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MATHEMATICAL COMMUNICATION AS A MEANING-MAKING PROCESS

A study of Preservice Class Teachers' Perspective

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ABSTRACT

Eunji Kim: Mathematical Communication as a Meaning-making Process: A study of Preservice Class Teachers' Perspective
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The importance of communication in mathematics has increased since empirical evidence pointing out the relationship between mathematical communication and meaningful learning emerged in the 2000s. However, the classroom still is a quiet and independent environment without a clear explanation, and previous studies mainly focused on the knowledge for teaching. Therefore, this study aimed to explore the perceptions of Preservice Class Teachers' (PCT) on communication as a meaning-making process in mathematics teaching and learning based on their past schooling experience and new experience in the Teacher Training Programme (TTP). Among PCTs in the University of Tampere and Kokkola University Consortium Cyrenius, 35 participants responded to the online survey from December 2021 to January 2022. The survey consisted of 41 Likert-scale items based on the previous instruments and 4 Open-ended items. Questionnaire analysis results using SPSS and coding processes were integrated into joint displays used in convergent mixed methods.

There are two major findings. First, most participants showed constructivist views on mathematical beliefs and had positive learning experiences in TTP. Nevertheless, they tended to focus on different components of communication for future teaching based on their previous experience. Second, PCTs regarded the meaning of evaluation in communication as a score and recognized the relationship between communication and academic achievement was weak. Therefore, this study supports existing studies showing positive mathematics experience in TTP can strengthen their constructivist mathematical beliefs. In addition, it suggests that creating environments for mathematical communication and assessing students' learning process is an essential part of the TTP.

Keywords: Mathematical communication, preservice class teacher, languaging

1 INTRODUCTION

Education always plays a crucial role in enhancing knowledge, skills, attitudes, and values to adjust students to this unknown society for a sustainable future (OECD, 2018). Among many subjects of education, mathematics education aims to improve logical thinking, mathematical understanding, and reasoning skills which are directly connected to solving the problem one could face in the future (Krzywacki et al., 2016; OECD, n.d.). Specifically, those skills need a mathematical communication process that is expressing one's thoughts in oral or written types (Hirschfeld-Cotton, 2008; Joutsenlahti & Kulju, 2017; Lee, 2015). Generally, communication used verbal or nonverbal means of sending and receiving messages, while mathematical communication requires logical and mathematical thinking using the multimodal languaging mode (Joutsenlahti & Kulju, 2017; Nordquist, 2021; Rohid et al., 2019). In addition, many researchers define mathematical communication as a tool for truly understanding mathematical concepts and meaningful learning (Carley, 2011; Hirschfeld-Cotton, 2008; Hoyles, 1985; Kaya & Aydin, 2016; Krzywacki et al., 2016; Lee, 2015; Teledahl, 2017).

However, the research on encouraging mathematical communication in the classroom is still needing further attention (Kaya & Aydin, 2016). American research by Lee (2015) pointed out that *communication* in mathematics has often been disregarded. By contrast, solving mathematics problems individually and silently has been considered effective for students. Moreover, mathematics learning in Finland has been also conducted quietly and independently with memorizing concepts and algorithms (Hirschfeld-Cotton, 2008; Joutsenlahti & Kulju, 2017). However, Viro et al. (2020) emphasise that communication as a new skill across Finnish school subjects should be prepared for students to live better lives. In addition, Finnish National Core Curriculum for Basic Education 2014 (FNCC, 2014) focuses on the support to develop pupils' skills of communication, social interaction, and cooperation. Therefore, without meaningful mathematical communication skills, students will not recognize their true understanding, and teachers are difficult to follow how students have thoughts through their solution to a mathematical problem (Carley, 2011; Joutsenlahti & Kulju, 2017).

In response to these problems, teachers are mostly required to become experts in how to promote students' mathematical communication skills rather than acquiring and memorizing facts or algorithms (Hirschfeld-Cotton, 2008). For example, previous research findings have shown that teachers are responsible for implementing various communication strategies effectively (Carley, 2011; Hirschfeld-Cotton, 2008; Kaya & Aydin, 2016; Krzywacki et al., 2016). On the other hand, according to a research review by Erath et al. (2021), language studies aimed at promoting students' learning in mathematics over the past 40 years have mainly been qualitative studies. The purpose of thesis studies was to develop teachers' Pedagogical Content Knowledge (PCK) of mathematics or instruction language through Teacher Education Programme (TEP). In addition, within other studies, the focus lay on In-services Teachers' perspectives on communication through personal experience or programs for improving professionalism (Brendefur & Frykholm, 2000; Sfard & Kieran, 2001; Steinbring, 2000). Therefore, they pointed out that research regarding how teachers change their teaching practices is necessary, for example, by asking how current teacher professional development programmes affect teachers' mathematics teaching.

Consequently, based on the research gap between the increasing importance of communication in mathematics teaching and previous research areas, this study investigated Preservice Class Teachers' (PCT) views on communication as a meaning-making process in mathematics teaching and learning, addressing specifically their previous and new learning experiences using the mixed methods.

2 CONCEPT OF MATHEMATICAL COMMUNICATION

This chapter addresses the three themes connected to the study. Firstly, the meaning of mathematical communication used in this study will be defined. Secondly, a general overview of the importance of mathematical communication will be illustrated followed by mathematics issues, teacher training programme, and teacher's learning autonomy in Finland. Thirdly, the main point of the role of teachers, students, a group of students in building meaningful communication in mathematics will be presented.

2.1 Meaning of mathematical communication

Literature on communication and mathematics has highlighted two perspectives. The first perspective is to learn to communicate to improve mathematics learning, and the second perspective is to learn mathematics to communicate (Mathematical Association of America, n.d.). For the first purpose, learning to communicate mathematically can be effective for written or oral expression. And for the second one, communication is regarded as a tool to reduce mathematical anxiety or assess their mathematical understanding. Eventually, the common reason for using communication in mathematics is that it is a process to make one's meaning based on learners' mathematical understanding. Therefore, the term mathematical communication in this paper represents the role of communication as a meaning-making process.

2.2 Importance of mathematical communication

According to the ProQuest database search (2021, January 21), there were only 217 journals, books, magazines, and dissertations addressing the topic of "mathematical communication" in the 1980s. However, we can see a continuous increase in the number of studies addressing the same topic in each decade (e.g., in the 1990s we had 2,600, 8341 in the 2000s, and 19688 in the 2010s). Lee (2015) explains that this remarkable change is due to the emphasis on mathematical communication in the field of education as an important tool to demonstrate students' understanding of mathematical thinking and mathematical concepts rather than their mathematical performance. Therefore, in many studies, mathematical communication is regarded as a tool to develop students' mathematical thinking, problem-solving skills, and mathematical understanding (Baxter et al., 2005; Hoyles, 1985; Kaya & Aydin, 2016; Kostos & Shin, 2010; Lee, 2015). On the other hand, general communication has three parts: the sender, the message, and the receiver. Messages can be presented in spoken or verbal type, non-verbal type, written type, and visualizations including graphs and charts. Regardless of similarity, however, mathematical communication is distinguished from general communication because it has a specific purpose, meaning, and effect and uses mathematical language. Kaya and Aydin (2016) describe the specific purpose of mathematical communication as follows:

> Mathematical communication is defined as planned interaction in a classroom setting, which includes strategies such as questioning, discussions, and group activities. The purpose of mathematical communication is to encourage students to express, share and reflect on their ideas. (p. 620)

This definition means that mathematical communication requires a classroom environment designed based on the plan and for students to express their ideas by the teacher. Therefore, the meaning of mathematical communication and its effects exist for both teachers and students. First, teachers can identify clues about students' true mathematical understanding, mathematical errors, or misunderstandings (Mooney et al., 2009). Moreover, it provides an opportunity for teachers to assess students' knowledge (Teledahl, 2017). Second, students also can get an opportunity to clarify their understanding or incorporate scattered mathematical ideas through other pupils' explanations (Hirschfeld-Cotton, 2008). For instance, expressing their mathematical thoughts in drawing, speaking, and writing enables students to enhance logical thinking skills (Lee, 2015). Third, writing strategies including pictures can improve the mathematical performance of students with poor academic performance or special needs (Baxter et al., 2005;

Kim & Jeon, 2019). Overall, rather than paying attention to whether they got the right answer, students find mathematics more enjoyable when participating in the communication process (Hirschfeld-Cotton, 2008).

2.2.1 Mathematical communication as meaning-making

When it comes to mathematical communication as a meaning-making process, it has been investigated as slightly separate ways as well as the importance of mathematical communication. Early studies about the mathematics of communication formulated theories based on the nature of language or emphasised clear teaching language and instruction (Baroody & Hume, 1991; Weaver, 1949). Up to date, several studies have shown the need for TPACK (Technological Pedagogical Content Knowledge) learning for teachers. For example, teachers can use new software in mathematics classes to teach meaningful communication (Chai et al, 2011; Mendoza & Mendoza, 2018; Patahuddin, 2013; Robinson et al, 2017). Moreover, Joutsenlahti and Kulju (2017) suggest that digital technology is the fourth mode in a multimodal language model. It can be connected to the symbolic mathematical language for the young generation, which is called the tactile language (Joutsenlahti & Rättyä, 2015). Consequently, previous studies have recognized communication in mathematics classes as part of learning and support for meaningful learning.

2.2.2 Mathematics learning issues in Finland

Despite the growing importance of mathematical communication, Finnish News recently reported that students' interest in mathematics is declining. Besides, Finland's mathematical achievement in PISA remained at the top during the two-decade, its performance has been declining in recent years, including the gender gap in mathematical performance (Ahonen, 2021). For example, according to an article by Paula interviewing Professor Erno of Turku University (2018, August 16), he was concerned that Finnish students' interest and skills in mathematics are gradually decreasing. She quoted Professor Erno as saying that this is because many people still consider mathematics to be a special thing used only in mathematics classes and assignments. Laura’s article (2015, September 16) has already addressed this issue, mentioning that it is quite difficult to study mathematics in many primary schools, and suggests changing the attitude towards mathematics into one language that can support students.

Before understanding these issues, it is necessary to fundamentally examine how mathematics education has developed in Finland and how teacher education has been conducted. According to the paper by Krzywacki et al. (2016), Finnish mathematical history began in the late 1980s, there was a voluntary informal committee. In the 1990s, mathematics teaching was centred on mathematics experts and teachers to discuss the future of mathematics education and the need for reform. Then, in 1995, the LUMA-project (LU means science and MA in mathematics) was launched as a national project. The main purpose of the project was to emphasise the significance of learning while providing a special interest in mathematics and science education. Simultaneously, the project aimed to strengthen students' knowledge and skills in both fields. From 2014 to 2019, in order to promote the teaching of mathematics and natural sciences in basic education, the Ministry of Education and Culture led a national development project. It was a project of the Luma Suomi network for educating preservice teachers and became an important project.

Regarding primary school teacher education, it has been required to obtain a master’s degree to become a teacher since 1979 (Saloviita & Tolvanen, 2017). The master's degree includes a total of 300 credits, based on the European Credit Transfer System (ECTS), which is used to compare the academic performance of university students across the European Union (EU). For instance, a master's degree includes a minimum of 60 credits in educational studies, and 60 credits in multidisciplinary studies including research on school subjects such as language, mathematics, art, and crafts. In addition, there are eight Finnish teacher preparation universities in Finland, the university course is divided into two parts: lecture practice conducted in university-related schools, and theory-oriented research conducted on university premises. After graduation, it is assumed that they have a deep understanding of mathematics as a teacher by forming high independence and autonomous responsibility (Krzywacki et al., 2016). Specifically, Finnish primary school teachers teach mathematics classes in general schools at almost two-thirds or more. Therefore, at the primary level, the responsibility of teachers is to implement communication strategies effectively for promoting mathematical thinking and to encourage mathematical communication by creating a classroom environment (Carley, 2011; Hirschfeld-Cotton, 2008).

As I have noted the issues in the first paragraph, Erno's interview in Paula's article (2018, August 16) suggests strengthening in-service teacher education, and special needs for flexible mathematics including mathematical thinking skills and problem-solving skills. As part of this solution, the Finnish government started new in-service teachers' programs for improving teachers' professionalism. LuMo Homegroup is also one of the in-service class teacher programmes at the University of Jyväskylä. It focuses on teaching mathematics and natural sciences, and its goal is to educate teachers to inspire students to learn mathematics and bring out the enchantment of the subject. On the other hand, LUMATIKKA (LUMA and matikka) is a continuing education programme in mathematics teaching funded by the National Board of Education. The project phase of the program is from 2018 to 2022, and the target group of education is all levels of teachers related to mathematics education. As a result, a lot of research and support has been focused on educating in-service teachers, which corresponds to the research of Earth et al. (2021).

2.3 Building communication as meaning-making in mathematics

The purpose of the research by Kaya and Aydin (2016) was to explore in-service teachers' perspectives and live experiences on using mathematical communication in their classes. It was a qualitative and phenomenological approach to gain deep insight into the essence of communication in mathematics. Based on the results, they categorized five topics throughout the interviews of nine participant teachers. The first topic was the definition of mathematical communication, and the answers were the usage of mathematics in a real-life context, mathematical language, and understanding mathematics. The second topic was strategies to enhance mathematical communication. They usually provided students with real-life examples or used activities and games, such as question-answer and peer learning techniques. The third topic was related to the purposes and benefits of mathematical communication. In the case of the teachers, it was possible to monitor the learning process of the students and provide a positive learning environment. The fourth topic was another impact of mathematical communication on mathematical thinking, problem-solving, reasoning, and critical thinking. The last topic was the limitations of implementing mathematical communication in mathematics teaching. For instance, the teachers as participants reported that they needed more time for communication in the mathematics classroom regardless of the curriculum. It also suggested that teachers' mathematical beliefs and positive attitudes toward teaching and learning mathematics remain important.

2.3.1 Teacher’s role

Scholars have defined the important role of teachers to improve communication as a meaning-making process in mathematics classrooms. (Bratina & Lipkin, 2003; Kim & Jeon, 2019; Vale & Barbosa, 2017). Vale and Barbosa (2017) define the role of teachers as constructing practices that lead students to communicate, reason mathematically, and use various visual forms of representation. Similarly, Kim and Jeon (2019) emphasised that teachers should present problems to explore together, create an environment for discussion, and continuously provide positive feedback so that students can actively participate in classes and feel that they are receiving attention. In addition, Kaya and Aydin (2016) recommend strategies for active interaction in mathematics class including questioning, discussions, and group activities. Such strategies motivate students to learn vigorously, and an opportunity to experience that doing mathematics or mathematical thinking is enjoyable (Hirschfeld-Cotton, 2008; McKenney, 2020). In brief, by creating open-minded classroom settings that encourage communication in mathematics, students can build their mathematical meanings using self-system (Hirschfeld-Cotton, 2008).

Consequently, although the meaning-making process occurs individually in that self-system (Malmivuori, 2001), there are also previous studies about teachers’ role and responsibility to construct a communicable classrooms environment (Bratina & Lipkin, 2003; Carley, 2011; Kaya & Aydin, 2016; Kim & Jeon, 2019; Krzywacki et al., 2016; Vale & Barbosa, 2017). Bratina and Lipkin (2003) also insist that all educators should teach communication skills such as reading, writing, speaking, and listening to students. They emphasise using specific language to develop students' vocabularies through a variety of experiences in different contexts. On the other hand, Finnish education requires teachers to have a high understanding skill of teaching and learning mathematics (Krzywacki et al., 2016). Therefore, the most important goal of the teacher's role is how to create communication as a meaning-making process for students. Furthermore, teachers should understand students' language of mathematics and help students express their mathematical thinking (Joutsenlahti & Kulju, 2017).

2.3.2 Role of students, and student groups

Students also need a process to connect previous experience with mathematics to new mathematical learning, called self-system (see Malmivuori, 2001, for more). Teachers can create positive classroom settings so that students can build an understanding of mathematical concepts by doing, talking, and discussing with other peers. On the other hand, making mathematical meaning in a group of students can be expressed as an "AHA" point that enriches the thinking process (Lee, 2015; McKenney, 2020). Lee (2015) described that when a teacher asks students about what they found and how they solved the problem, they responded commonly following that: "Oh, I just had it in my head" or "That made sense to me in my head." Albert (2000) also explains that the group activities include collaborative discourse and interaction, simultaneously, each pupil process individual thought such as self-application, self-regulation, and self-scaffolding.

Comprehensively, teachers play a vital role in communication as meaning-making in mathematics teaching that supports student-centred learning rather than a chairperson. Therefore, teachers should effort to understand the language of mathematics, as well as promote students to express their mathematical thinking in enjoying circumstances. In addition, students should also participate in doing, speaking, and discussing mathematics using a variety of methods that lead to an understanding of mathematics concepts. As a group of students, they should listen to each other and find meaningful learning points.

3 BELIEFS IN MATHEMATICS TEACHING AND LEARNING

This chapter explores teachers' beliefs toward teaching and learning mathematics, including the relationship among experience, beliefs, and behaviours in mathematics. This review is followed by a description of the study and research questions.

3.1 Teachers' mathematical beliefs

Without teachers' emphatically and enthusiastically acknowledgment that communication is essential to truly understand mathematical concepts, students hardly follow the well-planned lesson (Bratina & Lipkin, 2003). For decades, teachers' knowledge has been the main topic in educational research (Skott et al., 2018), however, it has been proposed that it is not able to distinguish beliefs from knowledge (Pehkonen, 1998). Furinghetti (1998) indicated that beliefs should be included as a component of knowledge regarding personal knowledge. Pehkonen (1998) explained the terms of knowledge and beliefs following this:

> Teachers' knowledge of mathematics teaching includes their knowledge of mathematics and pedagogy, as well as an understanding of pupils' cognitions. Always underlying teachers' knowledge is their beliefs. (p. 52)

Besides, Malmivuori (2001) categorized mathematical beliefs into the following four items: (1) Beliefs about mathematics, (2) Beliefs about self with mathematics, (3) Beliefs about mathematics learning and teaching, and (4) Beliefs about the social context or environments. Specifically, the third item includes beliefs as a means for mathematics learning and teaching, or problem solving, which is also similar to the first, and the second category. Eventually, beliefs about learning and teaching are regarded as the main categories of mathematical beliefs.

3.1.1 Developing teachers' beliefs

With the perception beliefs about learning and teaching mathematics, many researchers have indicated necessary to change preservice teachers' beliefs about mathematics more positively way (White et al., 2006). Hannula et al. (2005) investigated 269 Finnish preservice teachers' views on mathematics and their various belief profiles. Furthermore, Oksanen et al. (2015) explored the change in Finnish teachers' mathematical beliefs within four decades, the years from 1970 to 2010. The main point of them was to pay attention to learning mathematics as a constructive process. The process has a central concept of teaching that helps students build their meaning in learning.

As a result, there are two fundamental perspectives on beliefs about learning and teaching mathematics. One is a constructivist view, and the other is a transmissive view as in Table 1.

TABLE 1. Overview on the subscales of instrument measuring beliefs (Schmeisser et al., 2013)

Content area	Theoretical Learning Foundation	
	Transmissive	Constructivist
Nature of knowledge	Mathematics as a toolbox	Mathematics as a process
Learning and Teaching of Mathematics	Clarity of solution procedure Preceptive learning from examples and demonstrations Automatization of technical procedures	Independent and insightful discursive learning Confidence in the mathematical independence of students

According to research by Schmeisser et al. (2013), in the constructivist perspective, the learning and teaching processes are student-oriented, and teachers have the role of a learning environment creator to help students build knowledge. Therefore, learning mathematics is considered a process in this view. In contrast, the transmissive perspectives make it important for learners to learn knowledge by demonstration, repetition, and integration. Mathematical knowledge is an objective fixed set of facts and procedures in the transmissive view.

On the other hand, previous studies emphasised that teachers' beliefs toward teaching and learning mathematics influenced students' beliefs as well (Barcelos, 2003; Lindgren, 1998; Voss et al., 2013). For this reason, students learn different mathematical comprehension methods from different mathematics teachers, and they learn mathematics in diverse ways (Pehkonen, 1998). Furthermore, when the students teach someone with mathematics again in the future, they will teach with previous learning memories. For example, if there is a teacher who thinks mathematics learning works best in calculation tasks by doing, her/his teaching will focus on doing as many tasks as possible (Pehkonen, 1998). Lindgren (1998) called this process a "teaching-learning-teaching" cycle and explained that teachers' beliefs are connected to their teaching practice which influences the pupils' views of mathematical learning.

After extensive research into the importance of teacher beliefs, many studies have been interested in whether they can positively change teachers' beliefs. Schmeisser et al. (2013) mentioned that positive changes in theoretical learning beliefs might mean the growth of constructivist beliefs or a decrease in transmissive beliefs. That is consistent with the research of Alfaro Víquez and Joutsenlahti (2021) that found the nature of mathematics is dynamic and its learning necessarily follows a constructivist direction. In addition, Skott et al. (2018) presume that beliefs can be changed with considerable new experiences. However, it takes a long time. Therefore, Pehkonen (1998) insisted that beliefs have a vital role in change teaching, that is why we should focus on providing opportunities for change of beliefs instead of the question of change possibility.

3.1.2 Connection between experience and beliefs

Based on the Mathematics Learning Model (Malmivuori, 2001), mathematical beliefs and knowledge, effective schema, and mathematical behaviour patterns must interact with the self-system for prior experience in mathematics to be linked to new mathematical learning situations. Hourigan et al. (2016) exclaim that after preservice teachers experienced mathematics education programs, beliefs toward mathematics changed more positively. Especially in the field of mathematics, teachers' beliefs significantly influence both teaching implementation and students' beliefs on mathematics (Hourigan et al., 2016; Oksanen et al., 2015).

In the case of the past schooling experience, the ideas and beliefs formed by teachers in their mathematical experiences influence their teaching behaviour (Fernandes, 1995). For example, the results of a survey conducted by Jong and Hodges (2013) found that more than 80% of participants recognized their previous schooling experiences and mathematics course experiences as having a significant impact on expected teaching practices. Therefore, as new experiences of Teacher Training Programme, the purpose of educating Finnish teachers is to develop autonomous teachers who have obvious aims and a vision of long-term goals in their teaching (Tirri & Kuusisto, 2016). When they teach mathematics, finding the meaning of teaching the subject and reflecting on their teaching interests enable them to imagine future teaching with professional confidence. Teachers' stable perception of this teaching purpose is consistent with the teaching-learning-learning line emphasized by the Finnish National Curriculum (Tirri & Kuusisto, 2016), which has a significant impact on the formation of meaningful goals for student learning. However, Tirri and Kuusisto (2016) point out that mathematics and science preservice teachers lack confidence in vision and activities related to the purpose of teaching the subject. As a result, formative beliefs are not easily changed over the teacher training courses (Philippou & Christou, 1998), notwithstanding, the new experiences would help preservice teachers' attitudes about mathematics teaching support students' learning (White et al., 2013).

Consequently, enhancing their confidence in mathematics teaching is necessary throughout meaningfully connecting previous schooling experience in mathematics and their beliefs to new experiences and knowledge. Specifically, White et al. (2006) classified the core of preservice teachers' perspectives as follows: the belief of the difficulty of mathematics; and one's liking of mathematics. These categories were used as variables to examine past mathematics learning experiences in the survey.

4 THE PRESENT STUDY

Based on the research gaps presented, this study was established past schooling learning experience in mathematics and new experience in the teacher training program as variables. This chapter explains the research questions and two theories to be considered as the main theoretical framework.

4.1 Research questions

Research questions include two major ideas. The first idea is the relationship among past schooling learning experiences, new experiences, and mathematical beliefs. The second idea is which components are assumed as meaning-making in mathematics teaching. Figure 1 shows the relationship between the research questions.

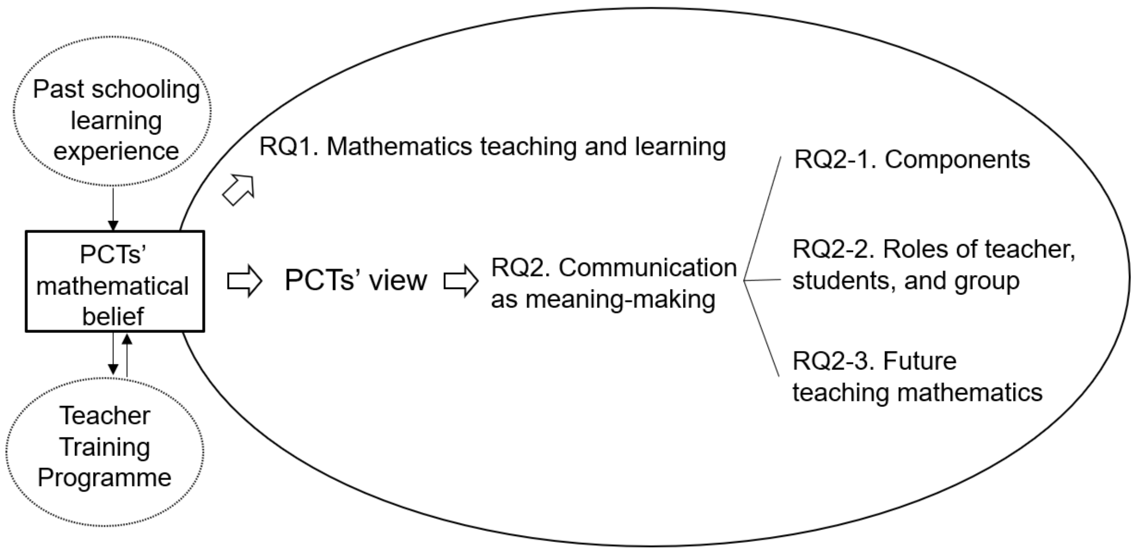

FIGURE 1. Research question 1 is about Preservice Class Teachers' beliefs toward teaching and learning mathematics. Research questions from 2 to 4 are consist of Preservice Class Teachers' view of communication as a meaning-making process in mathematics.

The study seeks answers to two main research questions and two sub-questions as follows:

1. What kinds of beliefs toward teaching and learning mathematics do Preservice Class Teachers have based on their experience?
2. What kinds of roles do Preservice Class Teachers view communication plays in meaning-making in mathematics?

   2-1. What are the components of building communication as meaning-making in mathematics teaching?

   2-2. What are the roles of teachers, students, and student groups for communication as meaning-making in mathematics teaching?

      1) What is the teacher's role for communication as meaning-making?

      2) What is the student's role for communication as meaning-making?

      3) What is the role of student groups for communication as meaning-making?

   2-3. How will Preservice Class Teachers apply communication as meaning-making in their future teaching mathematics?

To explore these research questions, Mathematics Learning Model by Malmivuori (2001) which assumes that PCTs' past mathematics experiences connect new mathematics situations through the self-system was adapted. On the other hand, Multimodal Languaging Model by Joutsenlahti and Kulju (2017) was used for taking into consideration mathematical communication as a meaning-making process. This model presents a theoretical basis for the definition and implementation of mathematical communication. Furthermore, more research (Hannula, 2006; Joutsenlahti & Rättyä, 2015; Lehtonen & Joutsenlahti, 2017; Skott et al., 2018) grounded these two theories was observed in the following chapter.

5 THEORETICAL FRAMEWORK

Since this study focuses on communication as meaning-making in mathematics teaching and learning, it is important to explore the relevant theoretical framework. In the first place, Mathematics Learning Model (Malmivuori, 2001) can explain the links between previous learning mathematics and new learning situations. In the second place, Multimodal Languaging Model (Joutsenlahti & Kulju, 2017) was explored for the meaning of communication in mathematics teaching and learning.

5.1 Mathematics Learning Model

A considerable amount of literature has been published about mathematical learning models (Bereiter, 1990; Fennema & Peterson, 1985; Goldin, 1992). Malmivuori (2001) analyzed a variety of models or theoretical frameworks for mathematics learning and concluded that, in a general perspective, pupils' mathematical processes are related to their unique learning or performance situations; the so-called self-system. The "self-system" links personal previous experience to new mathematics learning situations under mathematical context and socio-cultural context.

To activate and develop pupils' self-system process, it is necessary to explore their past mathematics experiences, personality, and mathematics learning situations as interpreted by them. Mathematical beliefs play an important intermediate role under the active process of mathematical learning. This view of mathematics learning in Malmivuori (2001) is illustrated in Figure 2.

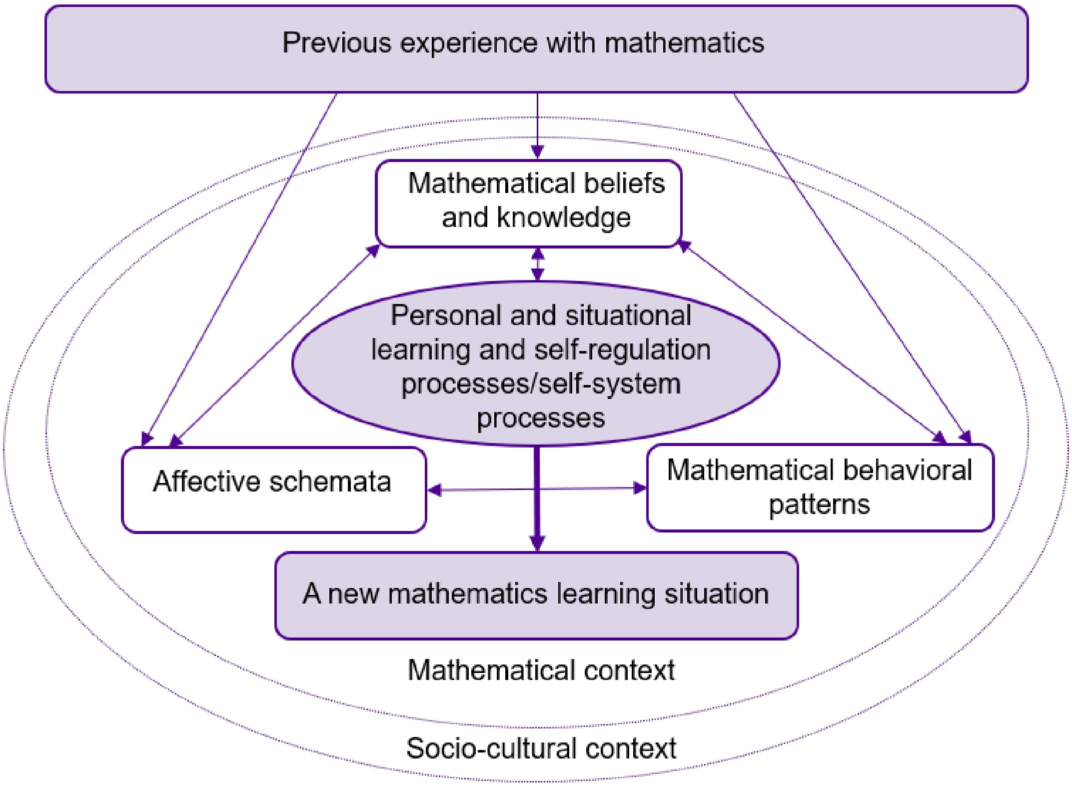

FIGURE 2. A view of the aspects and development of pupils' important mathematics learning components or self-systems within various contexts (Malmivuori, 2001, p31)

The academic literature on beliefs has revealed that there is no specific agreement about a definition of beliefs (Skott et al., 2018). Even mathematical beliefs are sometimes considered as attitudes (Malmivuori, 2001), however, Skott et al. (2018) indicate that the original research of beliefs in mathematics is kinds of expectation as a faith of how to teach and learn mathematics. Hannula (2006) defines that beliefs are personal knowledge, and for the individual, the beliefs are always true. An essential part of beliefs is one's self-image or self-concept. While an emotional attitude to mathematics is regarded as a rather ambiguous construct, mathematical beliefs are accepted as relatively stable concepts.

Furthermore, Skott et al. (2018) studied teacher beliefs and their impact on teaching instruction. Beliefs are as stable as attitudes and take time to develop, but they can be changed by new experiences (Skott et al., 2018). Ball (1990) also emphasised the importance of supporting preservice teachers including class teachers and subject teachers to develop mathematical skills into constructive belief systems, although it is not necessary to completely change their beliefs about mathematics teaching and learning. Therefore, in this study, Preservice Class Teachers' beliefs toward mathematics teaching and learning will be used as a particularly important starting point for preparing their teaching in the future.

5.2 Multimodal Languaging Model

Languaging (Kielentäminen in Finnish) has been developed since the 1990s when requiring several semiotic systems (Joutsenlahti & Kulju, 2017). Semiotics is the study of using systems of words, images, symbols, and actions to create meaning (Lemke, 1998). In semiotics study, scientific education is also described as a way of creating meaning for the goals of learning, nature, and technological phenomenon. In addition, the mathematics' language is useful to express quantitative relationships in scientific texts or talks. It works supporting meaning-making in spoken language. The language of mathematics is also relevant as a representation tool for expressing mathematical thinking itself. Joutsenlahti and Kulju (2017) illustrate that languaging in mathematics is used in sharing mathematical ideas and thoughts using modes in oral or written types. Albert (2000) further defined this process as follows:

> When students synthesize information, they organize more than one idea into a single concept. This process involves working with individual pieces of information and rearranging them in such a way as to construct a new pattern or structure. A way to accomplish this is through language in the form of conversations or writing. (p. 109)

Specifically, the multimodal languaging model is utilized in making meaning by using intentionally various modes, such as a natural language, a symbol language, and a pictorial language (Joutsenlahti & Kulju, 2017). First, the expression of mathematical thinking using the mother tongue is defined as a natural language, which is a fundamental factor of speaking forms. Second, when it comes to written types, it can be categorized into mathematical symbol language and pictorial language, which is an important mode at the basic education level. For example, research by Joutsenlahti and Kulju (2017) found that most students who participated in the group using only verbal expression showed one level of understanding when expressing division. By comparison, the group that expressed their understanding in speaking, writing, and drawing, showed a variety of comprehension understanding in division expressions. Albert (2000) also suggests writing to reflect students' thinking as a mode, and a drawing type has a positive effect on communication of students who are academically low-achieving or need special support. In Joutsenlahti and Rättyä's (2015) research, the tactile language such as hands-on materials have been added as the fourth to the previous modes. Therefore, they argued that the expression of various mathematical ideas can reinforce students' mathematical conceptual learning and support teachers' mathematics teaching.

On the other hand, those language modes relate to multiliteracies approaches to gain a profound understanding of the mathematical concepts. The Finnish core curriculum defined multiliteracies as the competence to interpret, produce and value judgments across different texts helping students to understand the various ways of cultural communication and form their identities (FNCC, 2014). In addition, it describes that the multiliteracy for Grades 1-2 is the following statement:

> Texts mean information presented by systems of verbal, visual, auditive, numeric, and kinaesthetic symbols and their combinations. (p. 137).

Therefore, the pupils are willing to be encouraged to use and create various texts, and to represent themselves through them. In the case of the instructions for Grades 3-6, the available information is expanded compared to the lower grades. For instance, they include this statement, "oral, audio-visual, printed, digital courses, search engines, and library services." (p. 165)

Consequently, the four types in the multimodal language model enhance students to express their ideas and problem-solving processes in mathematics, leading to the process of making meaning on their own. In addition, Finnish National Core Curriculum for Basic Education 2014 (FNCC, 2014) specified that using these kinds of modes in teaching and learning mathematics is aimed at understanding mathematical concepts and structures, which is one of the very crucial proficiencies in mathematics (Lehtonen & Joutsenlahti, 2017). Hence, it is important to explore Preservice Class Teachers' view of communication as a meaning-making process by the multimodal language model.

6 RESEARCH DESIGN

This study designed research to explore PCTs' views on mathematical communication using a mixed-method. Specifically, a convergent sequential model was used to sequentially merge quantitative data and qualitative data. This chapter address the research process with mixed methodology.

6.1 Mixed methodology

Mixed-method studies combine or incorporate quantitative and qualitative approaches as components of the entire study (Ponce & Pagán-Maldonado, 2015). According to Creswell and Creswell (2018), the field of mixing methods began in the late 1980s. It combines two different research methods as follows: Qualitative and quantitative research through researcher insight. Qualitative data includes open responses, while quantitative data generally uses closed responses with questionnaires or psychological tools. The origin of this research field begins with Campbell and Fisk's introduction (1959; as cited in Creswell & Creswell, 2018) to quantitative research incorporating the results of observations and interviews traditionally. Since then, mixed research has attracted attention as an alternative to supplementing the limitations of the two different research methods. And in the early 1990s, specific design methods that were systematically integrated were studied. Procedures for mixed methods are developed in three types of design as follows: Convergent mixed methods, Explanatory sequential mixed methods, and Exploratory sequential mixed methods. Eventually, a convergent model is suitable for research that analyse quantitative and qualitative data to derive research findings. Thus, using quantitative and qualitative approaches to examine the views of Preservice Class Teachers on communication as a meaning-making process. Appendix 1 shows the research design of the mixed-method process for the study.

6.2 Participants

According to Saloviita and Tolvanen (2017), Finland has eight universities aimed at high educational qualifications autonomy, there is no difference in educational background or regional background between universities. Among them, the University of Tampere, and Kokkola University Consortium Chydenius (University of Jyväskylä) have professors who are experts in mathematical communication. They teach PCTs with various mathematical expressions methods and students' language in the compulsory mathematical curriculum. Therefore, convenience sampling was used for collecting participants which is a part of the Teacher Training Program of two universities in Finland.

6.3 Data collection

The online survey included open-ended items and Likert-scale items, which required all participants to complete from December 2021 to January 2022. Written responses to open-ended questions were aimed at clarifying what participants think that mathematical communication means, how they see mathematical communication could be used to teach their future students as a meaningful process, and how PCTs' experience influenced their views. The target group was a total of 120 Preservice Class Teachers (PCT), and 50 PCTs answered the survey by Qualtrics program that was sent to them by mail, posting on the internet site, and introducing at the seminar. The original number of PCTs was 50, however, 15 of them had low percentages of progress (0~8%), so a list-wise deletion was applied, leaving a final sample of 35. They were studying at universities less than 1 year (34%), 1 - 2 years (40%), 3 - 4 years (9%), and more than 4 years (17%). And half of the participants (49%) had no experience of the previous teaching, while 18 participants (51%) had pre-teaching experience. All participants had similar mathematics courses at university as a lecture, in addition, numerous participants observed classrooms (60%), practiced teaching at school (66%), and 1 participant had an intensive course in mathematics. Table 2 showed the basic information of participants

TABLE 2. Participant distribution by basic information

Variable	Factor	Frequency (N)	Percent (%)
Gender	Female	28	80.0
	Male	6	17.1
	Prefer not to say	1	2.9
University	Tampere University	21	60.0
	Jyväskylä University	14	40.0
Studying years at the Universities	Less than 1 year	12	34.3
	1 - 2 years	14	40.0
	3 - 4 years	3	8.6
	More than 4 years	6	17.1
Pre-teaching experience	None	17	48.6
	Less than 5 years	14	40.0
	5 – 10 years	2	5.7
	More than 10 years	2	5.7
Experience at Universities	Lecture	27	77.1
	Class Observation	19	60.0
	Teaching Practice	23	65.7
	Intensive course	1	2.9
	None	3	8.6

Note. Experiences at universities as variables were selected by multiple choice.

Furthermore, participants were divided into 4 groups based on their previous experience in learning mathematics. There were 4 items regarding this issue and all responses were analysed by groups for deep exploring. Figure 3 presented the characteristics of the divided group based on the survey results.

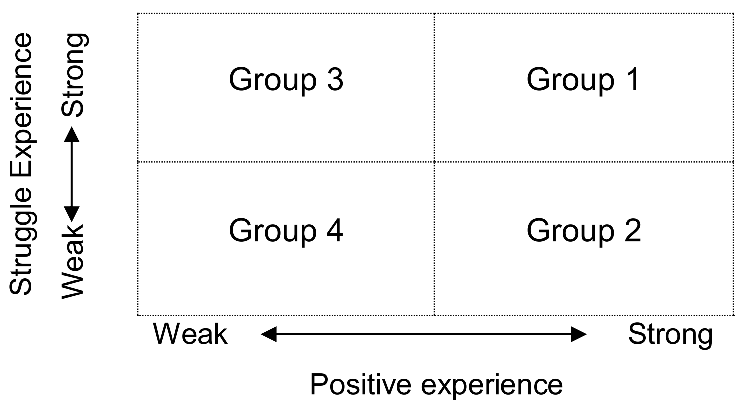

FIGURE 3. Participants' (N = 35) classifications for the survey based on their previous schooling experience in mathematics.

The statement on positive mathematics experience begins with “I have several positive experiences with mathematics during basic education.” and “I have several positive experiences with mathematics during general upper secondary school.”. In addition, the statement of struggling with mathematics means that there were some difficulties in solving mathematics problems. For example, Group1 had a strong positive experience but had some difficulties experience in mathematics learning, and Group 2 had a strong positive experience and had little difficulties. Otherwise, Group 3 had weak positive and strong struggle experience, also Group 4 had weak positive and weak struggle experience in mathematics learning.

6.3.1 Questionnaire

The questionnaire consists of four areas according to the research problem, each of which sets a subcategory. Part 1 is a category of items in teaching and learning mathematics, including (A) Mathematics teaching and learning, (B) Past experiences and beliefs. Part 2 is a category of items in important components for communication as meaning-making, including (A) Purpose, (B) Benefits, (C) Components. Part 3 is divided into 3 parts, including roles of teachers, students, the group of students. Part 4 is about future teaching mathematics, including (A) Mathematical experience for future teaching mathematics, (B) How to apply communication to future mathematics teaching. Table 3 shows the overview of questionnaire items.

TABLE 3. Category, Item content, and subcategory for questionnaire

Category	Item Content	Subcategory
Part 1	Beliefs in Teaching and Learning Mathematics (29 Likert scaled and 1 open-ended)	A. Mathematics teaching and learning (18 Likert scaled) B. Past experiences and beliefs (11 Likert scaled and 1 open-ended)
Part 2	Important components for communication as meaning-making (2 choosing items and 1 open-ended)	A. The purpose of communication (Choosing 3 items) B. The benefits of communication (Choosing 3 items) C. The components for communication as meaning-making (1 open-ended)
Part 3-1 3-2 3-3	Roles of teachers Role of students Roles of a group of students (12 Likert scaled)	A. Roles of teachers (6 Likert scaled) B. Roles of students (6 Likert scaled) C. Roles of the group of students (3 Likert scaled)
Part 4	Future teaching for communication as meaning-making in mathematics teaching (2 open-ended)	A. Mathematical experience for future teaching mathematics (1 open-ended) B. How to apply communication to the future mathematics teaching (1 open-ended)

For the Part 1 survey, the questionnaire survey of White et al. (2006) containing eighteen items in two subscales was used to measure beliefs about mathematics, mathematics learning, and mathematics teaching. Nine items are in the constructivist view and the other nine items are in the transmissive view, and “I think” statements are written to obtain more suitable responses. Based on the previous survey of White et al. (2006), several modifications were made to clarify the question in consultation with a mathematics education expert. For example, a long question asking about the perception of the teacher’s role was divided into two distinct sentences. In addition, the original statement of dynamic searching for order and pattern was deleted due to ambiguity of perspective. The revised explanation was reviewed through agreement with the supervisor. For all the five-point Likert scale of items (Nemoto & Beglar, 2014) requiring agreement or disagreement (SA = Strongly Agree, A = Agree, N = Neutral, D = Disagree, and SD = Strongly Disagree). The statement is shown in Appendix 3.

On the other hand, when it comes to the relationship between experience and mathematical attitude or future teaching mathematics for Part 1-B, the survey of Jong and Hodges (2013) would be valuable. They gathered fifteen similar surveys of mathematics and constructed a questionnaire including attitude and past experiences, teaching and learning, mathematics course experiences, diverse learners, and future teaching. It was an experimental research, so they used pre-survey and post-survey. There were fourteen items in the section on attitude and past experiences, and only eight items are extracted and revised some words, such as K-8 into basic education (1-9 grade), grade 9-12 into general upper secondary school, according to the Finnish school system. Questions 9-11 were added to explore whether the teacher training programmes were perceived as positive experiences, and only the participants corresponding to items 9, 10, and 11 responded. Therefore, the following questionnaire in Appendix 4 includes 11 Likert-scale items and 1 open-ended item.

Regarding Part 2 and Part 3, the lists were organized by literature review. For example, there are nine important purposes, and there are eight important benefits that can be obtained when using communication in the mathematics classroom in Part 2. And participants were supposed to choose the three most important items. Moreover, 1 open-ended item was added to Part 2 and Part 3 consisted of Likert-scaled items was about the roles of teachers, students, students' groups for communication. Part 4 had two open-ended items as follows: What they need most now for future mathematics teaching, how they will versatile communication in future mathematics teaching. The specific statement of Part 2, Part 3, and Part 4 are described in Appendix 5, Appendix 6, and Appendix 7.

6.3.2 Ethical Considerations

Several factors must be considered to ethically conduct mixed-method research. All participants voluntarily attended the survey and signed consent agreements for the use of their personal data. The consent agreement was elaborated under the lawful basis for processing under the EU's General Data Protection Regulation, Article 6 Paragraph 1, and the Personal Data Act, Section 4 (Office of the Data Protection Ombudsman, n.d.). The study also collected self-reported data on demographics of participants, such as gender, and the current year of the teacher training program (see Appendix 2). Demographics were used to evaluate the representability of the study and to analyse if these variables have any significant effect on the relationships explored.

Personal data used in the study were protected by means of username, password, registered use, and access control (physical facilities). For data security, there were four parts to consider. First, the security of information networks is as follows. The data controller only had personal usage rights to read and write data with usernames, passwords, and fingerprints. Confidential information was stored on the controller's server that was not connected to networks. Second, for physical security, the controller's computer was only in the personal house with a safety door lock. Third, software update, which requires critical updates for operating systems and programs. And they were installed as quickly as possible. Fourth, for virus protection, the controller's computer was used in the study regularly, and automatically updated antivirus software was installed.

The study does not impose any physical or psychological harm on the participants and does not entail any life-endangering interventions. However, asking about their own mathematics experience in the times of past schooling might evoke some uncomfortable feelings for some individuals. They could stop their participation in the study at any time until the questionnaire was completed without specifying a reason.

When it comes to the benefits and incentives to study participants, it was possible to understand how preservice class teachers perceive the meaning and role of communication and their future teaching in primary school mathematics classes. In addition, participants could reflect on their mathematical beliefs and mathematical learning and teaching. Participants will receive a summary of the findings after the study has been completed.

6.3.3 Validity and Reliability

Validity is based on determining whether the survey results are accurate from the standpoint of researchers, participants, and account readers (Creswell & Creswell, 2018). They explained that validity using the convergent approach is based on establishing both quantitative validity and qualitative validity for each database. Firstly, the reason for selecting two of the eight Finnish universities for quantitative validity is based on the background of equity and equality in Finnish education. Therefore, after graduating, it is assumed that PCTs have a strong dependence on independence and autonomous responsibility, which means to demand a universal understanding of teaching and learning mathematics (Krzywacki et al., 2016). Secondly, qualitative validity can be obtained in the process of analysing and integrating three data: quantitative survey results, answers to open questions, and literature review. This process, called Triangulation, allows three data to be checked for the association among the analysis results and the theoretical background, and the research questions. Thirdly, increased reliability by checking for obvious errors when cross-checking code developed by expert researchers.

For performing reliability analysis, Cronbach's alpha was calculated which measured the consistency of the scale based on inter-item correlations and the number of items (Platas, 2015). In Part 1-A, the origin alpha reliability of questionnaire about beliefs in teaching and learning mathematics (White et al., 2006) was 0.58, in this study, the transmissive subscale consisted of nine items ($\alpha = .58$), the constructivist subscale consisted of nine items ($\alpha = .56$). The positive past experience of Part 1-B was found to be highly reliable (4 items; $\alpha = .82$), the struggle past experience was shown to be reliable (2 items; $\alpha = .78$). The traditional teaching ways subscale consisted of 2 items ($\alpha = .77$), and the subscale of different ways in recent class consisted of 2 items ($\alpha = .70$). Cronbach's alphas for the fifteen roles of teacher, students, student's group items were .67.

6.4 Data analysis and integration

For analysing the convergence model, assembling results of the data analysis for quantitative methods and qualitative methods proceeds at the interpretation stage of a study (Gregory J. Cizek., 1999). Therefore, in this study, qualitative content analyses used data-driven open coding, and a priori codes deriving from the literature. All the data (i.e., Liker-type responses and the open coded) is analysed throughout frequencies and percentages by the Statistical Package for the Social Sciences (SPSS ver.27) software. In addition, a descriptive statistical analysis (Mean, SD, Min, Max, Skewness, Kurtosis) was illustrated for quantitative analysis of Likert-type items. Correlations and Paired Samples t-test were used for finding the statistically significant relationship between two variables. However, due to the limited number of participants, it was not possible to examine the statistical significance of most comparisons. Only the responses to mathematical schooling experience could be found the statistically significant difference between positive experiences and struggle experiences. In conclusion, the Joint display of data procedure is used in merging the different forms of data in a table or a graph at the last stage (Creswell & Creswell, 2018, p220). The table with main concepts on the horizontal axis and then each column including quantitative responses and qualitative responses to the concepts.

7 RESULTS

The purpose of this chapter is to describe and integrate the quantitative and qualitative results of the survey. It is divided into three parts, each of which is related to answering a research question. First, a questionnaire analysis using SPSS was used in the study of mathematical learning experiences and mathematical beliefs. In addition, all participants were divided into four groups based on the participants' past learning experiences. I analysed participants' open-ended responses to determine how their previous experiences affected their views on mathematic teaching and learning. Second, the Likert-scale and open-ended responses were analysed and integrated to understand the major components of communication. Third, I analysed open answers about what new mathematical experiences participants need in Teacher Training Programme, and how they would demonstrate mathematical communication in future teaching mathematics. Finally, quantitative data and qualitative data were effectively integrated into the joint display (see Appendix 10 and 11).

7.1 Research Question 1: Beliefs and experience in mathematics

Teachers emphatically and enthusiastically believe that communication is crucial for truly understanding mathematical concepts (Bratina & Lipkin, 2003), which enables students to engage in learning mathematics. Therefore, Pehkonen (1998) and Furinghetti (1998) pointed out that mathematical beliefs should be included as components of knowledge. Moreover, many researchers have explored the possibility of changing preservice teachers' beliefs about mathematics in a positive way or with a constructivist view. While this view emphasises a student-oriented process of teaching and learning mathematics, the transmissive perspective places importance on learners learning mathematical concepts and skills through demonstration, repetition, and clear instruction from the teacher. However, changing beliefs requires a long time and significant new experiences

(Skott et al., 2018). Therefore, instead of asking about the possibility of changing beliefs, RQ1 focused on what mathematical beliefs the Preservice Class Teachers (PCT) construct between previous and new learning experiences.

7.1.1 Beliefs about mathematics, mathematics learning, and teaching

For the Part 1-A survey, the survey included eighteen items on two subscales to measure beliefs about mathematics, mathematics learning, and mathematics teaching. One subscale has nine items from a constructivist point of view and the other subscale also has nine items from a transmissive point of view. I wrote the statement "I think" to obtain more appropriate responses. Agreement or disagreement is required for all 5 Likert-scaled items (i.e., SA = strongly agree, A = Agree, N = Neutral, D = Disagree, and SD = strongly disagree). The statements are referred to as Mathematics = Mathematics classroom at primary school, Students = Primary school students. Table 4 shows the result of responses to the Likert-scaled items, and descriptive statistics are presented in Appendix 8.

TABLE 4. Responses to the statements of mathematical beliefs (Part 1-A)

Item		SD *f* (%)	D *f* (%)	N *f* (%)	A *f* (%)	SA *f* (%)
PA1	BELIEVE01T	24 (68.6)	9 (25.7)	1 (2.9)	1 (2.9)	0 (0.0)
PA2	BELIEVE02T	9 (25.7)	18 (51.4)	5 (14.3)	3 (8.6)	0 (0.0)
PA3	BELIEVE03C	1 (2.9)	2 (5.7)	2 (5.7)	19 (54.3)	11 (31.4)
PA4	BELIEVE04T	14 (40.0)	15 (42.9)	4 (11.4)	1 (2.9)	1 (2.9)
PA5	BELIEVE05C	0 (0.0)	0 (0.0)	9 (25.7)	23 (65.7)	3 (8.6)
PA6	BELIEVE06C	2 (5.7)	6 (17.1)	10 (28.6)	16 (45.7)	1 (2.9)
PA7	BELIEVE07T	20 (57.1)	10 (28.6)	2 (5.7)	3 (8.6)	0 (0.0)
PA8	BELIEVE08C	0 (0.0)	2 (5.7)	4 (11.4)	21 (60.0)	8 (22.9)
PA9	BELIEVE09C	0 (0.0)	2 (5.7)	13 (37.1)	19 (54.3)	1 (2.9)
PA10	BELIEVE10T	1 (2.9)	7 (20.0)	6 (17.1)	17 (48.6)	4 (11.4)
PA11	BELIEVE11C	0 (0.0)	1 (2.9)	5 (14.3)	20 (57.1)	9 (25.7)
PA12	BELIEVE12C	0 (0.0)	0 (0.0)	4 (11.4)	15 (42.9)	16 (45.7)
PA13	BELIEVE13C	0 (0.0)	0 (0.0)	7 (20.0)	17 (48.6)	11 (31.4)
PA14	BELIEVE14T	7 (20.0)	12 (34.3)	9 (25.7)	7 (20.0)	0 (0.0)
PA15	BELIEVE15T	2 (5.7)	7 (20.0)	4 (11.4)	20 (57.1)	2 (5.7)
PA16	BELIEVE16T	0 (0.0)	3 (8.6)	2 (5.7)	23 (65.7)	7 (20.0)
PA17	BELIEVE17T	0 (0.0)	2 (5.7)	6 (17.1)	15 (42.9)	12 (34.3)
PA18	BELIEVE18C	1 (2.9)	1 (2.9)	11 (31.4)	11 (31.4)	11 (31.4)

Note. SA = Strongly Agree, A = Agree, N = Neutral, D = Disagree, and SD = Strongly Disagree. BELIEVE_C = Constructivist beliefs, BELIEVE_T = Transmissive beliefs.

Based on the results in Table 4, firstly, in the constructivist statement responses, supporting learning environment (PA12, 89%) with the statement the most, followed by the nature of mathematics as a beautiful human endeavour (PA3, 86%). The mathematics learning process includes the periods of uncertainty, conflict, confusion, and surprise (PA8, 83%), activities related to students' experiences (PA11, 83%) and problematic situations (PA13, 80%) were represented in the strong rationale for constructivist belief. Two items regarding mathematics knowledge based on students' experience (PA5, 74%) and mathematical knowledge based on cooperative learning environment (PA18, 63%) were positively indicated. Furthermore, participants agreed less with the statement of belief in students' capability of determining what is right or wrong (PA6, 48%) and much higher levels of mathematical thinking (PA9, 57%) than other responses.

Secondly, in transmissive statements, four items were shown to be much higher than other items. For example, the item that teachers should recognize students' errors and confusions from an adult point of view (PA17, 77%) was the highest, followed by the item that teachers should evaluate students' mathematical knowledge (PA16, 76%). The teacher's role to transmit mathematical knowledge to students (PA15) was selected by 63% of participants, and only half of the participants agreed that memorizing is a critical skill in mathematics learning (PA10, 50%). Five items that the authorities for what is right or wrong in teachers, textbooks (PA14, 20%), quickly solvable problems (PA2, 9%), quickly finding right answers (PA7, 9%), finding right answers (PA4, 5%), and calculation (PA1, 3%) showed a low agreement rate.

In order to analyse the results according to the theoretical learning basis of beliefs, the questionnaire items were classified into two subscales. In addition, only two scale variables (agree and strongly agree) were accepted as the sum variable for the statement. Within SPSS analysing, there was a nonsignificant correlation ($p = .259$) between transmissive belief and constructivist belief sum variables. Nevertheless, for comparing the mean value, the constructivist view ($M = 8.6$, $SD = 2.9$) of belief in theoretical learning foundation was slightly higher than the transmissive view ($M = 4.1$, $SD = 1.9$) by Paired Samples t-test.

7.1.2 Past learning experience in mathematics

The past learning experience in mathematics plays a crucial role to construct own's mathematical belief and it would be connected to a new experience through self-system (Malmivuori, 2001). Thus, it was important to examine how the PCTs perceived the mathematics learning experience in basic education, general higher education (Lukio in Finland), and TTP (Teacher Training Program) as new experiences in universities. Appendix 9 presents descriptive statistics of mathematical past experience and Table 5 shows the results of the responses.

TABLE 5. Responses to the statements of past experiences (Part 1-B)

Item	Statement variables	SD *f* (%)	D *f* (%)	N *f* (%)	A *f* (%)	SA *f* (%)
PB1	Positive Experience (Grade 1-9)	3 (8.6)	5 (14.3)	1 (2.9)	14 (40.0)	12 (34.3)
PB2	Positive Experience (Upper general school)	3 (8.6)	8 (22.9)	2 (5.7)	14 (40.0)	8 (22.9)
PB3	Struggle Experience (Garde 1-9)	14 (40.0)	7 (20.0)	3 (8.6)	7 (20.0)	4 (11.4)
PB4	Struggle Experience (Upper general school)	6 (17.1)	9 (25.7)	1 (2.9)	12 (34.3)	7 (20.0)
PB5	Today's different teaching ways (Grade 1-9)	0 (0.0)	5 (14.3)	3 (8.6)	22 (62.9)	5 (14.3)
PB6	Today's different teaching ways (Upper general school)	2 (5.6)	3 (8.6)	11 (31.4)	14 (40.0)	5 (14.3)
PB7	Traditions teaching ways (Grade 1-9)	0 (0.0)	0 (0.0)	1 (2.9)	8 (22.9)	26 (74.3)
PB8	Traditions teaching ways (Upper general school)	0 (0.0)	1 (2.9)	1 (2.9)	9 (25.7)	24 (68.6)
PB9	New positive experience at University (Mathematics course)	1 (2.9)	3 (8.6)	3 (8.6)	6 (17.1)	13 (37.1)
PB10	New positive experience at University (Teaching practice)	0 (0.0)	0 (0.0)	2 (5.7)	15 (42.9)	6 (34.3)
PB11	New positive experience at University (Intensive course)	0 (0.0)	1 (100.0)	0 (0.0)	0 (0.0)	0 (0.0)

Note: All participants (N = 35) responded to items 1-8. Items 9-11 were only answered by participants who have the same experience based on the answer to the previous experience at university. Item 9 (n = 26), Item 10 (n = 23), Item 11 (n = 1). SA = Strongly Agree, A = Agree, N = Neutral, D = Disagree, and SD = Strongly Disagree.

For example, Part B items were about the participants' (N = 35) opinions about positive learning experiences in mathematics (PB1, PB2), and struggle or difficult learning experiences in mathematics (PB3, PB4). Four items of PB5, PB6, PB7, and PB8 were about teaching methods between the past classroom and the recent classroom. Participants who experienced new learning experiences were able to respond regarding positive mathematics learning experiences in universities, including TTP (PB9, PB10, PB11). Statements of past learning experiences were divided into basic education and the upper general school. In addition, the description of the teaching method compares the current educational environment to when the participant was a student. Last, new mathematics learning experiences in university included TTP, internships, and other courses.

As a result, numerous PCTs had positive mathematics learning experiences during basic education (74%), and general upper secondary school (63%). In terms of learning experiences involving struggling with mathematics, only 31 % agreed with it during basic education while over a half of participants (54%) experienced mathematic learning as hard. Mathematics courses at universities were experienced positively by almost half of PCT (54%), moreover, the response to a positive teaching practice experience was selected by 77% of participants. Only one participant experienced the intensive mathematics course, and s/he disagreed in a positive statement.

7.1.3 Past and present teaching methods

Descriptive statistics of past and present teaching methods are shown in Table 6.

TABLE 6. Descriptive statistics of teaching methods by group

			Today is a different teaching way from the way I learned it.						I mostly learned mathematics in traditional ways (i.e., textbooks, worksheets, rules, lectures).					
			Disagree		Neutral		Agree		Disagree		Neutral		Agree	
Group		Total	*f*	%	*f*	%	*f*	%	*f*	%	*f*	%	*f*	%
A	G1-9	12	2	16.7	0	0.0	10	83.3	0	0.0	0	0.0	12	100.0
	Lukio	12	1	8.3	5	41.7	6	50.0	1	8.3	0	0.0	11	91.7
B	G1-9	14	3	21.4	2	14.3	9	64.3	0	0.0	0	0.0	14	100.0
	Lukio	14	3	21.4	3	21.4	8	57.1	0	0.0	0	0.0	14	100.0
C, D	G1-9	9	0	0.0	1	11.1	8	88.9	0	0.0	1	11.1	8	88.9
	Lukio	9	1	11.1	3	33.3	5	55.6	0	0.0	1	11.1	8	88.9
Total	G1-9	35	5	14.3	3	8.6	27	77.1	0	0.0	1	2.9	34	97.1
	Lukio	35	5	14.3	11	31.4	19	54.3	1	2.9	1	2.9	33	94.3

Note: Group C (n = 8) and Group D (n = 1) were merged due to the limited number of participants. Agree includes agree and strongly agree, disagree includes disagree and strongly disagree.

As a result, it was found that although each group had different past learning experiences, they almost completely agreed with the statement that they experienced traditional teaching methods (97% for grades 1-9, 94% for Lukio). These results are consistent with the present study problem pointing to the use of traditional methods in mathematics classrooms (Joutsenlahti & Kulju, 2017). Conversely, 77% of PCT agreed that the teaching methods used today in basic education are different from those used in the past, while only 54% agreed that the same is true in general upper secondary school. However, there was no significant statistical difference between groups.

To specifically explore personal learning experiences in mathematics, items PB1 to PB4 were recorded as scaled values and only responses of Agree and Strongly Agree were accepted. In Figure 5, the blue bar graph represents the sum variable of positive learning experience in the past schoolings, and a red one addresses the sum variable of struggle learning experience in the past schools. Participants coded from PCT01 to PCT35 and analysed by ID when analysing open-ended answers. Positive experience sum variable and struggle experience sum variable were significantly correlated (r = .79, p < .001) with SPSS analysing.

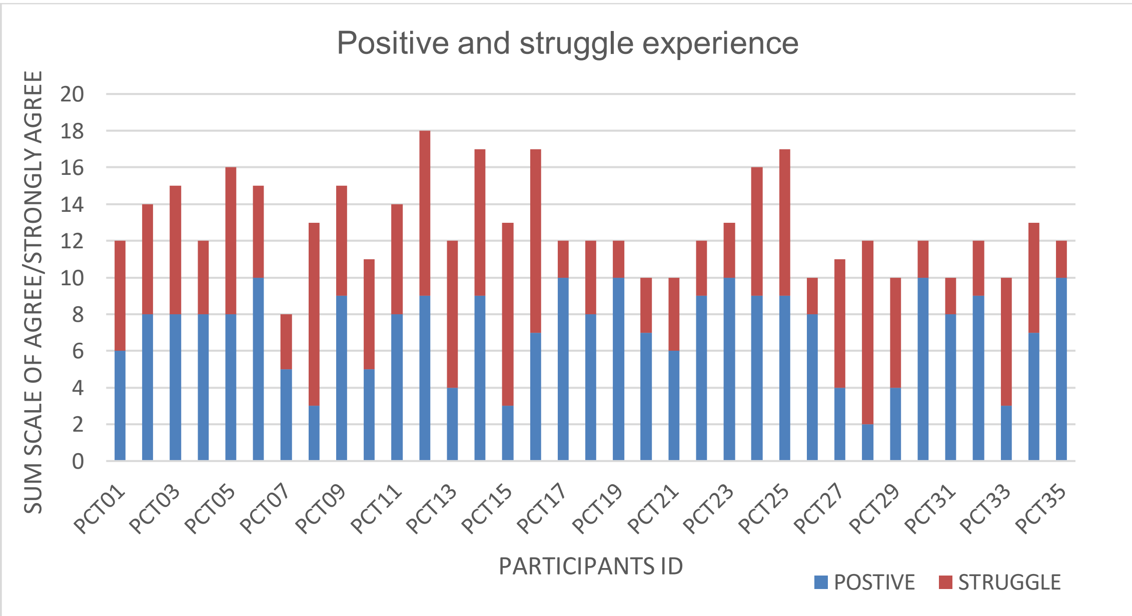

FIGURE 4. Personal Positive and Struggle experience in Mathematics

As mentioned in the 7.1.1 section, although the constructivist view (M=8.6, SD=2.9) was slightly higher than the transmissive view (M=4.1, SD=1.9), the individual figures showed interesting results. For example, among participants with similar levels of constructivism, there were cases in which the same level of transmissive views (i.e., PCT12, PCT14, PCT25) or relatively low transmissive views were observed (i.e., PCT09, PCT15, PCT29). In addition, some participants showed a relatively high transmissive view compared to the constructivist view (i.e., PCT17, PCT19, PCT23, PCT31, PCT35).

Nevertheless, since there is no statistical difference between the two perspectives, the focus has been on how the perspectives on learning and teaching mathematics. In other words, participants were divided into 4 groups to examine how past learning experiences might have affected current participants' mathematics teaching and learning. The criteria for dividing the group were set by the two variables, positive and facing difficulty in the mathematics learning experience. For instance, Group1 (n = 10) had strong positive experiences but a lot of difficult experiences in mathematics learning, Group 2 (n = 13) had positive learning experiences and fewer troubles in solving mathematics problems. Otherwise, Group 3 (n = 8) experienced struggling to solve the mathematics problems throughout less positive experiences, also Group 4 (n = 1) had a weak positive but little struggle in mathematics learning experiences. Figure 5 illustrates the distribution of participants based on positive experiences on the x-axis and struggle experiences on the y-axis. Three intermediate score data that did not respond to open-end items were excluded from grouping (PCT01, PCT02, PCT21). Participants' data representing median scores (sum = 6) were grouped by analyzing open responses. Participants who presented the same coordinates were presented with a large signal.

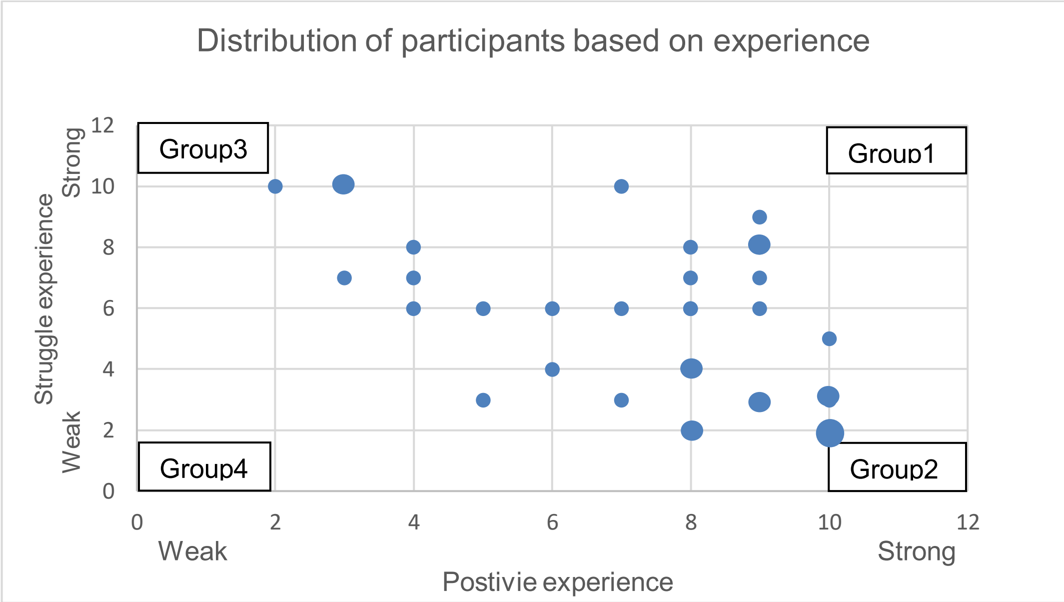

FIGURE 5. Distribution of participants based on positive and struggle experience

Consequently, the differences between the groups' (e.g., transmissive belief scale, constructivist belief scale, interval transmissive and constructivist belief scales) responses were not statistically significant. However, through previous learning experiences, similarities were found among participants in the same group. In other words, the participants' views on how their past learning experiences affected their present mathematics learning and how they would affect their future mathematics teaching were analysed in the next open answer. Therefore, RQ1. The results of the open-ended responses for each group on how their experience affected their thoughts about mathematics learning and teaching are presented in the following section.

7.1.4 Analysing open-ended responses by group

For finding the relationship between past experiences and their thoughts about teaching and learning mathematics, all participants could participate in the below open question, and 21 participants showed their opinion. A specific number of participants was 6 in Group 1, 9 in Group 2, 5 in Group 3, and 1 in Group 4.

P1-B12. What past experiences have influenced your thoughts about teaching and learning mathematics the most? How did it affect you?

All responses were analysed by a priori codes deriving from the theoretical framework by Malmivuori (2001), which included three criteria such as previous experience with mathematics (past), a new mathematics learning situation (new), and self-system (effect). In that framework, self-system links personal past experience to new mathematics learning situations complicatedly, hence, this study focused on how their past experience effect their mathematics learning and teaching, what kinds of new experiences did they have, and it did change their views, and how.

Group 1

Six of the participants who had a strong positive experience as well as strong struggle experience simultaneously in mathematics learning during past schooling responded to the open question. They all had more constructivist perspective (min = 33, mix = 38) on mathematical learning than transmissive view (min = 22, max = 28). In particular, the four participants (PCT03, PCT09, PCT12, PCT24) showed more 10 higher sum variables in the constructivist view than in the transmissive view. All participants had experiences in learning mathematics at university including internships. Moreover, it can be seen that these new experiences at university have positively influenced their thinking about teaching and learning mathematics. Table 7 shows the result of analysing open responses of Group 1 (n= 6).

TABLE 7. Result of analysing open responses of Group 1

ID	Past	New	Effect	Main Statement
PCT03	Basic education	University: positive	Changed: a variety teaching method	*I've also realized that a class of mathematics does not have to be just calculating and doing the exercises from the book.*
PCT09	Whole education: enjoy, memorization	University	Changed: to create a positive attitude in students	*I think we should strive to create a positive attitude in children relating to math and education in general.*
PCT11	High school: challenging	University: positive	Changed: to promote computing tasks, using textbook materials	*When it comes to teaching, you know that textbook materials have good illustrative support. The basics are strong enough to make it possible to progress to trickier computing tasks*
PCT12	High school: positive, traditional ways	University: positive, modern ways	Changed: traditional & modern ways are needed	*Both experiences were positive, and I came to realize that both styles are needed.*
PCT24	-	University: positive,	Changed: to expand own mathematical thinking	*I understood the importance of creating mathematical concepts between everyday life and school mathematics.*
PCT34	-	Internship: positive (inspired)	Changed: a variety of learning options instead of traditional ways	*This is inspired to consider a variety of options for studying mathematics instead of a traditional textbook/support*

Note: Past (What experience most influenced your thoughts?), New (What new things did you experience?), Effect (Did you change your thought? And how?), '-' represented the no explanation from responses.

Consequently, teaching methods were the most concerning issue for them, all participants strongly agreed with the statement they learned mathematics by traditional teaching, as with the results shown in Table 13. Nevertheless, five participants had positive experiences at university and 1 participant who had inspired experience at internship changed their thoughts into needing multimodal language (Joutsenlahti & Kulju, 2017), creating a positive attitude toward students, and using real-life problems. These ideas are connected with the constructivist view that teachers should support students to expand their mathematical thinking on their own. In addition, the answer that classroom mathematics is not all about solving and calculating exercises in textbooks shows that this group has a strong constructivist belief in mathematics.

Group 2

Nine of the participants in Group 2 who had a strong positive experience, but weak struggle experience responded to the open question. They had more the constructivist perspective (min = 26, mix = 38) on mathematical learning than the transmissive view (min = 15, max = 29). Specifically, PCT17 showed the strong constructivist view (sum = 36) than transmissive view (sum = 15), and PCT18 had both views as a low sum (Constructivist = 26, Transmissive = 24). The results of analysing data are shown in Table 8.

TABLE 8. Result of analysing open responses of Group 2

ID	Past	New	Effect	Main Statement
PCT06	High school: positive,	-	Mathematics is great, challenging is needed	*I know I can make up some difficult things.*
PCT17	Textbook oriented instruction	University: positive	Changed: a variety teaching method	*University studies convinced me that teaching mathematics can be much more (functionality, instruments).*
PCT18	-	-	-	*Functional teaching mathematics. Amazing experience!*
PCT22	-	University: positive	Changed: a variety teaching method	*I understood the importance of concrete examples and physical objects in mathematics.*
PCT23	Father: positive	-	positive attitude toward mathematics	*My fathers helped me develop a positive picture of mathematics.*
PCT30	-	-		*Teachers should know how to support different learners in diverse ways.*
PCT31	-	University: positive	Changed: a variety teaching method	*The importance of using tools in the development of mathematical thinking was also strengthened by pre-existing conditions.*
PCT32	Whole education: positive	-	Convince to own competence	*Mostly my mathematics teachers. They inspired us, gave us clear examples, and made us think that we can surely do math.*
PCT35	-	University: positive, mindset	Changed: a variety teaching method	*I think it is important to teach children mathematical thinking in a diverse way so that they understand what it is about.*

Note: Past (What experience most influenced your thoughts?), New (What new things did you experience?), Effect (Did you change your thought? And how?).

Many participants of Group 2 mentioned how to struggle did they do mathematics, and why the experience of success in mathematics and support for their challenges are important in teaching. Therefore, although learning mathematics was difficult, they had many experiences of success and showed strong confidence in solving difficult mathematical problems (see PCT06). Similar to Group 1, most participants realized that there was a variety of teaching methods, while they explained that a teacher's role to propose a clear solution or example for the diverse learner was crucial in teaching mathematics. They indicated that teachers need to know the different methods of mathematics teaching that will support learners at different levels. Furthermore, they mentioned that students must be confident in their abilities and strive to expand their mathematical thinking. On the other hand, PCT23 noted that her/his father was the most positive supporter of mathematics learning. Therefore, this indicated that non-school factors could play an important role among the past experiences that concrete mathematical beliefs. Which addresses that a further complex self-process is required (Malmivuori, 2006).

Group 3

Five of the participants in Group 3 responded to the open question, which group had a weak positive experience but strong struggle experience. In other words, they had a lot of difficulties solving mathematics problems and had a little positive learning experience. Most of them had more constructivist perspective (min = 29, mix = 40) on mathematical learning than transmissive view (min = 18, max = 32). Interestingly, PCT08 had both strong views (Constructivist = 37, Transmissive = 32), while PCT15 (Constructivist = 37, Transmissive = 18), PCT27 (Constructivist = 36, Transmissive = 15) had strong constructivist perspectives. PCT29 showed the same level of constructivist (sum = 29) and transmissive (sum = 29) perspectives. In the response to new experiences, experience as a sub-teacher was added, and various responses were obtained about the influence of the experience on their thoughts of mathematics teaching and learning. Table 9 presents the result of analysing open responses of Group 3 (n= 5).

TABLE 9. Result of analysing open responses of Group 3

ID	Past	New	Effect	Main Statement
PCT08	-	University: positive	Changed: a variety of teaching methods (digitalization, mixing with physical activities, programming), math is a nice subject	*I've been very happily surprised about all the new practices and ways of teaching mathematics that I haven't been able to experience myself while studying in elementary school. Before I was scared of teaching mathematics, now I think it is a nice subject to teach with an important meaning.*
PCT10	-	University: Negative, engineering	Not to give the negative feeling for future students	*Studying engineering in university in my previous studies. It was horrible. I don't want it to be horrible for my future students.*
PCT15	Very little functional study and speaking in mathematics (quiet working)	University: Positive, learning disabilities	Changed: a variety teaching method	*Mathematical instruments were only used when there were difficulties with counting, which at least made it embarrassing for me as a student. Now as a teacher, using tools with my students in math classes is commonplace for all pupils.*
PCT27	-	-	-	*Learners' own experience, everyday math, learning by doing, also through mistakes are the most perfect ways to learn and to design various kinds of teaching.*
PCT29	Basic education & Lukio: difficult to follow	Sijainen (substitute teacher): difficult	Unchanged: need time to think and make it	*I am slow in mathematics. I need time to think and that makes everything hard. The scariest part is to answer pupils' questions. BUT my grades in mathematics have been good thus no one has realized how bad I am in mathematics.*

Note: Past (What experience most influenced your thoughts?), New (What new things did you experience?), Effect (Did you change your thought? And how?).

As a result, the various teaching methods learned in TTP were surprising to them, but they simultaneously expressed anxiety about whether they could effectively use them in future mathematics teaching. Interestingly, PCT29's statement indicates that superficial mathematical scores and internal mathematical anxiety are inconsistent. Thus, although s/he her/himself perceives that the process of learning mathematics is slow, his teacher may not have noticed, and her/his anxiety may not have been outwardly expressed by her/his high mathematical scores. Therefore, encouraging students to express their mathematical thinking through words, writings, or drawings helps teachers to accurately understand the mathematics learning process of students.

Group 4

Only one participant belonging to Group 4 answered the open question. This group had a weak positive experience as well as a weak struggle experience in learning mathematics. This participant had stronger constructivist view (sum = 35) than transmissive view (sum = 25). This participant got much pressure to get a good grade from teachers no matter how great grades s/he got in the exam. That is related to negative feelings such as 'problematic, harmful' to self-image. The last statement indicated the strong motivation not to give a negative environment for future students. The results of analysing open responses of Group 4 are presented in Table 10.

TABLE 10. Result of analysing open responses of Group 4

ID	Past	New	Effect	Main Statement
PCT07	Basic education: Negative, great pressure to get good grades from teachers	-	Not to press future students to get good grades	*On my point, teachers always expected perfection towards math from good pupils, which is why, especially in primary school, I was under great pressure to get good grades so that teachers would not have been disappointed. As an adult like this, I have noticed how problematic and harmful this was, for example, for my self-image. Now I personally want to be a teacher in the future and act better than my own teachers.*

Note: Past (What experience most influenced your thoughts?), New (What new things did you experience?), Effect (Did you change your thought? And how?).

As a result, most participants changed their minds about various teaching methods for future mathematics teaching. The new experience in learning mathematics at the university was surprisingly different from traditional teaching methods, and most of the participants mentioned that these various teaching methods will be used in their future mathematics teaching. However, similar interesting statements were observed for each group divided according to previous mathematics learning experiences. In short, Group 1 expressed that both traditional and new methods are necessary for future education. Group 2 noted strong confidence in solving difficult problems on its own. Group 3 and 4 explained negative feelings about past mathematics learning experiences such as scary, difficult, terrible, too slow, problematic, and harmful.

7.2 Research Question 2-1: Communication as meaning-making

Part 2 of the survey contains the question about the purpose of communication and the benefits of communication for communication as meaning-making in mathematics teaching and learning. In addition, one open-ended question was added to explore the components of communication as meaning-making. All participants could select the most three important factors that construct communication as meaning-making in mathematics teaching and learning. The results were presented as a bar chart for all participants' responses and as the line chart for each group except for Group 4, which had only one participant. Nevertheless, the small number of participants in each group made it difficult to investigate significant similarities and differences between groups. Therefore, the results were explained only with the overall preference and the numerical characteristics between groups.

7.2.1 The purpose of communication

According to the literature review, there were nine statements explaining the purpose of communication. All participants should select the three most important purposes of communication as meaning-making among them. The purpose of this survey item was to identify the components of communication that PCTs consider most important. All statements were generally divided for the purpose of using mathematical communication on the student's side and the teacher's side. First, five statements related to mathematical thinking, problem-solving ability, mathematical understanding, critical thinking ability, and multiliteracy related to mathematical competence from the student's side were included, as well as a statement to improve mathematical achievement. Second, in terms of teachers, there was a statement that communication is used to provide an enjoyable learning environment for students. In addition, it included the statements that communication is used to monitor a student's learning process or to assess the mathematical understanding of students. In Figure 6, the number indicates the sum of respondents who selected the corresponding statement and is represented by line graphs of different colours for each group.

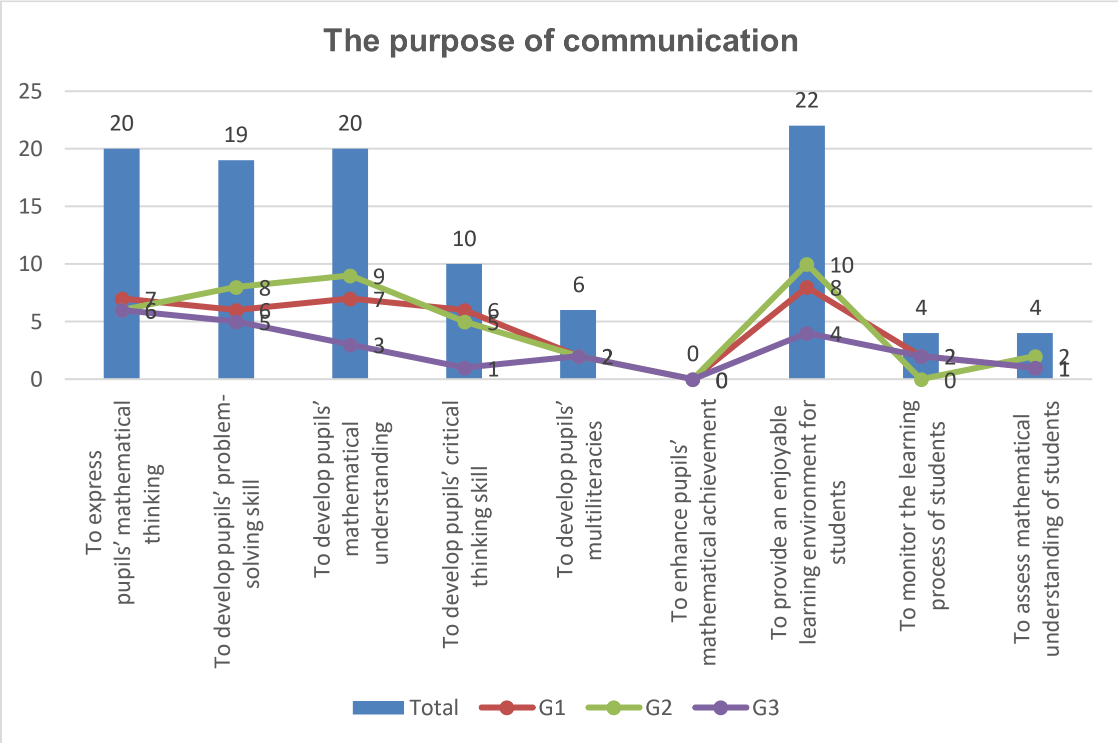

FIGURE 6. Response to the purpose of communication (Part 2-A)

The results in Figure 6, most participants considered providing an enjoyable learning environment (n = 22) to be one of the most important purposes to use communication, followed by expressing mathematical thinking (n = 20) and developing mathematical understanding statements (n = 20). In addition, developing problem-solving skills were highly regarded as a key purpose (n = 19). The results of each group were mostly consistent with the overall tendency. However, monitoring and assessing students' learning process or understanding was less considered as a crucial factor in using communication, moreover, none of them chose the statement of enhancing mathematical achievement.

7.2.2 The benefits of communication

Figure 7 illustrates the results of the survey of eight benefits of communication extracted from the literature review. All participants had to select the most important benefits of communication as meaning-making among them. The question of the benefits of communication was also constructed to understand the PCT's perspective on the components of communication.

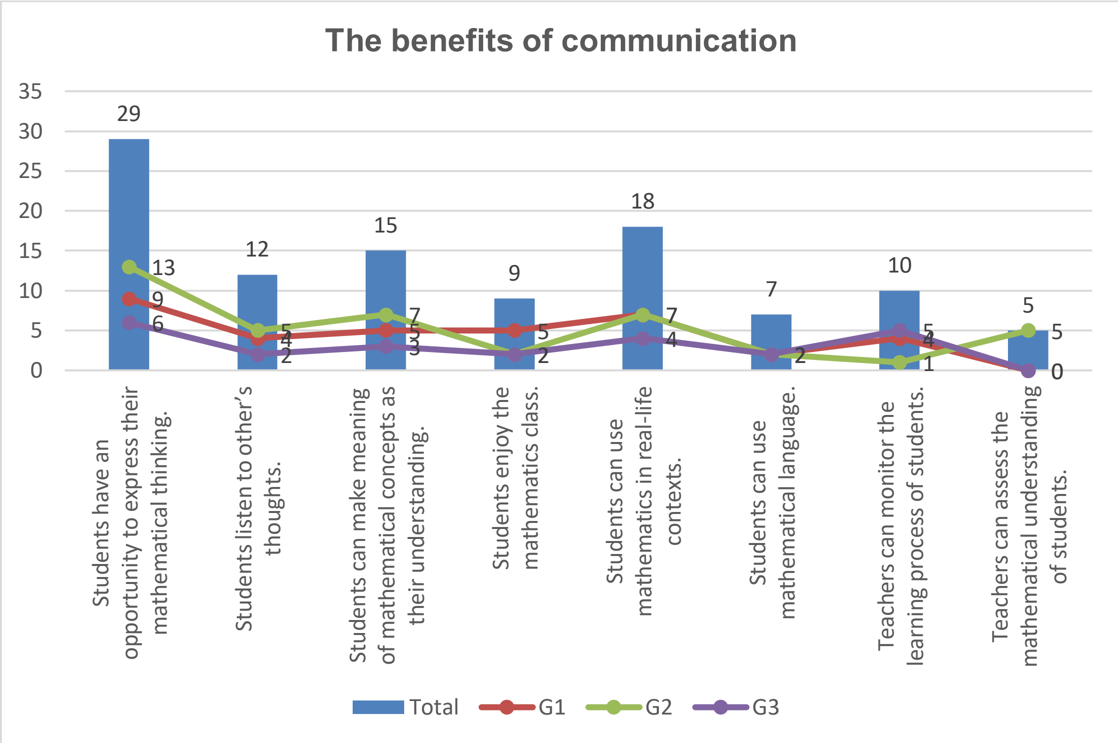

FIGURE 7. Response to the benefits of communication (Part 2-B)

In Figure 7, having an opportunity to express students' mathematical thinking was the most beneficial of communication among participants (n = 29). Moreover, numerous participants focused on connection to real-life text in mathematics (n = 18), followed by a capability of students who can make meaning as their understanding (n = 15), and students listen to other's thoughts (n = 12). Less than 10 participants chose the statement of teacher's monitor, students' enjoyment, and students' mathematical language. Otherwise, the statement of assessing students' mathematical understanding was selected (n = 5) only in Group 2. In other words, Group 2 had a positive mathematics learning experience and relatively weak mathematics learning difficulties. In brief, all of them identified that mathematical communication would be a useful tool for understanding students' mathematical understanding.

In brief, participants strongly agreed that communication as a process of making meaning was expressing their mathematical thinking and sharing ideas. When it comes to the components for communication as meaning-making, however, they recognized the relationship between mathematical communication and assessing, monitoring, the achievement was weak.

7.2.3 The components for communication as meaning-making

One open-ended item stated as “Part 2-C, for communication as meaning-making in the mathematics classroom, what are the most important components?” was questioned to the participants, and seventeen PCTs responded to the survey.

TABLE 11. Result of analysing P2-C using data-driven open coding (Part 2-C)

Category	*f*	%	Main Statement
Encourage environment	7	63.6	*‘Encourage to participate’* *‘Saying their thought processes aloud’* *‘Support atmosphere’*
Emotional environment	6	54.5	*‘Respect, appreciation for a different thing, motivation’* *‘Safe, secure’* *‘Understanding mistakes’* *‘Not being afraid of wrong answers,’*
Creative environment	4	36.4	*‘Create situations with instruments’* *‘Physical objects to help understand’* *‘Diversely options for solving the tasks’*
Teacher’s knowledge	2	18.2	*‘Knowing what they are struggling’* *‘Professional teaching’*

Note: Participants’ (n = 17) statements were multiple checked under category.

As a result of Table 11, various answers from creation environment to professional teaching appeared as communication components in a meaning-making process. As a detailed area of the environmental category, the environment that encourages students to express their thoughts accounted for the largest proportion (64%), followed by the emotional environment (55%) and the creative environment (36%). The answer to the emotional environment was a safe mathematics learning environment where students were not afraid to make mistakes and encouraged each other. The creative environment was to create creative problem-solving tasks using various teaching methods. Otherwise, two participants responded it was important to know what students were struggling with by teachers’ professional teaching knowledge.

7.3 Research Question 2-2: Roles of teacher, student, and a student group

For quantitative analysis, Likert-scale responses were analysed quantitatively in terms of frequencies and percentages using the Statistical Package for the Social Sciences (SPSS) 27 version software (IBM, Armonk, NY). In the case of 3A, 3B, and 3C items were only conducted for students who had more than 1 year at university including lecture experience. Therefore, there were 30 responses for 3A and additional 1 missing data (n = 29) who didn’t respond to 3B and 3C.

7.3.1 Teacher’s roles

Six roles of teachers were presented according to previous literature studies, and for 3A-1 and 3A-5, the statements were reversed to precisely explore the relationship between the student as the subject of learning and the teacher as a supporter of student learning. For example, 3A-5 indicated that teachers should continuously provide positive feedback so that students can actively participate in learning. Which can enable students to feel that they are receiving attention (Kim & Jeon, 2019). In addition, 3A-1 included the assertion that learning should be placed on students’ insightful discursive learning process (Schimeisser et al., 2013), not on the teacher’s position. In fact, teachers should monitor and support students’ learning process rather than control it. Teachers can also improve students’ learning through appropriate feedback. In another case, Joutsenlahti and Kulju (2017) emphasise that teachers should understand students’ language of mathematics (3A-2) and help students express their mathematical thinking (3A-3, Hirschfeld-Cotton, 2008). Effective strategies for active interaction in mathematics class included questioning, discussions, and group activities that can encourage communication in mathematics (3A-4, Kaya & Aydin, 2016; McKenney, 2020). Table 12 provides the results of PCTs' opinions in Teacher's roles for communication as a meaning-making process regarding six statements. These statements began with "In Mathematics classroom at primary school, a Teacher should~".

TABLE 12. Responses to the statements of teacher’s role (Part 3-A)

Item	Statement		Strongly disagree		Disagree		Neutral		Agree		Strongly agree	
“In Mathematics classroom at primary school, a Teacher should~”		Total	*f*	%	*f*	%	*f*	%	*f*	%	*f*	%
3A-1	Control all learning processes of students.	30	6	20.0	10	33.3	6	20.0	7	23.3	1	3.3
3A-2	Try to understand students’ mathematical language	30	0	0.0	0	0.0	1	3.3	8	26.7	21	70.0
3A-3	Encourage students to express their mathematical thinking	30	0	0.0	0	0.0	1	3.3	2	6.7	27	90.0
3A-4	Plan strategies, such as discussion, questioning, and group activities.	30	0	0.0	0	0.0	1	3.3	6	20.0	23	76.7
3A-5	Not give any feedback to students to get their own understanding of learning.	30	20	66.7	8	26.7	2	6.7	0	0.0	0	0.0
3A-6	Use mathematical problems related to real-life contexts.	30	0	0.0	0	0.0	1	3.3	9	30.0	20	66.7

Note. Missing value n=6 (less than 1 year).

Consequently, most participants (97%) considered encouraging students to express their mathematical thinking to be one of the most important roles for teachers. Their responses are exactly consistent with the definition of mathematical communication by studies of Hirschfeld-Cotton (2008), Joutsenlahti and Kulju (2017), and Lee (2015). Numerous participants also strongly agreed with the items on planning strategies (77%) and understanding students' mathematical language (97%). The item on using problems related to real-life contexts (97%) showed high agreement. 93% of the participants disagreed with the 3A-5 item that negatively explained the teacher's feedback. Some participants considered the feedback necessary for students' understanding of learning. Interestingly, controlling all learning processes of students (Item 3A-1) was accepted by 27% of participants, while half of them did not agree with this transmissive statement.

7.3.2 Student's roles

Based on the results of students' roles, the most important student's role in communication (Item 3B-1,2,4,5) can be found in Table 13. Other. All statements were related to the previous literature reviews, for instance, expressing their thinking in versatile ways (96%), listening carefully (93%), enjoying learning (93%), and understanding communication (93%).

TABLE 13. Responses to the statements of student's roles (Part 3-B)

Item	Statement		Strongly disagree		Disagree		Neutral		Agree		Strongly agree	
	"In Mathematics classroom at primary school, a Student can ~"	Total	*f*	%	*f*	%	*f*	%	*f*	%	*f*	%
3B-1	Express their mathematical thinking in versatile ways, such as speaking, writing, and drawing.	29	0	0.0	0	0.0	1	3.4	4	13.8	24	82.8
3B-2	Listen carefully to other students' explanations about their solutions to mathematical problems.	29	0	0.0	1	3.4	1	3.4	10	34.5	17	58.6
3B-3	Finally, find out a correct answer.	29	0	0.0	2	6.9	4	13.8	14	48.3	9	31.0
3B-4	Enjoy learning in mathematics class	29	0	0.0	0	0.0	2	6.9	3	10.3	24	82.8
3B-5	Understand that communication is a way to construct own meanings of mathematical concepts.	29	0	0.0	0	0.0	2	6.9	9	31.0	18	62.1
3B-6	Practice persuading their opinions to others.	29	0	0.0	4	13.8	7	24.1	12	41.4	6	20.7

Note. Missing value n=7 (less than 1year, suspended processing value n=1).

According to Table 13, 79% of participants responded finding out the correct answer was the most important role, by contrast, only 7% of participants disagreed with this transmissive statement. Nevertheless, many participants (62%) showed that practicing persuading their opinions to others was one of the students' roles, although practicing this process is not considered a component of mathematical communication according to the literature review. Moreover, this Item 3B-6 also belongs to the transmissive view.

7.3.3 Roles of a student group

As for the roles of a student's group, almost all participants agreed with 3C-1 (Listen to other's explanation, 97%), and 3C-2 (Find out the meaningful learning point, 100%) as detailed in Table 14.

TABLE 14. Responses to the statements of a student's group (Part 3-C)

Item	Statement		Strongly disagree		Disagree		Neutral		Agree		Strongly agree	
	"In Mathematics classroom at primary school, A student group can ~"	Total	*f*	%	*f*	%	*f*	%	*f*	%	*f*	%
3C-1	Listen to one student's explanation about his/her solution.	29	0	0.0	0	0.0	1	3.4	9	31.0	19	65.5
3C-2	Find out the meaningful learning point "Aha" moment.	29	0	0.0	0	0.0	0	0.0	7	24.1	22	75.9
3C-3	Point out the lack of other students' answers to mathematical problems.	29	7	24.1	6	20.7	6	20.7	9	31.0	1	3.4

Note. Missing value n=7 (less than 1year, suspended processing value n=1).

Nevertheless, only less than half of the participants disagreed with the statement about pointing out the lack of others' answers (45%) as the role of a group, followed by agreement (34%) and neutral (21%). In addition, the correct answer (3B-3) to the problem was determined (3A-1), and the process of teacher controlling (3C-3), persuading one's opinion (3B-6) can be linked to the transmissive perspective. In this perspective, it emphasises that teachers need to find the correct answer with guided teaching. However, mathematical communication is to express one's mathematical thoughts in various ways and listen to each other's opinions. Therefore, students are the centre of learning and teachers play a role in supporting them from a constructivist perspective.

7.4 Research Question 2-3: Future teaching mathematics

The aim of RQ 2-3 was to explore how to use PCT's mathematical experience and beliefs, knowledge of communication implementation in future teaching mathematics, and what experiences universities should provide.

7.4.1 How to apply communication to the future teaching mathematics

All participants could answer the below open question, and fifteen participants explained their future teaching mathematics (see Table 15).

P4-B. I imagine you are teaching mathematics in the future. How would you communicate in a versatile manner in your mathematics teaching?

TABLE 15. Result of analysing Part 4-B using data-driven open coding

Category	*f*	%	Main Statement
Using multimodal language	10	66.7	*'Kielentäminen (languaging)'* *'Using spoken and written language, pictures and drawings'*
Using variable materials	5	33.3	*'For communication with a variety of instruments (images, bodies, action...)'*
Asking about mathematical thoughts	3	20.0	*'I always ask the student to tell you how he has come to a solution.'*
Encouraging to express	3	20.0	*'Encourage students to communicate their thought process*
Give feedback	1	6.7	*'Supportive feedback is important'*
Functional teaching	1	6.7	*'The teaching of mathematics should be functional, and students should get concrete, everyday mathematics when studying mathematics.'*

Note: Participants' (n = 15) statements were multiple checked under category.

The results In Table 15, numerous participants answered that they would use multimodal language to teach mathematics (67%), followed by using a variety of teaching tools (33%), asking questions about mathematical thinking (20%), and encouraging them to express their thoughts (20%). One participant expressed that s/he will feedback to support students' mathematics learning and use functional mathematics teaching.

7.4.2 Mathematical experience for future teaching mathematics

All participants could participate in the below open question, and seventeen participants expressed their needs (see Table 16).

P4-A. I imagine you are teaching mathematics in the future. What mathematical experience do you most want to provide in university?

TABLE 16. Result of analysing Part 4-A using data-driven open coding

Category	*f*	%	Main Statement
Teaching methods	10	58.8	*'Variable ways of practicing mathematics'* *'Variable instruments and storytelling'*
Making learning environment	5	29.4	*'How to motivate students'* *'Enjoyment, A-ha moment, curious, happy, fun'* *'Safe (to share their thoughts regardless of mistakes)'* *'Challenging situation, support them'*
Mathematical knowledge	2	11.8	*'Teaching the basics (multiplication and dividing)'* *'Means modeling (Välineillä mallintaminen)'* *'Qualification of a mathematics subject teacher'*
Students' language	2	11.8	*'Understanding students' thinking'* *'Dialog'*
Other	1	5.9	*'I'd like to teach the Varga-Neményi -way.'*

Note: Participants' (n = 17) statements were multiple checked under category.

According to the results in Table 16, about 60% of participants mentioned that they needed various methods of teaching tools, and about 30% of participants presented that they wanted to know how to create a learning environment such as learning motivation, curiosity, happiness, joy, and safety. In addition, 12% of participants answered that they wanted universities to provide specific knowledge of the detailed areas of mathematics and experience in learning how to understand students' language and thoughts.

Consequently, this chapter was intended to explain the result of analysing data according to all responses to the survey including Likert-scale items and Open-ended items. Pre-service Class Teachers' perspectives on mathematical communication were formed by mathematical beliefs based on their past mathematics learning experiences. Their mathematical beliefs have been changed due to new experiences, which could affect cyclical behaviours of future mathematics teaching. A joint display of data procedure (Creswell & Creswell, 2018, p220) was used in merging the different forms of data at the last stage. Tables and columns with major concepts on the horizontal axis include quantitative and qualitative responses to concepts. Appendix 10 shows the results of a merging of experience learning mathematics and mathematical beliefs in the Joint display. Accordingly, all participants agreed with the statement that teachers used traditional teaching methods as textbooks and worksheets in their past mathematics learning experiences. However, various teaching methods are being used in the current classroom environment, and participants also wanted to learn new teaching methods that could teach students of various levels. In addition, participants could be divided into four groups according to their past learning experiences, and it was found that past experiences and present beliefs were closely related to future teaching mathematics as a result of analysing statements by group. Appendix 11 addresses the overall results of the components of communication as a meaning-making process in mathematics teaching and learning using distinct colors according to their similar concepts. Factors constituting communication as meaning-making were encouragement environment, emotional environment, creative environment, and teacher knowledge. Furthermore, the role of communication as meaning-making indicated a particularly low consent rate for items in which teachers can monitor and evaluate the student's learning process.

8 DISCUSSION

This study aimed to explore Preservice Class Teachers' views on communication as a meaning-making process in mathematics teaching and learning based on their previous schooling experiences and new learning experiences. A convergent sequential mixed method was used in the study and the survey including Likert-scaled and open-ended items was conducted. Research question 1 required to respond to how participants' past schooling experiences in learning mathematics were and what kinds of new experiences affected their mathematical beliefs. Research question 2 addressed participants' perception of communication as a meaning-making process in mathematics teaching and learning. The questionnaire included items addressing the components, purpose, benefits of communication as well as the roles of teacher, students, students' groups for consisting of communication. Several responses to their needs for future mathematics teaching were connected to the RQ2. A joint display of data procedure was used to merge different forms of data in the previous chapter, which will be discussed in this chapter followed by implications, limitations, and suggestions for future research.

8.1 The connection between mathematical experience and mathematical beliefs

According to the theoretical framework, the Mathematical learning model explains the self-system that connects past mathematical experiences with new experience situations (Malmivuori, 2001). This system actively interacts with mathematical beliefs, schemata, and mathematics behaviour patterns. In addition, the system needs to explore their past mathematics experiences, personality, and mathematics learning situations to activate and develop pupils' self-system process. In this study, factors that mathematics teaching, past mathematics experiences, and new learning experiences were investigated, excluding personality as personal factors.

8.1.1 Mathematics teaching methods influencing mathematics learning

As I have shown in the 7.1.3 section, PCTs almost perfectly agreed that traditional teaching methods by their teachers were used. This result was consistent with the research problem about using only traditional ways in mathematics classrooms (Joutsenlahti & Kulju, 2017). Although the Finnish National Core Curriculum for Basic Education 2014 (FNCC, 2014) also emphasised the support to develop pupils' skills of communication, it would have been difficult for teachers to learn various teaching methods at least 20 to 30 years ago in the educational field. However, about 77% of participants responded today's teaching methods are different from the traditional methods before in basic education. Besides, nearly half of PCTs answered the methods were different in upper general education which requires entrance preparations for higher education. Therefore, as the curriculum changes, the educational field also gradually accepts various teaching methods, on the contrary, there are still obstacles of limited time and numerical evaluation in the classroom. Research by Kaya and Aydin (2016) also indicated that teachers needed more time for communication in the mathematics classroom regardless of the curriculum or grading students on a numeric scale. Hence, changes in teaching methods were obviously linked to how to evaluate or assess students' mathematics skills not only for good grades. The findings of this relationship reveal that evaluation or assessing can be an obstacle to changing teaching methods. In conclusion, this result can suggest the possibility that a precise understanding on mathematical communication evaluation will have a positive effect on implementation of communication.

8.1.2 Mathematics learning experience influencing mathematical beliefs

Malmivuori (2001) categorized mathematical beliefs into four items, however, the constructivist view was only explained in the beliefs about natural mathematics categories. In the other categories of beliefs about self with mathematics, and beliefs about mathematics learning and teaching, those concepts are corresponding to White et al. (2006)'s categorize; belief of ones' talent, belief of the difficulty of mathematics, and one's liking of mathematics.

When it comes to previous experiences in learning mathematics, there were two parts, one was the positive learning experiences in mathematics during basic education and the upper general education. The other was the struggle experiences in mathematics during the same periods, which was related to difficulty in solving mathematics problems. All participants were divided into four groups depending on their experiences as variables. Additionally, all of them had new learning experiences, for example, a lecture, an observation, a teaching practice, an intensive course, or something else at universities. Most of them experienced positive mathematics related to their mathematical beliefs at universities including Teacher Training Programme. For the same reason, the constructivist belief showed slightly higher than the transmissive belief, even though there was a nonsignificant correlation between transmissive and constructivist belief sum variables. The results of having more constructivism are similar to previous findings (Hannula et al., 2005; Oksanen et al., 2015) that the mathematical beliefs of Finnish teachers have changed into constructive processes within 40 years. In other words, a majority of participants consider the mathematics teaching and learning process is student-oriented, and teachers play the role of learning environment creator.

8.1.3 "Teaching-learning-teaching" cycle

Specifically, Group 2 who had a strong positive and weak struggle learning experience in mathematics showed high confidence in solving difficult problems on their own, which was related to White et al. (2006)'s categorizes of beliefs. This group mentioned that they would provide challenging problems in their future teaching to give students strong motivation to learn mathematics. Group 1 who had strong positive and difficulty in mathematics learning expressed the traditional and the various teaching ways are needed using real-life contexts. On the other hand, Group 3 and Group 4 had weak positive experiences in mathematics expressed relatively strong negative emotions to self-image, such as 'scary, difficulty, horrible, too slow, problematic, harmful'. It is unknown why or what kinds of negative mathematical experiences affect their thoughts. However, one participant noted that s/he got too much pressure to get a good grade from previous teachers, and s/he would focus not to give those feeling to future students at all.

These findings can be interpreted through the previous research that the ideas and beliefs formed by teachers influence their teaching behaviour (Fernandes, 1995), or the “teaching-learning-teaching” cycle (Lindgren, 1998). It can be assumed that Group 3 and 4 will more concentrate on making a safe, secure, less stressful environment rather than Group 2 will prepare challenging tasks. While Group 1 will consider how to conduct two different teaching ways according to individual levels. Consequently, this study cannot comment on statistically significant differences in mathematical beliefs related to experience but can be explored that it is key to understand the detailed categories of mathematical beliefs formed by PCTs’ previous experiences.

8.2 Perception of a process of meaning-making in mathematics

As I have been noted in chapter 2, understanding mathematics concepts is defined as meaningful learning (Carley, 2011; Hirschfeld-Cotton, 2008; Hoyles, 1985; Kaya & Aydin, 2016; Krzywacki et al., 2016; Lee, 2015; Teledahl, 2017). At this point, the communication process helps students create meaning by their understanding of mathematics. Therefore, exploring PCTs’ perspective on the roles of communication as meaning-making in mathematics was a crucial bridge between Teacher Training Programme and future teaching mathematics.

8.2.1 Main components of communication as meaning-making

According to the results (see 7.2.3 section), there were four components which were categorized by open-ended answers as follows: Encourage environment, Emotional environment, Creative environment, and Teachers’ knowledge. Firstly, an encourage environment component is consistent with the purpose of mathematical communication to support or encourage students to express, share and reflect on their ideas (Kaya & Aydin, 2016). Moreover, having an opportunity to express mathematics thoughts was the most important benefit of using communication. Secondly, an emotional environment represented by respect, safety, appreciation, understanding mistakes, not being afraid of wrong answers, and enjoyment was a key component of communication. In addition, the most chosen answer for the purpose of using communication was to provide enjoyable learning. For creating an enjoyable learning environment, students and a student's group should listen carefully to others' explanations and find out the meaning point "Aha" moment. Thirdly, a creative environment meant using various strategies of discussion, questions, and group movements or expressing mathematical thinking in multiway. In addition, providing mathematical problems related to the actual context could be a component of the creative environment. Kaya and Aydin (2016) also emphasise that planned interaction in a classroom setting should be conducted for students to learn vigorously mathematical thinking (Hirschfeld-Cotton, 2008). Lastly, the teacher's knowledge of communication as meaning-making was required to understand the students' mathematical language. They recognized teachers as professionals who knew all kinds of components for meaning-making.

Overall, four components in communication as meaning-making are intimately connected to the teachers' roles and responsibility to construct a communicable classrooms environment (Bratina & Lipkin, 2003; Carley, 2011; Kaya & Aydin, 2016; Kim & Jeon, 2019; Krzywacki et al., 2016; Vale & Barbosa, 2017). Which enables students to express their thoughts, develop mathematical understanding, problem-solving skills, critical thinking skills, and multiliteracies.

8.2.2 Weak links between communication and assessment

Specifically, monitoring the learning process and assessing mathematical understanding was chosen as the purpose of communication by a small number. None of the participants selected the option to enhance mathematical achievement. Moreover, in the benefit of using communication, only five participants thought it was important. However, meaningful mathematical communication skills help teachers to follow how students have thoughts through their solution to mathematical problems (Joutsenlahti & Kulju, 2017), and teachers can identify clues about students' true mathematical understanding, or mathematical errors (Mooney et al., 2009). For instance, assessing the learning process is different from evaluating one's mathematical knowledge or calculating functional skills using pen and paper. Therefore, it is necessary to present the next steps to understand what PCTs knew and did not know, to see if there is a creative solution to the problem, and to support individual learning processes. The reason why the perspective on assessing showed somewhat different from the results of previous literature studies can be explained as follows. As mentioned in the previous section, most participants considered that assessing was regarded as a means of grading or as a tool for entering higher school. In other words, it can be said that it suggests that PCTs do not accurately recognize the purpose and benefits of using evaluation in mathematical communication rather than negatively recognizing the evaluation itself.

Summing up, PCTs considered that the evaluation of monitoring the process of learning had less relationship in mathematical achievement although it is one of the important purposes and benefits of using mathematical communication. These results contradict the findings in previous literature studies that mathematical communication helps improve the mathematical performance of students with poor academic performance or special needs (Baxter et al., 2005; Kim & Jeon, 2019). Therefore, it is necessary to help PCTs understand accurately the meaning of assessing in using mathematical communication.

8.2.3 What do PCTs need for future teaching mathematics

Regarding support for future teaching mathematics, the majority of participants expressed that they need various experiences on how to create a learning environment. For example, they wanted to understand students' mathematics language illustrated by speaking, writing, or drawing. For their future mathematics teaching, they showed strong confidence in using multimodal language to encourage students to express their ideas. As a result, it was found that the meaning of the essential concept of mathematical communication was effectively considered in future teaching mathematics for PCTs. However, none participants mentioned that understanding students' mathematics languages is an important process of evaluation.

8.3 Implications for Teacher Training Programme

Overall, the results of PCT's experience and mathematical beliefs suggest how to recognize the importance and role of communication as meaning-making in mathematics teaching and in what direction the Teacher Training Program (TTP) should develop. There are three implications for TTP.

First, it is difficult to change the natural belief in mathematics formed from previous experiences and the belief in learning mathematics on its own, but the new experience in TTP was generally positive and meaningful enough to strengthen mathematical knowledge. Nevertheless, it is also strongly necessary to examine their experiences and beliefs in detail and to consider the strengths and weaknesses of their perception. For example, in the case of PCTs with strong positive and weak difficulty in mathematics learning experiences, they tend to provide challenging tasks to future students. This case includes the advantages of scaffolding for an individual learner, but the disadvantages of the performance gap between learners, which are to be considered together. Conversely, in the case of PCTs with a very negative past experience, it is necessary to think about how to overcome it. For the same reason, I suggest that them think about how a teacher can help in a future classroom where students have the same concerns.

Second, it can be suggested that how to create a class environment that promotes meaningful communication be provided as a course of TTP. There are various levels of learners in one classroom, and PCTs want to learn new teaching methods other than the traditional methods of independently supporting individual learning. For instance, an emotional, creative, and encouraging learning environment can support students with low academic levels to participate in active learning activities.

Third, it is necessary to help PCTs redefine the meaning of assessing. This allows them to understand students' mathematical learning process and eventually enhance a positive belief that mathematical communication can develop students' mathematical abilities. Also, based on the existing research that mathematical communication has a positive relationship to the improvement of mathematical academic achievement, this study suggests that teaching on this in TTP needs to be provided.

9 CONCLUSION

This study has found the connection between mathematical experience and mathematical belief. Although the results of today's teaching methods differ from those of earlier traditional teaching methods, PCTs still aspire to learn a variety of teaching methods. Furthermore, they recognized that it was necessary to create an encouraging, creative, and emotional environment in order to realize mathematical communication, and pointed out that the professional knowledge of teachers is required. The result of their perception of the evaluation implied that it still needs to change the perspective of assessing the mathematics learning process. Therefore, understanding what PCTs knew and didn’t know, or whether they have creative solutions to the mathematics problem should be utilized for supporting the individual learning processes rather than grading. And the analysis of mathematical beliefs according to their previous mathematical experience laid the foundation for subsequent research data. All participants were divided into four groups based on their previous positive and struggle experiences in mathematics, which showed interestingly different findings among the groups. Moreover, it can be assumed that they will versatile mathematical communication in a variety of ways, for instance, challenging problems related to real-life contexts, or not giving students negative feelings.

Consequently, although four components (Encourage, Emotional, Creative environment, and Teacher’s knowledge) of communication as a meaning-making process are equally important factors, it should be considered that PCTs’ mathematical beliefs and experience could lead to focus on a specific component in their future teaching. This approach will prove useful in expanding our understanding of how the Teacher Training Programme supports positively PCTs’ new experiences. Therefore, this study suggests that further research will be able to explore a meaningful connection between theory and practice in mathematical communication.

9.1.1 Limitations

The study was limited in several ways. Firstly, a limited sample size (n = 35) of Preservice Class Teachers in two universities could have affected the results of this study. Although their open-ended responses could be analysed as a qualitative approach, I could not find significant statistical findings. However, it could be able to explore the answers to this study in depth through the high-quality answers. Secondly, language used in the survey was the limitation. While proficiency in the English language was enough for participants, some participants commented that there were hard-to-understand statements in English. Because all participants were native Finnish language speakers and had a varied level of fluency in English. To ensure that the intended meaning of participants' statements or selected responses, ID coding was performed on each participant, and individual data were analysed consistently. Thirdly, due to the one-time short-term research method, only weak connectivity could be found for the effect of their experiences and beliefs on future teaching activities. Nevertheless, it was possible to grasp the overall view of how to use their current mathematical beliefs and ideas for future teaching mathematics.

9.1.2 Future research

The issue of perspectives on mathematical communication is an intriguing one that could be usefully explored in further research. Further studies need to be carried out in order to validate PCTs' views on a larger scale for finding significant statistics factors. Using a broader range of PCTs' participants could shed more light on meaningful links among learning experience, mathematical beliefs, and behavior. In addition, further research can also be conducted to expand to In-service teachers and Teacher educators who charge the responsibility of mathematics education. That research can find out an effective solution to PCTs' struggle and confusion to use communication in real class. Which enable to find a meaningful connection between theory and practice for implementation. Finally, based on the results of this study, it can be developed into research on the effective use of mathematics teaching and learning as a meaning-making process in the classroom.

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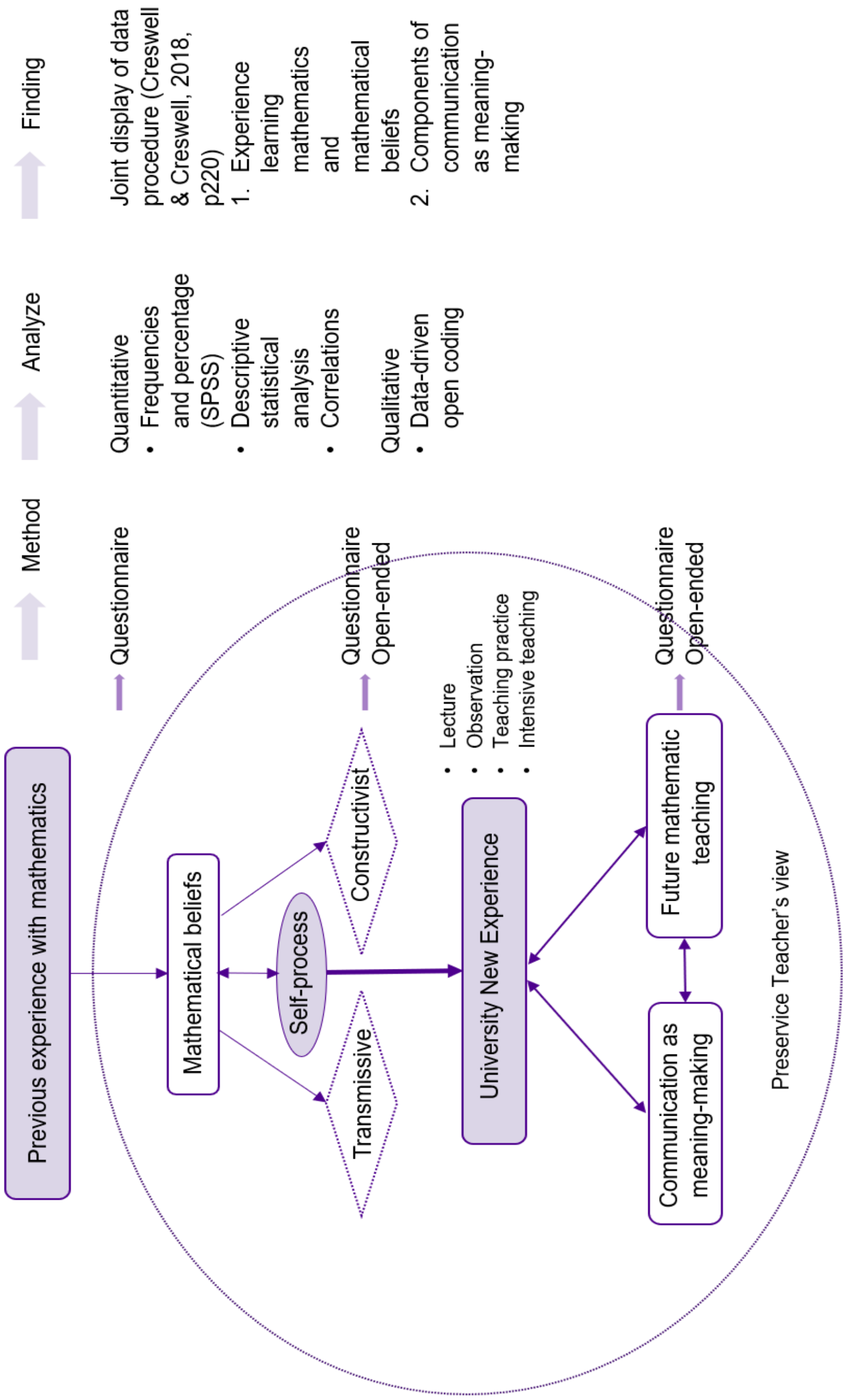
APPENDICES

Appendix 1: Research design of a mixed-methods process for study



Appendix 2: Background Information

1. Gender: ☐ Female ☐ Male ☐ Prefer not to say
2. University: ☐ Tampere University ▫ University of Jyväskylä (Kokkola University Consortium Chydenius)
3. How many years have you been studying in the Department of Teacher Education at the university?

   ☐ Less than 1 year ☐ 1-2 years ☐ 2-3 years ☐ more than 4 years

4. Which University Mathematics program experiences have you participated in? (Select all that apply)

   ☐ Mathematics lecture as a subject at University

   ☐ Mathematics classroom observation

   ☐ Mathematics teaching practice

   ☐ Broaden Mathematics Practice (Tampere University)

   ☐ None

   ☐ Other

5. How many years did you teach primary school students before entering the University?

   ☐ None

   ☐ Less than 5 years

   ☐ 5-10 years

   ☐ More than 10 years

Appendix 3: Part 1-A. Content of Beliefs about mathematics, mathematics learning and teaching (White et al., 2006)

No.	Statement
1	Mathematics is only calculation.
2	Mathematics problems given to students should be quickly solvable in a few steps.
3	Mathematics is a beautiful, creative, and useful human endeavour that is both a way of knowing and a way of thinking.
4	Right answers to mathematical problems are much more important in mathematics than how you get them.
5	Mathematics knowledge is the result of students interpreting and organising the information gained from their experiences.
6	Students are rational decision-makers capable of determining for themselves what is right and wrong in mathematical problems.
7	Mathematics learning is being able to get the right answers to mathematical problems quickly.
8	Periods of uncertainty, conflict, confusion, surprise are a significant part of the students' mathematics learning process.
9	Young students are capable of much higher levels of mathematical thinking than has been suggested traditionally.
10	Being able to memorise facts is a critical skill in mathematics learning.
11	Mathematics learning is enhanced by activities that are built upon and respect students' experiences.
12	Mathematics learning is enhanced by challenges in a supportive environment.
13	Teachers should provide mathematical activities which result in problematic situations for students.
14	Teachers and the mathematics textbook – not the student – are the authorities for what is right or wrong in answers to mathematical problems.
15	The mathematics teacher's role is to transmit mathematical knowledge to students.
16	The teacher's duty is to evaluate that student has mathematical knowledge.
17	Teachers should recognize that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding.
18	Teachers should negotiate social norms with the students in order to develop a cooperative learning environment in which students can construct their mathematical knowledge.

Appendix 4: Part 1-B. Content of past experiences and mathematics (Jong & Hodges, 2013)

No.	Statement
1	I had several positive experiences with mathematics during basic education (1-9 grade).
2	I had several positive experiences with mathematics during general upper secondary school.
3	I have struggled with mathematics during basic education (1-9 grade).
4	I have struggled with mathematics at upper secondary school.
5	The way mathematics is taught today is different from the way I learned it during basic education (1-9 grade).
6	The way mathematics is taught today is different from the way I learned it at upper secondary school.
7	During basic education, I mostly learned mathematics in traditional ways (i.e., textbooks, worksheets, rules, lectures).
8	During general upper secondary school, I mostly learned mathematics in a traditional manner (i.e., textbooks, worksheets, rules, lectures).
9*	I had several positive experiences with mathematics during Mathematics courses at University.
10*	I had several positive experiences with mathematics during teaching practice at University.
11*	I had several positive experiences with mathematics during an intensive mathematics course at University. (i.e., Broaden Mathematics Practice in Tampere University)
12	(open-ended) What past experiences have influenced your thoughts about teaching and learning mathematics the most? And how did it affect you?

Note. Item number 9–11(*) were added to explore whether the teacher training programs was perceived as a positive experience, and only the corresponding participants responded in item number 9, 10, and 11.

Appendix 5: Part 2. Content of Important components for communication as meaning-making in mathematics class

Subcategory	No.	Statement
A. The purpose of communication	1	On the list, choose the three most important purposes to use communication in the mathematics classroom. ● To express pupils' mathematical thinking ● To develop pupils' problem-solving skill ● To develop pupils' mathematical understanding ● To develop pupils' critical thinking skill ● To develop pupils' multiliteracies ● To enhance pupils' mathematical achievement ● To provide an enjoyable learning environment for students ● To monitor the learning process of students ● To assess mathematical understanding of students ● Other answers
B. The benefits of communication	1	On the list, choose the three most important benefits of using communication in the mathematics classroom. ● Students have an opportunity to express their mathematical thinking. ● Students listen to others' thoughts. ● Students can make meaning of mathematical concepts as their understanding. ● Students enjoy the mathematics class. ● Students can use mathematics in real-life contexts. ● Students can use mathematical language. ● Teachers can monitor the learning process of students. ● Teachers can assess the mathematical understanding of students. ● Other answers
C. The components for communication as meaning-making	1	(open-ended) For communication as meaning-making in the mathematics classroom, what are the most important components? (Multiple answers)

Appendix 6: Part 3. Content of Roles of teachers, students, and the group of students

Subcategory	No.	Statement (The four-point Likert-scale)
A. Teacher's roles "Teacher should-"	1	*Control all learning processes of students.
	2	Try to understand students' mathematical language
	3	Encourage students to express their mathematical thinking
	4	Plan strategies, such as discussion, questioning, and group activities.
	5	*Not give any feedback to students to get their own understanding of learning.
	6	Use mathematical problems related to real-life contexts.
B. Student's roles "Students can-"	1	Express their mathematical thinking in versatile ways, such as speaking, writing, and drawing.
	2	Listen carefully to other students' explanations about their solutions to mathematical problems.
	3	*Finally find out a correct answer.
	4	Enjoy learning in mathematics class
	5	Understand that communication is the way to make own meaning of mathematical concepts.
	6	*Practice persuading their opinions to others.
C. Roles of a student group "A student group can-"	1	Listen to one student's explanation about his/her solution.
	2	Find out the meaningful learning point "Aha" moment.
	3	*Point out the lack of other students' answers to mathematical problems.

Note. Each subcategory has 1 statement (*) based on the transmissive perspective. The results were analyzed within an extension of mathematical beliefs.

Appendix 7: Part 4. Content of Future teaching for communication as meaning-making in mathematics class

Subcategory	No.	Description
A. Mathematical experience for future teaching mathematics	1	(open-ended) I imagine you are teaching mathematics in the future. What mathematical experience do you most want to provide in University?
B. How to apply communication to the future mathematics class	2	(open-ended) I imagine you are teaching mathematics in the future. How would you communicate in a versatile manner in your mathematics class?

Appendix 8: Descriptive statistics of Mathematical Beliefs (Part 1-A)

Item	Statement	Mean	SD	Min	Max	Skewness	Kurtosis
PA1	Mathematics is only calculation.	1.40	0.695	1	4	2.056	4.866
PA2	Mathematics problems given to students should be quickly solvable in a few steps.	2.06	0.873	1	4	0.730	0.224
PA3	Mathematics is a beautiful, creative, and useful human endeavour that is both a way of knowing and a way of thinking.	4.06	0.938	1	5	-1.481	2.819
PA4	Right answers to mathematical problems are much more important in mathematics than how you get them.	1.86	0.944	1	5	1.413	2.600
PA5	Mathematics knowledge is the result of students interpreting and organizing the information gained from their experiences.	3.83	0.568	3	5	-0.031	0.056
PA6	Students are rational decision-makers capable of determining for themselves what is right and wrong in mathematical problems.	3.23	0.973	1	5	-0.694	-0.206
PA7	Mathematics learning is being able to get the right answers to mathematical problems quickly.	1.66	0.938	1	4	1.444	1.292
PA8	Periods of uncertainty, conflict, confusion, surprise are a significant part of the students' mathematics learning process.	4.00	0.767	2	5	-0.830	1.178
PA9	Young students are capable of much higher levels of mathematical thinking than has been suggested traditionally.	3.54	0.657	2	5	-0.496	0.064
PA10	Being able to memorize facts is a critical skill in mathematics learning.	3.46	1.039	1	5	-0.547	-0.498
PA11	Mathematics learning is enhanced by activities that are built upon and respect students' experiences.	4.06	0.725	2	5	-0.578	0.681
PA12	Mathematics learning is enhanced by challenges in a supportive environment.	4.34	0.684	3	5	-0.562	-0.680
PA13	Teachers should provide mathematical activities which result in problematic situations for students.	4.11	0.718	3	5	-0.174	-0.969
PA14	Teachers and the mathematics textbook – not the student – are the authorities for what is right or wrong in answers to mathematical problems.	2.46	1.039	1	4	0.121	-1.098
PA15	The mathematics teacher's role is to transmit mathematical knowledge to students.	3.37	1.060	1	5	-0.822	-0.353
PA16	The teacher's duty is to evaluate that student has mathematical knowledge.	3.97	0.785	2	5	-1.108	1.783
PA17	Teachers should recognize that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding.	4.06	0.873	2	5	-0.678	-0.088
PA18	Teachers should negotiate social norms with the students in order to develop a co-operative learning environment in which students can construct their mathematical knowledge.	3.74	1.094	1	5	-0.739	0.412

Note. SD = standard deviation, Min = minimum value, Max = maximum value.

Appendix 9: Descriptive statistics of Mathematical past experience (Part 1-B)

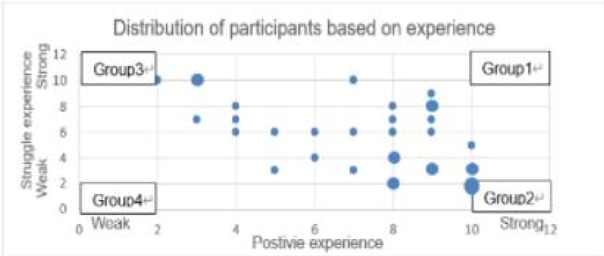
Item	Statement	Mean	SD	Min	Max	Skewness	Kurtosis
PB1	I had several positive experiences with mathematics during basic education (1-9 grade).	3.77	1.308	1	5	-0.971	-0.240
PB2	I had several positive experiences with mathematics during general upper secondary school.	3.46	1.314	1	5	-0.520	-1.022
PB3	I have struggled with mathematics during basic education (1-9 grade).	2.43	1.481	1	5	0.522	-1.267
PB4	I have struggled with mathematics at upper secondary school.	3.14	1.458	1	5	-0.202	-1.486
PB5	The way mathematics is taught today is different from the way I learned it during basic education (1-9 grade).	3.77	0.877	2	5	-0.907	0.373
PB6	The way mathematics is taught today is different from the way I learned it at upper secondary school.	3.49	1.040	1	5	-0.626	0.287
PB7	During basic education, I mostly learned mathematics in traditional ways (i.e., textbooks, worksheets, rules, lectures).	4.71	0.519	3	5	-1.644	2.002
PB8	During general upper secondary school, I mostly learned mathematics in a traditional manner (i.e., textbooks, worksheets, rules, lectures).	4.60	0.695	2	5	-2.056	4.866
PB9	I had several positive experiences with mathematics during Mathematics courses at University.	4.04	1.216	1	5	-1.091	0.132
PB10	I had several positive experiences with mathematics during teaching practice at University.	4.17	0.576	3	5	0.018	0.123
PB11	I had several positive experiences with mathematics during an intensive mathematics course at University. (i.e., Broaden Mathematics Practice in Tampere University)	4.00		4	4	-	-

Note. SD = standard deviation, Min = minimum value, Max = maximum value. PB = Part 1-B.

Appendix 10: Joint display of Experience learning mathematics and Mathematical beliefs

<table>
<tr><th colspan="3">Quantitative: Percentage showing agreement based on 5 Likert-scaled items (N = 35)</th><th colspan="2">Qualitative: Main statements of open-ended items using coding process (N = 21)</th></tr>
<tr><th>Teaching ways</th><th>Mathematical Beliefs</th><th>Distribution of participants based on experiences (Variable 1: Positive, Variable 2: Struggle)</th><th>Group (N)
New Experience</th><th>How did experiences affect your leaning and teaching mathematics?</th></tr>
<tr>
<td rowspan="4">Almost perfectly agreement of traditional teaching way by their teachers (97% during basic education, 94% during Lukio)

Different teaching way from the past is conducted today as agreement (77% in basic education, 54% in Lukio)</td>
<td rowspan="4">Nonsignificant correlation of .08 (p = n.s) between transmissive and constructivist belief sum variables.

Nevertheless, constructivist (M = 8.6, SD = 2.9) of belief showed more higher than transmissive (M = 4.1, SD = 1.9).</td>
<td rowspan="4">

Statements of variables 1 & 2 were recoded as a scaled value (Strongly Disagree = 1, Disagree = 2, Not to know = 3, Agree = 4, Strongly Agree = 5) and summed into the two parts. With SPSS analysing, positive experience sum variable and struggle experience sum variable average were significantly correlated (r = .79, p < .01).
<table>
<tr><th rowspan="2">Group</th><th rowspan="2">N</th><th colspan="2">Past experience</th></tr>
<tr><th>Positive</th><th>Struggle</th></tr>
<tr><td>1</td><td>12</td><td>Strong</td><td>Strong</td></tr>
<tr><td>2</td><td>14</td><td>Strong</td><td>Weak</td></tr>
<tr><td>3</td><td>8</td><td>Weak</td><td>Strong</td></tr>
<tr><td>4</td><td>1</td><td>Weak</td><td>Weak</td></tr>
</table>
Note. N = Total participants</td>
<td>G1 (N = 6)
n = 5 at university (positive)
n = 1 in Internship (positive)</td>
<td>All (n = 6) changed their thought toward the variety teaching ways for their future teaching.
Traditional ways and new ways are needed.
Creating mathematical concepts related to real-life.</td>
</tr>
<tr>
<td>G2 (N = 9)
n = 4 at university (positive)
n = 5 not mention</td>
<td>Respondents (n = 4) changed their thought toward the variety teaching ways for their future teaching.
Parent (n = 1) strong positively affected his/her mathematics learning.
Strong confidence to solve the difficult problems by themselves.</td>
</tr>
<tr>
<td>G3 (N = 5)
n = 2 at university (positive)
n = 1 at university (Negative, engineering)
n = 1 not mention
n = 1 other</td>
<td>Respondents (n = 2) changed their thought toward the variety teaching ways for their future teaching.
Unchanged (n = 1) participants mentioned to need more time to think and make it.
Negative feeling 'scary, difficulty, horrible, too slow' was expressed.</td>
</tr>
<tr>
<td>G4 (N = 1)
Not mention</td>
<td>This participant got much pressure to get a good grade from previous teachers. It is related to negative feelings such as 'problematic, harmful' to self-image.</td>
</tr>
</table>

Appendix 11: Joint display of Roles in Communication as Meaning-making

	Quantitative: Percentage showing agreement or disagreement based on 5 Likert-scaled items (N = 29)					**Qualitative: Main statements of open-ended items using coding process.** *Displayed by Category (%) 'main statements'		
	Purpose	Benefits	Teacher's role	Students' role	Group's role	Component (N=17)	Needing (N=17)	Future teaching (N=15)
high frequency ← Three most important purpose and benefits → Low frequency	To provide an enjoyable learning (n=22) To develop math understanding (n=20)	Have an opportunity to express math thoughts (n=29)	Encourage students to express (97%).			**Encourage environment** (64%) 'support' 'encourage'	**Making learning environment** (29%) 'motivate' 'enjoyment' 'Aha moment' 'curious' 'challenging' 'support'	Asking about mathematical thoughts (20%) Encouraging to express (20%)
	To express math thoughts (n =20) To develop problem-solving skills (n=19)	Can use math in real-life (n=18) Can make meaning of math concepts (n=15)		Listen carefully to other's explanations (93%). Enjoy learning (93%).	Listen to one's explanation (97%). Find out meaning point "Aha" moment (100%).	**Emotional environment** (55%) 'respect' 'safe' 'appreciation' 'Understanding mistakes' 'Not being afraid of wrong answers'		
	To develop critical thinking skill (n=10) To develop p multiliteracies (n=6)	Can listen to others (n=12) Can monitor the learning process (n=10) Can enjoy (n=9)	Plan strategies (discussion, questioning, and group activities) (97%). Use mathematical problem related to real-life contexts (97%).	Express mathematical thinking by versatile ways (97%). Understand that communication is a way to construct own meaning (93%).		**Creative environment** (36%) 'create' 'Physical objects' 'Diversly options'		Using variable materials (33%) Using multimodal language (67%) Functional teaching (7%)
	To monitor the learning process (n=4) To assess math understanding (n=4)	Can use math language (n=7)	Try to understand students' math language (97%). Control all learning process (27%).			**Teachers' Knowledge** (18%) 'professionalism' 'knowing'	Teaching methods (59%) Students' language (12%)	Give feedback (7%)
	To enhance math achievement (n=0)	Can assess (n=5)	Not give any feedback to students (disagree 94%).	Finally find out a correct answer (93%). Practice persuading their opinions to others (62%).	Point out the lack of other students' answers (disagree 45%, agree 34%).	Other		