

# A method for post-mission velocity and orientation estimation based on data fusion from MEMS-IMU and GNSS

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**Abstract**— INS and GNSS integrated systems have become widespread as a result of low-cost MEMS inertial sensor technology. However, the accuracy of computed velocity and orientation is not sufficient for some applications, e.g. performance and technique monitoring and evaluation in sports. Significant accuracy improvements can be made by post-mission data processing. The approach is based on fixed-lag Rauch-Tung-Striebel smoothing algorithm and provides a simple and effective solution to misalignment correction. The potential velocity accuracy is about 0.02 m/s and pitch/roll accuracy is about 0.02 deg. This algorithm was tested for walking and running. The proposed approach could also be used for accurate velocity and orientation estimation in other applications including different sports, e.g. rowing, paddling, cross-country and downhill skiing, ski jump etc.

## I. INTRODUCTION

Accurate determination of velocity and orientation is fundamental to many applications including studies of human locomotion in sports and ambulatory care settings. In most contemporary systems the approach for real-time computation of position, velocity and orientation is based on the fusion of data from IMU and GNSS and it has been implemented mainly through the use of different Kalman filter (KF) algorithms [1-5]. In this approach all of the possibilities for accuracy improvement through the enhancement of KF algorithms have been already used.

The Kalman filter is an optimal estimator only when the a priori information about the process noise and the update measurement noise as well as the models that describe the INS/GNSS system are accurate. However, the process and measurement noise depend on the application and the process dynamics, and therefore, they are not known accurately. Incorrect a priori filter statistics usually reduces the accuracy of the estimated filter states and can even introduce biases to their estimates. So, the approaches for achieving better accuracy of integrated IMU/GNSS systems are focused on the one hand on adaptive filtering algorithms that have more accurate a priori filter statistics [5], and on the other hand they are concerned with development of better error models that more accurately describe the IMU/GNSS system.

Accuracy can be also improved in post-processing (or potentially near real-time) when all of the data is available to the filter; the inertial sensor errors are also better estimated. There are many applications that only require the velocity and orientation for post mission analysis. Examples include analyses of human motion for sports or vehicle performance testing such as for racing cars or product testing. There are also other

applications that can benefit from the increased accuracy afforded by post-processed GNSS and low cost INS measurements and new applications can emerge because previously they were not practicable, mainly due to the cost of such technology. In some cases the post-processing smoothing is implemented only during GNSS outages [6,7], or in pipeline surveying [8] to minimize the position and velocity errors.

The post-processing approaches are often based on recursive KF type smoothing algorithm [6-11], for example, the Rauch-Tung-Striebel smoother (RTSS) or forward-backward combination of KF smoothing algorithms. Both of these smoothing algorithms are based on the maximum likelihood criterion and are mathematically equivalent in linear case. However, in navigation applications the RTSS is more effective and robust because it does not require implementation of the full-scale backward KF. Therefore, it can be regarded as an add-on correction to the KF unlike the forward-backward combination of KF smoothing algorithms that requires solutions of two independent KFs.

A smoothing framework for estimating sensor platform trajectories using an IMU and a dual-frequency GPS pseudorange and carrier-phase based on nonlinear least squares was described in [12]. In addition to the trajectory parameters this method can solve the carrier-phase integer ambiguities. Bernal et al. [13] proposed an algorithm for tight GNSS/INS integration based on linearly constrained least-squares problem to compute position and clock bias using a set of measured pseudoranges. In this approach the INS measurements are used to reduce the feasible variable space.

The objective of this paper is to introduce a novel approach for post-mission IMU/GNSS integration as an alternative to KF smoothing algorithms. Unlike the batch estimation algorithms described in [12,13] that estimate the entire optimal trajectory, we propose a parametric approach in which only a few parameters are computed as the solution of an optimization problem and then the entire trajectory including velocity and orientation is computed.

The main advantage of this approach is that it does not require the accurate a priori information about IMU and GNSS measurement error statistics and models. The algorithm is applied on fixed-length intervals of about 10-30 sec depending on the accuracy of gyroscopes. For such short duration the dominant source of the velocity error is the misalignment that can be used as an unknown parameter in a velocity error minimization problem. Unlike traditional recursive filtering methods that compute an optimal estimation of position, velocity and orientation as a weighted sum of a measurement and a filter prediction, our approach preserves the integrity and accuracy of a short-term IMU computed velocity and orientation.

The paper begins with a brief discussion of the INS velocity and orientation error propagation. It continues with the problem statement in the context of IMU/GNSS integration in real-time and for post-mission applications. Then it describes the proposed parametric method for estimation of attitude and velocity error and computation of corrected solution. Walking tests were performed to demonstrate the accuracy in estimation of velocity and orientation.

## II. PROBLEM STATEMENT

Our goal is to estimate the horizontal velocity and the angles that describe orientation of the sensor frame with respect to the geographic frame based on the IMU measurements and horizontal velocity measured by the GNSS receiver. The geographical frame that is used here is a local earth-fixed right-handed Cartesian coordinate system with X positive to the East (E), Y positive to the North (N), Z positive when pointing up (U).

For a low-cost MEMS INS that is nominally level with altitude compensation, and relatively short time of autonomous operation (less than 1 hour) the velocity and orientation errors can be approximated by the following equations [3]:

$$\begin{aligned}\delta\dot{V}_E &= a_N\phi_U - a_U\phi_N + a_E\mu_E + B_E \\ \delta\dot{V}_N &= a_U\phi_E - a_E\phi_U + a_N\mu_N + B_N \\ \dot{\phi}_N &= \frac{\delta V_E}{R} + \varepsilon_N \\ \dot{\phi}_E &= -\frac{\delta V_N}{R} + \varepsilon_E \\ \dot{\phi}_U &= \frac{\delta V_E}{R} \tan L + \varepsilon_U\end{aligned}\quad (1)$$

where  $\delta V_E, \delta V_N$  are the north and east components of velocity error,  $\phi_E, \phi_N$  are the horizontal tilt errors,  $\phi_U$  is the course error,  $B_E, B_N, \varepsilon_E, \varepsilon_N, \varepsilon_U$  are the projections of accelerometer biases and gyro drifts on the local-level frame,  $a_E, a_N, a_U$  are the acceleration vector components in the local-level frame and  $\mu_E, \mu_N$  are the accelerometer scale factor errors.

If the course error is small (less than 1 deg) and almost constant during at least one minute (or duration of the test) then the problem can be simplified by decoupling the North and the East channels. If the speed and dynamics are low (speed less than 10 m/s,  $\dot{V}_E, \dot{V}_N, \dot{V}_U \ll g$ , like in many sport applications) and accelerometer biases and misalignments are constant, then the horizontal tilts are the dominant sources of velocity error. Because the tilts cannot be separated from the accelerometer biases, only the combinations  $\phi_E + B_E, \phi_N + B_N$  can be estimated. This does not affect the accuracy of the horizontal velocity estimation, but it sets the limit for the tilt estimation. For factory calibrated accelerometers the scale factor error can be as low as 0.02%, making these error terms negligibly small compare to other terms in the equations for velocity error in most low dynamics applications.

The course error  $\phi_U$  can be accurately estimated only if there is substantial horizontal acceleration over a long period of time [14]. In most sport applications this condition is not fulfilled.

One solution to this problem is to fix the value for the course error and solve the following equations

$$\begin{aligned}\delta\dot{V}_E &= a_N\phi_U - a_U\phi_N + B_E \\ \delta\dot{V}_N &= a_U\phi_E - a_E\phi_U + B_N \\ \dot{\phi}_N &= \frac{\delta V_E}{R} + \varepsilon_N \\ \dot{\phi}_E &= -\frac{\delta V_N}{R} + \varepsilon_E\end{aligned}\quad (2)$$

where  $\phi_U$  is a parameter from a fixed set of values. In these equations the accelerometer scale factor error was neglected. Dmitriev et al. [4] proposed to use a posteriori density approximation based on partition of the domain of the most probable values  $\phi_U$  into a number of equal-length intervals with the center at the point  $\phi_U^j$  so that

$$\Omega = \bigcup_{j=1}^m (\phi_U^j - \Delta, \phi_U^j + \Delta), \text{ where } \Delta = \frac{\text{length}(\Omega)}{m} \quad (3)$$

A solution to this problem can be based on a bank of Kalman filters in which each KF is tuned to a specific value of  $\phi_U^j$  [4] and the measurement equation is described by

$$z = \begin{bmatrix} V_E^{INS} - V_E^{GNSS} \\ V_N^{INS} - V_N^{GNSS} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta V_E \\ \delta V_N \\ \phi_N \\ \phi_E \end{bmatrix} + \begin{bmatrix} n_E \\ n_N \end{bmatrix} \quad (4)$$

where  $n_E, n_N$  are the velocity measurement errors in a GNSS receiver that are assumed to be zero-mean Gaussian white noise.

## III. ALGORITHM

An alternative solution to Kalman filtering and smoothing is a post-mission parametric approach for estimation of orientation and velocity errors (the unknown parameters). The estimated errors are used to correct velocity and orientation that are computed using IMU output. This approach is based on numerical differentiation of the horizontal velocity errors that are computed using the IMU and GNSS measurements and Eq. 4. Once these derivatives are obtained these equations can be used to compute the misalignment (horizontal tilts errors) and velocity offsets.

In our approach the GNSS computed velocity is subtracted from the velocity that is computed based on IMU data (Eq.4). The typical behavior of low-cost MEMS IMU velocity errors is shown in Fig. 1. It can be observed that for short periods of time this plot can be very well approximated by a straight line, therefore, the dominant source of error is horizontal misalignment error, or in other words, the error in pitch and roll angles computed by an IMU. Accelerometer bias can also cause the same linear growing velocity error and it cannot be distinguished from the misalignment error. However, it is usually quite small and it does not affect the velocity error estimation accuracy.

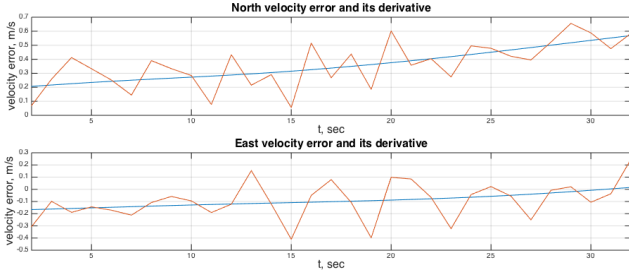


Figure 1: Typical velocity error in low-cost MEMS IMU obtained as a difference between IMU and GNSS computed velocities. The blue line shows the derivative of this error.

In our pedestrian system the body mounted IMU contains three gyros and three accelerometers that measure the projections of absolute angular rate and specific force on their sensitivity axes. The recorded data set is divided into intervals of about 8-15 sec. During this time the misalignment error is almost constant. The length of these intervals depends on the quality of the gyroscopes and can be longer for more accurate instruments. The angular misalignment can be computed as

$$\phi_{E,N} = \delta \hat{V}_{N,E} / \bar{a}_U \quad (5)$$

where  $\delta \hat{V}_{N,E}$  is the estimated rate of velocity error and  $\bar{a}_U$  is the measured vertical acceleration averaged over the time interval.

The computed misalignment and velocity offset can be used to correct the velocity computed based on IMU measured acceleration and orientation according to

$$V_{cor}(t_k) = \hat{V}_0 + \sum_{i=1}^k \Delta D \cdot D_N^S(t_i) \cdot \Delta V(t_i) \quad (6)$$

where  $\Delta V(t_i)$  is the velocity increment computed by the IMU in sensor frame,  $\hat{V}_0$  is the corrected velocity at time  $t_0$ ,  $D_N^S(t_i)$  is the transformation matrix from the sensor to global frame computed by the IMU,  $\Delta D$  is a skew-symmetric matrix for the horizontal misalignment correction that include the previously estimated pitch and roll errors

$$\Delta D = \begin{bmatrix} 1 & 0 & \phi_N \\ 0 & 1 & -\phi_E \\ -\phi_N & \phi_E & 1 \end{bmatrix} \quad (7)$$

#### A. Numerical Differentiation

Accurate numerical differentiation (ND) of a sequence of noisy measurements is an important part of the proposed approach for accurate velocity and orientation computation. We used an ND algorithm that was described in [15]. This algorithm is based on the state space model of the multiply-integrated stationary Wiener process. Derivatives are estimated using fixed-lag Rauch-Tung-Striebel smoothing implemented with numerically stable square-root formulas. The algorithm can treat independent simultaneous measurements and non-equally-spaced abscissas, and supports evaluation at abscissas other than data points (“dense output”). A maximum likelihood estimate of

all the state space models’ parameters, namely the initial state, driving noise intensity, and measurement noise variance, is computed using an extension of the expectation-maximization algorithm for state space model identification

## IV. FIELD TESTS

A measurement setup consist of a Raspberry Pi-2B board running Linux OS, Xsens MTi IMU, u-blox NEO-7P L1 GNSS receiver with SBAS capability and a power bank (Fig.2). The GNSS receiver is connected to the board through UART serial connection and Xsens IMU is connected through USB visible as a serial port on Linux. The GNSS antenna is embedded into receiver module and located in a close proximity to the IMU making the lever arm error negligible. The data from GNSS and IMU are synchronized with the timing error not exceeding 5 ms and stored on a memory card and in optional cloud storage (e.g. Google Drive). This is a small, completely autonomous and self-contained data logger that can be used in different field tests including walking and running tests.

In our outdoor walking tests the unit was attached to a torso and a test route was set in a parking area where the terrain is relatively flat. The test was preceded by a one minute alignment procedure during which the pitch and roll angles were computed based on the gravity vector and heading was estimated using magnetometer data. The algorithm for data fusion from IMU and GNSS is implemented in the geographical coordinate frame and then the velocity is computed in anatomical frame in which the x-axis is pointing in the anterior direction (direction of progression), the y-axis is vertical (parallel to the field of gravity) and points upwards and the z-axis is perpendicular to the x- and y-axes and pointing to the right direction.

The results of velocity computation in walking tests are presented in Figures 3-5. The blue curve in Fig. 3 and 4 shows the East and North velocity components computed by the IMU/GNSS fusion algorithm, the amber curve depicts the velocity computed by the IMU only and the asterisks show the velocity measured by the GNSS receiver. The disadvantage of GNSS receiver for velocity computation is quite clear: the measurements are sparse and noisy. The accuracy is not



Figure 2: The data logger comprising a Raspberry Pi-2B board, u-blox GNSS receiver and an Xsens IMU.

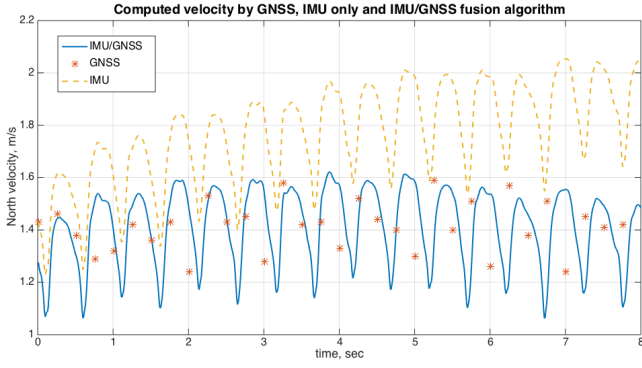


Figure 3: North velocity component computed by the GNSS receiver (asterisks), the IMU only and using the IMU/GNSS fusion algorithm

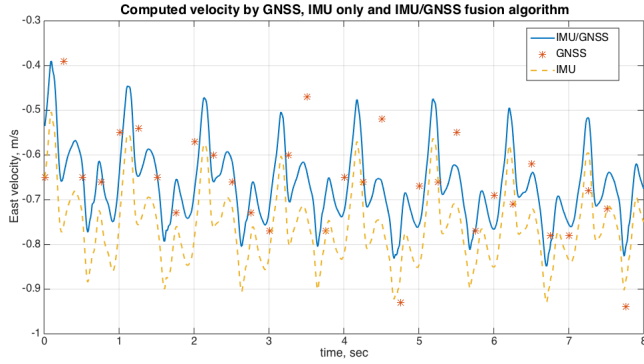


Figure 4: East velocity component computed by the GNSS receiver (asterisks), the IMU only and using the IMU/GNSS fusion algorithm

sufficient for gait analysis. In the case when velocity is computed only by IMU the velocity error can easily exceed 1 m/s.

According to our assessment of horizontal velocity and angles accuracy of IMU/GNSS fusion algorithms, the velocity error does not exceed 0.05 m/s and pitch and roll accuracy is about 0.03-0.05 deg. The value of horizontal tilt angles can be estimated, for example, for a stationary IMU by measuring the propagation of the position error over some period of time after the velocity measurements from GNSS stopped being provided to the fusion algorithm

$$\phi_E = \frac{2\delta N}{gT^2} \quad (8)$$

$$\phi_N = \frac{2\delta E}{gT^2}$$

where  $T$  is a time period over which the position errors grow by  $\delta N, \delta E$ . In this case all navigation parameters are computed using only IMU measurements.

To confirm these results we are planning to carry out new experiments and use a dual frequency differential GNSS receiver with RTK, a tactical grade IMU and Novatel Inertial Explorer software for IMU/DGNSS integration. This surveying grade equipment can provide very velocity and orientation reference and validate the accuracy of our approach.

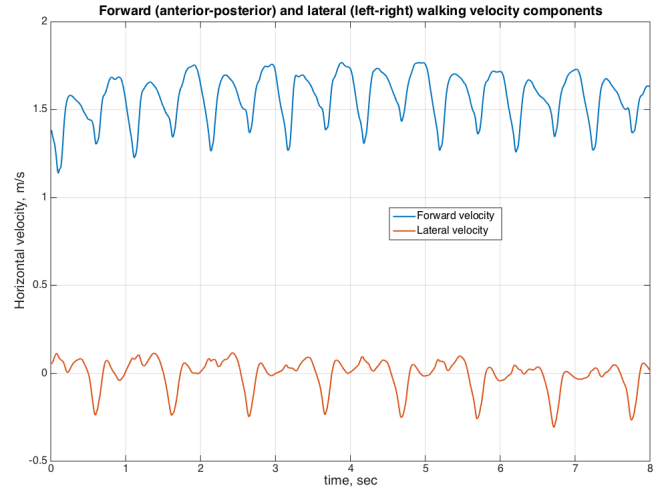


Figure 5: Horizontal velocity in anatomical frame (forward and lateral velocity components) estimated by the fusion algorithm during the walking test.

For kinesiology research and gait analysis it is more convenient to present the velocity measurements in anatomical frame with x-axis in forward direction, y-axis in lateral direction and z-axis in vertical direction. The velocity measurements in anatomical frame during the walking test are shown in Fig. 5. It can be seen that this measurements provide very accurate information about gait characteristics.

## V. CONCLUSION

This paper has demonstrated the performance of a post-mission approach for accurate velocity and orientation computation. The approach is based on fixed-lag Rauch-Tung-Striebel smoothing algorithm. Accuracy of velocity and orientation that are computed based on IMU and GNSS data can be improved in post-mission or near real-time processing using the data collected over some period of time, e.g. 10-30 sec. The potential velocity accuracy is about 0.02 m/s and pitch/roll accuracy is about 0.02 deg. This performance is better than for state-of-the-art MEMS INS/GNSS navigation systems available on the market today. This approach can be applied in application where real-time computations are not required and some delay of 15-30 sec in computed velocity and orientation is acceptable. In the future we are planning to compare the performance of our algorithm with state-of-the-art commercial INS/GNSS integration Kalman filter implementations.

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