

# Enhancement of single-shot lensless super-resolution phase imaging by preliminary observations

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**Abstract:** Algorithm enhancements are proposed for single-shot phase retrieval to reduce loss from ill-conditioning. The approach is based on prior observations, improved diffraction pattern up-sampling, and updated filters. High-quality super-resolved reconstructions are demonstrated in simulation. © 2021 The Author(s)

## 1. Introduction

The ill-posed problem of phase retrieval consists in the fact that through the observations only the intensity of the light radiation can be captured while the phase is lost. In the lensless case, the recorded intensity patterns are the summation of several diffraction orders. Reconstructions from these patterns using iterative methods become stagnant without the necessary extra information. To overcome this problem several techniques have been proposed in the past decade aiming to gain such information and be able to resolve complex objects. The most apparent solution is to capture multiple intensity patterns which might be generated by either moving optical elements, or multi-wavelength illumination, or special programmable devices. Contrary to these techniques, we successfully proved in our previous paper [1] that phase retrieval is achievable from a single pattern. The needed information is obtained using only a binary phase-modulation mask between the object and sensor planes, resulting in super-resolution complex object reconstruction. However, we found that the method is having difficulties to reconstruct small phase values and sensitive to the errors of the theoretically precise inputs. Enhancement techniques are embedded into the modified Super-Resolution Sparse Phase Amplitude Retrieval (SR-SPAR) [2] method to further improve the resolution of the reconstructions even with small phase values. We applied compensation by prior observations (*cpo*), improved diffraction pattern up-sampling, and updated filters.

## 2. Problem formulation and solution

The proposed setup is a lensless in-line holography scheme with illumination of a single wavelength  $\lambda$ , an arbitrary complex-valued object  $u_o$ , a binary phase modulation mask  $M$ , and sensor. The wavefront propagation is modelled by Angular Spectrum method with  $P_d$  forward propagation operator on the  $d_1$  object-mask and  $d_2$  mask-sensor distances. The intensity of the diffracted wavefront is captured on the sensor plane as

$$z = |P_{d_2} \{M \circ P_{d_1} \{u_o \circ u_b\}\}|^2, \quad (1)$$

where  $u_b$  represents the wavefront just before the object plane, while " $\circ$ " is the Hadamard product between the matrices. Due to the ill-conditioning even precise prior knowledge about these parameters are resulting in corrupted reconstructions. In the enhanced approach, we aim to reduce the loss from the ill-conditioning, hence correct the corruption using compensation by preliminary tests of the optical system. We define  $u_b = \sqrt{z_b} \circ cpo$ , in which  $z_b$  stands for the intensity captured by prior observing the illumination source (*beam*) only. We assume that our laser produces ideal coherent plane waves as a Gaussian-shaped beam with the same wavefront on any plane. To calculate *cpo* we define the wavefront on the object plane from the prior observation of the modulation mask  $z_m$  using Eq. (1) without the object ( $u_o = 1$ ). The difference between this corrupted wavefront  $u_b$  and the beam's wavefront determines the computational compensation as  $cpo = u_b / \sqrt{z_b}$ .

Although *cpo* significantly improves imaging, reconstruction from single diffraction pattern still needs effective filters to correct the remaining noises. Therefore, apodization, Gaussian filter, and updated Block-Matching 3D (BM3D) filter [3] are used. The latest version of BM3D filters firmly the correlated noises generated by the modulation mask.

The resolution of the reconstruction is also limited by the fact that the wavefront propagation modeling requires discretization of the continuous object by the computational pixel size of  $\Delta_c \times \Delta_c$ . Conventionally this size is

specified by the sensor pixel size as  $\Delta_c = \Delta_s$ . This case can be referred to as *pixel-wise* imaging, which means that the resolution is limited by this physical parameter. In SR-SPAR method, the input diffraction patterns are up-sampled by  $r_s = \Delta_s/\Delta_c$  super-resolution factor resulting in  $\tilde{z}_a$ ,  $\tilde{z}_m$  and  $\tilde{z}_b$  up-sampled images. If  $r_s > 1$  the limitation of the sensor is dissolved leading to computational *super-resolution* case. The up-sampling method affects on the resolution, therefore several techniques has been tested to find the proper way to up-sample the diffraction patterns. We found that using a staircase interpolation with Lanczos-3 kernel [4] the resolution increases compared to the previously used nearest-neighbor interpolation.

### 3. Results

The super-resolution reconstructions are demonstrated by simulations. USAF phase-target was used as an object with etch depth of  $100\text{ nm}$  ( $0.54\text{ rad}$ ) and smallest line-thickness of  $0.875\text{ }\mu\text{m}$ . We assumed Gaussian shaped laser illumination with a wavelength of  $\lambda = 0.532\text{ }\mu\text{m}$ , object-mask distance of  $d_1 = 1\text{ mm}$  and mask-sensor distance of  $d_2 = 8\text{ mm}$ . The diffraction patterns are generated with computational pixel size of  $\Delta_c = 0.4375\text{ }\mu\text{m}$  ( $r_s = 8$ ) and down-sampled by averaging every  $8 \times 8$  pixels to model a sensor with pixel size of  $\Delta_s = 3.5\text{ }\mu\text{m}$ . The resulted patterns are used as input in the modified SR-SPAR with super-resolution factor of  $r_s = 4$  and the phase reconstructions are shown in Fig. 1. The enhanced method provides a much clearer reconstruction with more than 2-times smaller Relative Root-Mean Square error (RRMSE) [2] than the previous method, while being able to resolve the smallest group elements with small etch-depth. The cross-section of the phase values demonstrates the super-resolution of the reconstructions. Additionally, the enhanced method is able to resolve the 6th element from group 9 with a line thickness of  $0.875\text{ }\mu\text{m}$ , which is 4-times smaller than the used sensor pixel size.

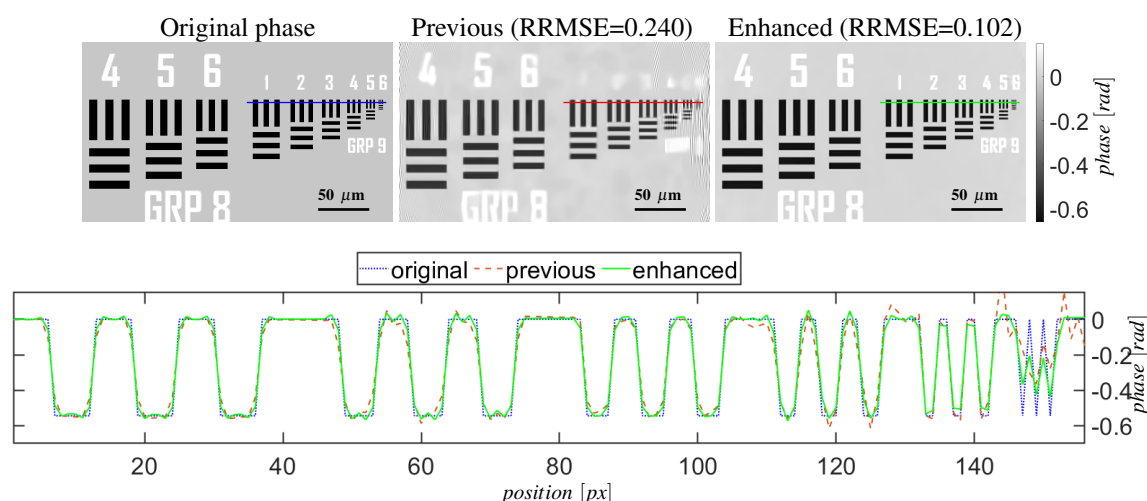


Fig. 1. Original phase image and reconstructions with previous [1] and enhanced methods. The resolutions are shown by cross-sections of the phase values.

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### References

1. P. Kocsis, I. Shevkunov, V. Katkovnik, and K. Egiazarian, "Single exposure lensless subpixel phase imaging: optical system design, modelling, and experimental study," *Opt. express* **28**, 4625–4637 (2020).
2. V. Katkovnik, I. Shevkunov, N. V. Petrov, and K. Egiazarian, "Computational super-resolution phase retrieval from multiple phase-coded diffraction patterns: simulation study and experiments," *Optica* **4**, 786–794 (2017).
3. Y. Mäkinen, L. Azzari, and A. Foi, "Collaborative filtering of correlated noise: Exact transform-domain variance for improved shrinkage and patch matching," *IEEE Transactions on Image Process.* **29**, 8339–8354 (2020).
4. K. Turkowski, "Filters for common resampling tasks," *Graph. Gems I* pp. 147–165 (1990).