



TAMPERE ECONOMIC WORKING PAPERS
NET SERIES

LABOUR INCOME UNCERTAINTY,
TAXATION AND PUBLIC GOOD PROVISION

Jukka Pirttilä
Matti Tuomala

Working Paper 16
September 2003
<http://tampub.uta.fi/econet/wp16-2003.pdf>

DEPARTMENT OF ECONOMICS
FIN-33014 UNIVERSITY OF TAMPERE, FINLAND

ISSN 1458-1191
ISBN 951-44-5767-6

Labour income uncertainty, taxation and public good provision^{*}

by

Jukka Pirttilä

and

Matti Tuomala[#]

1 September 2003

^{*} We are grateful to Hamish Low and Jim Mirrlees for most helpful comments and suggestions on an earlier version. Financial support for the first author from the Academy of Finland, The Finnish Cultural Foundation, University of Helsinki and Yrjö Jahnsson Foundation is gratefully acknowledged.

[#] Corresponding author. Address: Department of Economics, 33014 University of Tampere, Finland. Email: kmatuo@uta.fi

Abstract: This paper examines optimal non-linear income taxation, commodity taxation and public good provision under income uncertainty. Workers' income depends randomly on their effort, and effort is unobservable to the government. When income is taxed on a non-linear scale and commodities linearly, the consumption of commodities that are negatively (positively) associated with effort should be discouraged (encouraged). A similar rule is derived for public good provision. Conditions for when uniform commodity taxation and the first-best Samuelson rule for public good provision are desirable under income uncertainty are shown to be analogous to those derived in the conventional tax model. The paper also examines rules for optimal non-linear income and commodity taxation under income uncertainty.

Key words: income taxation, commodity taxation, public good provision, income uncertainty, hidden action

JEL classification: D82, H21

1. Introduction

The Mirrlees (1971) model treats differences in observed income as being due to unobserved differences in ability, which means that in his model the individual knows exactly what income he or she will receive at each possible level of effort. One might well argue that both high-income and low-income people do not owe their (un)success entirely to ability, but part of the income differentials are due to luck. In another approach to optimal income taxation introduced by Mirrlees (1974) and later used, among others, by Varian (1980), Tuomala (1984) and Low and Maldoom (2003), differences in observed income are mainly due to unobserved random factors. The critical question is whether differences in income come mostly from luck or from ability. If luck plays a substantial role in the determination of income it makes sense to have a progressive tax, creating a form of social insurance in which the lucky subsidize the unlucky.

Mirrlees (1976a) extends the standard optimal taxation model with differences in income-earning abilities by adding commodity taxation and public good provision. To our knowledge there is no similar analysis of optimal commodity taxation and public good provision in the social insurance set up.¹ Our aim in this paper is to analyze non-linear income taxation, commodity taxation that can be either linear or fully non-linear, and public good provision under income uncertainty.

¹ Implications of uncertainty on commodity taxation have been analysed by Cremer and Gahvari (1995) from a different angle. They consider a situation where the consumer has committed to a consumption of one good (e.g. housing) before the resolution of income uncertainty.

Income uncertainty and the need to deal with it by offering social insurance can be an important part of the concerns of real-world tax policy.² But analyzing commodity taxation and public good provision under income uncertainty is also interesting because of the theoretical connections between the standard income taxation model and the social insurance set-up. Both build on an asymmetric information based approach to tax policy, but from a different angle. In the standard optimal tax model, the income-earning ability is hidden information for the government, and therefore the tax policy is restricted by the self-selection constraints. In the income uncertainty, or moral hazard, case, individual effort is hidden action. The question then is how the properties of commodity taxation and public good provision should be revised when informational asymmetry arises from unobserved effort.

A key finding in commodity tax analysis is the famous separability result of Atkinson and Stiglitz (1976), according to which there is no need for (differentiated) linear commodity taxation, if income is taxed on a non-linear scale and consumer preferences are separable between consumption and leisure. With these separable preferences, the Samuelson rule for public good provision remains valid also in a second-best framework with distortionary taxation as well. In this paper we examine whether these remarkable results carry over to the framework with income uncertainty.

As much of the moral hazard literature, this paper utilizes the so-called first-order approach for solving the optimization problem. In this approach, the incentive compatibility constraint is captured by a local maximization constraint of the individuals in a way shown below. Mirrlees (1975, 1999) was the first to point out that this procedure may not necessarily be

² For a recent justification for why the social insurance point of view is important, see e.g. Barr (2001).

valid and, consequently, may fail to provide a global optimum. In this paper, we first present the tax rules assuming that the first-order approach is valid. We then characterize sufficient conditions for the validity of the approach. Earlier analysis of the first-order approach – such as Rogerson (1985) and Jewitt (1988) – has almost exclusively concentrated on restrictive, additively separable, preferences between consumption and effort. However, commodity taxation and public good provision rules turn out to depend on, as in earlier tax analysis, whether the goods are complements or substitutes with effort. To see this, it is crucial to start working with non-separable preference structure. Building on the work by Alvi (1997), we therefore analyze the conditions under which the first-order approach is a valid solution procedure in the present framework with non-separable preferences and commodity taxation.

Section 2 of the paper presents our main model of mixed taxation – one with non-linear income taxation but linear commodity taxation. It also analyses rules for optimal marginal income tax rates. Section 3 concentrates on linear commodity tax rules and examines conditions when the Atkinson-Stiglitz (1976) rule applies in the current setting with income uncertainty. Section 5 examines public good provision while section 6 derives rules for fully non-linear commodity taxation. Section 7 concludes.

2. The basic set up

Consider an economy where the worker-consumer does not know what income he or she will receive for each possible level of effort. Thus the worker's gross income, z , depends randomly on his or her effort, y . The government can use two tax instruments, income tax and commodity taxation. Furthermore it provides a public good.

Once income z is realized, a worker-consumer pays income taxes according to schedule $T=z-c(z)$ and uses the rest of his or her income to choose the bundle of commodities x by maximizing utility. The worker is risk averse with a partially indirect utility function $V(q, c(z), g, y)$, where $V_c > 0$, $V_{cc} < 0$, $V_y < 0$ and $V_{yy} < 0$. The consumer chooses a bundle x conditional on his choice of $y \subseteq (y_L, y_H)$ (effort belongs to an interval from low to high) and expenditure on commodities, c . q denotes consumer prices and g the public good.

Let $f(z, y)$ and $F(z, y)$ denote the continuous density and distribution functions of income z given that effort y is undertaken by the worker; it is assumed that they are continuously differentiable for all z and y . The worker-consumer chooses effort y to maximize private expected utility

$$\int V(q, c(z), g, y) f(z, y) dz, \quad (1)$$

The necessary condition of (1) is

$$\int (V_y + V f_y / f) f dz = 0, \quad (2)$$

where $f_y / f = h$. It measures the impact of unobservable effort on the log-likelihood of income.

The government maximizes expected utility with respect to $c(z)$, y and q subject to the incentive compatibility condition (2) that individuals choose their effort optimally. This approach, where the incentive compatibility condition is captured by (2), is called the first-order approach. Mirrlees (1975, 1999 and 1976b), Rogerson (1985) and Jewitt (1988) provide

sufficient conditions which justify the first-order approach (FOA) when preferences are separable between effort and consumption. Alvi (1997) in turn provides sufficient conditions for the validity of the FOA in nonseparable cases. We sketch a proof of the sufficient characteristics of these conditions to see that the FOA is valid in our problem with commodity taxation in the Appendix. The following tax rules are valid provided that the first-order approach is a proper solution procedure.

To reiterate, the government maximises

$$\int V(q, c(z), g, y) f(z, y) dz \quad (3)$$

$$\text{s.t. } \int (V_y + Vh) f dz = 0 \quad (2)$$

and a revenue constraint which, for large identical population with independent and identically distributed states of nature, can be written in the form

$$\int (z - px(q, c(z), g, y) - rg) f(z, y) dz = 0 \quad (4)$$

where p denotes producer prices of commodities and r is the production price of the public good.

The Lagrangean function of the optimisation problem is

$$L = \int \{V + \alpha(V_y + Vh) + \lambda(z - px - rg)\} f(z, y) dz \quad (5)$$

The first-order conditions with respect to $c(z)$ (point-wise maximization), y and q are

$$\frac{\partial L}{\partial c(z)} = V_c(1 + \alpha h) + \alpha V_{yc} - \lambda p x_c = 0 \text{ for all } z \quad (6)$$

$$\frac{\partial L}{\partial y} = \lambda \int (z - px - rg) f_y dz + \alpha \int (V_{yy} f + 2V_y f_y + V f_{yy}) dz = 0 \text{ for } y \in (y_L, y_H) \quad (7)$$

$$\leq 0 \text{ for } y = y_L$$

$$\geq 0 \text{ for } y = y_H$$

$$\frac{\partial L}{\partial q} = \int \{V_q(1 + \alpha h) + \alpha V_{yq} - \lambda p x_q\} f dz = 0 \quad (8)$$

Let us first consider how disposable income is related to earned income, i.e. what is the shape of $c(z)$. Further differentiate (6) through with respect to z

$$V_{cc} c'(1 + \alpha h)^2 = c'(\lambda p x_{cc} - \alpha V_{ycc})(1 + \alpha h) - (\lambda p x_c - \alpha V_{yc}) \alpha h' \quad (9)$$

Using (6) and rearranging, (9) can be rewritten as

$$c' = \frac{\mu h'}{V_c^{-1}(-V_{cc}/V_c p x_c + p x_{cc}) - \mu V_c^{-2}(V_c V_{ycc} - V_{cy} V_{cc})} \quad (10)$$

Recall that $r(c(z), y) = \frac{-V_{cc}}{V_c}$ denotes the worker-consumer's absolute aversion to income risk

and $-\frac{1}{V_c^2}(V_c V_{cyy} - V_{cy} V_{cc}) = \frac{\partial r}{\partial y} \geq 0$ (when absolute risk aversion increases with effort). Note

also that, by an adding-up condition, $px_c = (q-t)x_c = 1-tx_c$. Finally, for ease of interpretation, we make a simplifying assumption that $x_{cc} = 0$ (Engel-curves are linear)³.

With these in mind, the marginal income tax rate may be written as follows

$$MTR = 1 - c' = 1 - \frac{\mu h'}{V_c^{-1} r(1 - tx_c) + \frac{\partial r}{\partial y}} \quad (11)$$

The marginal tax rate depends, first, on the likelihood ratio (h'). The optimal marginal income tax rate increases, *ceteris paribus*, if the likelihood ratio falls, in other words, the connection between effort and income weakens. In some sense, the efficiency cost of taxation diminishes. Second, the marginal tax rate increases, if consumers become more risk averse (i.e. r goes up) and therefore their valuation of insurance increases. These impacts are familiar from the earlier literature on income taxation in the moral hazard framework.

In the present set-up, the marginal tax rate also hinges on how the taxpayer's risk aversion varies with effort. Since $\partial r / \partial y \geq 0$, the latter term in the denominator of (11) implies that *ceteris paribus*, the marginal tax rate is higher compared to a situation where risk aversion does not interact with effort. This channel has not been considered in earlier tax analysis, which has focused on the case with separable preferences.

³ This assumption is motivated by assumptions needed for the validity for the first-order approach. See appendix for details.

The presence of commodity taxation also affects the marginal income tax, through the term $(1 - tx_c)$. If commodity taxes are on average positive, the marginal *income* tax rate is smaller than in the case where no commodity taxation is applied. This is because what matters is the effective marginal tax rate that also includes the commodity tax burden. This result shows that the connection between the marginal labour income tax and commodity taxation is similar to the one that has been studied earlier in the mixed taxation framework with ability differences but without uncertainty (Mirrlees 1976a, Tuomala 1990).

4. Optimal commodity taxation

From Roy's identity, $V_q = -xV_c$. Therefore, we can write in (8) the cross-derivatives

$$V_{yq} = V_{qy} = -xV_{cy} - x_yV_c \quad (12)$$

Using (12) and the Slutsky equation we have

$$\int \{-x(V_c(1+h) + \alpha V_{cy} - \lambda px_c) - \alpha x_y V_c - \lambda px_q^c\} f dz = 0 \quad (13)$$

Using (6) in (13) we have

$$\int -px_q^c f dz = \mu \int V_c x_y f dz \quad (14)$$

Each row of the matrix/vector equation (14) is

$$\int \sum_k x_{ik}^c t_k f dz = \mu \int V_c \frac{\partial x_i}{\partial y} f dz \quad (15)$$

Equation (15) is analogous to equation (86) in Mirrlees (1976a). The left hand side of (15) is a measure of the extent to which commodity taxes encourage consumption of the different commodities. $\frac{\partial x_i}{\partial y} < 0$ implies that the consumption of commodity i should be discouraged and $\frac{\partial x_i}{\partial y} > 0$ in turn implies that it should be encouraged.

The redistributive income tax system involves distortions because of the reduced effort. The government wants to enhance effort, and one way to do so is to encourage consumption of goods whose valuation increases with effort. In other words, it is optimal to use differentiated commodity taxation (i.e. introduce an additional distortion), if the additional distortion reduces the overall distortions of the tax system.

Note that there is an interesting analogy to the commodity tax rule derived in the standard optimal taxation framework, where the government cannot observe the income-earning ability of individuals. In other words, income-earning ability is hidden information. In the standard model of Mirrlees (1976a), the consumption of goods for which, at a given income level, highly-skilled people have a relatively strong taste should be discouraged. This policy reduces the value of leisure, encourages labour supply and may reduce the overall distortions of the tax system. In the present framework, effort is unobservable to the government, i.e. it is hidden action. Therefore, the government seeks to encourage effort by its commodity tax policy. The analogy is therefore related to how the tax policy aims to reduce the efficiency losses stemming from asymmetric information: in standard optimal tax setting from hidden information (ability) and in the present, income uncertainty, framework, hidden action (effort).

Finally, from (15) we see that if preferences are separable between consumption and effort, i.e. the indirect utility takes the form $V[v(q, c(z)), g, y]$, there is no need to use differentiated commodity taxation. Income taxation alone or income taxation with uniform commodity taxation would be sufficient to achieve redistributive aims with minimal efficiency losses. The famous separability result of Atkinson and Stiglitz (1976), derived under varying income-earning ability, is therefore valid also in the case with income uncertainty.

The conditions (listed in the Appendix) for when the first-order approach is a valid solution procedure are rather stringent. However, it seems that for the Atkinson-Stiglitz separability result, these conditions are not needed.⁴ To see this, denote $v(q, c(z)) = w(z)$. Using Roy's identity we have $x = \xi(q, w(z))$. Now the worker/consumer minimises the expenditure on goods with respect to prices, $\int pxf(z, y)dz = \int p\xi(q, w(z))f(z, y)dz$, subject to the condition that the consumer attains the maximum expected utility, $\int V[v(q, c(z)), g, y]f(z, y)dz = V^*$. For these preferences, the consumer gains nothing from commodity taxes. Costs are minimised if the consumer pays the producer prices only, i.e. there are no commodity taxes ($q = p$). Therefore, the Atkinson-Stiglitz separability result appears to be valid in the income uncertainty case even without the assumptions required for the first-order approach.

6. Public good provision

The first order condition with respect to the public good is

⁴ We are grateful to Jim Mirrlees for suggesting this possibility.

$$\frac{\partial L}{\partial g} = \int \{V_g(1 + \alpha h) + \alpha V_{yg} - \lambda p x_g - \lambda r\} f dz = 0 \quad (16)$$

Let the expenditure function be $E(q, v, g, y)$. Then we can define the marginal willingness to

pay for the public good, σ , by the expression $\sigma = -\frac{\partial E}{\partial g} = E_g$. Noting that

$$-E_g = -E_v V_g \quad (17)$$

Then, since $E_v V_c = 1$,

$$\sigma = -E_g = V_g / V_c \quad (18)$$

In (17) and (18) we have used the fact that there is a duality between the prices of private commodities and the quantity of public good and the willingness to pay for the public good.

In (18) we have used the analogous of the derivative property of the expenditure function and Roy's identity for the public good. Now we can write in (16) the cross derivatives

$$V_{yg} = V_{gy} = -E_g V_{cy} - E_{gy} V_c \quad (19)$$

where $E_{gy} = \frac{\partial E_g}{\partial y}$.

Using (19) and the Slutsky equation for the public good $x_g = x_g^c + x_c E_g$ we have

$$\int \{-E_g(V_c(1 + h) + \alpha V_{cy} - \lambda p x_c) - \alpha E_{gy} V_c - \lambda p x_g^c - \lambda r\} f dz = 0 \quad (20)$$

Again using (6) in (20) and noting that

$$p.x_g^c = (q - t)x_g^c = E_g - t.(x_g + x_c E_g) = -t.x_g + (1 - t.x_c)E_g \quad (21)$$

we have

$$\int \left\{ \alpha V_c \frac{\partial \sigma}{\partial y} - \lambda p x_g^c - \lambda r \right\} f dz = 0 \quad (22)$$

After some manipulation in the second term in (22) we rewrite it in the following form

$$r = \int \sigma f dz - \int \sigma t x_c f dz + \int t x_g f dz + \mu \int \frac{\partial \sigma}{\partial y} f dz \quad (23)$$

Equation (23) in turn is analogous to eq. (117) in Mirrlees (1976a). The marginal rate of transformation at the left should be equated to the sum of the marginal rate of substitution (the first term at the right), producing the Samuelson rule for public good provision, and a number of additional terms. The second and third terms at the right capture the impact of public good provision on commodity tax revenues. The key addition is the last term at the right: If the marginal rate of substitution increases with effort, the overall valuation of the public good (the right-hand side) increases as well. Other things equal, the public good should therefore be overprovided if it increases the value of effort.

There is again an analogy to the standard optimal income taxation framework. In Mirrlees (1976a), the public good provision rule is related to how the marginal willingness to pay varies with income-earning ability. In the present framework, the public good rule is related to how the marginal willingness to pay varies with unobserved effort.

It is also natural to ask in this set up when (23) reduces to the first-best Samuelson rule even under asymmetric information. Clearly it is sufficient for the Samuelson rule to be valid with optimal mixed taxation that preferences take the form $V[v(q, c(z), g), y]$. In other words if

indirect utility takes this form then we have both the Atkinson-Stiglitz (1976) result and the Christiansen (1981) result in the income uncertainty framework. Namely from the result of the Atkinson and Stiglitz (1976) such preferences imply that all commodity taxes are zero at the optimum, so that at full optimum the second and the third term of (23) vanish.⁵

6. Fully non-linear taxation

In the analysis of fully non-linear taxation, one can proceed with standard direct utility functions of the form

$$\int u(x, y) f(z, y) dz, \quad (24)$$

where x is a vector of many goods. The necessary condition of (24) is

$$\int (u_y + u f_y / f) f dz = 0, \quad (25)$$

The government can directly control x for a given income. The Lagrangean and the first-order condition for x_i are therefore

$$L = \int \{u(x, y) + \alpha(u_y + u h) + \lambda(z - px - rg)\} f(z, y) dz \quad (26)$$

$$u_{x_i} (1 + \alpha h) + \alpha u_{yx_i} - \lambda p_i = 0 \quad (27)$$

Dividing the optimality rule for x_i with one for another good, x_j , gives

⁵ In a similar way to the validity of Atkinson-Stiglitz result, for proving the validity of the first-best Samuelson rule in the second-best framework, one does not require the sufficient conditions needed for the first-order approach. Minimisation of $\int (px - rg) f(z, y) dz$ subject to $\int V[v(q, c(z), g), y] f(z, y) dz = V^*$ suggest that costs are minimised if the public good is provided at the producer price, i.e. r .

$$\frac{u_{x_i}}{u_{x_j}} = \frac{p_i - \mu u_{yx_i}}{p_j - \mu u_{yx_j}}, \quad (28)$$

where $\mu = \alpha/\lambda$. Note first that with separable preferences, $u_{yx_i} = 0$. This means that there ought to be no distortion between goods, in other words the marginal rate of substitution between i and j at the left should be equal to the marginal rate of transformation at the right. Therefore, a counterpart of the Atkinson-Stiglitz result holds under nonlinear commodity taxation as well.

If i is more complementary⁶ with effort than j is – or if i is a complement and j is a substitute with effort, the right-hand side of (28) goes down ($u_{yx_i} > 0$ and $u_{yx_i} > u_{yx_j}$). This means that the relative price of i decreases at the margin. Therefore its consumption is encouraged by the tax system, i.e. its marginal tax rate is negative. Likewise, the consumption of goods which are substitutes with effort is discouraged by a positive marginal tax rate.

This result is again connected with one derived by Mirrlees (1976) for the fully non-linear case without income uncertainty. In his analysis, ‘the marginal tax rates should be greater on commodities the more able would tend to prefer’ (Mirrlees 1976, p. 337). The marginal tax rates in Mirrlees (1976) are related to the skill level (hidden information), whereas here they are related to the effort level (hidden action).

⁶ Complementarity here must be interpreted as increasing the value (or decreasing the discomfort) of effort, since $u_y < 0$.

6. Conclusions

This paper considered optimal taxation in a moral hazard framework initiated by Mirrlees (1974), where income differences arise from uncertain returns to variable individual effort (labour supply). The government provides social insurance through non-linear taxation of income, but redistribution is constrained by asymmetric information between the government and individuals about individual effort.

We analysed the role of commodity taxation and public good provision in this framework. These instruments can be used as additional devices for social insurance: The consumption of goods which are positively related to effort should be encouraged through commodity taxation. A public good should be overprovided, *ceteris paribus*, if the marginal willingness to pay is positively related to effort. Similarly, consumption of goods, or public goods, whose valuation is negatively related to effort should be discouraged, or under-provided. These policies enhance effort and therefore help reduce the distortions from redistribution.

Unlike much of the earlier analysis on income in the moral hazard situation, we concentrated on situation where consumer preferences are non-separable between consumption and effort. The marginal income tax now includes a new term capturing the way how risk aversion interacts with effort. Because of this term, the marginal tax rate was shown to be higher, *ceteris paribus*, than in the case with no interaction between risk aversion and effort. We also analysed conditions when the solution procedure applied, the so-called first-order approach, is valid under non-separable preferences.

An interesting theoretical analogy with the more standard tax model of Mirrlees (1971, 1976a) emerged. In both cases, commodity tax policy and public good provision are geared towards reducing distortions from information asymmetry. In the conventional model, the informational asymmetry is about hidden information (income-earning ability), whereas in the social insurance model, effort is hidden action. However, the conditions on the validity of the optimisation procedure (i.e. the conditions for the first-order approach) that enable one to arrive at this analogy are rather more stringent in the social insurance model than in the conventional model.

The results can have interesting implications for practical tax policy. If income uncertainty is a key concern, the government should encourage, or publicly provide, goods that increase effort. Examples might include taxing lightly goods that are used in conjunction with labour supply (work uniforms, tools etc.) or public provision of work-related infrastructure. Despite a different modelling framework, these recommendations are not very far from those based on standard tax model, according to which public or private goods that are complements to labour supply should be supported (e.g. Boadway and Keen 1993, Edwards et al 1994).

Calculations of optimal tax rates that have been done (e.g. Tuomala, 1990) do not include commodity tax effects. An important area for further numerical research would be to analyse the quantitative importance of commodity taxes both with income uncertainty and without income uncertainty.

References

- Alvi, E. (1997) 'First-Order Approach to the Principal-Agent Problems: A Generalization', *The Geneva Papers on Risk and Insurance Theory* 22, 59-65.
- Atkinson, A.B. and Stiglitz, J.E. (1976) 'The design of tax structure: direct versus indirect taxation'. *Journal of Public Economics* 6, 55-75.
- Barr, N. (2001) *The welfare state as a piggy bank*. Oxford: University Press.
- Boadway, R. and M. Keen (1993) 'Public goods, self-selection and optimal income taxation', *International Economic Review* 34, 463-478.
- Christiansen, V. (1981) 'Evaluation of public projects under optimal taxation', *Review of Economic Studies* 48, 447-57.
- Cremer, H. and Gahvari, F. (1995) 'Uncertainty, optimal taxation and the direct vs indirect taxation controversy', *The Economic Journal* 105, 1165-79.
- Deaton, A. and J. Muellbauer (1980) *Economics and consumer behaviour*. Cambridge: University Press.
- Edwards, J., Keen, M. and Tuomala, M. (1994) 'Income tax, commodity taxes and public good provision', *FinanzArchiv* 51, 472-87.
- Jewitt, I. (1988) 'Justifying the first order approach to principal-agent problems', *Econometrica* 56, 1177-90.
- Low, H. and Maldoom, D. (2003) 'Optimal taxation, prudence and risk-sharing', *Journal of Public Economics*, forthcoming.

- Mirrlees, J.A. (1971) 'An exploration in the theory of optimal income taxation', *Review of Economic Studies* 38, 175-208.
- Mirrlees, J.A. (1974), 'Notes on welfare economics, information and uncertainty', in Balch, McFadden and Wu (Eds.), *Essays on Economic Behaviour under Uncertainty*, Amsterdam: North Holland.
- Mirrlees, J.A. (1975,1999) 'The Theory of Moral Hazard and Unobservable Behaviour. Part I', *Review of Economic Studies* 66, 3-22.
- Mirrlees, J.A. (1976a) 'Optimal tax theory. A synthesis'. *Journal of Public Economics* 6, 327-58.
- Mirrlees, J.A. (1976b) 'The optimal structure of authority and incentives within an organization', *Bell Journal of Economics* 7, 105-31.
- Rogerson, W. (1985) 'The First-Order Approach to Principal-Agent Problems', *Econometrica* 53,1357-67.
- Tuomala, M. (1984) 'Optimal degree of progressivity under income uncertainty', *Scandinavian Journal of Economics*,87,184-93.
- Tuomala, M. (1990) *Optimal income tax and redistribution*, Oxford: Clarendon Press.
- Varian, H. (1980), 'Redistributive taxation as social insurance', *Journal of Public Economics* 14, 49-68.

Appendix

This appendix examines sufficient conditions for the first-order approach to be valid in the problem at hand. For this, we make the following five assumptions. Assumptions 1-4 are familiar from Alvi (1997), and assumption 5 is needed because of the inclusion of multiple commodities.

1. Monotone likelihood ratio condition, MLRC: $\partial h / \partial z > 0$. Income is increasing stochastically in effort, that is, higher output is more likely for higher effort than lower effort.

The MLRC also implies the stochastic dominance condition (SDC), $F_y(z, y) \leq 0$.

2. Convex distribution function condition, CDFC: $F_{yy}(z, y) > 0$. This is like a diminishing returns conditions, applied to the production of information of worker's action.

3. Normality of leisure, $\frac{\partial(-\frac{V_y}{V_c})}{\partial c} > 0$. This is implied by $V_{yc} \leq 0$ and $V_{cc} < 0$.

4. Increasing effort increases absolute risk aversion (IEIARA), $\frac{\partial r}{\partial y} \geq 0$, where

$r(c(z), y) = \frac{-V_{cc}}{V_c}$ is the coefficient of absolute risk aversion.

5. Engel curves are either convex or linear. (Engel curve condition ECC). An increase in income must not increase consumption in a diminishing way, i.e. $x_{cc} \geq 0$.⁷ This assumption is not needed if commodity taxes are zero.

For the first-order conditions to provide a unique optimal solution, the problem in (1) must be concave. For this, as an intermediate step, we need to show that consumption is an increasing function of income.

Rewrite (6)

$$\frac{px_c}{V_c} - \mu \frac{V_{yc}}{V_c} = \frac{1}{\lambda} + \mu h \quad (\text{A.1})$$

where $\mu = \alpha / \lambda$. We first show that $\mu > 0$. If μ is non-positive, c is non-increasing in z . But then the worker has no incentives to provide effort and he therefore chooses $y = y_L$. Therefore for $y > y_L$, μ must be positive.

Now since $\mu > 0$, and because (a) $-\frac{V_{yc}}{V_c}$ is a non-decreasing function of c ⁸ and (b) $V_{cc} < 0$, the second term at the left-hand side of (A.1) is increasing with respect to $c(z)$. Assumption 5 guarantees that the first term is non-negative with respect to increasing $c(z)$. Therefore, the left-hand side of (A.1) is increasing in $c(z)$. The assumption (a) can also be written

⁷ The so-called Gorman polar form is an example where no other restrictions are imposed on utility other than the linearity of Engel-curves (Deaton and Muellbauer 1980).

⁸ Mirrlees (1976) also makes this assumption.

$$\frac{\partial(-\frac{V_{yc}}{V_c})}{\partial y} = -\frac{1}{V_c^2}(V_c V_{ycc} - V_{yc} V_{cc}) \geq 0. \text{ The condition (a) is implied by assumption 4, i.e.}$$

IEIARA, $V_{ycc} \leq 0$ and $V_{yc} \leq 0$.

The MLRC guarantees that the right hand side of (A.1) is also increasing in z . Thus, $c(z)$ is increasing in z .

The optimisation problem (1) will be concave if

$$\int (V_{yy}f + 2V_y f_y + V f_{yy}) dz < 0 \quad (\text{A.2})$$

Integrating by parts we obtain

$$\int V_{yy}f dz - \int V_{yc} c' F_y dz - \int V F_{yy} dz \leq 0 \quad (\text{A.3})$$

The first term is non-positive by assumption of the utility function. Since we showed that $c' \geq 0$, and since $V_{yc} \leq 0$ (normality of leisure), the second term is non-positive. CDFC implies that the third term is non-positive. Expected utility is concave in y , thus validating the FOA in our problem.