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DECISIONS AND REDISTRIBUTION

Jukka Pirttilä
Matti Tuomala

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DEPARTMENT OF ECONOMICS
FIN-33014 UNIVERSITY OF TAMPERE, FINLAND

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Public versus private production decisions and redistribution *

by

Jukka Pirttilä[#] and Matti Tuomala

Abstract: We analyse the decision rules governing public employment policy and capital allocation between private and public sector in a simple two-type and two-sector optimal non-linear income tax model with endogenous wages. Results from a static framework indicate that to produce a given amount of consumption the government should employ more unskilled workers and less skilled workers than is necessary to minimize cost at the prevailing gross wage rate. The potential role of minimum wage as another policy variable is discussed as well. Extending the model into an OLG framework with public and private capital, we show that the discount rate that ought to be used in evaluating public sector projects is not the same as the return to capital in the private sector. In particular, public capital accumulation is favoured if capital in the private sector is complementary with high-skilled labour. Therefore, production efficiency holds neither in public employment decision nor capital allocation.

Key words: public production, redistribution, non-linear taxation, production efficiency

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[#] Corresponding author. Address: Prime Minister's Office, Secretariat of the Economic Council of Finland, Po Box 23, FIN-Government, Finland. Email: jukka.pirttila@vnk.vn.fi.

1. Introduction

Public-sector employment has grown in most OECD countries since the early post-war periods. But this growth was especially marked in Nordic countries. Consequently, the level of public sector employment in Nordic countries is much higher than in other OECD countries. For example, the public sector employment was 32 per cent of the labour force in the mid 1990s in Sweden, whereas the corresponding figures in Germany and UK were 15 per cent and 14 per cent. The other side of the high level of the public sector employment in Nordic countries is that the extent of public provision of goods such as education, health care and social services is also much larger than in other OECD countries. There also seems to be less inequality in countries where public production is widespread (Lundholm and Wijkander 2002). There are several explanations for public sector employment. One popular explanation in political economy literature is that governments have used public sector employment as a tool for generating and redistributing rents. In other explanations government employment has been seen as income security. In other words it can be used as a form of social insurance.

It is natural to also examine the use of public capital in connection to public production. As shown in several empirical studies (see e.g. Aschauer, 1990) the accumulated stock of public capital is an important factor in enhancing the productivity capacity of the economy. Thus, decisions on public investment taken by the government today have long-lasting effects both the well-being of present and future generations. Therefore the choice of discount rate that ought to be used in evaluating public projects is one of the most important decisions taken by the government. It is not only important for accepting or rejecting a specific project but, in particular, for the allocation of the resources between the public and the private sectors of the economy.

A key concern in recent optimal tax literature has been the role of public provision of private goods, such as education and healthcare, as redistributive devices (e.g. Boadway and Marchand (1995), Blomquist and Christiansen (1998) and Cremer and Gahvari (1997)).¹ These studies consider public funding of these goods and they do not focus on the question whether the goods are produced by the private or public sector. In contrast, the optimal tax literature has paid relatively little attention to the redistribution motive in public sector employment and the determination of optimal public capital stock. On public employment, a notable exception is a paper by Wilson (1982). He studies

¹ See Balestrino (1999) for a survey.

optimal employment policy in a linear income tax model with two types of labour. Wilson shows in the model where workers choose the type of labour they supply that the government should hire less unskilled labour and more skilled labour than is necessary to minimize costs at the prevailing gross wage rates. But this result is reversed when labour quantity is the choice variable.²

Similarly, there are surprisingly few studies of optimal public accumulation policy in an intertemporal optimal tax framework. The seminal paper by Pestieau (1974) is an exception. He shows that production efficiency is in general violated (interest rates in public and private sector differ). Our framework is more general than Pestieau's, as we allow for intragenerational heterogeneity and endogenous wage structure.

Our aim is to examine public provision – both public employment and capital allocation decisions – in a unified framework under non-linear income taxation. We use the self-selection approach to optimal non-linear taxation, along the lines of e.g. Stiglitz (1982) and Stern (1982), with two types of households that differ in their income earning abilities. The government cannot observe productivity and must design its redistributive policy subject to the self-selection constraint that the skilled households do not want to mimic the choice of the unskilled households. A key assumption of our approach is, following Naito (1999) and Pirttilä and Tuomala (2002), that the relative wage rates of different types of workers depend on their relative supplies. Another key assumption is that there are two production sectors in the economy (private and public) which both use low-skilled and skilled labour, but the factor intensities and complementarity with capital may differ.

Using this two-sector and two-type model of optimal taxation in a simple static framework, we first examine the use of public provision of private goods and public employment as instruments to address redistributive considerations. We show that the combination of public provision of private goods produced by public employment can be used as a means of relaxing the self-selection constraint in designing optimal redistribution policy. Indeed, several distortionary instruments are used as the means of relaxing the self-selection constraint. These instruments would not be used in the first best world. The intuition is that such a policy can increase the demand for low-wage workers and thereby reduce the wage differentials in the economy (which alleviates the redistributive burden on the income tax) and at the same time provide useful private goods.

² In a simultaneously and independently written paper, Lundholm and Wijkander (2002) address public production in a similar framework with non-linear taxes than ours. They consider public employment and conduct simulation analyses, but do not focus on the role of minimum wages and public sector capital.

Reasoning like this may explain the simultaneous use of public employment policy and public provision of private goods that is widely used in Nordic countries.

An interesting additional question in this context is the potential role of minimum wage as an additional instrument available to the government. The role of unemployment generating minimum wages as a tool of redistribution in a second best world is established by Guesnerie-Roberts (1984) and Allen (1987) for linear income taxation, and by Allen (1987), Marceau-Boadway (1994) and Boadway-Cuff (2001) under non-linear income taxation. Allen (1987) considered the desirability of the minimum wage policy in the context of a non-linear income tax model with general equilibrium effect on wage. He shows that the potential usefulness of the minimum wage policy does not hold in the case of underemployment. Marceau and Boadway (1994) in turn assume that the minimum wage causes involuntary unemployment. They show circumstances under which a minimum wage supplemented by unemployment insurance will be welfare improving. All these papers examined a one sector model. Our aim is to consider the potential usefulness of the minimum wage policy when public sector employment can be used as a redistributive tool at the same time.

Finally, we extend the previous static framework to an overlapping generations setting to examine optimal public investment policy. Production efficiency in capital allocation would imply that the discount rate used in public sector capital would be the same as the interest rate applied in the private sector. However, deviations from such simple policy rules can be warranted if introducing distortions to capital allocation assists in mitigating the distortions created by asymmetric information as a constraint for redistribution. Indeed, under certain conditions, public sector capital accumulation can be included in the policy package that enhances the demand for low-skilled labour in the private sector.

The structure of the paper is as follows. Section 2 introduces a two-type and two-sector optimal tax model. Section 3 examines how public sector employment combined with public provision of private goods such as education, health care and social services can be used as means of mitigating the incentive problem of the non-linear income tax system. Section 4 discusses the role of minimum wage in our two-sector model. Section 5, in turn, extends the previous static model to an overlapping generations setting. Section 6 concludes.

2. The static model

We examine a very general type of publicly provided good, denoted by g . It can be thought to be a pure public good or a private good that is provided by the government, i.e. a quasi-private good. The reason why the publicly provided good is modelled in a very general way is our focus on the production side of the public good.

There are two types of workers in the economy: Workers of type 1 are less skilled and earn income w_1 . The more skilled workers, type 2, earn a wage $w_2 (> w_1)$. The number of workers of each type is 1. Workers supply labour, denoted by l , and thus their budget constraint is $x = y - T(y)$, where $y = wl$ denotes workers' gross income, x their disposable income, and $T(y)$ is a non-linear tax schedule chosen by the government. In the case of a pure public good, the household uses all of its net income on a private good, denoted by c ($c = x$), whereas if the publicly provided good is a quasi-private good, the household can divide its income between c and its own purchases of the quasi-private good, denoted e.g. by z , and then $c + z = x$.

Consumer preferences are represented by a strictly monotone, strictly quasi-concave, and twice differentiable utility function, $v(x_i, l_i, g)$. Note that this is an indirect utility function, expressed in terms of observable variables, that is conditional to the households' optimal choice of using x . Workers maximise $v(x, l, g)$ with respect their labour supply, subject to a given tax schedule, and the budget constraint $x = y - T(y)$.

The production side of the economy is the following. Good c is produced in the private sector according to an aggregate, constant returns to scale, production function $F(l_1^c, l_2^c, g)$, where l_1^c and l_2^c denote the labour inputs in the private sector. In addition, we allow the public good to influence the productivity at the private sector. Good g , in turn, is produced according to the aggregate production function $G(l_1^g, l_2^g)$, where l_1^g and l_2^g are the labour inputs in the public sector. For simplicity, the producer prices for both goods are normalised to unity. This specification implies that the wage rates are endogenous in a similar way as in Stern (1982) or Stiglitz (1982). In the following, $\Omega = w_1/w_2$ depicts the relative wage of the low-skilled type. Assuming competitive labour market, the wage rate of each skill-type is equal to marginal productivity, i.e.

$\Omega = \frac{w_1}{w_2} = \frac{F_1(l_1, l_2, g)}{F_2(l_1, l_2, g)}$. Therefore, Ω is a function of l_1/l_2 and g . This modelling captures the idea that the relative wage rate of type 1, determined at the market, is a decreasing function of l_1/l_2 .

It is also assumed that the public sector minimizes costs with respect to the wages paid at the public sector, \tilde{w}_1 and \tilde{w}_2 . Thus $\frac{\tilde{w}_1}{\tilde{w}_2} = \frac{G_1(l_1, l_2)}{G_2(l_1, l_2)}$. If the government uses its policy instruments so that the relative labour supply of type 2 household rises – in other words low-skilled labour becomes scarcer in the private sector, it carries a redistributive benefit through an increase in the relative wage of low-skilled workers.³

Following the standard idea of Pareto-optimal taxation, the government maximises the utility of the less-skilled workers subject to the constraint that the skilled worker must stay at a given utility level. The government redistributes income by taxing income on a non-linear scale. It may also use a uniform public provision of g as a policy variable. We apply the information-based approach to tax policy by assuming that the government can observe labour income y , but it does not observe the income earning abilities (the wage rates) of the workers. Therefore, the government must select the tax schedule subject to the self-selection constraint that the skilled worker has an incentive to work $l_2 = y_2/w_2$, report income y_2 and consume x_2 instead of wishing to pretend to be the unskilled household, i.e. mimic, working $y_1/w_2 = w_1 l_1/w_2 = \Omega l_1$, reporting income y_1 , and consuming x_1 .

The government chooses the optimal tax schedule (or labour – after-tax income) bundles to the two different worker types subject to the constraint that the skilled worker be at a given utility level, the self-selection constraint of the skilled worker, and the resource constraint of the economy. In addition, labour market equilibrium conditions (that hours worked at the public and private sector add up to total labour supply) must hold. We concentrate here on the 'normal' case where redistribution occurs from the skilled workers to the unskilled ones. Thus the self-selection constraint of the skilled workers is binding. The Lagrangean of the government optimisation problem can therefore be written as follows:

³ See Stiglitz (1982) or Stern (1982) for details.

$$\begin{aligned}
L = & v(x_1, l_1, g) + \delta [v(x_2, l_2, g) - \bar{v}^2] \\
& + \lambda [v(x_2, l_2, g) - v(x_1, \Omega l_1, g)] \\
& + \rho_x [F(l_1^c, l_2^c, g) - x_1 - x_2] + \rho_g [G(l_1^g, l_2^g) - \phi g] \\
& + \alpha_1 [l_1 - l_1^c - l_1^g] + \alpha_2 [l_2 - l_2^c - l_2^g]
\end{aligned} \tag{1}$$

where ϕ is the number of goods provided: in the pure public good case, $\phi = 1$, and in the case of a publicly provided private goods, $\phi = 2$, since the number of households is 2. The first-order conditions are the following:

$$x_1 : v_x^1 - \lambda \hat{v}_x^2 - \rho_x = 0, \tag{2}$$

$$l_1 : v_l^1 - \lambda \hat{v}_l^2 \Omega + \alpha_1 = 0, \tag{3}$$

$$x_2 : (\delta + \lambda) v_x^2 - \rho_x = 0 \tag{4}$$

$$l_2 : (\delta + \lambda) v_l^2 + \alpha_2 = 0, \tag{5}$$

$$l_i^c : \rho_x F_i - \lambda \hat{v}_i^2 \frac{\partial \Omega}{\partial l_i} l_i - \alpha_i = 0, i = 1, 2 \tag{6}$$

$$l_i^g : \rho_g G_i - \alpha_i = 0, i = 1, 2 \tag{7}$$

where the hat terms refer to the so-called mimickers, i.e. type 2 workers when mimicking the choice of type 1.

We first note that standard results related to optimal tax structure hold. Combination of (4) and (5) and substitution for α from (6) can be used to show that the marginal tax rate for the high-skilled household is negative, as shown originally in this context with endogenous wages by Stern (1982) and Stiglitz (1982). The intuition is that by encouraging the work effort of the high-skilled, high-skilled labour becomes more abundant in the economy, giving rise to an increase of the relative wage of the low-skilled households. In a similar fashion, equations (2), (3) and (6) can be used to derive the standard result that the marginal tax rate for the low-skilled type is positive.

3. Public production and provision

The aim in this section is to examine the optimal choice of different skill levels for public production and the overall optimal provision rule for the publicly provided good. Suppose first that the government has chosen to produce a certain amount of consumption, g . Given this, suppose further that the government's income tax policy is optimal (following the way described above). Using (6) and (7) for the two skill types yields

$$\frac{w_1}{w_2} = \frac{\alpha_1 + \lambda \hat{v}_1^2 \frac{\partial \Omega}{\partial l_1} l_1}{\alpha_2 + \lambda \hat{v}_2^2 \frac{\partial \Omega}{\partial l_2} l_2} \leq \frac{\alpha_1}{\alpha_2} = \frac{\tilde{w}_1}{\tilde{w}_2} \quad (8)$$

From equation (8) we see that only in the case where the relative wage, Ω , is independent of labour supply, production efficiency holds i.e. $G_1/G_2 = F_1/F_2$. But we also note that in the general case the term $\lambda \hat{v}_1^2 \frac{\partial \Omega}{\partial l_1} l_1$ is negative and $\lambda \hat{v}_2^2 \frac{\partial \Omega}{\partial l_2} l_2$ positive. Thus, in the general the Diamond-Mirrlees (1971) efficiency theorem does not hold in this model. Given our assumptions about the public production function, equation (8) implies that the following result holds.

Proposition 1: To produce a given amount of consumption, the government should employ more unskilled workers and less skilled workers than is necessary to minimize cost at the prevailing gross wage rates.

This means that if the supply of low skilled workers becomes scarcer in the private sector, through hiring more of these workers into the public sector, the government can reduce the wage differentials of the workers. Thus indirect redistribution through public sector employment will Pareto-improve welfare by mitigating the incentive problem of the non-linear income tax system. Or, to put it in terms of envelope arguments: If in the beginning the production efficiency holds, then the marginal change in hiring more low skilled workers to the public sector has no first order welfare costs. It affects only relative wage of the low skilled workers.

Given the optimal income tax and employment policy, we may also use the envelope argument to detect the change in the social welfare from an increase in the level of the publicly provided good as follows:

$$\frac{dL}{dg} = v_g^1 + (\delta + \lambda)v_g^2 - \lambda\hat{v}_g^2 + \rho_x F_g - \lambda\hat{v}_l^2 \frac{d\Omega}{dg} l_1 - \phi\rho_g. \quad (9)$$

Our focus is more in the production side of the economy, and therefore we concentrate on the case with weakly separable (between consumption and labour (or leisure)) utility function. Rewriting (9) by substituting for ρ_x from (2) and (4) yields

$$\frac{dL}{dg} = \rho_x F_g - \lambda\hat{v}_l^2 \frac{d\Omega}{dg} l_1. \quad (10)$$

What is interesting in (10) is the link between the publicly provided good and the wage structure of the economy (the term $-\lambda\hat{v}_l^2 \frac{d\Omega}{dg} l_1$). If its provision leads to a relative increase in the wage rate for type 1 workers, then indirect redistribution through public provision will Pareto-improve welfare by mitigating the incentive problem of the non-linear income tax system.

Combining this result with the public employment decision says that the government can improve the relative position (their relative wage rate) of the low-skilled households both by public employment and by providing certain type of public or private goods⁴. Such goods can include e.g. public schooling, active labour market policies (training etc.) and deterring social exclusion. This may explain, for instance, why the Nordic countries have found it desirable to use a policy *package* of large-scale public provision combined with substantial public employment.

Our results complement the analysis of Naito (1999) that if wage rates are endogenous, redistribution devices that otherwise would not be applied – in Naito's case production inefficiency in public production and commodity taxation and in our case public sector employment combined with public provision of private goods – become welfare-improving.⁵

⁴ Note that it is not crucial whether provided good are public or private; what counts is impacts on the wage-earning capabilities of the low-skilled.

⁵ Gaube (2000) extends Naito's analysis and shows how production efficiency is violated not only in special cases, but in all potential regimes of second-best, two-sector, optimal taxation models.

4 The potential role of minimum wage and public sector employment

Guesnerie and Roberts (1984) and Allen (1987) assumed that the effect of minimum wage was to cause underemployment (reduced hours of work) of low skilled workers and that underemployment is equally shared by low income workers. They show how the imposition of a minimum wage can reduce tax-induced distortions in the case of linear income taxation. Allen (1987) shows that minimum wage is not desirable when the government redistributes income under non-linear income taxation from high income workers to low income workers. In our framework we can see this by setting $\rho_g = 0$ (no public sector). Now if underemployment due to the minimum wage is shared equally by all low income workers it is obvious that the minimum wage does not relax the self-selection constraint of the skilled worker. Namely a high income worker is still free to work less and earn the same income as a low income worker.

We relax both these assumptions. We have in our model two sectors and some low income workers may end up unemployed due to minimum wage. Now suppose the government imposes a minimum wage w_m ($F_l \geq w_m$). The government has a new control variable w_m in addition to the old ones (non-linear income taxation, public employment and public provision) and a new constraint $w_l \geq w_m$. So the minimum wage does not restrict labour supply of low income workers directly, but it raises the wage. The public sector hires more low skilled workers reducing the number of unemployed due to minimum wage in the private sector. If the public sector provides employment for all those low skilled workers who due to minimum wage face unemployment we have the same outcome as in the case we considered in the previous section. By employing more low income workers at the minimum wage than the private sector would, the public sector does not minimize cost, but the government can offset this efficiency loss by distributional gains.

5 Public investment policy

In this section, we extend the previous static analysis to an overlapping generations setting in order to characterise rules governing optimal capital allocation between the public and private sector. In particular, the interest is in the discount rate that ought to be used when assessing the accumulation of public capital.

We consider an economy where individual households live two periods, supplying labour at the first and consuming at the second period.⁶ We assume for simplicity that there is no population growth. Each generation consists of two households whose preferences and optimisation problem are similar to those in the static framework. We apply the following notation: c_i^t denotes the consumption of a household of type i born at time t , x_i^t the after-tax income of the same household, L_i^t household i 's labour supply, g^t denotes the publicly provided good, z_i^t refers to the potential private purchases of publicly provided good, and r is the interest rate. The household's utility function is thus $v = v(x^t, L^t, g^t)$ and its (intertemporal) budget constraint $z^t + \frac{1}{1+r_{t+1}}c^t = x^t$. Since the household consumes only at the second period, its savings are equal to the net income, x_i^t .

The production side of the economy consists again of two sectors. The production function at the private sector, $F(k^t, l_1^t, l_2^t, g^t)$, exhibits constant returns to scale. Private capital is denoted by k^t . The wage ratio between type 1 and type 2 wages, Ω^t , is now a function of l_1^t/l_2^t , g^t and k^t . The publicly provided good is produced using the production function $G(\kappa^t, m_1^t, m_2^t)$, where κ^t , m_1^t and m_2^t denote public capital, work at the public sector by type 1 household and type 2 household, respectively. Similarly to the static version, the government minimises costs of production at the public sector production subject to the shadow factor prices. The labour market equilibrium condition is $L_i^t = l_i^t + m_i^t$ for both types, 1 and 2.

We also need equations that determine production at the private and public sector. Production at the private sector must be sufficient for private consumption by the old living at the period as well as for investments. We assume that savings are provided by the private sector, and thus investments cover both investments at the public and the private sector. Therefore, private capital accumulation is determined by

$$F(k^t, l_1^t, l_2^t, g^t) = x_1^{t-1} + x_2^{t-1} + k^{t+1} - k^t + \kappa^{t+1} - \kappa^t. \quad (11)$$

Production at the public sector is just equal to public provision:

⁶ This section builds on the framework developed by Brett (1997) and Pirttilä and Tuomala (2001). We have dropped the first period consumption in order to keep the framework as similar as possible to the previous sections.

$$G(\kappa^t, m_1^t, m_2^t) = \phi g^t, \quad (12)$$

The government's problem is to maximise intertemporal social welfare that includes welfare comparison both within and between generations. We continue to assume that within generation, the government maximises the utility of type 1 household subject to a given utility to type 2 household. Across the generations it is perhaps easiest to assume that the government is utilitarian, but the utilities of future cohorts are discounted with a discount factor β^t . As above the government chooses the optimal tax schedules, optimal employment at the public sector, and the optimal level of public provision each generation, subject to the self-selection constraint of the skilled household, labour market equilibrium conditions and the production and capital market equilibrium conditions, given in (11) - (12).

The Lagrangean of this optimisation problem is written as follows:

$$\begin{aligned} L = & \sum_{t=1}^{\infty} \beta^t v(x_1^t, L_1^t, g^t) + \sum_{t=1}^{\infty} \delta^t [v(x_2^t, L_2^t, g^t) - \bar{v}_2^t] \\ & + \sum_{t=1}^{\infty} \lambda^t [v(x_2^t, L_2^t, g^t) - v(x_1^t, \Omega^t L_1^t, g^t)] \\ & + \sum_{t=1}^{\infty} \rho_x^t [F(k^t, l_1^t, l_2^t, g^t) - x_1^{t-1} - x_2^{t-1} + k^t - k^{t+1} + \kappa^t - \kappa^{t+1}] \\ & + \sum_{t=1}^{\infty} \rho_g^t [G(\kappa^t, m_1^t, m_2^t, g^t) - \phi g^t] \\ & + \sum_{t=1}^{\infty} \alpha_1^t [L_1^t - l_1^t - m_1^t] + \sum_{t=1}^{\infty} \alpha_2^t [L_2^t - l_2^t - m_2^t] \end{aligned} \quad (13)$$

The first-order conditions at an exemplary date t are as follows:

$$x_1^t : \beta^t v_{1,x}^t - \lambda^t \hat{v}_{2,x}^t - \rho_x^{t+1} = 0, \quad (14)$$

$$L_1^t : \beta^t v_{1,L}^t - \lambda^t \hat{v}_{2,L}^t \Omega^t + \alpha_1^t = 0, \quad (15)$$

$$x_2^t : (\delta^t + \lambda^t) v_{2,x}^t - \rho_x^{t+1} = 0, \quad (16)$$

$$L_2^t : (\delta^t + \lambda^t) v_{2,L}^t + \alpha_2^t = 0, \quad (17)$$

$$l_i^t : \rho_x^t F_i^t - \lambda \hat{v}_{2,L}^t \frac{\partial \Omega^t}{\partial l_i^t} L_1^t - \alpha_i^t = 0, i = 1, 2 \quad (18)$$

$$m_i^t : \rho_g^t G_i^t - \alpha_i^t = 0, i = 1, 2 \quad (19)$$

$$k^t : \rho_x^t F_k^t + \rho_x^t - \rho_x^{t-1} - \lambda^t \hat{v}_{2,L}^t \frac{\partial \Omega^t}{\partial k^t} L_1^t = 0, \quad (20)$$

$$\kappa^t : \rho_g^t F_\kappa^t + \rho_x^t - \rho_x^{t-1} = 0, \quad (21)$$

$$g^t : \beta^t v_{1,g}^t + (\delta^t + \lambda^t) v_{2,g}^t - \lambda \hat{v}_{2,g}^t + \rho_g^t F_g^t - \lambda \hat{v}_{2,L}^t \frac{d\Omega^t}{dg^t} L_1^t - \phi \rho_g^t, \quad (22)$$

We first note that results from the static framework carry over to the dynamic version of the model, as well. For the tax structure, (16) and (17), and (14) and (15), determine the marginal tax rates for the high-skilled and low-skilled households, respectively. Comparing (18) and (19) shows that production efficiency in the employment decision is violated so that the government favours employment of the low-skilled labour at the public sector. Finally, (22) is similar to the provision rule of the publicly provided good of the static version.

Our interest in this section is in the interest rate rules governing capital accumulation in the private and public sector. Equilibrium at the private sector requires that the interest rate is set equal to marginal productivity of capital, thus $F_k^t = r^t$. We denote the shadow interest rate, applied at the public sector by \tilde{r}^t and set $\tilde{r}^t = G_\kappa^t$. Combining (20) and (21) then yields

$$\rho_x^t r^t - \lambda^t \hat{v}_{2,L}^t \frac{\partial \Omega^t}{\partial k^t} L_1^t = \rho_g^t \tilde{r}^t. \quad (23)$$

Comparison is easiest in the case where the marginal value of private and public good are equal, and therefore $\rho_k^t = \rho_g^t$. In this case, (23) can be rewritten as follows:

$$r^t - \frac{\lambda^t \hat{v}_{2,L}^t}{\rho_x^t} \frac{\partial \Omega^t}{\partial k^t} L_1^t = \tilde{r}^t. \quad (24)$$

Equation (24) reveals that production efficiency in capital accumulation is violated if $\frac{\partial \Omega^t}{\partial k^t} \neq 0$, i.e.

if capital affects the wage ration between the low-skilled and the high-skilled labour at the private sector. The direction of the distortion depends on how the relative wage rate is affected. For

example, if $\frac{\partial \Omega'}{\partial k'} < 0$, the second term at the left is negative and hence $r' > \tilde{r}'$. This gives rise to the following proposition:

Proposition 2: Production efficiency in capital accumulation between the private and public sector is violated if the relative wage between low-skilled and high-skilled labour depends on the level of the private capital stock. The interest rate applied in determining public sector capital accumulation is smaller (higher) than the interest rate in the private sector if high-skilled labour and capital are complements (substitutes) at the private sector.

It is important to note that production efficiency only holds in a special case; in a general case where capital can be more or less complementary with high-skilled labour than low-skilled labour, deviations from production efficiency enhance welfare. The reason is, for instance in the case where private sector capital and high-skilled labour are complements, that by encouraging public sector capital accumulation with the use of an interest rate lower than the market interest rate, less capital accumulation occurs in the private sector, and therefore the relative labour market position and the relative wage rate of the low-skilled households improve. Because of the increase in the wage rate, there is less need for (distortionary) redistributive taxation, and overall distortions in the economy can be reduced.

6 Conclusion

In this study we analyse the role of public production and capital allocation between private and public sector as potential redistributive devices when non-linear taxation is available. Using a two-type and two-sector optimal non-linear income tax model with endogenous wages, we show that to produce a given amount of consumption the government should employ more unskilled workers and less skilled workers than is necessary to minimize costs at the prevailing gross wage rate. This policy reduces the relative supply of low-skilled labour at the private job market and contributes to higher market wages for the low skilled. In these circumstances the pressure to redistribute income using distortionary income taxes is reduced.

On the other hand, the reasons why public provision is desirable include the case where public provision increases the income-earning capacity of the low-skilled and thereby improves their

labour market positions. The rationale for both public provision and public production stems from the labour market considerations for the low skilled and may partly explain the combination of these policies in some countries, such as the Nordic welfare states. We also discussed the role of minimum wages with simultaneous public production. It appears that government employment may offset some of the harmful labour-market impacts of minimum wages by public employment of otherwise unemployed low-skilled workers.

With respect to capital allocation, the analysis based on an OLG framework shows that the discount rate that ought to be used in evaluating public sector projects is not the same as the return to capital in the private sector. In particular, public capital accumulation is favoured if capital in the private sector is complementary with high-skilled labour. Therefore, production efficiency holds neither in public employment decision nor capital allocation.

While the direction to which deviations in the public sector employment decisions should be taken (towards favouring low-skilled employment at the public sector) was clear, the direction of deviation in the capital accumulation can go either way. Whether the interest rate applied in evaluation of public capital accumulation should be lower or higher than the market interest rate is in the end an empirical question that depends on the production technology. In the current debate about the future of the welfare state, the case where capital accumulation and technological progress favours high-skilled labour has received a lot of attention (see e.g. the discussion in Atkinson 2000). Krusell et al (2000) present evidence, based on US data from 1963 to 1992, that skilled labour has been more complementary with capital than low-skilled labour. If indeed the relative worsening of the labour market position of low-skilled labour is a problem, part of the solution can involve an 'over-investment' on public projects / capital to improve their employment and income earning possibilities.

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