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Multi-Agent Formation Control under Switching Network and Input Constraints

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Abstract: This work discusses formation control of heterogeneous Multi-Agent Systems (MASs) under discrete time setting where its formation size is scalable by a scaling factor. The case with and without input constraints are discussed. The communication network is assumed to be jointly connected and the leader-follower network is assumed for the unconstrained case. Compared to the previous work on distributed scalable formation control for heterogeneous agents, this work considers input constraints in the formulation. The proposed algorithm is based on the discrete time version of the internal model principle and the constraint is handled by the command governor. Numerical examples are shown to illustrate the proposed method.

Keywords: Multi-agent formation, Internal Model Principle, Input Constraint.

1. INTRODUCTION

Multi-agent formation is one of the basic problems in cooperative control of MASs (see the work of Lafferriere et al. [2005], Oh and Ahn [2013], Peymani et al. [2014], Yang et al. [2014], Yaghmaie et al. [2017] and others). The shape of the formation is produced by specifying a bias to the consensus control law of each agent. Usually, the formation size cannot be altered during the operation of MAS, which makes the MAS unable to adapt to the environment or mission change.

Past works on formation control with size adjustment will be discussed in the following. Coogan in Coogan et al. [2011] and Coogan and Arcak [2012] solves scalable formation problem of double integrator agents. The scalable formation is achieved by introducing an auxiliary state that acts as a multiplier to the bias in the standard formation control law. In Coogan et al. [2011], the auxiliary state is updated using a consensus algorithm and the consensus value of the auxiliary state determines the formation scaling factor. Whereas in Coogan and Arcak [2012], the auxiliary state is updated by estimating the desired formation scaling factor (known only by leader agent(s)) by monitoring the relative position of agents' neighbor. Recent work of Djamari [2022] discusses scalable formation of single integrator and double integrator agents under fixed directed network. The approach is based on the estimation of agents position. Another recent work Djamari et al. [2022] solves scalable formation for single integrator agents where the formation scaling mechanism is based on the origin of the formation. Both scaling factor and the origin of the formation are adjustable by the leader agent.

Scalable formation control considering homogeneous linear system agents is discussed in Tran and Yucelen [2016]. In Tran and Yucelen [2016], a consensus-based scalable formation algorithm is used as a dynamic compensator. In addition to scalable formation, the formulation in Tran and Yucelen [2016] also allows for orientation adjustment of the formation. This additional adjustment is achieved by adding additional variable multiplied to the desired position in the dynamic compensator. Meanwhile, scalable formation considering heterogeneous agents is discussed in Djamari [2021]. The control input in Djamari [2021] is based on the internal model principle and H_∞; and the system is considered in continuous-time setting with switching graph. However, the work Djamari [2021] did not consider input constraint in the problem.

Unlike the literature in scalable formation described previously, this work considers heterogeneous discrete-time agents under input constraint. This problem has not been considered yet in any previous literature on scalable formation. The proposed control input is based on the internal model principle (see Wieland et al. [2011]), command

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governor, and projection-based consensus. Each agent has a reference output to be tracked, and the tracking under input constraint is made possible by the internal model principle and command governor (see Ong et al. [2020]). The difference of this work compared to Ong et al. [2020] is the reference output matrix. To achieve formation, the reference output matrix is specified differently for each agent (this is so that each agent tracks different output).

The rest of this paper is organized in the following. Section II discusses graph notations and the problem formulation. Section III discusses the control input without constraint, while section IV discusses the proposed control input with input constraint. Section V discusses numerical examples and section VI concludes this paper.

Notations
Non-negative and positive integer sets are indicated by $\mathbb{Z}_0^+$ and $\mathbb{Z}^+$ respectively. Let $M, L \in \mathbb{Z}^+$ with $M > L$. Then $\mathbb{Z}^M := \{1, 2, \cdots, M\}$ and $\mathbb{Z}_L^M := \{L, L+1, \cdots, M\}$. While $\mathbb{R}, \mathbb{R}^n, \mathbb{R}^{n\times m}$ refer respectively to the sets of real numbers, n-dimensional real vectors and n by m real matrices. I_n is the $n \times n$ identity matrix with 1_n being the n-column vector of all ones (subscript omitted when the dimension is clear). Given a set C, $|C|$ denotes its cardinality. The transpose of matrix M and vector v are indicated by M' and v' respectively. Additional notations are introduced when required in the text.

2. PRELIMINARIES

2.1 Graph Notations

The network of N nodes described by a time-varying directed graph is $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ with vertex set $\mathcal{V} = \{1, 2, \cdots, N\}$ and edge set $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$. The pair $(j, i) \in \mathcal{E}(t)$ if node i receives information from node j at time t. The set of neighbors of node i at time t is $\mathcal{N}_i(t) := \{j \in \mathcal{V} : (j, i) \in \mathcal{E}(t), i \neq j\}$. We define $d_i(t) = |\mathcal{N}_i(t)|$ to be the number of neighbors of agent i at time t.

The union of the graph sequence $\mathcal{G}(t)$ at the time interval $[t_a, t_b]$ is defined as $\mathcal{G}_{t_a}^{t_b} = (\mathcal{V}, \bigcup_{t_a}^{t_b} \mathcal{E}(t))$. The graph sequence $\mathcal{G}(t)$ is said to be uniformly connected if there exist a finite time horizon $T > 0$ such that for all t, the graph $\mathcal{G}_t^{t+T}$ contains a spanning tree, or that there exist a directed path from at least one node to every other nodes; and it is uniformly strongly connected if there exist a directed path between any two nodes.

The adjacency matrix $\mathcal{A}(t) = [a_{ij}(t)]$ of $\mathcal{G}(t)$ is the $N \times N$ matrix whose (i, j) entry is 1 if $(j, i) \in \mathcal{E}(t)$, and 0 otherwise. The Perron matrix $P(t) = [p_{ij}(t)]$ associated to graph $\mathcal{G}(t)$ is defined by

$$p_{ij}(t) = \begin{cases} 1 - \gamma \cdot d_i(t) & i = j \\ \gamma \cdot a_{ij}(t) & i \neq j \end{cases}$$

where $\gamma = \frac{1}{N}$, and this imply that the off-diagonal term of $P(t)$ is either $\frac{1}{N}$ or 0. By the above construction, it is easy to see that $P(t)$ is a nonnegative matrix and its row sum equals one, implying that $P(t)$ is a stochastic matrix, and also its diagonal entries are positive.

2.2 Problem Formulation

Consider N heterogeneous agents where each agent i is given by

$$x_i(t+1) = A_i x_i(t) + B_i u_i(t), i \in \mathbb{Z}^N, \quad (1)$$

$$y_i(t) = C_i x_i(t), i \in \mathbb{Z}^N, \quad (2)$$

where $x_i(\cdot) \in \mathbb{R}^{n_i}$, $y_i(\cdot) \in \mathbb{R}^p$ and $u_i(\cdot) \in \mathbb{R}^{m_i}$ are the state, output and control signal of agent i. The assumptions on the systems' matrices are:
(**A1**) The pair (A_i, B_i) and (A_i, C_i) are stabilizable and detectable for all $i \in \mathbb{Z}^N$.
Note that assumption (A1) is needed to guarantee the existence of stabilizing feedback matrix.

The objective in this work is to coordinate the N agents in a distributed manner to form a formation with variable size (scalable formation). Let $\delta_i \in \mathbb{R}^p$ and $\delta_j \in \mathbb{R}^p$ be the desired outputs of agents i and j defined on a local coordinate system, then the scalable formation objective is to achieve

$$\lim_{t\to\infty} (y_i(t) - y_j(t)) = \lambda(\delta_i - \delta_j), \ \forall i, j \in \mathbb{Z}^N.$$

Here, $\lambda \in \mathbb{R}^+$ is the scaling factor of the formation which will be determined through the process of consensus.

In this paper, we consider the following two cases:

(i) Unconstrained case where the communication network topology is given by a leader-follower network and without loss of generality agent N is chosen as the leader agent, i.e. $a_{Nj}(t) = 0$ for all $j \neq N$ and $t \geq 0$. Specifically, the communication network satisfies the following assumption:
(**A2a**) The graph $\mathcal{G}(t)$ is uniformly connected.
(ii) Constrained case where the communication network satisfies the following assumption:
(**A2b**) $\mathcal{G}(t)$ is uniformly strongly connected.

(A2a) is a standard assumption on consensus problem under directed time-varying graph (see for example Scardovi and Sepulchre [2008] and Moreau [2005]) which is applicable for the unconstrained case, while (A2b) is necessary for the constrained case (see Lin and Ren [2014]).

3. UNCONSTRAINED SCALABLE FORMATION

To achieve scalable formation, the output of agent i is made to track its reference-output defined in the following:

$$y_{i,ref}(t) = \delta_i \lambda_i(t) + Q w_i(t), \quad (3)$$

where λ_i is the formation scaling variable of agent i, w_i is the common trajectory variable of agent i, and Q is the output matrix of w_i dynamics. The variables λ_i and w_i are updated independently using consensus dynamics:

$$\lambda_i(t+1) = \textstyle\sum_{j=1}^N p_{ij}(t)\lambda_j(t), i \in \mathbb{Z}^N, \quad (4)$$

$$w_i(t+1) = S \textstyle\sum_{j=1}^N p_{ij}(t) w_j(t), i \in \mathbb{Z}^N. \quad (5)$$

where $p_{ij}(t)$ is the entry of the Perron matrix $P(t)$, S is the state matrix of the reference generator state w_i with the following assumption made on S:
(**A3**) Eigenvalues of S lie on the unit circle.

Following the leader-follower network setting, since $a_{Nj}(t) = 0$ for all j and $t \geq 0$, then $\lambda_N(t+1) = \lambda_N(t)$ and $w_N(t+1) = Sw_N(t)$. The following lemma shows that for leader-follower network with agent N as the leader agent, then the consensus value of λ is the initial state of agent N:

Lemma 1. Let node N be the leader node in the leader-follower graph $\mathcal{G}(t)$. Given system of N agents (4). Suppose (A2a) holds. Then $\lambda_i(t) \to \lambda_N(t_0)$ for all $i \in \mathbb{Z}^N$.

Since $a_{Nj}(t) = 0$ for all j and all $t \geq 0$, agent N do not update its λ or $\lambda_N(t) = \lambda_N(t_0)$ for all $t \geq 0$. Since consensus is achieved (due to (A2a)), then all λ_i approaches $\lambda_N(t_0)$. Since $\lambda_N(t_0)$ is the consensus value of λ, agent N can determine the consensus value by choosing its initial state.

From Scardovi and Sepulchre [2008] and Moreau [2005], under (A2a), the N agents (4)-(5) reach consensus exponentially, or that $\lambda_i(t) \to \lambda_\infty$ and $w_i(t) \to \bar{w}(t)$ exponentially for all $i \in \mathbb{Z}^N$, where $\bar{w}(t)$ is a solution to $w_0(t+1) = Sw_0(t)$ for some $w_0 \in \mathbb{R}^{\bar{n}}$. Since $\lambda_i(t) \to \lambda_\infty$ and $w_i(t) \to \bar{w}(t)$ for all $i \in \mathbb{Z}^N$, then $y_{i,ref}(t) \to \delta_i\lambda_\infty + Q\bar{w}(t)$ for all $i \in \mathbb{Z}^N$. From here, we can see that asymptotically $y_{i,ref}(t)$ contains two components. The first component, $\delta_i\lambda_\infty$, is the formation component scalable by λ_∞ and the second component, $Q\bar{w}(t)$, is the common (consensus) trajectory component.

If each y_i tracks $y_{i,ref}$ asymptotically, which will be shown in the sequel, then asymptotically the outputs of agents i and j, $i, j \in \mathbb{Z}^N$ will differ by $\lambda_\infty(\delta_i - \delta_j)$, i.e. $\lim_{t\to\infty}(y_i(t) - y_j(t)) = \lambda_\infty(\delta_i - \delta_j)$ for all $i, j \in \mathbb{Z}^N$. This implies that λ_∞ is the formation scaling factor.

The proposed control input to the i^{th} agent (1)-(2) is then designed based on the internal model principle approach discussed in Wieland et al. [2011] such that agents track $y_{i,ref}(t)$:

$$u_i(t) = K_i\hat{x}_i(t) + L_{\lambda_i}\lambda_i(t) + L_{w_i}w_i(t), i \in \mathbb{Z}^N, \tag{6}$$

where $K_i \in \mathbb{R}^{m_i \times n_i}$ is a feedback matrix designed such that $(A_i + B_iK_i)$ is Schur for all $i \in \mathbb{Z}^N$, while $L_{\lambda_i} \in \mathbb{R}^{m_i}$ and $L_{w_i} \in \mathbb{R}^{m_i \times \bar{n}}$ are the feedback vector and feedback matrix that will be elaborated shortly. The state $\hat{x}_i \in \mathbb{R}^{n_i}$ in (6) is the estimate of x_i obtained from

$$\begin{aligned}\hat{x}_i(t+1) &= A_i\hat{x}_i(t) + B_iu_i(t)\\ &\quad + H_i(\hat{y}_i(t) - y_i(t)), i \in \mathbb{Z}^N, \end{aligned}\tag{7}$$

$$\hat{y}_i(t) = C_i\hat{x}_i(t), i \in \mathbb{Z}^N, \tag{8}$$

where $\hat{y}_i$ is the estimated output, and $H_i \in \mathbb{R}^{n_i \times p}$ is feedback matrix designed such that $(A_i + H_iC_i)$ is Schur for all $i \in \mathbb{Z}^N$.

To ensure each agent tracks their own reference-output, the feedback vector L_{λ_i} and feedback matrix L_{w_i} are designed via the well-known internal model principle (Knobloch et al. [1993]). L_{λ_i} in (6) is feedback vector computed by $L_{\lambda_i} = \Gamma_{\lambda_i} - K_i\Pi_{\lambda_i}$, where Γ_{λ_i} and Π_{λ_i} are the solutions to the following Francis Equations:

$$A_i\Pi_{\lambda_i} + B_i\Gamma_{\lambda_i} = \Pi_{\lambda_i} \tag{9}$$

$$C_i\Pi_{\lambda_i} = \delta_i \tag{10}$$

On the other hand, L_{w_i} is feedback matrix computed by $L_{w_i} = \Gamma_{w_i} - K_i\Pi_{w_i}$, where Γ_{w_i} and Π_{w_i} are the solutions to the following Francis Equations:

$$A_i\Pi_{w_i} + B_i\Gamma_{w_i} = \Pi_{w_i}S \tag{11}$$

$$C_i\Pi_{w_i} = Q \tag{12}$$

To guarantee the solvability of (9)-(10) and (11)-(12) for any δ_i and Q, the following assumption is needed (see Knobloch et al. [1993]):

(**A4**) The matrices:

$$\begin{bmatrix} A_i - I & B_i \\ C_i & 0 \end{bmatrix} \text{ and } \begin{bmatrix} A_i - \xi I & B_i \\ C_i & 0 \end{bmatrix}$$

are full-row rank for all $i \in \mathbb{Z}^N$ and for every eigenvalue ξ of S.

Assumption (A4) implies that the systems $(A_i - I, B_i, C - i)$ and $(A_i - \xi I, B - i, C_i)$ must not have zero at the origin for all ξ, eigenvalue of S.

We can present the following theorem:

Theorem 1. Given system of N agents (1)-(2) under leader-follower network with agent N as the leader, with control input $u_i(t)$ given by (6), observer dynamics given by (7)-(8), reference generator dynamics given by (4)-(5), and reference-output $y_{i,ref}(t)$ given by (3). Let $\lambda_N(t_0)$ be the initial value of λ_N. Suppose Assumptions (A1), (A2a), (A3), and (A4) are satisfied, then:

(i) $y_i(t) \to y_{i,ref}(t)$ exponentially for all $i \in \mathbb{Z}^N$,
(ii) $y_{i,ref}(t) \to \delta_i\lambda_N(t_0) + Q\bar{w}(t)$ exponentially for all $i \in \mathbb{Z}^N$, and
(iii) $\lim_{t\to\infty}(y_i(t) - y_j(t)) = \lambda_N(t_0)(\delta_i - \delta_j)$ for all $i, j \in \mathbb{Z}^N$.

The auxiliary systems (4)-(5) are independent from the primary systems (1)-(2), so λ_i and w_i will reach consensus under (A2a) and part (ii) in Theorem 1 above is then guaranteed. Thr remaining problem is to track $y_{i,ref}(t)$ which is guaranteed by the internal model principle (assumptions (A1), (A3), and (A4)). Once $y_{i,ref}(t)$ is tracked, agents then formed a formation and their size is determined by the consensus value of λ_i.

4. SCALABLE FORMATION WITH INPUT CONSTRAINTS

In this section, the objective of the formation control is the following

$$\lim_{t\to\infty}(y_i(t) - y_j(t)) = \lambda(\delta_i - \delta_j), \forall i, j \in \mathbb{Z}^N \tag{13a}$$

such that

$$u_i(t) \in \mathcal{U}_i, i \in \mathbb{Z}^N, \forall t \geq 0 \tag{13b}$$

where λ is the formation scaling factor and $\mathcal{U}_i \subset \mathbb{R}^{m_i}$ is the constraint set for the control input, u_i. The assumption on $\mathcal{U}_i$ is:

(A5) $\mathcal{U}_i$ is a polytope and contains the origin in its interior for all i.

The proposed control input to solve scalable formation with input constraints is the following:

$$u_i(t) = K_i x_i(t) + L_i \rho_i(t), i \in \mathbb{Z}^N \tag{14a}$$
where
$$\rho_i(t) = \text{argmin}_q\{\|q - z_i(t)\|^2 : q \in Z_i(x_i)\}, \tag{14b}$$
$$z_i(t+1) = \text{Proj}_{Z^i_\infty}\left(\bar{S}\sum_{j=1}^N p_{ij}(t) z_j(t)\right), \tag{14c}$$

$p_{ij}(t)$ is the (i, j) entry of the Perron matrix at time t which represents the communication exchange among agents, $z_i = [\lambda_i, w_i]'$ is the stacked vector of the auxiliary states and is also the reference function for the command governor ρ_i (see Gilbert and Ong [2011] for the discussion on command governor), K_i is the stabilizing feedback matrix, $L_i = \Gamma_i - K_i\Pi_i$ is the feedback matrix with Γ_i and Π_i being the solutions to the Francis equations:

$$A_i\Pi_i + B_i\Gamma_i = \Pi_i\bar{S} \tag{15}$$
$$C_i\Pi_i = diag\{\delta_i, Q\}. \tag{16}$$

Meanwhile, $\text{Proj}_W(\mu) := \text{argmin}_v\{\|v - \mu\|^2 : v \in W\}$, $\bar{S} = diag\{1, S\}$, and the sets Z^i_∞ and $Z_i(x_i)$ will be explained next. Note that assumption (A4) is still needed to solve (15) and (16). Meanwhile, we only need (A_i, B_i) to be stabilizable (assumption A1).

The set Z^i_∞ is defined as (see Ong et al. [2020]):

$$Z^i_\infty := \{z : \left(H^i_x\Pi_i + H^i_z\right) z \leq 1\} \tag{17}$$

where H^i_x and H^i_z are the corresponding matrices to state x_i and z_i, respectively, in the invariant set $\mathcal{O}^i_\infty := \{(x_i(0), z_i(0)) : K_i x_i(t) + L_i z_i(t) \in \mathcal{U}_i\ \forall t\}$ with $x_i(t)$ and $z_i(t)$ given by the combined system

$$\begin{bmatrix} x_i(t+1) \\ z_i(t+1) \end{bmatrix} = \begin{bmatrix} \bar{A}_i & B_iL_i \\ 0 & \bar{S} \end{bmatrix} \begin{bmatrix} x_i(t) \\ z_i(t) \end{bmatrix}, \forall i \in \mathbb{Z}^N \tag{18}$$

where $\bar{A}_i = A_i + B_iK_i$. The invariant set can be computed and expressed as $\mathcal{O}^i_\infty := \{(x, z) : H^i_x x + H^i_z z \leq 1\}$, (see Gilbert and Tan [1991], Gilbert and Ong [2008], and Gilbert and Ong [2011]). The existence of the invariance set $\mathcal{O}^i_\infty$ for the system (18) with constraint $K_i x_i(t) + L_i z_i(t) \in \mathcal{U}_i\ \forall t$ is shown in Lemma 2 of Ong et al. [2020].

The motivation in using (18) to compute the invariant set is that when the system (14c) reaches consensus or that $\lim_{t\to\infty}(z_i(t) - z_j(t)) = 0$ for all i, then the system becomes $z_i(t+1) = \bar{S}z_i(t)$. The consensus discussion on $z_i(t)$ is discussed in Theorem 2 of (Ong et al. [2020]). The system (18) is the unconstrained system discussed in section III when the consensus of z_i is reached. The additional condition to reach consensus on $z_i(t)$ for the constrained case, in addition to (A2b) is
(**A6**) $\cap_i \text{int}(Z^i_\infty) \neq \emptyset$.
Assumption (A6) is needed to guarantee the existence of consensus value. The algorithm (14c) is devised such that z_i reaches consensus and z_i is inside the invariant set of the intended steady-state system (18) for all $t \geq 0$.

The set $Z_i(x_i)$ is defined as (see Ong et al. [2020]):

$$Z_i(x_i) := \{z : H^i_z z \leq 1 - H^i_x x_i\} \tag{19}$$

The set $Z_i(x_i)$ represents the feasible z for a given x_i (such that $u_i \in \mathcal{U}_i$). Thus, this set is used in the command governor (14b) to select feasible ρ_i (admissible to the input constraint $\mathcal{U}_i$). In other words, the command governor (14b) guarantee the computed control input u_i to be inside the set $\mathcal{U}_i$ for all $t \geq 0$.

Following the concept in section III, to reach the scalable formation, we need y_i to track the reference output given by

$$\bar{y}_{i,ref}(t) = diag\{\delta_i, Q\}z_i(t) = \delta_i\lambda_i(t) + Qw_i(t), \tag{20}$$

while satisfying (13b). The second equality in (20) is true since $z_i = [\lambda_i, w_i]'$. Notice that (20) is equal to (3) and the reference output notation is differentiated to avoid confusion between the constrained and unconstrained case. The tracking of reference output is done by the command governor (14b) combined with the internal model principle based control input (14a). The mathematical proof showing $y_i(t) \to \bar{y}_{i,ref}(t)$ is given in Theorem 2 of Ong et al. [2020].

The additional assumptions in the constrained case, following the work of Ong et al. [2020] are:
(**A7**) The states x_i for all i are measurable
(**A8**) $(\bar{S}, diag\{\delta_i, Q\})$ is observable for all i

The result on scalable formation with input constraints is concluded in the following theorem:

Theorem 2. Given system of N agents (1)-(2) with reference input given by (20). Suppose assumptions (A1), (A2b), and (A3) - (A8) are satisfied. Then, by using the control input (14a), the command governor (14b), and the projection based consensus (14c), the MAS reach the scalable formation satisfying (13a)-(13b).

When each y_i tracks $\bar{y}_{i,ref}$, then the scalable formation under input constraint is achieved. The scaling factor of the formation depends on the consensus value of λ_i, while the consensus value of λ_i depends on the command governor which select the new value of z_i every time step.

5. NUMERICAL EXAMPLES

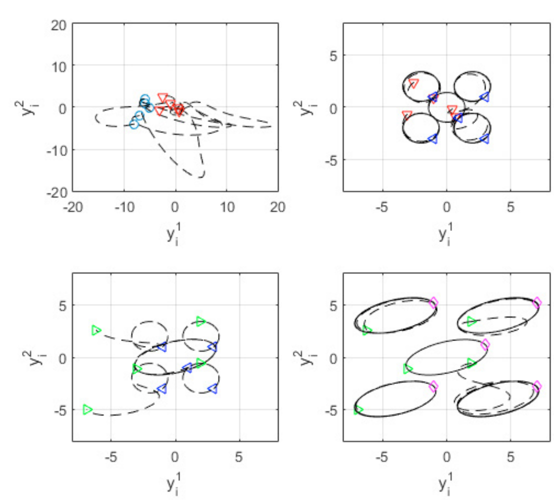

Fig. 1. Agents trajectories of unconstrained case; $\circ, \triangledown, \triangleleft, \triangleright$ and $\diamond$ shows agents position at $t = 0s$, $t = 15$s, $t = 150$s, $t = 165$s, and $t = 300$s, respectively.

In this section, numerical examples on the unconstrained and constrained case will be given. There are 5 agents or $N = 5$ and the agents' dynamics used in both cases are

$$A_j = \begin{bmatrix} 0_{2\times 2} & I_2 \\ I_2 & \begin{bmatrix} a_i & -1 \\ -0.5 & b_i \end{bmatrix} \end{bmatrix}, B_j = \begin{bmatrix} 0_{2\times 2} \\ I_2 \end{bmatrix}, C_j = [I_2 \ 0_{2\times 2}]$$

for $j \in \mathbb{Z}^5$, where $a_i = 1.5$, $b_i = 1$ for $i = 1, 2, 3$ and $a_i = 0.25, b_i = 2$ for $i = 4, 5, K_i$ and H_i for all $i \in \mathbb{Z}^5$ are computed such that $A_i + B_i K_i$ and $_iA + H_i C_i$ are Hurwitz. Meanwhile, agent 5 is the leader agent for the unconstrained case.

For both unconstrained and constrained cases, we use $Q = [1, 0, 0, 0; 0, 0, 1, 0]$, $\delta_1 = [2, -2]'$, $S = diag\{S_1, S_1\}$ with $S_1 = [0.9988, 0.05; -0.05, 0.9988]$, $\delta_2 = [2, 2]'$, $\delta_3 = [-2, 2]'$, $\delta_4 = [-2, -2]'$, and $\delta_5 = [0, 0]'$. For both unconstrained and constrained case, we will plot the output of agents in 2D space. Additionally, for the constrained case, we will observe the plot of u_i for all agents.

5.1 Unconstrained Case

In the unconstrained, the leader-follower graph $\mathcal{G}(t)$ switches between three graphs $\mathcal{G}_1$, $\mathcal{G}_2$, and $\mathcal{G}_3$; and following are the associated adjacency matrices: $a_{12}(1) = a_{23}(1) = a_{23}(2) = a_{34}(2) = a_{34}(3) = a_{45}(3) = 1$, with $a_{ij}(k)$ is the (i, j) entry of the adjacency matrix associated with the k-th graph for $k = 1, \cdots, 3$. The switching order of the graphs in the simulation is $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, \mathcal{G}_1, \cdots$.

At $t = 0$ s, the auxiliary state of agent 5 is set to $\lambda_5(0) = 1$ (which will make $\lambda_\infty = 1$), $w_5(0) = [\sqrt{2}, 0, 0, \sqrt{2}]'$, and it triggers the 5 agents to move in a circular trajectory, where each circle is centered at δ_i in the local coordinate system, and its trajectories are shown in Fig. 1 for $t = 0$s - 150s. At $t = 150$s the auxiliary state of agent 5 is set to $\lambda_5(150) = 2$ (which will make $\lambda_\infty = 2$), $w_5(150) = [\sqrt{8}, -\sqrt{2}, 0, -0.6\sqrt{8}]'$, and it triggers the 5 agents to move in a slanted ellipsoid trajectories, where each ellipsoid is centered at $2\delta_i$ in the local coordinate system, and its trajectories are shown in Fig. 1 for $t = 150$s - 300s. In Fig. 1, at $t = 0s$, $t = 15$s, $t = 150$s, $t = 165$s, and $t = 300$s; agents position are marked with $\circ, \nabla, \triangleleft, \triangleright$ and $\diamond$ respectively. As we can see from Fig 1, $\triangleleft$ and $\diamond$ shows that the relative position of agent i and j are $\lambda_\infty(\delta_i - \delta_j)$ for all $i, j \in \mathbb{Z}^5$.

5.2 Constrained Case

In the constrained, the graph $\mathcal{G}(t)$ switches between five graphs $\mathcal{G}_1, \mathcal{G}_2, \cdots, \mathcal{G}_5$, where $a_{12}(1) = a_{21}(1) = a_{23}(1) = a_{32}(1) = a_{34}(1) = a_{43}(1) = a_{23}(2) = a_{32}(2) = a_{34}(2) = a_{43}(2) = a_{45}(2) = a_{54}(2) = a_{12}(3) = a_{21}(3) = a_{23}(3) = a_{32}(3) = a_{34}(3) = a43(3) = a_{23}(4) = a_{32}(4) = a_{34}(4) = a_{43}(4) = a_{45}(4) = a_{54}(4) = a_{12}(5) = a_{21}(5) = a_{23}(5) = a_{32}(5) = a_{34}(5) = a_{43}(5) = 1$, with $a_{ij}(k)$ is the (i, j) entry of the adjacency matrix associated with the k-th graph for $k = 1, \cdots, 5$.

The set $\mathcal{U}_i$ for all $i = 1, \cdots, 5$ is $\mathcal{U}_i = \{|u_i^m| \leq 5, m = 1, 2\}$. Which also means that the elements of the control signal for all agents must be between -5 and 5. To compute the set Z_∞^i, we use the Invariant Set Toolbox developed by Eric Kerrigan, in MATLAB. The initial states for all agents are chosen such that (14b) is feasible. For the same

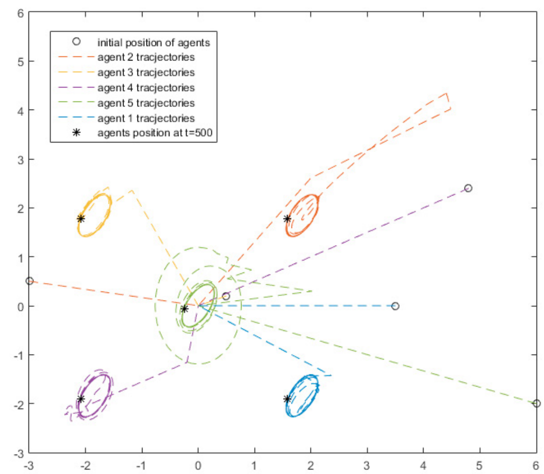

Fig. 2. Agents trajectories of constrained case set 1

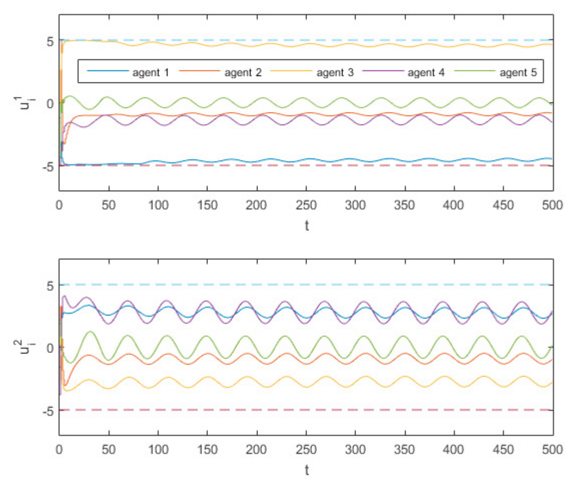

Fig. 3. Control input of constrained case set 1

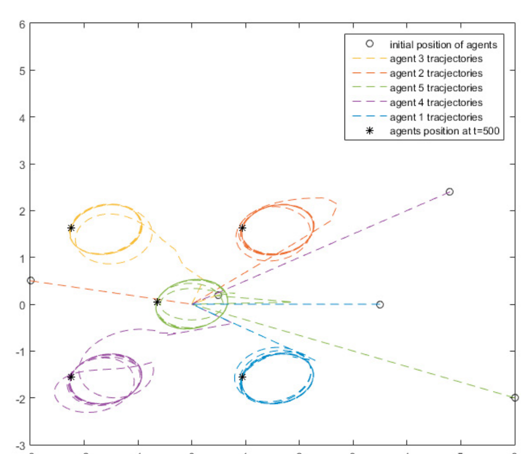

Fig. 4. Agents trajectories of constrained case set 2

initial states of x_i, two different initial auxiliary states z_i are chosen for two different simulation sets. We can see that u_i^1 of some agents are saturated and almost flat for set 1, but eventually agents can form the scalable formation. It can be observed that the circular trajectories for set 1 is smaller than set 2, although the formation size of the two sets are roughly equal.

The consensus value of λ is limited by the input constraint. This is to say that the formation size can be made as small as possible, but its size is bounded by the input constraint through the set Z^i_∞. We believe that the proposed strategy is suitable to solve the formation containment problem.

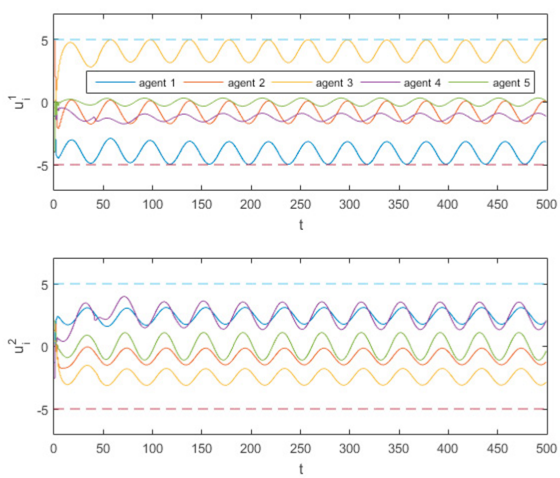

Fig. 5. Control input of constrained case set 2

6. CONCLUSION

Scalable formation for heterogeneous linear system agents under a directed time-varying network for unconstrained and constrained input has been presented in this paper. The scalable formation is achieved, via internal model principle and set projection, by having agents track a reference-output with bias. The bias has two components which are δ_i and λ_i. While the performance of constrained input case is good, agents need to be able to measure all states which is not practical.

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