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Sanna Tenhunen

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Optimal Tax Policy with Environmental Externalities and Heterogeneous Preferences*

Sanna Tenhunen†
University of Tampere and FDPE

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Abstract

This study considers simultaneously two important aspects of taxation, environmental policy and redistribution. Tax policy is constrained by the asymmetric information of agents' productivities and preferences. Two-dimensional heterogeneity affects the optimality of commodity taxation: it can be used to redistributional or environmental purposes, but there seems to be a trade-off between these objectives. However, the contradiction between the two aspects is not as clear as in the case with identical consumer preferences.

It is also shown that the Sandmo-Dixit result of the separability of environmental taxes fails with two-dimensional heterogeneity in the pooling optimum, but not in the separating optimum.The explanation to this is that there are too few policy instruments in the pooling equilibrium: commodity taxes should take care of both redistribution and externality internalisation.

Keywords: heterogeneous preferences, externalities, commodity taxation

JEL: H21, H23

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†E-mail: sanna.tenhunen@uta.fi

1 Introduction

In the recent literature of optimal taxation and redistribution the assumption of preference homogeneity is abandoned and households are allowed to have different tastes. Heterogeneity can arise from different tastes for leisure, different tastes for consumption or from some combination of these. Most of the literature concentrates on the former type of heterogeneity[1]. Introducing an additional dimension to individuals' characteristics makes the analysis more difficult: there are now four groups of households. To make the calculations more reasonable the number of household types is often reduced to three. Three groups are adequate in most cases to characterise the essential effects of heterogeneous preferences.

To design the optimal tax scheme, the direction of redistribution needs to be determined. It is no longer straightforward to determine which group should be treated the most gently. Sandmo (1993) discusses the problem of comparing utilities when tastes differ. He finds that even if utilities could be compared, a change in the parameter set describing the economy (such as prices) can reverse the order. Also the literature of social choice devotes attention to this question[2]. The social welfare approach aims at taking individuals' different preferences into account even when it means treating individuals with equal skill levels differently. According to the horizontal equity principle individuals should not be treated differently on the ground of their different preferences. However under asymmetric information this might be a costly or even an impossible requirement.

In a model with heterogeneous and usually unobservable preferences a possibility of screening is of special interest. Heterogeneous preferences have first been analysed in the framework considering the optimality of workfare e.g. by Besley and Coate (1995), Beaudry and Blackorby (1997) and Cuff (2000). Tarkiainen and Tuomala (1999) study the optimal tax policy numerically in an economy where households differ with respect to abilities and work preferences. Boadway et al. (2002) find that if there are some im-

[1] An exception is Blomquist and Christiansen (2004), where heterogeneity in preferences for consumption goods is also discussed.

[2] See Fleurbaey and Maniquet (1999) for a survey on this subject.

portant but unobservable characteristics affecting individual's choices, it can be optimal to abandon the income tax schedule based on equal weights on preference groups, even if the resulting tax scheme might seem regressive in terms of the observable characteristics, such as income.

With two-dimensional unobservable characteristics any action revealing characteristics under asymmetric information is valuable. Introducing an additional consumption good to the economy offers a possibility to study the role of indirect taxation. Saez (2002) finds that a tax on commodity is desirable when individuals with high income have relatively higher preference for the commodity or if the consumption of the commodity increases with leisure. Also Jordahl and Micheletto (2002) got parallel results: in a model with heterogeneous preferences the consumption of a good complementary to leisure does not need to be discouraged by taxation, and a commodity that is expected to be encouraged should not always be subsidized by a negative tax rate. Blomquist and Christiansen (2004) suggest that commodity taxation may get a new role as a device in differentiating between different groups. Contrary to the Atkinson-Stiglitz result of the redundancy of commodity taxation, when individuals have different preferences for commodity bundles imposing a tax on commodities might be desirable even when preferences are separable.

In the light of the earlier studies an assumption of heterogeneous preferences seems to affect the recommendations of the optimal tax policy. This study aims at combining two fields of research, the recent extension of the optimal tax models to heterogeneous preferences and a somewhat older discussion of the redistributional problems of environmental taxation[3]. A closer analysis of a term defining the valuation of environmental externality reveals that there are some terms induced by redistribution constraint that affect to opposite direction compared to environmental aims. Although the existence of such terms does not imply that environmental and redistributional

[3] An excellent survey of environmental policy as a part of the optimal taxation can be found from Bovenberg and Goulder (2002). For a survey of problems in combining environmental and redistributional aspects see e.g. Smith (1992), Harrison (1994) and OECD (2001). More recent research on the possible regressivity of environmental taxes can be found from Walls and Hanson (1999), Jacobsen, Birr-Pedersen and Wier (2003).

aspects are contradictory, it can be interpreted as one possible explanation to empirical findings of regressivity of environmental taxes.

The model used here assumes mixed taxation, i.e. a non-linear income tax and a linear commodity tax. The valuation for the environmental externality and the optimal commodity tax rule will be considered in two cases: in the separating equilibrium where each household choose their own income-consumption bundle and in the pooling optimum, where two of the households cannot be distinguished by their choice.

The paper is constructed as follows. In Chapter 2 we introduce the model. Chapter 3 derives the optimisation problem and the first order conditions. The valuation of the externality is derived in Chapter 4, where also the tradeoff between environmental and redistributional aspects is discussed. Chapter 5 concentrates on the commodity tax rules and Sandmo-Dixit principle. Finally, Chapter 6 concludes.

2 The model

The model here is very similar to the one used in Blomquist and Christiansen (2004) with the exception of harmful environmental externalities. It is based on Mirrlees (1976) type of optimal income tax model with heterogeneous households, differing with respect to productivities (as in Stiglitz, 1982; Stern, 1982) and preferences. A model with mixed taxation with one-dimensional heterogeneity and environmental externalities is used e.g. in Pirttilä and Tuomala (1997). Aronsson (2005) studies environmental policy and taxation with an emphasis on employment aspect. Cremer,Gahvari and Ladoux (1998) consider optimal taxation in a case of two dimensional heterogeneity and environmental externalities. However, they do not examine the redistributional aspect or the possibility of different types of optima.

To avoid too restrictive assumptions of utilities, we assume that heterogeneity results from the different preferences for leisure. However, unless preferences are completely separable, the amount of leisure is likely to affect also households' consumption and their assessment of the environmental quality. The relation depends on the complementarity of leisure and consumption and environmental quality. When leisure and environmental quality are complements, the household with stronger preference for leisure has also stronger preference for the environment. Thus heterogeneity is reflected also to preferences for consumption and environment.

Here it is assumed to be three types of households as characterised in Table 1 below.

		productivity	
		low	*high*
preference	*weak*	type 1	type 3
for leisure	*strong*	-	type 2

Table 1: Characteristics of household groups

Households supply labour L^h and receive an exogenous wage rate w^h, which reflects their productivities, i.e. $w^1 < w^2 = w^3$. There is a constant wage ratio $\Omega = \frac{w^1}{w^3}$. Labour income $Y^h = w^h L^h$ is taxed by the optimal non-linear income tax scheme $T(Y^h)$. We also assume that the labour income of type 1 households is the lowest and the gross income of type 3 is the highest. Households use all their net income $B^h = Y^h - T(Y^h)$ to consumption. There are two goods in the markets[4] denoted by a matrix $\mathbf{X} = \begin{bmatrix} X_c \\ X_d \end{bmatrix}$. X_c is the "clean" good, whereas X_d is the "dirty" good creating a harmful environmental externality. Both goods are assumed to be normal.

There is a linear commodity tax $\mathbf{t} = \begin{bmatrix} t_c \\ t_d \end{bmatrix}$ so that consumer prices can be denoted by a vector $\mathbf{q} = \mathbf{t} + \mathbf{p}$, where $\mathbf{p} = \begin{bmatrix} p_c \\ p_d \end{bmatrix}$ stands for producer prices. The demand $X_i^h\left(\mathbf{q}, B^h, L^h, E\right)$ of household h $(h = 1, 2, 3)$ for commodity i $(i = c, d)$ is a function of prices $\mathbf{q}$, after-tax income B^h, labour supply L^h and externality E, where $E = \sum_h X_d^h\left(\mathbf{q}, B^h, L^h, E\right)$.

Public sector has two preferences: more equal income distribution[5] and

[4] We omit production side of the economy because it does not have any effect on our results.

[5] We implicitly assume that the current distribution of income is sufficiently unequal

cleaner environment. It has two tax devices, non-linear income tax and linear commodity tax, to finance a constant revenue requirement $\bar{G}$. The redistribution from the high productivity households to the low productivity households is constrained by self-selection constraints as productivities are unobservable. Furthermore there is a problem in separating between the households receiving low income due to low productivity and the high productivity households earning less due to their strong preference for leisure. If income is redistributed with means of income taxation from type 3 households to both type 1 and type 2 households, the horizontal equity principle demanding an equal treatment for households with same characteristics is violated. Thus income taxation has to be designed so, that all households choose the combination of labour supply and net income meant for them rather than mimic the choice of other household type.

In the separating optimum all households choose a different point. The binding self-selection constraints are that firstly, type 3 household should not mimic type 2 household, and secondly, type 2 household should not mimic type 1 household. In mathematical form the constraints are given as $V^3(\mathbf{q}, B^3, L^3, E) \geq V^3(\mathbf{q}, B^2, L^2, E)$ and $V^2(\mathbf{q}, B^2, L^2, E) \geq V^2(\mathbf{q}, B^1, \Omega L^1, E)$.

Not all self-selection constraints are necessary binding. In the pooling optimum we assume that households of type 1 and 2 voluntarily choose the same point and thus income taxation cannot be used to differentiate between these two groups. However, they do not necessarily choose exactly the same consumption bundles, as they are assumed to have heterogeneous preferences. This raises a question of whether commodity taxes could be used as a screening tool to the redistributional purposes. There are again two self-selection constraints $V^3(\mathbf{q}, B^3, L^3, E) \geq V^3(\mathbf{q}, B^2, L^2, E)$ and $V^3(\mathbf{q}, B^3, L^3, E) \geq V^3(\mathbf{q}, B^1, \Omega L^1, E)$ but in the pooling case, as we have $B^1 = B^2$, $Y^1 = Y^2$ and $\Omega L^1 = L^2$, they represent exactly the same outcome. Naturally, when households 1 and 2 choose voluntarily the same income consumption bundle they do not have any incentive to mimic each other.

for redistribution to be desirable. However, the results derived here would be applicable also to opposite case.

3 The optimisation problem

Government is optimising the utility of the low productivity (type 1) household subject to two Pareto constraints

$$V^2\left(\mathbf{q}, B^1, L^1, E\right) - \bar{V}^2 \text{ and } V^3\left(\mathbf{q}, B^3, L^3, E\right) - \bar{V}^3 \tag{1}$$

In the separating optimum there are two self-selection constraints, given by

$$V^3\left(\mathbf{q}, B^3, L^3, E\right) \geq V^3\left(\mathbf{q}, B^2, L^2, E\right) \text{ and } V^2\left(\mathbf{q}, B^2, L^2, E\right) \geq V^2\left(\mathbf{q}, B^1, \Omega L^1, E\right) \tag{2}$$

whereas in the pooling case one constraint $V^3\left(\mathbf{q}, B^3, L^3, E\right) - V^3\left(\mathbf{q}, B^1, \Omega L^1, E\right)$ is sufficient to capture the restriction mimicking has. Another restriction comes from government's budget constraint requiring that the income from taxes equals the revenue requirement. Using consumer's budget constraint this can be rewritten as

$$\sum_h Y^h - \sum_h \mathbf{p}^T \mathbf{X}^h\left(\mathbf{q}, B^h, L^h, E\right) = \bar{G} \tag{3}$$

And finally the fifth constraint captures the effect of the externality

$$\sum_h X_d^h\left(\mathbf{q}, B^h, L^h, E\right) = E \tag{4}$$

Because the Lagrange function and the first order conditions are very similar in two optimums, we present only the pooling case here, whereas the optimisation conditions in the separating equilibrium appear in the appendix A. The Lagrangean of the optimisation problem in pooling case is given by

$$
\begin{aligned}
\Psi = {} & V^1\left(\mathbf{q}, B^1, L^1, E\right) \\
& + \delta_2\left[V^2\left(\mathbf{q}, B^1, \Omega L^1, E\right) - \bar{V}^2\right] + \delta_3\left[V^3\left(\mathbf{q}, B^3, L^3, E\right) - \bar{V}^3\right] \\
& + \lambda\left[V^3\left(\mathbf{q}, B^3, L^3, E\right) - V^3\left(\mathbf{q}, B^1, \Omega L^1, E\right)\right] \\
& + \gamma\left[\sum_h w^h L^h - \sum_h \mathbf{p}^T \mathbf{X}^h\left(\mathbf{q}, B^h, L^h, E\right) - \bar{G}\right] \\
& + \mu\left[E - \sum_h X_d^h\left(\mathbf{q}, B^h, L^h, E\right)\right] \qquad (5)
\end{aligned}
$$

The optimisation problem is to choose B^1, B^3, L^1, L^3 and $\mathbf{t}$ optimally. The first order conditions needed are

$$
V_L^1 + \delta_2 V_L^2 \Omega - \lambda \widehat{V}_L^3 \Omega + \gamma\left(\sum_{h=1,2} w^h - \sum_{h=1,2} \mathbf{p}^T \frac{\partial \mathbf{X}^h}{\partial L^1}\right) - \mu \sum_{h=1,2} \frac{\partial X_d^h}{\partial L^1} = 0 \qquad (6)
$$

$$
V_B^1 + \delta_2 V_B^2 - \lambda \widehat{V}_B^3 - \gamma \sum_{h=1,2} \mathbf{p}^T \frac{\partial \mathbf{X}^h}{\partial B^1} - \mu \sum_{h=1,2} \frac{\partial X_d^h}{\partial B^1} = 0 \qquad (7)
$$

$$
\left(\delta_3 + \lambda\right) V_L^3 + \gamma\left(w^3 - \mathbf{p}^T \frac{\partial \mathbf{X}^3}{\partial L^3}\right) - \mu \frac{\partial X_d^3}{\partial L^3} = 0 \qquad (8)
$$

$$
\left(\delta_3 + \lambda\right) V_B^3 - \gamma \mathbf{p}^T \frac{\partial \mathbf{X}^3}{\partial B^3} - \mu \frac{\partial X_d^3}{\partial B^3} = 0 \qquad (9)
$$

$$
\begin{aligned}
& V_E^1 + \delta_2 V_E^2 + \left(\delta_3 + \lambda\right) V_E^3 - \lambda \widehat{V}_E^3 \\
& \qquad - \gamma \sum_{h=1,2,3} \mathbf{p}^T \frac{\partial \mathbf{X}^h}{\partial E} + \mu\left(1 - \sum_{h=1,2,3} \frac{\partial X_d^h}{\partial E}\right) = 0 \qquad (10)
\end{aligned}
$$

$$V_{\mathbf{q}}^{1}+\delta_{2}V_{\mathbf{q}}^{2}+(\delta_{3}+\lambda)V_{\mathbf{q}}^{3}-\lambda\widehat{V}_{\mathbf{q}}^{3} \\ -\gamma\sum_{h=1,2,3}\mathbf{p}^{T}\frac{\partial\mathbf{X}^{h}}{\partial\mathbf{q}}-\mu\sum_{h=1,2,3}\frac{\partial X_{d}^{h}}{\partial\mathbf{q}}=0 \quad (11)$$

where the hat terms refer to mimickers, i.e. true type 3 household mimicking type 1 household.

4 The harmfulness of the externality

The valuation of the externality is an useful term for two reasons. First, a closer look at the terms in it enables a simultaneous consideration of environmental and redistributional aspects. Second, the optimal tax rules depend on the environmental quality via term defining the valuation of the externality. The valuation of the environmental externality tells how much harm externality produces and it is measured here by the shadow price. Here the form $\frac{\mu}{\gamma}$ is used, i.e. the shadow price is given relative to the government's tax revenues.

We notice that by taking a derivative of the Lagrangian with respect to B^2, we get an useful term $\left[\frac{\delta_2 V_B^2}{\gamma}-\mathbf{p}^T\frac{\partial\mathbf{X}^2}{\partial B^2}-\frac{\mu}{\gamma}\frac{\partial X_d^2}{\partial B^2}\right]=\frac{1}{\gamma}\frac{\partial\Psi}{\partial B^2}=\frac{\Psi_{B^2}}{\gamma}$. This term indicates the value given to the hypothetical increase in the net income of type 2 households. A similar term was utilised in Blomquist and Christiansen (2004). It can be interpreted to be negative if the desired direction of redistribution is from the high ability households towards the low ability households. The implicit form[6] the shadow price can be solved from the

[6] Written in this form Ψ_{B^2} contains the shadow price $\frac{\mu}{\gamma}$. In explicit form the harmfulness of the externality is given by

$$\frac{\mu}{\gamma}=\sigma^*\left\{\begin{array}{c}\sum_h MWP_{EB}^h+\left[\frac{\delta_2 V_B^2}{\gamma}-\mathbf{p}^T\frac{\partial\mathbf{X}^2}{\partial B^1}\right]\left[MWP_{EB}^2-MWP_{EB}^1\right] \\ -\lambda^*\left[M\hat{W}P_{EB}^3-MWP_{EB}^1\right]-\sum_h\mathbf{t}^T\frac{\partial\mathbf{x}^h}{\partial E}\end{array}\right\}$$

where $\sigma^*\frac{1}{1-\sum_h\frac{\partial x_d^h}{\partial E}+\frac{\partial X_d^2}{\partial B^2}\left[MWP_{EB}^2-MWP_{EB}^1\right]}$ and $\lambda^*=\frac{\lambda\widehat{V}_B^3}{\gamma}$.

equation (10)

$$\frac{\mu}{\gamma} = \sigma \begin{bmatrix} \sum\limits_h MWP_{EB}^h + \frac{\Psi_{B^2}}{\gamma}\left[MWP_{EB}^2 - MWP_{EB}^1\right] \\ -\lambda^* \left[M\hat{W}P_{EB}^3 - MWP_{EB}^1\right] - \sum\limits_h \mathbf{t}^T \frac{\partial \mathbf{x}^h}{\partial E} \end{bmatrix} \tag{12}$$

where $-\frac{V_E^h}{V_B^h} = MWP_{EB}^h$ is the marginal willingness to pay to avoid the externality, $\sigma = \frac{1}{1-\sum\limits_h \frac{\partial x_d^h}{\partial E}}$ is the environmental feedback parameter and $\lambda^* = \frac{\lambda \widehat{V}_B^3}{\gamma}$.

The exact sign of the shadow price cannot be determined from the form we have. However, as long as the externality is harmful, the shadow price can be assumed to positive. Environmental feedback parameter σ is known to be positive (Sandmo, 1980). Also the first term in brackets is positive implying the direct harm of the externality. The last term referring to government's tax revenues from commodity taxes depends on how externality affects the demand for goods. The first and the last term are similar to the ones in earlier literature (see Pirttilä and Tuomala, 1997; and Tenhunen, 2004), so we concentrate here on the two terms in the middle.

MWP_{EB}^h depends on how much household has leisure: it increases with leisure, when environmental quality and leisure are complements[7]. From the two terms in the middle with differences in MWP_{EB}^h, now only $\widehat{MWP}_{EB}^3 - MWP_{EB}^1$ refers to mimicking. When mimicking, high productivity households (type 3) have more leisure than true type 1 households with lower productivity. When the environmental harm is decreased by lowering the level of the externality, mimicking becomes more attractive. To prevent that, the income tax of type 3 households has to be lowered and redistribution comes more unequal. Thus environmental and redistributional terms lead to opposite directions implying a trade-off between the two government preferences

Here coefficient σ^* is positive; it only has an additional positive term in the denominator compared to ordinary environmental feedback parameter σ. The conclusions of the terms are identical, as $\Psi_{B^2} < 0$ implies that also coefficient $\frac{\delta_2 V_B^2}{\gamma} - \mathbf{p}^T \frac{\partial \mathbf{X}^2}{\partial B^1}$ is negative.

[7] Many of the conclusions in this paper are based on the assumption of environmental quality and leisure being complements. The analysis in the case of substitutes would go through analogously. As the former case seems more plausible to us, to avoid confusion we are concentrating on the case where harmfulness of the externality increases with leisure.

in this case. The same result was found earlier e.g. in Pirttilä and Tuomala (1997).

The second term in brackets, the difference in the marginal willingnesses to pay, $MWP_{EB}^2 - MWP_{EB}^1$ is positive, as both households 1 and 2 are choosing the same income level but type 2 household has higher productivity and thus more leisure. This term does not refer to mimicking nor to redistribution, because in pooling optimum these two household types cannot be differentiated and thus no redistribution can be made by income taxation. The term rather refers to the difference in the preferences for environmental quality. The fact that households are not equal affects the harmfulness of the shadow price. With negative Ψ_{B^2} the difference between MWP_{EB}^2 and MWP_{EB}^1 decreases the harmful effect of the externality. Thus this term affects to the same direction as the previous term referring to redistribution. If an increase in the level of the externality widens the difference between the marginal willingnesses to pay, a part of the harm from the externality is compensated by gains from redistribution. This trade-off between environmental quality and redistribution indicates that a higher level of the environmental externality can be used as a tool to deter mimicking and to redistribute income between otherwise indistinguishable households.

In the separating optimum the shadow price is given by[8]

$$\frac{\mu}{\gamma} = \sigma \left[\begin{array}{c} \sum\limits_h MWP_{EB}^h - \lambda_2^* \left[\widehat{MWP}_{EB}^2 - MWP_{EB}^1 \right] \\ -\lambda_3^* \left[\widehat{MWP}_{EB}^3 - MWP_{EB}^2 \right] - \sum\limits_h \mathbf{t}^T \frac{\partial \mathbf{x}^h}{\partial E} \end{array} \right], \tag{13}$$

where $MWP_{EB}^h = -\frac{V_E^h}{V_B^h}$ is the marginal willingness to pay to avoid the externality, $\sigma = \frac{1}{1 - \sum\limits_h \frac{\partial \mathbf{x}_d^h}{\partial E}}$ is the environmental feedback parameter and $\lambda_h^* = \frac{\lambda \widehat{V}_B^h}{\gamma}, h = 2, 3.$

The environmental feedback parameter σ and the sum of MWP_{EB}^h are also positive as before, and tax revenue term depends on how externality affects the demand. The two terms in the middle are again the interesting ones, because here the difference in the marginal willingnesses to pay (MWP_{EB}^h)

[8] For derivation, see Appendix B.

refers to redistribution question.

Type 2 household with higher productivity mimicking type 1 household gets more leisure, as they can do the work of type 1 faster and thus we have $\widehat{MWP}^2_{EB} > MWP^1_{EB}$. Thus the effect of the second term in the brackets in Equation (13) is negative when environmental quality is a complement with leisure. This means, that decreasing the level of the externality increases type 2 household's incentive to mimic. To deter that, the government needs to redefine income tax scheme by lowering type 2 households' taxes and increasing type 1 households' taxes, i.e. let the income differences between type 1 and 2 households rise.

The third term refers to difference $\widehat{MWP}^3_{EB} - MWP^2_{EB}$. Households 2 and 3 have same productivities and thus here the same conclusion as in previous case does not hold. The difference can be solved by thinking about the deviation in preferences. When environmental quality, i.e. the negative of the environmental externality, is complement with leisure, type 3 households prefer leisure less and hence they do not "care" of the environmental quality as much as type 2 with higher preference for leisure. This means that type 3 households are willing to pay less to avoid the externality than type 2 households, i.e. $\widehat{MWP}^3_{EB} < MWP^2_{EB}$ and the effect of the third term in Equation (13) is positive. Thus more equal income distribution between type 2 and 3 households decreases the effect of this term by making mimicking less attractive and lowers the valuation of the harmfulness of the externality. As a result, the following proposition holds.

Proposition 1 *When environmental quality and leisure are complements, the valuation of the harmfulness of the externality is decreased when income differences between households 2 and 3 decrease and income differences between type 1 and 2 households increase.*

The result implies that if the harmfulness of the externality increases with leisure, redistribution from high productivity households towards low ability households worsens environmental quality whereas redistribution from more working high productivity household towards less working high productivity household improves environmental situation. Now the contradiction between environmental and redistributional aspects found in the earlier literature is not so clear anymore, because of the effect of term $\widehat{MWP}^3_{EB} - MWP^2_{EB}$. It is possible, that the effect of the latter mimicking term is sufficiently large to compensate the negative effect of the first mimicking term so that the overall relation between the two aspects is positive. In that case environmental and redistributional aspects would be in accordance from government's point of view.

One possible interpretation for strong preference for leisure could be "laziness" and persons with a low preference for leisure as "hard working", as in Cuff (2000). The value of the harmfulness of the externality would be decreased most when redistribution is directed from hard working low productivity households and from hard working high productivity households to lazy high productivity households. This direction of redistribution may not be the most supported one by the majority.

It is also worth noticing that, both in the separating and pooling equilibrium, when the preferences are separable between leisure and environmental quality, the differences in MWP^h_{EB}, σ and the tax revenue term all are zero. In the case of separable preferences terms with the differences in MWP^h_{EB} will remain in the shadow price only when households have heterogeneous preferences directly for environmental quality.

5 Commodity taxation

In the presence of an externality commodity taxes aim at internalising the harmful effect. Sandmo's additivity property (Sandmo, 1975) and Dixit's principle of targeting (1985), referred here as the Sandmo-Dixit principle, states that the externality internalising part of the commodity tax should be separable from the other part of the tax rate and it should affect only the tax rate of that good, which creates the externality. There has been some discussion about the generality of Sandmo-Dixit principle (Kopczuk, 2003; Tenhunen 2004). Here we study how an additional dimension in heterogeneity affects the principle.

In our framework the internalisation of the externality is not the only objective of taxation. Commodity taxation might get a new role as a screening tool in a framework with heterogeneous preferences. Blomquist and Christiansen (2004) get a result that in an economy with one consumption good and no externalities commodity taxation has an effect on the redistribution and thus it can be used to mitigate self-selection constraint.

The optimal commodity tax rates[9] are given by

$$\mathbf{t} = \frac{\Psi_{B^2}}{\gamma}\mathbf{S}^{+}\left(\mathbf{X}^2 - \mathbf{X}^1\right) - \frac{\lambda \widehat{V}_B^3}{\gamma}\mathbf{S}^{+}\left(\widehat{\mathbf{X}}^3 - \mathbf{X}^1\right) + \frac{\mu}{\gamma}\begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{14}$$

where, $\mathbf{S}^{+}$ is an unique pseudoinverse[10] of $\sum_h \mathbf{S}^{hT}$.

This form corresponds the one received in Blomquist and Christiansen (2004) with the exception of the last term referring to externality. The second term implies that if mimicker consumes more goods, i.e. if leisure and consumption are complements, $\widehat{\mathbf{X}}^3 > \mathbf{X}^1$ and commodity tax can be used to deter mimicking. Increasing commodity taxes and decreasing income taxes of type 1 households leaves mimicker worse off and thus makes mimicking less attractive. If the difference in consumption is other way around, i.e. $\widehat{\mathbf{X}}^3 < \mathbf{X}^1$, a negative commodity tax has the same effect.

The first term in Equation (14) is interesting for two reasons. First it implies that if there is a difference in consumption between households of type 1 and 2, commodity taxation may also be used for redistribution. If type 2 households consume more, an increase in the commodity tax makes them worse off. In the opposite case a subsidy on commodities can be used. In the pooling equilibrium type 1 and 2 households were not observable, but now commodity taxes treat households differently.

The other important feature in the redistribution term is in the coefficient

[9] For derivation see appendix.

[10] Actually Slutsky substitution matrix $\mathbf{S}$ probably also has the ordinary inverse $\mathbf{S}^{-1}$, as it is negative semidefinite Slutsky matrix. However, a pseudoinverse exists also for singular and non-square matrices and it is equal to ordinary inverse in the case of non-singular square matrix. To ensure the existence of the inverse, we use here pseudoinverse $\mathbf{S}^{+}$.

$\frac{\Psi_{B2}}{\gamma}$. It includes the shadow price of the externality $\frac{\mu}{\gamma}$. This means, that when types 1 and 2 have unidentical demand for goods, the externality effect appears in the tax rates of both commodities, also in the tax rate of the clean good. Before we can be sure of the failure of the Sandmo-Dixit principle, it is worth noting that when the shadow price $\frac{\mu}{\gamma}$ from (12) is substituted in (14), there is a possibility that the coefficient for $\frac{\Psi_{B2}}{\gamma}$ cancels out to zero. The condition for this to happen is

$$\left(\mathbf{X}^{2T} - \mathbf{X}^{1T}\right) + \sigma\left(MWP_{EB}^{2} - MWP_{EB}^{1}\right)\sum_{h}\mathbf{s}_{d}^{h} = 0\,. \tag{15}$$

Without more precise functional forms for the model we cannot rule out the possibility of this term being zero. If in some special case the condition holds, the first terms in the right hand side of Equations (32) and (14) cancel out and the shadow price reduces to

$$\frac{\mu}{\gamma} = \sigma\left\{\sum_{h} MWP_{EB}^{h} - \lambda * \left[M\hat{W}P_{EB}^{3} - MWP_{EB}^{1}\right] - \sum_{h}\mathbf{t}^{T}\frac{\partial x^{h}}{\partial E}\right\} \tag{16}$$

However, in general case there is no need for the condition (15) to hold. Thus we can assume that generally term $\frac{\Psi_{B2}}{\gamma}$ remains in the redistribution term and the Sandmo-Dixit principle fails in this two dimensional case in the pooling optimum. The following proposition summarises the result.

Proposition 2 *The effect of the externality on commodity taxes can no longer be separated to affect only the good that creates the externality. Thus Sandmo-Dixit principle fails in the pooling optimum unless the consumption of the clean good is equal for pooling households.*

In the separating optimum the optimal commodity tax rule[11] corresponds the ones received in earlier literature (Blomquist and Christiansen, 2004). Sandmo-Dixit principle continues to hold and the ability to use commodity taxes to redistributional aims depends on the consumption behaviour of the mimickers. Under some assumptions (commodities are complements with

[11] For details see Appendix C.

leisure and substitutes with each other and S is non-singular) increasing the tax on commodities mitigates one self-selection constraint and tightens the other. Thus using commodity taxes to mitigate the self-selection constraints includes a trade-off: one of the constraints can be relaxed at the expense of the other and the effect of commodity taxation on mimicking terms is thus ambiguous.

The result of the generality of the Sandmo-Dixit principle is somewhat surprising. Our analysis suggests that assuming a three-type economy is not sufficient to make the principle fail, but in the pooling equilibrium the externality based part cannot anymore be separated from the rest of the tax rate. In the separating case Ψ_{B^2} is actually a first order condition which requires that marginal valuations for each groups' hypothetical increase in income are zero, whereas in pooling case this term is not (necessarily) zero. When the government wishes to redistribute away from type 2 i.e. $\Psi_{B^2} < 0$, in the pooling case the commodity taxation is the only tax instrument that separates type 2 households, as long as household groups in pooling choose unidentical consumption bundles. As the valuation of the hypothetical income depends on the externality created by an increased consumption of dirty good, also the optimal commodity tax of the clean good is affected by the externality.

The reason for the failure of the Sandmo-Dixit principle in the pooling case is the insufficient number of policy instruments. In the separating case income taxation takes care of the redistributional aims and commodity taxation internalises the externality. However, in the pooling case income taxes are not sufficient to handle redistribution, and the commodity tax has two policy objectives: redistribution and externality.

6 Conclusions

This study analyses the effect of a harmful environmental externality in an economy with two-dimensional heterogeneity. There are assumed to be three types of households that differ both with respect to their productivities and their preferences for leisure. The valuation of the externality and the optimal commodity tax rates are discussed in two cases: in the pooling optimum, where the low productivity household and the high productivity household with a strong preference for leisure are assumed to choose the same income-consumption bundle and in the separating optimum where each household chooses a different point.

The harmfulness of the externality measured by its shadow price is of the same form as in an economy with only one-dimensional heterogeneity. The term referring to mimicking suggests that there might be problems in combining environmental and redistributional preferences. The other term sourcing from the difference in preference for environmental quality may affect in either direction depending on the sign of its coefficient. However, with some assumptions of the desired direction of redistribution also this term indicates a contradiction between environmental and redistributional preferences. In the separating optimum one of the self-selection terms has negative effect implying problems in combining environmental and redistributional aspects, as in earlier literature. The other self-selection term has a positive effect. The valuation of the externality is decreased when income differences between high productivity households decreases and differences between high and low productivity households increases. Thus the contradiction observed in earlier case is not so clear anymore.

The optimal commodity taxes in the pooling equilibrium offer two important results. First is that commodity tax can be used as a tool to differentiate and redistribute income between the households behaving identically if these two household have unequal consumption of goods. If the household from which we want to distribute consumes more (less), a positive (negative) commodity tax makes them worse off and mitigates the self-selection constraint.

Another interesting question is the effect of the externality in commodity taxes. In the pooling optimum Sandmo-Dixit principle fails to hold, i.e. externality based part of the commodity tax cannot be anymore separated from the other part of the tax and it affects also the tax rate of the good not creating the harm. In the separating optimum we can generalize Sandmo-Dixit principle. The explanation to this is that whereas in the separating optimum optimisation is done with respect to all types, in the pooling optimum two household types are indistinguishable and the optimum is achieved with respect to two groups of households only. There are too few policy instruments in the pooling equilibrium: commodity taxes should take care of both redistribution and externality internalisation.

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Appendices

A Optimisation problem in the separating case

As a result of the similarity, the optimisation problem and the first order conditions of the separating optimum are presented only here. The Lagrange function of the optimisation problem is given by

$$
\begin{aligned}
L = {} & V^1\left(\mathbf{q}, B^1, L^1, E\right) \\
& + \delta_2\left[V^2\left(\mathbf{q}, B^2, L^2, E\right) - \overline{V}^2\right] + \delta_3\left[V^3\left(\mathbf{q}, B^3, L^3, E\right) - \overline{V}^3\right] \\
& + \lambda_2\left[V^2\left(\mathbf{q}, B^2, L^2, E\right) - V^2\left(\mathbf{q}, B^1, \Omega L^1, E\right)\right] \\
& + \lambda_3\left[V^3\left(\mathbf{q}, B^3, L^3, E\right) - V^3\left(\mathbf{q}, B^2, L^2, E\right)\right] \\
& + \gamma\left[\sum_h w^h L^h - \sum_h \mathbf{p}^T \mathbf{X}^h\left(\mathbf{q}, B^h, L^h, E\right) - \bar{G}\right] \\
& + \mu\left[E - \sum_h X_d^h\left(\mathbf{q}, B^h, L^h, E\right)\right] \qquad (17)
\end{aligned}
$$

The corresponding first order conditions with respect to L^h, B^h for $h = 1, 3$, E and $\mathbf{q}$ are[12]

$$V_L^1 - \lambda_2 \widehat{V}_L^2 \Omega + \gamma\left(w^1 - \mathbf{p}^T \frac{\partial \mathbf{X}^1}{\partial L^1}\right) - \mu \frac{\partial \mathbf{X}_d^1}{\partial L^1} = 0 \qquad (18)$$

$$V_B^1 - \lambda_2 \widehat{V}_B^2 - \gamma \mathbf{p}^T \frac{\partial \mathbf{X}^1}{\partial B^1} - \mu \frac{\partial \mathbf{X}_d^1}{\partial B^1} = 0 \qquad (19)$$

[12] Note that $\mathbf{X}^h$ is a function of L^h and B^h only, thus sums can be dropped out in the first six derivatives.

$$(\delta_2+\lambda_2)\,V_L^2-\lambda_3\widehat{V}_L^3+\gamma\left(w^2-\mathbf{p}^T\frac{\partial\mathbf{X}^2}{\partial L^2}\right)-\mu\frac{\partial\mathbf{X}}{\partial L^2}=0 \tag{20}$$

$$(\delta_2+\lambda_2)\,V_B^2-\lambda_3\widehat{V}_B^3-\gamma\mathbf{p}^T\frac{\partial\mathbf{X}^2}{\partial B^2}-\mu\frac{\partial\mathbf{X}_d^2}{\partial B^2}=0 \tag{21}$$

$$(\delta_3+\lambda_3)\,V_L^3+\gamma\left(w^3-\mathbf{p}^T\frac{\partial\mathbf{X}^3}{\partial L^3}\right)-\mu\frac{\partial\mathbf{X}_d^3}{\partial L^3}=0 \tag{22}$$

$$(\delta_3+\lambda_3)\,V_B^3-\gamma\mathbf{p}^T\frac{\partial\mathbf{X}^3}{\partial B^3}-\mu\frac{\partial\mathbf{X}_d^3}{\partial B^3}=0 \tag{23}$$

$$\begin{aligned}V_E^1+(\delta_2+\lambda_2)\,V_E^2+(\delta_3+\lambda_3)\,V_E^3-\lambda_2\widehat{V}_E^2-\lambda_3\widehat{V}_E^3&\\-\gamma\sum_h\mathbf{p}^T\frac{\partial\mathbf{X}^h}{\partial E}+\mu\left(1-\sum_h\frac{\partial\mathbf{X}_d^h}{\partial E}\right)&=0\end{aligned} \tag{24}$$

$$\begin{aligned}V_{\mathbf{q}}^1+(\delta_2+\lambda_2)\,V_{\mathbf{q}}^2+(\delta_3+\lambda_3)\,V_{\mathbf{q}}^3-\lambda_2\widehat{V}_{\mathbf{q}}^2-\lambda_3\widehat{V}_{\mathbf{q}}^3&\\-\gamma\sum_h\mathbf{p}^T\frac{\partial\mathbf{X}^h}{\partial\mathbf{q}}-\mu\sum_h\frac{\partial\mathbf{X}_d^h}{\partial\mathbf{q}}&=0\,,\end{aligned} \tag{25}$$

where again the hat terms refer to mimickers.

B The valuation of externality

B.1 The pooling case

To derive the shadow price first add and subtract terms $\lambda\hat{V}_B^3\frac{V_E^1}{V_B^1}$ and $\delta_2V_B^2\frac{V_E^1}{V_B^1}$ from Equation (10). Denoting $-\frac{V_E^h}{V_B^h}=MWP_{EB}^h$ and using Slutsky –type properties $\sum_h\mathbf{p}^T\frac{\partial\mathbf{X}^h}{\partial E}=\sum_h\mathbf{p}^T\frac{\partial\mathbf{x}^h}{\partial E}-\sum_h MWP_{EB}^h\mathbf{p}^T\frac{\partial\mathbf{X}^h}{\partial B^h}$ and $\sum_h\mathbf{p}^T\frac{\partial\mathbf{x}^h}{\partial E}=MWP_{EB}^h-\mathbf{t}^T\frac{\partial\mathbf{x}^h}{\partial E}$ subsequently we get

$$
\begin{aligned}
&- MWP^1_{EB}\left[\mathbf{p}^T\frac{\partial \mathbf{X}^2}{\partial B^1}+\frac{\mu}{\gamma}\frac{\partial X^2_d}{\partial B^1}\right]+MWP^2_{EB}\left[\mathbf{p}^T\frac{\partial \mathbf{X}^2}{\partial B^2}+\frac{\mu}{\gamma}\frac{\partial X^2_d}{\partial B^2}\right]\\
&\quad+\frac{\lambda\hat{V}^3_B}{\gamma}\left[M\hat{W}P^3_{EB}-MWP^1_{EB}\right]+\frac{\delta_2 V^2_B}{\gamma}\left[MWP^1_{EB}-MWP^2_{EB}\right]\\
&\quad\quad-\left[\sum_h MWP^h_{EB}-\sum_h \mathbf{t}^T\frac{\partial \mathbf{x}^h}{\partial E}\right]+\frac{\mu}{\gamma}\left(1-\sum_{h=1,2,3}\frac{\partial x^h_d}{\partial E}\right)=0 \quad (26)
\end{aligned}
$$

Because $B^1=B^2$, it must be that $\frac{\partial X^2}{\partial B^1}=\frac{\partial X^2}{\partial B^2}$. Thus we get

$$
\begin{aligned}
&\left[\frac{\delta_2 V^2_B}{\gamma}-\mathbf{p}^T\frac{\partial \mathbf{X}^2}{\partial B^1}-\frac{\mu}{\gamma}\frac{\partial X^2_d}{\partial B^1}\right]\left[MWP^1_{EB}-MWP^2_{EB}\right]\\
&\quad+\frac{\lambda\hat{V}^3_B}{\gamma}\left[M\hat{W}P^3_{EB}-MWP^1_{EB}\right]-\left[\sum_h MWP^h_{EB}-\sum_h \mathbf{t}^T\frac{\partial \mathbf{x}^h}{\partial E}\right]\\
&\quad\quad+\frac{\mu}{\gamma}\left(1-\sum_{h=1,2,3}\frac{\partial x^h_d}{\partial E}\right)=0 \quad (27)
\end{aligned}
$$

With help of term Ψ_{B^2} we get an implicit solution for the shadow price presented in Equation (12).

B.2 The separating case

Letting $-\frac{V^h_E}{V^h_B}=MWP^h_{EB}$ denote the marginal willingness to pay to avoid the externality and using the first order conditions in Equations (19), (21) and (23) we can write Equation (24) after some manipulations as

$$- MWP^1_{EB}\left[\gamma \mathbf{p}^T \frac{\partial \mathbf{X}^1}{\partial B^1} + \mu \frac{\partial X^1_d}{\partial B^1}\right] - MWP^2_{EB}\left[\gamma \mathbf{p}^T \frac{\partial \mathbf{X}^2}{\partial B^2} + \mu \frac{\partial X^2_d}{\partial B^2}\right]$$
$$- MWP^3_{EB}\left[\gamma \mathbf{p}^T \frac{\partial \mathbf{X}^3}{\partial B^3} + \mu \frac{\partial X^3_d}{\partial B^3}\right] + \lambda_2 \widehat{V}^2_B \left[\widehat{MWP}^2_{EB} - MWP^2_{EB}\right]$$
$$+ \lambda_3 \widehat{V}^3_B \left[\widehat{MWP}^3_{EB} - MWP^2_{EB}\right] - \gamma \sum_h \mathbf{p}^T \frac{\partial \mathbf{X}^h}{\partial E} + \mu \left(1 - \sum_h \frac{\partial X^h_d}{\partial E}\right) = 0 \tag{28}$$

Using similar Slutsky properties as in the pooling optimum and substituting these into Equation (28) and subtracting equivalent terms we get (13).

C The commodity tax rules

C.1 Derivation in the pooling case

Using Slutsky decomposition $\frac{\partial \mathbf{X}^h}{\partial \mathbf{q}} = \mathbf{S}^h - \frac{\partial \mathbf{X}^h}{\partial B^h}\mathbf{X}^{hT}$ and Roy's identity the first order condition in Equation (11) can be written as

$$- V^1_B \mathbf{X}^{1T} - \delta_2 V^2_B \mathbf{X}^{2T} - (\delta_3 + \lambda) V^3_B \mathbf{X}^{3T} + \lambda \widehat{V}^3_B \widehat{\mathbf{X}}^{3T}$$
$$- \gamma \sum_{h=1,2,3} \mathbf{p}^T \left(\mathbf{S}^h - \frac{\partial \mathbf{X}^h}{\partial B^h}\mathbf{X}^{hT}\right) - \mu \sum_{h=1,2,3} \left(s^h_d - \frac{\partial X^h_d}{\partial B}\mathbf{X}^{hT}\right) = 0 \tag{29}$$

Multiplying Equation (7) by $\mathbf{X}^{1T}$ and Equation (9) by $\mathbf{X}^{3T}$ we can substitute some terms. Furthermore we use derivative $\Psi_{B^2} = \delta_2 V^2_B - \gamma \mathbf{p}^T \frac{\partial \mathbf{X}^2}{\partial B^2} - \mu \frac{\partial X^2_d}{\partial B^2}$. With the help of the first order conditions and Ψ_{B^2} Equation (29) can be given as

$$- \delta_2 V_B^2 \left(\mathbf{X}^{2T} - \mathbf{X}^{1T}\right) + \lambda \widehat{V}_B^3 \left(\widehat{\mathbf{X}}^{3T} - \mathbf{X}^{1T}\right) - \gamma \sum_{h=1,2} \mathbf{p}^T \frac{\partial \mathbf{X}^h}{\partial B^1} \mathbf{X}^{1T}$$

$$- \mu \sum_{h=1,2} \frac{\partial X_d^h}{\partial B^1} \mathbf{X}^{1T} - \gamma \mathbf{p}^T \frac{\partial \mathbf{X}^3}{\partial B^3} \mathbf{X}^{3T} - \mu \frac{\partial X_d^3}{\partial B^3} \mathbf{X}^{3T}$$

$$- \gamma \sum_{h=1,2,3} \mathbf{p}^T \left(\mathbf{S}^h - \frac{\partial \mathbf{X}^h}{\partial B^h} \mathbf{X}^{hT}\right) - \mu \sum_{h=1,2,3} \left(s_d^h - \frac{\partial X_d^h}{\partial B} \mathbf{X}^{hT}\right) = 0 \quad (30)$$

Remembering that in the pooling equilibrium $B^1 = B^2$ and thus also $\frac{\partial X^h}{\partial B^1} = \frac{\partial X^h}{\partial B^2}$ we can simplify previous equation to the form

$$\left[-\delta_2 V_B^2 + \mu \frac{\partial X_d^2}{\partial B^2} + \gamma \mathbf{p}^T \frac{\partial \mathbf{X}^2}{\partial B^2}\right] \left(\mathbf{X}^{2T} - \mathbf{X}^{1T}\right)$$

$$+ \lambda \widehat{V}_B^3 \left(\widehat{\mathbf{X}}^{3T} - \mathbf{X}^{1T}\right) - \gamma \sum_{h=1,2,3} \mathbf{p}^T \mathbf{S}^h - \mu \sum_{h=1,2,3} s_d^h = 0 \quad (31)$$

With the help of a differential from consumer's budget constraint it can be noted that $\sum_h \mathbf{q}^T \mathbf{S}^h = 0$. Note also that $\sum_h \mathbf{s}_d^h = \begin{bmatrix} 0 & 1 \end{bmatrix} \sum_h \mathbf{S}^h$. Making use of the definition of Ψ_{B^2}, reorganising terms and dividing them by γ gives us

$$\sum_{h=1,2,3} \mathbf{t}^T \mathbf{S}^h = \frac{\Psi_{B^2}}{\gamma} \left(\mathbf{X}^{2T} - \mathbf{X}^{1T}\right) - \frac{\lambda \widehat{V}_B^3}{\gamma} \left(\widehat{\mathbf{X}}^{3T} - \mathbf{X}^{1T}\right) + \frac{\mu}{\gamma} \sum_{h=1,2,3} \mathbf{s}_d^h \quad (32)$$

Multiplying this from left by $\mathbf{S}^+$, an unique pseudoinverse (Moore-Penrose inverse) of $\sum_h \mathbf{S}^{hT}$, gives us optimal commodity tax rates in (14).

C.2 Commodity taxes in the separating optimum

The optimal commodity tax rate can be derived in the same way as in the pooling case. Using Roy's identity and Slutsky decomposition Equation (25) can be written as

$$
\begin{aligned}
&-V_B^1\mathbf{X}^{1T}-\left(\delta_2+\lambda_2\right)V_B^2\mathbf{X}^{2T}\\
&\qquad\qquad-\left(\delta_3+\lambda_3\right)V_B^3\mathbf{X}^{3T}+\lambda\widehat{V}_B^3\widehat{\mathbf{X}}^{3T}+\lambda\widehat{V}_B^2\widehat{\mathbf{X}}^{2T}\\
&\qquad\qquad-\gamma\sum_h\mathbf{p}^T\left(\mathbf{S}^h-\frac{\partial\mathbf{X}^h}{\partial B^h}\mathbf{X}^{hT}\right)-\mu\sum_h\left(\mathbf{s}_d^h-\frac{\partial\mathbf{X}_d^h}{\partial B^h}\mathbf{X}^{hT}\right)=0 \qquad (33)
\end{aligned}
$$

Substituting in Equation (19) multiplied by $\mathbf{X}^{1T}$, Equation (21) multiplied by $\mathbf{X}^{2T}$ and Equation (23) multiplied by $\mathbf{X}^{3T}$ and reorganizing gives us

$$
\sum_h\mathbf{t}^T\mathbf{S}^h=\sum_h\mathbf{q}^T\mathbf{S}^h-\lambda_2^*\left(\widehat{\mathbf{X}}^2-\mathbf{X}^1\right)^T-\lambda_3^*\left(\widehat{\mathbf{X}}^3-\mathbf{X}^2\right)^T+\frac{\mu}{\gamma}\left[\begin{array}{cc}0&1\end{array}\right]\mathbf{S}^h\,. \qquad (34)
$$

Taking a transpose, multiplying from left by $\mathbf{S}^+$, an unique pseudoinverse (Moore-Penrose inverse) of $\sum_h\mathbf{S}^{hT}$, gives us

$$
\mathbf{t}=-\lambda_2^*\mathbf{S}^+\left(\widehat{\mathbf{X}}^2-\mathbf{X}^1\right)-\lambda_3^*\mathbf{S}^+\left(\widehat{\mathbf{X}}^3-\mathbf{X}^2\right)+\frac{\mu}{\gamma}\left[\begin{array}{c}0\\1\end{array}\right]. \qquad (35)
$$