GDOP-based analysis of suitability of LEO constellations for future satellite-based positioning

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Abstract—There are efforts worldwide to build and launch new Low Earth Orbit (LEO) satellites for a multitude of communication and remote-sensing applications. The high number of LEO satellites soon to be available, their relative proximity to Earth compared to GNSS satellites, as well as the potential of Doppler-based positioning makes these LEO systems good candidates for future positioning solutions, to complement the existing GNSS and terrestrial navigation. LEO systems for Positioning, Navigation, and Time (PNT), briefly referred to as LEO-PNT, can be built either by reusing the existing constellations as signals of opportunity (SoO), or by building new LEO constellations optimized for the positioning purpose. The goal of this paper is to offer a comprehensive comparison in terms of code-based and Doppler-based Geometric Dilution of Precision (GDOP) between existing LEO systems and to discuss the optimization steps to follow in building novel LEO-PNT constellations for best positioning performance. We show that existing broadband LEO constellation with thousands or more satellites are good candidates for SoO in positioning and they can offer close to 100% coverage.

I. INTRODUCTION AND MOTIVATION

In the navigation community, it is a well-known fact that the existing Global Navigation Satellite Systems (GNSS) from Medium Earth Orbits (MEO) and Geo-stationary Orbits (GEO) have an increasing need of complementary navigation and positioning solutions in order to cope better with intentional and unintentional interferences [1] and to satisfy the demand for accurate indoor and urban navigation [2].

Various complementary solutions have been studied in detail so far, such as positioning solutions relying on cellular terrestrial networks, and in particular on 5G networks promising high accuracy, pseudolite-based positioning [2], WiFi-based positioning, sensor-based augmentation, positioning via long-range low-power Internet of Things (IoT) signals such as LoRa and NB-IoT, or Ultra Wide Band (UWB)-based positioning [3], [4], [5], [6]. Among the above-mentioned complementary techniques, the standalone systems able to achieve the sub-meter accuracy currently offered by GNSS are the 5G-based and UWB-based solutions, but both have limited coverage and a relatively large energy consumption at the receiver side. In recent years, as more and more effort has been put to develop various Low Earth Orbit (LEO) constellations for communication purposes, researchers have also started to investigate the possibility of using existing LEO signals for PNT purposes as well as of designing novel LEO constellations with good properties for target PNT metrics [7], [8], [9], [10], [11]. For example, in [7] the authors explored how LEO constellations in use for communication purposes can be explored for navigation purpose, based on the premises that LEO satellite signals are received at a much higher power than GNSS signals, due to their closer proximity to Earth, and are thus more capable to penetrate indoors and to offer good coverage in deep urban canyons. Four LEO satellite systems were selected, namely Iridium, Teledesic (nowadays cancelled), OneWeb, and SpaceX and detailed comparisons with GNSS were offered in terms of link budgets, number of satellites in view, Geometrical/Horizontal/Vertical Dilution of Precision (GDOP/HDOP/VDOP), and user range errors (URE). The conclusions were that the studied LEO constellations gave rise to better geometry than any of the GNSS constellations. The study in [7] did not explore the possibility of designing specifically a LEO-PNT constellation achieving a certain target GDOP with a minimum amount of LEO satellites in orbits, nor did it investigate other LEO satellite classes such as those used for Internet of Things (IoT) applications or for Earth Observation (EO) and Synthetic Aperture Radar (SAR).

The authors of [8] also explored alternative satellite systems structures to complement existing GNSS solutions and focused on low-MEO and some LEO architectures. A Pareto-based optimization approach was used, focusing on the deployment costs and payload power consumption. Their analysis concluded that LEO constellations will lead to significantly higher costs than current GNSS architectures and they proposed low-MEO orbits (e.g. around 8300 km altitude) as a good tradeoff for the next generation of satellite-based positioning. Nevertheless, in the section dealing with limitations of the study of [8] it was emphasized that more orbits and constellations need to be taken into account for more robust conclusions, leaving therefore place for further deeper investigations of more satellite constellations.

GDOP, as a metric related to both the coverage and the positioning accuracy of a system, has been largely used as an essential metric in the literature dedicated to navigation and positioning. Intuitively speaking, GDOP is a measure of how ‘good’ the distribution of the satellites on the sky is and it is inversely proportional to the tetrahedron formed with visible satellites together with the receiver as vertices. The smaller the GDOP value is, the better the accuracy of positioning solution is expected to be and the better the Earth coverage is [12], [13]. Traditionally, a GDOP value of 1 is considered ‘ideal’, but with the advent of an increased number of satellites in the sky, values below 1 have also become realistic. GDOP-based
Low Earth Orbit (LEO)-satellite systems classification

<table>
<thead>
<tr>
<th>Main characteristics</th>
<th>Examples</th>
<th>Medium-size constellations</th>
<th>Large-size constellations</th>
<th>Medium-to-large size constellations</th>
<th>Small-to-medium size constellations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oldest LEO constellations Typically in L and S bands (1-4GHz)</td>
<td>Amazon Kuiper (US)</td>
<td>Typically multiple orbits per constellation</td>
<td>High carrier frequencies: typically in Ka and Ku bands (17-20 GHz) or in mmWave bands (30-90 GHz)</td>
<td>Small-sized satellites (e.g., nanosatellites) fast to deploy</td>
<td>Many use low carrier carrier frequencies: typically in VHF/UHF (100-400 MHz)</td>
</tr>
<tr>
<td>Voice and other narrowband communications</td>
<td>Globalstar (US)</td>
<td>Target GDOP values of the new constellation were between 2 and 7.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1. Classification of current LEO satellite systems based on target applications, with illustrative examples and main characteristics.

optimization studies already exist in the literature, mostly in the GNSS context. For example, in [14], a GDOP-based minimization for GNSS receivers reached an average GDOP value of about 0.8 with at least 9 GNSS visible satellites, and a GDOP value of about 0.9 with at least 4 GNSS visible satellites. Horizontal, vertical, time, and position dilution of precision were also analyzed in [10], but only for StarLink LEO constellation. No comparison with other LEO systems was provided in [10]. A code-based GDOP optimization for LEO constellations suitable for positioning was also adopted in [11] and the authors proposed a new constellation with 240 satellites distributed in 3 orbital planes at an altitude of 1000 Km and orbital inclinations of 90°, 60°, and 35°, respectively (with higher inclinations orbits offering better coverage at poles and lower inclination orbits offering a greater coverage in equatorial areas). In [11], the target GDOP values of the new constellation were between 2 and 7. There was no comparison with other LEO systems provided in [11]. As one advantage of LEO satellites is the positioning capability with Doppler measurements. Doppler-based dilution of precision metrics, such as Doppler GDOP (DGDOP) become of interest and have been studied for example in [15].

As the above-mentioned studies show, there is still an unaddressed gap in the LEO-related literature regarding comprehensive comparisons between various emerging and existing LEO systems and their suitability for PNT purposes. Our paper aims to contribute to this gap, by offering a more comprehensive comparison between various classes of LEO systems than what exists in the current literature and by analyzing the concepts of code and Doppler-based GDOP performance. Our main contributions are two fold:

- Providing a comprehensive comparison between various LEO system classes in terms of their suitability for positioning;
- Introducing the generic framework of both code- and Doppler-based GDOP analysis and explaining the optimization steps to follow for building novel LEO-PNT constellations;

II. OVERVIEW OF CURRENT LEO SATELLITE SYSTEMS

The current LEO satellite systems can be broadly divided into four main categories, as illustrated in Fig. 1, based on their target applications: i) voice and other narrowband communications, which typically support data rates between few hundreds of kbits/s to few Mbits/s, such as the legacy Iridium and Globalstar; ii) broadband communications, aiming at data rates of tens to hundreds of Mbits/s, such as Amazon Kuiper and SpaceX StarLink constellations; iii) Internet of Things (IoT) low-power applications, such as Astrocast, Hiber, or Myriota; and iv) Earth Observation (EO) and Synthetic Aperture Radar (SAR), used in remote sensing applications. For our case studies, we selected at least one representative LEO constellation per class and analyzed them in a comparative framework.

Fig. 2. Comparison of coarse estimates of average Doppler shifts for MEO and LEO satellites.

Unlike MEO GNSS systems, LEO satellites operate at higher orbital speeds and often at higher carrier frequencies...
than GNSS and therefore they usually have larger Doppler shifts than MEO GNSS satellites. A basic comparison of coarse Doppler shifts estimates for various LEO and MEO constellations is shown in Fig. 2, revealing that most of LEO constellations (with the exception of LEO IoT ones, which typically operate in the VHF/UHF bands) have significantly higher Doppler shifts than MEO GNSS satellites.

III. BRIEF OVERVIEW OF POSITIONING WITH LEO SIGNALS

Similarly with any other wireless signal-based positioning, LEO signals can use time-based, angle-based, power-based, or Doppler-based measurements in order to form a positioning solution for an Earth-placed receiver. Time-based principles follow the general rules of GNSS-based positioning [12], with the main difference that the on-board LEO satellites clocks are much less stable than the on-board GNSS clocks, and therefore a GNSS on-board receiver is needed in order to perform the time synchronization between LEO satellites. Angle-based positioning for LEO has been so far very little studied, but it is worth to be further investigated as most LEO transmitters have multi-antenna arrays and have the potential of accurate angle-of-departure measurements.

Doppler-based positioning relies on measuring the Doppler frequency shifts from multiple satellites and combining the measurements in a non-linear system of equations solved for example via an Extended Kalman Filtering (EKF) [15]. The fast variation of the Doppler frequency due to the high speed of the LEO satellites makes the Doppler measurements a principal source of positioning information, which can be obtained even without demodulating the received signal. Since continuous tracking of the satellite signal is not necessary for Doppler-based positioning, then just Doppler acquisition of a strong signal is needed to produce the necessary measurements. The consequence is a low processing complexity which means also reduced time to produce the positioning solution, if the satellite positions are known and the accuracy requirement is not stringent.

IV. GDOP-BASED COVERAGE ANALYSIS

A. Code-GDOP analysis

In any satellite-based positioning system, the resulting position error is influenced by the relative geometry between transmitter and receiver [12]. This concept is called dilution of precision (DOP). DOP measurements can be expressed as:i) HDOP – Horizontal DOP; ii) VDOP – Vertical DOP; iii) PDOP – Position (3D) DOP; iv) TDOP – Time DoP; and v) GDOP – Geometric DOP. For calculating the code GDOP, we first define the measurement matrix \( \mathbf{H} \), which contains \( K \) range or code measurement residuals. It is obtained as the algebraic differentiation of the \( K \) code range measurement equations with respect to the user’s position and time:

\[
\mathbf{H} = \begin{bmatrix}
    h_{x1} & h_{y1} & h_{z1} & 1 \\
    h_{x2} & h_{y2} & h_{z2} & 1 \\
    \vdots & \vdots & \vdots & \vdots \\
    h_{xK} & h_{yK} & h_{zK} & 1
\end{bmatrix},
\]

where \( h_{xi} = \frac{x_{si} - x}{R_i} \), \( h_{yi} = \frac{y_{si} - y}{R_i} \) and \( h_{zi} = \frac{z_{si} - z}{R_i} \) are the components of a unit vector from the receiver to the \( i \)th satellite. \( x_{si}, y_{si}, z_{si} \) are the user coordinates and \( R_i \) is the pseudorange, defined as

\[
R_i = \sqrt{(x_{si} - x)^2 + (y_{si} - y)^2 + (z_{si} - z)^2}.
\]

The next step is to obtain the matrix \( \mathbf{Q} \) as

\[
\mathbf{Q} = (\mathbf{H}^T \mathbf{H})^{-1} = \begin{bmatrix}
    \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\
    \sigma_{12} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\
    \sigma_{13} & \sigma_{23} & \sigma_{33} & \sigma_{34} \\
    \sigma_{14} & \sigma_{24} & \sigma_{34} & \sigma_{44}
\end{bmatrix},
\]

Finally, the GDOP is defined as the square-root of the trace of \( \mathbf{Q} \), i.e. \( \sqrt{\text{sum}(\text{diag}(\mathbf{Q}))} \). Depending on the GDOP values we get, we can classify if we have a better or worse geometry based on the following rating:

<table>
<thead>
<tr>
<th>GDOP Value</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>\leq 1</td>
<td>Ideal</td>
</tr>
<tr>
<td>1 – 2</td>
<td>Excellent</td>
</tr>
<tr>
<td>2 – 5</td>
<td>Good</td>
</tr>
<tr>
<td>5 – 10</td>
<td>Moderate</td>
</tr>
<tr>
<td>10 – 20</td>
<td>Fair</td>
</tr>
<tr>
<td>&gt; 20</td>
<td>Poor</td>
</tr>
</tbody>
</table>

B. Doppler-GDOP analysis

Also in case of Doppler-based positioning, the magnitude of the positioning error is influenced by the geometrical disposition of the satellites in view w.r.t the user location. However, its contribution to the total positioning error changes according to the changed impact of the satellites positions in the measurement equations. In order to compute the new matrix \( \mathbf{H}_{DP} \), it is worth recalling the expression of the delta range residual function \( p_i(\hat{x}) \) for a generic \( i \)th satellite in view:

\[
p_i(\hat{x}) = v_i \cdot \frac{r_i - \hat{r}_u + \hat{d} - \hat{q}_i}{r_i - \hat{r}_u}, \quad (4)
\]

where: \( v_i, \forall i = 1, \ldots, K \), are the satellite relative velocity vectors; \( r_i \) are the satellite position vectors; \( \hat{d} \) is the delta range measurement; \( \hat{r}_u \) is the estimated position vector; \( \hat{q}_i \) is the estimated receiver clock drift. The matrix \( \mathbf{H}_{DP} \) is obtained through an algebraic differentiation of the function \( p(\hat{x}) \) as in the following:

\[
\mathbf{H}_{DP} = \frac{\partial p}{\partial \hat{x}} \in \mathbb{R}^{K \times 4},
\]

According to [16], by applying the derivative of (4) in (5) we can obtain the following expression for the \( \mathbf{H}_{DP} \) matrix:

\[
\mathbf{H}_{DP} = \begin{bmatrix}
    \left\{ \frac{r_1 - \hat{r}_u}{R_1} \times \left\{ \frac{r_1 - \hat{r}_u}{R_1} \times v_1 \right\} \right\}^T & 1 \\
    \left\{ \frac{r_2 - \hat{r}_u}{R_2} \times \left\{ \frac{r_2 - \hat{r}_u}{R_2} \times v_2 \right\} \right\}^T & 1 \\
    \vdots & \vdots \\
    \left\{ \frac{r_K - \hat{r}_u}{R_K} \times \left\{ \frac{r_K - \hat{r}_u}{R_K} \times v_K \right\} \right\}^T & 1
\end{bmatrix}.
\]
Similarly to (3) it is possible to calculate the matrix $Q_{DP} = \left( H_{DP}^T H_{DP} \right)^{-1}$. Eventually the Doppler GDOP is obtained by the square root of the trace of the $Q_{DP}$ matrix.

V. SIMULATION-BASED RESULTS

A. Selected existing LEO constellations

The selected constellations which were compared via simulations are listed in Table II. Table II comprises LEO constellations for diverse applications as illustrated earlier in Fig. 1. Global Internet broadband constellations contains the highest number of satellites in orbit, e.g., more than 3000 and close to 12000 for Amazon Kuiper and SpaceX StarLink, respectively. These constellations are spread in different orbital altitudes (3 and 8 orbital altitudes for Amazon Kuiper and SpaceX StarLink, respectively) and different orbital planes per each altitude. The frequency band used for these constellations comprises upper cm-wave bands and mm-wave bands, such as K-band (18 – 26.5 GHz), Ka-band (26.5 – 40 GHz) and even V-band (40 – 75 GHz). Next constellations with a high number of satellites are IoT constellations, with 80, 600 and 50 for Astrocast, Hiber and Myriota, respectively. These constellations are only at a single altitude, typically lower than for broadband communication satellites. These constellations use frequency bands typically employed for terrestrial IoT communications too, such as VHF (30 – 300 MHz), UHF (300–3000 MHz), L-band (1–2 GHz) and S-band (2–4 GHz). Iridium Next belongs to the narrowband communications category, it is distributed in a single altitude, and comprises about 66 satellites transmitting in L-band and Ka-band. EO/SAR LEO constellations are constantly observing the Earth, which guarantees global coverage. The selected constellations in Table II are spread in four altitudes and 60 satellites for BlackSky Global and one altitude and 18 satellites in one altitude for ICEYE constellation. The frequencies used for these constellations are in the X-band (8 – 12 GHz), which is typically assigned to EO/SAR purposes.

B. Code-GDOP-based analysis

Comparative results for the eight constellations shown in Table II are given in Figs. 3, 4, and 5, for GDOP distribution per Earth point, average, minimum and maximum GDOP over $10^5$ uniformly distributed Earth points, and percentage of Earth points with more than 4 LEO satellites in view, respectively. As seen in Fig. 3, Amazon Kuiper, Myriota, Astrocast and Iridium have low or no coverage at very high and very low latitudes, while Hiber is better optimized for extreme latitudes than for middle ones. As expected, StarLink
## TABLE II

SUMMARY OF COMPARED LEO CONSTELLATIONS.

<table>
<thead>
<tr>
<th>Constellation</th>
<th>Application</th>
<th>Altitude per orbit (Km)</th>
<th>Mean Velocity per orbit (Km/s)</th>
<th>Period (minutes)</th>
<th>Orbital Planes per Orbit</th>
<th>Satellites per Plane</th>
<th>Total Number of Satellites</th>
<th>Inclination (degrees)</th>
<th>Eccentricity</th>
<th>Frequency Bands/ Downlink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amazon Kuiper</td>
<td></td>
<td>590</td>
<td>7.4681</td>
<td>96</td>
<td>28</td>
<td>28</td>
<td>3236</td>
<td>51.9</td>
<td>0</td>
<td>K-band: 17.75-17.85 GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>610</td>
<td>7.4076</td>
<td>97</td>
<td>36</td>
<td>36</td>
<td>11943</td>
<td>42</td>
<td>33</td>
<td>18.8-19.3 GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>630</td>
<td>7.3485</td>
<td>97</td>
<td>34</td>
<td>34</td>
<td></td>
<td></td>
<td>19.25-18.45 GHz</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>19.7-20.2 GHz</td>
<td></td>
</tr>
<tr>
<td>SpaceX StarLink</td>
<td>Global</td>
<td>335.9</td>
<td>7.5576</td>
<td>91-112</td>
<td>9</td>
<td>277</td>
<td>11943</td>
<td>42</td>
<td>0</td>
<td>K-band: 17.8-18.5 GHz</td>
</tr>
<tr>
<td></td>
<td>Internet Broadband</td>
<td>346.8</td>
<td>7.5839</td>
<td></td>
<td>7</td>
<td>354</td>
<td></td>
<td>48</td>
<td>35</td>
<td>18.8-19.3 GHz</td>
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<tr>
<td></td>
<td></td>
<td>345.6</td>
<td>7.6098</td>
<td></td>
<td>9</td>
<td>283</td>
<td></td>
<td>53</td>
<td>33</td>
<td>19.7-20.2 GHz</td>
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<tr>
<td></td>
<td></td>
<td>550</td>
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<td>24</td>
<td>66</td>
<td></td>
<td>53</td>
<td>74</td>
<td>V-band: 37.5-42.0 GHz</td>
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<tr>
<td></td>
<td></td>
<td>1110</td>
<td>7.2139</td>
<td></td>
<td>32</td>
<td>50</td>
<td></td>
<td>74</td>
<td>70</td>
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<td>1130</td>
<td>7.2696</td>
<td></td>
<td>8</td>
<td>50</td>
<td></td>
<td>81</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1275</td>
<td>7.2128</td>
<td></td>
<td>5</td>
<td>75</td>
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<td>70</td>
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<tr>
<td></td>
<td></td>
<td>1325</td>
<td>7.1679</td>
<td></td>
<td>6</td>
<td>75</td>
<td></td>
<td></td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Astrocast</td>
<td>IoT</td>
<td>600</td>
<td>7.555</td>
<td>96</td>
<td>8</td>
<td>10</td>
<td>80</td>
<td>97.8</td>
<td>0</td>
<td>L-band 400-401 MHz</td>
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<tr>
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<td></td>
<td></td>
<td></td>
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<td>400.15-401 MHz</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>399-403 MHz</td>
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<tr>
<td>Hiber</td>
<td>IoT</td>
<td>600</td>
<td>7.6105</td>
<td>96</td>
<td>10</td>
<td>60</td>
<td>600</td>
<td>97</td>
<td>0</td>
<td>156-165 MHz</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>1.616-1.63 GHz</td>
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<td></td>
<td></td>
<td></td>
<td>19.3-19.7 GHz</td>
<td></td>
</tr>
<tr>
<td>Myriota</td>
<td>IoT</td>
<td>600</td>
<td>7.5558</td>
<td>96</td>
<td>5</td>
<td>10</td>
<td>50</td>
<td>97</td>
<td>0</td>
<td>UHF: 400.15-401 MHz</td>
</tr>
<tr>
<td>Iridium Next</td>
<td>Narrowband Communications</td>
<td>780</td>
<td>7.4628</td>
<td>97</td>
<td>6</td>
<td>11</td>
<td>66</td>
<td>86</td>
<td>0</td>
<td>L-band: 1.616-1.63 GHz</td>
</tr>
<tr>
<td></td>
<td>Earth Observation</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>800.15-8.4 GHz</td>
<td></td>
</tr>
<tr>
<td>BlackSky Global</td>
<td>Earth Observation</td>
<td>450</td>
<td>7.5825</td>
<td>94-96</td>
<td>2</td>
<td>6</td>
<td>60</td>
<td>97.59</td>
<td>0</td>
<td>X-band: 8.025-8.4 GHz</td>
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<tr>
<td></td>
<td></td>
<td>460</td>
<td>7.5632</td>
<td></td>
<td>2</td>
<td>6</td>
<td>45</td>
<td>97.73</td>
<td>0</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>550</td>
<td>7.5014</td>
<td></td>
<td>2</td>
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<td>45</td>
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<td></td>
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<td>580</td>
<td>7.5119</td>
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<td>4</td>
<td>6</td>
<td>45</td>
<td>97.73</td>
<td>0</td>
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</tr>
<tr>
<td>ICEYE</td>
<td>Earth Observation</td>
<td>570</td>
<td>7.5715</td>
<td>95</td>
<td>1</td>
<td>6</td>
<td>18</td>
<td>97.69</td>
<td>0</td>
<td>X-band: 8.025-8.4 GHz</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9.65 GHz</td>
<td></td>
</tr>
</tbody>
</table>

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**Fig. 5.** Percentage of points on Earth with more than 4 satellites in view belonging to a certain constellation.

The constellation has excellent GDOP values all over the Earth, thanks to their huge constellation of 11943 satellites. In Fig. 4, the GDOP statistics show that the average GDOP is generally better for the constellations with a high number of satellites than for the constellations with a low number of satellites. A relatively small GDOP (2.24) is obtained with Blacksky and only 60 satellites, but only in 0.47% cases 4 or more satellites are simultaneously in view in BlackSky case, as shown in Fig. 5. This points out towards the fact that a good average GDOP does not give the full information, as GDOP is only computed on points that 'see' at least 4 satellites, and therefore the plots in Figs. 4 and 5 should be considered together in order to understand the suitability of a LEO constellation for positioning. According to Fig. 5, StarLink is the best among existing LEO constellations to be used also as signals of opportunity for positioning purposes, as it would offer 100% coverage and excellent GDOP.

**C. Doppler-GDOP results**

By leveraging on equation (5) we have also computed the Doppler GDOP for the eight constellations under investigation for over $10^6$ uniformly distributed Earth points. However, due to lack of space in the paper, we limit the discussion to the two constellations with the highest number of visible satellites: Amazon Kuiper and SpaceX StarLink, respectively. For those two constellations, the results of Doppler GDOP are shown in Fig. 6 and compared with the traditional code GDOP for users located along an Earth meridian. As expected, the values of Doppler GDOP are remarkably bigger (almost two orders of magnitude) than the ones related to the code GDOP. This is a clue that the position solution obtained through a pure Doppler-based positioning method is not as accurate as the one obtained through code-ranging measurements. However, the advantages of using a Doppler-based positioning approach is in the simplicity of the method rather than in the accuracy of the...
estimated position. Moreover, such technique can be applied to satellite systems that are not designed for PNT purposes or that broadcast signals on a bandwidth not wide enough to allow ranging measurements. In addition, Doppler-based method can be used in combination with ranging signals to compute a valid position estimation also in case of visibility of only 1 or 2 LEO satellites [17], [18] or it can be used to provide an initial assessment of the user’s position in case of "snapshot-based" techniques [19].

VI. DISCUSSION ON GDOP-BASED OPTIMIZATION

In order to design novel LEO-PNT constellations, one must first define the parameters to optimize and the optimization criterion. We propose that the following parameters can be optimized in order to define a new constellation: the number of orbital altitudes, the number of orbital planes per altitude (e.g., a simplifying assumption can be that there is the same number of orbital planes at each altitude), the number of satellite per each orbital plane, and the satellites inclination. The optimization metric can be the the average code-GDOP, Doppler-GDOP or a joint code-Doppler GDOP metric. The average values can be computed over a sufficiently high number of Monte Carlo simulations with receivers distributed all over the Earth surface (e.g., by assuming a uniform distribution). Brute-force approaches can be used, as well as generic algorithms for the optimization process. In order to simplify the multi-dimensional search, one can also impose some additional constraints on some of the parameters that defined the constellation, such as the number of orbital altitudes (typically more than one altitude offers better positioning capabilities), the altitude heights (e.g., following certain design constraints in launching new LEO-PNT satellites), and the satellite inclination at each altitude. Typically, the satellites with a lower altitude should have a higher inclination than the satellites at a higher orbit altitude in order to be able increase the coverage at both high and low latitudes, e.g. as shown in [11].

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