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RESEARCH ARTICLE

Multi-Site Wind Farms Dependence Structure Using Vine Copulas: Impacts of Dataset Sizes and Employed Copulas

AMIR SHAHIRINIA[1], (Senior Member, IEEE), ZEINAB FARAHMANDFAR[2], MOHAMMAD TAVAKOLI BINA[3], (Senior Member, IEEE), HOSSEIN HAFEZI[4], (Senior Member, IEEE), AND VINCENT TANOE[1]

[1]School of Engineering and Applied Sciences, University of the District of Columbia, Washington, DC 20008, USA
[2]Department of Civil and Systems, Johns Hopkins University, Baltimore, MD 21218, USA
[3]Faculty of Electrical Engineering, K. N. Toosi University of Technology, Tehran 16317-14191, Iran
[4]Faculty of Information Technology and Communication Sciences, Tampere University, 33100 Tampere, Finland

Corresponding author: Hossein Hafezi (hossein.hafezi@tuni.fi)

ABSTRACT Using copulas in statistics evaluates the dependence between random variables. Copula modeling has significantly been used in many areas, especially in the search for multivariate distributions. As wind energy rapidly becomes an important renewable energy source, it is very important to deeply evaluate any potential existing dependencies among the data. This study introduces a comprehensive application of vine copulas in modeling multi-site wind speed dependencies for different sizes of datasets, offering a more flexible approach than traditional correlation-based methods. Unlike previous studies, this work systematically evaluates the impact of dataset sizes on the selection of the best-performing vine copula structures, providing valuable insights for improving wind forecasting and grid stability. These evaluations are studied using R-vine, C-and D-vine models by applying pair copulas families and the pairwise empirical Kendall's τ values, where the appropriate model is selected based on the Akaike Information Criteria (AIC), the Bayesian Information Criteria (BIC), and the likelihood method (Log-likelihood). This study finds out the best vine copula model for three different wind speed datasets, namely hourly (large), daily (medium), and small (weekly). Also, using all pair copulas families (Clayton, Frank, Gumbel, Student's t, and Gaussian), vine copula simulations provide guidelines for conceptual understanding of mutual impacts and correlations among multi-site wind farms. Simulations also show that the R-vine copula is the best structure for both large and small datasets, while the C-vine copula is the best structure for medium datasets.

INDEX TERMS R-vine copulas, C-and D-vine copulas, Kendall's τ, frank, Gaussian, clayton, Gumbel, student's t, AIC, BIC.

LIST OF ABBREVIATIONS

AIC	Akaike Information Criterion.
BIC	Bayesian Information Criterion.
CDF	Cumulative Distribution Function.
GoF	Goodness of Fit.
KS	Kolmogorov-Smirnov.
PDF	Probability Density Function.
R-Vine	Regular Vine Copula.
C-Vine	Canonical Vine Copula.
D-Vine	Drawable Vine Copula.
MLE	Maximum Likelihood Estimation.
KDE	Kernel Density Estimation.
τ (Tau)	Kendall's Tau (Rank Correlation Coefficient).
LL	Log-Likelihood.
PIT	Probability Integral Transform.
EVT	Extreme Value Theory.

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I. INTRODUCTION

Copulas have widely been used as one of the greatest statistical tools to study the relationships among several random variables by focusing on the joint cumulative density of quantiles of marginals. This is important since the relationship of the random variables can be directly studied while the idiosyncratic features of marginal distributions are bypassed. Although the copula models are widely applied in various fields, including finance, where they are used for risk management and asset dependency modeling in trading strategies, also a wide range of researchers, including wind speed analysts, have been exploiting copulas for the past decade to capture the dependency between mostly random variables [1], [2]. Although, the application of copula is common among stock market traders while exploiting the statistical arbitrage for the pairs trading framework, also a wide range of researchers, including wind speed analysts, have been exploiting copulas for the past decade to capture the dependency between mostly random variables [1], [2]. However, copulas are not limited to only two dimensions, expandable to arbitrarily larger dimensions than two [3], [4].

However, the weakness on the practical side is that there is a limited number of created Copulas with the larger dimensions available with a closed-form analytical expression [5], [6]. So, for only two dimensions, the existing bivariate Copulas can model the relationship between a pair of random variables reasonably well. However, for higher dimension models, the Copulas tend to lose a lot of useful details and become rigid [5]. Therefore, the higher dimensional probabilistic modeling of multiple random variables problem can be addressed by adopting vine copulas (VC) [7]. In multivariate statistical modeling, a conditional dependence structure refers to the way dependencies among multiple variables evolve when conditioned on certain known variables. Unlike simple correlation measures, conditional dependence structures allow for a more refined understanding of interrelationships by considering how variables influence each other in different conditions. A pair copula decomposition is a technique used in vine copula modeling, where complex multivariate dependencies are broken down into simpler, two-variable relationships. This decomposition allows for more flexible modeling, capturing dependencies that traditional methods might overlook, particularly in cases involving nonlinear or asymmetric relationships. By using pair copulas, we can systematically construct higher-dimensional dependency models with improved accuracy and interpretability. Hence, using VCs enables one to first decompose the probability density into conditional probabilities and then the conditional probabilities into bivariate Copulas. Therefore, instead of using d-dimensional Copula, the pdf is decomposed into marginal densities and bivariate Copulas [8] and [9]. Pair-copulas are multivariate copula that is constructed from a set of bivariate ones. The construction of pair-copulas decomposes a multivariate probability density into bivariate copulas, where each pair-copula can be independently selected from the others. This allows for wide flexibility of the dependence modeling. Asymmetries and tail dependence can be considered as well as independence to build more parsimonious models [8].

Representing a joint distribution as univariate margins, copulas allow the separation of the problems of estimating univariate distributions from the problems of estimating dependence. Vine copulas combine the advantages of multivariate copula modeling. The necessity of this study is related to obtaining correct correlations among multi-site wind farms by vine copulas in comparison with multivariate copula modeling using bivariate ones. In [9] authors have developed from vines copulas two statistical inference techniques for the two classes of Canonical Vine (C-Vine), and Drawable Vine (D-Vine). By analyzing, the special dependence in the wind and optimal wind power allocation, authors in [10] used a copula analysis of stochastic dependencies of the wind speed for a large data set of German on-and offshore weather stations and find that these dependencies turn out to be highly nonlinear but constant over the time. Using copula models helped them determine the value at risk of energy production for given allocation sets of wind farms and derive an optimal allocation plan. Authors in [11] discussed the application of multivariate T-copula in software project management and analyzed some risk factors that may lead to software project failure. Good performance of VCs is shown in [12] and [13] compared to alternative multivariate copulas in two comparative analyses. authors in [14] focus on the structure of the spatial dependence of wind speed using bivariate copula for a large number of wind speed sites. authors in [15] emphasize the importance of modeling wind speed uncertainty in applications such as power flow studies using Bayesian inference to estimate the copula parameter for each wind farm pair under weekly, daily, and hourly wind speed observations. and shows that a smaller volume of data will have more uncertainty in copula parameter estimation. Authors in [16] use vine copula to model the multivariate distribution of multiple soil parameters and show better performances than the conventional Gaussian model. In [17], the authors review the basic ideas underlying the copula model and present estimation and model selection approaches. Authors in [18] used the C-vine copula model to study the risk dependence structure of internet money funds and then introduces the time varying T-copula model to analyze the risk spillover of diverse IMFs. In [20], a new algorithm was developed to encode complete and truncated vines in a matrix, enabling the storage of vine information in a virtual environment. The conditional independence structure encoded by a vine can be graphically represented. When a perfect elimination ordering of a vine structure is given, it can be uniquely represented with a matrix. In [21], the vine copula model is employed to analyze the complex interdependencies among different segments of the technology industry. The results indicate that the C-vine model demonstrates superior effectiveness in capturing the dependence structures within the dataset, outperforming both the R-vine and D-vine structures.

Reference [22] reveals moderate to strong positive dependence, best modeled by the C-vine structure, with Microsoft as a central variable exerting influence on the remaining companies. In [23], the study examines the dynamic upper and lower tail dependence across rare earth metals, clean energy, gold, world equity, base metals, and crude oil markets at various time scales. The results show that, for raw returns, the rare earth market moderates the positive dependence between world equity and clean energy markets. At the short-term time scale, unlike other pairwise dependencies, rare earth eases the dependency between clean energy markets. In [24], a multivariate probabilistic framework using an R-vine copula is proposed for nuclear plants' fault detection and diagnosis (FDD). Instead of feature extraction, it uses variables' dependence descriptions for monitoring. In [25], quantile regression related to D-vine copulas is utilized. This approach is highly data-driven and allows for adopting more general dependence structures compared to the state-of-the-art zero-truncated ensemble output statistic (tEMOS) model.

Authors in [19] apply dynamic R-vine copulas to capture the evolving mortality dependence. Dynamic R-vine is obtained by allowing time-varying dependence parameters for the bivariate copulas associated with an R-vine. Multivariate distributions can be expressed in terms of simpler expressions using conditional probability Suppose $\boldsymbol{f}$ denotes the joint pdf for two continuous random variables $\boldsymbol{X_1} = \boldsymbol{x_1}$ and $\boldsymbol{X_2} = \boldsymbol{x_2}$ as follows:

$$\begin{aligned} f\left(x_1, x_2\right) &= f_{1|2}\left(x_1|x_2\right) f_2\left(x_2\right) = f_{2|1}\left(x_2|x_1\right) f_1\left(x_1\right) \\ &= \frac{\partial^2 C_{1,2}\left(F_1\left(x_1\right)\ F_2\left(x_2\right)\right)}{\partial x_1 \partial x_2} \\ &= c_{1,2}\left(F_1\left(x_1\right),\ F_2\left(x_2\right)\right) f_1\left(x_1\right) f_2\left(x_2\right), \end{aligned} \quad (1)$$

where F_1, F_2 denote the CDFs while $f_1(x) = \frac{\partial F_1(x)}{\partial x}$, and $f_2(x) = \frac{\partial F_2(x)}{\partial x}$, and $c_{1,2}$ is a bivariate density with support $[0.1]\times$ [1, 0]. Hence, the probability subspace is conditioned on the third random variable, $X_3 = x_3$, can be written as below:

$$\begin{aligned} f_{1|2,3}\left(x_1|x_2, x_3\right) &= \frac{f\left(x_1, x_2|x_3\right)}{f_{2|3}\left(x_2|x_3\right)} \\ &= c_{1,2|3}\left(F_{1|3}\left(x_1|x_3\right), F_{2|3}\left(x_2|x_3\right)\right) \\ &\quad \times f_{1|3}\left(x_1|x_3\right), \end{aligned} \quad (2)$$

where, $F_{1|3} = P\left(X_1 \leq x_1 | X_3 = x_3\right)$, and $c_{1,2|3}$ is bivariate copula with the conditioned two random variables X_1 and X_2 on $X_3 = x_3$. This approach decomposes the probability density into a bivariate copula and marginal densities. Using conditional probabilities for a three-dimensional pdf $f\left(x_1, x_2, x_3\right)$, there are six ways to decompose it; for example, one of the decompositions is given as below:

$$\begin{aligned} f\left(x_1, x_2, x_3\right) &= f_{1|2,3}\left(x_1|x_2, x_3\right) f_{2|3}\left(x_2|x_3\right) f_3\left(x_3\right) \\ &= c_{1,2|3}\left(F_{1|3}\left(x_1|x_3\right), F_{2|3}\left(x_2|x_3\right)\right) f_{1|3}\left(x_1|x_3\right) \\ &\quad \times f\left(x_2, x_3\right), \end{aligned} \quad (3)$$

where combining (1) and (2) with (3) results in:

$$\begin{aligned} f\left(x_1, x_2, x_3\right) &= f_3\left(x_3\right) \times c_{2,3}\left(F_2\left(x_2\right), F_3\left(x_3\right)\right) \\ &\quad f_2\left(x_2\right) c_{1,3}\left(F_1\left(x_1\right), F_3\left(x_3\right)\right) f_1\left(x_1\right) \\ &\quad \times c_{1,2|3}\left(F_{1|3}\left(x_1|x_3\right), F_{2|3}\left(x_2|x_3\right)\right), \end{aligned} \quad (4)$$

Although many models have been applied to analyze the dependencies between the wind speeds at multiple locations, the holistic important aspect of evaluating the appropriate VC models for the dependence analysis of the wind speed data needs to be addressed. It must be noted that traditional correlation-based methods, such as Pearson correlation, assume a linear relationship between variables and are limited in their ability to capture complex dependencies, particularly in cases where nonlinear interactions or tail dependencies exist. In contrast, vine copula models provide a flexible framework that allows for the modeling of both linear and nonlinear dependencies, as well as asymmetric tail dependencies. This advantage is particularly important in wind speed data, where extreme events (such as sudden drops or spikes in wind speed) may exhibit stronger dependencies that traditional correlation measures fail to capture. By using vine copulas, we can better model these intricate relationships and improve the accuracy of dependency structures, ultimately leading to more robust probabilistic wind forecasts and risk assessments.

This current research aims to use different classes of VCs, namely Regular R-Vine, C-Vine, and D-Vine, to select the consistent class with different sizes of wind speed data. The application of the VC techniques on wind speed data is essential since it helps to find the degree to which a pair of these wind speed data variables are linearly related. For instance, by analyzing the first tree of the VC outcome, one can find any severe wind speed fluctuations in one wind farm, which would conclude the most likely wind speed fluctuations in other wind farms under study [26]. Moreover, the VC could be used in power flow analysis when several random data sources are present in the power system, which could be used in future research. The contribution of this paper includes finding the impacts of dataset size and the type of copulas on modeling correlation structure in multi-site wind farms. As a case study, wind speed data of eight wind farms are simulated using the VC, and the correlation modeling is shown by different trees. Interpretation of outcome trees introduces how changes on one combination of sites result in variation on other combinations. The remainder of the paper is organized as follows: Section II provides the proposed stepwise methodology. In Section III, the data and methodology are covered. Section IV presents the simulations and analysis of the data, and the results are interpreted. Section VI concludes the paper and future works.

II. THE PROCEDURE OF PERFORMED STUDY

This paper will analyze wind speed dependencies in the following steps by working on eight wind speed data variables from the NREL databases.

TABLE 1. Descriptive statistics of three different wind speed datasets m/*Sec*., (a) Hourly, (b) Daily, and (c) Weekly.

US site	HUMBOLDT (H)	NED POWER (NP)	CRITERION WIND PARK (CWP)	ROTH ROCK (RR)	FREY FARM (FF)	BEAR CREEK (BC)	MOUNTAINEER (M)	LOCUST RIDGE (LR)
Index	1	2	3	4	5	6	7	8
Minimum	0.12	0.15	0.15	0.1	0.04	0.21	0.16	0.05
Mean	7.656	7.664	8.23	7.046	5.731	7.665	8.755	5.407
Median	7.355	7.15	7.605	6.585	5.35	7.27	7.68	4.85
Maximum	26.66	23.28	26.02	22.3	22.17	28.51	29.1	26.23

(a)

US site	HUMBOLDT (H)	NED POWER (NP)	CRITERION WIND PARK (CWP)	ROTH ROCK (RR)	FREY FARM (FF)	BEAR CREEK (BC)	MOUNTAINEER (M)	LOCUST RIDGE (LR)
Index	1	2	3	4	5	6	7	8
Minimum	1.82	1.96	2.26	1.6	1	2.14	2.17	1.19
Mean	7.656	7.664	8.23	7.046	5.732	7.665	8.755	5.408
Median	7.3	7.1	7.46	6.49	5.32	7.37	8.1	5
Maximum	17.5	17.9	21.31	16.49	14.34	18.77	25.76	18.87

(b)

US site	HUMBOLDT (H)	NED POWER (NP)	CRITERION WIND PARK (CWP)	ROTH ROCK (RR)	FREY FARM (FF)	BEAR CREEK (BC)	MOUNTAINEER (M)	LOCUST RIDGE (LR)
Index	1	2	3	4	5	6	7	8
Minimum	4.03	4.01	4.22	3.52	2.97	4.25	3.75	2.86
Mean	7.718	7.758	8.325	7.136	5.737	7.665	8.788	5.407
Median	7.5	7.47	7.88	6.67	5.66	7.61	8.46	5.21
Maximum	11.72	13.43	14.64	12.63	9.59	11.87	17.34	9.84

(c)

- Use the correlation matrix to evaluate the relationship between each size of the data and the boxplots to see how data are spread out.
- Transform the wind speed data to copulas data to have margins that are distributed as standard normal and use the boxplots on the transformed data (The standard normal transformation ensures that all marginal distributions have a common scale, allowing copula-based dependency modeling to be independent of the original data distribution. This approach is widely used in copula applications to facilitate statistical inference and improve model consistency)
- Apply Kendall's τ to measure the dependencies among the different sizes of the wind speed copulas data
- Apply each copulas model (R-vines, C-Vines, and D-Vines) on the copulas data using the vine tree structure
- Archimedean Copulas and Elliptical copulas are used as pair copula families fitted in R-vine, C-, and D-vine
- Apply the AIC, the BIC, and log-likelihood to select the best vine copulas model

Evaluate how efficient the selected vine copula family is in predicting the samples of the joint distribution compared to the real data.

III. METHODOLOGY AND DATA

This paper uses three different sizes of wind speed datasets from eight different sites across the US downloaded from NREL. The descriptive statistics of all datasets are shown in Table 1, while the following subsections show assessing tools for the available data. The assumption of independent and identically distributed (iid) data is a common limitation in copula modeling. Wind speed data, however, often exhibit seasonal trends and temporal dependencies. To mitigate these effects, the marginal distributions were examined for stationarity before applying the copula transformation. Additionally, a de-trending process was performed where necessary to remove seasonal biases, ensuring that the dependency structures captured by the vine copula models were not artifacts of long-term trends. Future studies may explore dynamic copula approaches that explicitly account for time-varying dependencies.

It must be noted that there are several software packages, such as the "VineCopula *R*" package and other established libraries, available for estimating and selecting vine copula models. These tools provide automated methods for determining the best-fitting copula structure based on statistical criteria such as AIC and BIC. However, we encountered computational challenges in handling a large number of wind farms using the "***R***″ package. The complexity of our model, especially in managing multiple wind farm sites, required more scalable computational capabilities. To overcome these limitations, we developed custom Python-based implementations, allowing us to efficiently analyze large-scale dependency structures. Furthermore, this study contributes to the literature by systematically evaluating the impact of dataset size on vine copula selection, which has not

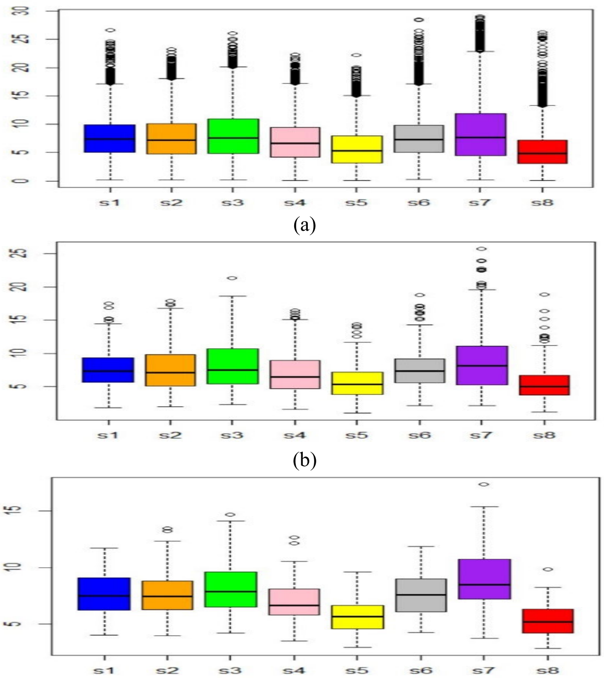


FIGURE 1. Boxplot of the three wind speed data gathered from NREL, (a) exact hourly data, (b) exact daily data, and (c) exact weekly data.

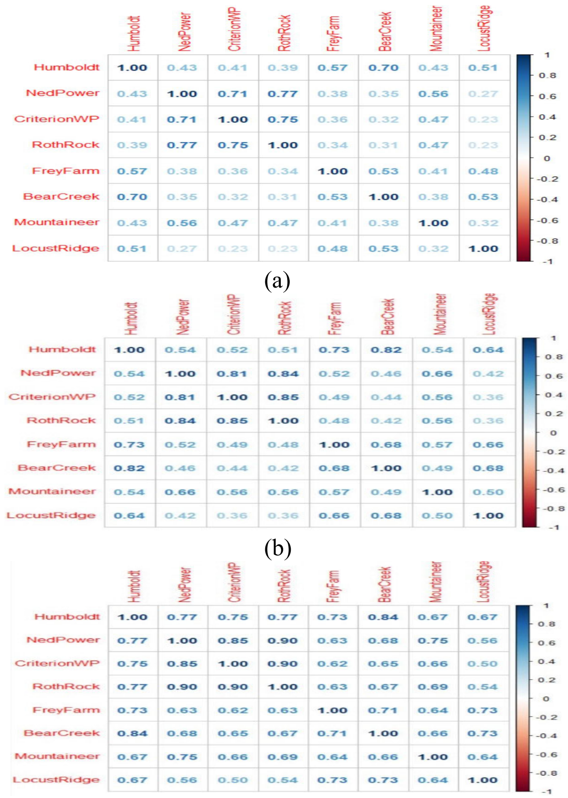


FIGURE 2. Kandall's τ dependencies of copula wind speed data, (a) hourly, (b) daily, and (c) weekly.

been comprehensively explored in previous works. By examining hourly (large), daily (medium), and weekly (small) datasets, this study provides insights into how dependency structures vary with temporal resolution, a key consideration for wind power forecasting and grid stability. t also should be noted that to assess the similarity of wind speed variations across locations, we performed the KS test on the first-differenced wind speed data. This approach ensures that statistical comparisons are not biased by long-term trends or seasonal fluctuations. The use of first-differencing helps transform the data to a stationary form, making it more suitable for dependency modeling and hypothesis testing.

A. CORRELATION MATRIX AND KENDALL'S τ

The correlation matrix was used in this study to select the correlation coefficients among the sets of wind speed variables. The correlation measures the degree of associated linearity between two continuous variables. For better visualization of the data, a boxplot is introduced to illustrate a good indication of how the values in the data are spread out. It can be seen from Fig. 1 that all variables contain outliers, and the data are not normally distributed. The data was then transformed by copulas for normalization. This will help us to model the margins separately from the dependence structure. In other words, all variables are transformed to be uniform to capture the strong dependence structures between wind speed data variables without any effect on the margins.

Figures 1 (b), (d), and (f) show the transformed data do not have outliers, and the data are well distributed. To analyze the dependencies within the wind speed data, the pairwise empirical Kandall's $\boldsymbol{\tau}$ has been applied. This is a non-parametric measure of dependencies that ranked data. The range of values is the interval $[-1, 1]$. Kendall's $\boldsymbol{\tau}$, denoted by $\boldsymbol{\tau}$, is the probability of concordance minus the probability of the discordance of two random variables $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ which is defined as:

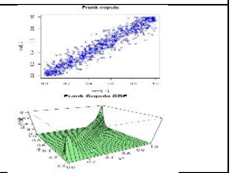
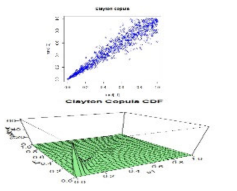
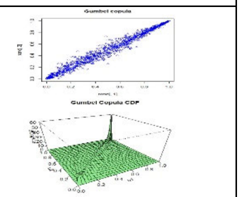
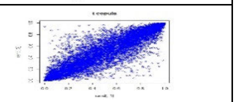
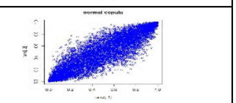
$$\tau\left(X_1, X_2\right) = P\left(\left(X_{11} - X_{21}\right)\left(X_{12} - X_{22}\right) > 0\right) - P\left(\left(X_{11} - X_{21}\right)\left(X_{12} - X_{22}\right) < 0\right), \qquad (5)$$

Hence, the dependencies for the three datasets are worked out, where Figs. 2 (a) – (c) shows the resulting τ matrices for the three datasets (large, medium, and small).

B. COMPARING REGULAR VINE MODELS

To compare the fitness of more specified regular vine copula for a given copula dataset, the vine tree structure $\mathcal{V}$ and the set of pair copula families $\mathcal{B}(\mathcal{V}) = \{\mathcal{B}_e | i = 1, \ldots, d-1; e \in E_i\}$ is considered. The set of corresponding copula parameters are denoted by $\theta(\mathcal{B}(\mathcal{V}))$. Using a random sample $u = (u_1^T, \ldots, u_n^T)^T$ of size n from the specifications of the model with the Kth observation $u_k \in [0, 1]^d$, the associated likelihood for the parameters $\theta(\mathcal{B}(\mathcal{V}))$ in a regular vine model with a vine structure $\mathcal{V}$ and pair copula family set $\mathcal{B}(\mathcal{V})\}$ is

TABLE 2. Different types of both the AC and EC.

Copulas	Features	Illustration of dependence feauture
AC: Frank	Symmetrical tail $0 < \Theta < \infty$ dim = 2 $-\infty < \Theta < \infty$ dim ≠ 2	
AC: Clayton	high lower tail dependence $0 < \Theta < \infty$ dim ≠ 2 $-1 < \Theta < \infty$ dim = 2	
AC: Gumbel	high upper tail dependence $1 < \Theta < \infty$	
EC: Student's *t*	showing elliptical diagonal symmetry $0 < \Theta < 1$	
EC: Normal (Gaussian)	showing extreme tail independence $0 < \Theta < 1$	

defined as:

$$\begin{cases} \ell\left(\theta\left(\mathcal{B}\left(\mathcal{V}\right)\right);u\right) = \prod_{k=1}^{n} \ell_k\left(\theta\left(\mathcal{B}\left(\mathcal{V}\right)\right);u_k\right), \text{ where} \\ \ell_k\left(\theta\left(\mathcal{B}\left(\mathcal{V}\right)\right);u_k\right) := \prod_{i=1}^{d-1} \prod_{e\in E_i} \\ \qquad c_{a_e,b_e;D_e}, \left(C_{a_e|D_e}\left(u_{k,a_e} \mid u_{k,D_e}\right),\right. \\ C_{b_e|D_e}\left(u_{k,a_e} \mid u_{k,D_e}\right) \end{cases} \tag{6}$$

where $\ell_k\left(\theta\left(\mathcal{B}\left(\mathcal{V}\right)\right);u_k\right)$ is the likelihood contribution of the kth observation u_k. This study uses the classical-based model AIC to compare regular vine copulas.

For a random copula dataset sample u of size n from s regular vine copula with triplet $\mathcal{V}(\mathcal{B}\left(\mathcal{V}\right), \theta(\mathcal{B}\left(\mathcal{V}\right)))$, the AIC for the complete R-vine copulas specification is defined as:

$$AIC_{\mathrm{RV}} = -2\sum_{k=1}^{n} \ln(\ell_k(\hat{\theta}\left(\mathcal{B}\left(\mathcal{V}\right);u_k\right) + 2k, \tag{7}$$

and the BIC is

$$BIC_{\mathrm{RV}} = -2\sum_{k=1}^{n} \ln(\ell_k(\hat{\theta}\left(\mathcal{B}\left(\mathcal{V}\right);u_k\right) + 2k + \ln(n)K, \tag{8}$$

where k is the number of model parameters, i.e., the length of the vector $\theta(\mathcal{B}\left(\mathcal{V}\right))$ and $\hat{\theta}(\mathcal{B}\left(\mathcal{V}\right))$ is the estimation of $\theta(\mathcal{B}\left(\mathcal{V}\right))$.

C. ARCHIMEDIAN COPULAS (AC)

Archimedean copulas are an important class of copulas because of the simple way in which they can be built and the nice properties they possess. In bivariate Archimedean copulas, let Ω be the set of all continuous, strictly monotone decreasing, and convex functions $\varphi : I \to [0, \infty]$ with $\varphi\left(1\right) = 0$. Let $\Omega\varphi \in$, then:

$$C\left(u_1, u_2\right) = \varphi^{[-1]}(\varphi\left(u_1\right) + \varphi\left(u_2\right)), \tag{9}$$

where the function C is called a bivariate Archimedean copula with the generator. If $\varphi\left(0\right) = \infty$, then the generator is called *strict*. Here $\varphi^{[-1]}$ is the pseudo-inverse of φ, which is defined as:

$$\varphi^{[-1]}\left(t\right) := \begin{cases} \varphi^{-1}\left(t\right), & \leq t \leq \varphi(0) \\ 0, & \varphi\left(0\right) \leq t \leq \infty \end{cases}, \quad \varphi^{[-1]} : [0, \infty] \to [0, 1], \tag{10}$$

This study uses three commonly Archimedean copulas: Clayton, Gumbel, and Frank. Table 2 describes these copulas regarding their features, mathematical relationships, and typical outcomes that illustrate dependence features.

D. ELLIPTICAL COPULAS (EC)

The elliptical copula is defined via Sklar's theorem as the dependence structure of related elliptical distribution. It is obtained from the respective multivariate distribution function by standardizing the univariate marginal laws. The commonly used elliptical distributions are the multivariate normal (Gaussian) and Student's *t* distributions. In this paper, both the Archimedean and the Elliptical copulas are applied as pair copula families (see Table 2). The R-vine, C-, and D-vine are fitted sequentially with these pair copula families; both AIC and BIC criteria and likelihood methods are applied to select the best structure and model for the pair copula families.

IV. SIMULATIONS AND ANALYSIS OF DATASETS

The primary objective of fitting vine copula models is to gain insights into the dependency structure among wind speed data across multiple sites. By capturing both linear and nonlinear dependencies, the models enable a more accurate representation of wind farm correlations, which is crucial for short-term forecasting and risk assessment. The fitted dependency structures reveal how wind speed fluctuations at one site influence others, providing valuable information for grid operators in managing wind power integration. Additionally, understanding these dependencies aids in resource allocation and investment planning for wind energy projects. To better analyze the dependence structure among the normalized wind speed data (Copula's wind data), the R-vine, C-, and D-vine copulas are examined for all datasets. These vine copulas are fitted sequentially with the pair copulas

TABLE 3. Comparing three datasets for finding out the best vine structure according to the AIC, BIC and log-likelihood.

Pair copula group	Vine structue	Large dataset			Vine structue	Medium dataset			Vine structue	Small dataset		
		log-likelihood	AIC	BIC		log-likelihood	AIC	BIC		log-likelihood	AIC	BIC
a	R-vine	38143.48	-76220.96	-75987.39	D-vine	2637.29	-5208.57	-5079.88	R-vine	465.07	-870.13	-811.03
a	C-vine	37640.67	-75211.34	-74963.61	C-vine	2637.55	-5209.11	-5080.41	C-vine	439.37	-816.74	-755.66
b	R-vine	38015.06	-75950.12	-75667	D-vine	2617.58	-5161.17	-5016.87	R-vine	454.96	-821.92	-735.22
b	C-vine	37682.14	-75280.27	-74983	C-vine	2633.71	-5191.43	-5043.23	C-vine	453.12	-828.24	-751.4
c	R-vine	37649.49	-75242.97	-75044.79	D-vine	2590.44	-5124.88	-5015.68	R-vine	455.72	-855.44	-800.27
c	C-vine	37404.13	-74752.26	-74554.07	C-vine	2592.16	-5128.32	-5019.13	C-vine	449.69	-843.39	-788.22

TABLE 4. Z different R-vine dependence structures for large datasets with selected copula and tree diagram.

families Archimedean (Clayton, Gumbel, and the Franck) and Elliptical (Gaussian and Student's t).

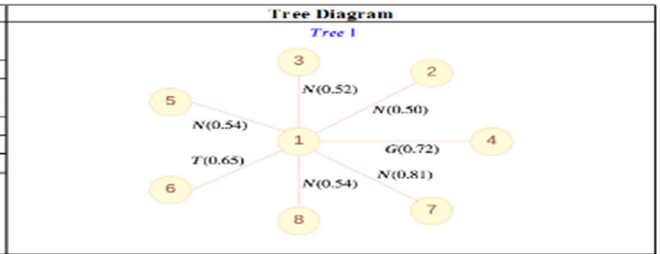
A. R-VINE, D-VINE, AND C-VINE COPULAS

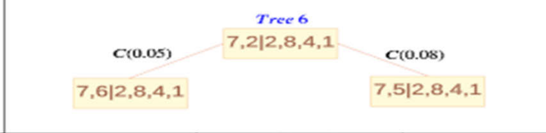
Here, three groups are considered for implementing pair copula families among copulas, as listed in Table 2, for analyzing three vine structures. The first group (group 'a') includes five copulas, three Archimedean, and two Elliptical copulas: Clayton, Frank, Gumbel, Gaussian, and Student's t. The second group (group 'b') includes three copulas, one Archimedean and two Elliptical copulas: Frank, Gaussian, and Student's t. The third group (group 'c') includes Archimedean and Elliptical copulas, namely, Frank and Gaussian.

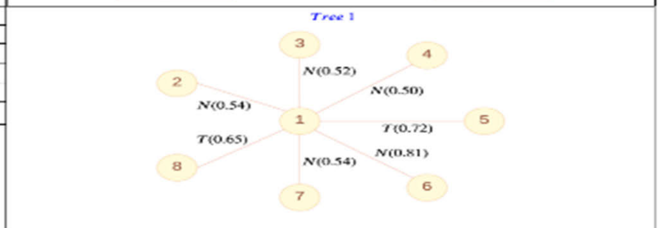
TABLE 5. Different C:-Vine dependence structure for medium dataset with selected copula and tree diagram.

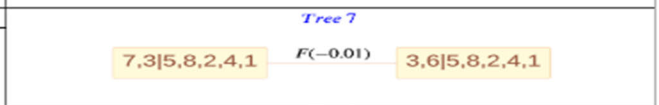
Pair copula group	Sites	Copula	Tau (τ)
'a'	1,5	Gumbel(3.51)	0.72
	1,3	Normal(0.73)	0.52
	1,6	Normal(0.96)	0.81
	1,2	Normal(0.75)	0.54
	1,7	Normal(0.75)	0.54
	1,4	Normal(0.71)	0.5
	8,1	T(0.85,9.7)	0.65
'a'	7,6|2,8,4,1	Gumbel90(-1.04)	-0.04
'b'	1,6	Normal(0.96)	0.81
	1,3	Normal(0.73)	0.52
	1,5	T(0.91,5.92)	0.72
	1,2	Normal(0.75)	0.54
	1,7	Normal(0.75)	0.54
	1,4	Normal(0.71)	0.5
	8,1	T(0.85,9.7)	0.65
'b'	7,6|3,5,8,2,4,1	Frank(-0.09)	-0.01
'c'	1,6	Normal(0.96)	0.81
	1,3	Normal(0.73)	0.52
	1,5	Normal(0.9)	0.72
	1,2	Normal(0.75)	0.75
	1,7	Normal(0.75)	0.54
	1,4	Normal(0.71)	0.5
	1,8	Normal(0.85)	0.65
'c'	7,6|3,5,8,2,4,1	Frank(0.36)	-0.02

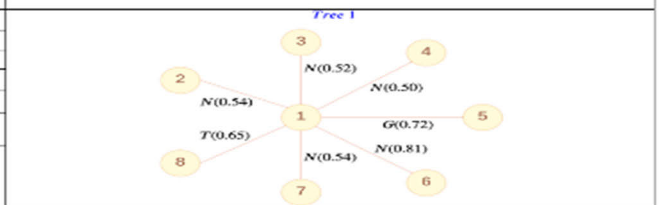
Tree Diagram











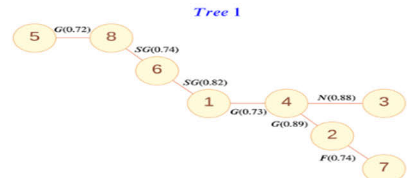
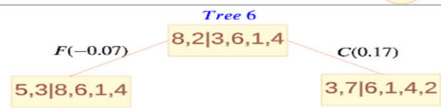
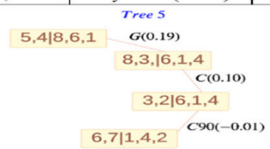
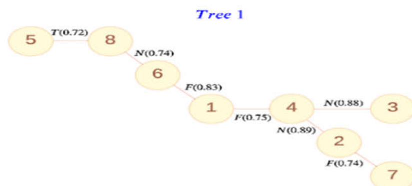
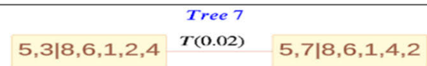
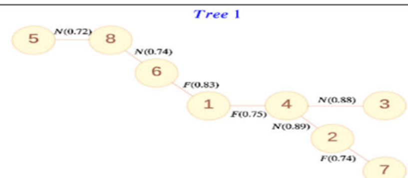
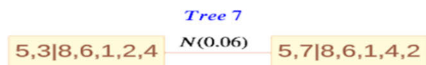
1) LARGE DATASET

First, simulations were performed on the large dataset, i.e., the hourly wind speed data (8760 data). Three simulations were carried out using pair copulas implemented via three groups 'a', 'b' and 'c', resulting in the selection of R-vine structure among R-vine and D-vine; also, comparing R-vine with C-vine shows R-vine is the best structure according to the AIC, BIC and log-likelihood (see Table 3). R-vine structure in Table 4 illustrates the resultant tree sequence plot in which each branch provides the selected pair copula family and its fitted Kendall's τ dependency value when pair copula families in three groups are allowed.

It can be seen from Table 4, Tree 1 (first row) with selected copula from group 'a', that most of the pair copulas are of student's t copulas; site 1 (Humboldt) shows strong dependencies with sites 2 (NedPower), 5 (FreyFarm) and 6 (BearCreek); similarly, site 2 (NedPower) shows strong dependencies with sites 1 (Humboldt), 7 (Mountaineer) and 4 (RothRock). Further, Tree 1 of group 'a' (the first row of Table 4) shows that while sites 1, 2, 4, and 6 connected at least to two different sites with considerable dependencies, sites 3 (Criterion Wind Park), 8 (Locust Ridge), 7 and 5 are correlated to only one site (located in the corners of this Tree diagram). In other words, (3,5) and (7, 8) are insignificant and need to be conditioned to changes observed in other sites; for example, any wind speed changes in sites (7, 8) could be correlated conditionally through possible changes in sites (1, 2, 5, 6).

The second row of Table 4 gives the weakest Kendall τ dependency (0.02) relating the changes of sites (3, 5) conditionally to the changes of sites (1, 2, 4, 7), denoted by (3, 5; 1, 2, 4, 7) as shown by tree 6 in the second row. This conditional combination (3, 5; 1, 2, 4, 7) of Tree 6 is described by Tree 5, in the second row, in which pair copula Gumbel correlates combinations (3, 1; 4, 7, 2) with (4, 5; 7, 2, 1). Note that each

TABLE 6. Different R-vine dependence structure for small dataset with selected copula and tree diagram.

Small dataset				Tree Diagram
Pair copula group	Sites	Copula	Tau (τ)	
'a'	2,7 2,4 3,4 1,4 1,6 8,5 8,6	Frank(13.6) Gumbel(8.96) Normal(0.98) Gumbel(3.7) Sur.Gumbel(5.52) Gumbel(3.55) Sur.Gumbel(3.8)	0.74 0.89 0.88 0.73 0.82 0.72 0.74	
'a'	3,7|6,1,4,2	Clayton90(-0.02)	-0.01	
				
'b'	2,7 2,4 4,3 1,4 1,6 8,5 6,8	Frank(13.6) Nomal(0.98) Normal(0.98) Frank(14.2) Frank(21.7) T(0.9,3.93) Normal(0.92)	0.74 0.89 0.88 0.75 0.83 0.72 0.74	
'b'	7,3|5,8,6,1,2,4	T(0.04,30)	0.02	
'c'	2,7 2,4 3,4 1,4 6,1 8,5 8,6	Frank(13.6) Normal(0.98) Normal(0.98) Frank(14.2) Frank(21.7) Normal(0.91) Normal(0.92)	0.74 0.89 0.88 0.75 0.83 0.72 0.74	
'c'	5,7|8,6,1,4,2	Frank(0.48)	0.05	
				

combination is rooted to other combinations in Tree 4, then Tree 3 down to Tree 1.

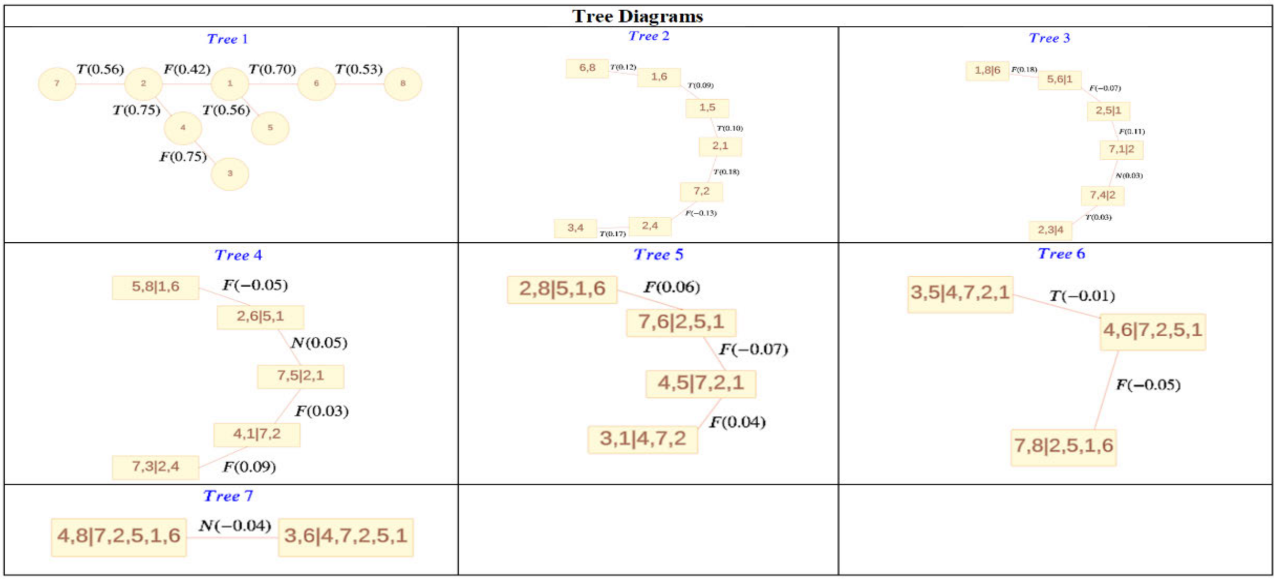
Interestingly, if pair copula is allowed from group 'b,' while the R-vine remains the best structure for the large dataset, then simulations in the third row of Table 4 show strong dependencies among the same sites as those of the first row with allowed pair copula from group 'a'; still, student's t is the selected pair copula. Moreover, allowing pair copula from group 'c' (see the fifth row of Table 4) results in R-vine as the best structure with strong dependencies among the same sites as those of the first and the third rows. In other words, no matter which group is chosen for analysis, the best vine structure is independent of the pair copula group.

Moreover, examining Tree 1 of groups 'b' and 'c' (the third and the fifth rows of Table 4) shows that sites 3, 8, 7, and 5 are located in the corners of these Tree diagrams. Thus, weak dependency between these sites is still expected to be conditionally related to changes in other sites. The fourth row of Table 4 gives the weakest Kendall τ dependency (-0.01) belonging to the changes of sites (3, 6) conditionally related to changes of sites (1, 2, 4, 5, 7), denoted by (3, 6; 1, 2, 4, 5, 7) as shown by tree 7 in the fourth row (for group 'b'). This conditional combination (3, 6; 4, 7, 2, 5, 1) in Tree 7 is described by Tree 6, in the fourth row, in which pair copula student's t correlates combinations (3, 4; 4, 7, 2, 1) with (4, 6; 7, 2, 5, 1). Note that each of these combinations is rooted to other combinations in Tree 5, then Trees 4 down to 1. Similarly, the sixth row of Table 4 provides the weakest dependency (0.04) for sites (4, 5) conditionally related to changes of sites in the sixth row, in which pair copula Frank correlates combinations (4, 1; 7, 2) with (7, 5; 2, 1).

2) MEDIUM DATASET

Second, let us concentrate on simulations on the medium dataset; i.e. the daily wind speed data (365 data). Three simulations were carried out using pair copulas implemented via three groups 'a', 'b' and 'c', resulting in the selection of C-vine structure as the best structure among R-vine, D-vine,

TABLE 7. A typical dependence structure with selected copula and all tree diagrams for 28 combinations.



and C-vine according to the AIC, BIC, and log-likelihood (see Table 3). C-vine structure in Table 5, like those of Table 4, provides different Trees related to the selected pair copula family from the three groups and its fitted Kendall's τ dependency values.

It can be seen from Table 5 that Tree 1 (first row) illustrates most of the selected copula from group 'a' is of Normal Gaussian copulas; site 1 has strong dependencies with sites 2, 3, 4, 5, 6, 7, and 8. The second row of Table 5 gives the weakest Kendall τ dependency (-0.04) relating the changes of sites (7, 6) conditionally to the changes of sites (2, 8, 4, 1), denoted by (7, 6; 2, 8, 4, 1) as shown by tree 6 in the second row. This conditional combination (7, 6; 2, 8, 4, 1) of Tree 6 is described by Tree 5, in the second row, in which pair copula Gumbel90 correlates combinations (7, 2; 8, 4, 1) with (2, 6; 8, 4, 1). Note that each of these combinations is rooted to other combinations in Tree 4, then Trees 3, Tree 2, and Tree 1.

Interestingly, if pair copula is allowed from groups 'b' and 'c,' while the C-vine remains the best structure for the medium dataset, then simulations both in the third and the fifth rows of Table 5 show strong dependencies among the same sites as those of the first row with allowed pair copula from group 'a'; is still Normal Gaussian is the most selected pair copula. In other words, no matter which group is chosen for analysis, the best vine structure is independent of the pair copula group.

The fourth row of Table 5 gives the weakest Kendall τ dependency (-0.01) relating the changes of sites (7, 6) conditionally to the changes of sites (3, 5, 8, 2, 4, 1), denoted by (7, 6; 3, 5, 8, 2, 4, 1) as shown by tree 7 in the fourth row (for group 'b') (built up from two conditional site dependences (7, 3; 5, 8, 2, 4, 1) and (3, 6; 5, 8, 2, 4, 1) with pair copula Frank). Similarly, the sixth row of Table 5 shows the weakest dependency when allowing pair copula from group 'c', where the same Tree is depicted as that of the fourth row except the weakest dependency is equal to (-0.02).

3) SMALL DATASET

Third, simulations on the small dataset are considered; i.e. the weekly wind speed data (52 data). Simulations were performed on three groups 'a', 'b' and 'c', resulting in the selection of R-vine structure as the best structure according to the AIC, BIC, and log-likelihood (see Table 3) for the small dataset. It can be seen from Table 6 that Tree 1 (first row) illustrates most of the selected copula from group 'a' is of Gumbel copula; site 4 has strong dependencies with sites 1, 2, and 3, where site 1 shows strong dependencies with sites 1 and 6. The second row of Table 6 gives the weakest Kendall τ dependency (-0.01) relating the changes of sites (3, 7) conditionally to the changes of sites (6, 1, 4, 2), denoted by (3, 7; 6, 1, 4, 2) as shown by tree 6 in the second row. This conditional combination (3, 7; 6, 1, 4, 2) of Tree 6 is described by Tree 5, in the second row, in which pair copula Clayton90 correlates combinations (3, 2; 6, 1, 4) with (6, 7; 1, 4, 2). Note that each of these combinations is rooted to other combinations in Tree 4, then Trees 3 down to 1.

Interestingly, if pair copula is allowed from groups 'b' and 'c', while R-vine remains the best structure for the small dataset, then simulations both in the third and the fifth rows of Table 6 repeat strong dependencies among the same sites as those of the first row with allowed pair copula from group 'a'; here Normal Gaussian is the most selected pair copula. Again, the three copula groups do not change the best vine structure in the analysis and simulations.

The fourth row of Table 6 gives the weakest Kendall τ dependency (0.02) relating the changes of sites (7, 3) conditionally to the changes of sites (5, 8, 6, 1, 2, 4), denoted by (7, 3; 5, 8, 6, 1, 2, 4) as shown by tree 7 in the fourth row (for group 'b') (built up from two conditional site dependences (5, 3; 8, 6, 1, 2, 4) and (5, 7; 8, 6, 1, 4, 2) with pair copula student's t). Similarly, the sixth row of Table 6 shows the weakest dependency when allowing pair copula from group 'c', where the same Tree is depicted as that of the fourth row except the weakest dependency with pair copula Frank is equal to (0.05).

Now it can be concluded that the selection of vine copula structures was guided by their ability to effectively capture dependencies across different dataset sizes. R-vine was chosen for both large and small datasets due to its higher flexibility in modeling complex dependencies, allowing for diverse dependency patterns across wind farm sites. This flexibility is particularly important for large datasets where multiple dependencies exist and for small datasets where capturing sparse but strong dependencies is critical. On the other hand, C-vine was preferred for medium-sized datasets as it structures dependencies hierarchically, making it more interpretable when the dataset is neither too large to require highly flexible dependency modeling nor too small to suffer from overfitting. The hierarchical approach of C-vine is particularly useful in moderate dataset sizes where dependencies exhibit more structured relationships. These selections were validated using statistical criteria such as AIC, BIC, and log-likelihood, confirming that R-vine consistently outperformed other structures for large and small datasets, while C-vine provided the best performance for medium-sized datasets.

It is worth mentioning that the key challenge in vine copula modeling is the potential for different vine structures to result in the same overall dependency model. Identifiability issues can arise when multiple vine decompositions capture similar dependency structures, making it difficult to determine a uniquely optimal configuration. While statistical criteria such as AIC and BIC guide model selection, it is important to acknowledge that alternative vine structures with similar goodness-of-fit metrics may exist. This highlights the need for further research into structural stability and robustness in vine copula applications.

V. DISCUSSIONS

When different trees are generated for wind farms by vine copula simulations, one can generally find out the existing dependencies among wind speed data of eight sites (e.g. seven trees shown in Table 7). Considering a group of different pair copulas (wind speeds of sites are independent variables), the vine structure introduces the first tree as a basic tree that selects the seven strongest unconditional dependencies between each two-site combination among 28 combinations. Thus, the remaining 21 combinations would be included in other trees. The second tree shows two unrelated sites in Tree 1 that are correlated conditionally through another common site; for example, Tree 1 relates common site 6 to unrelated sites 8 and 1 as two independent variables for pair copula student's t. Tree 2 depicts six combinations, where 15 remaining dependencies must become up with other trees. As expected, the second tree shows much weaker dependencies than the first tree. The third tree continues the same procedure by relating two unrelated combinations through a common combination; for example, Tree 2 relates common combination (2,4) to unrelated combinations (4,3) and (7,2), where this is illustrated by two conditionally related combinations (2,3;4) and (7,4;2) in Tee 3. Here the two independent variables are wind speeds of sites 3 and 7 that are unrelated sites in Tree 1. Tree 3 includes five dependencies out of the remaining 15 dependencies. The remaining trees provide four, three, two and one combinations in Trees numbered 4, 5, 6, and 7, respectively; all seven trees describe all 28 possible dependence structure for eight wind farms using the selected vine structures.

While the vine copula framework successfully models dependency structures across multiple wind farms, it is important to consider the underlying assumptions that may affect the generalizability of the results. Vine copula models assume that dependencies remain stable over time and that the selected marginal transformations adequately represent the underlying distribution of wind speed data. However, real-world wind dynamics exhibit seasonal variations, temporal non-stationarity, and spatial heterogeneity, which may introduce additional complexities not captured in a static vine structure. For instance, the dependency patterns identified in different trees might vary across seasons due to changing atmospheric conditions or local terrain influences. This could affect the strength and direction of dependencies, making a static model less representative of long-term behavior. Additionally, certain extreme events, such as storms or wind gusts, may temporarily alter the dependency relationships between wind farms, leading to deviations from the established tree structures. Future research could extend this study by incorporating dynamic copula models that adjust dependency structures over time, allowing for adaptive modeling of wind farm correlations under varying conditions. Such an approach would enhance the robustness of dependency modeling, improving wind power forecasting and risk assessment in renewable energy systems.

The KS test results confirm that wind speed variations follow statistically similar patterns across multiple locations. The D-statistic values range from 0.0275 to 0.0659, with all p-values exceeding 0.05, indicating that we fail to reject the null hypothesis. This suggests that changes in wind speed across locations are likely influenced by common meteorological factors. These findings have practical implications for wind power forecasting, as predictive models trained on one site may generalize well to other sites. This insight can enhance turbine placement strategies and optimize energy yield predictions for large-scale wind farm operations.

VI. CONCLUSION

Understanding the dependency structure among wind farms is crucial for improving the reliability and efficiency of renewable energy integration. This study demonstrates that selecting the appropriate vine copula structure significantly enhances dependency modeling, enabling a more precise representation of multi-site wind speed interactions. Unlike traditional correlation-based methods, which assume linear dependencies, vine copulas provide a more flexible and realistic approach to capturing complex and nonlinear relationships between wind speed variations at different sites. The findings of this study have important implications for wind power forecasting, grid stability, and risk assessment. Improved dependency modeling translates into better short-term forecasting accuracy, reducing the uncertainty in wind energy production estimates. This, in turn, enhances operational planning and risk mitigation for wind farm operators, ensuring a more stable integration of wind power into the grid. Furthermore, by more accurately capturing correlated wind fluctuations, system operators can optimize energy dispatch strategies, improving overall grid stability and reliability. Beyond technical applications, the results offer valuable insights into investment and policy decisions in the renewable energy sector. A better understanding of multi-site wind dependencies enables more informed decision-making for investors and policymakers, reducing financial and operational risks associated with wind energy projects. The ability to quantify uncertainty and dependencies across wind farms can aid in the development of more resilient energy infrastructure, ultimately supporting the transition to a more sustainable and reliable power system. These findings highlight the critical role of advanced statistical modeling in addressing challenges in renewable energy forecasting and integration. Future research could explore the application of these techniques in other renewable energy domains, such as solar power dependency modeling, or extend them to hybrid energy systems to further enhance grid reliability.

To promote reproducibility and further research, the dataset and code used in this study will be made available upon request. Interested researchers may contact the corresponding author for access.

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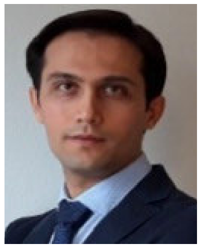
AMIR SHAHIRINIA (Senior Member, IEEE) received the B.Sc. and M.Sc. degrees from the K. N. Toosi University of Technology, Tehran, Iran, and the Ph.D. degree in electrical engineering from the University of Wisconsin–Milwaukee, USA. He has also performed postdoctoral studies with the Power Electronics Group, Rockwell Automation (Allen Bradly). He is currently the Director of the Smart Grids and Artificial Intelligence (SGAI) Laboratory and the Center of Excellence for Renewable Energy (CERE), UDC. His research interests include power systems, power electronics and motor drives, artificial intelligence applications in smart grids, and optimal planning and operation of renewable energy grid integration systems (REGIS). His work also delves into data science and Bayesian statistics, addressing contemporary challenges in energy systems, and beyond.

ZEINAB FARAHMANDFAR received the B.S. degree in civil engineering from Shahid Beheshti University, the M.S. degree in civil engineering from the K. N. Toosi University of Technology, Tehran, Iran, and the Ph.D. degree in civil engineering from Clemson University, USA. She was an Associate Research Scientist with the Department of Civil and Systems Engineering, Johns Hopkins University, where she also collaborates with the Community Resilience Group, National Institute of Standards and Technology (NIST). She is currently an Environmental Engineer with U.S. Nuclear Regulatory Commission (NRC), where she focuses on analyzing risks associated with external hazards. Her research interests include probabilistic analysis, uncertainty modeling, data-driven approaches to model complex systems, multi-hazard resilience of underground infrastructure, and optimal planning and operation of integrated infrastructure systems.

MOHAMMAD TAVAKOLI BINA (Senior Member, IEEE) received the B.Sc. degree in power electronics and power system utility applications from the University of Tehran, in 1988, the M.Sc. degree in power electronics and power system utility applications from the University of Ferdowsi, Iran, in 1991, and the Ph.D. degree in power electronics and power system utility applications from the University of Surrey, Guildford, U.K., in 2001. From 1992 to 1997, he was a Lecturer with the K. N. Toosi University of Technology, Tehran, where he was involved in power systems. He joined the Faculty of Electrical and Computer Engineering, K. N. Toosi University of Technology, in 2001, where he is currently a Professor of electrical engineering. He is engaged in teaching and researching power electronics and utility applications. His research interests include power electronics, flexible power systems, and power quality.



HOSSEIN HAFEZI (Senior Member, IEEE) received the B.Sc. degree in electrical engineering from the K. N. Toosi University of Technology, Tehran, Iran, the M.Sc. degree in electrical engineering from Dokuz Eylul University, İzmir, Türkiye, and the Ph.D. degree in electrical engineering from the Energy Department, Politecnico di Milano, Milan, Italy. His employment experience includes a Postdoctoral Research Fellow with the Energy Department, Politecnico di Milano, and as an Assistant Professor with the School of Technology and Innovations, University of Vaasa, Finland. He was a University Lecturer with Tampere University, Faculty of Information Technology and Communication Sciences, from September 2020 to August 2023. Since August 2023, he joined General Electric (GE) Grid Solutions as a Lead NPI (Research and Development) Engineer. At the same time, he continues as a Visiting Teacher with the Faculty of Information Technology and Communication Sciences, Tampere University. His research interests include power electronics converters design and control and renewable energy resources integration into smart grid systems.

VINCENT TANOE received the Ph.D. degree in computer science and engineering from the University of the District of Columbia, Washington, DC, USA. He is a data science and artificial intelligence expert with extensive experience across government, international organizations, and academia. Currently, he is a Data Scientist with the Federal Government, he leads initiatives in predictive modeling, AI/ML applications, and data-driven decision-making. Previously, he was an Economist Consultant with the African Development Bank, analyzing macroeconomic trends across Sub-Saharan Africa, and held positions at the IMF and World Bank, contributing to global economic analysis and policy development. His research portfolio includes impactful studies on Bayesian and non-Bayesian regression for renewable energy, econometric and deep learning analysis of financial data, and machine learning applications in financial inclusion.