Optimal Joint Radar and Communications Beamforming for the Low-Altitude Airborne Vehicles in SAGIN

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***Abstract*—A symbolic feature that integrates the space, air and ground network components for service in challenging and remote areas is being envisaged with continuity and high mobility of the 6G mobile system. Simultaneously providing sensing and connectivity over the radio signal becomes essential to support the management of low-space air crafts in the mobile system with limited spectrum resources. In this paper, we investigate the optimal joint radar and communications beamforming scheme with the presence of the clutter to support the low-space airborne vehicles, e.g. unmanned aerial vehicles or drones that are essential components of Non-Terrestrial Networks. The proposed scheme achieves the optimal signal-to-clutter-plus-noise ratio of the sensing function while maintaining the performance of the pre-defined communications. The novel application of approximations and rank-reduction algorithms in this work maximizes the joint radar and communications performance, for a system model similar to the one that is solved with a local optimum solution in a previous work. The numeric simulation results show that our approach maintains low complexity while guaranteeing the global optimum beamforming solution.**

I. Introduction

To provide high-speed and reliable service for users in challenging and remote regions in the foreseeable sixth-generation (6G) system, researchers have shifted the vision to the space-air-ground integrated network (SAGIN). Via the interoperating of spatial, aerial and terrestrial network segments, SAGIN is expected to be more flexible on balancing the coverage, latency, bandwidth, and resilient to infrastructure dysfunction caused by the natural disasters and malicious attack. Compared with terrestrial networks and space networks (e.g. satellites networks in geostationary, medium and low earth orbits), and the aerial network, which treats low-altitude aircraft (incl. drones, unmanned aerial vehicles, balloons) as network components [1] or service objects [2] is a comparable new research genre. To support the deployment of aerial networks and ensure the onboard safety of low-altitude crafts, mobility management and unmanned aircraft system traffic management (UTM) become the essential network functions for the SAGIN to handle the floating or flying network components or service objects. Usually, obtaining the position and motion information used in mobility management relies on the external infrastructure (e.g. global navigation satellite system) or auxiliary onboard inertial measurement unit (IMU, e.g. accelerometer and gyroscope). To improve the autonomy level of the aerial network, it is expected to sense the mobility information and keep the connectivity simultaneously with the internal resources, which means performing joint radar and communications (JRC) in the aerial networks.

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JRC system is an emerging research topic among spectrum sharing problems [3] [4]. Common properties of signals used in both systems allow using a type of signal suited for one task in another, and common circuitry in both systems reduces the cost of remodeling them. However, it is a necessity to implement suitable beamforming methods to allow for the optimization of the primary task and fulfillment of the performance requirements of the secondary task. Performance of the beamforming algorithm is of even greater importance when operation in millimeter wave (mmWave) is considered since path loss to be dealt with is even greater than in the case of operating in the microwave. Many beamforming techniques have been proposed to satisfy radar and communication performance requirements in multiple-input multiple-output (MIMO) JRC systems, including those operating in mmWave.

Reformulating the beamforming problem as a sparse reconstruction problem as it was done in [5] is a viable approach in the context of mmWave operation and large antenna arrays. Likewise, the work in [6] considers systems of a similar type and emphasizes the effectiveness of utilizing beam steering solutions instead of more complex precoding methods, narrowing the possible methods for achieving the optimal beamforming algorithm. A recent work approaches JRC beamforming from an optimization problem perspective and utilizes Cramér-Rao Bound (CRB) in problem definition to achieve closed-form optimal solution for a single user, and asserts that globally optimal solutions can be achieved for multiple use cases by employing the proposed approach [7]. Another aspect of the JRC beamforming problem is addressed in [8], where the convex optimization approach is applied to determine the optimal antenna selection strategy. However, multi-path possibilities which can emerge due to clutter in radar operation and reflectors in radio communication were not considered in the system model that [8] is based on. They are considered in the system model of [9], even though the formulated problem is non-convex in that work. Thus, global optimality cannot be claimed with the solution proposed there. In this paper, we consider a model with a single communication user and single clutter source and suggest a method to achieve the global optimal solution for maximizing radar signal-to-clutter plus noise ratio (SCNR) while keeping the communication requirements satisfied. The results demonstrate that JRC performance can be reliably maximized by our algorithm with low computational complexity.

Notation: We use boldface lowercase letters for column vectors, boldface uppercase letters for matrices, and normal font for scalars. Superscript $(.)^H$ represents Hermitian transpose, and $\text{Tr}(.)$ stands for the trace of a matrix. The $N \times N$-dimensional complex Euclidean space is expressed as $\mathbb{C}^{N\times N}$. Absolute value and Euclidean norm are denoted by $|.|$ and $\|.\|_2$, respectively.

II. System Model and Problem Formulation

We describe the signal and system models in this section before defining the optimization problem statement.

A. System setup

The single-user mmWave JRC system illustrated in Fig. 1 depicts a data transmission from the transmitter side to the receiver side, which occurs through a radar target, a clutter source, and a channel modeled by vector $\boldsymbol{h} \in \mathbb{C}^{N\times 1}$. Additive white Gaussian noise (AWGN) at the radar receive array and communication user receiver is modeled by $\boldsymbol{n} \in \mathbb{C}^{N\times 1}$ and $\nu \in \mathbb{C}$, respectively. The returning signal from the target and clutter is received and combined at the transmitter side again, where half of the antennas are allocated for this task. The beamforming process consists of weighting and phase shifting the transmit signal and returned signal from the target, such that SCNR from the return signal is maximized while keeping the signal-to-noise ratio (SNR) of the communication signal above a parametric threshold.

B. Optimization problem

The radar return signal received back by the antenna array is modeled by multiplying the transmit waveform x by transmit weight vector $\boldsymbol{u} \in \mathbb{C}^{N\times 1}$, transmit and receive steering vectors $\boldsymbol{a}_t(\theta) \in \mathbb{C}^{N\times 1}$ and $\boldsymbol{a}_r(\theta) \in \mathbb{C}^{N\times 1}$ directed at radar target θ_t and clutter θ_c, scaled by reflection factors of target σ_t and clutter σ_c, which can be expressed as

$$\boldsymbol{r} = \sigma_t \boldsymbol{a}_r(\theta_t)\boldsymbol{a}_t^H(\theta_t)\boldsymbol{u}x + \sigma_c \boldsymbol{a}_r(\theta_c)\boldsymbol{a}_t^H(\theta_c)\boldsymbol{u}x + \boldsymbol{n}. \quad (1)$$

The signal received by the communication user is given by

$$\boldsymbol{y}_c = \boldsymbol{h}^H\boldsymbol{u}x + \nu. \quad (2)$$

Multiplying (1) by Hermitian of receive weight vector $\boldsymbol{w}^H \in \mathbb{C}^{1\times N}$ from left yields the receive combined radar return signal. Doing the substitution $\boldsymbol{A}(\theta) = \boldsymbol{a}_r(\theta)\boldsymbol{a}_t^H(\theta)$, it could be written as

$$y_r = \boldsymbol{w}^H\boldsymbol{r} = \boldsymbol{w}^H\boldsymbol{A}(\theta_t)\boldsymbol{u}x + \boldsymbol{w}^H\boldsymbol{A}(\theta_c)\boldsymbol{u}x + \boldsymbol{w}^H\boldsymbol{n}. \quad (3)$$

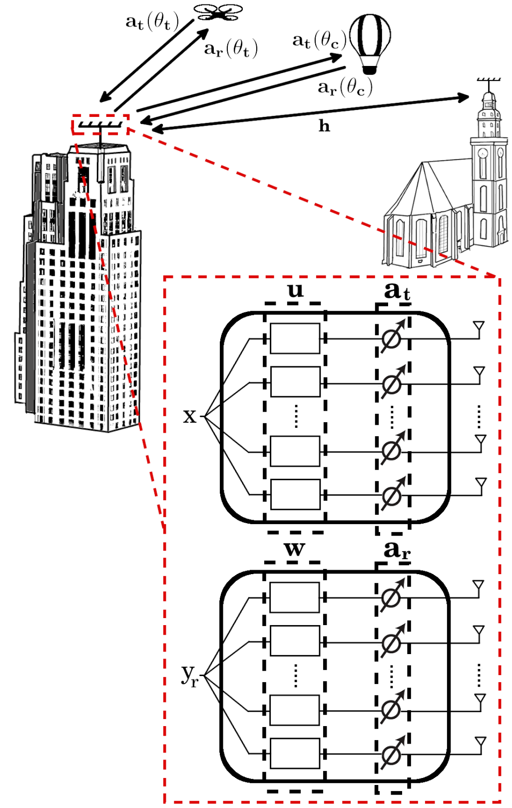

Fig. 1. Visualization of JRC operation and simplified block diagram of the transmitter side.

Average SCNR expression is acquired by the ratio of target return signal power to the clutter return signal power plus noise power as

$$\gamma_r = \frac{\sigma_t^2|\boldsymbol{w}^H\boldsymbol{A}(\theta_t)\boldsymbol{u}|^2}{\sigma_c^2\boldsymbol{w}^H(\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c) + \boldsymbol{I}\frac{N_0'}{\sigma_c^2})\boldsymbol{w}}, \quad (4)$$

and average communication SNR is expressed similarly as

$$\gamma_c = \frac{|\boldsymbol{h}^H\boldsymbol{u}|^2}{N_0'}, \quad (5)$$

where noise has a variance of N_0' and thus could be shown as an identity matrix scaled by N_0'. The ratio between squares of target and clutter reflection factors will be denoted as $\sigma_t^2/\sigma_c^2 = \sigma$, and noise variance divided by squared clutter reflection factor as $N_0'/\sigma_c^2 = N_0$ from now on. Maximizing γ_r by setting $\boldsymbol{w}$ and $\boldsymbol{u}$ vectors while keeping total transmit power $||\boldsymbol{u}||_2^2$ under an upper bound and keeping γ_c above a lower bound constitutes an optimization problem stated as

$$\begin{aligned} \underset{\boldsymbol{w},\boldsymbol{u}}{\text{maximize}}\ & \gamma_r = \sigma\frac{|\boldsymbol{w}^H\boldsymbol{A}(\theta_t)\boldsymbol{u}|^2}{\boldsymbol{w}^H(\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c) + \boldsymbol{I}N_0)\boldsymbol{w}}, \quad (6)\\ \text{subject to}\ & C1 : ||\boldsymbol{u}||_2^2 \leq P_t,\\ & C2 : \boldsymbol{w}^H\boldsymbol{w} = ||\boldsymbol{w}||_2^2 = 1,\\ & C3 : \gamma_c = \frac{|\boldsymbol{h}^H\boldsymbol{u}|^2}{N_0'} \geq \Gamma_c. \end{aligned}$$

The objective of the problem described in (6) is to maximize the average SCNR for the radar functionality of the JRC system. Constraint $C1$ guarantees that the total transmitted power is no more than the maximum allowed transmit power, and constraint $C2$ ensures that the gain for the radar target direction is set to unity. In contrast, the total array output power is kept unchanged, and constraint $C3$ makes sure that the average SNR for the communication operation satisfies the performance requirements. This problem is non-convex, due to its objective function and $C3$ constraint being non-convex, and hence difficult to solve in its current state. It is possible to describe a convex problem that is a close approximation of (6) though, which is done in the following section.

III. Proposed Solution

A series of mathematical equivalencies and approximations are applied in this section to solve the problem defined in the previous section. In this regard, we begin by using minimum variance distortionless response (MVDR) weights [10] to express $\boldsymbol{w}$ in terms of $\boldsymbol{u}$ and $\boldsymbol{A}(\theta)$ as follows

$$\boldsymbol{w}_{\text{MVDR}} = \frac{\boldsymbol{M}^{-1}(\boldsymbol{u})\boldsymbol{A}(\theta_t)\boldsymbol{u}}{\boldsymbol{u}^H\boldsymbol{A}^H(\theta_t)\boldsymbol{M}^{-1}(\boldsymbol{u})\boldsymbol{A}(\theta_t)\boldsymbol{u}}, \tag{7}$$

where $\boldsymbol{M}(\boldsymbol{u}) \in \mathbb{C}^{N\times N}$ is spatial covariance matrix given by

$$\boldsymbol{M}(\boldsymbol{u}) = \boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c) + \boldsymbol{I}N_0. \tag{8}$$

Substituting $\boldsymbol{w}$ with $\boldsymbol{w}_{\text{MVDR}}$ in γ_r yields the new problem statement:

$$\begin{aligned}\underset{\boldsymbol{u}}{\text{maximize}}\ & \gamma_r = \sigma\boldsymbol{u}^H\boldsymbol{A}^H(\theta_t)\boldsymbol{M}^{-1}(\boldsymbol{u})\boldsymbol{A}(\theta_t)\boldsymbol{u}, \\ \text{subject to}\quad & C1: ||\boldsymbol{u}||_2^2 \le P_t, \\ & C2: \gamma_c = \frac{|\boldsymbol{h}^H\boldsymbol{u}|^2}{N_0} \ge \Gamma_c.\end{aligned} \tag{9}$$

By introducing an auxiliary variable $y \ge 0$, the above problem can be equivalently written as

$$\begin{aligned}\underset{\boldsymbol{u}}{\text{minimize}}\quad & -y, \\ \text{subject to}\quad & C1: \sigma\boldsymbol{u}^H\boldsymbol{A}^H(\theta_t)\boldsymbol{M}^{-1}(\boldsymbol{u})\boldsymbol{A}(\theta_t)\boldsymbol{u} \ge y, \\ & C2: ||\boldsymbol{u}||_2^2 \le P_t, \\ & C3: \gamma_c = \frac{|\boldsymbol{h}^H\boldsymbol{u}|^2}{N_0} \ge \Gamma_c.\end{aligned} \tag{10}$$

As a further step in solving the problem described in (10), we reformulate it by stating and proving the following lemma.

Lemma 1: An optimization problem within the form of (10) can be equivalently written as

$$\begin{aligned}\underset{\boldsymbol{u}}{\text{minimize}}\quad & -y, \\ \text{subject to}\quad & C1: \boldsymbol{X} \succeq 0, \\ & C2: ||\boldsymbol{u}||_2^2 \le P_t, \\ & C3: \gamma_c = \frac{|\boldsymbol{h}^H\boldsymbol{u}|^2}{N_0^{'}} \ge \Gamma_c.\end{aligned} \tag{11}$$

Proof: See Appendix A.

After rewriting (10) in the form of (11), the only remaining factor that keeps this problem from being convex is the quadratic expression of $\boldsymbol{u}\boldsymbol{u}^H$ which can be written as the matrix $\boldsymbol{U} \in \mathbb{C}^{N\times N}$. It is also required to change the other expressions with $\boldsymbol{u}\boldsymbol{u}^H$ in the problem statement to equivalent statements with $\boldsymbol{U}$ for the sake of convexity. This results in the problem statement

$$\begin{aligned}\underset{\boldsymbol{U}}{\text{minimize}}\ & -y, \\ \text{subject to}\ & C1: \boldsymbol{X} = \begin{bmatrix} A(\boldsymbol{U}) & B(\boldsymbol{U}) \\ B^H(\boldsymbol{U}) & C(\boldsymbol{U})\end{bmatrix} \succeq 0, \\ & C2: tr(\boldsymbol{U}) \le P_t, \\ & C3: \gamma_c = \frac{\text{Tr}(\boldsymbol{U}\boldsymbol{H})}{N_0^{'}} \ge \Gamma_c.\end{aligned} \tag{12}$$

$\boldsymbol{U}$ should be defined as a Hermitian matrix in the solver tool since it is acquired by the multiplication of $\boldsymbol{u}$ by its Hermitian. A suitable approach here is to solve the stated problem without imposing any constraints on the rank of $\boldsymbol{U}$, and then process the resulting $\boldsymbol{U}$ through a rank-reduction algorithm to get a rank one matrix that can be decomposed as $\boldsymbol{U} = \boldsymbol{u}\boldsymbol{u}^H$ where $\boldsymbol{u}$ will be the weight vector to be used in beamforming as shown in Fig. 1.

Solving the optimization problem defined by (12) results in the minimum possible value of y, and values of $\boldsymbol{U}$, $\boldsymbol{X}$, γ_c that provide the satisfaction of conditions for acquiring that result. Let the values acquired from the problem solution be represented with an asterisk such as $\boldsymbol{U}^*$. The elements of matrix $\boldsymbol{X}$ with solved values can be shown as:

$$\text{Tr}(\boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_t)\boldsymbol{U}^*) = \frac{N_0}{\sigma}(a^* + y^*), \tag{13}$$

$$\text{Tr}(\boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_c)\boldsymbol{U}^*) = tr(\boldsymbol{A}^H(\theta_c)\boldsymbol{A}(\theta_t)\boldsymbol{U}^*) = b^*, \tag{14}$$

$$\text{Tr}(\boldsymbol{A}^H(\theta_c)\boldsymbol{A}(\theta_c)\boldsymbol{U}^*) = \frac{\sigma}{N_0}(c^* - N_0). \tag{15}$$

There are two more equations relating $\boldsymbol{U}^*$ to other values acquired by optimization problem solution:

$$\text{Tr}(\boldsymbol{U}^*) = P^*, \tag{16}$$

$$\text{Tr}(\boldsymbol{U}^*\boldsymbol{H}) = \gamma_c^* N_0^{'}. \tag{17}$$

Now the task of performing the rank one decomposition $\boldsymbol{U}^* = \boldsymbol{u}\boldsymbol{u}^H$ is equivalent to finding a vector $\boldsymbol{u}$ that satisfies the equations (13) to (17) when $\boldsymbol{U}^*$ is substituted by $\boldsymbol{u}\boldsymbol{u}^H$ in those. Among these equations, (14) could be eliminated by approximating $\boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_c)$ as 0. This is due to the fact that steering vectors directed to different angles approach orthogonality with each other as the number of antennas approaches infinity, as stated in Corollary 2 in [6]. Since this approximation describes an equation where $\boldsymbol{U}^*$ is multiplied by zero, which will be satisfied as $b^* = 0$ no matter the value of $\boldsymbol{U}^*$, there are only 4 non-trivial equations that define $\boldsymbol{u}$.

The problem of finding a vector $\boldsymbol{u}$ such that,

$$\begin{aligned}\text{Tr}(\boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_t)\boldsymbol{u}\boldsymbol{u}^H) &= \text{Tr}(\boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_t)\boldsymbol{U}^*) \\ &= \frac{N_0}{\sigma}(a^* + y^*),\end{aligned} \tag{18}$$

$$\begin{aligned}\text{Tr}(\boldsymbol{A}^H(\theta_c)\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H) &= \text{Tr}(\boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_c)\boldsymbol{U}^*) \\ &= \frac{\sigma}{N_0}c^* - N_0,\end{aligned} \tag{19}$$

$$\text{Tr}(\boldsymbol{u}\boldsymbol{u}^H) = \text{Tr}(\boldsymbol{U}^*) = P^*, \tag{20}$$

$$\text{Tr}(\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{H}) = \text{Tr}(\boldsymbol{U}^*\boldsymbol{H}) = \gamma_c^* N_0^{'}, \tag{21}$$

is solvable by utilizing the Theorem 2.3 in [11] which is based on theorems that were previously established in [12] and [13]. Solving the system of linear matrix equations defined by (18) to (21) yields a reasonably close approximation of the optimal transmit beamforming vector $\boldsymbol{u}$.

IV. Simulation Results

Simulations aimed to observe the tradeoff between performances of radar and communication operations were performed on cases with varying numbers of antenna array elements and lower thresholds for communication operation SNR. The noise and transmit power parameters had the most critical effect on the system performance due to their ratios to each other being directly related to SCNR. In the simulations whose results are presented in this subsection, the ratio of transmit power to the noise power was set as 100. Constant values of clutter and target reflection factors were set as $\sigma_c = \sigma_t = 1$ during the simulations. The direction of the target was set towards $30°$, while the clutter direction was set as $60°$. Distance between antenna elements was assumed to be $d = \lambda/2$ when modeling steering vectors $\boldsymbol{a}_t(\theta)$ and $\boldsymbol{a}_r(\theta)$, where λ is the transmit signal wavelength. The convex optimization problem for each one of the cases was solved with a new randomized channel vector for each iteration before performing the rank reduction to acquire the $\boldsymbol{u}$ vector for each case. Simulations were repeated 20 times with different randomly generated channel vectors every time, and the values of radar SCNR and communication SNR were averaged over all 20 results. PICOS [14] was used to specify and solve the convex optimization problems in the simulation.

Another set of simulations was performed to inspect how the approximation made for (14) affected the results. Even though it is theoretically known that steering vectors directed at different angles are orthogonal to each other as the number of antenna elements approaches infinity, there will be a smaller number of antenna elements in real applications. This results in $\boldsymbol{u}\boldsymbol{u}^*$ producing a different result than $\boldsymbol{U}^*$ would produce when substituted for it in (14). This implies that $\boldsymbol{u}$ is slightly different than the optimal vector, and the difference is expected to get smaller as the number of antennas increases. An extra pair of conditions, $\mathrm{Tr}(\boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_c)\boldsymbol{U}^*) = 0$, and $\mathrm{Tr}(\boldsymbol{A}^H(\theta_c)\boldsymbol{A}(\theta_t)\boldsymbol{U}^*) = 0$ were added to the optimization problem to enforce b^* actually being equal to 0 for this set of simulations. Expected trends of inversely related radar SCNR and communication SNR, in addition to better radar SCNR with a larger number of antennas, can be observed from Fig. 2. Additionally, it can be observed from Fig. 3 that difference in performance between cases where b^* is approximated to be 0, and cases, where b^* is constrained to be 0, is smaller for cases with a higher number of antenna elements. This is due to the fact that b^* is already quite close to 0 for a large number of antennas, and extra conditions do not put any serious restriction on the problem that would cause a non-negligible change in the optimal result.

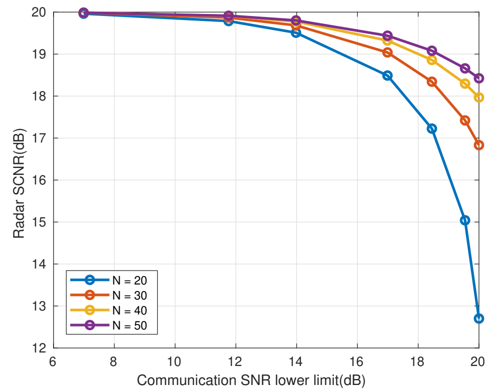

Fig. 2. Change in average radar SCNR as different values of average communication SNR are ensured by modifying problem constraints. N is the number of antennas.

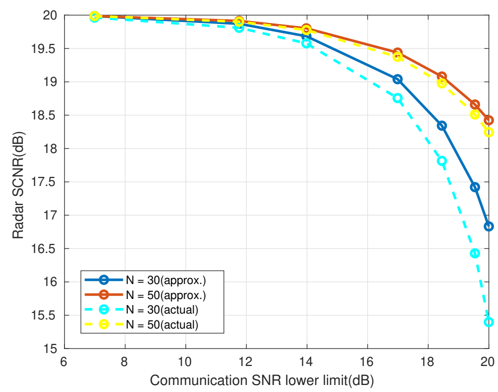

Fig. 3. Average radar SCNR versus average communication SNR plots with a different number of antenna elements, for cases where b^* is approximated to be zero and for cases where b^* is zero.

V. Conclusion

This paper proposes a novel algorithm for achieving the optimal radar SCNR while satisfying the power constraints and desired minimum communication SNR for JRC, which can be potentially used for sensing and communicating purposes for the low-altitude airborne from the ground stations in SAGIN systems. The assumed system setting involves a single non-negligible clutter source and no direct path between the transmitter and communication user. Our method consists of defining an optimization problem, using several mathematical relations to transform it into a convex optimization problem, and finally performing a rank-reduction procedure on the matrix and equations acquired from the problem to get the final vector of optimal beamforming weights. We proceed to show that the expected tradeoff between radar SCNR and communication SNR values can be observed on the beamforming vectors obtained from this algorithm, in addition to clarifying that the non-ideality of steering vectors directed at two different directions not being orthogonal does only have a negligible effect on the results, thus having no considerable negative effect on the performance of our algorithm. Our future works include i). developing the schemes to optimize the trajectory and energy consumption of aerial crafts; ii). optimal distribution, robust topology maintenance and routing schemes for the low-altitude crafts for the SAGIN; iii). drone involved/enhanced content distribution and traffic offloading.

Appendix A
Proof of Lemma 1

We begin by converting the inverse matrix expression $\boldsymbol{M}^{-1}(\boldsymbol{u})$ to a linear expression by utilizing Sherman-Morrison formula [15]

$$(\boldsymbol{N}+\boldsymbol{a}\boldsymbol{b}^T)^{-1}=\boldsymbol{N}^{-1}-\frac{\boldsymbol{N}^{-1}\boldsymbol{a}\boldsymbol{b}^T\boldsymbol{N}^{-1}}{1+\boldsymbol{b}^T\boldsymbol{N}^{-1}\boldsymbol{a}}, \tag{22}$$

where $\boldsymbol{N}\in\mathbb{C}^{N\times N}$ is an invertible square matrix and $\boldsymbol{b},\boldsymbol{a}\in\mathbb{C}^{N\times 1}$ are column vectors, and $1+\boldsymbol{b}^T\boldsymbol{N}^{-1}\boldsymbol{a}\neq 0$. $\boldsymbol{N}$ is an identity matrix scaled by N_0 in the case of this problem, and $\boldsymbol{a}=\boldsymbol{b}=\boldsymbol{A}(\theta_c)\boldsymbol{u}$. Substituting those in (22) yields

$$\begin{aligned}\boldsymbol{M}^{-1}(\boldsymbol{u})&=(\boldsymbol{I}N_0+\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c))^{-1} \qquad (23)\\&=\boldsymbol{I}\frac{1}{N_0}-\frac{\frac{1}{N_0}\boldsymbol{I}\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c)\frac{1}{N_0}\boldsymbol{I}}{1+\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c)\frac{1}{N_0}\boldsymbol{I}\boldsymbol{A}(\theta_c)}\\&=\frac{1}{N_0}(\boldsymbol{I}-\frac{\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c)}{N_0+tr(\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c))}).\end{aligned}$$

Hence, the left-hand side of $C1$ in (10) can be written as:

$$\begin{aligned}\gamma_r=\frac{\sigma}{N_0}&\boldsymbol{u}^H\boldsymbol{A}^H(\theta_t)(\boldsymbol{I}-\dots \qquad (24)\\&\dots\frac{\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c)}{N_0+tr(\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c))})\boldsymbol{A}(\theta_t)\boldsymbol{u}.\end{aligned}$$

Now that the expression for SCNR is finalized, one more manipulation is required to be done on the relevant optimization constraint. The $\gamma_r\geq y$ constraint can be rewritten as $\gamma_r-y\geq 0$ in order to treat γ_r-y as a 1×1 positive semi-definite matrix and utilize Schur complement to further transform the problem. Let us define a matrix $\boldsymbol{X}\in\mathbb{C}^{2\times 2}$,

$$\boldsymbol{X}=\begin{bmatrix}A(\boldsymbol{u}\boldsymbol{u}^H) & B(\boldsymbol{u}\boldsymbol{u}^H)\\ B^H(\boldsymbol{u}\boldsymbol{u}^H) & C(\boldsymbol{u}\boldsymbol{u}^H)\end{bmatrix}, \tag{25}$$

$$\begin{aligned}A(\boldsymbol{u}\boldsymbol{u}^H)&=\boldsymbol{u}^H\boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_t)\boldsymbol{u}-y, \qquad (26)\\&=\frac{\sigma}{N_0}\operatorname{Tr}(\boldsymbol{A}(\theta_t)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_t))-y,\\B(\boldsymbol{u}\boldsymbol{u}^H)&=\boldsymbol{u}^H\boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_c)\boldsymbol{u}, \qquad (27)\\&=\operatorname{Tr}(\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_t)),\\B(\boldsymbol{u}\boldsymbol{u}^H)^H&=\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c)\boldsymbol{A}(\theta_t)\boldsymbol{u}, \qquad (28)\\&=\operatorname{Tr}(\boldsymbol{A}(\theta_t)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c)),\\C(\boldsymbol{u}\boldsymbol{u}^H)&=\frac{N_0}{\sigma}(N_0+\operatorname{Tr}(\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c))). \qquad (29)\end{aligned}$$

We can then state that, as long as C is positive definite, then $\boldsymbol{X}$ is positive semi-definite if and only if C and its Schur complement $\boldsymbol{X}/C$ are both positive semi-definite [16]:

$$\boldsymbol{X}\succeq 0\Leftrightarrow C\succeq 0, \boldsymbol{X}/C=A-BC^{-1}B^H\succeq 0. \tag{30}$$

$C(\boldsymbol{u}\boldsymbol{u}^H)$ is positive definite since it is an expression of the addition of two non-zero real numbers. Positive semi-definiteness of one of $\boldsymbol{X}/C$ or $\boldsymbol{X}$ imply the positive semi-definiteness of the other, and $\boldsymbol{X}/C$ is equal to:

$$\begin{aligned}\boldsymbol{X}/C&=A-BC^{-1}B^H, \qquad (31)\\&=\frac{\sigma}{N_0}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_t)(\boldsymbol{I}-\dots \qquad (32)\\&\dots\frac{\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c)}{N_0+tr(\boldsymbol{A}(\theta_c)\boldsymbol{u}\boldsymbol{u}^H\boldsymbol{A}^H(\theta_c))})\boldsymbol{A}(\theta_t)\boldsymbol{u}-y,\\&=\gamma_r-y. \qquad (33)\end{aligned}$$

Since γ_r-y is equal to $\boldsymbol{X}/C$ and it being larger than or equal to 0 implies the positive semi-definiteness of X and vice versa, the optimization problem constraint involving γ_r and y can be rewritten as a constraint enforcing the positive semi-definiteness of X,

$$\begin{aligned}&\underset{\boldsymbol{u}}{\text{minimize}}\ -y, \qquad (34)\\&\text{subject to}\quad C1:\boldsymbol{X}\succeq 0,\\&\qquad C2:||\boldsymbol{u}||_2^2\leq P_t,\\&\qquad C3:\gamma_c=\frac{|\boldsymbol{h}^H\boldsymbol{u}|^2}{N_0'}\geq\Gamma_c.\end{aligned}$$

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