Guyau, J. M. (1895). *Les problèmes de l'esthétique contemporaine*. Paris: Félix Alcan.
Jensen, J. V. (1913). *Jäätikön poika. Tarukertomus Pohjolan jääkaudelta*. Trans. I. Sirkka, Kustannusosakeyhtiö Kirja, Helsinki.
Jensen, J. V. (1925). *Bræen. Myter om istiden og det første menneske*. København: Gyldendalske Boghandel – Nordisk Förlag.
Kelly, L. (ed.) (1994). *The City as Art: Interrogating the Polis*. Belfast: A.I.C.A (Irish Section).
Le Corbusier (1924). *Vers une architecture*. Paris: Les Editions G. Crès.
Le Corbusier (1987). *The Decorative Art of Today*. Trans. J. Dunnet, Cambridge, Massachusetts: The MIT Press.
Mumford, L. (1934). *Technics and Civilization*. New York: Harcourt, Brace and Company.
Mustelin, O. (1962). *Euterpe. Tidskriften och kretsen kring den*. Nr 398 Helsingfors: Svenska litteratursällskapet i Finland.
Niiniluoto, I. & Th. Wallgren (eds.) (2017). *On the Human Condition: Philosophical Essays in Honour of the Centennial Anniversary of Georg Henrik von Wright. Acta Philosophica Fennica*, Vol. 93. Helsinki: Societas Philosophica Fennica.
Niiniluoto, I. (2021). *Tekniikan filosofia*. Helsinki: Gaudeamus Oy.
Rhodes, R. (1988). *The Making of the Atomic Bomb*, Penguin Books, London.
Salokorpi, A. and Kärkkäinen M. (eds.) (1983). *abacus*, Yearbook 3. Helsinki: Museum of Finnish Architecture.
Sarje, K. (1999). "Aristokraattinen radikalismi". *Ajatus* 56, 103–117.
Sarje, K. (2000). *Sigurd Frosteruksen modernin käsite. Arkkitehtuuri ja maailmankatsomus, Dimensio* 3. Finnish National Gallery Research Series. Helsinki: Valtion taidemuseo.
Sarje, K. (2014). "The Rational Modernism of Sigurd Frosterus. A Nordic Interpretation". In A. Erjavec & T. Miller (2014), 193–218.
Sarje, K. (2017). "Finnish Impulses in Georg Henrik von Wright's Thinking – The Journals *Euterpe* and *Nya Argus*". In I. Niiniluoto & Th. Wallgren (2017), 403–419.
Sarje, K. (2024). "Essee kritiikin muotona". In Frosterus S. (2024), 6–18.
Wells, H. G. (1914). *The World Set Free*. Leipzig: Bernhard Tauchnitz.
Wirtanen, A. (1958). "Sigurd Frosterus, skriftställaren". *Nya Argus* 1-2, 1958, 13–17.
Wirtanen, A. (1960). "Finlandssvenska humanister II". *Nya Argus* 7, 94–96.

The Legacy of Leibniz's Intensional Logic

JARI PALOMÄKI

Die großen Denker studiert man nicht allein aus historischem Interesse, nicht nur, um ihre Gedanken wieder zu denken, sondern um sie weiter zu denken. Dies trifft auf eine besondere Weise für die Denkarbeit von Leibniz zu.

Raili Kauppi (1920–1995)

1. Leibniz

Gottfried Wilhelm von Leibniz (1646–1716) had a program of a universal science, *scientia universalis*, for coordinating all human knowledge as a framework within which each of the sciences would stand in relation to the other sciences. This program comprised two parts: Firstly, a universal character or notation, *characteristica universalis*, by use of which every item of information can be recorded in a simple and systematical way, and secondly, a formalized method or calculus for reasoning, *calculus ratiocinator*, by which the knowledge recorded in a computational way will reveal the logical consequences of any item and its relations with other items. This conception of a *calculus ratiocinator* led Leibniz to develop a mathematically inspired system for reasoning in a way that makes him the founder of modern symbolic logic.

Leibniz's other fascination was the oneness of all things by virtue of the universality of reason, which led him to an interest in ecumenism, *i.e.*, a religion derived from reason and hence accessible to all, Christians, and non-Christians alike. Especially, Leibniz worked on a theological system to bridge the gaps between Protestants and Catholics both for spiritual and political reasons. (See, e.g., Antognazza, 2009, 90-92, 233-247).

2. Saarnio

These two projects by Leibniz were reasons behind Uuno Saarnio (1896–1977) entering university to study philosophy and mathematics at the newly founded University of Turku in 1921. Eino Kaila (1890–1958) was the first professor of philosophy and Kalle Väisälä (1893–1968) a professor of mathematics. Saarnio was interested in theosophy, and thus knew Leibniz being an ecumenist. He also had talents for mathematics and logic and studied, e.g., Whitehead & Russell's *Principia Mathematica*. Accordingly, Saarnio's master's thesis was on Leibniz, written in Finnish, in 1926: *The Significance of Leibniz's Mathematical Innovations in his Philosophy* (unpublished).[1] In this thesis Saarnio sees Leibniz as a forefather of modern mathematical logic as well as logical empiricism, where Leibniz's *scientia generalis* is seen as a kind of "Einheitswissenschaft" in the sense of Moritz Schlick (1882–1936), to whom Saarnio in his thesis refers to.

In 1930, when Kaila was elected as the professor of theoretical philosophy at the University of Helsinki, Saarnio began to develop a word-theory in Leibnizian spirit, *characteristica universalis*, with linguist Aarni Penttilä (1899–1971), who had also studied philosophy, under Kaila. Saarnio and Penttilä were able to publish an article on this topic in *Erkenntnis* in 1934: "Einige grundlegende Tatsachen der Worttheorie nebst Bemerkungen über die sogenannten Unvollständige Symbole". In the following year, Saarnio published his doctoral thesis *Untersuchungen zur symbolischen Logik I: Kritik des Nominalismus und Grundlegung der logistischen Zeihentheorie (Symbologie)*, which was the first volume of the *Acta Philosophica Fennica*, published by the Philosophical Society of Finland. By following Kaila's advice not to include the treatment of Grelling's paradox in his thesis, Saarnio started to have an extensive correspondence on it with Kurt Grelling (1886–1942).[2]

[1] *Leibniz'in matemaattisten keksintöjen merkityksestä hänen filosofiassaan* (1926, julkaisematon).

[2] Kurt Grelling was a member of the group which Hans Reichenbach (1891–1953) organized on scientific philosophy in Berlin that developed out of the Society for Empirical Philosophy, *Die Gesellschaft für empirische Philosophie*, which was founded on 27. February 1927 by Josef Petzoldt (1862–1929) and became known as the Berlin Group. Its core members were Kurt Grelling

In their word-theory Penttilä and Saarnio distinguished between three kinds of words: spoken, written, and thought. They called the minimal semantic units of them *phoneme*, *grapheme*, and *psycheme*, respectively. The word-theory is founded on Russell's type theory including two non-logical binary relations: a *set-membership*-relation, *i.e.*, $\in$-relation, and a *signification*-relation, *i.e.*, *S*-relation. Words are the concatenations of graphemes, phonemes, or psychemes, which are physical items. Only they belong to the domain of *S*-relation, and they are called *words of type 0*, and are denoted as: *word t0*. A class of *words t0*, in turn, forms an extension of the concept 'word', *i.e.*, {*word t0*} = *word 1t*, where the word *word 1t* on the right side of the formula is a non-physical *concept 'word'*.

Next, two meanings of the word "word" are distinguished. Accordingly, we make a distinction between a "word" and a word, in which with the word "word" we mean – in this case – that physical sign of the word itself, whereas the word word signifies (*significatio*), depending on context, either a particular word (*copulatio*) or a class of words (*suppositio*). The difference between "word" and word can be now denoted type-theoretically as follows: *word 0t* = "word" and *word t0* = word. Following that kind of construction by means of an $\in$-relation, and an *S*-relation, we can present the words "word" as *word 0t*, *word 1t*, *word 2t*, etc., the extension of concepts 'word', in turn, as {*word t0*}, {*word t1*}, {*word t2*}, etc., the concepts 'word': *word t0*, *word t1*, *word t2*, etc. Moreover, A *language* is an ordered sequence of words of type *0* with a syntax, that is, a *language* $=_{df}$ $[\mathit{word}]_{s}$, where a *particular language* is a subclass of language, e.g., *Latin* = $[\mathit{Verbum}]_{s1}$; *English* = $[\mathit{word}]_{s2}$; *Polish* = $[\mathit{słowo}]_{s3}$;

(1886–1942), Walter Dubislav (1895–1937), and Carl Gustav Hempel (1905–1997). After Uuno Saarnio got married with Charlotte "Carola" Hollberg (1897–1972) from Berlin in 1923, they made several trips to Berlin (1926, 1928, 1930, 1932, 1934, 1935, 1937) to visit, e.g., Charlotte's sister Elisabeth Lemke in Berlin. Thus, attending the group's meetings, he could be personally acquainted with several members of the Berlin Group, especially with Hans Reichenbach, Kurt Grelling, and collaborators Walter Dubislav, and Carl Hempel, but also Paul Hertz (1881–1940), Richard von Mises (1883–1953), Heinrich Scholz (1884–1956), Jørgen Jørgensen (1894–1969), Karl Dürr (1888–1970), Paul Oppenheim (1885–1977), Charles W. Morris (1901–1979) and others.

Finnish = [*sana*]$_{s4}$; *German* = [*Wort*]$_{s5}$; etc. (see Penttilä & Saarnio 1934, Saarnio 1935).

In 1944 Saarnio published a paper, "Über den Begriff des Intensionalen in der Logik," in which he studied the possibility of an intensional logic as proposed by Leibniz, comparing it with an extensional logic. In the works on logic it has been customary to distinguish between extension and intension, and also to dispute whether logic should be mainly concerned with intensions or extensions. In his *La Logique de Leibniz*, Louis Couturat (1868–1914), who stressed the fundamental importance of logic for Leibniz's philosophy, wrote that logic could only be built up from the standpoint of extension (Couturat 1901, 387), and that Leibniz, instead of trying to work on intensional logic, should have preferred to work on logic from the extensional point of view (*ibid.*, 24). Further doubts about the possibility of developing of intensional logic were raised by C. I. Lewis (1883–1964): "Whoever studies Leibniz, Lambert and Castillon cannot fail to be convinced that a consistent calculus of concepts in intension is either immensely difficult or, as Couturat has said, impossible" (Lewis 1918, 14).[3]

3. Kauppi

In 1938 Raili Kauppi (1920–1995) enrolled at the University of Helsinki to study general history but soon changed to philosophy and mathematics, especially to follow Eino Kaila's lectures. In 1942 she attended the philosophy club organized at the Helsinki Public Library, where Saarnio was a director. Kauppi wrote her master's thesis on the concept of history in 1947, and then turned to intensional logic, completing her licentiate's thesis *Über die Grundbegriffe der Inhaltslogik* in 1957.

[3] There are several reasons to separate an intensional logic from an extensional logic. For instance: i) intensions determine extensions, but not conversely; ii) whether a thing belongs to a set is decided primarily by intension; iii) a concept can be used meaningfully even when there is not yet, nor ever will be, any individuals belonging to the extension of the concept in question; iv) there can be many non-identical but co-extensional concepts; v) the extension of a concept may vary according to context; and vi) it follows from Gödel's two incompleteness theorems that intensions cannot be wholly eliminated from the extensional set theory (Palomäki 1994, Palomäki & Kangassalo 2012).

However, at that time Saarnio showed her the aforementioned passage in Couturat's book, according to whom Leibniz should not have tried to develop an intensional logic, which was impossible. Kauppi thought that this could not be true, and so she began to study Leibniz's original manuscripts both in Hannover as well as in Leibniz-Forschungsstelle at the University of Münster.

As a result, Kauppi defended her dissertation *Über die Leibnizsche Logic mit besonderer Berücksichtigung des Problems der Intension und der Extension* in 1960 and became the first female in Finland to doctorate in philosophy. The main outcome in her thesis was that Leibniz worked out a variety of both intensional and extensional treatments of the logic of predicates, i.e., concepts, but preferring the intensional approach (Kauppi 1960, 220, 251, 252). Furthermore, she also wanted to show in a preliminary way that it was possible to further develop an intensional logic outlined by Leibniz. Subsequently, Kauppi published her intensional concept theory in the book *Einführung in die Theorie der Begriffssysteme* in 1967. In 1969 she was appointed to a permanent position as a professor of philosophy at the University of Tampere, being the first female professor in philosophy in the Nordic countries.[4] Although Raili Kauppi retired as an *emerita* in 1985, she was still actively taking part in philosophical life in Finland, especially in Tampere. In this context, it is appropriate to gain some understanding of Kauppi's work on logic, so in the next section I will present her basic intensional concept theory and, briefly, its background.

4. Kauppi's Intensional Concept Theory

A famous logic text, the *Port Royal Logic*, composed by two leaders of the Port Royal movement Antoine Arnauld (1612–1694) and Pierre Nicole (1625–1695) in 1662, made a distinction

[4] Raili Kauppi was one among those who was interviewed for a series of ten outstanding female researchers to celebrate the 350th Anniversary of the University of Helsinki in 1990. In that interview, Kauppi emphasized that she does not see any essential difference in the thinking of men and women and remarked that she and Leibniz have had a similar problem with an intensional negation (*Naisia tutkijoina*, Raili Kauppi. Työryhmä M. Engman, A. Korppi-Tommola et al., VHS Video Cassette, 18 min, DiArt Oy, 1991).

between the comprehension, *comprehension*, and the extension, *étendue* or *extension*, of an idea. The comprehension of an idea consists of "the attributes which it includes in itself, and which cannot be taken away from it without destroying it." The extension of an idea consists of "the subjects with which that idea agrees," or which contain it. Both the comprehensions of ideas and the extensions of ideas are used in the *Port Royal Logic* in justifying the basic rules of traditional logic (Adams 1994, 58). Leibniz, in turn, distinguished these two types in terms of ideas, *secundum ideas* or *per ideas*, on one hand, and in terms of instances, *secundum individua* or *per exempla subjecta*, or individuals belonging to the terms, *per individuis terminorum*, on the other hand (*ibid.*, 59). Nowadays this distinction is usually made in terms of "the intension of a concept" and "the extension of a concept."

In the *Port Royal Logic* "the extension of an idea" constituted both the species and the individuals that fall under it, whereas in Leibniz the extensional treatment is almost always in terms of individuals that fall under the idea (Kauppi 1960, 43). Nowadays "the extension of a concept" is taken to be a class (or a set) of all those individuals which fall under it. However, nowadays there are at least two different ways to interpret "the comprehension of an idea", i.e., either as "the intension of a concept" or as "the conceptual content of a concept", which, however, are to be distinguished (see Palomäki 1997).

4.1. Items Connected to a Concept

Firstly, there are some basic items connected to a concept, and one possible way to locate them is as follows: A *term* is a linguistic entity. It *denotes* things and *connotes* a concept. A concept, in turn, has an *extension* and an *intension*. The extension of a concept is a *set* (or a *class*, being more exact) of all those things that *fall under* the concept; see Diagram 1 (Palomäki 1994, 81, 82). Now, there may be many different terms which denote the same things but connote different concepts. That is, these different concepts have the same extension, but they differ in their intension, and thus being different but co-

extensional concepts.[5] By an intension of a concept we mean something which we must "understand" or "grasp" in order to use the concept in question correctly. Hence, we may say that the intension of concept is that information content of it which is required to recognize a thing belonging to the extension of the concept in question (cf. Kangassalo 1992/93, Palomäki 1994).

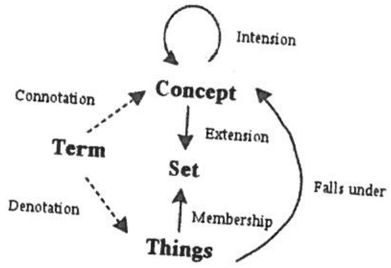

Diagram 1

Let $U = < V, C, F >$ be a universe of discourse, where i) V is a universe of (possible) individuals, ii) C is a universe of concepts, iii) $V \cap C \neq \{\}$, and iv) $F \subseteq V \times C$ is the "falls under" -relation. Now, if a is a concept, then for every (possible) individual i in V, either i falls under the concept a or it doesn't, *i.e.*,

(1) if $a \in C$, then $\forall i \in V$: $iFa \vee \sim iFa$.

The extension-relation E between the set A and the concept a in V is defined as follows:

(2) $E_U(A,a) =_{df} (\forall i)\ (i \in A \leftrightarrow i \in V \wedge iFa)$.

The extension of concept a may also be described as follows:

(3) $i \in E_U'(a) \leftrightarrow iFa$,

where $E_U'(a)$ is the extension of concept a in V, *i.e.*, $E_U(a) = \{i \in V \mid iFa\}$.

[5] For example, the concepts of 'human', 'rational animal', and 'featherless bipeds with flat nails' are non-identical but co-extensional concepts, the extension of which consists of a set of human beings.

4.2 *An Intensional Containment Relation*

The relations between concepts enable us to make conceptual structures. The basic relation between concepts is an intensional containment-relation, and it is this intensional containment relation between concepts that we are calling the Is-IN -relation.[6] More formally, let there be given two concepts a and b. When a concept a contains intensionally a concept b, we may say that the intension of the concept a contains the intension of the concept b, or that the concept a intensionally entails the concept b, or that the intension of the concept a entails the intension of the concept b. This intensional containment-relation is denoted as follows,

(4) $a \geq b$.

It was subsequently observed by Kauppi in (1967) that

(5) $a \geq b \rightarrow (\forall i)\ (iFa \rightarrow iFb)$,

that is, that the transition from intensions to extensions reverses the containment relation, *i.e.*, the intensional containment-relation between concepts a and b is converse to the extensional set-theoretical subset-relation between the sets of their extension. Thus, by (3),

(6) $a \geq b \rightarrow Eu'(a) \subseteq Eu'(b)$,

where "$\subseteq$" is the set-theoretical subset-relation, or the extensional inclusion relation between sets. Or, if we put $A = Eu'(a)$ and $B = Eu'(b)$, we will get,

(7) $a \geq b \rightarrow A \subseteq B$.

For example, if the concept of a dog contains intensionally the concept of a quadruped, then the extension of the concept of a quadruped, *i.e.*, the set of four-footed animals like elephants, contains extensionally as a subset the extension of the concept

[6] It is important to notice that the Is-IN -relation is to be clearly distinguished from the Is-A -relation, which is usually applied, e.g., in computer science. For instance, in a sentence: "Socrates is a human", the relation between "Socrates" and "human" is an Is-A -relation, whereas in a sentence: "humanity is in Socrates", the relation between "Socrates" and "humanity" is Is-In-relation, (cf. Palomäki & Kangassalo 2012).

of a dog, *i.e.*, the set of dogs; see Diagram 2. Observe, though, that we can deduct from concepts to their extensions, *i.e.*, sets, but not conversely, because for every set there may be many different concepts whose extension that set is.

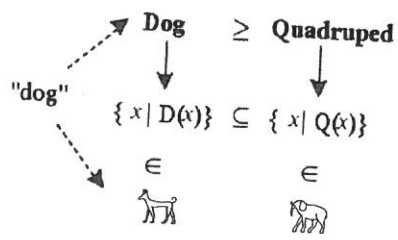

Diagram 2

4.3 *An Intensional Concept Theory* KC

Based on the intensional containment relation between concepts, Raili Kauppi presented her axiomatic intensional concept theory in Kauppi (1967), which is further studied in (Palomäki 1994). This axiomatic concept theory was inspired by Leibniz's logic, where the intensional containment relation between concepts formalises an "*inesse*"-relation[7] in Leibniz's logic.[8]

An intensional concept theory, denoted by *KC*, is presented in a first-order language *L* that contains individual variables *a*, *b*, *c*,..., which range over the *concepts*, and one non-logical 2-place *intensional containment relation*, denoted by "≥". I will first

[7] Literally, "*inesse*" is "being-in", and this term was used by Scholastic translators of Aristotle to render the Greek "huparchei", i.e., "belongs to" (Leibniz 1997, 18, 243).

[8] In a letter to Arnauld on 14 July 1786, Leibniz wrote: "[I]n every affirmative true proposition, necessary or contingent, universal or singular, the notion of the predicate is contained in some way in that of the subject, *praedicatum inest subjecto* [the predicate is included in the subject]. Or else I do not know what truth is" (*ibid.*, 62).

present four basic relations between concepts defined by "$\geq$", and then, briefly, the basic axioms of the theory.[9]

Two concepts a and b are said to be *comparable*, denoted by $a \mathrm{H} b$, if there exists a concept x which is intensionally contained in both.

$$\mathrm{Df_H} \quad a \mathrm{H} b =_{\mathrm{df}} (\exists x)(a \geq x \wedge b \geq x).$$

If two concepts a and b are not comparable, they are *incomparable*, which is denoted by $a \mathrm{I} b$.

$$\mathrm{Df_I} \quad a \mathrm{I} b =_{\mathrm{df}} \sim a \mathrm{H} b.$$

Dually, two concepts a and b are said to be *compatible*, denoted by $a \perp b$, if there exists a concept x which contains intensionally both.

$$\mathrm{Df_\perp} \quad a \perp b =_{\mathrm{df}} (\exists x)(x \geq a \wedge x \geq b).$$

If two concepts a and b are not compatible, they are *incompatible*, which is denoted by $a \mathrm{Y} b$.

$$\mathrm{Df_Y} \quad a \mathrm{Y} b =_{\mathrm{df}} \sim a \perp b.$$

The two first axioms of *KC* state that the intensional containment relation is a *reflexive* and *transitive* relation.

$$\mathrm{Ax_{Refl}} \quad a \geq a.$$

$$\mathrm{Ax_{Trans}} \quad a \geq b \wedge b \geq c \rightarrow a \geq c.$$

Two concepts a and b are said to be *intensionally identical*, denoted by $a \approx b$, if the concept a intensionally contains the concept b, and the concept b intensionally contains the concept a.

$$\mathrm{Df_\approx} \quad a \approx b =_{\mathrm{df}} a \geq b \wedge b \geq a.$$

The intensional identity is clearly a reflexive, symmetric and transitive relation, hence an equivalence relation.

A concept c is called an *intensional product* of two concepts a and b, if any concept x is intensionally contained in c if and only if it is intensionally contained in both a and b. If two concepts a and b have an intensional product, it is unique up to the intensional identity and we denote it then by $a \otimes b$.

[9] For more complete presentations of the theory, see Kauppi (1967) and Palomäki (1994).

$$\mathrm{Df_\otimes} \quad c \approx a \otimes b =_{\mathrm{df}} (\forall x)(c \geq x \leftrightarrow a \geq x \wedge b \geq x).$$

The following axiom $\mathrm{Ax_\otimes}$ of *KC* states that if two concepts a and b are comparable, there exists a concept x which is their intensional product.

$$\mathrm{Ax_\otimes} \quad a \mathrm{H} b \rightarrow (\exists x)(x \approx a \otimes b).$$

It is easy to show that the intensional product is idempotent, commutative, and associative.

A concept c is called an *intensional sum* of two concepts a and b, if the concept c is intensionally contained in any concept x if and only if it contains intensionally both a and b. If two concepts a and b have an intensional sum, it is unique up to the intensional identity, and we denote it then by $a \oplus b$.

$$\mathrm{Df_\oplus} \quad c \approx a \oplus b =_{\mathrm{df}} (\forall x)(x \geq c \leftrightarrow x \geq a \wedge x \geq b).$$

The following axiom $\mathrm{Ax_\oplus}$ of *KC* states that if two concepts a and b are compatible, there exists a concept x which is their intensional sum.

$$\mathrm{Ax_\oplus} \quad a \perp b \rightarrow (\exists x)(x \approx a \oplus b).$$

The intensional sum is idempotent, commutative, and associative.

The intensional product of two concepts a and b is intensionally contained in their intensional sum whenever both sides are defined.

$$\mathrm{Th\ 1} \quad a \oplus b \geq a \otimes b.$$

Proof: If $a \otimes b$ exists, then by $\mathrm{Df_\otimes}$, $a \geq a \otimes b$ and $b \geq a \otimes b$. Similarly, if $a \oplus b$ exists, then by $\mathrm{Df_\oplus}$, $a \oplus b \geq a$ and $a \oplus b \geq b$. Hence, by $\mathrm{Ax_{Trans}}$, the theorem follows.

A concept b is an *intensional negation* of a concept a, denoted by $\neg a$, if and only if it is intensionally contained in all those concepts x, which are intensionally incompatible with the concept a. When $\neg a$ exists, it is unique up to the intensional identity.

$$\mathrm{Df_\neg} \quad b \approx \neg a =_{\mathrm{df}} (\forall x)(x \geq b \leftrightarrow x \mathrm{Y} a).$$

The following axiom $\mathrm{Ax_\neg}$ of *KC* states that if there is a concept x which is incompatible with the concept a, there exists a concept y, which is the intensional negation of the concept a.

Ax$_\neg$ $(\exists x)(x \curlyvee a) \rightarrow (\exists y)(y \approx \neg a)$.

It can be proved that a concept a contains intensionally its intensional double negation if it exists.

Th 2 $a \geq \neg\neg a$.

Proof: By Df$_\neg$ the equivalence (1): $b \geq \neg a \leftrightarrow b \curlyvee a$ holds. By substituting $\neg a$ for b to (1), we get $\neg a \geq \neg a \leftrightarrow \neg a \curlyvee a$, and so, by Ax$_{Refl}$, we get (2): $\neg a \curlyvee a$. Then, by substituting a for b and $\neg a$ for a to (1), we get $a \geq \neg\neg a \leftrightarrow a \curlyvee \neg a$ and hence, by (2), the theorem follows.

Also, the following forms of the *De Morgan's formulas* can be proved whenever both sides are defined:

Th 3 i) $\neg a \otimes \neg b \geq \neg(a \oplus b)$,

ii) $\neg(a \otimes b) \approx \neg a \oplus \neg b$.

Proof: First we are to prove the following important lemma:

Lemma 1: $a \geq b \rightarrow \neg b \geq \neg a$.

Proof: From $a \geq b$ follows $(\forall x)(x \curlyvee b \rightarrow x \curlyvee a)$, and thus by Df$_\neg$ the Lemma 1 follows.

i) If $a \oplus b$ exists, then by Df$_\oplus$, $a \oplus b \geq a$ and $a \oplus b \geq b$. By Lemma 1 we get $\neg a \geq \neg(a \oplus b)$ and $\neg b \geq \neg(a \oplus b)$. Then, by Df$_\otimes$, Th 3 i) follows.

ii) This is proved in four steps as follows:

1. $\neg(a \otimes b) \geq \neg a \oplus \neg b$.

Since $a \geq a \otimes b$, it follows by Lemma 1 that $\neg(a \otimes b) \geq \neg a$. Thus, by Df$_\oplus$, 1 holds.

2. $\neg(\neg\neg a \otimes \neg\neg b) \geq \neg(a \otimes b)$.

Since $a \geq \neg\neg a$, by Th 2, it follows by Df$_\otimes$ that $a \otimes b \geq \neg\neg a \otimes \neg\neg b$. Thus, by Lemma 1, 2 holds.

3. $(\neg\neg a \otimes \neg\neg b) \geq \neg(\neg a \oplus \neg b)$.

Since $(a \oplus b) \geq a$, it follows by Lemma 1 that $\neg a \geq \neg(a \oplus b)$, and so, by Df$_\otimes$, it follows that $(\neg a \otimes \neg b) \geq \neg(a \oplus b)$. Thus, by substituting $\neg a$ for a and $\neg b$ for b to it, 3 holds.

4. $\neg a \oplus \neg b \geq \neg(a \otimes b)$.

Since $\neg a \oplus \neg b \geq \neg\neg(\neg a \oplus \neg b)$, by Th 2, and from 3 it follows by Lemma 1 that $\neg\neg(\neg a \oplus \neg b) \geq \neg(\neg\neg a \otimes \neg\neg b)$, and by Ax$_{Trans}$ we get, $\neg a \oplus \neg b \geq \neg(\neg\neg a \otimes \neg\neg b)$. Thus, by 2 and by Ax$_{Trans}$, 4 holds.

From 1 and 4, by Df$_\approx$, the Th 3 ii) follows.

If a concept a is intensionally contained in every concept x, the concept a is called a *general concept*, and it is denoted by G. The general concept is unique up to the intensional identity, and it is defined as follows:

Df$_G$ $a \approx G =_{df} (\forall x)(x \geq a)$.

The next axiom of *KC* states that there is a concept which is intensionally contained in every concept.

Ax$_G$ $(\exists x)(\forall y)(y \geq x)$.

When the axiom of the general concept is adopted, it follows that all concepts are to be comparable. Since the general concept is compatible with every concept, it has no intensional negation.

A *special concept* is a concept a, which is not intensionally contained in any other concept except for concepts intensionally identical to itself. Thus, there can be many special concepts.

Df$_S$ $S(a) =_{df} (\forall x)(x \geq a \rightarrow a \geq x)$.

The last axiom of *KC* states that for any concept y there is a special concept x in which it is intensionally contained.

Ax$_S$ $(\forall y)(\exists x)(S(x) \wedge x \geq y)$.

Since the special concept s is either compatible or incompatible with every concept, the *law of excluded middle* holds for s so that for any concept x, which has an intensional negation, either the concept x or its intensional negation $\neg x$ is intensionally contained in it. Hence, we have

Th 4 $(\forall x)(S(s) \rightarrow (s \geq x \vee s \geq \neg x))$.

A special concept, which corresponds to Leibniz's complete concept of an individual, *notio completa seu perfecta substantiae singularis*, would contain one member of every pair of mutually incompatible concepts.

5. A Model of *KC* – and the Work in Progress

It was 1990 when I planned to write my doctoral thesis on Bertrand Russell's (1872–1970) concept of proposition, advised by Veikko Rantala. However, I also met professor of computer sciences, Hannu Kangassalo, who had applied Kauppi's intensional concept theory to database designing (see Kangassalo 1992/93). He introduced me to Raili Kauppi, who, in turn, gave me her book *Einführung in die Theorie der Begriffssysteme* (1967), remarking that an intensional relational theory was still a work in progress. Thus, I changed my topic from the concept of proposition to the concept of concept. I defended my thesis *From Concepts to Concept Theory: Discoveries, Connections, and Results* in 1994, trying, among other things, to find an algebraic model for Kauppi's basic intensional concept theory, *KC*. This attempt may be summarized as follows.

From the completeness theorem it is known that every consistent first-order theory has a model. Accordingly, in (Palomäki, 1994, 94-97), a model of $KC + Ax_{\neg\neg}$ is found to be a *complete semilattice*, where every concept $a \in C$ defines a *Boolean algebra* $B_a = <\downarrow a, \otimes, \oplus, \neg, G, a>$, where $\downarrow a$ is an ideal, known as the *principal ideal generated by* a, *i.e.*, $\downarrow a =_{df} \{x \in C \mid a \geq x\}$, and the intensional negation of a concept $b \in \downarrow a$ is interpreted as a *relative complement of* a.

Let us have two concepts, 'man' and 'woman', as an example of a conceptual structure based on the intensional concept theory *KC*. The common concept which contains intensionally both the concepts of 'man' and of 'woman', and so is their intensional conceptual product, is the concept of 'human'. The concept of 'human' contains intensionally the concept of 'animal', which, in turn, contains intensionally the general concept '*G*'. Then, the concept in which both the concepts of 'man' and of 'woman' are intensionally contained is the concept of 'androgyne', which in this conceptual system is a special concept containing intensionally all these concepts. For a visual presentation of this conceptual structure, see the Diagram 3. Note that no intensional negation of any concept appears in this example.[10]

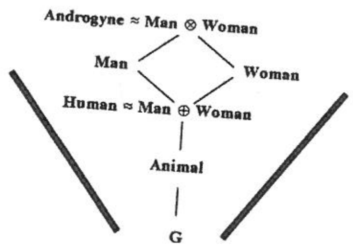

Diagram 3

Since then, I have tried to further develop an intensional concept theory in the spirit of Leibniz, Saarnio and Kauppi, e.g., in Palomäki (1994, 2004, 2012, 2014) – and the work is still in progress. Great thinkers are always posthumously productive.

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References

Adams, R. M. (1994). *Leibniz: Determinist, Theist, Idealist*. New York, Oxford: Oxford University Press.

Antognazza, M. R. (2009). *Leibniz: An Intellectual Biography*. Cambridge, et al.: Cambridge University Press.

Couturat, L. (1901). *La Logique de Leibniz: D'après des Documents Inédits*. Paris: F. Alcan.

[10] That is a very difficult problem indeed, a problem that also troubled both Leibniz and Kauppi, and which is studied more thoroughly in Palomäki (2012).

Kangassalo, H. (1992/93). "COMIC: A system and methodology for conceptual modelling and information construction". *Data and Knowledge Engineering* 9, 287-319.

Kauppi, R. (1960). *Über die Leibnizsche Logic mit besonderer Berücksichtigung des Problems der Intension und der Extension*. Acta Philosophica Fennica, Fasc. XII. Helsinki: Societas Philosophica Fennica.

Kauppi, R. (1967). *Einführung in die Theorie der Begriffssysteme*. Acta Universitatis Tamperensis, Ser. A. Vol. 15. Tampere: Tampereen yliopisto.

Leibniz, G. W. (1997). *Philosophical Writings*. Ed. G. H. R. Parkinson. Trans. M. Morris and G. H. R. Parkinson. London: The Everyman Library.

Lewis, C. I. (1918). *A Survey of Symbolic Logic*. Berkeley: University of California Press.

Palomäki, J. (1994). *From Concepts to Concept Theory: Discoveries, Connections, and Results*. Acta Universitatis Tamperensis, Ser. A. Vol. 416. Tampere: Tampereen yliopisto.

Palomäki, J. (1997). "Three Kinds of Containment Relations of Concepts". *Information Modelling and Knowledge Bases VIII*. Eds. H. Kangassalo, J.F. Nilsson, H. Jaakkola, and S. Ohsuga. *Frontiers in Artificial Intelligence and Applications*. Amsterdam, Berlin, Oxford, Tokyo, Washington, DC: IOS Press, 261-277.

Palomäki, J. (2004). "Reasoning with Improper Concepts". (Abstract.) *The Bulletin of Symbolic Logic* 2, 293.

Palomäki, J. (2012). "God, Intensional Negation - and the Devil(s)". *Philosophy Study* 5 (Vol. 2), 323-336.

Palomäki, J. (2014). "An Axiomatic Approach to Relational Concepts". *Information Modelling and Knowledge Bases, XXVI*. Eds. B. Thalheim, H. Jaakkola, Y. Kiyoki, and N. Yoshida. Amsterdam, Berlin, Oxford, Tokyo, Washington, DC.: IOSPress, 355-360.

Palomäki, J. & Kangassalo, H. (2012). "That Is-IN Isn't Is-A: A Further Analysis of Taxonomic Links in Conceptual Models". *Advances in Knowledge Representation*. Ed. C. Ramirez. InTech: Rijeka, 3-18.
Available from: http://www.intechopen.com/books/advances-in-knowledge-representation/that-is-in-isn-t-is-a-a-further-analysis-of-taxonomic-links-in-conceptual-modelling

Penttilä, A. & Saarnio, U. (1934). "Einige grundlegende Tatsachen der Worttheorie nebst Bemerkungen über die sog. unvollständigen Symbole". *Erkenntnis* IV, 28–45, 139–159.

Saarnio, U. (1926). *Leibniz'in matemaattisten keksintöjen merkityksestä hänen filosofiassaan*, Master's Thesis. Turun yliopisto.

Saarnio, U. (1935). *Untersuchungen zur symbolischen Logik, Teil I: Kritik des Nominalismus und Grundlegung der logistischen Zeichentheorie (Symbolologie)*. Acta Philosophica Fennica I. Helsinki: Societas Philosophica.

Saarnio, U. (1944). "Über den Begriff des Intensionalen in der Logik". In *Archiv für Rechts- und Sozialphilosophie* XXXVII, Berlin-Braunschweig-Wien, 27–58.