Abstract—The Siwakoti-H flying-capacitor inverter (sFCI) is a potential candidate for photovoltaic applications, specifically for the transformerless grid-connected systems. One of the main challenges in the control of a sFCI is to maintain the flying capacitor voltage within prescribed limits while balancing the voltages on the three flying capacitors. This paper proposes an indirect model predictive control strategy for a three-phase sFCI connected to the grid via an LCL-filter. By linearizing the system model, the nonlinearities introduced due to the dynamics of the flying capacitor are neglected. Moreover, by not directly controlling the switches, but rather manipulating the modulating signal, the optimization problem can be formulated as a quadratic program (QP) and solved in a computationally efficient manner. The explicit solution computed by the controller makes the real-time implementation feasible by employing a carrier-based pulse width modulator (CB-PWM). The presented results illustrate the steady-state and dynamic performance of the controller.

I. INTRODUCTION

RENEWABLE sources of energy, such as solar and wind energy, are being increasingly used as alternatives to conventional energy resources. The primary energy of these sources varies widely in its nature due to various factors like weather conditions, irradiance level, wind speed, etc [1]. Therefore, high efficiency power electronic converters are required to interface these sources with the grid. Specifically for solar power, there has been an increasing trend to use transformerless converters with multilevel output [2]. Among the recently proposed topologies, the Siwakoti-H inverter [3], [4] is a common-ground-type transformerless inverter which works on the principle of a flying capacitor and consists of only four switches. To mitigate the problem of leakage currents, the sFCI allows direct connection between the grid-neutral and negative-pole of the PV system [5].

Previously, the control of the grid-connected sFCI was presented using a state-feedback controller [5], [6]. However, the drawback of using linear control schemes on nonlinear systems is that they can produce uneven performance throughout the dynamic range. On the contrary, nonlinear control techniques like model predictive control (MPC) can offer faster dynamic response, favorable steady-state performance over the whole range of operating points and a provision to handle various nonlinearities including system constraints [7]. MPC is broadly classified into two methods, i.e., direct MPC (also referred to as finite control set MPC—FCS-MPC) and indirect MPC (also known as continuous control set MPC—CCS-MPC) [7], [8].

Direct MPC (DMPC) exploits the finite number of the switching states of a power converter and combines the control and modulation into one stage [9]. However, DMPC suffers from the problem of variable switching frequency and becomes computationally demanding as the number of switching states increases [9]. Compared to the enumeration based DMPC, indirect MPC usually has a lower computational cost due to the fact that the optimization problem is a quadratic program (QP) and not an integer program. Owing to this, indirect MPC can more effectively address long-prediction horizon problems [7]. Moreover, indirect MPC employs a modulator to generate the desired switching signals and therefore the system operates at a fixed switching frequency.

In this paper, a current controller based on indirect MPC is designed for a three-phase sFCI connected to the grid with an LCL-filter. Since a modulator is introduced, the optimization variable is the modulating signal. As a result, the optimization problem can be cast as a QP, for details see [10, Section 5.2]. This scheme allows the use of long prediction horizons and simplifies real-time implementation since it can effectively be solved with off-the-shelf solvers. The main contributions of this paper are: 1) the system modeling of the three-phase sFCI; 2) the formulation of optimization problem and design of the indirect MPC scheme.

II. SYSTEM MODELING

Fig. 1 shows the phase-leg of a grid-connected sFCI with an LCL-filter which is based on the inverter topology proposed in [3], [4]. The converter employs a common capacitor $C_{in}$ for the input dc-link with a constant voltage $u_{dc}$, and a separate flying capacitor ($C_{FC}$) for each phase. The LCL-filter consists of a main inductor $L_m$ and a grid-side inductor $L_g$, with internal resistances $R_m$ and $R_g$, respectively, and the filter capacitance $C_f$ with parasitic resistance $R$. The LCL-filter provides sufficient attenuation to the switching frequency harmonics present in the main inductor current $i_m$. In the remainder of this section, the continuous- and discrete-time models of the system are introduced. It is assumed that the amplitude and phase of the grid voltage remains constant for the model and the direction of the current flow is
from the converter to the grid. The following assumptions are introduced for the purpose of modeling:

**Assumption (A.1)** The grid voltage $u_g(t)$ is considered as a system state. It has a positive magnitude, and a constant angular frequency $\omega_g > 0$.

**Assumption (A.2)** The switching behavior of the converter bridge is neglected, i.e. the nonlinear behavior of the flying capacitor due to its interaction with the switches is not considered in the modeling stage. For details of nonlinear behavior see [5].

**Assumption (A.3)** The bridge voltage $u_m(t)$ is assumed to be constant during $kT_s < t < (k + 1)/T_s$, where $T_s$ is the sampling interval [11].

### A. Continuous-time mathematical model

To simplify the modeling and ease the control design, the variables are expressed in the stationary ($\alpha\beta$) reference frame instead of the three-phase system (abc). A variable $\xi_{abc} := [\xi_a \ \xi_b \ \xi_c]^T$ can be transformed to a variable $\xi_{\alpha\beta} := [\xi_\alpha \ \xi_\beta]^T$ in the $\alpha\beta$ system through $\xi_{\alpha\beta} = T_c \xi_{abc}$, where $T_c$ is the Clarke transformation matrix, given as

$$T_c = \frac{2}{\sqrt{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}. \quad (1)$$

Based on the assumption (A.1), the grid voltage is considered as a system state and its behavior can be modeled with an oscillator. Assuming the voltages $u_{g,\alpha}$ and $u_{g,\beta}$ to be sinusoidal signals of fixed frequency $\omega_g = 2\pi f$, an oscillator can be modeled using a two-state linear system as

$$\frac{d}{dt} \begin{bmatrix} x_{osc1} \\ x_{osc2} \end{bmatrix} = \begin{bmatrix} 0 & \omega_g \\ -\omega_g & 0 \end{bmatrix} \begin{bmatrix} x_{osc1} \\ x_{osc2} \end{bmatrix}. \quad (2)$$

The system states include the main inductor current, the grid current, the filter capacitor voltage and the grid voltage. Thus, the state vector is

$$x = [i_{m,\alpha\beta}^T \ u_f^{T,\alpha\beta} \ i_{g,\alpha\beta}^T \ u_{g,\alpha\beta}^T]^T \in \mathbb{R}^8.$$  

The three-phase modulating signal $s_{abc} \in S^3$ is considered as the input to the system, where the vector $s_{abc} = [s_a \ s_b \ s_c]^T$ and $S^3 = [-1, 1]^3 \subset \mathbb{R}^3$. Invoking assumptions (A.1), (A.2), Kirchhoff’s current and voltage laws, the continuous-time system model in the $\alpha\beta$ reference frame can be written as

$$\frac{d}{dt} x(t) = F x(t) + G T_c s_{abc}(t) \quad (3a)$$

$$y = C x(t). \quad (3b)$$

The matrices of the system model are

$$F = \begin{bmatrix} -R_1^R -R_2^R & -\frac{1}{L_2} I_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{R_1^R}{L_1} I_1 & \frac{R_1}{L_1} I_2 & -\frac{R_1^R}{L_1} I_2 & 0 & 0 & 0 & 0 & 0 \\ \frac{R_2^R}{L_1} I_1 & \frac{R_2}{L_1} I_2 & -\frac{R_2^R}{L_1} I_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4)$$

$$G = \begin{bmatrix} \frac{u_{g\alpha} + u_{g\beta}}{2L_f} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = [I_6 \ 0_{4 \times 2}],$$

$$\text{where } I_n \in \mathbb{R}^{n \times n} \text{ is an identity matrix and } 0_n \in \mathbb{R}^{n \times n} \text{ is a zero matrix.}$$

### B. Discrete-time mathematical model

Considering the converter bridge output $u_m(t)$ to be piecewise-constant within each sampling interval and invoking assumption (A.3), the continuous-time model (3), (4) can be discretized using zero-order-hold method. The exact discrete-time model is written as

$$x(k + 1) = Ax(k) + Bs_{abc}(k) \quad (5a)$$

$$y(k) = Cx(k), \quad (5b)$$

where $k \in \mathbb{N}$ is the discrete-time index, and the discretized system matrices are

$$A = e^{FT_s}, \quad B = \left( \int_0^{T_s} e^{FT_e} \omega_s \tau d\tau \right) GT_c. \quad (6)$$

### III. CONTINUOUS CONTROL SET MODEL PREDICTIVE CONTROL

The main objective of the predictive controller is to regulate the grid current, main inductor current and filter capacitor voltage along their references. Additionally, the flying capacitor voltage must be maintained within the allowable limits. To achieve the mentioned goals, the control algorithm calculates the optimal modulating signal, which is then applied to the CB-PWM block for subsequent generation of the gating signals.

**A. Control and optimization problem**

The discrete-time model (5), (6) is used to predict the output $y(k)$ of the system. At time-step $k$, the cost function that penalizes the error of the output variables and the switching effort over the finite prediction horizon of $N_p$ time steps is written as

$$J(k) = \sum_{l=k}^{k+N_p-1} \|y_{ref}(l + 1) - y(l + 1)\|_Q^2 + \lambda_s \|\Delta s_{abc}(l)\|_2^2. \quad (7)$$

In (7), $y_{ref} \in \mathbb{R}^6$ encompasses the references for the LCL-filter states. The first term of the cost function implements the objective of reference tracking with $Q \in \mathbb{R}^{6 \times 6}$ as the weighting factor matrix. The second term implements the
minimization of control effort, where penalization is carried out using a nonnegative weighting factor $\lambda_s$.

The dynamic evolution of the discrete-time prediction model (5) and (6) can be included in the cost function (7) by reformulating the cost function in vector form. Therefore, the remainder of this subsection closely follows the derivation of the optimization problem provided previously in [10].

1) Derivation of optimization problem in vector form: By successively using the state equation (5a) (omitting the subscripts for signals in the $abc$ frame) over the prediction horizon, we can write

$$x(k+1) = Ax(k) + Bs(k)$$
$$x(k+2) = Ax(k+1) + Bs(k+1)$$
$$\vdots$$
$$x(k+N_p) = A^{N_p}x(k) + \cdots + A^0Bs(k+N_p-1).$$

The output trajectory vector for a prediction horizon of $N_p$ is defined as

$$Y = [y^T(k+1) \ y^T(k+2) \ \cdots \ y^T(k+N_p)]^T. \quad (9)$$

Substituting (9) in (5b) yields

$$Y(k) = \Gamma x(k) + \Psi S(k), \quad (10)$$

where $S(k) = [s^T(k+1) \ s^T(k+2) \ \cdots \ s^T(k+N_p-1)]^T$ is the sequence of modulating signals that the controller has to decide upon. Finally, the matrices $\Gamma$ and $\Psi$ are given in the appendix.

If we define the output tracking error as $\Delta y = y_{ref} - y$, the first term in the cost function can be written as

$$J_1 = \sum_{l=k}^{k+N_p-1} \frac{\|\Delta y(l+1)\|^2}{\xi_Q} \quad (11a)$$
$$= \sum_{l=k}^{k+N_p-1} \Delta y^T(l+1)Q \Delta y(l+1) \quad (11b)$$
$$= \Xi^T(k) Q \xi \Xi(k) = \|\Xi(k)\|^2_{\xi_Q}, \quad (11c)$$

where $Q_{\xi} = \text{diag}(Q, \ldots, Q)$ is a diagonal matrix and $\Xi(k)$ is the output error trajectory. Substituting (10) into $\Xi(k) = Y_{ref}(k) - Y(k)$ in (11c), we get

$$J_1 = \|Y_{ref}(k) - \Gamma x(k) - \Psi S(k)\|^2_{\xi_Q}. \quad (12)$$

In a similar way the second term of the cost function (7) can be rewritten as

$$J_2 = \lambda_s \|WS(k) - Zs(k-1)\|^2_2, \quad (13)$$

with the matrices

$$W = \begin{bmatrix} I_3 & 0_3 & \cdots & 0_3 \\ -I_3 & I_3 & \cdots & 0_3 \\ 0_3 & -I_3 & \cdots & 0_3 \\ \vdots & \vdots & \ddots & \vdots \\ 0_3 & 0_3 & \cdots & I_3 \end{bmatrix}, Z = \begin{bmatrix} I_3 \\ 0_3 \\ \vdots \\ 0_3 \end{bmatrix}. \quad (14)$$

Combining (12) and (13) yields the cost function in vector form [10]:

$$J(k) = \|Y_{ref}(k) - \Gamma x(k) - \Psi S(k)\|^2_{\xi_Q} + \lambda_s \|WS(k) - Zs(k-1)\|^2_2. \quad (15)$$

The first term in (15) penalizes the reference tracking error, while the second term penalizes the control effort. After some further algebraic manipulations (see [10, Appendix 5.B]) the cost function can be written in a compact form

$$J(k) = S^T(k)HS(k) + 2\Theta^T(k)S(k) + \theta(k), \quad (16)$$

with

$$H = Y^T(k) Q_{\xi} Y + \lambda_s W^T W \quad (17a)$$
$$\Theta(k) = -(Y_{ref}(k) - \Gamma x(k))^T Q_{\xi} \xi - \lambda_s (Zs(k-1))^T W \quad (17b)$$
$$\theta(k) = \|Y_{ref}(k) - \Gamma x(k)\|^2_{\xi_Q} + \lambda_s \|Zs(k-1)\|^2_2. \quad (17c)$$

The cost function (16) consists of three terms. The first term is quadratic in the control sequence $S(k)$. The Hessian matrix $H$ is time-invariant if the system parameters are time-invariant. Also, the Hessian is positive definite for $\lambda_s > 0$. The second term is linear in the control sequence $S(k)$. Here the time-varying vector $\Theta(k)$ is a function of the state vector at time step $k$, the output references $Y_{ref}(k)$, and the previously chosen modulating signal $s(k-1)$. The third term is a time-varying scalar that has the same arguments as $\Theta(k)$ [10].

By completing the squares, (16) can be rewritten as

$$J(k) = (S(k) + H^{-1}\Theta(k))^T H(S(k) + H^{-1}\Theta(k)) + \text{const}. \quad (18)$$

The constant term in (18) is independent of $S(k)$ and will not have any effect on the optimal solution. Omitting the constant term, we can reformulate the optimization problem as [10]

$$S_{opt}(k) = \arg \text{minimize}_{S(k)} J(k) \quad (19a)$$
$$\text{subject to } S(k) \in [-1, 1]^{3N_p}. \quad (19b)$$

2) Solution in terms of the unconstrained minimum: The optimization problem (19) is solved by minimizing after neglecting any constraints (hence termed as unconstrained minimum). According to [10], the unconstrained minimum is the optimal solution of the problem (19), and can be calculated at every time step $k$ as

$$S_{unc}(k) = -H^{-1}\Theta(k). \quad (20)$$

$S_{unc}(k)$ contains a sequence of values of the modulating signal for operating the inverter at the next time steps. In order to provide feedback and control the system, only the first three-phase modulating signal $S_{unc,opt}(k)$ is used for further application to the modulator, as per the receding horizon principle [10].
The modulator generates the requisite gating signals inside the CB-PWM block (for details the reader is referred to [6]). The frequency minimization factor is set to $10^{-4}$. Despite the variation in the input dc supply, the grid current remains sinusoidal with an average total harmonic distortion.

### B. Controller block diagram

The scheme of the proposed indirect MPC is illustrated in Fig. 2. As can be observed, the control block takes in all the measurements and predicts the evolution of the states of the system, i.e., grid current, main inductor current, and the filter capacitor voltage. The reference signals are calculated separately and fed to the optimization stage where the optimal filter capacitor voltage. The reference signals are calculated for the system, i.e., grid current, main inductor current, and the measurements and predicts the evolution of the states of the system, i.e., grid current, main inductor current, and the filter capacitor voltage. The reference signals are calculated separately and fed to the optimization stage where the optimal solution is generated using the optimization problem (19). The output of the controller is the three-phase modulating signal $s_{abc,ref}$, which is supplied to the modulator for generation of switching signals. Additionally, the feedforward compensation of dc-link voltages, termed as dc voltage feedforward compensation (DVFC), is used for scaling of the modulating signal inside the CB-PWM block (for details the reader is referred to [6]). The modulator generates the requisite gating signals $u_{abc}$ and ensures desired system performance.

### C. Discussion

From the above derivations we can observe that an MPC scheme based on this approach can be used to implement long horizons with less computational effort. Most of the matrices required for the calculation of the unconstrained solution can be calculated offline. This removes the enormous computational burden on the controller compared to the enumeration based MPC approach. Additionally, meeting the grid standards, e.g., [12], is possible since the output current spectrum is deterministic, i.e., only nontriplen integer harmonics are expected. Both of these advantages make this control scheme feasible for real-time implementation.

### IV. Performance evaluation of indirect MPC

In this section, simulation results are presented to demonstrate the performance of the proposed indirect MPC scheme. The system under consideration is a three-phase sFCI rated at 5 kW. Table I contains the system parameters and the controller data for the indirect MPC scheme. A level shifted CB-PWM with a carrier frequency of 40 kHz is employed to generate the switching signals. The weighting factor matrix is set to $Q = \text{diag}(10, 10, 1, 1, 90, 90)$ and the switching frequency minimization factor is set to $\lambda_s = 10^{-4}$.

### A. Steady-state performance

For a real-power reference of 4.9 kW and zero reactive power, the grid-current reference (peak) $i_{g,ref}$ is set to 10 A. The steady-state performance of the three-phase sFCI is shown in Figs. 3(a) and (b). The voltage of all the flying capacitors is maintained below the allowable limit of 450 V, as seen in the Fig. 3(a). Additionally, the flying capacitors discharge below the input dc-link voltage and hence the sFCI works as desired.

Despite the variation in the input dc supply, the grid current remains sinusoidal with an average total harmonic distortion.
Fig. 3: Steady-state performance of the grid-connected three-phase sFCI with indirect MPC for a prediction horizon of $N_p = 15$.

(THD) of 2.3\% (see Fig. 4 for the frequency spectrum). Unlike the state-feedback control of sFCI, see [5], [6], where the grid current deviates from the sinusoidal behavior due to an uneven bump around zero crossing, the current response of the indirect MPC is superior with better THD.

B. Dynamic performance

The dynamic performance of the indirect MPC strategy is shown in Figs. 5, 6, and 7. At 0.005 s the real power demand is stepped to 4.9 kW and at 0.025 s the reactive power demand is set to 1.5 kVar (see Fig. 7). Fig. 5 compares the three controlled variables $u_f$, $i_m$, and $i_g$ with their respective references. As can be seen, the indirect MPC has excellent transient response with insignificant overshoot and very short settling time. In addition, all the variables follow their references implying superior reference tracking. The grid current tracking performance is shown in Figs. 6(a) and (b). It can be observed that, as soon as the reference value changes the controller is able to follow with minimal overshoot. The capability of the sFCI to deliver both real and reactive power is clearly demonstrated in Figs. 6(a) and (b). Fig. 7 highlights the voltage control of flying capacitors during system transients. Additionally, it also depicts the reference tracking of the desired real and reactive power references.

Fig. 4: Frequency spectrum of grid currents. The THD is 2.3\%.
A grid-connected three-phase sFCI with LCL model predictive control using long-horizon approach for a Moreover, the switching frequency is fixed and therefore the becomes easy as most of the calculation can be done offline. Using this method the implementation of long horizon control (QP)—is designed to compute the optimal modulating signal. To overcome the limitations of enumeration-based MPC, an LCL states of the a better dynamic and steady-state performance, all the three response, and the grid voltages.

Fig. 6: Reference tracking of the grid currents.

(a) Real power reference is set to 4.9 kW at 5 ms.

(b) Reactive power reference is set to 1.5 kVar at 0.025 s.

Fig. 7: Closed-loop simulation results of the sFCI with indirect MPC depicting the capacitor voltages, the output power response, and the grid voltages.

V. CONCLUSION

This paper presents a control strategy based on indirect model predictive control using long-horizon approach for a grid-connected three-phase sFCI with LCL-filter. To achieve a better dynamic and steady-state performance, all the three states of the LCL-filter are controlled using reference tracking. To overcome the limitations of enumeration-based MPC, an optimization problem—formulated as a quadratic program (QP)—is designed to compute the optimal modulating signal. Using this method the implementation of long horizon control becomes easy as most of the calculation can be done offline. Moreover, the switching frequency is fixed and therefore the control step time and sampling time of the system can be set to practically feasible values. Hence, this control scheme is feasible for real-time implementation.

Based on the presented results, it can be concluded that the presented MPC scheme has a better dynamic performance compared to a linear controller (state-feedback control), with shorter settling times, and also achieves superior reference tracking. As can be observed, thanks to the long horizon approach the proposed controller is relatively more robust to system transients compared to deadbeat control [12]. Moreover, the voltage balancing of the three flying capacitors is also accomplished as desired.

VI. APPENDIX

The matrices used in (10) are the following:

\[
\Gamma = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix}, \quad \Upsilon = \begin{bmatrix} CB & 0_{6 \times 3} & \ldots & 0_{6 \times 3} \\ CAB & CB & \ldots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N_p} - 1 B & CA^{N_p} - 2 B & \ldots & CB \end{bmatrix}
\]

REFERENCES


