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# Optimization of City Size

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**Abstract:** *Club theoretic analysis of migration between asymmetric cities shows that centralized policy intervention is necessary to ensure the efficient allocation of people between cities. Quantity rationing and equalizing lump-sum tax-transfers are compared as policy instruments of central government. These instruments are found to differ in their effects on residential allocation and welfare. This is because lump-sum tax/transfers pool the welfare-creating potentials of cities thus affecting the efficiency condition. Therefore, lump-sum tax/transfers are superior to quantity rationing, and they also activate rather than stabilize migration.*

*Key words: agglomeration economies, club theory, lump-sum taxes and transfers*  
*JEL Classification: H77, R51*

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# 1 Introduction

It is quite indisputable that urban surroundings offer the most fertile soil for the modern market economy. Even in the present stage of technological and social development spatial proximity still is vital in lowering transaction costs and in creating externalities that have direct and indirect effects on people's welfare. City size is also important, because increasing size yields agglomeration economies, which eventually turn into agglomeration diseconomies with consequent effects on welfare.

Cities being so central in the market economy, it should be natural that the market would also be able to steer the formation of cities efficiently. Besides firms' location decisions, the mobility of people is the main private element in the formation of cities. However, since migration cannot alone lead to optimal city size, cities as collectives of their residents must complement the market mechanism in that respect (Laurila, 2004).

The economic intuition of the Tiebout type equilibrium is that the competitive forces imposed by migration make cities adopt the best practices, just as highly competitive firms are compelled to achieve technical efficiency. Therefore, the cities become homogenous for homogenous residents like firms begin to apply identical production technologies in the same industries. Differences exist between industries and, likewise, between cities with residents having different preference patterns. In spite of these differences, free migration should ensure that the experienced welfare levels are equal everywhere.

It is commonly understood that centralized policy intervention over cities is needed only if there are notable inter-city spillovers. This paper aims to show, however, that centralized policy intervention

may be necessary to secure the efficiency of the city system even without spillovers. This happens when there are profound differences between cities that constraint their welfare creation potentials.

In practice, considerable differences in cities' capability to create welfare exist because of variation in geography and climate, natural resources, industrial structure, national infrastructure and networks, administrative hierarchy, transport and trade connections etc. These mostly exogenous circumstances create absolute and comparative advantages that explain migration and affect the interregional allocation of people in the same manner as they affect the pattern of trade.

This paper builds on the paper by Laurila (2004), which shows that cities' competition for residents improves technical efficiency in the economy but leaves open the question about long-run allocative efficiency. Section 2 of the paper sets the club theoretic model of systematic migration of people between two cities that are asymmetric in their long-term capability to create welfare. Section 3 investigates the need and effects of centralized policy by comparing administrative and economic instruments. Section 4 derives the optimal policy rule. Section 5 concludes the findings.

## **2 Migration between asymmetric cities**

The main economic rationale of migration arises from the fact that people's experiences of welfare are highly place dependent. Utility maximizing people thus continuously seek those places that they expect to serve them best in terms of welfare, composed of the net sum of private benefits and costs experienced in everyday life. Agglomeration economies and diseconomies affect both the benefits and the costs so that the welfare depends on city size.

The welfare that a city can provide to its individual residents can be approximated by average welfare, that is total welfare divided by the number of residents. This simplification is possible,

when commuters are excluded, or net commutation is assumed zero. Otherwise, the calculation of average welfare would underestimate it depending on if more people commute into the city than out of it, and vice versa. The assumption is practical also because only the residents can use voice in collective decision-making.

The theoretical concept of average welfare is empirically observable from the pattern of systematic migration: systematic emigration from a city simply indicates lower expected average welfare than elsewhere and vice versa. It is also assumed that the expectations are realized rapidly so that people respond to possible overestimations by instant relocation so that the average welfare measures the actual, not the expected level.<sup>1</sup> While systematic migration reflects the choices of representative individuals, the decisions of the non-representative households fall into the purely stochastic category of the migration pattern.

Figure 1 below illustrates the economy consisting of two cities, city A and city B. The total population in the economy is fixed at  $n$ , measured by the length of the horizontal axis. The population in city A,  $n_A$ , is measured rightwards from  $O_A$  and that in city B,  $n_B$ , leftwards from  $O_B$  so that  $n_A + n_B = n$ .

(Figure 1 here)

In Figure 1, the  $AW$  curves present the size-dependent average welfares in the two cities reflecting the exogenously constrained long run potentials not able to be improved by local policies. The curves are superimposed on each other so that the average welfare in city A denoted by  $AW_A$  is

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<sup>1</sup> Welfare is here treated simply in a static sense. In real life, welfare may be more of dynamic nature, because a large part of it is relative (i.e. one's income relative to other people's incomes) and adaptive (i.e. in respect to increasing expectations of service levels and quality).

drawn from left to right and that in B denoted by  $AW_B$  is drawn from right to left. The curves are of inverse U shape because agglomeration economies dominate at earlier stages of city growth, but agglomeration diseconomies start to dominate at the culmination points of the curves. Assuming that city A has an absolute/comparative advantage in generating welfare, the  $AW_A$  curve reaches a higher top value than the  $AW_B$  curve. The  $MW$  schedules present the marginal effects of migration to individual welfare, and they dissect the respective  $AW$  curves from above at their top points.

In Figure 1, the  $AW$  curves are drawn to intersect twice to present a rich set of possible solutions. Start from point  $a$ , where  $n_A = n^a$ ,  $n_B = n - n^a$ , and the average welfare is equal in both cities,  $AW_A^a = AW_B^a$ . With this allocation of population, agglomeration economies dominate in the small but potentially prosperous city A while the diseconomies dominate in the bigger but less prosperous city B. Suppose that a stochastic movement occurs from A to B so that the allocation of population changes to the left from point  $a$ . On impact, a welfare gap opens in favor of B thus motivating systematic migration from A to B until A becomes totally deserted. If the initial stochastic movement occurs from B to A thus shifting the allocation rightwards from  $a$ , the welfare gap opens in favor of A, and systematic migration starts to draw the solution further to the right. Therefore, the solution in point  $a$  is not stable and it thus cannot be a market outcome of migration only.

In Figure 1, systematic migration left from point  $a$  leads to a corner solution and to a fall in social welfare. The path rightwards from  $a$  seems more promising, and along this path there are several interesting positions worthy of closer analysis. First, at the point  $b$ , migration from B to A makes  $AW_A$  reach its maximum with  $n_A = n^b$ ,  $n_B = n - n^b$ . Supposing that the current residents of city A collectively optimize on size by the within-club rule (Ng, 1973; Cornes & Sandler, 1999), the solution becomes stable. In practice, an in-migration city is well able to stop immigration by

planning, dimensioning of service provision and social housing, public transport etc. thus making the welfare gap  $AW_A^b - AW_B^b$  sustainable. The welfare in the whole economy is measured by

$$(1) \quad W^b = n^b AW_A^b + (n-n^b) AW_B^b.$$

Second, if city A does not, for some reason, implement the optimizing policy, systematic migration continues beyond point  $b$  in response to the existing welfare gap. From the point of view of city B, continuing migration eventually leads to its welfare maximum at point  $c$ . However, city B lacks ample instruments to stabilize the situation – it is much more difficult to stop emigration than immigration by local acts. Therefore, migration will evidently continue beyond the point  $c$ .

Third, there is the second intersection point  $d$ , where  $n_A = n^d$ ,  $n_B = n-n^d$ , and  $AW_A^d = AW_B^d$ . Systematic migration stops here and the solution is stable: any stochastic movement left or right from point  $d$  triggers systematic migration drawing the solution back to  $d$ . Total welfare amounts to

$$(2) \quad W^d = n^d AW_A^d + (n-n^d) AW_B^d = n AW_A^d.$$

However, neither of the possible solutions (1) and (2) is optimal from the point of view of the whole economy. The socially optimal solution is at point  $e$ , where the  $MW$  curves intersect reflecting the economy-wide optimal mix of agglomeration economies utilized in city B and agglomeration diseconomies suffered in city A. The optimal population allocation is  $n_A = n^e$ ,  $n_B = n-n^e$ , saying that a welfare gap  $AW_A^e > AW_B^e$  exists. Social welfare is measured by

$$(3) \quad W^e = n^e AW_A^e + (n-n^e) AW_B^e.$$

The social optimum (3) is superior to (1) by the welfare gain measured by the area  $efb$ , and superior to (2) by the area  $egh$  in Figure 1. The welfares in (1) and (2) cannot be compared with each other. The conclusion is that because the socially optimal solution is reached neither by migration alone nor by migration complemented by local policy-making, and particularly because the optimum involves a welfare gap, it is obvious that centralized policy is needed in the economy.

### 3 Policy considerations

Microeconomic policy instruments are usually divided to administrative and economic measures. Administrative measures concern quantitative terms thus operating along the horizontal axis while economic instruments concern monetary issues thus operating along the vertical axis in the setting of Figure 1. The conventional wisdom is that the effects should be equal so that the market solution is affected symmetrically by either instrument.

In making the optimal allocation  $n_A=n^e$ ,  $n_B=n-n^e$  sustainable in Figure 1, administrative quantity rationing can be tried by rules, standards, constraints and other such regulative tools. Practical implementation of the policy means that some people are forced to move so that the optimal allocation is reached irrespective of the welfare gap  $AW_A^e > AW_B^e$ . This is feasible if the central government is able to rule out the local policy of city size optimization (in the regime left from point  $e$  in Figure 1), to persuade people to move against their rationale (in the regime right from point  $e$  in the Figure), and to stabilize the situation by fixing the allocation to  $n_A=n^e$ ,  $n_B=n-n^e$ .

There is good reason, however, to doubt the feasibility of administrative rationing. Use of force fits badly into the spirit of free market economy and causes considerable enforcing costs. The fact that the policy divides people into better-off and worse-off groups living in more and less prosperous

cities violates the highly esteemed principles of residential liberty and regional equity. These principles have a constitutional status, for example in Finland.

The economic instruments are more usable in the present context because they aim rather to steer (by *carrot*) than force (by *stick*) people's rational choices. This means affecting local welfares by taxes, subsidies etc. in order to manipulate systematic migration and thus the spatial allocation of population. Under this kind of policy, the welfare equalizing migration solutions are sustainable without further enforcement. Figure 2 illustrates the effects of such policy.

(Figure 2 here)

In Figure 2 it is assumed that free migration has driven the solution to point *d*. For a simple benchmark, assume that the central government is able to calculate the welfare gap at the optimal allocation  $n_A=n^e$ ,  $n_B=n-n^e$ . In order to drive the migration solution to the efficient allocation, it then suffices to levy a non-distorting lump sum tax on the residents of city A so that

$$(4) \quad T = AW_A - AW_B$$

with  $T=T^e$ ,  $AW_A=AW_A^e$ ,  $AW_B=AW_B^e$ . The tax moves the  $AW_A$  curve downwards by the amount  $T^e$  at every population allocation thus yielding the  $AW_A^T$  curve in the Figure. On impact, a welfare gap equal to  $T^e$  is opened thus causing systematic migration from city A to city B. Migration continues until the intersection of the curves  $AW_A^T$  and  $AW_B$  is reached at point  $e^T$ . This stable solution is also efficient, because social welfare, including the tax revenue  $n^e T^e$ , equals  $W^e$  of equation (3).

The above analysis leaves some open questions, however. One concerns the tax revenue, which remains in the pocket of the central government thus contributing to social welfare only very implicitly. Another critical point is that the government is supposed to be omniscient in setting its policy, which is seldom the case. Third, the active role of local policy-making is ignored. Therefore, a more realistic exercise is worth trying. Figure 3 illustrates some elaboration.

(Figure 3 here)

In Figure 3 it is assumed that the only thing that the central government must be aware of is that city A is too big compared to B. Ignoring the concept of the optimal tax  $T^e$ , the government can impose a modest lump-sum tax  $t$  on the residents of city A. Furthermore, to make the welfare measure of the tax revenue explicit, assume that the government returns the tax revenue as a lump-sum transfer  $s$  to the residents of city B. Denoting the welfare gap by  $T$ , and assuming no implementation costs

$$(5) \quad t = T/2 = s.$$

Assuming that taxes and transfers appear as mirror images in people's budget constraints, the average welfare schedule shifts downwards in A by  $t$  to  $AW_A^t$  and upwards in B by  $s$  to  $AW_B^s$  so that the budget constraint (5) holds at any allocation of population. On impact, a welfare gap is opened, and systematic emigration from A to B continues until the point  $d'$  is reached. Since the population allocation still is beyond the optimum, taxes and transfers must be increased. The iterating process continues until the solution  $d^e$  and the respective optimal population allocation is reached so that  $t^e = T^e/2 = s^e$ .

In Figure 3, the solution  $d^e$  brought up by policy-steered migration is stable. Furthermore,  $W^{d^*} = W^e$ , because the present policy simply splits  $T^e$  evenly between the people in both cities. Therefore, the solution should be efficient, too. However, this aspect deserves closed attention.

## 4 Optimal policy

The efficiency of the above policy-induced solution is reconsidered in Figure 4, which clearly indicates that Pareto superior solutions are attainable by expanding the scale of the policy.

(Figure 4 here)

If the tax-transfers are increased from  $t^e = T^e/2 = s^e$ , the migration solutions move leftwards from  $d^e$  along a rising path in Figure 4. The incremental increases of tax-transfers form the full solution path  $AW^e$  in the Figure, along which the policy equalizes welfare in the cities thus resulting in a joint average welfare curve just between the original  $AW_A$  and  $AW_B$  curves. Clearly, points along the rising part of the curve left from  $d^e$  are beneficial to both A and B and are therefore Pareto superior.

The optimal population allocation under the lump-sum tax-transfer policy can be found by maximizing  $AW^e = AW_A - t$  and recalling (4) and (5). The optimality condition yields

$$(7) \quad dAW_A/dn_A = -dAW_B/dn_A$$

saying that the optimal point  $\varepsilon$  is in the culmination point of  $AW^e$ , and  $t^*=T^*/2=s^*$ . The average welfares in both cities are maximized at the same time. Under the policy, the marginal welfare curves in city A and city B shift, too. At the optimum, the curves  $MW_A^e$  and  $MW_B^e$  intersect at the top of the joint average welfare curve  $AW^e$ .

The point  $\varepsilon$  can be reached by incremental increases in tax-transfers. At the top point, the solution is stable: migration draws towards  $\varepsilon$  from both sides of it in Figure 4. However, the solutions would be stable also beyond  $\varepsilon$  up to the point  $E$ , in which the scale of the tax-transfer policy is maximized ( $T^{max}$ ), and which divides the solution path to stable (right from  $E$ ) and unstable (left from  $E$ ) sections. Both features arise from the maximization of (4), which yields

$$(8) \quad dAW_A/dn_A = dAW_B/dn_A$$

saying that relative slopes of  $AW_A$  and  $AW_B$  change at  $E$ . The stable solution could thus be steered by central policy all the way from  $d$  to  $E$  along  $AW^\varepsilon$ .

How then is the socially efficient equilibrium point  $\varepsilon$  reached? The aspect of local optimization on city size now enters into the picture. Since the population allocation  $n^\varepsilon$  is optimal for both cities, they would thus fix that once achieved allocation by local policy if possible. In particular, if the government should increase taxes from  $T^*$ , city B would close doors for further immigration, because the tax would put them on the falling regime of  $AW_B$  again. At  $n^\varepsilon$  this would also violate the government's budget constraint thus making the expansion of the policy impossible. The local optimizing behavior can be taken as signal for the central government to stop the policy iteration.

In Figure 4, the new optimum  $\varepsilon$  clearly is to the left from  $e$ . This is because the optimality condition given by the intersection of the  $MW$  curves is affected by the tax-transfer policy, and city A should be smaller and city B bigger than without policy. At the optimal allocation of people,  $n_A=n^\varepsilon$  and  $n_B=1-n^\varepsilon$ , both cities are of optimal size and the social welfare is

$$(9) \quad W^e = n^e A W_A^e + (n - n^e) A W_B^e.$$

In particular,  $W^e = n A W_A^e > n A W_A^{d^*} = W^e$ . This is because the equalizing lump-sum tax-transfer policy in effect redistributes the absolute advantages so that the existing resources and other preconditions come to be utilized efficiently in the economy. The reshaping of the  $AW$  curves means that agglomeration economies are boosted and diseconomies depressed in the economy.

## 5 Conclusions

When cities are asymmetric in their capacity to offer welfare, an efficient market solution of population allocation is achievable neither by migration alone nor by migration complemented by collective optimization by local policy. The first conclusion of the paper is that centralized policy intervention may be necessary even if there are no inter-city spillovers.

Two instruments of centralized policy are studied, namely administrative measures in the form of quantity rationing and economic measures in the form of equalizing lump-sum transfers. It is shown that both measures can produce a superior outcome to that yielded by migration alone or by migration complemented by local level policy-making.

The use of administrative policy measures implies that the efficient allocation of people is maintained in spite of the existing welfare differentials between cities. The principle of regional equity is thus violated. Furthermore, these measures may also be difficult to implement in a market economy because they necessitate constraints on free mobility.

Lump-sum transfers from more prosperous to less prosperous cities can be used to equalize the welfare differentials thus making the free migration solution sustainable. A surprising result of the

paper is that the optimal allocation of population yielded by lump-sum transfers differs from that of the administrative alternative. Moreover, lump-sum transfers also tends towards higher social welfare than administrative quantity rationing. This is because the policy effectively pools together the welfare potentials of the cities thus changing the efficient residential pattern from that without the tax-transfers.

To sum up, the results of this paper contradict the conventional wisdom that administrative and economic measures should have symmetric effects. It can also be concluded that, contrary to the conventional wisdom, the use of equalizing lump-sum transfers activates, rather than stabilizes, migration of people.

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Figure 1: Possible population allocations

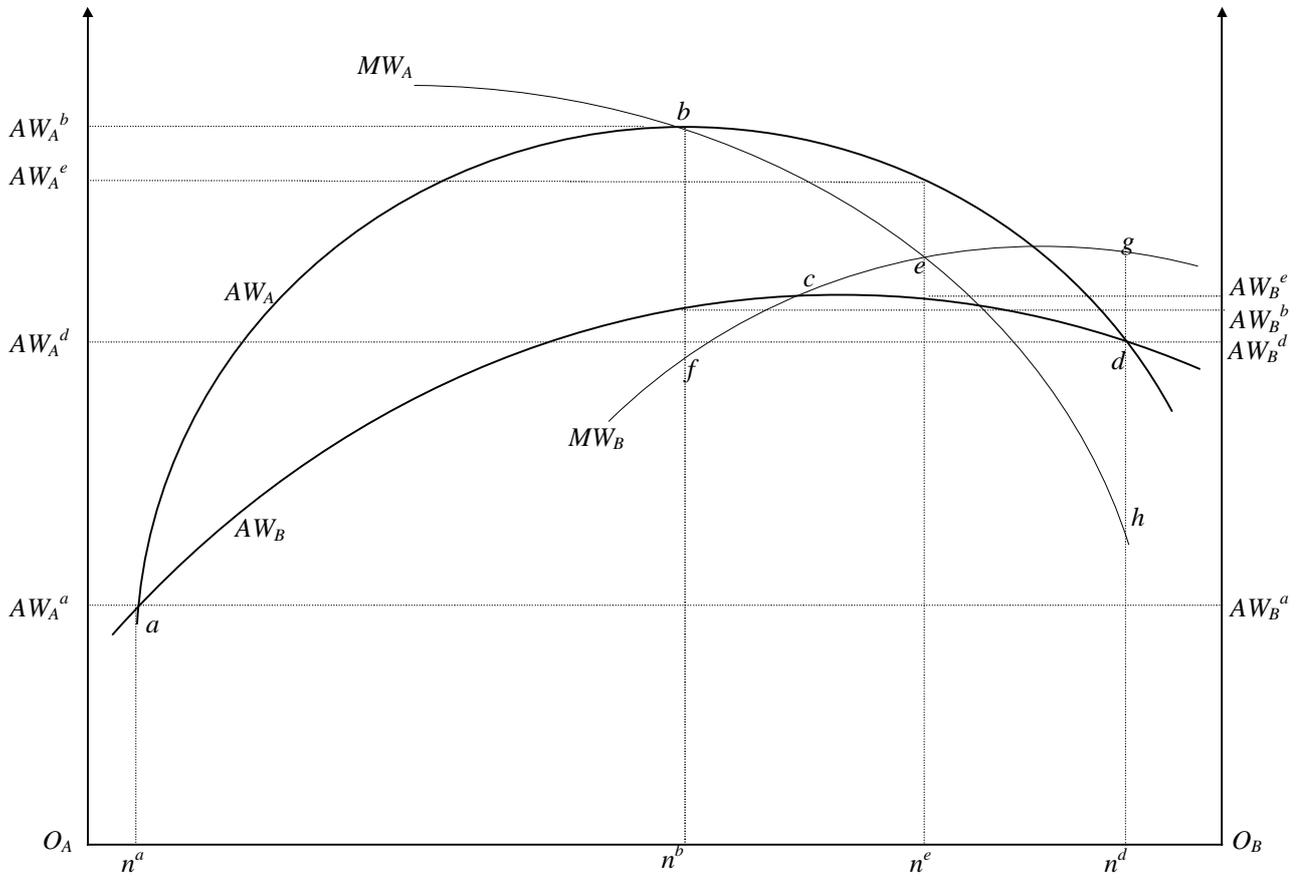


Figure 2: Lump-sum taxes and migration

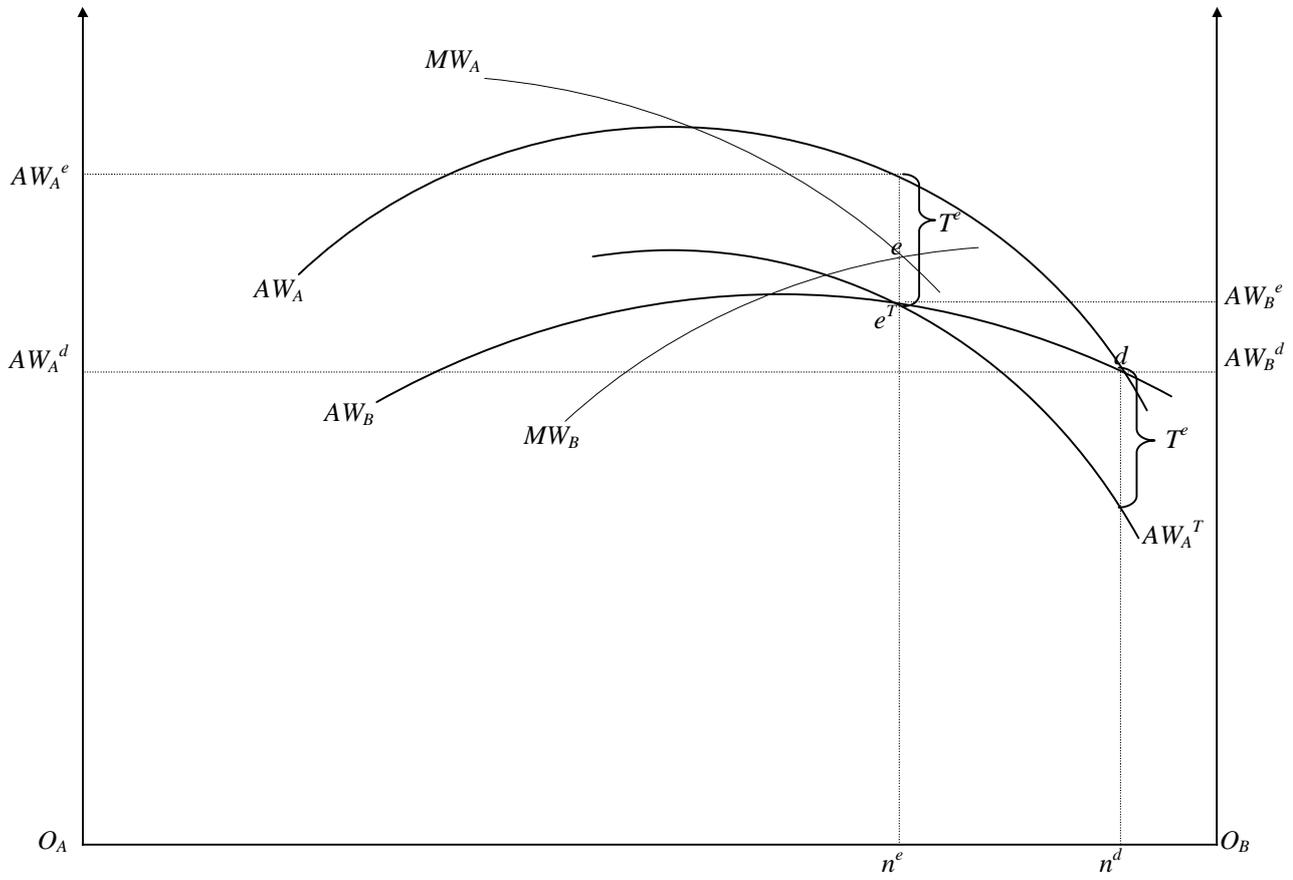


Figure 3: Taxes and transfers

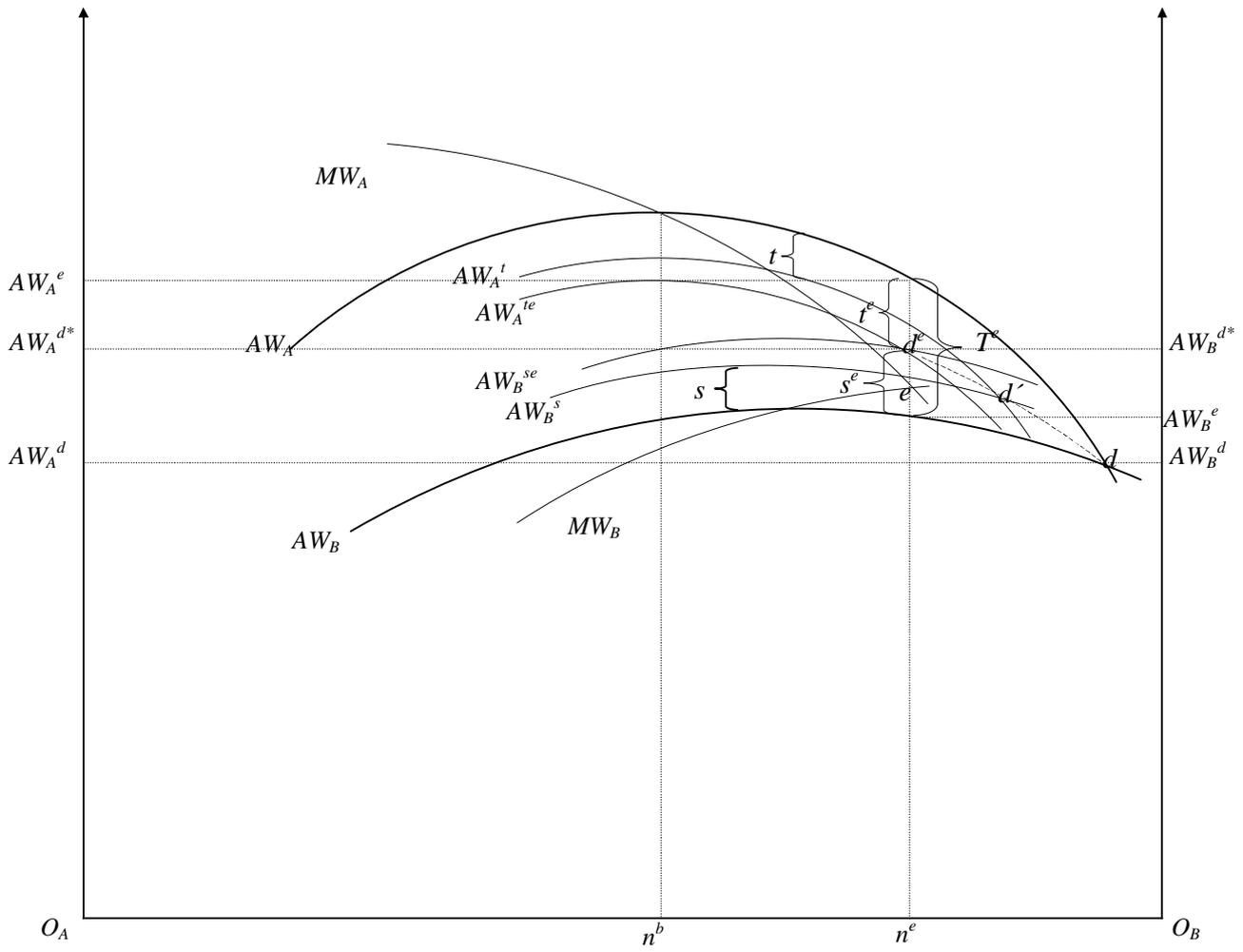


Figure 4: Optimal inter-city transfer policy

