l_1/l_2$ . Hence, if the government uses its policy instruments

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<sup>4</sup> The derivative  $u_z$  refers to the derivative with respect to the third argument in the utility function.

so that the relative labour supply of type 2 household rises, it carries a redistributive benefit through an increase in the relative wage of low-skilled household.<sup>5</sup> The second key feature of our framework is that the overall level of the publicly provided good is allowed to affect not only consumers directly, but also the production side of the economy by influencing the productivity and the wage rates in the economy. Thus  $e$  has a positive externality feature.

Following the standard idea of Pareto-optimal taxation, the government maximises the utility of the less-skilled households subject to the constraint that the skilled household must stay at a given utility level. The government redistributes income by taxing income on a non-linear scale. It may also use a uniform public provision of  $g$  as a policy variable. We apply the information-based approach to tax policy by assuming that the government can observe the labour income  $y$ , but it observes neither the income earning abilities (the wage rates) of the households or their consumption allocation decisions (between  $c$  and  $z$ ). Therefore, the government must select the tax schedule subject to the self-selection constraint that the skilled household has an incentive to work  $l_2 = y_2 / w_2$ , report income  $y_2$  and consume  $x_2$  instead of wishing to pretend to be the unskilled household, i.e. mimic, working  $y_1 / w_2 = w_1 l_1 / w_2 = \Omega l_1$ , reporting income  $y_1$ , and consuming  $x_1$ . The government chooses the optimal tax schedule (or labour – after-tax income) bundles to the two different household types subject to the constraint that the skilled household be at a given utility level, the self-selection constraint of the skilled household, and the resource constraint of the economy. The Lagrangean of the government optimisation problem can therefore be written as

$$\begin{aligned} \underset{x_1, l_1, x_2, l_2}{Max} \quad & L = v(x_1, l_1, g) + \delta [v(x_2, l_2, g) - \bar{v}^2] \\ & + \lambda [v(x_2, l_2, g) - v(x_1, \Omega l_1, g)] \\ & + \rho [F(l_1, l_2, e) - x_1 - x_2 - 2g] \end{aligned} \quad (4)$$

The first-order conditions for the tax choice are the following:

$$x_1 : v_x^1 - \lambda \hat{v}_x^2 - \rho = 0, \quad (5)$$

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<sup>5</sup> See Stiglitz (1982) or Stern (1982) for details.



$$l_1 : v_l^1 - \lambda \hat{v}_l^2 \left( \frac{\partial \Omega}{\partial l_1} l_1 + \Omega \right) + \rho w_1 = 0, \quad (6)$$

$$x_2 : (\delta + \lambda) v_x^2 - \rho = 0, \quad (7)$$

$$l_2 : (\delta + \lambda) v_l^2 - \lambda \hat{v}_l^2 \frac{\partial \Omega}{\partial l_2} l_1 + \rho w_2 = 0, \quad (8)$$

where the hat terms refer to the so-called mimickers, i.e. type 2 households when mimicking the choice of type 1. Stern (1982) and Stiglitz (1982) show that in this framework, the marginal tax rate for the skilled household is negative and the marginal tax rate for the less-skilled household is positive.<sup>6</sup> Given this income tax schedule, we may use the envelope theorem to detect the change in the social welfare from an increase in the level of the publicly provided good as follows:

$$\frac{dL}{dg} = v_g^1 + (\delta + \lambda) v_g^2 - \lambda \hat{v}_g^2 - \lambda \hat{v}_l^2 \frac{d\Omega}{dg} l_1 + \rho F_e \frac{de}{dg} - 2\rho. \quad (9)$$

Rewriting (9) by substituting for  $\rho$  from (5) and (7) yields

$$\frac{dL}{dg} = (v_g^1 - v_x^1) + (\delta + \lambda)(v_g^2 - v_x^2) + \lambda(\hat{v}_x^2 - \hat{v}_g^2) - \lambda \hat{v}_l^2 \frac{d\Omega}{dg} l_1 + \rho F_e \frac{de}{dg}. \quad (10)$$

The first three terms in (10) restate the Boadway and Marchand (1995) result (their proposition 1) that public provision of private good is welfare improving, if the mimicker becomes crowded out (i.e. his private purchases,  $z$ , fall to zero) while households of type 1 and type 2 that report their types honestly are not yet crowded out.<sup>7</sup> The intuition is that pushing  $g$  above the point where the private purchases of the mimicker fall to zero makes the mimicker worse off, but if households of type 1 and 2 are not crowded out, it increases social welfare by relaxing the self-selection constraint (that a type 2 household honestly reporting its income must be better off than a mimicker). Boadway and Marchand also note that in the case

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<sup>6</sup> This result can be obtained by combining (7) and (8), as well as (6) and (5).

<sup>7</sup> This may be seen by using the properties in (2) and combining (10) with the households' first-order conditions in (1).

where the publicly provided good enters the households' utility functions, if preferences are weakly separable between goods and leisure, the mimicker and a true type-1 representative become crowded out at the same point, because of which there is no benefit from having a positive provision of the publicly provided good.

Our focus is, however, more in the production side of the economy, and therefore we concentrate on the case with separable preferences, implying that the three first terms of (10) are zero. With regard to it, the following result emerges:

*PROPOSITION 1: If preferences are weakly separable between goods and leisure, public provision of private good is welfare improving if it (a) increases productivity and (b) reduces the wage differentials of the households.*

The results can be seen from the last two effects in (10). The direct productivity effect (last term in (10)) is usually positive (if it is negative, then it offsets part of the potential benefits for households, ie the original Boadway-Marchand part). What is more interesting, however, is the link between the publicly provided private good and the wage structure of the economy

(the term  $-\lambda v_i^2 \frac{d\Omega}{dg} l_1$ ). If its provision leads to a relative increase in the wage rate for type 1

households, then indirect redistribution through public provision will Pareto-improve welfare by mitigating the incentive problem of the non-linear income tax system. Intuitively this would be a case where the publicly provided good augments the productivity of type 1 labour supply more than that of type 2 labour supply. This effect is similar to the impact of labour supply on the wage rates, originally analysed by Stiglitz (1982) and Stern (1982). The result is also important in that it bears resemblance to the interesting recent findings by Naito (1999) that if wage rates are endogenous, redistribution devices that otherwise would not be applied –

in Naito's case public inputs and commodity taxation and in our case public provision of private goods – become welfare-improving.

### 3. Publicly provided education

In this section we focus on public provision of education<sup>8</sup>. Similarly to the previous section, the household may supplement publicly provided education from the private markets. The household optimisation now changes somewhat from the general case (presented above), and therefore we explain it in this section again.<sup>9</sup> The household utility now depends only on private consumption,  $c$ , and labour,  $l$ . Education itself does not enter the utility function; it affects household wages through its influence on productivity. Let us denote the wage rate for household  $i$  as  $\omega^i = \omega(l^1, l^2, e^i)$ , where  $e^i = z^i + g$ . The utility function is now

$u(c, l) = u\left(x - z, \frac{y}{\omega}\right)$ . The first-order conditions with respect to  $z$  are now:

$$-u_c - u_l y \frac{\omega_e}{(\omega)^2} \leq 0; \quad z \left( -u_c - u_l y \frac{\omega_e}{(\omega)^2} \right) = 0, \quad (11)$$

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<sup>8</sup> In public economics there is quite a large literature on optimal provision of education, beginning with Arrow's (1971) paper. The interaction of optimal income taxation and educational choices has been examined among others by Ulph (1977), Hare and Ulph (1979) and Tuomala (1986). Following Arrow the first two papers assume that ability to benefit from education is observed by the education authorities. However, this information is not available to the tax officials. The most recent literature on public provision departs from early contribution by assuming that public provision cannot be related to an individual's ability. Tuomala (1986) in turn focuses on the question of how educational choices affect the progressivity of the optimal income tax.

<sup>9</sup> The modelling of the household optimisation of this section is based on the framework in Boadway and Marchand (1995).

where  $\omega_e = \frac{\partial \omega}{\partial e}$ . Now, the following properties hold for the conditional indirect utility

function  $v(x, l, g)$ :

$$v_x = u_c, \quad v_l = u_l, \quad v_g = -u_l y \frac{\omega_e}{(\omega)^2}. \quad (12)$$

The second step of consumer optimisation gives a marginal tax rate condition similar to the one given in (3) in the set-up in the section above. Another change is that the production function now depends not only on the labour supply of the households, but also on the level of their education. To simplify the production function somewhat, we assume that it takes a specific, two factor, form with education-weighted effective labour supplies of both household types. The production function may then be written as  $F(e^1 l^1, e^2 l^2)$ . This also means that

$\Omega = \frac{\omega^1}{\omega^2}$  depends on  $g$ .

Since the government optimisation remains the same as in the previous section, the optimality conditions with respect to the tax rates are also the same. Given the optimal tax schedule, the change in the social welfare with respect to public provision of education is

$$\frac{dL}{dg} = v_g^1 + (\delta + \lambda)v_g^2 - \lambda \hat{v}_g^2 + \rho \left( F_1 l^1 \frac{de^1}{dg} + F_2 l^2 \frac{de^2}{dg} \right) - \lambda \hat{v}_l^2 \frac{d\Omega}{dg} l_1 - 2\rho. \quad (13)$$

Rewriting (13) by substituting for  $\rho$  from (5) and (7) yields

$$\frac{dL}{dg} = (v_g^1 - v_x^1) + (\delta + \lambda)(v_g^2 - v_x^2) + \lambda(\hat{v}_x^2 - \hat{v}_g^2) + \rho \left( F_1 l^1 \frac{de^1}{dg} + F_2 l^2 \frac{de^2}{dg} \right) - \lambda \hat{v}_l^2 \frac{d\Omega}{dg} l_1. \quad (14)$$

Boadway and Marchand show that without endogenous wages, public provision of education is welfare improving, if mimicker becomes crowded out before the true-type households. This effect can be seen from the first three terms in expression (14). In our case, where the production function is more general, there are also additional impacts. The following proposition summarises the effects:

*PROPOSITION 2: Public provision of education tends to be welfare improving if it (a) renders mimicker to be crowded-out first, (b) increases productivity and (c) reduces the wage differentials of the households.*

As discussed above, part (a) has already been shown by Boadway and Marchand (1995). Part (b) is due to the fourth term in (14) and measures the value of additional production arising from an increase in publicly provided education. Unlike in Proposition 1, the productivity effect is unambiguously non-negative, and positive, if private education purchases are not crowded out. Part (c) is due to the last term in expression (14), with a similar interpretation than before: if publicly provided education reduces wage differentials, its provision becomes welfare improving, other things being equal.

Boadway and Marchand also show that if the mimicker becomes crowded out at the same point as the true-type households, there is nothing to be gained from the public provision of education. More precisely, this is the case if the elasticity of the wage rate towards education is similar for the mimicker and true type 1 representative. The similar result holds in our framework as well: if also type 1 and type 2 households become crowded out, the fourth term in (14) vanishes. In addition, with an equal elasticity of the wage rate towards education, additional education raises the wage rates of both types in a similar manner, and therefore no impacts arise from the fifth term in (14). Note that when the additional impacts we introduce refer to the production side, from where the potential effects in the original framework also come from, the separability issue highlighted in Proposition 1 does not change the role of public provision in the framework of this section.

#### **4. Public provision in an OLG framework**

In this section, we extend the previous analysis to a dynamic, overlapping generations, setting to address the intergenerational effects of public provision. These are clearly essential in public provision of pensions, for example. However, important implications also arise in the context of education: public provision of education not only affects efficiency of redistribution within one cohort, it can also have lasting effects because of accumulation of knowledge in

the economy. These impacts are of key interests in this section that focuses on the role of education in an OLG framework<sup>10</sup>.

We consider an economy where individual households live two periods, acquiring education and supplying labour at the first and consuming at the second period.<sup>11</sup> We assume for simplicity that there is no population growth. Each generation consists of two households that have different productivity at the labour market. We apply the following notation:  $c_i^t$  denotes the consumption of a household of type  $i$  born at time  $t$ ,  $x_i^t$  the after-tax income of the same household,  $l_i^t$  household  $i$ 's labour supply,  $z_i^t$  refers to the private purchases of education, and  $r$  is the interest rate. The household's utility function is thus  $u = u(c^t, l^t)$  and the combined budget constraint  $z^t + \frac{1}{1+r_{t+1}}c^t = x^t$ . As above, the household wage rate depends, among

other things, on its labour supply and the aggregate level of its education (sum of public and private purchases). As before, in the first stage of household optimisation, the household chooses the level of education for a given labour supply. Maximisation of

$u(c^t, l^t) = u\left((1+r_{t+1})(x^t - z^t), \frac{y^t}{\omega^t}\right)$  produces the first-order conditions

$$-(1+r_{t+1})u_c - u_l y \frac{\omega_e}{(\omega)^2} \leq 0; \quad z \left( -(1+r_{t+1})u_c - u_l y \frac{\omega_e}{(\omega)^2} \right) = 0, \quad (15)$$

where  $\omega_e = \frac{\partial \omega}{\partial e}$ . Now, the following properties hold for the conditional indirect utility

function  $v(x, l, g)$ :

$$v_x = (1+r_{t+1})u_c, \quad v_l = u_l, \quad v_g = -u_l y \frac{\omega_e}{(\omega)^2}. \quad (16)$$

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<sup>10</sup> For an early contribution in the intertemporal context, see Hare and Ulph (1981). The crucial element in their model is imperfect capital markets.

<sup>11</sup> This section builds on the framework developed by Brett (1997) and Pirttilä and Tuomala (2001). We have dropped the first period consumption in order to keep the framework as similar as possible to the previous sections.

The second step of consumer optimisation gives a condition for the marginal tax rate similar to (3) in the first section.

The production side of the economy utilises three factors: capital denoted by  $k^t$  and the two kinds of effective labour. In addition, the productivity of the economy depends on the cumulated level of education (or human capital)  $e^t$  in the economy. Hence, we write the production function as  $F = F(k^t, e_1^t l_1^t, e_2^t l_2^t, e^t)$ . The production function is assumed to exhibit constant returns to scale with respect to labour and capital under perfect competition. Factor prices are then determined by the marginal productivities as follows:  $F_k^t = r^t$ ,  $F_{l_1^t} = \omega_1^t$  and  $F_{l_2^t} = \omega_2^t$ . The relative wage between the two types is again denoted by  $\Omega^t = \frac{\omega_1^t}{\omega_2^t}$ .

The government's problem is to maximise intertemporal social welfare that includes welfare comparison both within and between generations. We continue to assume that within generation, the government maximises the utility of type 1 household subject to a given utility to type 2 household. Across the generations the government is assumed to be utilitarian, but the utilities of future cohorts are discounted with a discount factor  $\beta^t$ . As above the government chooses the optimal tax schedules and the optimal level of public provision of education for each generation, subject to the self-selection constraint of the skilled household.<sup>12</sup>

Finally, we need equations that determine the path of human capital and physical capital in the economy. The key underlying idea in this section is that the publicly provided good has potential durable impacts over generations. This is modelled here by assuming that the productivities of different generations are linked via the term  $e^t$ . Therefore, human capital at time  $t$  is given by

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<sup>12</sup> An additional policy instrument would be a tax levied on savings (or second period consumption). Its usefulness as a policy instrument in OLG framework has been addressed in e.g. Brett (1997) and Pirttilä and Tuomala (2001).

$$e^t = \alpha e^{t-1} + \sum_i (z_i^t + g^t), \quad (17)$$

i.e., it is a sum of the human capital at the previous period (weighted with parameter  $\alpha$ ) plus the increase of education (private and public) at period  $t$ . The idea is that future generations can utilise the knowledge accumulated by the previous generations.

The evolution of physical capital is determined by the resource constraint of the economy and may be written as

$$k^{t+1} = F(k^t, e_1^t l_1^t, e_2^t l_2^t, e^t) + k^t - z_1^t - z_2^t - c_1^{t-1} - c_2^{t-1} - 2g^t. \quad (18)$$

Note that the production at each period must finance consumption of those that are old and the education expenses of the young at that period.

If we ignore the generation that is old at period 1,<sup>13</sup> and write the resource constraint in terms of observables using the household budget constraint, we are in a position to write the Lagrangean for the government optimisation problem as follows:

$$\begin{aligned} L = & \sum_{t=1}^{\infty} \beta^t v(x_1^t, l_1^t, g^t) + \sum_{t=1}^{\infty} \delta^t [v(x_2^t, l_2^t, g^t) - \bar{v}_2^t] \\ & + \sum_{t=1}^{\infty} \lambda^t [v(x_2^t, l_2^t, g^t) - v(x_1^t, \Omega^t l_1^t, g^t)] \\ & + \sum_{t=1}^{\infty} \rho^t \left[ F(k^t, e_1^t l_1^t, e_2^t l_2^t, e^t) + k^t - x_t^1 - x_t^2 + \frac{1}{1+r^{t+1}} (c_t^1 + c_t^2) - c_1^{t-1} - c_2^{t-1} - 2g^t - k^{t+1} \right] \\ & + \sum_{t=1}^{\infty} \mu^t \left[ \alpha e^{t-1} + \sum_i (z_i^t + g^t) - e^t \right] \end{aligned} \quad (19)$$

The development of the stock of the human capital is captured by the last constraint, with the multiplier referring to the social marginal value of the stock at period  $t$ . An important assumption to note is that the government possesses perfect control over the capital stock. As

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<sup>13</sup> It is a usual feature of dynamic optimal taxation exercises that it is optimal for the government to collect the bulk of the revenue by confiscating the capital stock at the outset. We want to abstract from these complications by leaving out the generation 0. For a good overview on these issues, see Domeij and Klein (1998).



shown by e.g. Atkinson and Sandmo (1980), this implies that only one from the resource constraint and government budget constraint need to be taken into account. Because of this, together with the observation that the resource constraint may be derived from combining the government's and the households' budget constraints in the Mirrleesian fashion, the multiplier associated with the resource constraint,  $\rho^t$ , may be interpreted as the shadow price of government's revenue. For a given level of publicly provided education, the first-order conditions at an exemplary date  $t$  revealing the optimal tax structure are the following:

$$k^t : \rho^t (F_k^t + 1) - \rho^{t-1} - \lambda^t \hat{v}_{2,l}^t \frac{\partial \Omega^t}{\partial k^t} l_1^t = 0, \quad (20)$$

$$x_1^t : \beta^t v_{1,x}^t - \lambda^t \hat{v}_{2,x}^t - \rho^t = 0, \quad (21)$$

$$l_1^t : \beta^t v_{1,l}^t - \lambda^t \hat{v}_{2,l}^t \left( \Omega^t + \frac{\partial \Omega^t}{\partial l_1^t} l_1^t \right) + \rho^t \omega_1^t = 0, \quad (22)$$

$$x_2^t : (\delta^t + \lambda^t) v_{2,x}^t - \rho^t = 0, \quad (23)$$

$$l_2^t : (\delta^t + \lambda^t) v_{2,l}^t - \lambda^t \hat{v}_{2,l}^t \frac{\partial \Omega^t}{\partial l_2^t} l_1^t + \rho^t \omega_2^t = 0. \quad (24)$$

These conditions yield the optimal marginal income tax rates for the two household types for each period. For each generation, these are similar to the tax rules in a static economy.<sup>14</sup> Our main interest is however on the role of public provision in this framework. The conditions that determine the welfare impact of public provision are the following:

$$\frac{dL}{dg} = \beta^t v_{1,g}^t + \delta^t v_{2,g}^t + \lambda^t (v_{2,g}^t - \hat{v}_{2,g}^t) + \rho^t \left( F_1^t l_1^t \frac{de_1^t}{dg} + F_2^t l_2^t \frac{de_2^t}{dg} \right) - 2\rho^t + 2\mu^t \leq 0, \quad (25)$$

$$e^t : \lambda^t \hat{v}_{2,l}^t \frac{\partial \Omega^t}{\partial e^t} l_1^t + \rho^t F_e^t + \mu^{t+1} \alpha - \mu^t = 0. \quad (26)$$

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<sup>14</sup> See Pirttilä and Tuomala (2001) for details.

Use (21) and (23) to rewrite equation (25) as follows:

$$\begin{aligned} \frac{dL}{dg} = & \beta^t (v_{1,g}^t - v_{1,x}^t) + (\delta^t + \lambda^t) (v_{2,g}^t - v_{2,x}^t) + \lambda^t (\hat{v}_{2,x}^t - \hat{v}_{2,g}^t) \\ & + \rho^t \left( F_1^t l_1^t \frac{de_1^t}{dg} + F_2^t l_2^t \frac{de_2^t}{dg} \right) + 2\mu^t \leq 0 \end{aligned} \quad (27)$$

Define the social marginal benefit of public provision of education within a period  $t$  (similar to the expression given in (14) within the static framework) as

$$\begin{aligned} SMB_g^t \equiv & \beta^t (v_{1,g}^t - v_{1,x}^t) + (\delta^t + \lambda^t) (v_{2,g}^t - v_{2,x}^t) + \lambda^t (\hat{v}_{2,x}^t - \hat{v}_{2,g}^t) \\ & + \rho^t \left( F_1^t l_1^t \frac{de_1^t}{dg} + F_2^t l_2^t \frac{de_2^t}{dg} \right) + 2\lambda^t \hat{v}_{2,t}^t \frac{\partial \Omega^t}{\partial e^t} l_1^t + 2\rho^t F_e^t \end{aligned} \quad (28)$$

Combining (28) and (26) enables us to rewrite (27) as

$$\frac{dL}{dg} = SMB_g^t + 2\mu^{t+1} \alpha \quad (29)$$

To arrive at our final expression for the public provision, lead (29) first by one, then two and so on periods and combine the conditions.<sup>15</sup> The next proposition summarises the rule for public provision in this setting at time  $t$ :

*PROPOSITION 3: Given an optimal income tax scheme, the dynamic version of the welfare effects of public provision of education is given by:*

$$\frac{dL}{dg} = SMB_g^t + 2 \sum_{\tau=1}^{\infty} \alpha^\tau \left( \lambda^{\tau+t} \hat{v}_{2,t}^{\tau+t} \frac{\partial \Omega^{\tau+t}}{\partial e^{\tau+t}} l_1^{\tau+t} + \rho^{\tau+t} F_e^{\tau+t} \right)$$

Proposition 3 shows how the social value of public provision is the sum of the present period effects, captured by  $SMB_g^t$ , plus all the discounted benefits that accrue to future generations.

The interpretation of  $SMB_g^t$  is straightforward, given our knowledge of the previous section.

The contemporaneous effect of  $g$  in the OLG framework contains similar effects than in the

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<sup>15</sup> In other words, we simply add the first-order conditions for the public good evaluated at different periods together to obtain the present value of an increase in the public good.

static case: terms capturing the effect on self-selection (the first line in (28)), and impacts that address productivity and wage differences (the first two terms at the second line). There is also one additional term (the last term in (28)) that is related to the overall, externality type, impact of education. In sum, the within-period impacts of public provision arise both from the preference and from the production side.

What is novel in the OLG set-up are the inter-temporal productivity effects that arise from the presence of human capital in the model and that are captured by the rest of the terms in Proposition 3. The level of education, or human capital, that is obtained at a given period  $t$  partially preserves to the following periods according to (17). In the subsequent periods, therefore, present investments in education affect first, the wage structure (the first term in the brackets above) and second, the overall productivity of the economy (the second term in the brackets). These impacts are not taken into account by the individuals themselves in their educational decisions. Therefore, besides the present period effects given by  $SMB'_g$ , there can be intergenerational reasons for public provision of education.<sup>16</sup>

If future productivity effects are large enough, public provision can be welfare improving even if we consider the case where no impacts arise from the (static) crowding out mechanism. Finally, it is interesting to note that some of these impacts may of course partially offset each other: there can be trade-offs between within-period impacts and benefits to the future generations. Balancing these requires the comparison of the shadow value of public funds ( $\rho'$ ) and the importance of the self-selection constraint ( $\lambda'$ ) in different periods, because the valuation of future benefits depends, among other things, on these terms.

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<sup>16</sup> It is interesting to compare the dynamic provision rule of the publicly provided private good to the dynamic Samuelson rule of a pure public good, analysed in Pirttilä and Tuomala (2001). The key difference is that if a public good is durable, its effects on the utilities of future generations must be taken into account directly, whereas the publicly provided private good affects directly only the utilities of current generation. The dynamic impacts in the private good case arise from the potential production side implications.

## 5 Conclusion

Earlier literature on public provision of private goods in an optimal taxation framework has recently addressed the role of public provision as a mechanism to relax the self-selection constraint that hinders redistribution. This paper focuses on two sets of impacts that have been neglected in this literature, but that can yet be important in practice. These include, first, impacts arising from the production side of the economy (that has been before analysed in a very simple way). We model the production side in a versatile way, allowing the production capabilities to depend on labour supply of different households – because of which wage rates are endogenous – and also on the level of the publicly provided good. In these circumstances we first show that there can be a role for public provision because of productivity and wage-structure effects, even if standard separability assumptions of household utility assumptions are applied. Second, we deal with a specific example of public provision of education that provides an intuitively appealing case for the production side impacts.

In the second part of the analysis, we address another extension of the framework and consider the role of public provision in a dynamic, overlapping generations, economy, whereby public provision may affect efficiency and social costs of redistribution of future generations as well. We believe that in the case of many publicly provided goods, both the production side impacts (e.g. education) and intergenerational concerns (e.g. pensions) can be sizeable and indeed at the heart of the political decisions of public provision.

There are various important issues that are not developed here but could be interesting to examine in further work. Our first framework (whereby public provision affects utility directly) could be extended by allowing the private and publicly provided goods to be produced using different technology with different intensity of low and high skilled workers. Arguments similar to those presented by Naito (1999) might be used to show that public provision could be welfare improving if its technology is unskilled-labour intensive, even if standard separability assumptions of the utility function applied. In addition, our treatment of the dynamic issues of public provision leaves many important issues aside, including

examination of pensions in this framework and political economy aspects when looking at intergenerational comparisons.

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