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Design of Two-Channel IIR Filterbanks with Rational Sampling Factors

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***Abstract*—This paper concentrates on designing two-channel filterbanks (FBs) with rational sampling factors based on using infinite-impulse response (IIR) filters. In order to generate FBs having a low implementation complexity, a special structure for the IIR filters is utilized, which enables one to evaluate the recursive part of the IIR filters at the subband sampling rate, that is, at the lowest sampling rate in the FB. A method for designing such FBs is proposed and the implementation efficiency of these FBs is demonstrated by means of examples.**

I. Introduction

In a multirate filterbank (FB), an input signal is divided into two or more subband signals with each of these subband signals containing only a part of the frequency-domain information involved in the input signal. Processing these subband signals gives in most applications a better overall performance compared with that of directly processing the input signal. Moreover, the subband signals are usually decimated in the FB which results in a smaller amount of data to be processed [1].

For dividing the signal into subbands, in most cases, a uniform FB is used. In these FBs, each subband is down-sampled with the same decimation factor, and consequently, all subbands have equal bandwidths. However, it has turned out that in many applications such a division does not result in the best achievable performance. For example, in speech processing [2], a non-uniform band division is preferable over the uniform one. The simplest way to design a nonuniform FB is to use two-channel FBs in a tree structure. If two-channel FBs with rational sampling factors (rational FBs) are used, then any desired band division can be achieved either exactly or at least approximated well enough. In a rational FB, the decimation factors are represented as q/p, where q and p are positive integers. Here, the decimation factor denoted by q/p means that the signal is, first, up-sampled by a factor of p and then down-sampled by a factor of q [3], [4].

There exist two alternative approaches for designing two-channel rational FBs. In the first approach, introduced in [5]–[9], two sets of filters are used. One set separates the signal into the lowpass and highpass part, whereas the second set removes aliasing (imaging) errors due to the sampling rate alteration. In the second approach, described in [10]–[12], only one set of filters is used for both purposes. The second approach is used in this paper and is described in more detail in Section II.

In the past, when building rational FBs, finite-impulse response (FIR) filters have been used in most cases. This is due to the facts that FIR filters are easier to design than their infinite-impulse response (IIR) counterparts and there exist efficient implementations for rational sampling rate converters based on FIR filters [13], [14]. This paper shows how rational FBs can be designed by utilizing IIR filters. In order to generate a FB having an efficient implementation, in addition to the constraints related to the FB, constraints on the filter structure are imposed. The goal is to use IIR filters for which the recursive part can be implemented at the subband sampling rate, thereby reducing the amount of calculations in the FB. For designing such FBs, an optimization method is suggested and the usefulness of the proposed FBs is demonstrated by means of examples.

The outline of this paper is as follows. Section II describes the basic properties of two-channel rational FBs considered in this paper with the main emphasis laid on the IIR based FBs. A design method for IIR FBs is proposed in Section III, whereas Section IV discusses the design and implementation complexity of these FBs. Finally, in Section V, the benefits of the proposed FBs are illustrated by means of examples.

II. Two-Channel IIR Filterbanks with Rational Sampling Factors

This section considers the most important properties of the two-channel rational IIR FBs under considerations. More detail can be found, for example, in [12].

A. Filterbank Structure and Relations

The two-channel rational FB considered in this paper is shown in Figure 1. This FB consists of an analysis FB and a synthesis FB separated by a processing unit. It is assumed that the processing unit does not alter the signals. The channel with signals $x_0[n]$ and $y_0[n]$ $\{x_1[n]$ and $y_1[n]\}$ is referred to as the lowpass {highpass} channel. The sampling rate conversion

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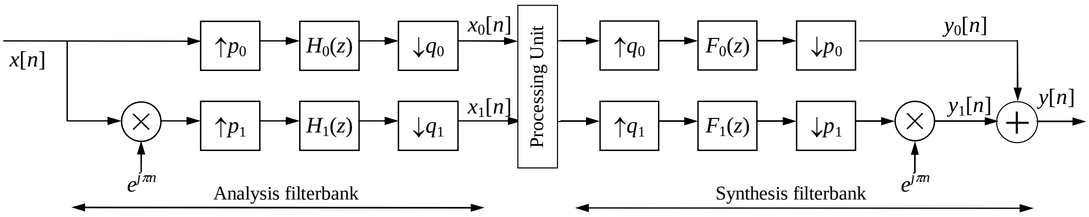

Figure 1. Two-channel FB with rational sampling factors.

factors q_k and p_k for $k=0,1$, can be any positive integers as long as $p_k<q_k$. Moreover, for a critically sampled FB, following relation has to be satisfied [12]:

$$\frac{p_0}{q_0}+\frac{p_1}{q_1}=1, \tag{1}$$

which implies that $q_1=q_0$ and $p_1=q_0-p_0$. Such a FB separates the input signal into two parts, as illustrated in Figure 2. Furthermore, the highpass channel is modulated by $e^{j\pi}$ in the analysis and synthesis FB in order to enable the implementation of FBs for all combinations of sampling rate factors p_k and q_k [3], [5], [12].

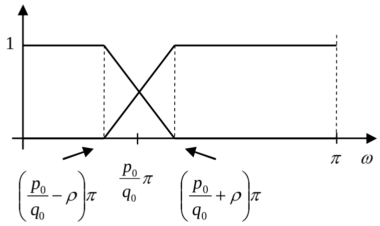

Figure 2. The band division of the analysis FB in Figure 1

The relation between the output and the input signal for the system shown in Figure 1 can be expressed in the *z* domain as [12]

$$Y(z)=T_0(z)X(z)+\sum_{l=1}^{q_0-1}T_l(z)X(zW_{q_0}^{lp_0})=$$
$$=\sum_{k=0}^{1}\left\{\frac{1}{p_kq_k}\sum_{q=0}^{q_k-1}\sum_{p=0}^{p_k-1}\left\{F_k(z^{1/p_k}W_{p_k}^{p})\right.\right. \tag{2a}$$
$$\left.\left.H_k(z^{1/p_k}W_{p_k}^{p}W_{q_k}^{q})X(zW_{q_k}^{p_kq})\right\}\right\}.$$

Here, $T_0(z)$ is the FB distortion transfer function, $T_l(z)$'s for $l=1,2,\ldots,q_0-1$ are the alias transfer functions, and

$$W_p^q=e^{-2\pi jq/p}. \tag{2b}$$

B. *Filterbanks Utilizing IIR Filters*

In this paper, the transfer functions of IIR filters used for building the FB are of the form

$$H_k(z)=\frac{B_k^{(h)}(z)}{A_k^{(h)}(z^{q_k})}=\frac{\sum_{n=0}^{N_k^{(h)}}b_k^{(h)}[n]z^{-n}}{1+\sum_{m=1}^{M_k^{(h)}}a_k^{(h)}[m]z^{-mq_k}}, \tag{3}$$

for $k=0,1$. The filters in the synthesis bank have the same form as the filters in the analysis bank.

There are two reasons for using IIR filters with such transfer functions. First, these filters can be efficiently implemented in a rational FB as illustrated in Figure 3. In this structure, the recursive part of the IIR filter has been moved to the subband sampling rate, that is, the lowest sampling rate in the system, whereas the numerator of the IIR filter can be efficiently implemented as discussed in [13]. Second, by comparing various IIR FBs, it has turned out that FBs with these transfer functions provide systems with the best ratio between the number of required multiplications per input sample and the FB properties in comparison with other IIR based FBs.

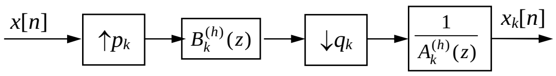

Figure 3. Implementation of the transfer function $H_k(z)$ in a rational FB.

III. FILTERBANK DESIGN

In order to design filters appropriate for the FBs considered in this paper, the following optimization problem is considered: Given the decimation factors p_0 and q_0, the desired FB delay D, the numerator and denominator filter orders $N_k^{(h)}, M_k^{(h)}, N_k^{(f)}$, and $M_k^{(f)}$ for $k=0,1$, the stopband and passband edge parameter ρ (see Figure 2), the desired passband ripple δ_p, and the alias and amplitude distortions, denoted by δ_d and δ_a, minimize

$$\delta=\max\left\{\delta_0^{(h)},\delta_1^{(h)},\delta_0^{(f)},\delta_1^{(f)}\right\} \tag{4a}$$

subject to

$$\left|T_0(e^{j\omega})-e^{-j\omega D}\right|\le\delta_d \quad \text{for} \quad \omega\in[0,\pi] \tag{4b}$$

$$\left|T_l(e^{j\omega})\right|\le\delta_a \quad \text{for} \quad \omega\in\left[0,\omega_s^{(k)}\right], l=1,2,\ldots q_0-1 \tag{4c}$$

$$\text{roots}\left\{A_k^{(h)}(z^{q_k})\right\}<1 \text{ for } k=0,1 \tag{4d}$$

$$\text{roots}\left\{A_k^{(f)}(z^{q_k})\right\}<1 \text{ for } k=0,1. \tag{4e}$$

Here,

$$\delta_0^{(h)}=\begin{cases} w_{s,0}^{(h)}\left|H_0(e^{j\omega})\right| & \text{for } \omega\in\left[\omega_s^{(0)},\pi\right] \\ w_{p,0}^{(h)}\left|\left|H_0(e^{j\omega})\right|-1\right| & \text{for } \omega\in\left[0,\omega_p^{(0)}\right] \\ w_{t,0}^{(h)}\left|H_0(e^{j\omega})\right|-1 & \text{for } \omega\in\left[\omega_p^{(0)},\omega_s^{(0)}\right]. \end{cases} \tag{4f}$$

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The remaining $\delta_1^{(h)}$, $\delta_0^{(f)}$, and $\delta_1^{(f)}$ are evaluated in the same way as $\delta_0^{(h)}$ by replacing $H_0(e^{j\omega})$ by $H_1(e^{j\omega})$, $F_0(e^{j\omega})$, and $F_1(e^{j\omega})$, respectively. The weighting parameters w can be used to appropriately control the behavior of the filter magnitude response in the passband, transition band, and stopband regions. Furthermore, $T_0(z)$ and the $T_l(z)$'s are defined by (2) whereas the passband and stopband edge frequencies are given by

$$\omega_p^{(k)} = \left(\frac{1}{q_k} - \frac{1}{p_k}\rho\right)\pi \tag{4g}$$

$$\omega_s^{(k)} = \left(\frac{1}{q_k} + \frac{1}{p_k}\rho\right)\pi \tag{4h}$$

for $k = 0, 1$, respectively [12].

The above design problem has been formed in such a way that weighted error functions for the magnitude responses of all the filters are minimized simultaneously. The weighting factors enable one to control the magnitude response of each filter in various regions as well as to emphasize the importance of the four filters involved in the optimization. The reconstruction property of the FB is controlled by the parameters δ_a and δ_d. Finally, it is ensured that the resulting filters are stable by constraining their poles to lie inside of the unit circle.

The above problem can be solved by using a standard non-linear optimization routine, for example, the function `fminimax` included in the Optimization Toolbox [15] provided by MathWorks, Inc. In order to use a numerical optimization routine, the interval $[0, \pi]$ has to be discretized into points $\omega_r \in [0, \pi]$ for $r = 0, 1, \ldots, R-1$. All the functions included in (4a)–(4f) are then evaluated on the resulting grid of frequencies. If after solving the problem the stopband attenuations (or passband ripples) of the resulting filters do not satisfy the design requirements, then the filter orders have to be increased, and the design procedure has to be repeated.

IV. Filterbank Design and Implementation Complexity

In a nonlinear optimization problem, the design complexity Y is proportional to the number of design parameters, that is, the number of unknowns. The method proposed in this paper has the following number of unknowns:

$$Y = Y_0 + Y_1, \tag{5a}$$

where

$$Y_k = M_k^{(h)} + N_k^{(h)} + M_k^{(f)} + N_k^{(f)} + 2 \tag{5b}$$

for $k = 0, 1$.

The implementation complexity C, defined as the number of multiplications per input sample, is given by

$$C = C_0 + C_1, \tag{6a}$$

where C_0 and C_1 are the implementation complexities for the lowpass and the highpass branch, respectively. They can be evaluated as

$$C_k = \frac{N_k^{(h)} + 1}{q_k} + M_k^{(h)}\frac{p_k}{q_k} + \frac{N_k^{(f)} + 1}{q_k} + M_k^{(f)}\frac{p_k}{q_k} \tag{6b}$$

for $k = 0, 1$.

V. Examples

This section shows the efficiency of the proposed IIR FBs by comparing them with some other FIR and IIR based rational FBs.

Example 1: It is desired to synthesize a two-channel rational FB with the following parameters: $p_0 = 2$, $q_0 = 3$, $\rho = 0.1$, $D = 15$, and $\delta_d = \delta_a = 0.001$. By selecting the filter orders as $N_0^{(h)} = N_0^{(f)} = 38$, $M_0^{(h)} = M_0^{(f)} = 1$, $N_1^{(h)} = N_1^{(f)} = 15$, and $M_1^{(h)} = M_1^{(f)} = 1$, a FB with some responses shown in Figure 4 is generated. In this figure, the responses of the filter transfer functions $H_1(-z)$ and $F_1(-z)$ are shown, instead of $H_1(z)$ and $F_1(z)$, in order to make them more distinguishable from those of the filters in the lowpass channel.

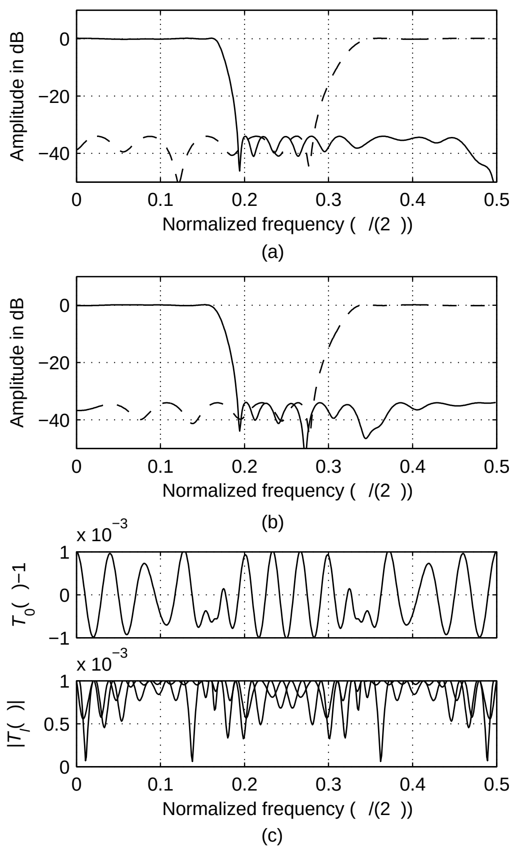

Figure 4. Various responses for the FB in Example 1. (a) Analysis filters. (b) Synthesis filters. (c) Amplitude and alias distortions.

In order to show the benefits provided by the IIR FBs over their FIR counterparts, various FBs have been generated such that the number of poles in these FBs vary, whereas all other FB design parameters, including the numerator orders, are identical. Figure 5 shows the relation between the filter stopband attenuations and the number of poles. The corresponding numerical data is provided in Table I. As seen from the figure and table, a few poles considerably improve the performance, that is, better FB selectivity is achieved by slightly increasing the overall implementation complexity. Moreover, it is also observed there that after a certain number of poles, it is not worth adding more poles because this results only in a very marginal improvement in the attenuation. It is worth emphasizing that due to the form of the transfer function, as given by (3), the number of poles in the transfer function can be changed only by integer increments of q_0. In order to generate FIR FBs having similar properties as their IIR equivalents, FIR filters of higher order have to be used, which consequently increases the design and implementation complexity. As shown in Table I, by using lower order numerators and adding 3 (6) poles, the implementation complexity reduces from 51.3 (59.3) to 41.3 (43.3) multiplications per input sample compared with the corresponding FIR FBs. This corresponds to a 20% (25%) savings.

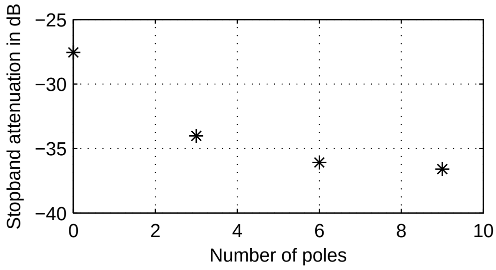

Figure 5. Filter stopband attenuation for various numbers of poles.

TABLE I FB DESIGNS IN EXAMPLE 1

$N_0^{(h)}$	$N_1^{(h)}$	#poles	SB_{Att}	Y	C
38	15	0	27.5	118	39.3
		3	34.0	122	41.3
		6	36.1	124	43.3
		9	36.6	128	45.3
50	25	0	34.0	154	51.3
58	29	0	36.2	178	59.3

TABLE II FB DESIGNS IN EXAMPLE 2

	Proposed	[8]
Filter orders	38/5, 57/5	10/10, 11/11
D	19	43
Attenuation [dB]	40	32.2
δ_d	0.01	0.015
Y	198	44
C	40.8	88

Example 2: In this example, it is desired to design a FB with the following parameters: $p_0=2$, $q_0=5$, $\rho=0.1$, and $\delta_d=\delta_a=10^{-2}$. FBs generated by using these parameters can be easily compared with FBs designed in Example 1 in [8]. The properties of both FBs are summarized in Table II. As seen from this table, the proposed FB has a higher number of design unknowns, but it generates a FB that has a lower FB delay (19 vs. 43), a lower FB distortion (0.01 vs. 0.015), a better channel selectivity (40 dB vs. 32.2 dB) and a more efficient implementation (40.8 vs. 88). It should be pointed out that the FB delay and implementation complexity of the design in [8] is actually larger than shown Table II because of the existence of additional filters between the up-sampling and down-sampling stage.

VI. CONCLUSION

In this paper, a method has been proposed for designing IIR filters used in two-channel FBs with rational sampling factors. In order to design FBs with good overall properties as well as to ensure that their implementation requires a small number of multiplications per input sample, a special structure for the IIR filters has been utilized that enables one to implement the recursive part at the subband sampling rate. The efficiency of the proposed design technique has been illustrated by means of examples.

REFERENCES

[1] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[2] E. Zwicker and H. Fastl, *Psychoacoustics*. New York: Springer, 1990.

[3] J. Kovačević and M. Vettereli, “Perfect reconstruction filer banks with rational sampling factors,” *IEEE Trans. Signal Process.*, vol. 41, pp. 2047–2066, June 1993.

[4] K. Nayebi, T. P. Barnwell III, and M. J. T. Smith, “Non-uniform filter banks: Reconstruction and design theory,” *IEEE Trans. Signal Process.*, vol. 41, pp. 1114–1127, March 1993.

[5] B. Liu and L. T. Bruton, “The design of non-uniform-band maximally decimated filter banks,” in *Proc. IEEE Int. Symp. Circuits Syst.*, 1993, pp. 375–378.

[6] S. Wada, “Design of non-uniform division multirate FIR filter banks,” *IEEE Trans. Circ. Syst. II*, vol. 42, Feb. 1995, pp. 115–121.

[7] J.-H. Lee and D.-C. Tang, “Minimax design of two-channel nonuniform-division FIR filter banks,” *IEE Proc.- Vis. Image Signal Process.*, vol. 145, pp. 88–96, April 1998.

[8] J.-H. Lee and I.-C. Niu, “Design of two-channel IIR filter banks with arbitrary group delay,” *IEE Proc.- Vis. Image Signal Process.*, vol. 147, pp. 534–542, Dec. 2000.

[9] J.-H. Lee and Y.-H. Yang, “Minimax design of two-channel nonuniform-division filterbanks using IIR allpass filters,” *IEEE Trans. Signal Process.*, vol. 42, pp. 3227–3240, Nov. 2004.

[10] T. Watanabe, Y. Shibahara, T. Kida, and N. Sugino, “Design of non-uniform FIR filter banks with rational sampling factors,” in *Proc. Asia-Pacific Conf. Circ. Syst.*, 1998, pp. 69–72.

[11] T. Blu, “A new design algorithm for two-band orthonormal rational filter banks and orthonormal rational wavelets,” *IEEE Trans. Signal Process.*, vol. 46, pp. 1494–1504, June 1998.

[12] R. Bregović, Y. C. Lim, and T. Saramäki, “Frequency-response masking based design of nearly perfect-reconstruction two-channel FIR filterbanks with rational sampling factors,” to appear in *IEEE Trans. Circ. Syst. II*.

[13] R. E. Crochiere and L. R. Rabiner, *Multirate Digital Signal Processing*, Englewood Cliffs, NJ: Prentice-Hall, 1983.

[14] R. Bregović, T. Saramäki, Y. J. Yu, and Y. C. Lim, “An Efficient Implementation of Linear-Phase FIR Filters for a Rational Sampling Rate Conversion,” in *Proc. IEEE Int. Symp. Circuits Syst.*, Island of Kos, Greece, May 2006, pp. 5395–5398.

[15] T. Coleman, M. A. Branch, and A. Grace, *Optimization Toolbox User’s Guide*, Version 2, MathWorks, Inc., Natick, MA, 1999.