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Working Paper 135
August 2022

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Abstract

Cash money can be a rational devise of saving as an insurance against external uncertainty. Liquid money, controlled by a stable and trustworthy central bank, offers an insurance against stock market crashes, bankrupts and other economic turmoils that endanger the yield of illiquid saving modes. In turbulent times, the value-carrying property of money is accentuated, and the recent dark episodes including the last financial crisis, the pandemic and the war in Ukraine have made the public in Europe respond to uncertainty by increasing their cash holdings. The paper constructs a simple life cycle framework for the analysis of rational and irrational motives to save money, answers to questions about the effects of saving liquid money on illiquid saving and education and examines the inherent cost of the use of cash as a saving mode. The main findings of the paper are the following. The insurance motive to save money increases total savings by replacing deposit saving more than one-to-one. The share of deposit savings depends positively on the expected interest rate, while the share of cash savings is the higher the less there is inflation. Deposit saving correlates positively and education negatively with the expected market interest rate thus affecting their relative proportion, but education does not affect the implicit price paid for cash insurance. Incorporating money illusion adds an internal bias to life-time optimization. Misjudgment of the inflation rate makes consumers save excessively in cash at the cost of market deposits and increases the cost of using cash as rational insurance against external uncertainty.

Key words: compensating variation, deposit saving, expected yield, liquid money, uncertainty

1 Introduction

The traditional monetary theory tells that, since the nominal price mechanism is in the core of the market economy, there must be liquid money in the circulation enough to meet the transaction requirements. The supply of money is organized by the banking system, where the central bank prints money in its balance sheet and lends it to the commercial banks, which deliver it to the market. Thus, the dwell of money is based on a twofold debt relationship, where the central bank lends the money to the commercial banks, and the commercial banks lend it to the firms and public in the market.

Previously, the asset account of the central bank's balance sheet was largely based on the gold standard, like in the Bretton Woods -system between 1944–1971. The hub of the system was USA with both its gold reserves and its position as the world's economic leader. However, the European monetary union resembles the Bretton Woods -system because the stability of both the currency and the economy in the Eurozone is connected to leading economic power, that is Germany (Blyth, 2017).

By the classical quantity theory of money, the price level in the economy depends directly on the amount of active money in the circulation. In its turn, the activity of money depends on its velocity, which is constant in the short term. The quantity theory of money omits the inactive nature of money, but there are certainly reasons for holding cash money out of circulation. Alfred Marshall (1890) highlighted precautionary saving, and John Maynard Keynes (1930) emphasized that money is a stone of value and thus a vehicle of saving.

By Miquel Sidrauski (1967), holding cash money yields utility just like the consumption of goods and leisure. That is, money with real purchasing power is one argument in consumers' utility function with standard properties, including diminishing marginal utility and positive cross effects. The latter property means that a marginal increase in one argument causes the marginal utility from another argument to increase. So, a bit more of money should make the marginal utility form consumption of goods or leisure rise. This may be at odds with common sense, but not so with standard economic theory.

Ayse Imrohoroglu (1992) noted that cash reserves provide insurance against the accidental volatility of illiquid assets. The expected returns of alternative saving methods, like fixed deposits or other illiquid modes are prone to higher risk and uncertainty, from which cash provides protection. Carl Walsh (2013) added that money with real value creates utility by providing so called transaction services thus making the acquisition and use of other utility producing items more convenient. For example, motoring is a lot more carefree, when one knows that the accidentally broken car can right away be replaced without sacrifice of other consumption. That can be guaranteed either by an insurance company or by a fat wallet. In any case, one can enjoy consumption and leisure more relaxed, which means that the enjoyment should also be more luxurious.

From the insurance perspective, holding cash money can be a rational choice for a part of savings, especially in turbulent times. In fact, both the 2008 financial crisis and the 2020 pandemic have shown that people have responded to uncertainty by increasing their cash holdings. In Europe, the increase has generally been notable, while the response in Finland has been somewhat milder. Still, there has been a slightly growing trend in cash holdings of the Finnish public. However, money attracts people also by irrational motives. People's irrationality has for long been broadly studied in behavioral economics (Tversky & Kahneman, 1992), while this paper restricts to money illusion, which refers to a tendency to think in terms of nominal rather than real monetary values. By Irving Fisher (1928), nominal monetary values should have no role in rational market behavior.

The aim of the paper is twofold. First, the aim is to construct a simple life cycle framework for the analysis of rational and irrational motives to save money. Second, the paper aims to answer to some questions about the consequences of holding cash in people's life cycle decisions under uncertainty, like the effects of saving liquid money on illiquid saving and investment in education. In particular, the inherent cost of the use of cash as a saving mode is examined.

The utility creating property of money can be modelled in a standard way without plugging it in the utility function. This paper constructs an illustrative model of intertemporal optimization where lifetime incomes can be timed to consumption according to one's time preference, forwards by saving or backwards by lending. For practical reasons, this paper considers the case of a representative saver. In the model, money enters the budget constraint just like any savings do. It means that the utility and cost from holding cash money as a part of total savings can be measured indirectly, that is in terms of compensating variation. The analysis starts with assuming uncertainty for reasons that are external to the saver and ends with also considering her internal biases, namely money illusion.

The main findings of the paper are the following. In the elementary model without education, it is found that the rational insurance motive to save money increases total savings by replacing deposit saving more than one-to-one. Yet, the share of deposit savings depends positively on the expected interest rate. On the other hand, the share of cash savings is the higher the less there is fully anticipated inflation, which means that as smaller inflation makes cash insurance cheaper, consumers purchase better security. Incorporating education into the model shows that deposit saving correlates positively and education negatively with the expected market interest rate thus affecting their relative proportion, but education does not affect the implicit price paid for cash insurance. Incorporating money illusion, that is misjudgment of the inflation rate makes consumers save excessively in cash at the cost of market deposits. More importantly, money illusion increases the cost of using cash as insurance. The extra cost equals the discounted opportunity cost of the overly saved amount of money.

The paper proceeds as follows. Section 2 constructs the basic life cycle model of cash as rational insurance with perfect foresight on inflation but external uncertainty about the yield of market deposits. Section 3 adds education into the model with external uncertainty about its yield. Section 4 incorporates consumer's internal bias into the analysis, namely irrationality caused by money illusion. Section 5 discusses and concludes the findings.

2 Money as Insurance

In an elementary life cycle consumption model, the representative consumer allocates her lifetime incomes to consumption according to her time preference (Burda & Wyplosz, 2017). For simplicity, the consumer's life cycle is divided into two parts, period 1 and period 2, and all decisions are made at the beginning of period 1. Moreover, time use in both periods is fixed to acquiring labor income so that leisure time is not an argument in the utility function. Thus, the representative consumer maximizes:

$$u(q_1, q_2)$$

In the utility function, q_1 is consumption in the first period, and q_2 is consumption in the second period. The standard assumptions $u_1, u_2 > 0$; $u_{11}, u_{22} < 0$; $u_{12}, u_{21} > 0$ hold, where the subscripts denote first and second order derivatives according to the two arguments of the utility function. Moreover, the utility function is assumed homothetic, meaning that plain income effects do not change the consumer's time preference in the short term. This is a reasonable assumption in the static model.

The holding of liquid cash money is reasoned by external uncertainty concerning the returns of illiquid fixed deposits. The amount of cash saving is accounted in the budget constraints so that it becomes a choice variable just like all saving. Assuming that the consumer's productivity remains unchanged over the life cycle and normalizing both time and labor income to unity in both periods, the income endowment is $E = (1,1)$. The periodic budget constraints read:

$$q_1 = 1 - m - s \qquad (1)$$

$$q_2 = 1 + \alpha m + (1+r)s \qquad (2)$$

In equation (1), 1 is labor income, m is cash saving and s is illiquid saving. In equation (2), $\alpha = 1/(1+\pi)$ where π is the inflation rate between the periods and $0 < \alpha \leq 1$. So, αm is the real return from holding cash. On the other hand, $r \geq 0$ is the expected real market rate of return and $(1+r)s$ is the expected return from illiquid saving. Solving for s from (1) and substituting to (2) produces the consumer's life-time budget constraint:

$$q_2 = 1 + \alpha m + (1+r)(1-m) - (1+r)q_1 \qquad (3)$$

So, the optimization problem of the representative consumer reads:

$$Max\ u(q_1, q_2)\ \text{s.t.}\ q_2 = 1 + \alpha m + (1+r)(1-m) - (1+r)q_1$$

The solution to the optimization problem can be analyzed in two steps. First, the consumer chooses the amount of cash saving in order to protect herself from unpleasant surprises in the return of illiquid savings. The rule for optimal cash holdings is derived by plugging (1) and (2) into the utility function and solving optimal m. This produces:

$$\frac{u_1}{u_2} = \alpha \qquad (4)$$

Rule (4) tells that, at the optimum, the slope of the indifference curve $\frac{u_1}{u_2}$, which measures the consumer's time preference, must be equal to the marginal rate of return α of cash saving.

Second, the consumer decides on the optimal timing of her consumption by deciding on illiquid saving along the life-time budget constraint, which is now restricted by the simultaneous choice of m. The rule for optimal illiquid saving is derived by plugging (3) into the utility function and solving for q_1. This produces:

$$\frac{u_1}{u_2} = (1 + r) \qquad (5)$$

Rule (5) tells that, at the optimum, the slope of the indifference curve $\frac{u_1}{u_2}$ must be equal to the expected return of illiquid savings $(1+r)$.

Note that $\alpha = (1+r)$ only if $\alpha = 1$ and $r = 0$. If so, all savings would be in cash. When $\alpha \neq (1+r)$, the rules (4) ja (5) hold in optimal solutions, which occur on different utility levels depicted by the set of homothetic indifference curves portraying the consumer's time preference.

Figure 1 illustrates the analytical treatment of the representative consumer's simultaneous life-time optimization.
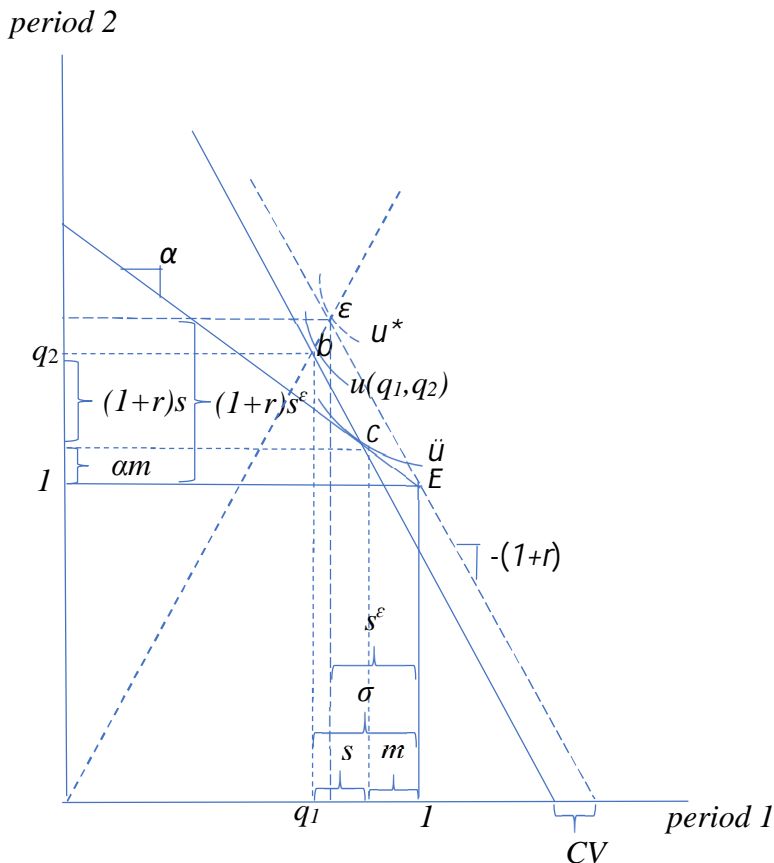

Figure 1: Money as insurance against uncertainty

Figure 1 illustrates the representative consumer's simultaneous life-cycle decision making, which can be analyzed in two steps. First, the consumer chooses the amount of her cash savings. Assuming that inflation is perfectly anticipated, consumption possibilities can be transferred forwards by cash money along the return line, which starts from point E and rises leftwards. The slope of the line is $0 < \alpha < 1$ due to inflation so that the return from saving money is negative. The optimum condition (4) holds at point c, where the highest possible indifference curve $\ddot{u}$ from the set of the homothetic utility function touches the return line. The optimal amount of cash saving is m, which also measures the amount of working time devoted to the accumulation of cash money. Thus, the utility level $\ddot{u}(1-m, 1+ \alpha m)$ is the bottom safeguard for life-time consumption utility. Note that the external uncertainty may concern not only the market rate of return r, but also the return of capital so that $\ddot{u}$ is the safeguard against total vanish of the deposits.

Second, the consumer decides on her illiquid saving according to its expected return *1+r*. Since the cash saving *m* is now given, the labor income that can be used to market saving in period 1 is restricted. Therefore, the life-time budget constraint with the slope -*(1+r)* is shifted inwards so that it goes through point *c* instead of point *E*. The optimum condition (5) for illiquid saving holds in point *b*, where the highest possible indifference curve from the set of the homothetic utility function touches the solid life-time budget constraint. The optimal amount of illiquid deposit saving is *s*, which also determines the amount of work needed for the accumulation of deposit savings.

The consumer's simultaneous life-time optimization at the beginning of period 1 produces total savings $\sigma = m + s$ in period 1 to be consumed in period 2. The return from cash saving is αm in terms of secure consumption in period 2, and the return from deposit saving is *(1+r)s* in terms of expected consumption in period 2. Thus, consumption in period 1 is $q_1 = 1-m-s$, and the expected consumption in period 2 is $q_2 = 1+\alpha m+(1+r)s$.

In figure 1, the consumer's expected utility at the final optimum point *b* is $u(q_1,q_2)$. For comparison, a dashed life-time budged constraint going through point *E* is drawn to illustrate the hypothetical case, where there would be no uncertainty about the yield of illiquid deposits and thus no need for insurance. In that case, the optimization would include only one analytical step according to rule (5). The optimum would occur at point ε, where the highest possible indifference curve from the set of the homothetic utility function touches the dashed life-time budget constraint. The consumer would then reach to the higher utility level u^* by depositing s^ε and receiving the warranted $(1+r)s^\varepsilon$ in return.

The analysis shows that, since $m + s = \sigma > s^\varepsilon$, the rational insurance motive to save money increases the amount of total savings compared to the case of perfect foresight. So, secure cash saving replaces insecure deposit saving more than one-to-one. It can also be inferred from figure 1 that the higher is the expected market interest rate *r* the larger is the share of deposit savings in total savings σ/s, and vice versa. The results are clear under the assumption of homothetic preferences saying that the consumer's time preference is not affected by pure income effects.

Another important observation can be deduced by considering the value of the inflation parameter α. From figure 1 one can infer that the closer the value of α is to unity that is the steeper is the yield line for cash the further leftwards from *E* point *c* along the solid life-time budget constraint settles. This is because of the substitution effect caused by the fall in the relative price of cash saving. The final optimum would then be below point *b* along the dashed line from the origin implying lower expected consumption and utility. The implication is that both cash and deposit saving would increase, but the share of *m* in σ would grow, because less inflation means higher return to cash savings.

The comparison between the cases of perfect foresight and external uncertainty reveals the implicit price of the consumer's insurance against misfortunes. It can be measured by compensating variation (*CV*), that is by the distance between the solid and dashed budget constraints in figure 1. The horizontal difference between the constraints can be calculated in terms of period 1 consumption by solving q_1 from (3) in both cases. The point of intersection of the solid constraint with $m > 0$ along the horizontal axis is:

$$q_1 = 1-m + (1+\alpha m)/(1+r)$$

The respective intersection point of the dashed constraint with $m = 0$ reads:

$$q_1 = (2+r)/(1+r)$$

Subtracting the former expression from the latter one produces, after some manipulation:

$$CV = (1 - \frac{\alpha}{1+r})m > 0 \qquad\qquad (6)$$

The compensating variation *CV* in (6) measures the amount of period 1 consumption that would compensate the discrepancy between *u* and *u**. Thus, it gives the cost that the consumer willingly pays for secured consumption. Expression (6) tells that the price of the insurance *1-α/(1+r)* per one unit of *m* depends negatively on *α* and positively on *r*. Since *α* is the inverse measure of inflation, the conclusion is that both lower inflation and lower expected interest rate mean a lower implicit insurance price, and vice versa.

Yet, figure 1 also reveals that the lower the inflation rate the higher is the voluntary cost *[1-α/(1+r)]m* of the insurance. This may sound surprising, because one might presume the opposite due to better yield for cash savings. However, the homothetic utility function is uncompromising: lower inflation presented by a higher value of *α* would make point *c* travel north-west from point *c*, which implies a broader *CV* difference along the horizontal axis. The economic intuition is that, as smaller inflation makes the unit price of cash insurance cheaper and, as leisure time is omitted, the consumer purchases broader security for any unforeseen contingencies. In that case, consumption utility *u* would be lower but the safeguard level *ü* would be higher than depicted in figure 1.

The static life-cycle optimization at the start of period 1 is based on expectations, and the securement against unwanted surprises is irrespective of whether the possible misfortune occurs or not. As a matter of fact, this does not differ much from normal insurances. The main difference is that while the pricing of commercial insurance products is distorted by adverse selection and moral hazard, cash saving can be individually planned. Another difference is that the commercial *force majeure* - insurances do not usually cover total unpredictability or "black swans". In that respect, cash can be a better option.

3 Money, Education and Saving

Basic life-cycle models usually also include the possibility to invest in one's own productivity (Burda & Wyplosz, 2017). A simplifying assumption is that one can use part of her time in period 1 in education to cultivate her skills and enjoy the product of the investment in period 2. Another reasonable assumption is that the time used in education fosters productivity and thus wages with diminishing marginal returns. Denoting the time used in education in period 1 by e, the expected product from the investment in period 1 to period 2 is $w_2 = w(e)$ with $w' > 0$, $w'' < 0$, where the sub-primes denote first and second order derivatives. With these amendments, the representative consumer's periodic budget constraints read:

$$q_1 = 1 - e - m - s \tag{7}$$

$$q_2 = w_2 + \alpha m + (1+r)s \tag{8}$$

Solving s from (7) and plugging into (8) produces the life-time budget constraint:

$$q_2 = w_2 + [\alpha - (1 + r)]m + (1+r)(1-e) - (1+r)q_1 \tag{9}$$

The consumer solves the problem:

$$Max\ u(q_1,q_2)\ \text{s.t.}\ q_2 = w_2 + [\alpha - (1 + r)]m + (1+r)(1-e) - (1+r)q_1$$

Now, the solution to the representative consumer's simultaneous optimization problem can be analyzed in three steps.

The first step is again to decide on cash saving, which necessitates time use in work to make money in period 1. Plugging (7) and (8) into the utility function and solving m yields:

$$\frac{u_1}{u_2} = \alpha \tag{10}$$

Again, rule (10) tells that the slope of the highest possible indifference curve from the set of homothetic utility function must be equal to the marginal rate of return of cash saving.

The second step concerns the optimization of education time. This is done by maximizing the life-time income from the right-hand side of (9) with respect to e. Recalling the expected yield $w_2 = w(e)$, the optimum condition is:

$$w' = (1 + r) \tag{11}$$

Rule (11) says that, in the optimum, the marginal rate of return from the time spent in education must equal the slope of the life-time budget constraint, which measures the expected opportunity cost of education in terms of deposit saving.

Finally, the consumer optimizes the timing of consumption along the life-time budget constraint. Plugging (9) into the utility function and solving for q_1 produces:

$$\frac{u_1}{u_2} = (1 + r) \tag{12}$$

Rule (12) determines the optimal amount of deposit saving. When the secure yield of cash savings differs from the expected market yield, $\alpha \neq (1+r)$, the conditions (10) and (12) refer to solutions that occur on different utility levels among the set of indifference curves describing the homothetic utility function. On the other hand, (11) and (12) depict separate education and consumption optimums along the expected budget constraint thus obeying the Fisherian condition:

$$w' = (1+r) = \frac{u_1}{u_2} \tag{13}$$

Figure 2 illustrates the representative consumer's simultaneous optimization including cash saving, education, and the timing of consumption via deposit saving. The consumer is assumed to be perfectly

aware of inflation, while the product of education and the market yield from savings are uncertain. Thus, there is a rational motive for saving cash money.
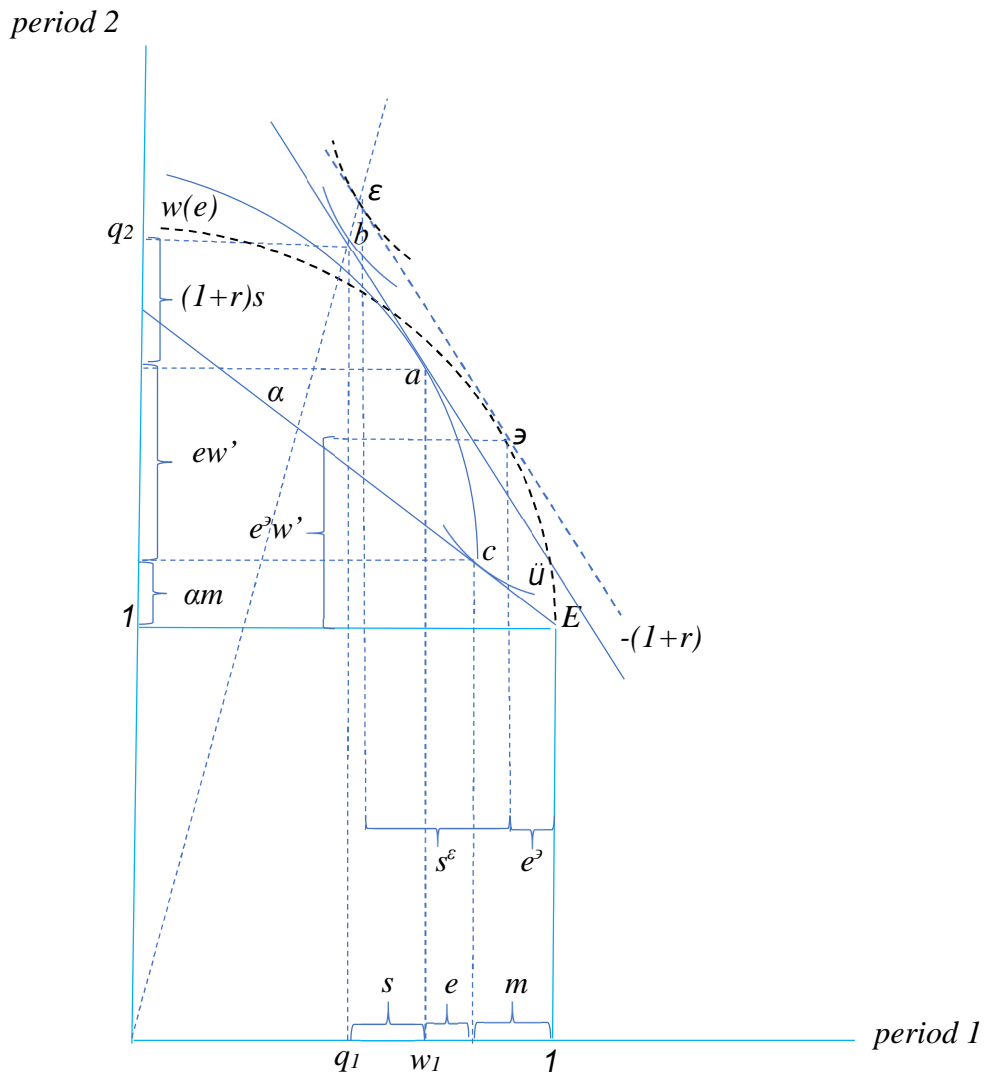

Figure 2: Life-cycle optimization with education and cash insurance

In figure 2, the first step of the consumer's simultaneous optimization is to decide on the amount of cash saving. This is done by devoting part of the time available in period 1 to work so that the accumulated labor income covers the chosen amount of cash. By (10), the optimum is at point c and the optimal amount of cash holdings is m, which also measures the time used in work to accumulate the needed cash. The cash saving m is transferred to period 2 as αm, where $0 < \alpha < 1$. This secures the utility level $\ddot{u}$ as protection for any external uncertainties.

On the second step, the consumer optimizes on education time in period 1. Due to the use of time in work necessitated by cash saving, the time available for other purposes is now $1 - m$. Therefore, the time investment in education can start from point c, and the expected product of the investment develops leftwards along the deceleratingly rising solid curve $w(e)$. Condition (11) holds in point a,

where the expected marginal product of education equals its expected opportunity cost. Labor income in period 1 is w_1 and the expected labor income in period 2 is $w_2 = ew'$.

The final analytical step concerns optimal timing of consumption by deposits. Since the optimization of education has shifted the life-time budget constraint outwards to go through point a, condition (12) holds in point b. Deposit saving in period 1 is s with the expected return $(1+r)s$ in terms of period 2 consumption. Thus, consumption in period 1 is $q_1 = (1-e) - (m+s)$ and the expected consumption in period 2 is $q_2 = \alpha m + ew' + (1+r)s$.

The analysis yields the following notions. First, the introduction of education does not affect cash insurance saving. Rule (10) says that the optimization on the amount of cash m depends on the perfectly anticipated inflation parameter α, which has nothing to do with either education or deposit saving. Therefore, $\ddot{u}$ depicts the bottom safeguard for the worst possible scenario, where the deposits would be wiped out and employment corresponding to the chosen education would not be available in period 2.

Second, to put it the other way round, the rational cash insurance does not affect the consumer's choice of education, because the model does not include leisure time thus omitting the opportunity cost of leisure. Figure 2 provides a comparison to the case of perfect foresight, where the dashed product curve of education starts from point E, and education and saving decisions are made along the budget line tangent to the curve. Then, cash saving makes the start of the product curve $w(e)$ shift north-west from point E to point c. Since the curve retains its shape, rule (11) holds at point $ɘ$, and the point pair E and $ɘ$ produces the same amounts of e and ew' as the point pair c and a with cash insurance. Namely, $e^ɘ = e$ and $e^ɘw' = ew' = w_2$ in figure 2.

Third, figure 2 shows again that uncertainty makes the consumer save more in total compared to the case of perfect foresight, that is $s + m = \sigma > s^\varepsilon$. The effect remains qualitatively the same as in figure 1 because of the neutrality of cash saving on education. Yet, figure 2 also reveals that a fall in the expected market interest rate would make the investment in education e grow at the expense of deposit saving s. From the macroeconomic point of view, the partial replacement of s by e would mean replacement of physical capital accumulation by human capital accumulation. In the case of a rise in the expected interest rate, the effects would be reversed.[1]

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[1] The comparative static properties of the model can be checked by totally differentiating (11) and (12) with recall of (13) and taking m fixed in (7) and (8). Denoting $-u_{11} + 2(1+r)u_{12} - (1+r)^2 u_{22} = A > 0$ and $u_{12} - (1+r)u_{22} = B > 0$ and using Cramer's rule produces:

$$\frac{de}{d\alpha} = 0$$

$$\frac{ds}{d\alpha} = -m\frac{B}{A} < 0$$

$$\frac{de}{dr} = \frac{1}{w''} < 0$$

$$\frac{ds}{dr} = -\left(\frac{1}{w''} + s\frac{B}{A}\right) + \frac{u_2}{A} > 0$$

The first three effects are straightforward. In the fourth formula, the first term in the parentheses on the right-hand side depicts the composite income effect with $1/w'' < 0$ and $sB/A > 0$ and the second term presents the substitution effect $u_2/A > 0$. The income effect is also positive due to homothetic preferences, which dictate that a marginal change in r induces a change in education and a smaller opposite change in deposit saving. Since $1/w'' + sB/A < 0$, also the first term on the right-hand side is positive.

Lastly, the above findings pave path to the conclusion that the introduction of education into the model does not affect the voluntary implicit price paid for the cash insurance. In technical terms, the *CV* measure of the model should equal the measure derived from figure 1. This can be verified by recalling (9) and calculating the horizontal intersection point with money and education:

$$q_1 = (1\text{-}e\text{-}m) + (w_2 + \alpha m)/(1+r)$$

This refers to the solid life-time budget constraint in figure 2. The respective intersection of the dashed budget line that depicts the case with $m = 0$ is:

$$q_1 = (1\text{-}e) + w_2/(1+r)$$

Subtracting the former from the latter gives:

$$CV = (1 - \frac{\alpha}{1+r})m > 0$$

Since the result is the same as in (6) before, the cost of the cash insurance in terms of period 1 consumption is independent from the decision on education. The economic intuition is that, since the yield from both education and deposit saving decision is uncertain, the optimal amount of cash savings m that secures $\ddot{u}$ provides insurance for all negative surprises, including those that concern the yield of education.

4 Money Illusion

Above, the assumption has been that the consumer faces only external uncertainty concerning the yields of deposit saving and education. In those circumstances, cash is a rational insurance mode of saving. Still, there also exist various internal biases that make people hoard money more than what would be reasoned by rational arguments. Money illusion is one such psychological factor, which is broadly accepted in economic theory and confirmed by many empirical verifications (Fisher, 1928; Blanchard, Amighini & Giavazzi, 2010; Begg, Vernasca, Fischer & Dornbusch, 2014). Money illusion makes people concentrate on the nominal value of money instead of its real purchasing power. By Akerlof & Schiller (2009), the misjudgment of inflation causes market anomalies because it distorts the market by making the simultaneous price mechanism sticky.

Money illusion distorts especially long-term decisions, in which expected inflation should naturally be accounted. In static sense, the implications of money illusion can be dealt with the life-cycle model presented in the previous section, and figure 3 illustrates the effects of the matter. For simplicity, the representative consumer is assumed to suffer from utmost money illusion.
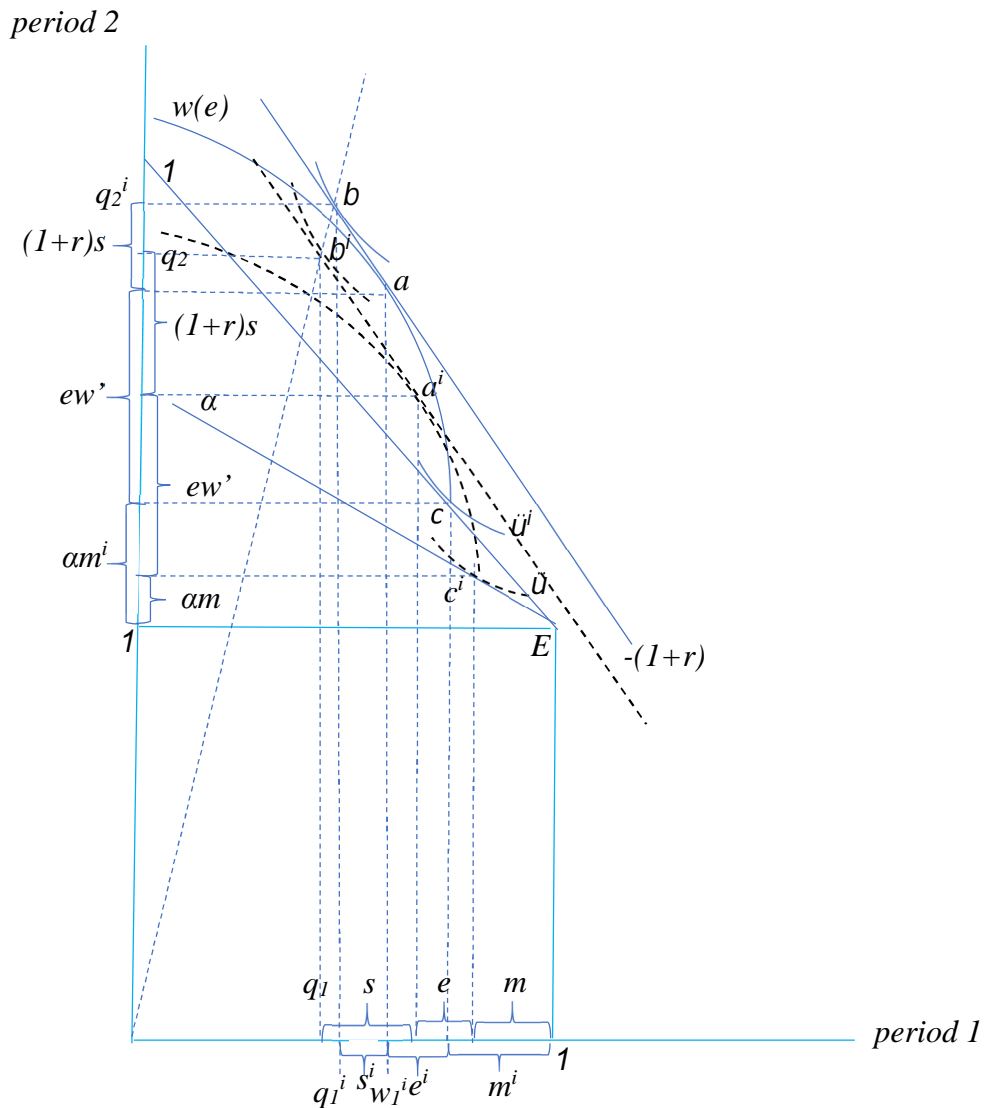

Figure 3: Absolute money illusion in the life-cycle model

Absolute money illusion means that consumers falsely presume one-to-one yield from saving money forwards. Figure 3 presents this so that the representative consumer presumes $\alpha = 1$ instead of the fact that $0 < \alpha < 1$, where the demarcation is quite notable for clarity. The consumer's choices under money illusion are presented by subscript i.

On the first step of her simultaneous optimization, the internally biased consumer chooses the amount of money along the yield line that rises leftwards from E by the slope 1. As a result, money illusion misleads the consumer to choose m^i according to the point c^i instead of choosing the rationally motivated m according to point c. This means that the difference $m^i - m > 0$ is due to irrationality.

On the second step, the consumer optimizes on education taken that the choice m^i has made the yield curve of education start from point c^i instead of point c. By (11), the choice of education is not distorted by money illusion, because it is based on the expected real interest rate that is externally given to the consumer. Thus, the consumer chooses education according to the point a^i instead of choosing it according to point a. Nevertheless, $e^i = e$ and $e^i w' = ew'$ because the model omits the opportunity cost of leisure.

The third step of the consumer's simultaneous optimization is based on (12), but the homotheticity of the utility function causes money illusion to affect the choice. The consumer chooses s^i according to b^i instead of choosing s according to b, which means that $s^i < s$. While excess cash saving m^i cuts available working time thus placing point c^i north-west from point c, homothetic preferences keep points b^i and b on the dashed line starting from the origin.

The analysis produces the following findings. First, money illusion misleads the representative consumer to save excessively in cash and shortly in market deposits. This is because the consumer underestimates the cost of holding money and errs to take an overly covering insurance m^i against uncertainty based on false beliefs. Moreover, albeit total savings are smaller under money illusion than under perfect foresight, that is $m^i + s^i = \sigma^i < \sigma = m + s,$ the share of cash of total savings is larger under money illusion, $m^i/\sigma^i > m/\sigma$. The qualitative effects of changes in the expected market interest rate remain the same as discussed in the previous sections.

Second, the static life-cycle model, where all decisions are made at the beginning of period 1, shows only the optimal plan but not the true outcome in period 2. Yet, an external observer can evaluate the cost of money illusion by the means of the *CV* analysis. From (9), the calculation of the horizontal intersection point of the dashed constraint in figure 3 produces:

$$q_1 = 1 - m - e + (w_2 + \alpha m)/(1+r)$$

This presents the case without money illusion. In the case with absolute money illusion, (9) turns to $q_2 = w_2 + m^i + (1+r)(1 - m^i - e - q_1)$ where m^i denotes the cash saved in period 1 and anticipated to be recollected as such in period 2. Calculation and manipulation produce the horizontal intersection point of the solid constraint:

$$q_1 = (1-e) + (w_2 - r m^i)/(1+r)$$

Subtracting the latter formula with money illusion from the former with no such internal bias, that is calculating the horizontal difference between the dashed and solid budget constraints of figure 3 produces:

$$CV^i = -\left(1 - \frac{\alpha}{1+r}\right)m + \frac{r}{1+r}m^i \qquad\qquad (14)$$

Note that the CV^i in (14) is calculated to catch the fact that money illusion lures the consumer to reach beyond true consumption possibilities thus motivating excessive saving of money. On the other hand, the CV in (6) measures what would be the cost of rational precaution to external uncertainty, that is the horizontally measured inward shift of the dashed constraint caused by holding m. Therefore, the total cost of holding m^i amounts to $CV + CV^i$. Since the CV in (6) cancels out the first term of the CV^i in (14), the remainder is:

$$\frac{r}{1+r} m^i > 0$$

This measures the excess burden caused by money illusion. Thus, in terms of period 1 consumption, the excess cost caused by money illusion equals the discounted expected marginal opportunity cost of the overly saved amount of money m^i. The result is economically intuitive and, while the analysis considered absolute money illusion, the qualitative result holds under partial money illusion, too.

5 Conclusions

This paper examined the role of cash money as a mode of saving. A simple life cycle consumption model was constructed, where people live for two periods and allocate their life-time incomes into life-time consumption to maximize utility from consumption in the two periods. In the static model, all decisions are made at the start of the life cycle. The optimal allocation of consumption is guided by constant time preferences depicted by a homothetic utility function, and income can be timed to consumption forwards by saving or backwards by lending. For practical reasons, the paper focused on the decision making of a representative saver. In the model, life-time income is also endogenous, because consumers can cultivate their skills in the first period to improve their productivity and thus wages in the second period.

Both rational and irrational motives for holding cash money were studied. The rational motive was assumed to arise from uncertainty caused by reasons that are external to the decisionmaker. The external uncertainty was assumed to blur the expected yield from illiquid savings and investment in education. In both cases, the worst scenario was that not only the interest but also the invested capital would vanish. Thus, saving cash money can provide a rational insurance against misfortunes of any kind. As an irrational motive for saving cash, a factor internal to the decisionmaker was introduced, namely the psychological bias caused by money illusion.

The consumer's intertemporal optimization was first studied in an elementary case, where life-time income was fixed, and the optimization problem was to decide on optimal saving from period 1 to period 2. In particular, the optimization concerned the composition of total savings consisting of liquid cash money and illiquid market deposits, while the market yield of the deposits is uncertain. Under these circumstances, it is rational to save cash as an insurance for future consumption according to the time preference. It was found that the cash insurance increases total savings compared to the case of perfect foresight. Moreover, it was found that the share of deposit savings in total savings depends positively on the expected market rate of return, and that the share of cash depends negatively on inflation.

The implicit cost of the consumer's rational cash insurance against uncertainty was measured by compensating variation, which gives the amount of extra consumption that would compensate the discrepancy from the utility level under perfect foresight. Calculating in terms of period 1 consumption, the cost of holding money was found to depend on both perfectly anticipated inflation and expected market interest rate. Quite intuitively, higher anticipated inflation rates or higher expected interest rates mean a higher cost of holding one unit of money, and vice versa. Moreover, the calculus of compensating variation gives the implicit price that consumers are willing to pay for secured and therefore more satisfying consumption.

Introducing investment in education into the model showed that rational cash insurance does not affect the consumer's choice of education, assuming that the opportunity cost of leisure is omitted. This implied that the findings concerning liquid and illiquid saving remained qualitatively the same as without education. Quite expectedly, it was found that a lower expected market interest rate would make the investment in education grow at the expense of deposit saving, and vice versa. It was also concluded that the introduction of education into the model does not affect the voluntary implicit price paid for the cash insurance. That is, the respective compensating variation remained unaltered. The economic intuition is that, since the yields from both education and deposit saving decisions are uncertain, the optimal amount of cash savings provides insurance for all kinds of negative surprises

in period 2, including total vanish of the deposits and inability to find a job that corresponds to the chosen education.

Third, the implications of introducing money illusion into the model were studied. It was found that money illusion misleads the representative consumer to save excessively in cash and shortly in market deposits, because she underestimates the cost of holding money and errs to take an overly covering insurance against external uncertainty. Moreover, albeit total savings are smaller under money illusion than under perfect foresight, the share of cash of total savings is larger. The qualitative effects of changes in the expected market interest rate were not changed. Most importantly, the effect of money illusion on the cost of the rational insurance was calculated and compared to the case where inflation is accounted perfectly. The excess cost caused by the internal psychological bias in holding cash saving was found to bel the discounted expected opportunity cost of the overly saved amount of money. In general, money illusion becomes the costlier the broader is the deviance from the true inflation rate. The findings are relevant, because empirical evidence shows that money illusion is a persistent feature in people's behavior with distortive consequences in the economy, particularly under creeping inflation.

To conclude, people can use money as a rational insurance against external uncertainties, but human psychology may internally delude them to excessive cash saving. The paper managed to present a simple and illustrative life cycle model for the analysis of holding cash money for rational and irrational reasons. The main pitfall of the model is that it omits leisure, which undeniably is an important complement to consumption. Moreover, the static model is too rigid to present the final outcomes neither at the end of the life span nor happenings along the span. In real life, people presumably learn about their disappointments in the long run, and they surely err and correct their decisions repeatedly during the run, too. Dynamic life-cycle models might be more revealing in that respect. However, the static viewpoint is not too far from reality, where people usually buy expensive insurances from insurance companies against sad incidences, which they wish never to happen.

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