PAPER • OPEN ACCESS

Optimization procedure of a new type counter–flow heat exchanger

To cite this article: R Karvinen 2021 *J. Phys.: Conf. Ser.* **2116** 012093

View the article online for updates and enhancements.

Optimization procedure of a new type counter–flow heat exchanger

R Karvinen

Tampere University (TAU), Finland

E-mail: karvinenreijo@gmail.com

Abstract. Plenty of studies exist in books and archival journals dealing with different types of heat exchangers. In the paper an analytical approach to evaluate the overall heat transfer coefficient of a new type heat exchanger is presented. Derived equations are applied to multi-objective optimization of a very large economizer of a recovery boiler, when the exchanger mass and size should be small but simultaneously heat transfer rate high.

1. Introduction and description of the problem

Heat exchangers have been widely studied but the analysis of an exchanger in figure 1 does not exist even counter -flow exchangers are dealt in the literature [1,2,3,4]. Figure 1 shows one channel of a very huge economizer of a recovery boiler, the key component of pulp and paper industry, where water is heated by flue gases formed in combustion of black liquor. High pressure water is heated in pipes which are connected together with welded fins. Typically the wall length L can be even 30 m and the width of one wall some meters containing tens of pipes. The walls are located in a parallel position forming clusters of many walls. The total weight can be 10^6 kg. The distance between the walls in figure 1 must be large enough to make possible mechanical cleaning with pressure steam.

In optimization it is enough to consider only one channel. The flow distribution is uniform and similar at every channel entrance. In actual practise the pipe wall thickness t_p in figure 2 is small compared to the pipe diameter d , and its heat conductivity k_p is high so that we can solve conduction in a pipe using one dimensional analysis as in the traditional treatment of fins. In our problem heat transfer coefficients on the water and gas side (h_w, h_g) and also mean temperatures ($T_w(x), T_g (x)$) are different on both sides of the pipe. Because gas and water flows are turbulent heat transfer coefficients depend only slightly on the wall temperature distribution. Thus we can use correlations of fully developed turbulent flows. For evaluation of heat transfer coefficents and friction factors   in a pipe and in a channel for instance using correlations of Gnielinsky and Petukhov are found in the literature. The proper evaluation of gas side heat transfer is challenging , and the validation requires experimental data.

## 2. Temperature distribution of the fin and pipe in a channel wall

First heat transfer between gas and liquid flows must be solved.   We approximate the combination of a pipe and fin in figure 2a with one quarter marked with a dotted line in figure 2b. If heat transfer coefficients from gas to the outside pipe surface h_g and from water inside the pipe h_w are known, the pipe temperature distribution T_p is governed by equation

$$k_p t_p \frac{d^2 T_p}{dx_p^2} + h_g (T_g - T_p) - h_w (T_{p} - T_w)= 0 \tag{1}$$
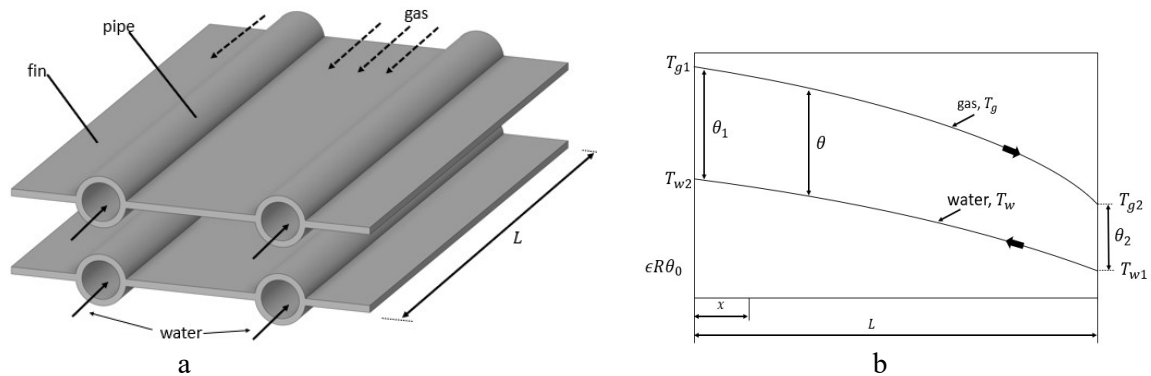
Figure 1. Schematic of heat exchanger channel walls formed of pipes and fins (a) and mean temperature distributions of water and gas (T_g, T_w) in the flow direction (b).
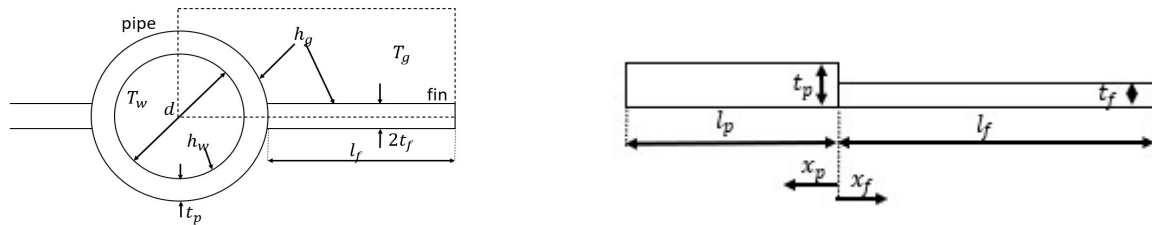
Figure 2. Gross-section of a channel wall (a) and approximated geometry (b).

where T_w and T_g are local liquid and gas mean temperatures in a wall cross-section at the distance x in figure 1b. Equation (1) differs from the traditional fin equation but if we define a new reference temperature

$$T_{ref} = \frac{h_g T_g + h_w T_w}{h_g + h_w} \tag{2}$$

we get using variable $\theta_p = T_p - T_{ref}$ a similar type of equation and boundary conditions as in the treatment of fins (see figure 3 for notations)

$$\frac{d^2\theta_p}{dx_p^2} = m_p^2 \theta_p , \quad \frac{d\theta_p(l_p)}{dx_p} = 0, \quad \theta_p(0) = \theta_{po} \tag{3}$$

An analytical solution and heat flux at the base $x_p = 0$ are found in every text book [1,2]

$$\frac{\theta_p(x_p)}{\theta_{po}} = \frac{\cosh m_p(l_p - x_p)}{\cosh m_p l_p}, \rightarrow \quad \emptyset'_p = -k_p t_p \frac{d\theta_p(0)}{dx_p} = k_p t_p m_p \tanh(m_p l_p)\theta_{po}, \quad m_p^2 = \frac{h_g + h_w}{k_p t_p} \tag{4}$$

Equation (4) includes temperature difference θ_{po} , which is not known but depends also on the fin temperature and must be solved simultaneously with the pipe equation (3). The fin temperature distribution T_f and heat flux at the connecting point $x_f = x_p = 0$ are using $\theta_f = T_f - T_g$

$$\frac{\theta_f(x_f)}{\theta_{fo}} = \frac{\cosh m_f(l_f - x_f)}{\cosh m_f l_f}, \rightarrow \quad \emptyset'_f = k_f t_f m_f \tanh(m_f l_f)\theta_{fo} , \quad m_f^2 = \frac{h_g}{k_f t_f} \tag{5}$$

Heat fluxes $\emptyset'_p$ and $\emptyset'_f$ must be equal which gives ($\theta_{po} - \theta_{fo} = T_g - T_{ref}$, θ_{fo} is negative)

$$\frac{\theta_{po}}{-\theta_{fo}} = \frac{t_f k_f m_f \tanh(m_f l_f)}{t_p k_p m_p \tanh(m_p l_p)} \tag{6}$$

3. Heat transfer between the flows
We can solve the local total heat transfer between water and gas flows either by solving heat transfer

from the pipe   to water by substituting the temperature distribution (4) into equation

$$\emptyset'_w = \int_0^{l_p} h_w\,(T_p - T_w\,)dx_p \;=\; \frac{h_w}{m_p}\tanh(m_p\,l_p)\,\theta_{po} + h_w(T_{ref} - T_w\,)l_p \tag{7}$$

or from gas to the   water pipe and fin using also result (5)

$$\emptyset'_g = \int_0^{l_p} h_g\,(T_g - T_p)dx_p + \int_0^{l_f} h_g\,(T_g - T_f)dx_f = h_g(T_g - T_{ref})l_p \; -\frac{h_g}{m_p}\tanh(m_p\,l_p)\,\theta_{po} -$$
$$\frac{h_g}{m_f}\tanh(m_f\,l_f)\,\theta_{fo} \tag{8}$$

Unknowns θ_{po} and θ_{fo} are solved by noting that $T_{po} = T_{fo}$ and $\emptyset'_w = \emptyset'_g$ from above equations. After some algebraic manipulations the result is same as equation (6).

If heat transfer is between liquid and gas as in many applications the heat transfer coefficient h_w on the liquid side is much higher than h_g on the gas side, and in addition   the thermal conductivity of pipe material is high. Thus the pipe is almost isothermal and its temperature is close to T_{po}. Equations (7) and (8) give limit values $\emptyset'_{wi}$ and $\emptyset'_{gi}$ for a high thermal conductivity isothermal pipe. When $m_p \rightarrow$ 0 and $T_p = T_{po}$ they give, because $\displaystyle\lim_{m_p\to 0}\frac{h_w}{m_p}\tanh(m_p\,l_p)\,\theta_{po} = h_w l_p \theta_{po}$

$$\emptyset'_{wi} = h_w(T_{po} - T_w\,)l_p \tag{9}$$

$$\emptyset'_{gi} = h_g(T_g - T_{po})l_p\; + k_f\,t_f\,m_f\tanh(m_f l_f)\theta_{fo} \tag{10}$$

4. Heat exchanger performance

Usually the behaviour of a heat exchanger is expressed using the overall heat transfer coefficient

$$U = \emptyset'\,/\,(T_g - T_w) \tag{11}$$

A simple equation of the overall heat transfer coefficient U is obtained to a high thermal conductivity pipe from equations (9) and (10)

$$1/U = 1/(h_g l_p + k_f t_f\,m_f\tanh(m_f\,l_f) + 1/h_w\,l_p \tag{12}$$

which is obtained easily also by forming the energy balance of a constant temperature pipe so that the heat flux from the fin to pipe + from gas to pipe is the same as the heat flux from pipe to water.

Using the notations in figure 1b the local heat transfer between gas and water in the element dx at the distance x is

$$d\emptyset = \dot{C}_g dT_g = \dot{C}_w dT_w = U\theta dx \tag{13}$$
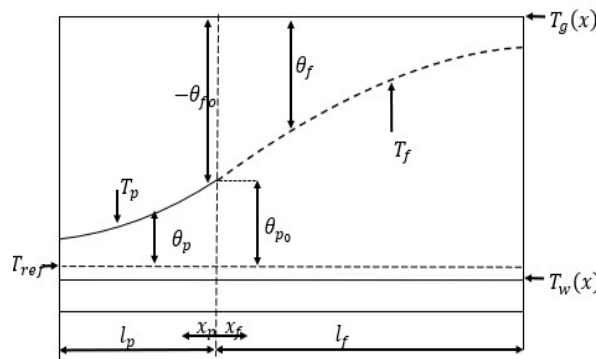
Figure 3. Pipe and fin temperature distributions of a channel wall (T_p, T_f) at the location x in Fig.1a
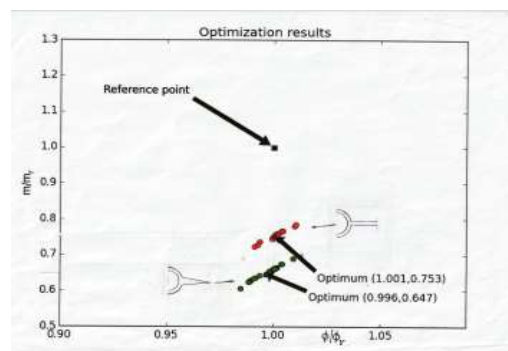
Figure 4. Pareto-optimal weight vs. heat transfer of straight and triangular fin walls.

where $\dot{C_w} = \rho_w \dot{m}_w$ and $\dot{C_g} = \rho_g \dot{m}_g$. U is the overall heat transfer coefficient and θ is the difference between local mean temperatures in figure1b. The change of θ in an element at the distance x is

$$d\theta = dT_w - dT_g = \frac{U\theta dx}{\dot{C_g}}\left(\frac{\dot{C_g}}{\dot{C_w}} - 1\right) = -z(1 - R)\ \text{dx} \tag{14}$$

It gives when integrated over the exchanger length L the ratio of flow mean temperature differences θ_1 and θ_2 at the inlet and outlet in figure.1b

$$\frac{\theta_2}{\theta_1} = e^{-z(1-R)} \quad , \quad z = \frac{UL}{\dot{C_g}} \quad , \quad R = \frac{\dot{C_g}}{\dot{C_w}} \tag{15}$$

When the performance of the heat exchanger is expressed using the concept of heat transfer effectiveness ϵ, which gives the ratio of real heat flux rate to the maximum value, equation (15) can be arranged using the effectiveness concept and notations in figure 1b

$$\epsilon = \frac{T_{g1} - T_{g2}}{T_{g1} - T_{w1}} = \frac{1 - e^{(R-1)z}}{1 - Re^{(R-1)z}} \tag{16}$$

5. Multi-objective optimization

If the goal is to maximize heat transfer rate and minimize weight we have plenty of variables $(d, t_p, t_f, l_p, l_f, L)$. Manufacturing and standards give some limits in practise. Several methods can be used for the optimization such as gradient -based, gradient- free and population- based methods. We have used population based method (PSO) [8], where heat transfer is evaluated using above equations. Figure 4 presents a practical example on how heat transfer rate and weight of the economizer change in optimization compared to an existing reference boiler in practise. Performance of the reference equipment was in a reasonable agreement with calculations. If straight fins could be replaced by triangular fins in economizer the weight is about 15 % less, but manufacturing is difficult. Weight of a triangular fin is 41% lighter than a plate fin if the geometry is optimized [1,2], but optimization of a fin geometry is more difficult than e.g. in electronics due to manufacturing difficulties [9]. Use of non-dimensional variables in Section 4 helps to understand the performance of the exchanger.

6. Conclusions

In the paper analytical equations are given for a new type heat exchanger. Developed theoretical model is applied to a rapid multi-objective optimization (PSO) when the weight and heat transfer capacity of the economizer are conflicting objective functions. It was observed that existing design instructions can be modified and increased understanding of basic phenomena improves cost effectiveness. Using optimization new ideas can be found how the ratio of heat transfer/weight is increased.

References
[1] Eckert E R G and Drake R M 1972 *Analysis of heat and mass transfer* (McGraw-Hill)
[2] Sunden B 2012 *Introduction to heat transfer* (WIT Press)
[3] Shah R K and Bell K J 2000 *Heat Exchangers* (Handbook of Thermal Engineering) ed F Kreith (CRC Press)
[4] Laskowsky R 2015 *Heat Mass Transfer* **51** pp 1111-1119
[5] Faheri A 2007 *ASME J. Heat Transfer* **129** pp 1268-1276
[6] Bahadori A 2011 *Appl Energy* **88** pp 4191-4196
[7] Pirttiniemi M 2016 Optimization of a recovery boiler economizer *Master`s thesis* (TUT,Finland)
[8] Coello C, Pulido G and Lechuga M 2004 *IEE Trans Evol Comput* **8(3)** pp 256-279
[9] Lampio K and Karvinen R 2018 *Heat Mass Transfer* **54** pp 2571-2580