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Varieties of Ideal Language Philosophy

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Introduction

In the honorable tradition of analytic philosophy,[1] it has been common to distinguish two subordinate traditions, "Ideal Language Philosophy" and "Ordinary Language Philosophy."[2] The latter obviously denotes philosophy which focuses on natural languages. The former refers to the kind of philosophy that utilizes artificial formal languages and emphasizes their importance for philosophy. This tradition has often had a somewhat critical attitude toward colloquial languages. Paradigmatic representatives of this approach include Alfred Tarski and especially Rudolf Carnap. What is distinctive in Ideal Language Philosophy is the central role of the language of new formal logic, due to Frege and Russell. The new logic was initially developed to serve as a tool in the philosophy of mathematics, namely, to enable rigorous gap-free inferences and precise definitions in their attempts to reduce arithmetic conclusively to logic. However, it subsequently achieved a much more general and philosophically pivotal role in the tradition at stake here.

In what follows, I aspire to track how new formal logic became so enormously central to early analytic philosophy. I will look at the beginnings of Ideal Language Philosophy in Frege's and Russell's work, very briefly discuss the role of early Wittgenstein, and review the relationship of its key representatives—Carnap and Tarski—to ordinary language, and by doing so I aim to shed light on why these philosophers gave such an important place to artificial formal languages in their whole philosophy.

There is already a rich scholarly literature on each of these philosophers. I do not want to pretend that anything I am saying here is big news for scholars. However, I think it is fruitful to provide a comparative overview, a synoptic picture, or a lengthwise cross-section, and collate these highly influential and original thinkers and consider how they viewed the role of artificial formal languages in philosophy. We can then see more clearly both differences and some continuity and similarities, as well as how certain ideas about the relations of colloquial languages and artificial formal languages evolved within this tradition.[3] (I must

[1] The nature and scope of this tradition is a subject of some debate; see Raatikainen 2013a for my own, somewhat unorthodox view.

[2] The distinction, with these very labels, was influentially propagated by Richard Rorty in his widely read introduction to the collection *Linguistic Turn* he edited (Rorty 1967).

[3] The only overview with a somewhat similar concentration on the ideal language tradition I am aware of is Hylton 2018; however, his emphasis is quite different (the specific Russellian idea of a logically perfect language), and his focus is more on later Quine and even Lewis than on the earlier figures discussed in this paper. Thus, I think that the present paper and Hylton 2018 nicely complement each other. My understanding of Russell specifically here is, though, indebted to an earlier paper by Hylton (2007).

necessarily set aside many interesting details, including various changes of mind made by the philosophers discussed, in order to keep the size of the paper reasonable.)

Background: Leibniz, *Characteristica Universalis* and *Calculus Ratiocinator*

An important background figure for the tradition under consideration here is the 17th-century polymath, mathematician, and philosopher Gottfried Wilhelm Leibniz (1646–1716). Aristotle and his followers over centuries had assumed that natural languages reflect quite well forms of logical reasoning and other logical relationships, and perhaps even the structure of reality. Leibniz, on the other hand, thought that everyday words do not adequately reflect reality, and that a new artificial language, modeled after algebra and arithmetic, which would undistortedly mirror the reality and its structure, should therefore be constructed.

Leibniz put forward the idea of a universal logical ideal language, "a Characteristica Universalis," which would reflect the structure of the whole world without distortion, and the "Calculus Ratiocinator," a precise and comprehensive system of logical reasoning that would facilitate reasoning by making it entirely mechanical and thus enable the derivation of all truths from simple thoughts.[4] The universal language should include, for any simple thought, a sign designating it unambiguously. It would represent all the logical structure of the world. On the one hand, according to Leibniz himself, it is not possible for man to know the latter, so even in Leibniz's view, a perfect universal language would ultimately be impossible for man. On the other hand, Leibniz did have high hopes for the universal language: for example, he believed that his universal language would help to resolve disputes that had been entrenched in the wars of religion between Catholics and Lutherans, among others. Leibniz's more concrete attempts in logic were extensive but far from completed.[5] His ideas, however, were not completely forgotten either, at least in the German-speaking world,[6] where they were kept alive by some of the less well-known thinkers.

Frege and his concept-script

Leibniz's vision had begun to come true—insofar as it was at all possible—in the work of German mathematician and philosopher Gottlob Frege (1848–1925), the founder of modern logic, who is generally considered to be the greatest logician since Aristotle.[7] Frege undertook to construct a new logical language that would implement both of Leibniz's ideas: Frege's logical ideal language was intended to be both a universal medium of expression, a

[4] For Leibniz, these were not two separate projects, but two aspects of the broader project of general science; see, e.g., Peckhaus 2004.

[5] For a rather comprehensive overview of Leibniz's work on logic, see Lenzen 2004.

[6] Recall that Leibniz was—although he wrote in Latin and French—German.

[7] A more complete story should certainly also discuss at least George Boole and the Boolean tradition in logic.

"*Lingua Characterica*,"[8] and a system of rules of logical reasoning, a *Calculus Ratiocinator*, as he interpreted them.

Frege referred to the rather little-known German logician Adolf Trendelenburg, who had written earlier a review of Leibniz's idea of universal language (Trendelenburg 1857). In Trendelenburg's text, the ideal language of Leibniz was called "a concept-script," which Frege adopted as the name of his own ideal language. Trendelenburg peculiarly interpreted Kant as a developer of the Leibnizian ideal language project. As is well-known, Kant distinguished sharply conceptual and empirical components of thought. According to Trendelenburg, Leibniz's original goal is impossible to achieve, but he interpreted Kant's distinction as resulting in the more realistic goal of an ideal language: the task is no more to try to represent in an ideal language all the properties of objects, but only the conceptual properties.[9] Frege adopted this picture of the relationship between Leibniz and Kant. He left empirical objects outside his ideal language and focused on the study of formal concepts. Indeed, Frege sometimes used the label "formula language of pure thought" with a definite Kantian ring of his logical ideal language. (See Sluga 1980, Haaparanta 1985, Beaney 1996, Gabriel 2013.)

For Leibniz, thought and perception were distinguished only by the degree of clarity and distinctness. Kant, on the other hand, made a sharp distinction between the faculties of sensibility and understanding. Frege followed Kant by sharply distinguishing between reason as the source of logical knowledge, perception as the basis of empirical knowledge, and *a priori* intuition as the basis of synthetic *a priori* knowledge. For Leibniz the rationalist, after all, all knowledge was in the end *a priori* and analytic, and an ideal language would make it at least in principle possible to achieve all truths. For Frege, in contrast, the use of the envisioned ideal language was much more limited, as it was restricted to form and hence to logic. Frege, on the other hand, followed Leibniz in that he, too, took a quite critical stance toward natural language: he considered it ambiguous, unclear, and contaminated with erroneous (including psychologistic) philosophy, and did not trust it as a basis for logical knowledge. Frege's concept-script was intended as a new universal language logically superior to natural language. The language of his new logic was published in his first book *Begriffsschrift* ("Concept-script"; Frege 1879).

Frege was indeed dissatisfied with the philosophical theories of his time about mathematical truths and our knowledge of them. Frege took as his vocation to reduce arithmetic to logic. In this way he wanted to demonstrate for good, on the one hand, that the various then-fashionable empiricist and psychologistic theories of mathematics were totally wrong and that knowledge in arithmetic is *a priori*, and, on the other hand—contrary to Kant's claim—that arithmetic was not synthetic but analytic. He soon found traditional Aristotelian logic hopelessly inadequate for this program and developed single-handedly modern propositional logic and quantification theory.[10] Frege's view in the philosophy of mathematics that at least

[8] Frege and Trendelenburg (see below) called the Leibnizian idea "Lingua Characterica," not "Characteristica Universalis," as Leibniz himself had named it. The former likely derives from Erdmann's influential edition of Leibniz's works (1839–40) which also employed that formulation.

[9] Trendelenburg in turn cited Ludwig Benedict Trede (1811), who had earlier put forward somewhat similar ideas. Frege certainly knew about Trede at least through Trendelenburg's summaries.

[10] Beaney 2016 contains an accessible summary of the benefits of Frege's new logic.

arithmetic is reducible to the truths of logic is commonly called "Logicism."[11] This idea, too, was inherited from Leibniz.

Frege's goal was to show that all the truths of arithmetic can be proved on the ground of "laws of thought that transcend all particulars." Frege states in the Preface to his *Concept-script* that in order to prevent anything intuitive from sneaking in imperceptibly, he sought to keep the chain of inferences free of gaps:

> In striving to fulfill this requirement in the strictest way, I found an obstacle in the inadequacy of language: however cumbersome the expressions that arose, the more complicated the relations became, the less the precision was attained that my purpose demanded. Out of this need came the idea of the present [concept-script]. It is thus intended to serve primarily to test in the most reliable way the validity of a chain of inference and to reveal every presupposition that tends to slip in unnoticed, so that its origin can be investigated. (Frege 1879, 48–49)

Frege's logical notation was intended to express all the content of any judgment that is relevant to the logical reasoning in which it occurs. It is intended to be a tool for assessing the validity of any inference on any subject matter and for preventing any presuppositions from creeping in. Once our inference is expressed in the concept-script, the expectation is that it is a purely mechanical task to determine whether a given inference is valid and free of gaps, or whether it requires a hidden premise. It must be possible to see by examination whether or not a given claim is a logical law and whether the transition from one claim to another follows the logical rules put forward by Frege.

It follows from the above that not everything that can be expressed in natural language can be expressed in Frege's ideal language. Frege says he has chosen to refrain from expressing anything that is irrelevant to the chain of inferences. He calls what his ideal language expresses "conceptual content." Two judgments from which exactly the same consequences can be deduced are said to have the same conceptual content. The intuitive difference between what the words "and" and "but" express in natural language is a classic example of something that his notation cannot express, and Frege himself mentions it in *Concept-script*.

It might be tempting to assume that Frege's concept-script is only a version of natural language from which additional content that would obscure logical connections has been removed. However, this would be a mistake, as it ignores important differences between the purposes of concept-script and natural language. In the preface to his *Concept-script*, Frege writes:

> I believe I can make the relationship of my [concept-script] to ordinary language clearest if I compare it to that of the microscope to the eye. The latter, due to the range of its applicability, due to the flexibility with which it is able to adapt to the most diverse circumstances, has a great superiority over the microscope. Considered as an optical instrument, it admittedly reveals many imperfections, which usually remain unnoticed only because of its intimate connection with mental life. But as soon as scientific purposes place great demands on sharpness of resolution, the eye turns out to be inadequate. The microscope, on the other hand, is perfectly suited for just such purposes, but precisely because of this is useless for all others. (1879, 49)

[11] On the other hand, Frege agreed with Kant on his conviction that geometry is synthetic *a priori*, and as such, is not reducible to logic (which is analytic). Russell (see below), in contrast, advocated all-encompassing logicism with respect to all of mathematics.

The microscope does not filter out irrelevant details from the images we see. Rather, the sharpness of resolution makes it possible to see what cannot be seen with the naked eye. Frege therefore believes that the concept-script has expressive power that natural language does not have. In other respects, its expressive power is weaker. Like a microscope, an ideal language is perfectly suited to certain needs, but that is why it is also "useless for all others." The concept-script is a device developed for certain specific scientific purposes and should not be condemned, according to Frege, because it is not suited to some other purposes (ibid.).

For scientific purposes, natural language is deficient. However, these logical faults are, according to Frege, necessary for natural language to serve its own purposes. Elsewhere, Frege also compared natural language to a hand:

> The shortcomings [of ordinary language] stressed are rooted in a certain softness and instability of [ordinary] language, which nevertheless is necessary for its versatility and potential for development. In this respect, [ordinary] language can be compared to the hand, which despite its adaptability to the most diverse tasks is still inadequate. We build for ourselves artificial hands, tools for particular purposes, which work with more accuracy than the hand can provide. And how is this accuracy possible? Through the very stiffness and inflexibility of the parts the lack of which makes the hand so dexterous. Word-language is inadequate in a similar way. We need a system of symbols from which every ambiguity is banned, which has a strict logical form from which the content cannot be escape. (Frege 1882, 86)

If we are interested in something that serves the purposes of natural language, then the logical notation of the concept-script is inadequate. It would therefore be a mistake to describe Frege's logical language as a properly functioning version of natural language. Frege's notation is intended not to be a perfect language but a *logically* perfect language.[12]

As a philosopher, Frege still belonged to the earlier broadly Kantian epistemological tradition to a significant extent, and he did not yet—like Wittgenstein or Carnap later (see below)—put forward any radical general theses about the aims and the scope of philosophy. Still, with hindsight it is difficult not to see the following words in the preface to *Concept-script* as anticipating and grounding what was to come:

> If it is a task of philosophy to break the power of words over the human mind, by uncovering illusions that through the use of language often almost unavoidably arise concerning the relations of concepts, by freeing thought from the taint of ordinary linguistic means of expression, then my [concept-script], further developed for these purposes, can become a useful tool for philosophers. (Frege 1879, 50–51)

Russell and logically perfect language

Along with Frege, another early key figure in Ideal Language Philosophy is the influential British philosopher Bertrand Russell (1872–1970). He knew Leibniz's thought firsthand—after all, he had published the book *A Critical Exposition of the Philosophy of Leibniz* (1900). Russell too found Leibniz's original universal language project in its entirety unrealistically ambitious, but he believed in its feasibility in the area of mathematics (Russell 1903a). From

[12] Some further ideas of Frege which he shared with Russell will be discussed in the next section.

Leibniz, Russell also adopted the idea that there is no specifically mathematical method but that mathematics reduces to logical truths, i.e., the core idea of logicism.

Russell reported in retrospect that "the most important year in his intellectual life" was the year 1900, when he attended an international mathematics conference in Paris and in particular heard Giuseppe Peano, an Italian mathematician and logician (Russell 1944, 12). Russell was greatly impressed by the artificial symbolic language developed by Peano, which seemed to him to provide a new powerful method for the study of the foundations of mathematics. Peano also explicitly saw his symbolic language as an extension of Leibniz's program. In the early years of the 20th century, Russell then also delved into Frege's work and it certainly influenced him. However, Russell had already ended up with many of the key ideas independently.[13]

Russell later said that he was not really interested in meaning until 1918. All the same, certain previous philosophical investigations by him had a tremendous impact on the development of analytic, language-centered philosophy as a whole. The most important was the classical analysis of definite descriptions presented by Russell as early as 1905 in his classic article "On Denoting." In Ramsey's words, the analysis Russell presented there formed a "paradigm of philosophy" (Ramsey 1931, 263). Definite descriptions are descriptions of the form of "the so-and-so" which apply to at most one individual; for example, "the oldest man in the world," or "the current president of Finland." Frege had not clearly distinguished between proper names and definite descriptions, but treated the latter in a way as a subcategory of simple individual names. This has certain undesirable consequences: If a definite description (e.g., "the current king of France") is not realized by any entity in the world, a sentence containing it seems to have no truth value. However, this is ill-suited for classic logic with "the law of the excluded middle"—the thesis that every meaningful (declarative) sentence is either true or false—to which both Frege and Russell were officially committed.[14]

Russell put forward a more sophisticated analysis and sought to show that sentences containing definite descriptions can be converted into sentences with the same meaning in which definite descriptions do not occur at all, i.e., that they can always be eliminated. Russell argued that a sentence with a definite description, such as "the current king of France is bald," actually means the same as the conjunction of the following three sentences:

(1) There is at least one entity that is the current king of France;

(2) There is no more than one entity who is the present king of France; and

(3) any object that is the current king of France is also bald.[15]

This much more complex sentence is thus, according to Russell, an analysis of the meaning of the original sentence in written out form and reveals its true logical form. Since the sentence (1) is false, the whole long combined sentence (1)–(3) and thus also (according to Russell) the original sentence "the current king of France is bald," which means the same, are also false. This is a clear improvement over Frege's simpler intuitive approach, e.g., in

[13] Korhonen 2013 is a rich and useful source on the earlier Russell.

[14] I do not intend to suggest that this is the route through which Russell actually ended up with his theory of descriptions. The complicated story can be found in Makin 2000.

[15] More formally, the sentence has the logical form: $(\exists x)(K(x) \wedge (\forall y)(K(y) \rightarrow x{=}y) \wedge B(x))$.

that it is compatible with the law of the excluded middle. (However, all the tools of Russell's analysis were already included in Frege's logic. Frege just never figured to take the decisive step.)

Russell clearly came up with a new kind of idea of analysis here: In the earlier thought of Russell and Moore, "analysis" had meant the *metaphysical analysis* of reality into its fundamental building blocks—literally division into parts. The analysis now envisaged, on the other hand, focuses on language and sentences, and the sentence to be analyzed is transformed sometimes into a very different form. This sort of analysis has been called "transformative analysis" (see Beaney 2002, 2007b).[16]

In fact, such an analysis already appears in Frege's work in his analysis of the concept of number, although it did not yet at that time become a more general model for doing philosophy. Namely, Frege and also early Russell put forward an analysis of the concept of the natural number as follows: they suggested that a sentence involving a certain natural number, say 4, for example, "Jupiter has four moons," should be analyzed as "the concept *moon of Jupiter* has four instances." In other words, the sentence does not actually predicate the property *has four moons* of Jupiter, but rather predicates a (second order) property *has four instances* of the (first-order) concept *moon of Jupiter*. The purpose of such an analysis is to reveal the "real" logical form of the sentence to be analyzed.

One might perhaps argue that the idea of the unsatisfactory quality of natural language from the point of view of logic, and also the idea of transformative analysis, is already contained in the rejection of the subject-predicate structure of natural language and replacement of it by the function-argument structure (a starting-point of Frege's new logic), and in particular in the thesis of the ambiguity of "is" which was at the heart of the new logic of Frege and Russell.[17] Namely, Frege and Russell suggested that the following different meanings could be distinguished in the verb "is":

Meaning:	Example:
1. The *is* of identity	Saul is Paul
2. The *is* of predication (copula)	Paul is an apostle
3. The *is* of class inclusion	A vole is a mammal
4. The *is* of existence	God is

According to Frege and Russell, these are logically utterly different things, even though natural language uses the same verb "is" for all of them. This thesis is built into the whole new logic developed by Frege and Russell, for in it all the above things are expressed in quite different notation—in contrast to the ambiguous natural language:[18]

1. $a = b$ 2. $P(a)$ 3. $(\forall x)\,[(P(x) \rightarrow Q(x)]$ 4. $(\exists x)\,(x = a)$

[16] The same thing is sometimes (including Beaney himself) alternatively called "interpretive analysis," and also "logical analysis"; but I personally find "transformative analysis" a more descriptive and apt label.

[17] See Haaparanta 1985, 1986 (these focus on Frege).

[18] I am using throughout this paper the familiar notation now common in logic; not Frege's quite idiosyncratic two-dimensional notation nor Russell's notation which he adopted from Peano (the latter is closer to the modern one).

In all these cases (i.e., the analysis of the concept of number, the ambiguity of "is," and in Russell's case, also the analysis of definite descriptions), it seems that the surface form of ordinary language is unreliable and can mislead us and result in confusions, and only an analysis in terms of the constructed ideal language reveals the true logical form of the sentences at stake and dissolves confusions.

In 1902, Russell found a contradiction in Frege's grand system of logic, what is now called "Russell's paradox." Russell himself sought to develop a paradox-free general logic, which he began to call "the theory of types" (Russell 1908; the basic idea appears already in Russell 1903b).[19] The comprehensive presentation of the system was the ponderous three-part *Principia Mathematica* co-written with Whitehead (Russell & Whitehead 1910–1913). In the theory of types, predicates have their own restricted ranges of significance, properties have their own "types," and known paradoxes are ungrammatical and hence impossible to formulate. Its language clearly distinguishes, at the grammatical level, first-order predicates related to the properties of individual objects, predicates related to the properties of such properties (second-order properties), etc. In Russell's view, natural language is deficient in this case too, as it does not distinguish between them but makes the properties of different orders appear to be on an equal footing, which then results in contradictions.

Inspired by these phenomena, it was quite natural to think that perhaps at least some of the eternal problems of philosophy that seemed unsolvable would be revealed in logical analysis to be ungrammatical and thus meaningless (e.g., Carnap; see below). This vision has played an important role in making new formal logic such an integral part of contemporary philosophy. Russell's idea of the ranges of significance of the concept also influenced Ordinary Language Philosophy, where philosophical problems were sometimes interpreted to result from "category mistakes" (esp. Gilbert Ryle 1938, 1949).[20]

In more detail and explicitly, Russell described his own conception of a logical ideal language in his lectures on the philosophy of logical atomism from 1918 (they already show clearly Wittgenstein's influence):

> In a logically perfect language the words in a proposition would correspond one by one with the components of the corresponding fact, with the exception of such words as "or", "not", "if", "then", which have a different function. In a logically perfect language, there will be one word and no more for every simple object, and everything that is not simple will be expressed by a combination of words, by a combination derived, of course, from the words for the simple things that enter in, one word for each simple component. A language of that sort will be completely analytic, and will show at a glance the logical structure of the facts asserted or denied.
>
> The language which is set forth in *Principia Mathematica* is intended to be a language of that sort. It is a language which has only syntax and no vocabulary whatsoever. Barring the omission of a vocabulary I maintain that it is quite a nice language. It aims at being the sort of a language that, if you add a vocabulary, would be a logically perfect language. Actual languages are not logically perfect in this sense, and they cannot possibly be, if they are to serve the purposes of daily life. A logically perfect language, if it could be constructed, would not only be intolerably prolix, but, as regards its vocabulary, would be very largely private to one speaker. (Russell 1918, 197–198)

[19] See Urquhart 2006 for an overview.

[20] Carnap's famous 1931 paper on overcoming metaphysics (see below) may perhaps have also been an influence; it discusses (briefly) examples very similar to those of Ryle, under the label "type confusions" with an explicit reference to the Russellian theory of types.

When Russell says here that the language is analytic, it does not mean that the sentences in the language are analytically true but that all the sentences in the language are fully, completely analyzed sentences. The structure of such a language thus undistortedly reflects the metaphysical structure of the world. The distinctions and categories of language are thus also the distinctions and categories of the world, metaphysical categories. The structure of the world can be read directly from the structure of the ideal language.

According to Russell, in a logically perfect language, communication from one speaker to another is impossible, except for matters of logic. Since ordinary language is not logically perfect, a philosopher who wants to find out the true logical form of a statement must analyze the statement and transform it into some, perhaps very different, sentence of a logically perfect language. Apparently, Russell also assumed that the fully analyzed form corresponds to the structure of a thought expressed by the unanalyzed sentence in ordinary language and corresponds to something psychologically real. For Russell, thought is more primary than linguistic expression, and ordinary language often only imperfectly expresses the thought.

In his introduction to the English edition of Wittgenstein's *Tractatus* (see below), Russell wrote:

> A logically perfect language has rules of syntax which prevent nonsense, and has single symbols which always have a definite and unique meaning. Mr Wittgenstein is concerned with the conditions for a logically perfect language – not that any language is logically perfect, or that we believe ourselves capable, here and now, of constructing a logically perfect language, but that the whole function of language is to have meaning, and it only fulfills this function in proportion as it approaches to the ideal language which we postulate.
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> ... The first requisite of an ideal language would be that there should be one name for every simple, and never the same name for two different simples. (Russell 1922, 8–9)

From his pivotal article on definite descriptions from 1905 onwards, Russell considered "the principle of acquaintance," as he called it, to be the very central guiding rule for constructing the ideal language:

> Every proposition which we can understand must be composed wholly of constituents with which we are acquainted.[21]

According to Russell, we can all be acquainted with the same abstract objects. Therefore, we can communicate about logic and mathematics, even if each of us spoke a logically perfect language. In contrast, we are not acquainted with physical objects or other minds. For example, in 1912, Russell thought that we can only be acquainted with the following: sense data, inner data, and memories of such things—and "perhaps" of the Self. Interpreted in this way, the principle of acquaintance imposes very severe conditions on the nature of fully analyzed sentences, and thus also on a logically perfect language: The sentences of the language involve only logical constants, abstract objects or universals, and data from internal and external senses and memories of such—besides abstract objects, these are data of no more than one subject. I know my own sense data, you know yours, etc. Russell's logically perfect language is thus essentially a language of one person. With the sole exception of abstract objects, the sentences of my logically perfect language contain only words that refer

[21] This formulation is from Russell 1912; there is a slight variation in formulations in different works of Russell.

solely to objects that no one else but I know and can be acquainted with. The logically perfect language outlined by Russell was indeed the paradigm of private language criticized by Wittgenstein in his later philosophy (cf. Hylton 2007).

After finishing *Principia*, the exhausted Russell moved in the 1910s from logic and the philosophy of mathematics to work mainly in epistemology. In addition to the principle of acquaintance, he was now guided by Occam's razor and his "supreme maxim in scientific philosophizing," formulated in 1914: "Whenever possible, inferred entities must be replaced by logical constructions." Still, in 1912, Russell had regarded the material objects of everyday life—i.e., rocks and trees, cats and dogs, tables and chairs—as inferred entities which explain and cause sense data. However, this opened the door to skepticism, and Russell did not tolerate the situation for long. He began to think that material objects should be given a treatment similar to the one he had given to numbers: words that seem to refer to material objects should be defined in terms of words that refer to things with which we are acquainted. From 1914 onwards, Russell thought that material objects were mere "logical constructions" out of sense data. By 1914, Russell's conception of philosophy also seemed to become more austere. He now wrote: "Every philosophical problem, when it is subjected to the necessary analysis and purification, is found to be not really philosophical at all, or else to be, in the sense in which we are using the word, logical" (1914, 42). Although this was not yet quite Wittgenstein's full-blown radical view of philosophy (see below), it certainly looks like a move in that direction, paving the way for it.

Be that as it may, Russell's matured position has in fact truly radical consequences for analysis and a logically perfect language: A fully analyzed form of even simple everyday sentences would be thus astronomically complex and practically humanly unattainable—as would be a logically perfect language. A complete analysis would only be possible in logic and mathematics. One might think that a philosopher benefits already from partial, incomplete analyses—that we can at least get closer to a fully analyzed sentence. However, given the large-scale transformations the sentences go through in such an analysis, there is no good reason to assume that each intermediate step would be closer in logical form to the actual logical form than the previous one. The conclusion is quite discouraging for philosophy (cf. Hylton 2007).

Wittgenstein and the Logico-Philosophical Treatise

In 1911, Ludwig Wittgenstein (1889–1951), a young Austrian student of engineering who had become interested in philosophical problems in mathematics, sought to become a student of Russell at Cambridge. Soon the ingenious student began to influence his already famous teacher. Wittgenstein's early philosophy culminated in a small book with a downright cult reputation, *Tractatus Logico-Philosophicus* (1921).[22] The idea of an ideal logical language played an important role also in Wittgenstein's thought during that period.[23] It was familiar

[22] For illuminating discussions on the aims and arguments of this short but difficult tractate, see Ricketts 1996, Kremer 2013.

[23] I shall simply put aside the difficult question of the correct interpretation of *Tractatus* as a whole, and how the relation of its quite skeptical conclusions and more constructive parts should be understood; I will only summarize how the theme of this paper appears in *Tractatus*. I leave it for Wittgenstein scholars to dispute

to him from Frege's work and from Russell through both his writings and their personal conversations. Right away in the introduction to *Tractatus*, he states that philosophical problems are based on a "misunderstanding of the logic of our language." Later in the book, he argues that "[m]ost questions and propositions of the philosophers result from the fact that we do not understand the logic of our language" (4.003).[24]

According to early Wittgenstein too, ordinary language is a source of confusion: "In the language of everyday life it very often happens that the same word signifies in two different ways—and therefore belongs to two different symbols—or that two words, which signify in different ways, are apparently applied in the same way in the proposition." (3.323) Wittgenstein immediately gives, as an example, the ambiguity of the expression "is" emphasized by Frege and Russell: "Thus the word 'is' appears as the copula, as the sign of equality, and as the expression of existence"; and "In the proposition 'Green is green'—where the first word is a proper name as the last an adjective—these words have not merely different meanings but they are different symbols." And this, in Wittgenstein's mind, has a philosophical significance: "Thus there easily arise the most fundamental confusions (of which the whole of philosophy is full)." (3.324)

Therefore, according to Wittgenstein, an ideal language is needed: "In order to avoid these errors, we must employ a symbolism which excludes them, by not applying the same sign in different symbols and by not applying signs in the same way which signify in different ways. A symbolism, that is to say, which obeys the rules of logical grammar—of logical syntax." (3.325) Wittgenstein adds that "[t]he logical symbolism of Frege and Russell is such a language, which, however, does still not exclude all errors."

Indeed, Wittgenstein arrives at a very radical conception of the nature of philosophy: "All philosophy is 'Critique of language' … Russell's merit is to have shown that the apparent logical form of the proposition need not be its real form." (4.0031) Here Wittgenstein is gesturing, of course, toward Russell's analysis of sentences containing definite descriptions.[25] This statement of Wittgenstein incidentally marked the start of the whole orthodox language-focused analytic philosophy.

Although he did not much develop an ideal language more formally, Wittgenstein was not an unoriginal follower of Frege and Russell either; his philosophical interpretation of an ideal language differed in certain important respects from theirs. For example, Frege and (earlier) Russell viewed also logical constants (such as "¬" and "∨") as names that—in order to be meaningful—must have some kind of abstract logical objects as their referents. According to Wittgenstein, in contrast, logical constants do not denote anything: they are not the names of any objects or complexes of objects. Wittgenstein, for example, suggested

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whether and in what sense those statements are in the end themselves nonsensical and devoid of meaning, as the final mysticist paragraphs of the book suggest.

[24] In full:

"4.003 Most propositions and questions, that have been written about philosophical matters, are not false, but senseless. We cannot, therefore, answer questions of this kind at all, but only state their senselessness. Most questions and propositions of the philosophers result from the fact that we do not understand the logic of our language.
(They are of the same kind as the question whether the Good is more or less identical than the Beautiful.) And so it is not to be wondered at that the deepest problems are really no problems."

[25] For much more about Russell's "merit" here, see Kremer 2012.

that the sentences "*P*" and "¬¬*P*" say the same thing or have the same content—if "¬" were a name, however, they would have radically different meanings.

However, the key difference between them in relation to natural languages and artificial ideal languages is the following: Frege and Russell thought that natural languages are logically flawed because they contain vague words and misrepresent the object of logic. Wittgenstein, in contrast, argued that "All propositions of our colloquial language are actually, just as they are, logically completely in order" (5.5564). The sentences of natural language are not, according to him, less logically correct or more logically confused than the sentences formed in the ideal languages of Frege or Russell. (Of course, the correct logical form of sentences is easier to see in an ideal language.) For Wittgenstein, logic is a prerequisite for all meaningfulness. Thus, nothing like illogical language can simply exist. If a sign has a sense at all, it must be logically in order. Thus, natural languages only seem to be logically flawed, according to Wittgenstein.[26]

Carnap: **From rational reconstruction to explication**

Rudolf Carnap (1891–1970), a German-born logician-philosopher and one of the central logical positivists of the Vienna Circle, has often been considered the paradigmatic representative of Ideal Language Philosophy.[27] Carnap continued the tradition of Frege and Russell and believed in the superiority of artificial formal languages in conducting philosophical research and used them essentially in his own philosophical investigations. He had attended Frege's lectures in 1914 and was greatly impressed by Russell's logical work. He was also deeply influenced by Wittgenstein's *Tractatus*. Carnap wanted to replace natural language even in everyday communication with a better artificial substitute: he was an active advocate of Esperanto.

Early Carnap continued in many ways from where Russell had left off. However, as a radical empiricist, he did not allow abstract objects or "the inner sense" that would enable acquaintance with such, as Russell had done. As a logical positivist, he also took a very negative view of all metaphysics and thus did not think that an ideal language would reflect the structure of any external reality. In this sense, he did not believe in any logically perfect language in the sense that Russell did. However, Carnap initially thought that Russell's logical system provided more or less the only possible and absolutely correct language of logic.

In his early classic work *Der logische Aufbau der Welt* ("The Logical Structure of the World") (1928), Carnap refers at the outset to Russell's supreme maxim in scientific philosophizing: whenever possible, inferred entities must be replaced by logical constructions. Indeed, in this work Carnap seeks to carry through in detail the program outlined by Russell of the logical construction of our knowledge of physical reality with mere sense data as a starting point (few believe that his attempt succeeded).[28] Carnap also

[26] For recent discussions on Wittgenstein and the limits of language, see the various essays in Appelqvist 2020.

[27] Leitgeb & Carus 2020 gives a rather encompassing review of Carnap's thought.

[28] There are, however, some substantive differences between Carnap's approach and that of Russell; see, e.g., Beaney 2004.

refers at the beginning of this work to Leibniz's idea of an ideal language. At the time, he called his project "rational reconstruction"—i.e., he aimed to clarify old concepts by giving them new, more precise definitions.

Among other things, his article on the rejection of metaphysics (Carnap 1931)—famous for its critique of Heidegger, albeit it represents only a brief interphase in Carnap's thought—clearly shows his sour attitude toward natural language. According to Carnap, metaphysical statements are devoid of meaning either because they contain meaningless words, or because they combine meaningful words in a way that violates the logical syntax, i.e., the rules of sentence formation. Carnap called the latter type of apparent statements "pseudo-statements": They look like statements, but in reality do not state anything and do not express true or false statements. Carnap writes: "The fact that natural languages allow the formation of meaningless sequences of words without violating the rules of grammar, indicates that grammatical syntax is, from a logical point of view, inadequate. If grammatical syntax corresponded exactly to logical syntax, pseudo-statements could not arise." (Carnap 1931, 68) "It follows," Carnap continues, that "metaphysics could not even be expressed in a logically constructed language. This is the great philosophical importance of the task, which at present occupies the logicians, of building a logical syntax." (Ibid.)

According to him, "perhaps the majority" of the logical errors that underlie pseudo-statements are based on the ambiguity of the expression "to be" (or "is") in natural language (an apparent gesture toward Frege and Russell). Another very common violation of the correct logical syntax is, according to Carnap, "type confusions" of concepts, i.e., a natural language sentence which conflicts the sentence-formation rules and meaningful ranges of significance of predicates in Russell's theory of types.

Soon, however, Carnap abandoned the whole idea of one correct logical language[29] and adopted his famous Principle of Tolerance:

> Principle of Tolerance: It is not our business to set up prohibitions, but to arrive at conclusions. [Carnap 1934, §17]
>
> In logic, there are no morals. Everyone is at liberty to build his own logic, i.e. his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments. [Carnap 1934, §17]

Thus, a wide variety of alternative logical languages were now equally permitted. However, they must be presented with precise grammatical rules. Therefore, they cannot be natural languages but must be alternative artificial, formal languages. From now on, the choice of language system for Carnap was a pragmatic question: one language may be more useful for one purpose, another for another purpose. However, the choice of a language is not meaningfully a question of right and wrong, or true and false. In the 1930s, Carnap's philosophical inquiries were restricted to syntax, and for him all autonomous and legitimate philosophy that did not reduce to empirical sciences—such as psychology—and was not meaningless metaphysics was limited to studying the logical syntax of the language of science: "Philosophy is to be replaced by the logic of science" (Carnap 1934).

[29] Carnap's quite sudden and radical change of view is tracked in Awodey & Carus 2007.

However, the syntactic perspective soon proved too restrictive, and in particular under the influence of Tarski (see below), Carnap expanded his conceptual framework to include the semantics of language, i.e., the meaning relations of language to the world and its objects. However, Carnap's analyses still focused on artificial formal languages. Indeed, after his "semantic turn," Carnap made a distinction between pure and descriptive semantics (see Carnap 1942, 11–15). Descriptive semantics is concerned with historically given natural languages, such as German, and is based on empirical investigation. Pure semantics, on the other hand, is an analysis of semantical systems with artificial languages which are stipulatively defined. It is entirely analytic and without factual content. "Here we lay down definitions for certain concepts, usually in the form of rules, and study the analytic consequences of these definitions. In choosing the rules we are entirely free," he explains (Carnap 1942, 13). Philosophy then, according to Carnap, must confine itself to pure semantics. For Carnap, pure and descriptive semantics seem to be independent and autonomous projects.

At this point, a new kind of conception of analysis began to emerge more and more clearly in Carnap's thought: Instead of rational reconstruction, he started to talk about "explication." He borrowed the term from Husserl, even though these two philosophers meant somewhat different things with it. Carnap's explication relies essentially on artificial formal languages. Explication is clarification or "refining" of meaning. The criterion for the goodness of its results may be their ability to clarify the meaning of the old term in a way that highlights one of its key "meanings" (ambiguous terms) or covers "clear cases" in the extensions of the original inaccurate term, and creates and clarifies links with other scientific concepts. They are expected to have not only "usability" but also "theoretical fertility" and "systematic strength."[30]

In his groundbreaking work on the semantics of intensional logic and possible worlds semantics, *Meaning and Necessity* (Carnap 1947), Carnap says he seeks to clarify the concept of *meaning*. At the same time, he describes the idea of explication as follows:

> The task of making more exact a vague or not quite exact concept used in everyday life or in an earlier stage of scientific or logical development, or rather if replacing it by a newly constructed, more exact concept, belongs among the most important tasks of logical analysis and logical construction. We call this the task of explicating, or of giving an explication for the earlier concept: this earlier concept, or sometimes the term used for it, is called the *explicandum*; and the new concept, or its term, is called an *explicatum* of the old one. (Carnap 1947, 7–8)

Although Carnap's actual project here is an explication of the concept of *meaning*, he gives as an example of explication the analysis of the concept of *number* by Frege and Russell:

> Thus, for instance, Frege and, later, Russell, took as an *explicatum* the term "two" in the not quite exact meaning in which it is used in everyday life and in applied mathematics; they proposed as an *explicandum* for it an exactly defined concept, namely, the class of pair classes. (Carnap 1947, 8)

Carnap mentions, as other examples of explication, Russell's analysis of definite descriptions and Tarski's semantic analysis of the concept of truth (see below). He adds:

[30] The fullest presentation of his conception of explication is in Carnap 1950. Beaney 2004 includes a thorough discussion of it.

> Generally speaking, it is not required that an *explicatum* have, as nearly as possible, the same meaning as the *explicandum*; it should, however, correspond to the *explicandum* in such a way that it can be used instead of the latter. (Carnap 1947, 8)

Were the analyses of Frege and Russell then cases of explication in Carnap's sense, or just analyses of already existing meanings? On the one hand, their own explicit comments may suggest that the latter is the case. On the other hand, their critical views concerning natural languages make it somewhat difficult to understand how it could be. Therefore, Carnap may be on the right track when he is suggesting that the concept of explication he presents describes better what they were actually doing: Frege and Russell may not have sufficiently distinguished between the two.[31]

Later, Carnap puts forward four requirements for a good *explicatum*: 1) it is to be similar to the *explicandum* in such a way that it can be used in most cases in which the *explicandum* has so far been used; 2) the rules of its use are to be given in an exact form, in conjunction with other scientific concepts; 3) it is to be a fruitful concept, i.e., useful for the formulation of many universal statements; and 4) it should be as simple as possible, given the more important requirements (1)–(3) (Carnap 1950, 7).

Carnap advocated to the end the fundamental thesis—inherited from Wittgenstein—that philosophy is primarily an activity of clarifying language, and makes no claims and presents no theories. More specifically, it came to mean to him that all legitimate philosophy amounts to the activity of explicating concepts by means of artificial formal languages.

Tarski and the inconsistency of natural language

The Polish logician Alfred Tarski (1901–1983) can also be naturally viewed as a representative of Ideal Language Philosophy. He was the father of logical semantics and one of the most significant logicians of our time. He is known in philosophy especially for his theory of truth based on the tools of formal logic (Tarski 1933/1935, 1944). Tarski's influence was also essential when Carnap turned from the syntactic approach to the semantic one in the late 1930s. Tarski was primarily a logician, and unlike Wittgenstein or Carnap, he did not put forward any general theses on the task and the nature of philosophy. In practice, however, his work on truth has been one of the best-known examples of the ideal language tradition.[32]

Tarski contended that truth can only be defined in formal languages and only one at a time. Natural languages, he suggested, are "semantically closed," meaning they can talk about their own truth and other semantical properties. This in turn leads to many paradoxes and contradictions, e.g., "the liar paradox." Therefore, according to Tarski, the concept of truth can be unequivocally defined only for languages which are "semantically open" and which have precisely defined rules of grammar. (See Tarski 1944.) Tarski's great influence on Carnap may suggest that their philosophical attitudes are also more or less the same. However, there are in fact some interesting differences between them.

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[31] In fact, in a relatively late lecture (Frege 1914), Frege sketches a notion of definition which is not that different from Carnap's idea. What is more, Carnap attended that lecture of Frege in Jena. See Beaney 2004.

[32] Gómez-Torrente 2019 gives a good overview of Tarski's work. For more about philosophical aspects of Tarski's thought, see, e.g., Woleński 1993, Mancosu 2009, Patterson 2012.

To begin with, for Tarski, the "formal languages" whose truth is under consideration must always be interpreted languages, not purely formal, as he repeatedly emphasized:

> It remains perhaps to add that we are not interested here in 'formal' languages and sciences in one special sense of the word 'formal', namely sciences to the signs and expressions of which no meaning is attached. For such sciences the problem here discussed has no relevance, it is not even meaningful. We shall always ascribe quite concrete and, for us, intelligible meanings to the signs which occur in the languages we shall consider. (Tarski 1933/1935, 166–67)

Nor was this just an occasional philosophical opinion for Tarski; it was quite an integral part of Tarski's whole approach to truth that the meanings of the object language must be given and fixed. Only in this way can the definition of truth (applied to sentences) make any sense at all:

> For several reasons it appears most convenient to apply the term "true" to sentences, and we shall follow this course.[footnote omitted] Consequently, we must always relate the notion of truth, like that of a sentence, to a specific language; for it is obvious that the same expression which is a true sentence in one language can be false or meaningless in another. (Tarski 1944, 342)
>
> We shall also have to specify the language whose sentences we are concerned with; this is necessary if only for the reason that a string of sounds or signs, which is a true or a false sentence but at any rate meaningful sentence in one language, may be a meaningless expression in another. (Tarski 1969, 64)
>
> . . . the concept of truth essentially depends, as regards both extension and content, upon the language to which it is applied. We can only meaningfully say of an expression that it is true or not if we treat this expression as a part of a concrete language. As soon as the discussion concerns more than one language the expression "true sentence" ceases to be unambiguous. If we are to avoid this ambiguity we must replace it by the relative term "a true sentence with respect to the given language." (Tarski 1933/1935, 263)

Tarski made a distinction, which resembled Carnap's distinction of descriptive and pure semantics, between descriptive semantics and theoretical semantics (Tarski 1944). By "descriptive semantics," he refers to the totality of the study of semantic relations in natural languages. "Theoretical semantics" apparently means to Tarski the kind of research he does himself.

It is true that Tarski constantly stressed that natural languages drift into semantic paradoxes, and that truth can be unequivocally defined only for formal languages. This has led many to assume that Tarski, like Carnap, would have liked to limit his "theoretical semantics" to artificial formal languages only—that it could not be applied at all to real-life natural languages. In the case of Tarski, however, the matter is more complicated. We have already seen above that, formal or not, the languages in question must, for Tarski, be "concrete" and already interpreted, that is to say, already provided with "concrete" meanings. This alone makes them quite different from artificial formal languages in the conventional sense.

Tarski also thought that his formal semantic tools could be applied to the limited languages of various special sciences, such as chemistry, so long as they did not contain semantic vocabulary. Furthermore, Tarski suggested that theoretical semantics is, after all, applicable to natural languages, albeit "only with certain approximation" (Tarski 1944, 365). Namely: "the approximation consists in replacing a natural language (or a portion of it in which we are interested) by one whose structure is exactly specified, and which diverges from the

given language 'as little as possible'" (Tarski 1944, 347). Tarski also writes that "[t]he results obtained for formalized language also have a certain validity for colloquial language ... if we translate into colloquial language any definition of a true sentence which has been constructed for some formalized language, we obtain a fragmentary definition of truth which embraces a wider or narrower category of sentences" (Tarski 1933/1935, 165). In fact, at one point, Tarski stressed that when he used the term "formal language," he did not "have in mind anything essentially opposed to natural languages"; he continues, "[on]n the contrary, the only formal languages that seem to have real interest are fragments of natural languages (fragments provided with complete vocabularies and precise syntactic rules) or ones that can at least be sufficiently translated into natural languages" (Tarski 1969, 68). Tarski's attitude toward natural language was thus in fact somewhat less hostile than that of Carnap.

Afterword

The new formal logic developed by Frege and Russell, or rather the first-order logic contained in it as a proper part, has become an established and familiar tool of philosophers. Carnap's work has been an important point of departure in both the philosophy of language and the philosophy of science, and Tarski's formal theory of truth is a mandatory basic theory in all philosophical theorizing of truth. The tools of formal logic continue to play a central role in philosophy, especially in the philosophy of mathematics and the philosophy of science, and in some respects also in the philosophy of language, and even in metaphysics.

However, few still believe, like Carnap did, that all that philosophy can legitimately do is engage in clarificatory activity focusing on language. Today, philosophers incontinently present arguments with conclusions and advocate philosophical theories. At the same time, the idea that formalized languages would play such a central role in all philosophical activity as suggested by Carnap and some others has become quite rare. The attitude of philosophers toward the tools of formal logic today is usually more pragmatic: they have their own fruitful applications, but often recourse to them is neither necessary nor useful. Many follow in practice Quine's humorous "maxim of shallow analysis": "where it doesn't itch don't scratch" (Quine 1960, 160).

It is perhaps natural to end the present overview with a quote from Saul Kripke (1940–2022), who is arguably one of the most important philosophers of our time. What is interesting here is that he has significantly followed in the footsteps of Carnap and Tarski in the study of intensional logic and the logical theory of truth and has been one of the most brilliant logicians in contemporary philosophy. According to him, the use of the tools of formal logic is sometimes useful in philosophy, but it must be informed by a sensitivity to the philosophical significance of the formalism and by a generous admixture of common sense. Kripke stated: "It should not be assumed that formalism can grind out philosophical results in a manner beyond the capacity of ordinary philosophical reasoning. There is no mathematical substitute for philosophy." (Kripke 1976, 416)[33]

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[33] This paper is a somewhat revised and modified translation of my earlier Raatikainen 2013b (in Finnish). I am very grateful to Leila Haaparanta for her valuable comments on an earlier version of this paper.

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