Tampere University

Zuhair ul Haq

# HYBRID SPATIAL INTERPOLATION

## RSS Based Indoor Localization

Master of Science Thesis
Faculty of Information Technology and Communication Sciences
Examiners: DSc (Tech) Joni Kämäräinen
and DSc (Tech) Jukka Talvitie
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# ABSTRACT

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GNSS is a constellation of satellites that provides global positioning, navigation, and tracking in outdoor spaces. However, due to complex infrastructure, the satellite signals become weak in the indoor environment, and therefore, GNSS cannot provide reliable positioning. The indoor environment comes packed with radio signals generated by WIFI and Bluetooth access points. The RSS of the radio signals in indoor spaces can be used to provide accurate indoor positioning. Furthermore, radio access points deployment is increasing steadily in indoor spaces, which makes it ideal for indoor positioning.

RSS-based indoor localization is a two-step process, the first step being RSS fingerprinting, where RSS measurements are recorded along with reference location coordinates to generate radio maps. The second step is the positioning step, where real-time RSS measurements are collected and compared with radio maps to estimate the user's location. However, fingerprinting is an arduous task that requires time and workforce. This leads to the need for methods that can generate radio maps from little recorded radio measurements.

The goal of the thesis was to analyze various interpolation and extrapolation methods in traditional RSS fingerprinting and investigate their effects on overall indoor positioning. The advantage of these extrapolation and interpolation methods is to reduce the overhead of collecting data and covering those areas which are not accessible to users. In addition, these methods can also help automate the process of fingerprinting, leading to a much wider deployment of indoor positioning services at a lower cost. The thesis evaluates three different interpolation and extrapolation methods based on five evaluation parameters: mean error, maximum error, building detection, floor detection, and consistency of indoor positioning.

For evaluation purposes, actual RSS measurements were recorded using smartphones in an indoor environment. The experimental building was a multistory office space consisting of complex indoor infrastructure. The test RSS measurements were classified into edge and non-edge measurements and studied separately. Out of three methods compared, a hybrid method that combines Delaunay triangulation and RSS-based spatial interpolation performed the best.

The hybrid method harnesses the advantages of two interpolation and extrapolation methodologies; Delaunay triangulation with linear interpolation and spatial interpolation. The use of Delaunay triangulation makes the process simpler with very little computational complexity. The RSS-based spatial interpolation uses a physical radio path loss model that makes it feasible for deployment in diverse indoor environments.

Keywords: RSS, interpolation, extrapolation, spatial, delaunay

# PREFACE

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In the end, words might not be enough to thank my parents. But, I am where I am today because of them. Their endless support unceasingly pushed me forward to face challenges, always making me a better version of myself.

Tampere, 1st October 2021

Zuhair ul Haq

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# LIST OF SYMBOLS AND ABBREVIATIONS

| | |
|---|---|
| $\mathbf{C}$ | Path Loss |
| $\mathbf{\Psi}$ | Noisy radio signal |
| $\alpha$ | alpha - Path Loss Constant |
| $\beta$ | beta - Path loss penalty for complex indoor infrastructure |
| $\mu$ | Standard deviation of RSS distribution |
| $\sigma$ | RSS noisy signal |
| $\mathbf{c}$ | Speed of Light |
| $\mathbf{m}$ | Gradient of straight line |
| $\mathbf{n}$ | RSS noise |
| AOA | Angle of Arrival |
| AP | Access point |
| BLE | Bluetooth Low Energy |
| dBM | decibel milliWatt |
| FDOA | Frequency Difference of Arrival |
| GNSS | Global Navigation Satellite System |
| GPS | Global Positioning System |
| HIRM | HERE indoor radio mapper |
| IPS | Indoor Positioning System |
| LOS | Line of Sight |
| PDF | Probability distribution function |
| RSS | Received Signal Strength |
| RSSI | Received Signal Strength Indication |
| TDOA | Time Difference of Arrival |
| TOA | Time of Arrival |
| UWB | Ultra Wide Band |
| WLAN | Wireless local area network |

# 1. INTRODUCTION

## 1.1 Background

The 21st century has brought a revolution in the field of wireless communication and global connectivity. The latest wireless technologies allow us to use the existing radio signals to provide location-based services. Some of the famous use cases include satellite-based and network-based location services, where the radio signals are used to determine user location.

The GNSS implements a satellite-based localization methodology that includes groups of satellites that are orbiting around the earth and transmitting radio signals that enable us to determine the user position [1]. Initially, the GPS, a component of GNSS, was deployed by the US Department of Defense for military use; however, after its civilian accessibility, a vast number of global use cases have been observed, from global shipments tracking to roads navigation [2]. Furthermore, with the increasing need for location-aware and tracking applications on smartphones in the last decade, GNSS has become a multi-billion-dollar industry. It is estimated that by the end of 2029, the GNSS industry revenue will reach € 325 billion [3].

The GNSS satellites are continuously transmitting radio signals towards earth, which are picked by receivers in electronic devices, and user location is determined with an average accuracy of 5 meters [3]. However, near and in deep indoor environmental setups, the radio signals reception becomes very weak, and it becomes impossible to determine indoor location with reasonable accuracy [4, 5, 6]. For example, a positioning error of greater than 2 meters in two-dimensional indoor space can guide a user to the wrong hallway or room. Therefore, it is essential to have an accurate representation of the Indoor position. Secondly, the GNSS does not perform well in congested urban environments, and the indoor environment is full of such complex infrastructures [5]. Furthermore, the GNSS system is not very energy efficient and requires a lot of battery power. Therefore, it is evident that GNSS has some shortcomings, and multiple positioning technologies are required to fulfill the need for accurate and seamless positioning in indoor and outdoor environments.

The Indoor Location-based services market has seen a sudden increase in the last decade due to its increasing need [7]. The advance and smart infrastructures of big shopping malls, airports, skyscrapers, hospitals, high technology parks, and offices have led to the fast and increased implementation of indoor location services. Most indoor location-based applications on Google's Play Store and Apple's App store provide indoor tracking, assistance, and security surveillance. However, IPS use cases are not limited to these only [8]. Nowadays, more and more businesses are adapting to indoor positioning services and harnessing its advantages in their products [9, 10].

In the past few years, many indoor localization systems have been proposed [11]. These Indoor localization systems can be classified into Ultrasonic, Infrared, and Radio based systems [11, 12]. Among them, the radio-based system is most famous and is widely used one [7]. In this latest era of wireless communication, we have widespread deployment of WLAN in offices, homes, and public places, and the low infrastructure cost of BLE beacons has led to their increased deployment. Therefore, many location service providers use RSS of the existing WLAN/BLE signals to provide indoor location services. The main advantage is that no new infrastructure or special hardware is required to measure the RSS value of these signals on smartphones [7]. Secondly, the demand for wireless communication infrastructure is constantly increasing, resulting in vast coverage areas [10].

## 1.2 Thesis Objectives

A typical indoor positioning algorithm consists of two phases, the training phase, and the positioning phase. In the training phase, radio data is collected, and radio maps are generated. Then, real-time radio data is compared with radio maps in the positioning phase, and user location is estimated. The collection of data is referred to as fingerprinting and is an uphill task. Fingerprinting cannot be done on all areas of the experimental site as some areas are inaccessible or covered with indoor structures like furniture. In this thesis, the focus was to study the various forms of interpolation and extrapolation methods used during the fingerprinting step. The advantage of these extrapolation and interpolation methods is to reduce the overhead of collecting data and covering those areas which are not accessible to users.

The thesis is organized as follow:

- Chapter 2 discusses the various technologies with which indoor positioning is performed these days. It also discusses the various challenges and uses cases of indoor positioning.
- Chapter 3 focuses on RSS-based indoor positioning methods. The second part of this chapter explains how radio models are generated and how the positioning is done during the estimation phase.

- Chapter 4 mentions general interpolation and extrapolation methods used in the fingerprinting stage of indoor positioning.
- Chapter 5 explains the process of data collection and analysis done for this thesis. The second part of the chapter mentions some hybrid methods that can be used for interpolation and extrapolation. In the last part, we compared the performance of these hybrid methods based on five different evaluation parameters.

# 2. INDOOR POSITIONING SYSTEMS

Indoor positioning system or IPS can be referred to as a service that provides accurate location of a user or object in an indoor space, such as an office, apartments, and indoor shopping areas [13]. IPS can be categorized based on the infrastructure they are using. For example, IPS using radio infrastructure for localization are referred to as wireless technologies. However, some IPS also perform localization without the need for any special infrastructure. This chapter focuses on various types of wireless and non-wireless positioning technologies commonly used these days. Nowadays, various wireless technologies are used to perform indoor positioning. Some IPS try to leverage the existing indoor wireless systems to perform indoor positioning, while others require specialized hardware to position in an indoor environment. Usually, IPS which uses specialized devices, provide a more accurate position.

The field of WIFI-based indoor positioning is relatively mature, and many real-time indoor positioning systems use this technology. The reason for that is directly associated with the ongoing popular demand for smartphones. Smartphones come with built-in WIFI modules and, therefore, have a better built-in capability for WIFI-based positioning [14]. The main advantage is that the user does not need to carry a separate WIFI receiving module. On the service provider side, most indoor spaces have WIFI access points available, whose signal measurements can be used for positioning. In some cases, WIFI-based positioning is fused with sensors to further increase the accuracy [14]. In the next section, WIFI positioning systems have been divided into range-based and range-free categories.

## 2.1 Range-Free WIFI Positioning Systems

Range-free indoor positioning involves the process of fingerprinting. Fingerprinting can be categorized into offline and online modes. A wireless map is established in the offline mode, which consists of the received signal strength of multiple WIFI access points and the corresponding location coordinates [14]. In the online mode, real-time RSS is compared with the wireless map collected in offline mode to estimate the location. Based on how the wireless map is collected and the kind of model being used to compute the position in online mode, range-based WIFI positioning can be categorized further based on deterministic and probabilistic methods they are using.

### 2.1.1 Deterministic

Deterministic algorithms stores the signal strength of WIFI access points as the location fingerprint information in the Wireless map. The location fingerprint contains the RSS value and the global latitude and longitude of that location. The location fingerprint is given as

$$\mathbf{F} = [\mathbf{x}, \mathbf{y}, \mathbf{RSS}] \tag{2.1}$$

In equation 2.1, $\mathbf{x}$ and $\mathbf{y}$ represent the global coordinates, $\mathbf{RSS}$ is the received signal strength. The Indoor positioning system then uses a deciding algorithm to obtain the location information by comparing the feature fingerprints received on mobile devices to wireless maps in the database. The most common deterministic method to compute the user location is euclidean distance.

The RSS value at a fixed location changes significantly with the change in the surrounding environment. A robust method has been introduced to update the wireless map in the database through numerous hardware modules connected to the wireless network [15]. This method eliminated the need to update the RSS wireless maps with the change of environment. The WIFI receiver in smartphones varies highly, therefore, causing changes in the RSS values. The procrustes method [16] can be used to change the fingerprints obtained from different devices to a standard format. Using the Weighted k-nearest neighbor method while estimating the positioning on these standard fingerprints can help to obtain higher accuracy [16].

In the estimation or so-called online phase, any localization algorithm's performance depends on the collected fingerprints. However, fingerprinting is an demanding task, and not all the spaces in indoor locations can be fingerprinted. In recent years, models have been developed to control the overhead of fingerprinting in non-accessible indoor locations. A vector regression model was introduced, which can estimate the unmeasured RSS based on the neighboring values [17, 18]. This model helps not only increases the coverage area, but the performance accuracy is also increased.

### 2.1.2 Probabilistic

The probabilistic models provide a higher level of accuracy than the deterministic model at the cost of high computational complexity. The idea behind probabilistic models is to compute the joint probability distribution function of each WIFI access point. Once the PDF for each access point is computed, they are joined together to obtain a combined distribution function [14]. The combined distribution function acts as a fingerprint in the database. In the online phase, the real-time received RSS value is compared to the values of the joint distribution function, and location is estimated.

Working with probabilistic models, one of the main issues faced is the size of the finger-prints database. This problem restricted the use of probabilistic models to use in real-time systems. A spectral compression system was introduced to eliminate this problem [19]. The methods got rid of noise from the fingerprints based on the correlation between the nearby Fingerprints and saved the valid information.

### 2.1.3 Fusion Technologies

Fusion technologies in positioning refer to combining two or more sensors or modules to achieve better positioning results [14]. Smart Devices these days combine various sensors e:g Bluetooth, cameras, and inertial sensors. Data from these sensors can be fused with WIFI modules which have proven to be very effective in increasing the accuracy of Indoor Positioning. For the sake of discussing this topic in more detail, this topic is discussed in the section 2.5.

## 2.2 Range-Based WIFI Positioning Systems

The idea behind range-based indoor positioning is to find the distances between the transmitting and receiving device. The best case for finding the distance is the transmitter and receiver being in direct LOS; however, due to complex indoor infrastructure, the radio signals are prone to standard path loss phenomenon e:g, diffraction, reflection, etc. The distance can be found in a number of ways, as discussed below:

### 2.2.1 Time of arrival:

In TOA, the distance between the transmitting module and receiving module is calculated based on the signal's propagation time transmitted from the sender [20]. The speed of the signal transmitted is assumed to be equal to the speed of light. Figure 2.1 shows the basic setup for TOA Setup. The distance is calculated based on the following equation:

$$\mathbf{distance} = \mathbf{c} * \mathbf{TOA} \tag{2.2}$$

In equation 2.2 $\mathbf{c}$ is the speed of light, and $\mathbf{TOA}$ is the time taken by the signal to reach from transmitting device to receiving device. Multiple distances are calculated from each access point to increase the accuracy of positioning. The algorithm requires a very stern synchronization of time between the transmitter and receiver.

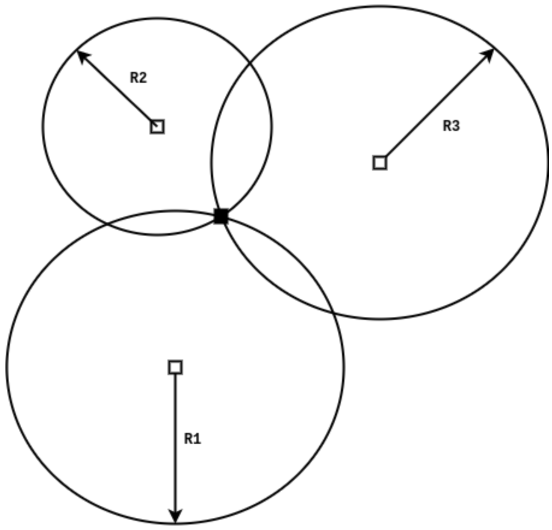


***Figure 2.1.*** *Setting for indoor positioning based on TOA [20].*

### 2.2.2 Time difference of arrival:

The accuracy of TOA is highly dependent on the time synchronization between the transmitter and receiver. TOA requires absolute time synchronization between devices to provide good accuracy. However, in real-time systems, there are usually errors. In TDOA, the receiver receives signals from multiple transmitters and calculates the time difference of arrival of signals [20]. As a result, the distance difference between the transmitters is obtained. This method requires simultaneity between the transmitters. Figure 2.2 shows TDOA setup in a cellular system. The formula calculates TDOA in the equation 2.3:

$$(\mathbf{d_1} - \mathbf{d_2}) = \mathbf{c} * \mathbf{TDOA} = \mathbf{c} * (\mathbf{TOA1} - \mathbf{TOA2}) \tag{2.3}$$

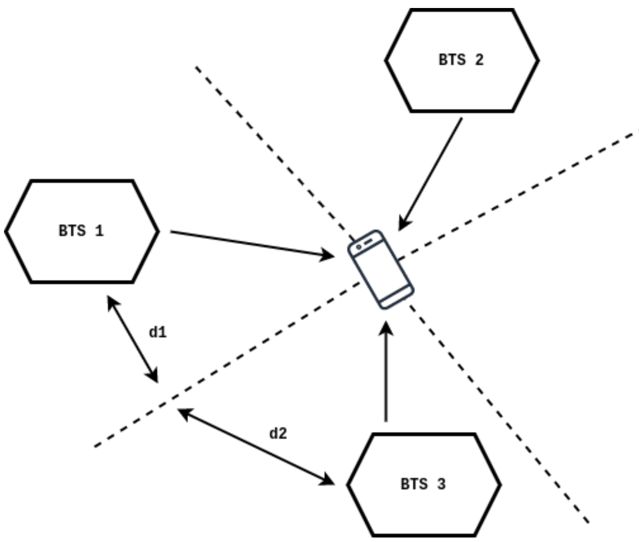


***Figure 2.2.*** *Setting for indoor positioning based on TDOA [20].*

where $\mathbf{d1}$ and $\mathbf{d2}$ are the distance between the receiver and first and second transmitter, respectively. $\mathbf{c}$ is the speed of light. $\mathbf{TOA1}$ and $\mathbf{TOA2}$ are the time of arrival of signals from transmitter one and transmitter two, respectively.

### 2.2.3 Angle of arrival:

To implement AOA, signals are sent by a mobile station, and at least two access points are required to receive those signals. Having two access points makes it easier to obtain those incident lines between the access points and mobile station by the angle of the transmitted signal. The intersection of these lines is used to measure the location of the mobile station [14]. Figure 2.3 shows AOA algorithm scenario in mobile phone cases.

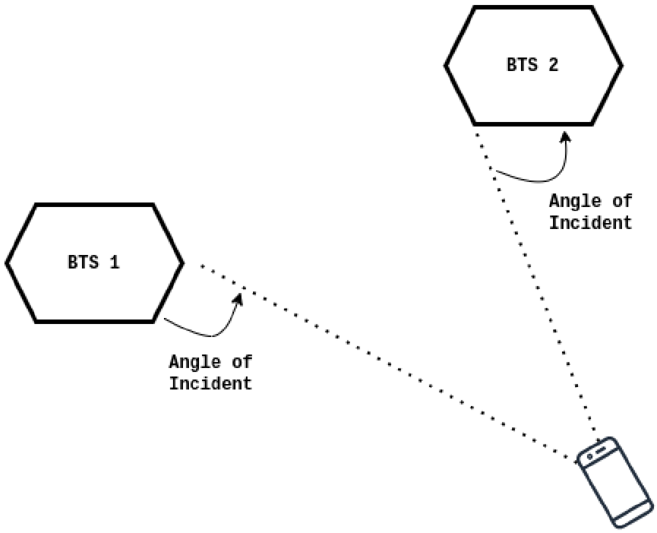


***Figure 2.3.*** *Setting for indoor positioning based on AOA.*

A single AOA measurement combining with TOA or RSS measurement can also be used to estimate the mobile station's location [20]. This method requires complex and high-cost hardware components to measure the angle of the signal transmitted by the mobile station.

### 2.2.4 Frequency Difference of Arrival:

In the FDOA algorithm, the speed of the mobile station is used to calculate the location. Due to speed changes between the base stations and mobile stations, the frequency of the received signal by mobile stations changes with the Doppler effect [14]. FDOA is a challenging algorithm since the user movements in the indoor environment are negligible.

## 2.3 UWB Based Positioning

Ultra-wideband uses a higher bandwidth (>500 MHz) for transmitting information across devices. In recent years, research has been focused on using the UWB range for indoor positioning [21]. The difference between a traditional indoor positioning system and a UWB is that UWB allows transmission of radio signals without interfering with other frequencies in the same radio bandwidth. Moreover, the transmission speed is relatively fast, making it ideal for a real-time positioning and tracking system. UWB based positioning systems can provide cm-level accuracy [21]. The downside for UWB based indoor positioning is that special tags are to be placed on devices, and a particular UWB transmitter should also be installed in an indoor location. However, many big smartphone companies announced their latest mobile devices with preinstalled tags, e.g., Samsung and Apple. Figure 2.4 shows UWB client-based setup in a cellular system.

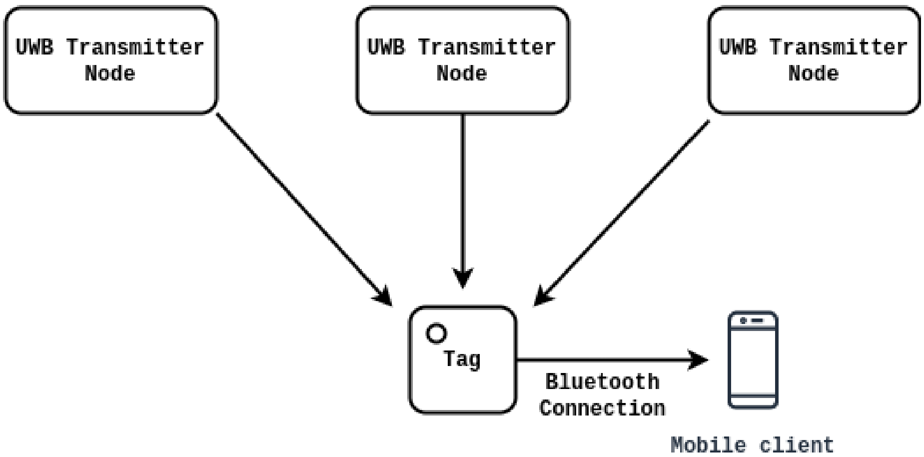


***Figure 2.4.*** *Setting for indoor positioning based on UWB.*

## 2.4 Bluetooth Low Energy

Bluetooth low energy is currently the hot topic in the field of indoor positioning. BLE is a part of the Bluetooth 4.0 release [22]. The significant difference from its predecessors is the power consumption, both at the transmitting and receiving end. BLE beacons have a low range of transmission; however, they can last up to 100 times than standard Bluetooth devices with their low power consumption [22]. For indoor positioning, several Bluetooth beacons are installed in indoor premises, and their RSSI value is used for fingerprinting. This thesis is based on BLE beacons for data collection, and this topic is discussed in more detail in the following chapters. In BLE-based indoor positioning, the RSSI method is used to develop fingerprints.

RSS value is the received power of the signal triggered by the receiving device. In RSSI, the receiving device is the network adapter usually found in smartphones for receiving WIFI and Bluetooth signals. Currently, the RSS-based positioning is famous because of its adaptability. It is usually used for tasks of human tracking and human detection [14]. The RSS value can either calculate the distance between the transmitter and receiver or develop a fingerprint database. This thesis is based on RSS-based indoor positioning; therefore, this topic will be discussed in more detail in the following chapters.

## 2.5 Fusion Technologies

This section describes the fusion of various positioning technologies. The fusion can either be between wireless technologies or a combination of wireless and sensor data. There are multiple ways to perform hybrid positioning; however, we will only mention currently being discussed in the research.

### 2.5.1 Magnetic Positioning

The geomagnetic field is distributed in space all around the world. The flux value of the geomagnetic field is different at different places, which makes it ideal for indoor positioning [14]. In the outdoor environment, the magnetic field is relatively stable; however, in indoor spaces, due to complex infrastructure, the magnetic field is constantly changing. These changes are location-dependent and are highly affected based on the indoor environment. The WIFI signals can be combined with continuously changing geomagnetic field values for indoor positioning.

The advantages of magnetic field for indoor positioning include no deployment of extra infrastructure; it is available everywhere. The magnetic field has three main components; Inclination angle, declination angle, and the horizontal component [23]. However, magnetic positioning also has some downsides; only three elements can be used during fingerprinting for data collection, making it a bit unreliable [24]. Multiple magnetic sensors can be used to solve this problem. However, that adds complexity to the positioning system. Further, the change in indoor infrastructure significantly affects the magnetic flux values [24]. Most real-time indoor positioning systems that use magnetic technology often combine it with some standard positioning technology e.g., WIFI, Bluetooth [14].

### 2.5.2 Inertial Measurements

Smartphones these days come packed with various sensors. In recent years, research has been carried out to use these sensors to improve indoor positioning accuracy. For example, accelerometers and gyroscopes can be used to provide indoor inertial navigation [14]. However, using raw data from these sensors cannot provide accurate results. These measurements often come with an error, and using these measurements as a function of time leads to error accumulation. Therefore, often data from these sensors are combined with standard WIFI-based positioning to improve its accuracy further.

A hybrid positioning system can be developed by fusion of RSS-based indoor positioning and acceleration sensor. The system uses WIFI positioning as a base combined with the number of steps, speed for the accelerometer, and gyroscope to make point estimates more precise [25].

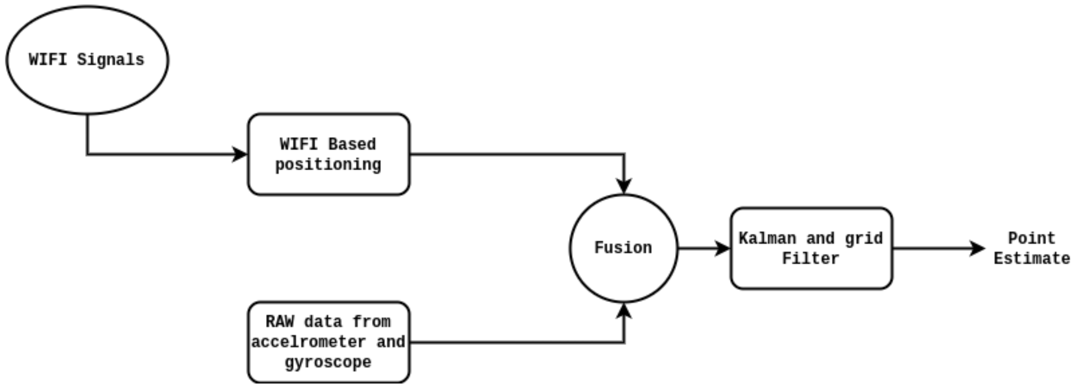


***Figure 2.5.*** *KAFG setup for hybrid positioning using accelerometer and gyroscope [25].*

The data from accelerometers and gyroscope contains errors, and these errors can accumulate over time, thus causing irregular paths and changes in trajectory. To solve this problem, KAFG [26] used a Kalman filter and Grid filter after the fusion of data. The research proved that such filters could improve the position estimate and reduce the error overhead from raw sensors. Figure 2.5 shows a hybrid positioning system based on KAFG research.

### 2.5.3 Visual Positioning

Visual positioning refers to using a camera module for localization. A lot of research and effort has been put into this positioning technology recently. The image data from the camera can be used for localization and for creating 3D indoor maps [14]. Working only with images can provide good positioning accuracy; however, it can only work in Line of sight. Combing camera data with WIFI-based positioning can not only increase the positioning accuracy but also the coverage area can be increased [14]. Creating a database of images also creates a need for more significant memory segments, and therefore, more processing power is required to extract features from images. As a result, the computational cost of querying from an extensive database also increases.

WIFI-based indoor positioning and visual positioning can be fused in a parallel method to get better tracking [27]. Such a system is shown in the figure 2.6. Contrary to this, a camera sensor can also be used to create depth maps. These depth maps help identify the human body and provide better positioning results [28].

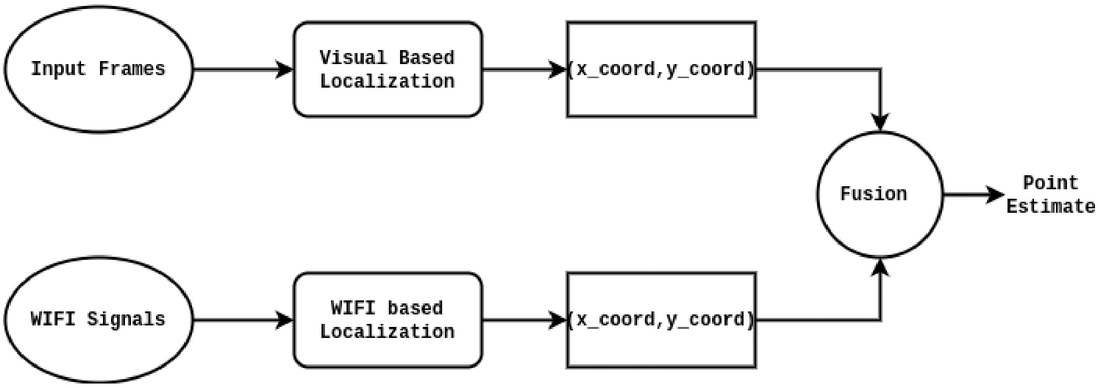


***Figure 2.6.*** *Hybrid positioning system based on WIFI and visual positioning [14].*

## 2.6 Applications of Indoor Positioning

The applications of indoor positioning systems are increasing day by day. Especially with the wide range of use of smartphones, indoor positioning applications increased. Some of the applications are mentioned below.

### 2.6.1 Marketing and Customer support

The skyscrapers are getting higher each day, and the shopping malls are increasing in size. This makes indoor positioning an essential part of the indoor customer experience. Indoor positioning makes it easier to navigate to the right shops using mobile applications [29]. On the other hand, the office spaces also benefit from indoor positioning. For example, most of the major airports of the world use indoor positioning so that the passengers can reach their flight gates in time. Usually, WIFI-based positioning is used in malls and airports.

### 2.6.2 Health Sector

The health sector is highly benefiting from indoor positioning these days. The indoor positioning system helps the medical staff to reach their patient and assist them in no time [29]. For example, in an emergency, Indoor positioning makes it easier to navigate to their patient's rooms. Further, indoor positioning also keeps track of patients so that their safety is not compromised.

### 2.6.3 Security

One of the vital use of indoor positioning these days is in the security domain. For example, the access of employees in sensitive areas can be restricted with indoor positioning. Indoor positioning systems can also help in the deployment of security officers in security-sensitive areas [29].

## 2.7 Challenges in Indoor Positioning

This section discusses the common challenges in indoor positioning these days.

### 2.7.1 Multipath Effect

Multipath signal propagation poses a significant challenge in indoor positioning. The signal strength of radio signals changes over time at a fixed location due to physical phenomena such as reflection, refraction, dispersion, complex indoor structure, and environment. Figure 2.7 shows the concept of multipath. All of these phenomena cause the amplitude and phase of radio signals to change and scatter [20].

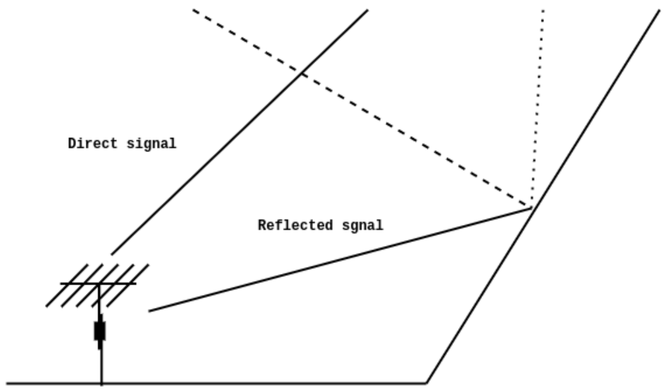


***Figure 2.7.*** *Concept of multi path effect [20].*

Therefore, it is impossible to obtain a single signal from a single radio transmitter. The multipath effect can be taken into account by introducing proper stochastic models utilizing, for example, Rayleigh and Nakagami distribution [20]. These models can be used to reduce the negative effects of multipath, but they make the IPS much more complex.

### 2.7.2 Security and Privacy

Privacy is also an essential challenge in indoor positioning. The current positioning system does not care much about privacy since a global standard for indoor positioning is not yet available [29]. However, not everyone wants to share their location, especially the indoor location. As a result, smartphone manufacturers are putting more restrictions on how third-party applications can use hardware modules such as WIFI and Bluetooth modules in mobile phones.

### 2.7.3 Cost

Cost is one of the most important factor in an indoor positioning system. The lower the cost, the more the service provider will have leverage in the market. Some positioning systems require additional modules to provide better accuracy [29]. Therefore, it is of utmost importance for companies to keep the cost of hardware and software low in IPS.

# 3. RSS BASED INDOOR POSITIONING

The scope of this thesis is in RSS-based indoor positioning; therefore, in this chapter, we will discuss the RSS-based positioning in more detail. In the first part of the chapter, the parametric and non-parametric models of RSS positioning are discussed. In the later part, various access points and radio models are discussed. Further, we also analyze how experimental RSS values look like on floors and buildings and how these RSS value are used during estimation phase of indoor positioning.

## 3.1 Traditional Fingerprinting

In the previous chapter, we discussed range-based positioning methods that compute the distance between transmitter and receivers; however, in fingerprinting, a dataset of RSS value from each transmitter and their reference location is recorded, and radio maps are generated. This process of generating the radio maps is often called the offline stage. In the online stage, the real-time RSS value is compared to RSS values in radio maps generated through the offline phase. The best match of RSS value is found, and its reference location is returned as the mobile station's location. This combined step of the offline and online phase is called Fingerprinting. Fingerprinting consists of two steps, Training Phase or Data Collection Phase and Positioning Phase. Figure 3.1 shows the training and positioning phases in traditional fingerprinting.

### 3.1.1 Training Phase

The first phase in the process of fingerprinting is called the training phase. During this phase, radio measurements are collected at various locations in the experimental site. These radio measurements are collected more or less randomly at various locations. In the post-processing step, these measurements are processed and map into grids format. Often, there is more than one radio measurement at a single grid point, and hence the mean of all radio values is used as the RSS measurement at that point. With the traditional fingerprinting, very few grid points are filled with radio measurements, and empty grid points are filled in post-processing steps. The reference point consists of (x,y,z) - 3D information, where x and y are the local x,y coordinate at a specific location, and z represents the floor level. The measurements at each reference grid point usually consist of an array of RSS values received from multiple access points [20]. The received array of RSS values at a specific reference location is called its fingerprint. Thus, collecting fingerprints at multiple reference points leads to a database of fingerprints and is referred to as a radio map [20]. The radio map depicts the distribution of RSS value over the experimental site along with the reference information.

The quality of a radio map is often determined by the number of RSS values heard at a specific reference location. Thus, the higher the number of access points at an experimental site, the better the generated radio map will be [30]. Collecting a fingerprint database is a laborious task that requires workforce and time. In recent years, work has been put to automate this laborious task by using robots to perform fingerprinting. For example, self guided robots with mobile devices have been used in the past [30]. Another dynamic way of collecting fingerprints is through crowdsourcing. In crowdsourcing, instead of having a dedicated site survey, users already at the experimental site contribute to the collection of radio measurements. Later on, these collected radio measurements are processing altogether and convert into radio grids [31, 32].

### 3.1.2 Positioning Phase

The second stage in traditional fingerprinting is called the positioning phase. In this phase, the user's position at an unknown location is estimated using a positioning algorithm. The RSS values obtained at the unknown location are compared with the RSS values in the radio map collected during the training phase. The closest matching fingerprint can be found by finding the minimum difference of RSS values at the unknown location with the one stored in the radio map. There are several ways to determine the difference, the most common being the euclidean distance. In this method, Euclidean distance is found between the received RSS vector and the vectors stored in the RSS map for each reference location [20]. The minimum difference fingerprint is returned as the user's location. The equation 3.1 shows the formula for computing the minimum difference.

$$\mathbf{location_x} = \mathbf{argmin_{rss}}\left\{\sum_{\mathbf{n=1}}^{\infty}(\mathbf{rss_x} - \mathbf{rss_{n*m}})^{\mathbf{2}}\right\} \tag{3.1}$$

where, $\mathbf{x}$ is the unknown location and $\mathbf{n{,}m}$ represents the local reference points in radio maps. This method is also known as the 1-nearest neighbor estimation method. However, this method is prone to produce significant errors [33]. If an error is found, the minimum error is estimated to be equal to the distance between two consecutive reference locations [33]. A way to minimize the error is to use the k-nearest neighbor averaging method. The user location averages k neighboring reference points starting with the minimum difference reference fingerprint as the base [20]. The distance from the base point to the nearest K neighbors can be find with euclidean distance as in equation 3.1. The

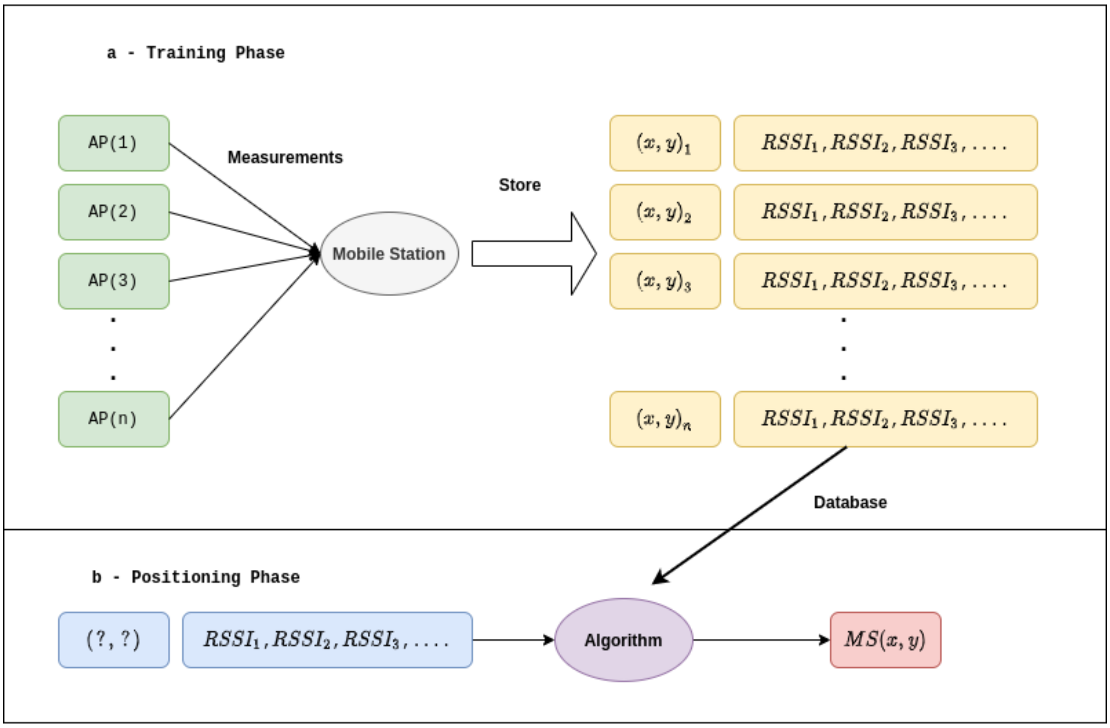


***Figure 3.1.*** *Training and positioning in traditional fingerprinting [20].*

average value of K points which have the minimum distance is used as the user location. This method cannot provide good accuracy; however, it reduces the possibility of error overhead to a low level. In addition, the RSS value is affected by fading and interference with the environment and the k-nearest neighbor reduces the errors caused by these phenomenons [33].

## 3.2 RSS based Path-loss Model

As mentioned in the previous section, traditional fingerprinting is a time-consuming uphill task requiring a workforce to generate complete radio maps. To overcome these issues, a parametric model can be used to generate radio maps. Since we are dealing with radio signals, a natural choice is parametric path loss models. These path loss models compute parameters for each access point based on the collected fingerprints; however, they require fewer fingerprint samples than traditional fingerprinting methods. Instead of directly comparing the received RSS measurements with actual ones in the radio map database, path loss models used computed parameters during the data collection phase and RSS vector to estimate the user's location [20]. Just like traditional Fingerprinting, this method also has a training and positioning phase. Figure 3.2 represents the training and positioning phase of RSS based path loss models.

### 3.2.1 Training Phase

The training phase of this method also requires collecting fingerprints. However, fewer samples are required as a complete radio map is not generated covering all the areas of the experimental site. After collecting fingerprints, specific parameters of the path loss model are estimated based on collected samples. Based on these estimated parameters, a path loss model is generated for each access point [20]. The path loss model connects the distance value to RSS values. There are multiple path loss models available to be used these days [33]; however, for the sake of simplicity, we are going to mention only two of them. The RSS from $\mathbf{n}$ access point at unknown location $\mathbf{z} = (\mathbf{x}, \mathbf{0})$ is given by

$$\psi^*_{\mathbf{n}}(\mathbf{z}) = \mathbf{C} + \alpha\mathrm{log}_{\mathbf{10}}(||\mathbf{z} - \mathbf{z}^{\mathbf{AP}}_{\mathbf{n}})||)) \tag{3.2}$$

Equation 3.2 is known as Hyperbolic model [33]. In equation 3.2, $\mathbf{z}^{\mathbf{AP}}_{\mathbf{n}}$ is the location of AP $\mathbf{n}$ to $\mathbf{x}$ in three dimensional space. Variable $\alpha$ is constant which is assumed to be equal to 10. $\mathbf{C}$ is path loss at 1 reference point away from from $\mathbf{z}^{\mathbf{AP}}_{\mathbf{n}}$. Another pathloss model is mixture model [33] given by equation 3.3 as follow:

$$\psi^*_{\mathbf{n}}(\mathbf{z}) = \mathbf{C} + \alpha\mathrm{log}_{\mathbf{10}}(||\mathbf{z} - \mathbf{z}^{\mathbf{AP}}_{\mathbf{n}})||) + \beta(||\mathbf{z} - \mathbf{z}^{\mathbf{AP}}_{\mathbf{n}})||)) \tag{3.3}$$

where, $\mathbf{z}^{\mathbf{AP}}_{\mathbf{n}}$ is the location of AP $\mathbf{n}$ to $\mathbf{x}$ in three dimensional space. Variable $\alpha$ is constant which is assumed to be equal to 10. $\mathbf{C}$ is path loss at 1 reference point away from from $\mathbf{z}^{\mathbf{AP}}_{\mathbf{n}}$. The difference between the hyperbolic and mixture model is use of term $\beta$, which takes care of effects caused by complex indoor architecture e:g walls and furniture. The reference parameters are the (x,y,z) coordinates of the mobile station relating the distance to transmitter. The parameters of path loss models can be computed by using linear regression using the collected fingerprints. Furthermore, the location of the access points can also be computed by averaging the highest RSS values [20].

### 3.2.2 Positioning Phase

During the positioning phase, complete RSS grids are generated using the parameters and location of access points computed during the training phase [20]. In addition, the creation of radio maps can also be done during the offline phase to provide better throughput by real-time systems. After creating a complete radio map, positioning is done similarly to traditional fingerprinting methods. The received RSS of the user is directly compared with the RSS stored in radio maps, and the user's location is estimated. Another way is to use various algorithms to compute the user location without generating the radio maps. These methods usually involve methods likes the Bayesian approaches [20].

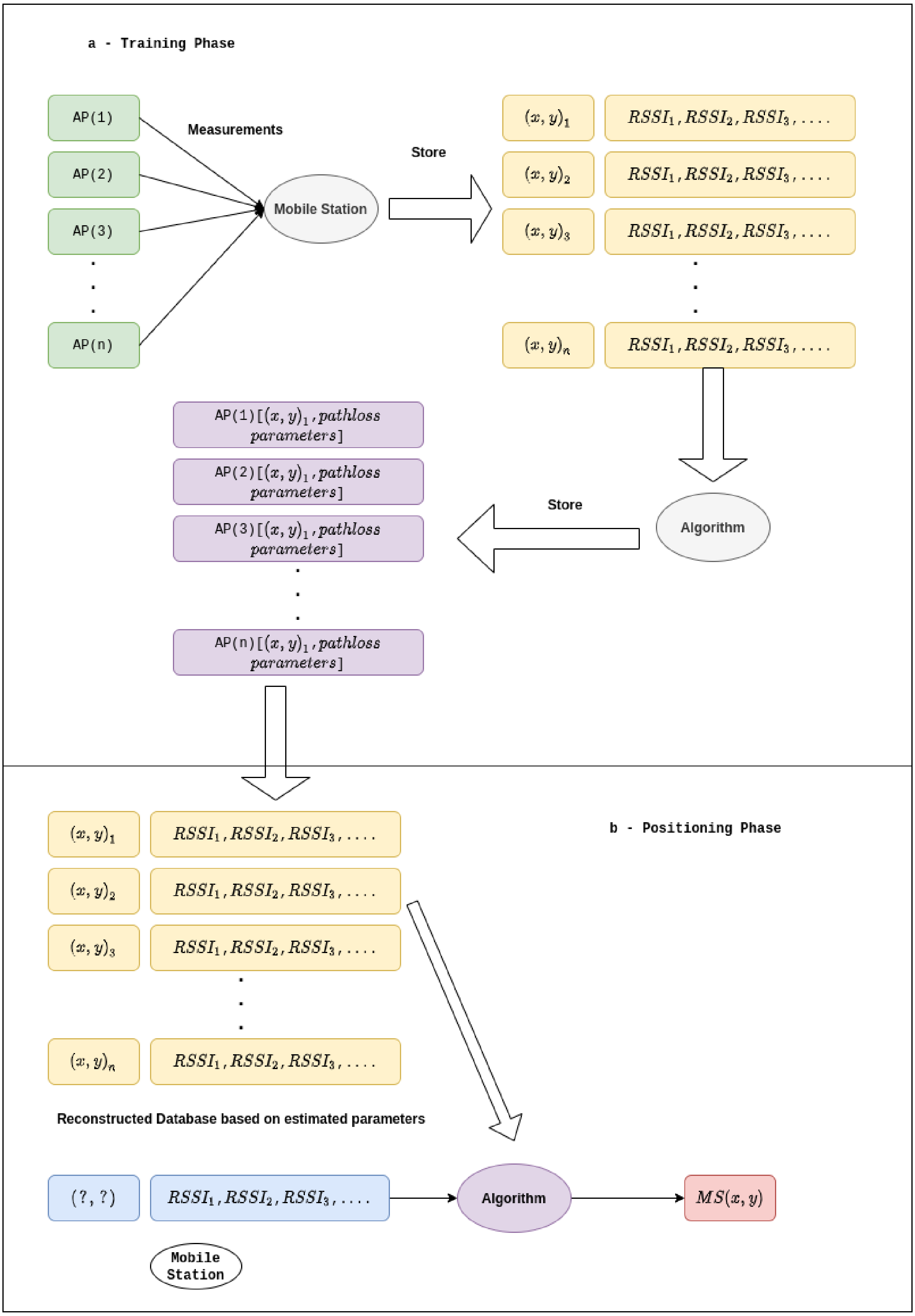


***Figure 3.2.*** *Training and positioning in RSS based path loss models [20].*

## 3.3 Non-parametric models

Contrary to parametric models that use environmental conditions to learn propagation parameters, non-parametric models use the collected fingerprints to model the radio propagation behavior. Learning from the collected fingerprints makes it possible for non-parametric models to adjust themselves to adjust to different environments and solve the problems of temporal variation [34]. Non-parametric models can be implemented similar to parametric models using radio propagation models; however, instead of pre-configuring the parameters, they are configured from learned data. Non-parametric models can be configured either using probabilistic or deterministic methods.

Just like parametric models, the non-parametric models also have training phase and estimation phase. In the training phase, radio maps are generated using non-parametric methods. In position estimation phase, instead of using the distribution of data, deterministic models use the statistical mean of data resulting in a lesser need for data recording and faster processing.

Contrary to deterministic models, probabilistic methods use the distribution of data to locate the user. In the offline phase, the distribution is stored as radio maps. Gaussian processes are an ideal way to generate radio maps probabilistically. There are many advantages of using the Gaussian process as they are non-parametric. They do not need the representation of space for generating radio maps, and they can ideally represent the RSS likelihood models. The idea behind probabilistic models is to compute the joint probability distribution function of each WIFI access point and join those joint distribution functions to make a combined distribution function. In the location estimation phase, there is one on one comparison of received RSS value with radio maps using maximum likelihood estimators. During the estimation stage, the maximum likelihood estimator can be used to compute the user location.

Similar to the radio model generated in parametric models, the user can be localized at any location, even where no RSSI measurement is available, which is not possible with traditional fingerprinting [34]. The probabilistic and deterministic models give more or less the same accuracy when positioning a static object. However, the probabilistic models tend to perform better than deterministic models to perform continuous positioning of moving objects [34].

## 3.4 Access Points

In terms of indoor positioning, an access point is defined as a wireless device that can transmit a signal to a specified range of areas. Most of the indoor positioning solution these days relies on pre-deployed infrastructure and provide good accuracy. The two most used access points are WIFI access points and Bluetooth beacons.

WIFI access points are the wireless device that transmits radio signals following a standard data protocol. For example, the RSS value of transmitted signals is used to create radio maps in indoor positioning. The range of WIFI access points depends upon the transmitter module of access points. Most of the buildings have pre-installed WIFI access points for internet access, which makes it the most common source of the access point in indoor positioning technology [35]. Figure 3.3 shows an example of a Bluetooth beacon commonly used these days.

***Figure 3.3.*** *Bluetooth beacon [36].*

Another access point source is BLE beacons. The research in this thesis has used BLE beacons for positioning. BLE beacons need to be installed once in a building, and a Bluetooth-enabled smartphone is enough to carry out indoor positioning training and estimation phase [35]. BLE beacons are suitable for indoor positioning since many smartphone devices come with Bluetooth modules in them. The cost of BLE beacons is meager, and they go up to years due to their low battery power consumption [35]. BLE beacons are small and are usually placed at a distance of 8 to 10 meters apart. Figure 3.3 shows a BLE beacon commonly used these days.

The positioning algorithm that has been discussed in this thesis majorly focus on Bluetooth based indoor positioning technology. The algorithm relies on the RSS values from BLE beacons. They have many advantages over the other localization techniques mentioned as follow:

- Bluetooth technology is supported by all major smartphones.
- Bluetooth beacons are quite compact, cheap, and they can last for years with small batteries.
- Bluetooth beacons deployment is rapidly increasing due to fast growing field of “Internet of Things”.
- There is no special receiver required on smartphones to receive the RSS values from these beacons.
- Major mobile phone companies are moving towards total wireless practices that leads to continuous use of Bluetooth technology. For example, removal of headphone jacks and use of Bluetooth headphones leads to continuous usage of Bluetooth.

The Beacons used for this research projects have properties shown in table 3.1.

***Table 3.1.*** *Bluetooth beacon properties.*

| Property | Specification |
|---|---|
| Transmitted Power | 0 dbm |
| Advertising Interval | 852 milliseconds |
| Antenna | Omnidirectional |
| Placement | 8-10 meters apart |
| Environmental Protection | P65 |
| Security | Authentication for modifying beacons configurations |

### 3.4.1 Access Point Radio Model

Access point radio models contain information about what RSS can be found in different points of the 3D space from a single access point. The information is usually stored in a 2-dimensional data structure, generally a 2D matrix. Each matrix refers to a particular floor of a building. The indexes of the matrix are mapped to the latitude and longitude global coordinates. The elements of the matrix represent the RSS value of a particular access point. Multiple floor matrix stacked together makes their arbitrary representation in 3D space, making a radio model for a single access point. Figure 3.4 shows the scatter plot of single AP radio model. The RSS values from the access point can be heard in two different buildings on multiple floors. The highest RSS value is heard on 3rd floor of the right building. The dimension of the plot are latitude, longitude and floor ID.

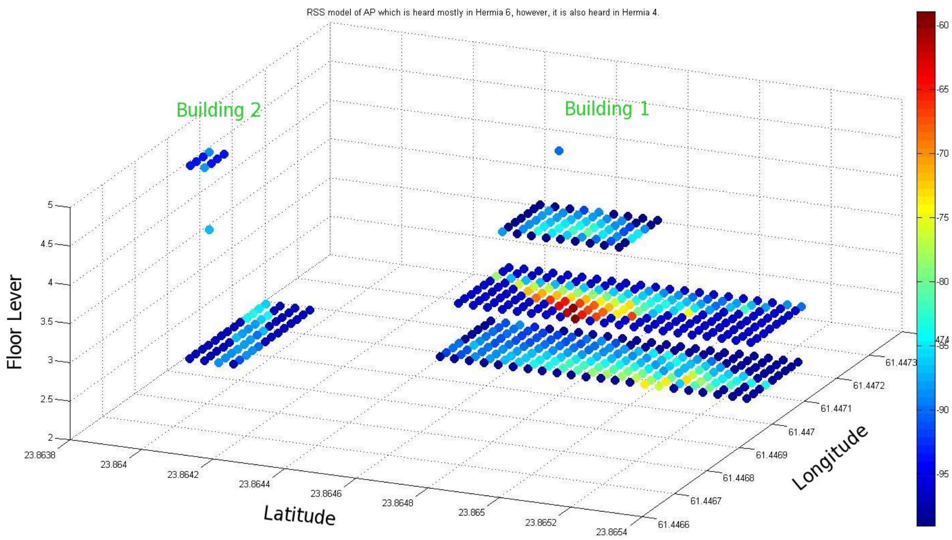


***Figure 3.4.*** *Access point radio model.*

### 3.4.2 Radio Map

The RSS measurements are prune to environmental factors; therefore, a single access point cannot be used to obtain a satisfactory positioning. The reason for that is with one access point we usually get one reference RSS value at a specific grid point. This RSS value is not unique and therefore cannot depict a single location. Further, a single access point in a big building is not enough to cover all the experimental area. Therefore, multiple access points are needed to cover the area under consideration and perform satisfactory positioning. A radio map typically consists of RSS values from various access points placed on different floors. Each building has a single radio model associated with it. Since saving access points models from various AP's can take up a lot of memory, some compression methodologies are also used to generate radio maps.

### 3.4.3 RSS Noise

RSS value varies in time, even at one physical location. There could be many factors involved which cause the change in the RSS measurement. Therefore, RSS measurements are assumed to contain noise and therefore given as

$$\mathbf{\Psi_n^t} = \mathbf{f_n(x)} + \mathbf{n_n^t} \tag{3.4}$$

where, $\mathbf{f_n(x)}$ is noiseless signal and $\mathbf{n_n^t}$ is the noise. The noise is normally distributed with zero mean and standard distribution. The figure 3.5 shows the RSS noise with a standard deviation equal to 4.73 dBm.

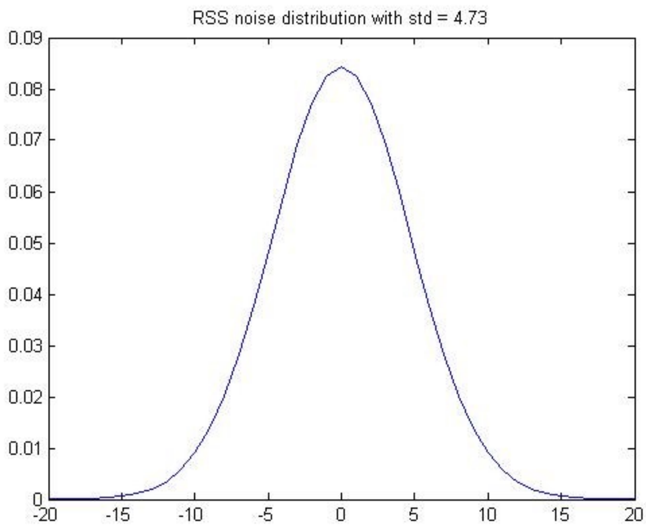


***Figure 3.5.*** *RSS noise distribution.*

## 3.5 Likelihood Estimation

There are many likelihood estimation methods available to compare the received RSS value with the access point models; however, for the sake of this thesis and the fact that RSS measurements follow a gaussian distribution. We will only discuss the maximum likelihood estimator, which is a probabilistic method. The probability of observing a single value x, that is generated from a normal distribution is given by:

$$\mathbf{P}(\mathbf{x};\mu,\sigma) = \frac{\mathbf{1}}{\sigma\sqrt{\mathbf{2}\pi}} * \mathbf{exp}^{(-\frac{(\mathbf{x}-\mu)^{\mathbf{2}}}{\mathbf{2}\sigma^{\mathbf{2}}})} \tag{3.5}$$

where $\mu$ and $\sigma$ is the mean and standard deviation of the distribution. In the following sections, we will analyze the likelihood of single access point and total likelihood of the radio model.

### 3.5.1 Single Access Point Likelihood

Given a single access point model and the RSS noise distribution, it is possible to calculate the likelihood of detecting certain RSS of an AP at the points of the 3D space. For example, given the AP's model and RSS noise presented in the figure 3.4, the likelihood of detecting RSS equal to -75 is shown in the figure 3.6.

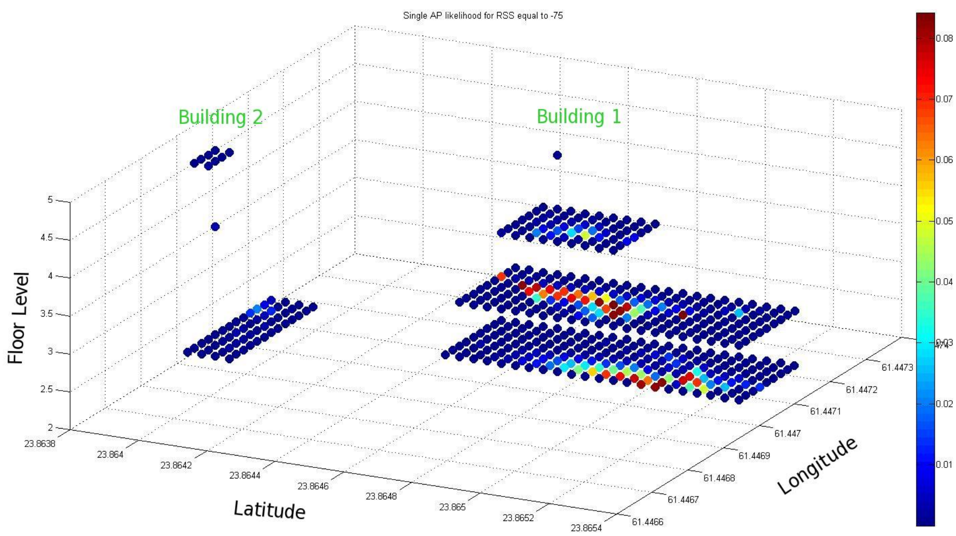


***Figure 3.6.*** *Likelihood scatter plot for RSS = 75.*

From the scatter plot above it is seen that likelihood is high in the points, where AP's model has RSS values close to -75 dBms, and likelihood is low where AP's model has RSS values that are varies considerably from -75 dBms. The formula for calculating single

AP's likelihood at each point for given measured RSS is:

$$\mathbf{l_{i,j,k}} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}\pi\sigma^{\mathbf{2}}_{\mathbf{rss}}}} * \mathbf{e}^{(-\frac{\mathbf{rss_{i,j,k}-rss}}{\mathbf{2}\sigma^{\mathbf{2}}_{\mathbf{rss}}})} \tag{3.6}$$

Where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the spatial indexes of the point, $\mathbf{rss_{i,j,k}}$ is the RSS value in the point that is detected according to AP's model, $\mathbf{rss}$ is the measured RSS. $\sigma$ is used as equal to rss noise defined in previous section.

### 3.5.2 Total Likelihood

As seen from the previous figure , single likelihood for one AP does not provide accurate positioning information, since the likelihood is high in grid points that are spread on different floors and occupy large area. In most of the cases, radio scan contains signals from several AP's, and positioning is based on several AP's. Having more than one AP in the scan, radio map likelihood over all AP's can be calculated by multiplying element wise the single likelihoods of the AP's. The formula for calculating the radio map likelihood 3.7 is given by

$$\mathbf{L_{i,j,k}} = \prod_{\mathbf{s=1}}^{\mathbf{N^{AP}}} \mathbf{l^{s}_{i,j,k}} \tag{3.7}$$

where $\mathbf{l^{s}_{i,j,k}}$ is the likelihood of single access point at index $\mathbf{s}$.

# 4. INTERPOLATION AND EXTRAPOLATION OF AP GRIDS

In the previous chapter, we analyzed the steps of fingerprinting to develop radio maps. However, fingerprinting cannot be performed on all the locations, leading to holes in the access points grids. This chapter will first analyze the need for interpolation and extrapolation methodologies in access point grids. In the second part, we will go through various interpolation and extrapolation techniques that can be used to fill holes in access point grids.

## 4.1 Need of Interpolation and Extrapolation

When fingerprinting is performed, a user moves to the experimental site and records the RSS values from WIFI access points or Bluetooth beacons. This task requires a lot of workforce and time. In addition, all the locations in buildings are not accessible to users. These non-accessible location leads to empty holes in the RSS grids. Further, All grids from all access points should be equal to calculate the total likelihood as described in the chapter 3. The radio grid, which contains the original data from fingerprinting without the addition of interpolated or extrapolated grid points is called the synthetic grid. Figure 4.1 shows how a typical synthetic grid looks like. The colored dots shows the fingerprints collected and the empty spaces among them depicts the holes.

***Figure 4.1.*** *Synthetic radio grid with empty holes.*

To calculate the total likelihood during the position estimation stage as shown in equation 3.7, all grids should be equal in size and have no empty holes left. The filling of empty holes between synthetic fingerprints is referred to as the interpolation of radio grids. The increase of the size of radio grids by creating empty grids and filling them is referred to as the extrapolation of radio grids. Figure 4.2 shows how a typical radio grid looks like after interpolation and extrapolation are performed on it. In the next section, we will analyze various types of interpolation and extrapolation methods used in the thesis to process the radio grids.

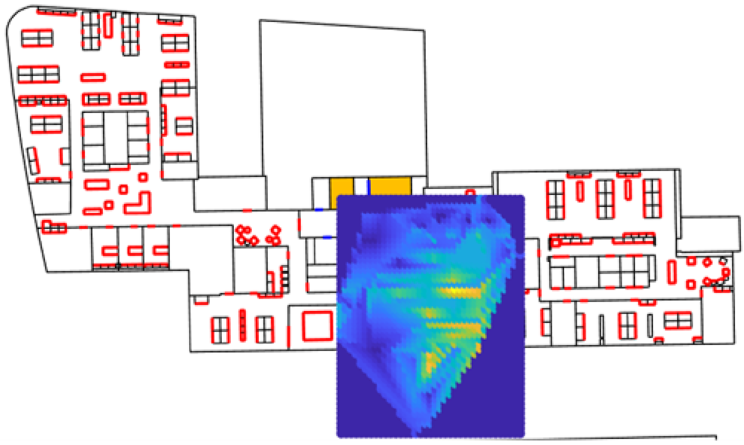

***Figure 4.2.*** *Interpolated and extrapolated radio grid.*

## 4.2 Types of Interpolation and Extrapolation

Interpolation is an estimation method for calculating the value of unknown data points based on known data points. Contrary to this, extrapolation estimates unknown data points beyond the limit of known data points. In this section, we are going to analyze various kinds of interpolation and extrapolation methodologies. The idea behind interpolation and extrapolation is to find a function that can pass through the points to interpolate and extrapolate the unknown data point [37].

### 4.2.1 Linear Interpolation

Linear interpolation is the process of fitting a curve using first degree polynomials. As this is an interpolation methodology, the new data points can only be formed in the range of known data points. Two data points $(x_0, y_0)$ and $(x_1, y_1)$ are given in the coordinate frames. In order to find a function that passes between these data points, the straight-line equation can be used [37]. Equation 4.1 shows the straight-line equation

$$f(x) = y = mx + c \tag{4.1}$$

$\mathbf{m}$ is the gradient of a straight line and $\mathbf{c}$ is y-intercept. Here the y-intercept is the actual value of $\mathbf{y}$ at $\mathbf{x = 0}$. Variables $\mathbf{m}$ and $\mathbf{c}$ are given by the equations 4.2 and 4.3 respectively

$$\mathbf{m} = \frac{(\mathbf{y_1} - \mathbf{y_0})}{(\mathbf{x_1} - \mathbf{x_0})} \tag{4.2}$$

$$\mathbf{c} = \mathbf{y_1} - \mathbf{m}\mathbf{x_1} \tag{4.3}$$

substituting value of $\mathbf{m}$ and $\mathbf{c}$ from equation 4.2 and 4.3 leads us to a linear interpolation function given by equation 4.4

$$\mathbf{m} = \frac{(\mathbf{y_1} - \mathbf{y_0})}{(\mathbf{x_1} - \mathbf{x_0})}(\mathbf{x_1} - \mathbf{x_0}) + \mathbf{y_0} \tag{4.4}$$

Figure 4.3 shows the illustration of linear interpolation based on function given equation 4.4.

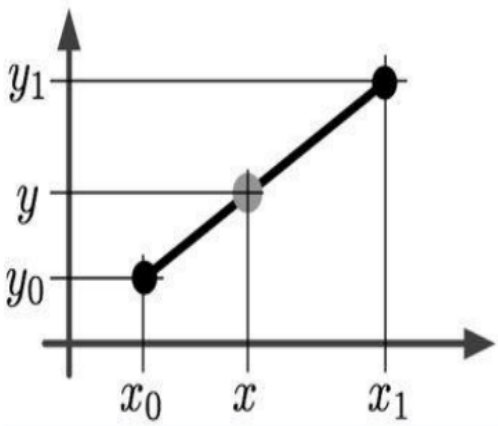


***Figure 4.3.*** *Illustration of 1-D linear interpolation [38].*

This method performs linear interpolation in one dimension, leaving empty holes in radio grids since they are two-dimensional. To overcome this issue, bilinear interpolation, an extension of linear interpolation, can be used. Bilinear interpolation can be performed by doing linear interpolation in one direction and then in the other one [39]. Each linear interpolation performed in bilinear interpolation is linear, although bilinear interpolation as a whole is quadratic.

### 4.2.2 Piecewise Interpolation

Piecewise interpolation is similar to linear interpolation, except it can have any number of points. Piecewise interpolation can make a straight line passing through consecutive data points, contrary to linear interpolation, which makes a smooth curve. Linear interpolation of estimate of $\mathbf{y}$ is given by equation 4.5.

$$\mathbf{y} = \mathbf{f_k}(\mathbf{x}) = \mathbf{y_k} + \frac{(\mathbf{y} - \mathbf{x_k})}{(\mathbf{x_{k+1}} - \mathbf{x_k})}(\mathbf{y_{k+1}} - \mathbf{y_k}) \tag{4.5}$$

where, $\mathbf{N}$ is the number of data points and $\mathbf{k} = \mathbf{N} - \mathbf{1}$. In piecewise interpolation, for each successive interval of data points, a separate function is fitted which makes it a continuous function. Figure 4.4 shows piecewise interpolation done by function equation 4.5.

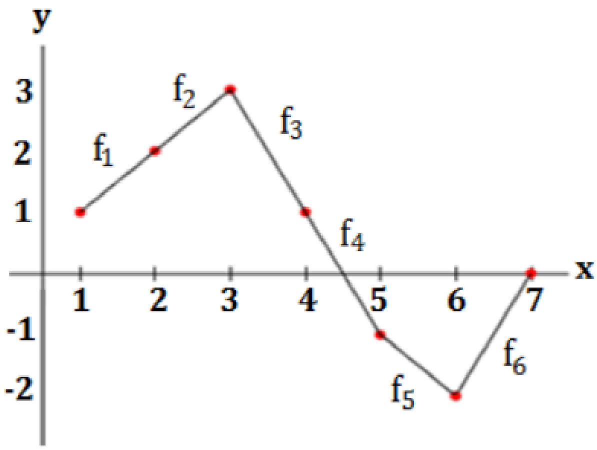


***Figure 4.4.*** *Illustration of piecewise interpolation [38].*

### 4.2.3 Nearest Neighbor Interpolation

The nearest neighbor interpolation uses the values of the nearest known data point as the value of the unknown data point. In order to understand the concept of nearest-neighbor interpolation consider two consecutive data points $\mathbf{x_k}$ and $x_{k+1}$. This methodology finds the mid-value of these data points to use as reference. The values of $\mathbf{x}$, which is smaller than the reference value, leads to the value of $\mathbf{y_k}$ and values that are larger than the reference value lead to the value of $\mathbf{y_{k+1}}$. The function of nearest-neighbor interpolation is given by equation 4.6. This methodology is faster then linear interpolation. Figure 4.5 shows interpolation(Red Dots) done using nearest neighbor method.

$$\mathbf{f_k}(\mathbf{x}) = \begin{cases} \mathbf{y_k} & \mathbf{x} \leq \frac{1}{2}(\mathbf{x_k} + \mathbf{x_{k+1}}) \\ \mathbf{y_{k+1}} & \mathbf{x} > \frac{1}{2}(\mathbf{x_k} + \mathbf{x_{k+1}}) \end{cases} \tag{4.6}$$

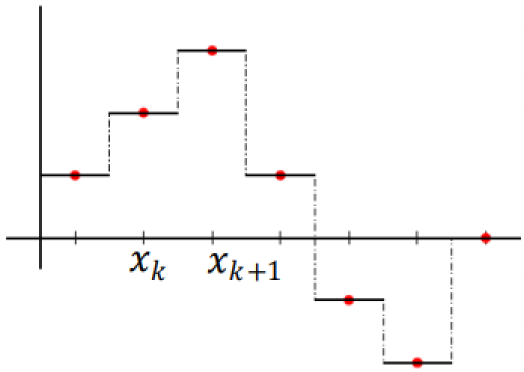


***Figure 4.5.*** *Illustration of nearest neighbor interpolation [37].*

### 4.2.4 Minimum and Maximum Value Extrapolation

Minimum and maximum value extrapolation use a given input's minimum and maximum values as the value of unknown data points. Thus, the method goes through all known data points and finds the minimum and maximum values of $\mathbf{x}$ and $\mathbf{y}$ in the coordinates plane. This method can use either interpolation or extrapolation since it uses a single hard value for all unknown data points. Since a single hard value is used for unknown data points, specific peaks and falls can be seen in the interpolated data points. Minimum and maximum value interpolation or extrapolation is given by equation 4.7 and 4.8 respectively.

$$\mathbf{f_k} = \begin{cases} \mathbf{x_k} = \mathbf{min}(\mathbf{x_n}) \\ \mathbf{y_k} = \mathbf{min}(\mathbf{y_n}) \end{cases} \tag{4.7}$$

$$\mathbf{f_k} = \begin{cases} \mathbf{x_k} = \mathbf{max}(\mathbf{x_n}) \\ \mathbf{y_k} = \mathbf{max}(\mathbf{y_n}) \end{cases} \tag{4.8}$$

where, $\mathbf{k}$ is the index of unknown data points points and $\mathbf{n}$ is the number of known data points. Similar to minimum and maximum value interpolation, a predefined value can be used to fill the unknown data points. Minimum value interpolation and extrapolation is a common way of filling RSS grids. The reason behind that is when we move away from the access points; the RSS values start to go down, and eventually, there comes the point where the RSS value from the specific access point is not heard. So there are two possibilities on the points where we don't hear the RSS value; either we are out of range of the access point or the RSS values are minimal. So, filling those points with minimum values fills up the empty holes, and their contribution to the overall radio grid remains minimal.

### 4.2.5 Delaunay triangulation and linear interpolation

Delaunay triangulation is a method for creating triangles given a set of D discrete data points so that no point in D is inside the circumcircle of the triangles created. It is standard to use Delaunay triangulation with linear interpolation, especially in a two-dimensional grid format [40]. The algorithm creates triangles for an unknown data point by creating lines between known and unknown data points. The triangle is created in such a way that the edges of any triangle are not intersected with another triangle [40]. Thus, the method results in triangular nodes over the grid data. Figure 4.6 shows triangulation with the circumcircles. The black circles in the circumcircles created through the known data points. The red dots are the center of the circumcircles.

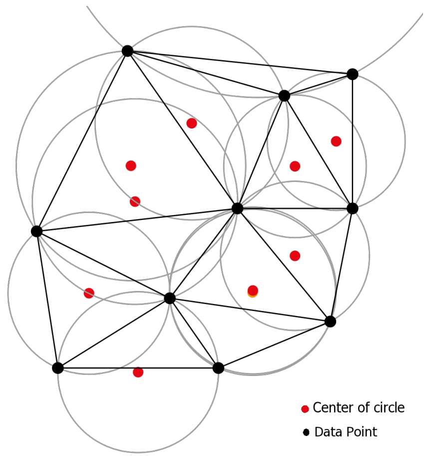


***Figure 4.6.*** *The Delaunay triangulation with all the circumcircles and their centers [40].*

The center of circles are joined with unknown data points through linear interpolation as shown in figure 4.7.

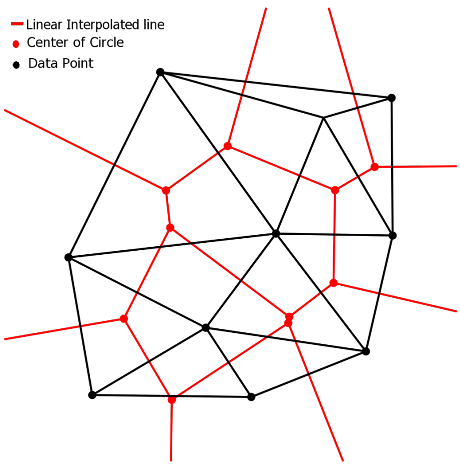


***Figure 4.7.*** *Connecting the centers of the circumcircles [40].*

This method works best when the data is distributed evenly in a grid format. Data grids with extensive sparse areas lead to the distinct face of triangles.

### 4.2.6 Spatial Interpolation

In RSS-based spatial interpolation, the first step is to estimate the trend based on the data points collected through fingerprinting. To estimate the global trend of the RSS data, a pathloss model can be used.

$$\mathbf{PL = A - 10 * n * \log_{10}(d) - Lf} \tag{4.9}$$

where,

$\mathbf{A}$ = RSS at 1m distance to the AP,
$\mathbf{n}$ = Pathloss exponent,
$\mathbf{d}$ = Distance to the AP,
$\mathbf{Lf}$ = Floor losses in total

Equation 4.9 shows the pathloss model used for characterizing the propagation of radio waves as a distance function between the antennas of transmitter and receiver in spatial interpolation. Since RSS values follow the trend of normal distribution, the mean value is not zero. After the estimation of the trend function, the trend is removed from the synthetic RSS values. This step normalizes the RSS values so that they have zero mean and one standard deviation. The step makes the data points as standard normally distributed. This residual obtained from removing the trend function can be considered as a zero-mean Gaussian process with a specific spatial (inter-sample) correlation function.

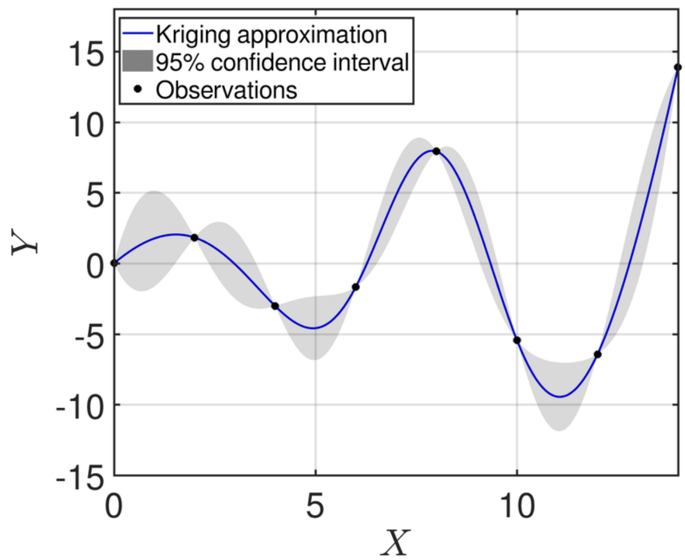


***Figure 4.8.*** *1-D kriging process [41].*

Kriging is a process to predict the value of a random variable over a spatial region and is governed by covariances of random variables. Given several measurements at a set of locations in the spatial region, Kriging creates values throughout the region. Since Kriging can predict values in between known data points and beyond the last knows values, it can take care of both interpolation and extrapolation of data points. In the case of RSS-based Kriging or spatial interpolation, we use the residual RSS value for estimation.

Figure 4.8 shows one-dimensional spatial interpolation. The black dots show the actual data points. The blue line passing through the data points depicts the kriging interpolation. The gray area shows the confidence interval between two data points.

Since, Kriging is performed for the residual RSS values, the estimated trend value is added on top of the interpolated values to obtain the final RSS estimate.

# 5. DATA ANALYSIS

In this chapter, data collection, processing, and testing has been discussed.

## 5.1 Data Collection

To study the effects of interpolation and extrapolation methods described in the chapter 4 on the overall performance of indoor positioning, we study one reference venue: HERE Tampere shown in figure 5.1. The venue had strong coverage of radio signals provided by Bluetooth beacons installed 8 meters apart. HERE Tampere was a multistory office space consisting of solid indoor infrastructure. The building consists of hallways, restaurants, and office spaces. For data collection, google pixel devices were used, which were equipped with the latest android OS. HERE Indoor Radio map [42] was installed on the device, which is available on play store. The app was used for collecting fingerprints and test tracks.

***Figure 5.1.*** *HERE Tampere - Indoor office space [43].*

### 5.1.1 Radio Mapping

As mentioned in the previous chapter, the positioning algorithm consists of two phases training and testing phase. The training phase consists of collecting radio data in the venue and producing a radio model. Radio Mapping was done for the mentioned venue, and the maximum accessible area was covered to have uniform data collection on all the floors. The radio data was collected through HERE Indoor Radio Mapper (HIRM), and the collected data was inspected through the HERE admin portal. Figure 5.2 shows the interface of HIRM while collecting radio data on the 3rd Floor in HERE Tampere office. The process of collecting radio data using HIRM consists of following steps:

- Choose a venue to collect data.
- Marking your location on the venue map.
- Pressing the start button and moving along a straight line.
- Pressing the stop button and marking your location on the venue map.
- If the collection goes as planned, pressing the save button and starting with another collection.
- Once the whole area is covered, export the radio data in a .txt file that can later be used to generate radio models.

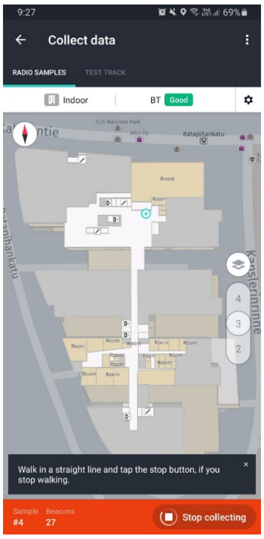


***Figure 5.2.*** *Test track collection using HIRM.*

It is essential to check the quality of radio data that has been collected. If the data collected at a specific location is not good enough, the performance of positioning will be significantly affected. The admin portal associated with HERE indoor radio mappers provides tools to access the quality of radio data.

Figure 5.3 shows the quality of radio data collected for HERE Tampere office. The black lines on top of the venue depict the radio data collected by moving in straight lines. The green color shows that the coverage and data collected are good enough. The yellow coverage area represents that it is better to collect some more data in these areas. The red coverage area (usually the end/edges of venue maps) depicts that it is strongly recommended to collect more data in these areas. Figure 5.3 clearly shows good amount of data collected for the 4th floor of HERE Tampere. Similarly, radio data was collected for other floors too.

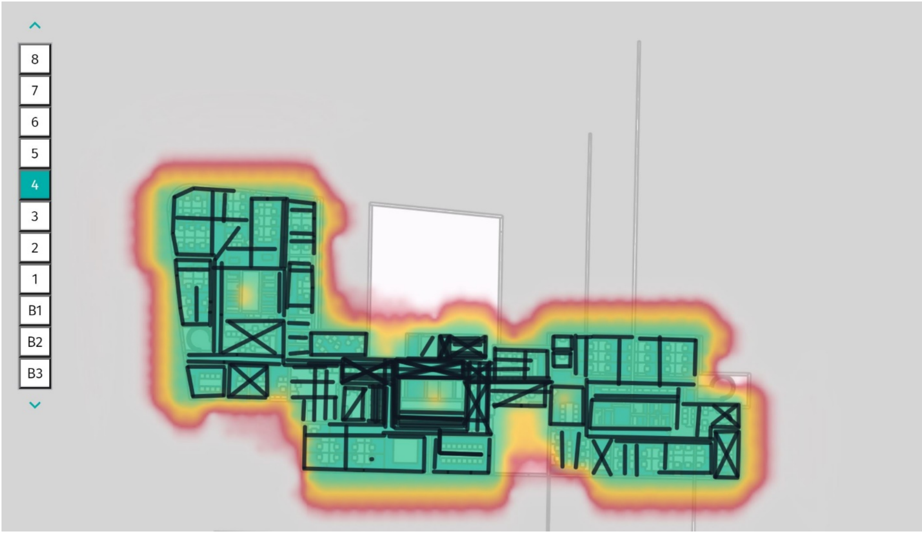


***Figure 5.3.*** *Quality of radio data collected indoors.*

### 5.1.2 Test Track Collection

For the testing phase of the positioning algorithm, multiple test tracks were collected using HIRM. The collection of test tracks in HIRMU is similar to the collection of radio data. The dataset of test tracks consisted of 9 test tracks with a total of 278 measurements. The collected tracks have been classified into Near-Edge and Non-Edge test tracks. The radio tracks which are near the edges of the experimental site are categorized as edge test tracks. Contrary to this, tracks in the middle of the experimental site are categorized as non-edge test tracks. Table 5.1 below describes the classification of the number of test tracks collected.

***Table 5.1.*** *Summary of collected test tracks.*

| Venue | Description | Edge Measurements | Non-edge Measurements | Total Measurements |
|---|---|---|---|---|
| HERE Tampere | Office Space | 176 | 102 | 278 |

All of the data collected was later on exported in a .txt file so that test tracks can be used to test the performance of the radio models in MATLAB.

## 5.2 Data Processing

As mentioned in chapter 3, indoor positioning includes two steps; training and positioning phase. The processing of both steps takes place in MATLAB, which is an easy-to-use programming language. The generated fingerprint logs are fed to the MATLAB simulator, which generates a radio model that can be used for positioning later on. The training phase also contains various data compression techniques that helps save data storage and reduces the computational complexity. The radio model is generated with an extension of .mat.

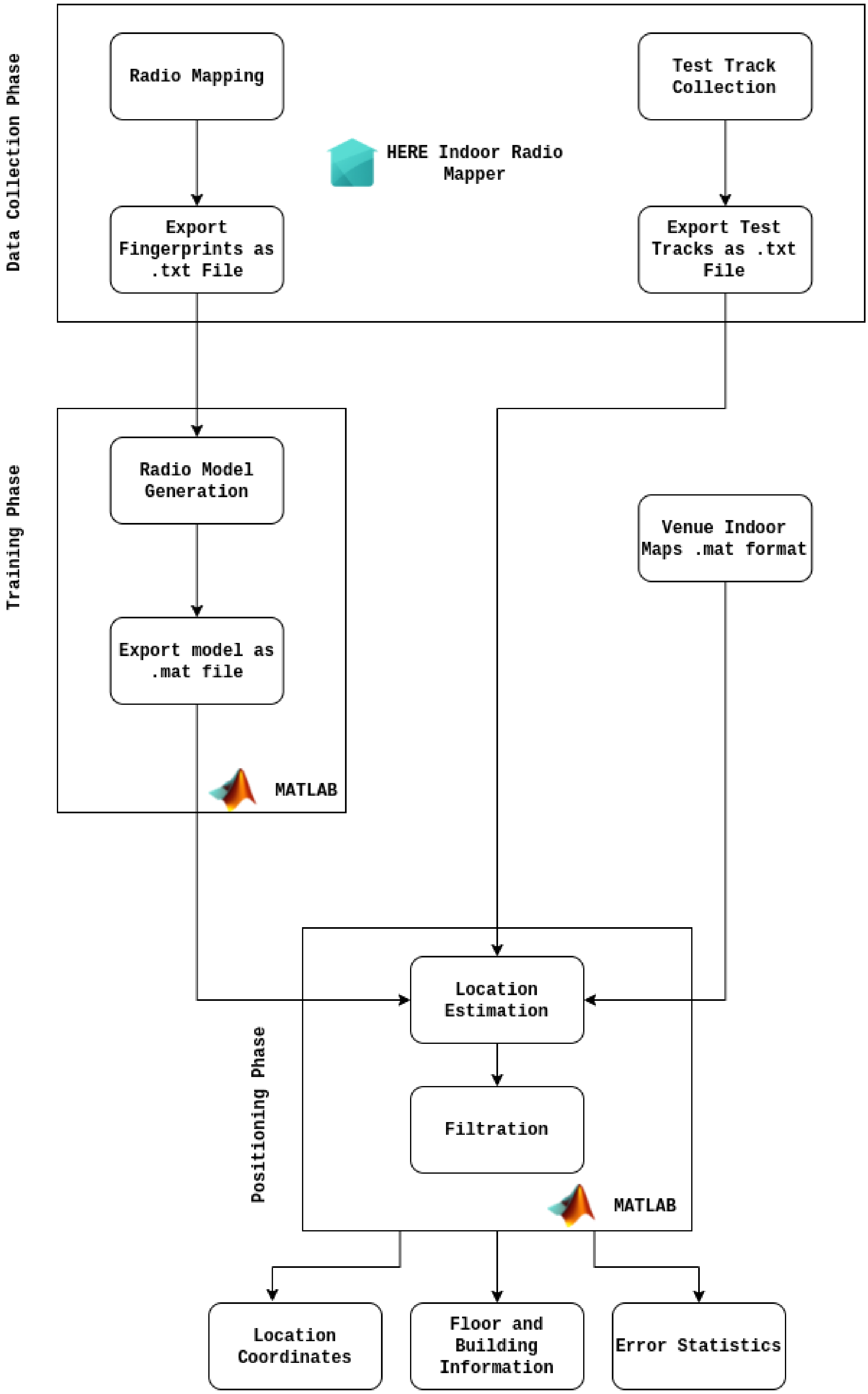


***Figure 5.4.*** *Block diagram of data collection and processing.*

Once the radio model is generated, the model can be tested with the test tracks collected through the HERE indoor app. The radio model generated during the training phase and the test tracks generated during data collection phase are fed to the MATLAB simulator for the positioning phase. The positioning phase also consists of various filters to further increase the overall positioning accuracy. The MATLAB simulator generates error statistics and gives indoor location coordinates with floor and building information. Figure 5.4 shows the overall data processing steps.

## 5.3 Selected Interpolation and Extrapolation Techniques

In order to analyze the results of various interpolation and extrapolation methods on indoor positioning accuracy, we chose three main techniques to make a comparison.

### 5.3.1 Default Method

The first technique is a combination of Delaunay triangulation and linear interpolation. Since Delaunay triangulation can only be done for a given set of points, another extrapolation method is needed to fill the remaining empty holes. Minimum value extrapolation is used to fill the remaining empty grid points. This method also helps make the radio grids in rectangles which can then be easily used for position estimation. The method involves the following steps:

- Collect synthetic data through Fingerprinting.
- Perform Delaunay Triangulation with linear interpolation to fill the holes.
- Use a minimum value in dBs to fill the remaining holes that lead to rectangular radio grids.

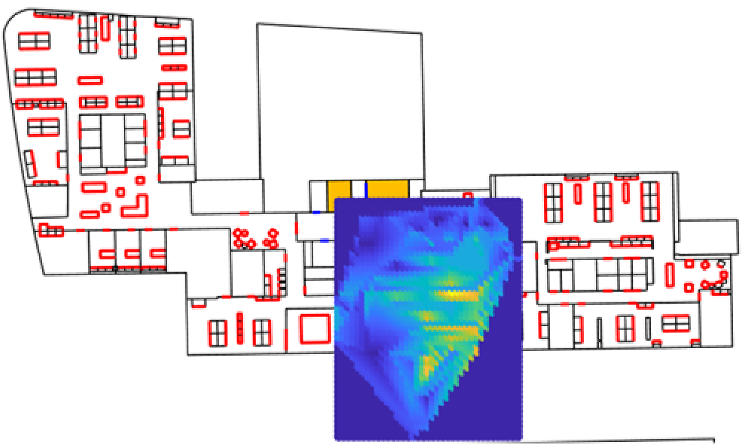

***Figure 5.5.*** *Interpolation with default method.*

Figure 5.5 shows the interpolation done through default method. The dark blue data points shows the grid points filled with minimum value extrapolation. The light blue and yellow data points shows linear interpolation done through Delaunay triangulation with linear interpolation.

### 5.3.2 Hybrid Spatial Interpolation

Another method is the combination of Default method and Spatial Interpolation. The idea behind hybrid interpolation is to interpolate the grids with Delaunay triangulation with linear interpolation and fill the maximum grid points. Unlike the default method, filling remaining grid points with minimum value interpolation, we use spatial interpolation. Therefore, all the grids points have radio data collected through fingerprinting or generated through a function based on the synthetic data. The hybrid spatial interpolation involves the following steps:

- Spatially interpolate the synthetic grids.
- Perform Delaunay interpolation on synthetic data.
- Match the coordinates of Delaunay interpolated grids and spatially interpolated grids, and wherever holes are found, fill them with the spatially interpolated data.
- The area for interpolation is chosen as the default size of radio grids generated by the default method.

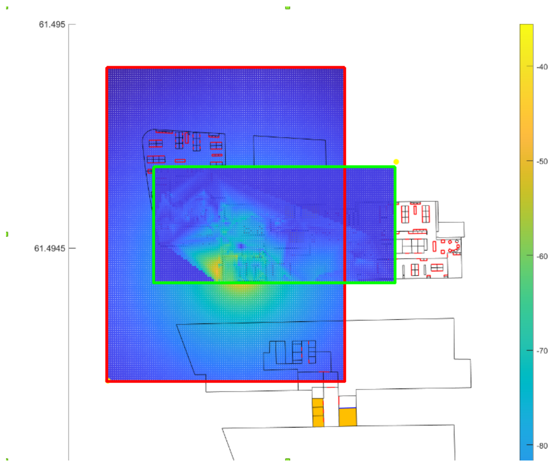


***Figure 5.6.*** *Hybrid spatial interpolation.*

Figure 5.6 shows Delaunay interpolated grid (green) and spatially interpolated grid (red) placed on top of each other. The holes in Delaunay interpolated grid are filled with RSS values of matching spatially interpolated grid points.

### 5.3.3 Hybrid Spatial Interpolation and increase in grid size

Usually, the radio signals along the edges are weak that dramatically affects the accuracy of indoor positioning. In order to combat this issue, the size of radio grids can be increased, which will drag the point estimates more towards the edges when the position is estimated. For this purpose, the same hybrid spatial interpolation method described above is followed. In addition, extra empty holes are created on the edges of radio grids to increase the size. These holes are filled with spatially interpolated data.

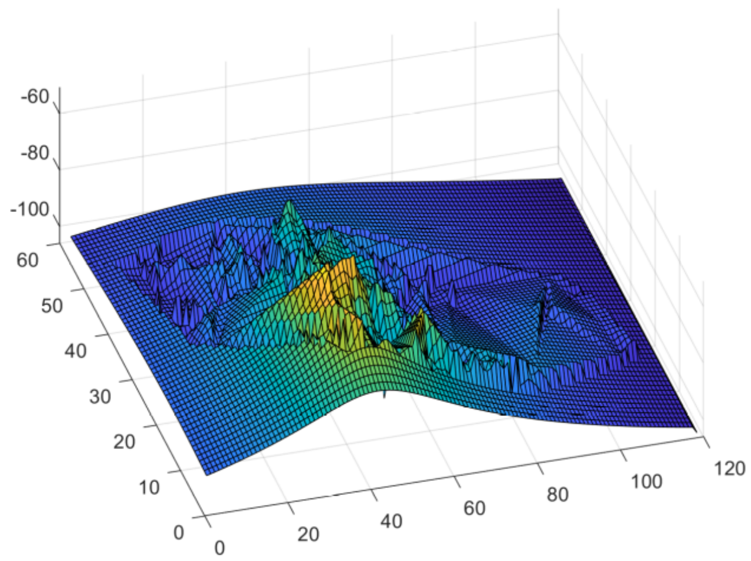


***Figure 5.7.*** *Surf plot - Hybrid spatial interpolation and increase in size of radio grid.*

Figure 5.7 shows the radio grid spatially interpolation and the size increased by creating holes on the edges.

## 5.4 Comparison and Results

In order to make a comparison of various interpolation and extrapolation methods, specific evaluation parameters are used, which are explained below:

**Mean error** is the average MSE between the calculated position estimates and reference estimates for a complete test track.

**Maximum error** is the maximum MSE between the calculated position estimates and reference estimates for a complete test track.

**Building detection** measures how many estimates are correctly identified in a specific building by the algorithm given the reference estimates in that building.

**Floor detection** measures how many estimates are correctly identified on a specific floor by the algorithm given the reference estimates on that Floor.

**Consistency** is a measure of mean error and its standard deviation to calculate the stability of positioning. The higher the jumps between position estimates, the lower the consistency will be and vice versa.

Figure 5.8 shows a general reference plot and estimated plot. Figure 5.9 shows the CDF of mean for the track for the corresponding figure.


***Figure 5.8.*** *Example estimates in a test track.*

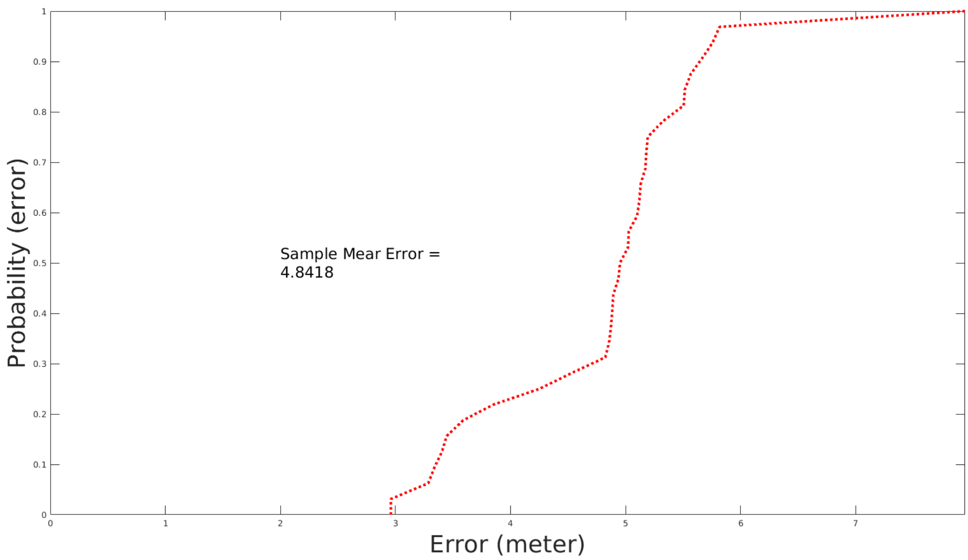


***Figure 5.9.*** *Error CDF of a test track.*

In order to better understand the effects of interpolation and extrapolation on radio-based indoor positioning, we evaluated the test tracks separating them as edge cases and non-edge cases.

### 5.4.1 Near-Edge Cases

Near edge cases consist of radio test tracks collected near the wall of the experimental site. Often these areas have little to no radio coverage and are hard to cover during fingerprinting. A total of 176 measurements were collected in HERE experimental sites near edges. Table 5.2 shows the comparison of interpolation methods discussed above. Floor detection and building detection remain unaffected by the change in interpolation methods. Mean error is also the same for all three methods, with a slight change in decimal points that can be considered negligible to none. The maximum error remains lowest with the default method and about one meter higher in Hybrid Spatial Interpolation and Hybrid Spatial Interpolation with an increase in grid size. Consistency was also highest in the default method and a little lower in Hybrid Spatial Interpolation with an increase in the grid and default Hybrid Spatial Interpolation, respectively.

***Table 5.2.*** *Comparison for edge cases.*

| Method | Building Detection(%) | Floor Detection(%) | Mean Error(m) | Maximum Error(m) | Consistency |
|---|---|---|---|---|---|
| Default | 100 | 100 | 3.916 | 6.826 | 54.632 |
| Hybrid Spatial interpolation | 100 | 100 | 3.945 | 7.261 | 52.624 |
| Hybrid + size increase | 100 | 100 | 3.959 | 7.333 | 53.967 |

Figure 5.10 shows a comparison of near edge method for all three method discussed above.

### 5.4.2 Non-Edge Cases

Non-edge cases consist of radio test tracks collected away from the walls of the experimental site. These areas usually have strong radio coverage with the detection of multiple access points. A total of 102 measurements were collected in HERE experimental sites in non-edge areas. Table 5.3 shows the comparison of interpolation methods discussed above for non-edge cases. Building detection remains good for default and hybrid method with the increase in size. Although for Hybrid spatial interpolation, it falls to 99%, which can still be considered good. Floor detection remains unaffected by the change in interpolation methods.

We see a significant change of one meter on average for mean error with the hybrid spatial interpolation method. There is also a big reduction in maximum error with both spatially interpolated methods compared to the default method. Following the trend, consistency is also improved by almost 10% for the hybrid spatial interpolation method. There is also an improvement in consistency for the spatial interpolation method with an increase in grid size.

***Table 5.3.*** *Comparison for Non-edge cases.*

| Method | Building Detection(%) | Floor Detection(%) | Mean Error(m) | Maximum Error(m) | Consistency |
|---|---|---|---|---|---|
| Default | 100 | 100 | 3.725 | 11.130 | 55.135 |
| Hybrid Spatial interpolation | 99.230 | 100 | 2.874 | 5.588 | 64.594 |
| Hybrid + size increase | 100 | 100 | 3.411 | 7.884 | 59.002 |


***(a)*** *Default method.*


***(b)*** *Hybrid spatial interpolation.*


***(c)*** *Hybrid method and increase in size.*

***Figure 5.10.*** *Comparison of a test track in edge cases.*


***(a)*** *Default method.*


***(b)*** *Hybrid spatial interpolation.*


***(c)*** *Hybrid method and increase in size.*

***Figure 5.11.*** *Comparison of a test track in non-Edge cases.*

# 6. CONCLUSION

GNSS is a satellite-based localization methodology that provides global navigation and tracking with reliable positioning accuracy. However, satellite signal strength becomes weak in the indoor environment due to complex infrastructure, and GNSS positioning cannot be considered decisive. Therefore, the pre-existent radio signals in the indoor environment can be used to obtain reliable indoor positioning. The thesis focuses on using WIFI and Bluetooth signals in an indoor environment for accurate indoor positioning. The typical indoor positioning algorithm consists of two steps, the training phase and the positioning phase. RSS measurements are recorded along with reference location during the training phase, and radio maps are generated. In the positioning phase, real-time RSS measurements are compared with radio maps, and user location is estimated.

Collecting the RSS measurements, also known as the process of fingerprinting, is an uphill task requiring time and workforce. Furthermore, not all areas in an indoor environment can be fingerprinted due to inaccessibility and complex indoor infrastructure. The goal of the thesis was to analyze various interpolation and extrapolation methods in traditional RSS fingerprinting. The interpolation and extrapolation methods were evaluated on Mean Error, Maximum Error, Floor Detection, Building Detection, and Consistency of indoor positioning.

The RSS measurements were collected in Technopolis Tampere, which is a complex indoor environment. The building had strong radio coverage provided with BLE beacons. The building consists of multiple stories and is segmented into two blocks. Firstly, fingerprinting was performed on multiple floors, and the maximum accessible area was covered. Next, the fingerprints were processed and converted into radio grids leading to holes in areas where no radio measurements were collected. Furthermore, for evaluation purposes, multiple test tracks were recorded in near-edge and non-edge cases. A near-edge case is one where radio measurements are collected near the edges of the experimental building. Contrary to this, a non-edge case is anywhere in the middle of the experimental site.

For analysis of interpolation and extrapolation methods, three methods were chosen and compared based on five different evaluation parameters. The first method was Delaunay triangulation with linear interpolation, referred to as the default method in this thesis. The

***Table 6.1.*** *Combined summary of edge and non-edge tracks.*

| Method | Building Detection(%) | Floor Detection(%) | Mean Error(m) | Maximum Error(m) | Consistency |
|---|---|---|---|---|---|
| Default | 100 | 100 | 3.874 | 7.783 | 54.744 |
| Hybrid Spatial interpolation | 99.829 | 100 | 3.707 | 6.889 | 55.284 |
| Hybrid + size increase | 100 | 100 | 3.832 | 7.455 | 55.086 |

second method combined the default method with RSS-based spatial interpolation, referred to as the Hybrid spatial interpolation in this thesis. The third method was similar to the Hybrid method, with the only difference being the size of radio grid was increased by creating empty holes on the edges. For the edge cases, the floor and building detection remain perfect for all the three methods mentioned. The mean error remained almost the same for the three methods with a slight difference in decimal figures. The minimum max error and the highest consistency were given by the default method. For the non-edge cases, the floor and building detection again remained perfect for all the interpolation and extrapolation methods. The minimum mean and max error was given by hybrid spatial interpolation. The maximum consistency was also generated by the hybrid method. For the non-edge cases, the reduction in mean and maximum error by the hybrid spatial interpolation was significant compared to the other two methods.

Table 6.1 shows the combined results of all three methods described above. For all three methods, the building detection and floor detection remain reliable. All three methods generated almost the same results in terms of mean error. For maximum error, we observe a significant change of one meter on average in hybrid spatial interpolation compared to the other two methods. The highest consistency was also given by hybrid spatial interpolation.

The current data was collected in a complex indoor office space. For future work, we can test these methods in other indoor locations with different infrastructures, e.g., apartments, parking spaces, shopping malls. A more extensive and diverse dataset will powerfully depict the practicality of these methods in real indoor scenarios. Another addition could be to make changes in the interpolations and extrapolation methods; for example, a different path loss models that takes into account the indoor infrastructure can be used while calculating the trend function in Hybrid spatial interpolation.

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