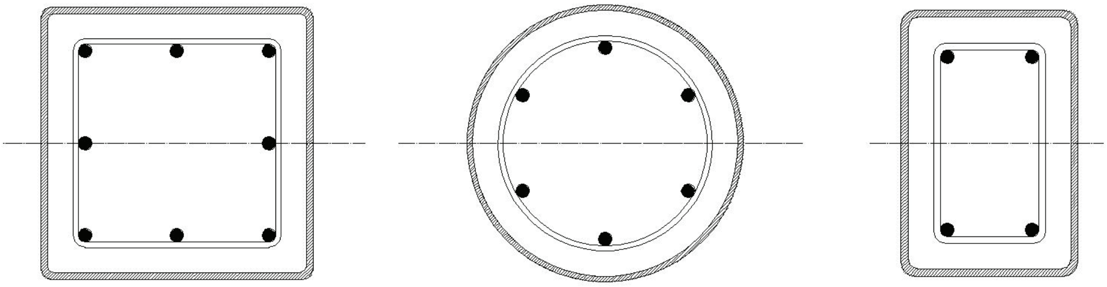
Tampereen teknillinen yliopisto. Rakennustekniikan laitos. Rakennetekniikka.
Tutkimusraportti 147
Tampere University of Technology. Department of Civil Engineering. Structural Engineering.
Research Report 147

Karol Bzdawka

Composite Column - Calculation Examples



TAMPEREEN TEKNILLINEN YLIOPISTO
TAMPERE UNIVERSITY OF TECHNOLOGY

Tampereen teknillinen yliopisto. Rakennustekniikan laitos. Rakennetekniikka. Tutkimusraportti 147
Tampere University of Technology. Department of Civil Engineering. Structural Engineering. Research Report 147

Karol Bzdawka

Composite Column – Calculation Examples

Tampereen teknillinen yliopisto. Rakennustekniikan laitos
Tampere 2010

ISBN 978-952-15-2321-2 (printed)
ISBN 978-952-15-2803-3 (PDF)
ISSN 1797-9161

PREFACE

This study was done in Research Centre of Metal Structures in Hämeenlinna unit. The financial support of Hämeenlinnan kaupunki, Hämeen Ammattikorkeakoulu and Rautaruukki Oyj is gratefully acknowledged. The discussions with Mr. Arto Sivill, Finnmap Consulting Oy, and Mr. Aarne Seppänen, Rautaruukki Oyj, were very helpful when completing the study.

This study is part of on-going research dealing with optimization of load bearing structures of metal based buildings. The previous report of Karol Bzdawka dealt with design of WQ-beams and cost estimations of different framing systems for office buildings. In this study the design of composite columns is presented based on EN standards. The design methods are implemented into MATLAB enabling in the future the applications of mathematical optimizations tools to be used to find better solutions for load bearing structures than now are available using “manual” optimization.

Hämeenlinna 10.1.2010

Markku Heinisuo

Professor of Metal Structures

COMPOSITE COLUMN – CALCULATION EXAMPLES

TABLE OF CONTENTS

1 ABSTRACT

Composite columns are often used in structures due to the ease and speed of erection, and high performance in fire situation. Concrete filled tubes are steel tubes that are in site filled with reinforced concrete. In normal situation column works as composite while in fire situation majority of the load is carried by the reinforced concrete core. There are numerous publications about this type of columns but all of them use very simplified methods. Especially regarding the calculation of the neutral axis of circular columns. Also the shear resistance of the column is often omitted.

The reason for this document is to present the calculation of concrete-filled tubes in details. Also to investigate the influence of the bars arrangement in circular columns on their bending resistance. Calculations have been carried for ambient and fire temperatures.

This report includes calculation examples and results in different design situations, for a number of various column cross-sections.

2 SQUARE COLUMN

2.1 Ambient design

Data for the calculation:

$col := 250mm$	width and height of the steel tube
$t := 6mm$	wall thickness of the steel tube
$n := 4$	number of reinforcing bars
$fi := 20mm$	diameter of reinforcing bars
$fi_s := 6mm$	diameter of stirrups
$s := 300mm$	stirrups spacing
$u_s := 35mm$	distance from the inner surface of the tube to the surface of the bar
$L_{eff} := 3600mm$	effective length of the column

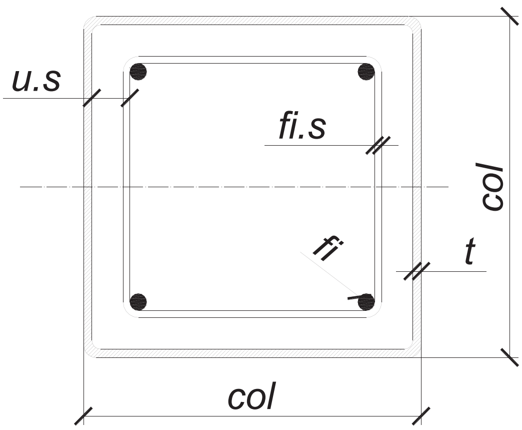


Figure 1. Symbols used for the basic design variables

Material properties

Steel	Concrete	Reinforcement	
$f_y := 355MPa$	$f_{ck} := 40MPa$	$f_{sk} := 500MPa$	characteristic strength
$gamma_a := 1.1$	$gamma_c := 1.35$	$gamma_s := 1.15$	material safety factor
$f_{ad} := \frac{f_y}{gamma_a}$	$f_{cd} := \frac{f_{ck}}{gamma_c}$	$f_{sd} := \frac{f_{sk}}{gamma_s}$	design strengths
$f_{ad} = 322.7MPa$	$f_{cd} = 29.6MPa$	$f_{sd} = 434.8MPa$	
$E_a := 210000MPa$	$E_{cm} := 35000MPa$	$E_s := 210000MPa$	Young modulus

Forces in the column in normal situation:

$N_{Sd} := 1842.0 kN$

$M_{Sd1} := 131.5 kN \cdot m$ bigger bending moment (upper or lower end)

$M_{Sd2} := -113.0 kN \cdot m$ smaller bending moment (upper or lower end)

(bending moments on the same side of the column have the same signs)

$V_{Sd} := \dfrac{M_{Sd1} - M_{Sd2}}{L_{eff}}$ $V_{Sd} = 67.9 kN$ constant shear force

Characteristics of the cross section

Steel tube

$A_a := 4 \cdot (col - t) \cdot t$ $A_a = 5856 mm^2$ tube cross-section area

$I_a := \dfrac{col^4}{12} - \dfrac{(col - 2 \cdot t)^4}{12}$ $I_a = 5.814 \times 10^7 mm^4$ tube second moment of area

$W_{pa} := \dfrac{col^3}{4} - \dfrac{(col - 2 \cdot t)^3}{4}$ $W_{pa} = 5.359 \times 10^5 mm^3$ plastic modulus

Reinforcement

$A_s := \dfrac{n \cdot fi^2 \cdot \pi}{4}$ $A_s = 1257 mm^2$ cross-section area of the reinforcement

$I_s := \left(\dfrac{col}{2} - t - u_s - \dfrac{fi}{2}\right)^2 \cdot A_s$ $I_s = 6.881 \times 10^6 mm^4$ second moment of area of the reinforcement

Concrete core

$A_{cgross} := (col - 2 \cdot t)^2$ $A_{cgross} = 56644 mm^2$ gross cross-section area of the concrete

$A_c := A_{cgross} - A_s$ $A_c = 55387 mm^2$ nett cross-section area of the concrete

$I_{cgross} := \dfrac{(col - 2 \cdot t)^4}{12}$ $I_{cgross} = 2.674 \times 10^8 mm^4$ gross second moment of area of the concrete

$I_c := I_{cgross} - I_s$ $I_c = 2.605 \times 10^8 mm^4$ nett second moment of area of the concrete

Check if the reinforcement meets the requirement of the reinforcement level

$\rho := \dfrac{A_s}{A_{cgross}}$ $\rho_{max} := 6.0 \cdot \%$ > $\rho = 2.2\%$ > $\rho_{min} := 1.5 \cdot \%$ Requirement met

Maximum design bending resistance for the cross-section is given by equation:

$$M_{max.Rd} := W_{pa} \cdot f_{ad} + W_{ps} \cdot f_{sd} + \frac{W_{pc}}{2} \cdot f_{cd}$$

Where:

W_{pa} is the plastic section modulus of the steel tube

W_{ps} is the plastic section modulus of the reinforcing bars

W_{pc} is the plastic section modulus of the conctrete, in above equation only half is taken into account since concrete does not carry tension.

$$W_{pa} := \frac{col^3}{4} - \frac{(col - 2 \cdot t)^3}{4} \qquad W_{pa} = 535.9\,cm^3$$

$$W_{ps} := A_s \cdot \left(\frac{col}{2} - \frac{fi}{2} - u_s - t\right) \qquad W_{ps} = 93.0\,cm^3$$

(reinforcement area times the distance from the midline)

$$W_{pc} := \frac{(col - 2 \cdot t)^3}{4} - W_{ps} \qquad W_{pc} = 3277.3\,cm^3$$

Thus:

$$M_{max.Rd} := W_{pa} \cdot f_{ad} + W_{ps} \cdot f_{sd} + \frac{W_{pc}}{2} \cdot f_{cd} \qquad M_{max.Rd} = 261.9\,kN \cdot m$$

For a column cross-section in pure bending the neutral axis is not the same as the axis of symmetry of the cross-section. New neutral axis has to be calculated from the equations of equilibrium. For details of used symbols see Figure 2.

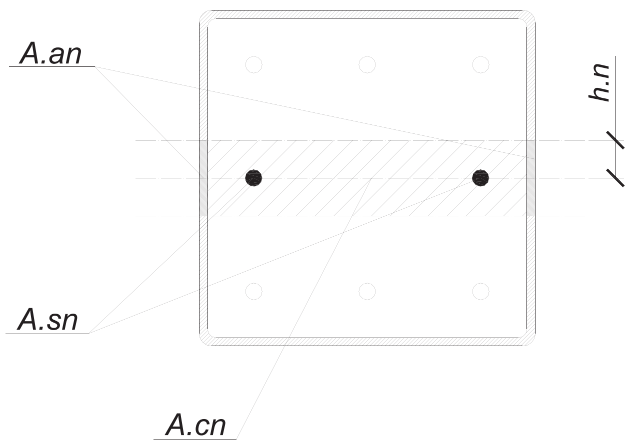


Figure 2. Symbols used in calculation of plastic bending resistance

Design axial resistance of the concrete only is equal to:

$$N_{pmRd} := A_c \cdot f_{cd} \qquad N_{pmRd} = 1641\,kN$$

Equilibrium equation:

$$0.5 \cdot N_{pmRd} - (col - 2 \cdot t) \cdot h_n \cdot f_{cd} = A_{sn} \cdot f_{sd} + 4 \cdot t \cdot h_n \cdot f_{ad}$$

A_{sn} is the cross-section area of the reinforcement in range - h_n to h_n from the centerline of the column cross-section. In this case equal to zero because there are no bars lying on the centerline.

$$A_{sn} := 0mm^2$$

After rearranging the equilibrium equation, distance between the centerline and the neutral axis is given:

$$h_n := \frac{N_{pmRd} - A_{sn} \cdot \left(2 \cdot f_{sd} - f_{cd}\right)}{2 \cdot \left[(col - 2t) \cdot f_{cd} + 4t \cdot f_{ad}\right]} \qquad h_n = 55.453\,mm$$

Difference between maximum plastic bending resistance $M_{max.Rd}$ and plastic bending resistance around the neutral axis $M_{pl.Rd}$ is equal to the bending resistance of the part of the cross section that is in distance h_n from the centerline.

$$M_{pl.Rd} := M_{max.Rd} - W_{pan} \cdot f_{ad} - W_{psn} \cdot f_{sd} - \frac{W_{pcn}}{2} \cdot f_{cd}$$

Where:

$$W_{pan} := 2 \cdot \frac{t \cdot \left(2 \cdot h_n\right)^2}{4} \qquad W_{pan} = 36.9\,cm^3$$

$$W_{psn} := 0 \cdot cm^3$$ there are no bars in considered region

$$W_{pcn} := \frac{(col - 2 \cdot t) \cdot \left(2h_n\right)^2}{4} - W_{psn} \qquad W_{pcn} = 731.9\,cm^3$$

$$M_{pl.Rd} := M_{max.Rd} - \left(W_{pan} \cdot f_{ad}\right) - \left(W_{psn} \cdot f_{sd}\right) - \left(\frac{W_{pcn}}{2} \cdot f_{cd}\right) \qquad M_{pl.Rd} = 239.2\,kN \cdot m$$

Values of the resistance to axial force (EN 1994-1-1 [3], 6.7.3.2)

$$N_{plRd} := A_a \cdot f_{ad} + A_c \cdot f_{cd} + A_s \cdot f_{sd} \qquad N_{plRd} = 4077\,kN$$ design resistance

$$N_{plR} := A_a \cdot f_y + A_c \cdot f_{ck} + A_s \cdot f_{sk} \qquad N_{plR} = 4923\,kN$$ characteristic resistance

Effective flexural and axial stiffnesses (EN 1994-1-1, 6.7.3.3)

$$EI_{eff} := E_a \cdot I_a + 0.6 \cdot E_{cm} \cdot I_c + E_s \cdot I_s \qquad EI_{eff} = 19125.4\,kN \cdot m^2$$

$$N_{cr} := \frac{\pi^2}{L_{eff}^2} \cdot EI_{eff} \qquad N_{cr} = 14564.8\,kN$$

Check if the long term effect need to be taken into account

$$\delta := \frac{A_a \cdot f_{ad}}{N_{plRd}} \qquad \delta = 0.464$$

is the contribution ratio of the steel tube, has to be between 0.2 and 0.9 - only then column works as a composite

$$\lambda := \sqrt{\frac{N_{plR}}{N_{cr}}} \qquad \lambda = 0.581$$

is the relative slenderness of the column, has to be smaller than 2 (EN 1994-1-1 6.7.3.3)

$$\lambda_{vert} := \frac{0.8}{1 - \delta} \qquad \lambda_{vert} = 1.491$$

According to [1], if λ is smaller than λ_{vert}, long term effects do not need to be taken into account.

Calculation of the long term effect

$$E_{c.eff} := E_{cm} \cdot \frac{1}{1 + \left(\frac{N_{G.Sd}}{N_{Sd}}\right) \cdot \phi_t}$$

Where:

$\phi_t := 1.50$ is the creep coeff. acc. to EN 1992-1-1 [4], 3.1.4, calculated for relative humidity 80%, loading time 30 days, concrete C 40/50

$N_{G.Sd}$ is the permanent part of the load, in this case assumed $0.8 \cdot N_{Sd}$

$$E_{c.eff} = 15.9\,GPa$$

If the long term effect has to be taken into account, effective flexural stiffness is equal:

$$EI_{eff} := E_a \cdot I_a + 0.6 \cdot E_{c.eff} \cdot I_c + E_s \cdot I_s \qquad EI_{eff} = 16141.5\,kN \cdot m^2$$

And the elastic critical normal force:

$$N_{cr} := \frac{\pi^2}{L_{eff}^2} \cdot EI_{eff} \qquad N_{cr} = 12292.5\,kN$$

Design value of the effective flexural stiffness, used to determine the sectional forces. (EN 1994-1-1 6.7.3.4)

$$EI_{effII} := 0.9\cdot\left(E_a\cdot I_a + 0.5\cdot E_{cm}\cdot I_c + E_s\cdot I_s\right) \qquad EI_{effII} = 16392.3kN\cdot m^2$$

$$N_{cr.eff} := \frac{\pi^2}{L_{eff}^2}\cdot EI_{effII} \qquad N_{cr.eff} = 12483.4kN$$

Including the long term effects:

$$EI_{effII.long} := 0.9\cdot\left(E_a\cdot I_a + 0.5\cdot E_{c.eff}\cdot I_c + E_s\cdot I_s\right) \qquad EI_{effII.long} = 14154.4kN\cdot m^2$$

$$N_{cr.eff.long} := \frac{\pi^2}{L_{eff}^2}\cdot EI_{effII} \qquad N_{cr.eff.long} = 12483.4kN$$

Stiffness to axial compression, used to determine the sectional forces.:

$$EA := E_a\cdot A_a + E_s\cdot A_s + E_{cm}\cdot A_c \qquad EA = 3432.2MN$$

Axial resistance (EN 1994-1-1, 6.7.3.5)

$$N_{Rd} := \chi\cdot N_{plRd}$$

Where:

χ is the reduction factor for the relevant buckling mode, in terms of relative slenderness λ. (EN 1993-1-1 [5], 6.3.1.2)
if λ =< 0.2 or N.Sd/N.plRd =< 0.1 than χ = 1.

$$\chi := \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}} \qquad \text{but =< 1}$$

$$\lambda = 0.581$$

$$\Phi := 0.5\cdot\left[1 + \alpha\cdot\left(\lambda - 0.2\right) + \lambda^2\right] \qquad \Phi = 0.762$$

Where:

α is an imperfection factor, equal to 0.49 (buckling curve C)

$$\chi := \min\left(\frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}}, 1\right) \qquad \chi = 0.796$$

$$N_{Rd} := \chi\cdot N_{plRd} \qquad N_{Rd} = 3247.1kN$$

Utilization check:

$$\frac{N_{Sd}}{N_{Rd}} = 0.567 \qquad < \qquad 1.0$$

Increse of the bending moment due to second order effects (EN 1994-1-1, 6.7.3.4)

Second order effects may be allowed for by multiplying the bigger of the M_{Sd} by a factor k, where k is geater or equal to 1.0.

$$k := \frac{\beta}{1 - \frac{N_{Sd}}{N_{cr.eff}}}$$

Where:

β is an equivalent moment factor (EN 1994-1-1, Table 6.4)

$$\beta := \max\left[\left[0.66 + 0.44\cdot\left(\frac{M_{Sd2}}{M_{Sd1}}\right)\right], 0.44\right] \qquad \beta = 0.440$$

$$k := \frac{\beta}{1 - \frac{N_{Sd}}{N_{cr.eff}}} \qquad k = 0.516$$

since k is smaller than 1.0 => $k := \max(k, 1)$ $k = 1.0$

Thus:

$$M_{Sd} := k\cdot M_{Sd1} \qquad M_{Sd} = 131.5\,\mathrm{kN\cdot m}$$

Transverse shear

Resistance of the steel tube (EN 1993-1-1, 6.2.6)

$$A_v := A_s\cdot\frac{col}{(2\cdot col)} \qquad A_v = 628.3\,\mathrm{mm}^2$$ area in shear for RHS

$$V_{plaRds} := \left(\frac{A_v\cdot f_y\cdot\sqrt{3}}{gamma_a}\right) \qquad V_{plaRds} = 351.2\,\mathrm{kN}$$ plastic shear resistance of the tube.

Resistance of the concrete (EN 1992-1-1, 6.2)

$$d := col - 2\cdot t - u_s - \frac{fi}{2} \qquad d = 193.0\,\mathrm{mm}$$ effective depth of the cross-section

$$b_w := col - 2\cdot t \qquad b_w = 238.0\,\mathrm{mm}$$ smallest width of the cross section in the tensile area

$$A_{sl} := \frac{A_s + A_{sn}}{2} \qquad A_{sl} = 628.3\,mm^2$$

area of the tensile reinforcement, properly anchored

$$\rho_1 := \frac{A_{sl}}{b_w \cdot d} \qquad \rho_1 = 0.014$$

ratio of the longitudinal reinforcement

Resistance of the concrete without shear reinforcement. (6.2.2)

$$\sigma_{cp} := \min\left(\frac{N_{pmRd}}{N_{plRd}} \cdot \frac{N_{Sd}}{A_c}, 0.2 \cdot f_{cd}\right) \qquad \sigma_{cp} = 5.9\,MPa$$

σ_{cp} is the compressive stress in concrete due to axial loading. Part of the axial loading that is coming into concrete is taken as the proportion of the concrete plastic resistance (N_{plRd}) and cross-section plastic resistance (N_{plRd}) times axial load.

Design value of the shear resistance is given by:

$$V_{Rdc} := \left[C_{Rdc} \cdot \left[\left(100 \cdot \min\left(\rho_1, 0.02\right) \cdot f_{ck}\right)^{\frac{1}{3}}\right] + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

With the minimum value of:

$$V_{Rd.c.min} := \left[\left(v_{min} + k_1 \cdot \sigma_{cp}\right) \cdot b_w \cdot d\right]$$

Calculation of design shear resistance:

$$C_{Rdc} := \frac{0.18}{gamma_c} \qquad C_{Rdc} = 0.133$$

recommended value

$$k_1 := 0.15$$

recommended value

$$V_{Rdc} := \left[C_{Rdc} \cdot \left[\left(100 \cdot \min\left(\rho_1, 0.02\right) \cdot \frac{f_{ck}}{MPa}\right)^{\frac{1}{3}}\right] + k_1 \cdot \frac{\sigma_{cp}}{MPa}\right] \cdot b_w \cdot d \cdot MPa \qquad V_{Rdc} = 64.1\,kN$$

Minimum value of the shear resistance:

$$k := \min\left[1 + \left(\frac{200mm}{d}\right)^{0.5}, 2\right] \qquad k = 2.0$$

$$v_{min} := 0.035 \cdot k^{\frac{3}{2}} \cdot \left(\frac{f_{ck}}{MPa}\right)^{0.5} \qquad v_{min} = 0.626$$

recommended value

$$V_{Rd.c.min} := \left[\left(v_{min} + k_1 \cdot \frac{\sigma_{cp}}{MPa}\right) \cdot b_w \cdot d\right] \cdot MPa \qquad V_{Rd.c.min} = 69.6kN$$

In the end design shear resistance is the bigger of the velues V_{Rdc} and $V_{Rd.c.min}$.

$$V_{Rd.c} := \max\left(V_{Rdc}, V_{Rd.c.min}\right) \qquad V_{Rd.c} = 69.6kN$$

Resistance of concrete cross-section with shear reinforcement (6.2.3)

The shear resistance of a member with perpendicular shear reinforcement is equal to:

$$V_{Rds} := \frac{A_{sw}}{s} \cdot z \cdot \frac{f_{sd}}{\tan(\theta)}$$

but not greater than:

$$V_{Rdmax} := \alpha_{cw} \cdot b_w \cdot z \cdot \nu_1 \cdot \frac{f_{cd}}{(\tan(\theta) + \cot(\theta))}$$

Where:

$\theta := 45deg$ — angle between concrete compression strut and column axis [degrees]. If not proven otherwise equal to 45 degrees.

$$z := 0.9 \cdot d \qquad z = 173.7mm$$

$$\nu_1 := 0.6 \cdot \left(1 - \frac{f_{ck}}{250MPa}\right) \qquad \nu_1 = 0.504$$

ν_1 is the strength reduction factor for concrete cracked in shear

α_{cw} is the coefficient taking account of the state of the stress in the compression chord, it depends on σ_{cp}.

$$\sigma_{cp} := \min\left(\frac{N_{pmRd}}{N_{plRd}} \cdot \frac{N_{Sd}}{A_c}\right) \qquad \sigma_{cp} = 13.4MPa$$

$$\alpha_{cw} := \begin{cases} 1 & \text{if } \sigma_{cp} = 0 \\ 1 + \dfrac{\sigma_{cp}}{f_{cd}} & \text{if } 0 < \sigma_{cp} \le 0.25 \cdot f_{cd} \\ 1.25 & \text{if } 0.25 \cdot f_{cd} < \sigma_{cp} \le 0.5 \cdot f_{cd} \\ 2.5 \cdot \left(1 - \dfrac{\sigma_{cp}}{f_{cd}}\right) & \text{if } 0.5 \cdot f_{cd} < \sigma_{cp} \end{cases} \qquad \alpha_{cw} = 1.25$$

The maximum effective cross-sectional area of the shear reinforcement, calculated for $\cot(\theta) = 1$, is given by:

$$A_{swmax} := (0.5 \cdot \alpha_{cw} \cdot \nu_1 \cdot f_{cd}) \cdot b_w \cdot \frac{s}{f_{sd}} \qquad A_{swmax} = 1532.7\,mm^2$$

Cross sectional area of a single stirrup.

$$A_{sw} := 2 \cdot fi_s^{\,2} \cdot \frac{\pi}{4} \qquad A_{sw} = 56.5\,mm^2$$

Resistance of stirrups in tension:

$$V_{Rds} := \frac{A_{sw}}{s} \cdot z \cdot \frac{f_{sd}}{\tan(\theta)} \qquad V_{Rds} = 14.2\,kN$$

Resistance of concrete chords in compression:

$$V_{Rdmax} := \alpha_{cw} \cdot b_w \cdot z \cdot \nu_1 \cdot \frac{f_{cd}}{(\tan(\theta) + \cot(\theta))} \qquad V_{Rdmax} = 385.8\,kN$$

Resistance of cross-section requiring shear reinforcement

$$V_{Rd.reinf} := \min(V_{Rds}, V_{Rdmax}) \qquad V_{Rd.reinf} = 14.2\,kN$$

Resistance of the column to transverse shear.

If number of longitudinal reinforcing bars is 0, resistance to shear is calculated from:

$$V_{Rd} := V_{Rd.c} + V_{plaRds}$$

Otherwise it is equal to:

$$V_{Rd} := \max(V_{Rd.c}, V_{Rd.reinf}) + V_{plaRds} \qquad V_{Rd} = 420.8\,kN$$

Influence of shear on the resistance to bending has to be taken into account if the part of the shear force that is coming to the steel tube $V_{a.Sd}$ is greater than half it's resistance V_{plaRds}
It is done by reduction of the design resistance of the steel tube, calculated using reduced yield strength in the part of the tube that carries shear - A_v (EN 1993-1-1, 6.2.8).

$$\left(1 - \rho_{shear}\right) \cdot f_{yd}$$

Where:

$$\rho_{shear} := \left(2 \cdot \frac{V_{a.Sd}}{V_{plaRds}} - 1\right)^2$$

Where:

V_{plaRds} is the plastic design resistance of the steel tbe only

$V_{a.Sd}$ is the part of the shear force that is coming to the steel tube, assuming that the resistance of the concrete core to shear is fully utilized.
$V_{a.Sd}$ is calculated as follow:

$$V_{a.Sd} := \max\left(V_{Sd} - \max\left(V_{Rd.c}, V_{Rd.reinf}\right), 0\right)$$

To simplify the calculation and avoid the reduction of the yield strength, limitation has been imposed that:

$$\frac{V_{a.Sd}}{V_{plaRds}} \le 0.5$$

In the considered case:

$V_{a.Sd} = 0.0\,N$ => $\frac{V_{a.Sd}}{V_{plaRds}} = 0.0$

Utilization of the cross-section in shear.

$\frac{V_{Sd}}{V_{Rd}} = 0.161$ < 0.5

For purposes of conditioning all the restraints to be < 1.0, above condition has been rewritten in another form:

$\frac{2 \cdot V_{Sd}}{V_{Rd}} = 0.323$ < 1.0 (this is how the program checks utillities, thanks to this all the constraints are penalized with correct proportions)

Shear does not affect the bending resistance.

Resistance of column for bending (EN 1994-1-1, 6.7.3.3)

$$M_{Rd} := \alpha_M \cdot \mu \cdot M_{pl.Rd}$$

To determine the bending resistance of the cross-section, which is depending on the axial force, value of μ coefficient has to be callculated. A graphic method is used to determine μ. The method is explained in Figure 3. Symbols that are used are evaluated below.

$$\chi_{pm} := \frac{N_{pmRd}}{N_{plRd}} \qquad \chi_{pm} = 0.402$$

$$\chi := \frac{N_{Rd}}{N_{plRd}} \qquad \chi = 0.796$$

$$\chi_d := \frac{N_{Sd}}{N_{plRd}} \qquad \chi_d = 0.452$$

$$\chi_n := \chi \cdot \frac{\left(1 - \frac{M_{Sd2}}{M_{Sd1}}\right)}{4} \qquad \chi_n = 0.370$$

$$\frac{M_{max.Rd}}{M_{pl.Rd}} = 1.095$$

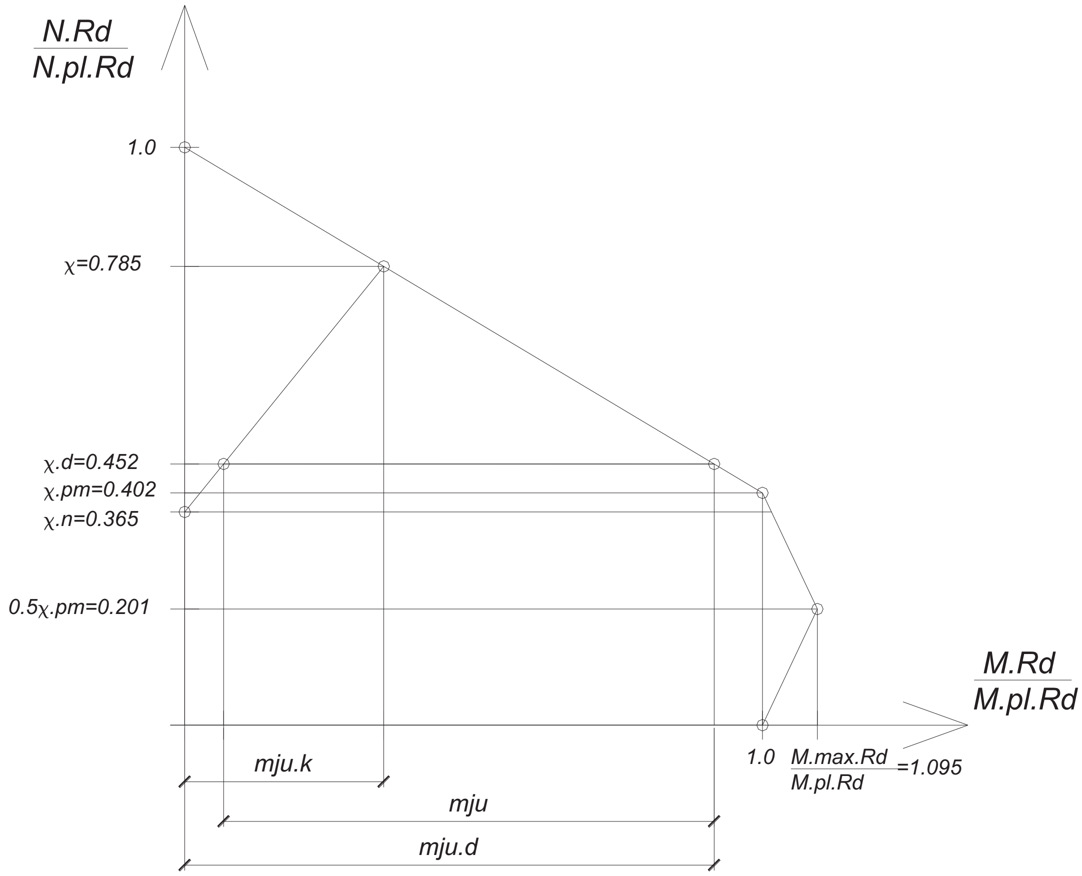


Figure 3. Graphic method of calculating μ.

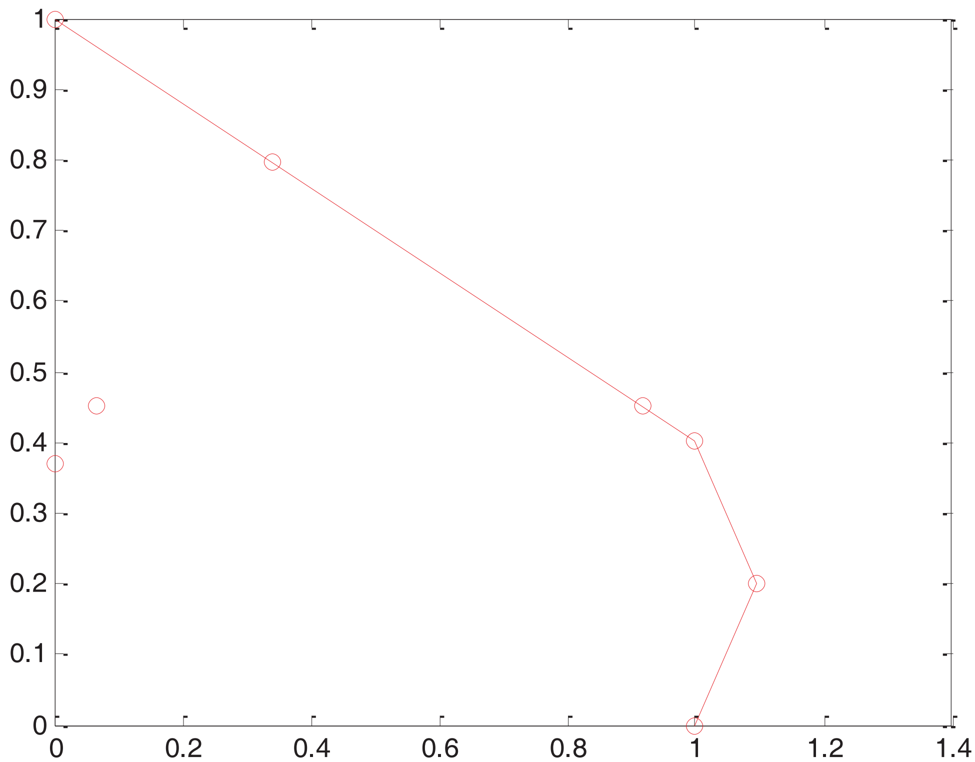


Figure 4. Graphic method using Matlab.

For steel grades S235 - S355 the coefficient α_M is taken equal to 0.9

For steel grades S420 - S460 the coefficient α_M is taken equal to 0.8

In this case:

$$\mu := 0.7436$$

Coefficient μ greater than 1.0 may be used only when bending moment M_{Ed} in column depends directly on the action of the normal force N_{Ed}. Otherwise the cross-sectional force that is causing the increase of the resistance should be reduced by 20% ($\gamma_F := 0.8$ acc. to EN 1994-1-1, 7.2.1.(7))

It has been checked, in this research, that reduction of the N_{Ed} not always leads to the reduction of M_{Rd}. Especially in fire situation. Thus the μ value has been modified so that only the amount that is above 1.0 is reduced by 20%. It has been done according to the folllowing expression.

$$\mu := \begin{cases} \mu & \text{if } \mu \le 1.0 \\ (\mu - 1)\cdot\gamma_F + 1 & \text{if } \mu > 1.0 \end{cases}$$

$$\mu = 0.7436$$

Thus:

$$\frac{M_{Sd}}{\mu \cdot M_{pl.Rd}} = 0.739 \qquad < \qquad \alpha_M := 0.9$$

Results:

$$EA = 3432.2MN$$

$$EI_{effII} = 16.4MN \cdot m^2$$

$$N_{plRd} = 4077.4kN$$

$$N_{pmRd} = 1641.1kN$$

$$N_{Rd} = 3247.1kN$$

$$M_{pl.Rd} = 239.2kN \cdot m$$

$$M_{max.Rd} = 261.9kN \cdot m$$

$$\mu \cdot M_{pl.Rd} = 177.9kN \cdot m$$

$$V_{Rd} = 420.8kN$$

2.2 Fire design (R60)

Material properties at elevated temperatures in fire situation.

Temperatures for the steel tube, concrete core and the reinforcement of the concrete core were taken from: Betonitaytteisen terasliitto-pilarin suunnitteluohje [1].
The temperatures depend on:

- the fire class - in this case R60
- the dimensions of the steel tube cross-section
- for reinforcement - on the distance from the inner surface of the tube to the axis of the rod of the longitudinal reinforcement. Denoted: u_s + fi/2

Also according to [1], u_s is equal to 35 mm if the outer tube dimension (for rectangular tubes - lesser of the dimensions) is not greater than 300 mm. Otherwise u_s = 45 mm.

In considered case:

$u_s = 35.0\text{mm}$

$col = 250.0\text{mm}$

$t = 6.0\text{mm}$

$n = 4$

$fi = 20.0\text{mm}$

Effective lenght of the column in fire situation is equal to half the total column length:

$L_{eff.\theta} := 0.5L_{eff}$

Temperatures read from [1]:

$T_a := 911\text{deg}$ tube temperature, (Appendix 4)

$T_c := 475\text{deg}$ concrete temperature, (Appendix 3)

$T_s := 488.5\text{deg}$ steel temperature, (Appendix 4, value interpolated for u_s+fi/2 = 45 mm)

Reduction factors for steel tube are taken from Table 6 [1].

Design yield strength in fire:

$k_{y.\theta} := 0.0578$ and $f_{a.\theta} := f_y \cdot k_{y.\theta}$ => $f_{a.\theta} = 20.5\text{MPa}$

Young modulus in fire:

$k_{Ea.\theta} := 0.065025$ and $E_{a.\theta} := k_{Ea.\theta} \cdot E_a$ => $E_{a.\theta} = 13.7\text{GPa}$

Reduction factors for concrete are taken from Table 7 [1].

Design compressive strength in fire:

$k_{c.\theta} := 0.6375$ and $f_{c.\theta} := f_{ck} \cdot k_{c.\theta}$ => $f_{c.\theta} = 25.5\,\mathrm{MPa}$

Young modulus in fire:

$\varepsilon_{cu.\theta} := 0.01375$ and $E_{csec.\theta} := k_{c.\theta} \cdot \dfrac{f_{ck}}{\varepsilon_{cu.\theta}}$ => $E_{csec.\theta} = 1854.5\,\mathrm{MPa}$

Reduction factors for reinforcement are taken from Table 8 [1].

Design compressive strength in fire:

$k_{s.\theta} := 0.70105$ and $f_{s.\theta} := f_{sk} \cdot k_{s.\theta}$ => $f_{s.\theta} = 350.5\,\mathrm{MPa}$

Young modulus in fire:

$k_{Es.\theta} := 0.4184$ and $E_{s.\theta} := E_s \cdot k_{Es.\theta}$ => $E_{s.\theta} = 87864.0\,\mathrm{MPa}$

Forces in the column in fire situation:

$N_{Sd.\theta} := 807.0\,\mathrm{kN}$

$M_{Sd1.\theta} := 11.5\,\mathrm{kN \cdot m}$ bigger bending moment (upper or lower end)

$M_{Sd2.\theta} := -3.0\,\mathrm{kN \cdot m}$ smaller bending moment (upper or lower end)

(bending moments on the same side of the column have the same signs)

$V_{Sd.\theta} := \dfrac{M_{Sd1.\theta} - M_{Sd2.\theta}}{L_{eff}}$ $V_{Sd.\theta} = 4.0\,\mathrm{kN}$ constant shear force

Maximum design bending resistance for the cross-section:

Is calculated similarily to ambient situation but with material resistances in fire situation

$M_{max.Rd\theta} := W_{pa} \cdot f_{a.\theta} + W_{ps} \cdot f_{s.\theta} + \dfrac{W_{pc}}{2} \cdot f_{c.\theta}$ $M_{max.Rd\theta} = 85.4\,\mathrm{kN \cdot m}$

Design axial resistance of the concrete only is equal to:

$N_{pmRd.\theta} := A_c \cdot f_{c.\theta}$ $N_{pmRd.\theta} = 1412\,\mathrm{kN}$

Calculation of the new neutral axis in fire is done using the equilibrium equation:

$$0.5 \cdot N_{pmRd.\theta} - (col - 2 \cdot t) \cdot h_{n.\theta} \cdot f_{c.\theta} = A_{sn} \cdot f_{s.\theta} + 4 \cdot t \cdot h_{n.\theta} \cdot f_{a.\theta}$$

Same as previoously A_{sn} is equal to zero because there are no bars lying on the centerline.

$A_{sn} := 0\,\mathrm{mm}^2$

After rearranging the equilibrium equation, distance between the centerline and the neutral axis is given:

$$h_{n.\theta} := \frac{N_{pmRd.\theta} - A_{sn} \cdot \left(2 \cdot f_{s.\theta} - f_{c.\theta}\right)}{2 \cdot \left[(col - 2t) \cdot f_{c.\theta} + 4t \cdot f_{a.\theta}\right]} \qquad h_{n.\theta} = 107.627\,mm$$

Plastic bending resistance of the cross section is equal:

$$M_{pl.Rd.\theta} := M_{max.Rd\theta} - W_{pan.\theta} \cdot f_{a.\theta} - W_{psn.\theta} \cdot f_{s.\theta} - \frac{W_{pcn.\theta}}{2} \cdot f_{c.\theta}$$

Where:

$$W_{pan.\theta} := 2 \cdot \frac{t \cdot \left(2 \cdot h_{n.\theta}\right)^2}{4} \qquad W_{pan.\theta} = 139.0\,cm^3$$

$W_{psn.\theta} := 0 \cdot cm^3$ there are no bars in considered region

$$W_{pcn.\theta} := \frac{(col - 2 \cdot t) \cdot \left(2h_{n.\theta}\right)^2}{4} - W_{psn.\theta} \qquad W_{pcn.\theta} = 2756.9\,cm^3$$

$$M_{pl.Rd.\theta} := M_{max.Rd\theta} - W_{pan.\theta} \cdot f_{a.\theta} - W_{psn.\theta} \cdot f_{s.\theta} - \frac{W_{pcn.\theta}}{2} \cdot f_{c.\theta} \qquad M_{pl.Rd.\theta} = 47.4\,kN \cdot m$$

Values of the resistance to axial force (EN 1994-1-1, 6.7.3.2)

$N_{plRd.\theta} := A_a \cdot f_{a.\theta} + A_c \cdot f_{c.\theta} + A_s \cdot f_{s.\theta}$ $\quad N_{plRd.\theta} = 1973\,kN$ design resistance

$N_{plR.\theta} := A_a \cdot f_{a.\theta} + A_c \cdot f_{c.\theta} + A_s \cdot f_{s.\theta}$ $\quad N_{plR.\theta} = 1973\,kN$ characteristic resistance

Effective flexural and axial stiffnesses (EN 1994-1-1, 6.7.3.3)

$$EI_{eff.\theta} := \left(0.9 \cdot E_{a.\theta} \cdot I_a + 0.8 \cdot E_{csec.\theta} \cdot I_c + 0.9 E_{s.\theta} \cdot I_s\right) \qquad EI_{eff.\theta} = 1645.2\,kN \cdot m^2$$

$$N_{cr.\theta} := \frac{\pi^2}{L_{eff.\theta}^2} \cdot EI_{eff.\theta} \qquad N_{cr.\theta} = 5011.6\,kN$$

Long term effects in fire situation are not taken into account.

Design value of the effective flexural stiffness, used to determine the sectional forces.

$$EI_{effII.\theta} := 0.9 \cdot \left(E_{a.\theta} \cdot I_a + 0.5 \cdot E_{csec.\theta} \cdot I_c + E_{s.\theta} \cdot I_s \right) \qquad EI_{effII.\theta} = 1476.1 kN \cdot m^2$$

$$N_{cr.eff\theta} := \frac{\pi^2}{L_{eff.\theta}^2} \cdot EI_{effII.\theta} \qquad N_{cr.eff\theta} = 4496.5 kN$$

Long term effects in fire situation are not taken into account.

Stiffness to axial compression, used to determine the sectional forces.:

$$EA_{\theta} := E_{a.\theta} \cdot A_a + E_{s.\theta} \cdot A_s + E_{csec.\theta} \cdot A_c \qquad EA_{\theta} = 293.1 MN$$

Axial resistance

$$N_{Rd.\theta} := \chi_{\theta} \cdot N_{plRd.\theta}$$

Where:

$$\chi_{\theta} := \frac{1}{\Phi_{\theta} + \sqrt{\Phi_{\theta}^2 - \lambda_{\theta}^2}} \qquad \text{but } =< 1$$

$$\lambda_{\theta} := \sqrt{\frac{N_{plR.\theta}}{N_{cr.\theta}}} \qquad \lambda_{\theta} = 0.627$$

$$\Phi_{\theta} := 0.5 \cdot \left[1 + \alpha \cdot \left(\lambda_{\theta} - 0.2 \right) + \lambda_{\theta}^2 \right] \qquad \Phi_{\theta} = 0.802$$

Where:

α is an imperfection factor, equal to 0.49 (buckling curve C)

$$\chi_{\theta} := \min\left(\frac{1}{\Phi_{\theta} + \sqrt{\Phi_{\theta}^2 - \lambda_{\theta}^2}}, 1 \right) \qquad \chi_{\theta} = 0.769$$

$$N_{Rd.\theta} := \chi_{\theta} \cdot N_{plRd.\theta} \qquad N_{Rd.\theta} = 1517.2 kN$$

Utilization check:

$$\frac{N_{Sd.\theta}}{N_{Rd.\theta}} = 0.532 \qquad < \qquad 1.0$$

Increse of the bending moment due to second order effects. (EN 1994-1-1, 6.7.3.4)

Second order effects may be allowed for by multiplying the bigger of the $M_{Sd.\theta}$ by a factor k_θ, where k_θ is greater or equal to 1.0.

$$k_\theta := \frac{\beta_\theta}{1 - \frac{N_{Sd.\theta}}{N_{cr.eff\theta}}}$$

Where:

$$\beta_\theta := \max\left[\left[0.66 + 0.44\left(\frac{M_{Sd2.\theta}}{M_{Sd1.\theta}}\right)\right], 0.44\right] \qquad \beta_\theta = 0.545$$

$$k_\theta := \frac{\beta_\theta}{1 - \frac{N_{Sd.\theta}}{N_{cr.eff\theta}}} \qquad k_\theta = 0.664$$

$$k_\theta := \max\left(k_\theta, 1\right) \qquad k_\theta = 1.0$$

Thus:

$$M_{Sd.\theta} := k_\theta \cdot M_{Sd1.\theta} \qquad M_{Sd.\theta} = 11.5 kN \cdot m$$

Transverse shear

Resistance of the steel tube (EN 1993-1-1, 6.2.6)

$$V_{plaRds.\theta} := \left(A_v \cdot f_{a.\theta} \cdot \sqrt{3}\right) \qquad V_{plaRds.\theta} = 22.3 kN$$

Resistance of the concrete (EN 1992-1-1, 6.2)

Resistance of the concrete without shear reinforcement. (6.2.2)

$$\sigma_{cp.\theta} := \min\left(\frac{N_{pmRd.\theta}}{N_{plRd.\theta}} \cdot \frac{N_{Sd.\theta}}{A_c}, 0.2 \cdot f_{c.\theta}\right) \qquad \sigma_{cp.\theta} = 5.1 MPa$$

Design value of the shear resistance is given by:

$$V_{Rdc.\theta} := \left[C_{Rdc} \cdot \left[\left(100 \cdot \min\left(\rho_1, 0.02\right) \cdot f_{c.\theta}\right)^{\frac{1}{3}}\right] + k_1 \cdot \sigma_{cp.\theta}\right] \cdot b_w \cdot d$$

With the minimum value of:

$$V_{Rd.c.min.\theta} := \left[\left(v_{min.\theta} + k_1 \cdot \sigma_{cp.\theta}\right) \cdot b_w \cdot d\right]$$

Calculation of design shear resistance:

$$C_{Rdc} := 0.18$$

$$V_{Rdc.\theta} := \left[C_{Rdc} \cdot \left[\left(100 \cdot \min\left(\rho_1, 0.02\right) \cdot \frac{f_{c.\theta}}{MPa}\right)^{\frac{1}{3}}\right] + k_1 \cdot \frac{\sigma_{cp.\theta}}{MPa}\right] \cdot b_w \cdot d \cdot MPa \qquad V_{Rdc.\theta} = 62.2kN$$

Minimum value of the shear resistance:

$$v_{min.\theta} := 0.035 \cdot k^{\frac{3}{2}} \cdot \left(\frac{f_{c.\theta}}{MPa}\right)^{0.5} \qquad v_{min.\theta} = 0.500$$ recommended value

$$V_{Rd.c.min.\theta} := \left[\left(v_{min.\theta} + k_1 \cdot \frac{\sigma_{cp.\theta}}{MPa}\right) \cdot b_w \cdot d\right] \cdot MPa \qquad V_{Rd.c.min.\theta} = 58.1kN$$

Design shear resistance.

$$V_{Rd.c.\theta} := \max\left(V_{Rdc.\theta}, V_{Rd.c.min.\theta}\right) \qquad V_{Rd.c.\theta} = 62.2kN$$

Resistance of concrete cross-section with shear reinforcement (6.2.3)

The shear resistance of a member with perpendicular shear reinforcement is equal to:

$$V_{Rds.\theta} := \frac{A_{sw}}{s} \cdot z \cdot \frac{f_{s.\theta}}{\tan(\theta)}$$

but not greater than:

$$V_{Rdmax.\theta} := \alpha_{cw.\theta} \cdot b_w \cdot z \cdot \nu_{1.\theta} \cdot \frac{f_{c.\theta}}{\left(\tan(\theta) + \cot(\theta)\right)}$$

Where:

$$\nu_{1.\theta} := 0.6 \cdot \left(1 - \frac{f_{c.\theta}}{250MPa}\right) \qquad \nu_{1.\theta} = 0.539$$

$\alpha_{cw.\theta}$ depends on $\sigma_{cp.\theta}$.

$$\sigma_{cp.\theta} := \min\left(\frac{N_{pmRd.\theta}}{N_{plRd.\theta}} \cdot \frac{N_{Sd.\theta}}{A_c}\right) \qquad \sigma_{cp.\theta} = 10.4MPa$$

$$\alpha_{cw.\theta} := \begin{cases} 1 & \text{if } \sigma_{cp.\theta} = 0 \\ 1 + \dfrac{\sigma_{cp.\theta}}{f_{c.\theta}} & \text{if } 0 < \sigma_{cp.\theta} \le 0.25 \cdot f_{c.\theta} \\ 1.25 & \text{if } 0.25 \cdot f_{c.\theta} < \sigma_{cp.\theta} \le 0.5 \cdot f_{c.\theta} \\ 2.5 \cdot \left(1 - \dfrac{\sigma_{cp.\theta}}{f_{c.\theta}}\right) & \text{if } 0.5 \cdot f_{c.\theta} < \sigma_{cp.\theta} \end{cases} \qquad \alpha_{cw.\theta} = 1.25$$

The maximum effective cross-sectional area of the shear reinforcement, calculated for $\cot(\theta) = 1$, is given by:

$$A_{swmax.\theta} := \left(0.5 \cdot \alpha_{cw.\theta} \cdot \nu_{1.\theta} \cdot f_{c.\theta}\right) \cdot b_w \cdot \frac{s}{f_{s.\theta}} \qquad A_{swmax.\theta} = 1749.1\,mm^2$$

Resistance of stirrups in tension:

$$A_{sw} = 56.5\,mm^2$$

$$V_{Rds.\theta} := \frac{A_{sw}}{s} \cdot z \cdot \frac{f_{s.\theta}}{\tan(\theta)} \qquad V_{Rds.\theta} = 11.5\,kN$$

Resistance of concrete chords in compression:

$$V_{Rdmax.\theta} := \alpha_{cw.\theta} \cdot b_w \cdot z \cdot \nu_{1.\theta} \cdot \frac{f_{c.\theta}}{\left(\tan(\theta) + \cot(\theta)\right)} \qquad V_{Rdmax.\theta} = 355.0\,kN$$

Resistance of cross-section requiring shear reinforcement

$$V_{Rd.reinf.\theta} := \min\left(V_{Rds.\theta}, V_{Rdmax.\theta}\right) \qquad V_{Rd.reinf.\theta} = 11.5\,kN$$

Resistance of the column to transverse shear.

If number of longitudinal reinforcing bars is 0, resistance to shear is calculated from:

$$V_{Rd.\theta} := V_{Rd.c.\theta} + V_{plaRds.\theta}$$

Otherwise it is equal to:

$$V_{Rd.\theta} := \max\left(V_{Rd.c.\theta}, V_{Rd.reinf.\theta}\right) + V_{plaRds.\theta} \qquad V_{Rd.\theta} = 84.5\,kN$$

To simplify the calculation and avoid the reduction of the yield strength, limitation has been imposed that:

$$V_{a.Sd.\theta} := \max\left(V_{Sd.\theta} - \max\left(V_{Rd.c.\theta}, V_{Rd.reinf.\theta}\right), 0\right)$$

$$\frac{V_{a.Sd.\theta}}{V_{plaRds.\theta}} \le 0.5$$

In the considered case:

$$V_{a.Sd.\theta} = 0.0\,N \qquad \Rightarrow \qquad \frac{V_{a.Sd.\theta}}{V_{plaRds.\theta}} = 0.0$$

Utilization of the cross-section in shear.

$$\frac{V_{Sd.\theta}}{V_{Rd.\theta}} = 0.048 \qquad < \qquad 0.5$$

Shear does not affect the bending resistance.

Resistance of column for bending (EN 1994-1-1, 6.7.3.3)

$$M_{Rd.\theta} := \alpha_M \cdot \mu_\theta \cdot M_{pl.Rd.\theta}$$

To determine the bending resistance of the cross-section, which is depending on the axial force, value of μ coefficient has to be callculated. A graphic method is used to determine μ. (see Figure 5.) Symbols that are used are evaluated below.

$$\chi_{pm.\theta} := \frac{N_{pmRd.\theta}}{N_{plRd.\theta}} \qquad \chi_{pm.\theta} = 0.716$$

$$\chi_\theta := \frac{N_{Rd.\theta}}{N_{plRd.\theta}} \qquad \chi_\theta = 0.769$$

$$\chi_{d.\theta} := \frac{N_{Sd.\theta}}{N_{plRd.\theta}} \qquad \chi_{d.\theta} = 0.409$$

$$\chi_{n.\theta} := \chi_\theta \cdot \frac{\left(1 - \frac{M_{Sd2.\theta}}{M_{Sd1.\theta}}\right)}{4} \qquad \chi_{n.\theta} = 0.242$$

$$\frac{M_{max.Rd\theta}}{M_{pl.Rd.\theta}} = 1.802$$

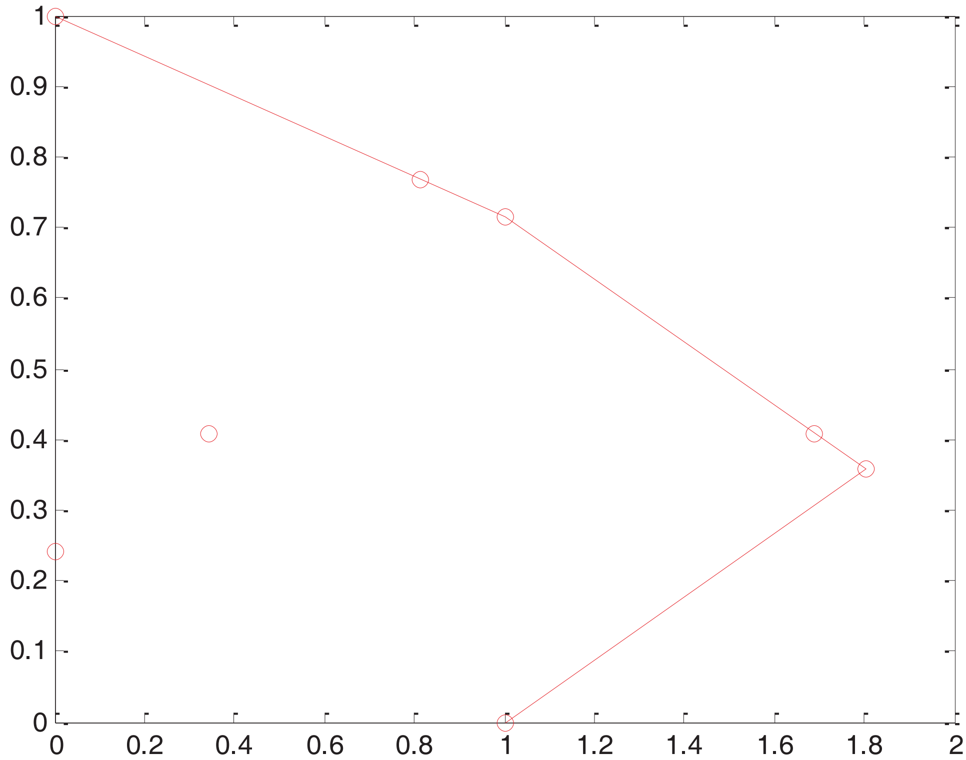


Figure 5. Calculation of μ in fire situation.

The bending resistance in fire situation is checked as follows:

$$\mu_\theta := 1.4256$$

$$\mu_\theta := \begin{cases} \mu_\theta & \text{if } \mu_\theta \le 1.0 \\ (\mu_\theta - 1)\cdot\gamma_F + 1 & \text{if } \mu_\theta > 1.0 \end{cases}$$

$$\mu_\theta = 1.3405$$

Thus:

$$\frac{M_{Sd.\theta}}{\mu_\theta \cdot M_{pl.Rd.\theta}} = 0.181 \quad < \quad \alpha_M := 0.9$$

Results:

$$EA_\theta = 293.1\,MN$$

$$EI_{effII.\theta} = 1476.1\,kN\cdot m^2$$

$$N_{plRd.\theta} = 1973.0\,kN$$

$$N_{pmRd.\theta} = 1412.4\,kN$$

$$N_{Rd.\theta} = 1517.2\,kN$$

$$M_{pl.Rd.\theta} = 47.4\,kN\cdot m$$

$$M_{max.Rd\theta} = 85.4\,kN\cdot m$$

$$\mu_\theta \cdot M_{pl.Rd.\theta} = 63.5\,kN\cdot m$$

$$V_{Rd.\theta} = 84.5\,kN$$

2.3 Fire design at 20ºC

Material properties at room temperatures in fire situation.

Temperatures of steel tube, and concrete core is 20 degrees Celsius.

Material properties are taken at temperature 20 degrees.

Steel tube material charcteristics in fire situation at 20 deg.:

$k_{y.\theta 20} := 1.000$ and $f_{a.\theta 20} := f_y \cdot k_{y.\theta 20}$ => $f_{a.\theta 20} = 355.0\text{MPa}$

$k_{Ea.\theta 20} := 1.000$ and $E_{a.\theta 20} := k_{Ea.\theta 20} \cdot E_a$ => $E_{a.\theta 20} = 210.0\text{GPa}$

Concrete charcteristics in fire situation at 20 deg.:

$k_{c.\theta 20} := 1.000$ and $f_{c.\theta 20} := f_{ck} \cdot k_{c.\theta 20}$ => $f_{c.\theta 20} = 40.0\text{MPa}$

$\varepsilon_{cu.\theta 20} := 0.00250$ and $E_{csec.\theta 20} := k_{c.\theta 20} \cdot \dfrac{f_{ck}}{\varepsilon_{cu.\theta 20}}$ => $E_{csec.\theta 20} = 16.0\text{GPa}$

Reinforcement material charcteristics in fire situation at 20 deg.:

$k_{s.\theta 20} := 1.000$ and $f_{s.\theta 20} := f_{sk} \cdot k_{s.\theta 20}$ => $f_{s.\theta 20} = 500.0\text{MPa}$

$k_{Es.\theta 20} := 1.000$ and $E_{s.\theta 20} := E_s \cdot k_{Es.\theta 20}$ => $E_{s.\theta 20} = 210.0\text{GPa}$

Effective lenght of the column in fire situation is equal to the column length:

$L_{eff.\theta 20} := L_{eff}$

Forces in the column in fire situation:

$N_{Sd.\theta 20} := 1807.0\text{kN}$

$M_{Sd1.\theta 20} := 21.5\text{kN}\cdot\text{m}$ bigger bending moment (upper or lower end)

$M_{Sd2.\theta 20} := -8.0\text{kN}\cdot\text{m}$ smaller bending moment (upper or lower end)

(bending moments on the same side of the column have the same signs)

$V_{Sd.\theta 20} := \dfrac{M_{Sd1.\theta 20} - M_{Sd2.\theta 20}}{L_{eff}}$ $V_{Sd.\theta 20} = 8.2\text{kN}$ constant shear force

Maximum design bending resistance for the cross-section:

Is calculated similarily to ambient situation but with material resistances.

$$M_{max.Rd\theta20} := W_{pa} \cdot f_{a.\theta20} + W_{ps} \cdot f_{s.\theta20} + \frac{W_{pc}}{2} \cdot f_{c.\theta20} \qquad M_{max.Rd\theta20} = 302.3\,kN \cdot m$$

Design axial resistance of the concrete only is equal to:

$$N_{pmRd.\theta20} := A_c \cdot f_{c.\theta20} \qquad N_{pmRd.\theta20} = 2215\,kN$$

Calculation of the new neutral axis is done using the equilibrium equation:

$$0.5 \cdot N_{pmRd.\theta20} - (col - 2 \cdot t) \cdot h_{n.\theta20} \cdot f_{c.\theta20} = A_{sn} \cdot f_{s.\theta20} + 4 \cdot t \cdot h_{n.\theta20} \cdot f_{a.\theta20}$$

Same as previoously A_{sn} is equal to zero because there are no bars lying on the centerline.

$$A_{sn} := 0mm^2$$

After rearranging the equilibrium equation, distance between the centerline and the neutral axis is given:

$$h_{n.\theta20} := \frac{N_{pmRd.\theta20} - A_{sn} \cdot \left(2 \cdot f_{s.\theta20} - f_{c.\theta20}\right)}{2 \cdot \left[(col - 2t) \cdot f_{c.\theta20} + 4t \cdot f_{a.\theta20}\right]} \qquad h_{n.\theta20} = 61.405mm$$

Plastic bending resistance of the cross section is equal:

$$M_{pl.Rd.\theta20} := M_{max.Rd\theta20} - W_{pan.\theta20} \cdot f_{a.\theta20} - W_{psn.\theta20} \cdot f_{s.\theta20} - \frac{W_{pcn.\theta20}}{2} \cdot f_{c.\theta20}$$

Where:

$$W_{pan.\theta20} := 2 \cdot \frac{t \cdot \left(2 \cdot h_{n.\theta20}\right)^2}{4} \qquad W_{pan.\theta20} = 45.2cm^3$$

$$W_{psn.\theta20} := 0 \cdot cm^3$$ there are no bars in considered region

$$W_{pcn.\theta20} := \frac{(col - 2 \cdot t) \cdot \left(2h_{n.\theta20}\right)^2}{4} - W_{psn.\theta20} \qquad W_{pcn.\theta20} = 897.4cm^3$$

$$M_{pl.Rd.\theta20} := M_{max.Rd\theta20} - W_{pan.\theta20} \cdot f_{a.\theta20} - W_{psn.\theta20} \cdot f_{s.\theta20} - \frac{W_{pcn.\theta20}}{2} \cdot f_{c.\theta20}$$

$$M_{pl.Rd.\theta20} = 268.3\,kN \cdot m$$

Values of the resistance to axial force (EN 1994-1-1, 6.7.3.2)

$$N_{plRd.\theta 20} := A_a \cdot f_{a.\theta 20} + A_c \cdot f_{c.\theta 20} + A_s \cdot f_{s.\theta 20} \qquad N_{plRd.\theta 20} = 4923\,\text{kN}$$ design resistance

$$N_{plR.\theta 20} := A_a \cdot f_{a.\theta 20} + A_c \cdot f_{c.\theta 20} + A_s \cdot f_{s.\theta 20} \qquad N_{plR.\theta 20} = 4923\,\text{kN}$$ characteristic resistance

Effective flexural and axial stiffnesses (EN 1994-1-1, 6.7.3.3)

$$EI_{eff.\theta 20} := \left(E_{a.\theta 20} \cdot I_a + 0.8 \cdot E_{csec.\theta 20} \cdot I_c + E_{s.\theta 20} \cdot I_s\right) \qquad EI_{eff.\theta 20} = 16989.3\,\text{kN} \cdot \text{m}^2$$

$$N_{cr.\theta 20} := \frac{\pi^2}{L_{eff.\theta 20}^{\;2}} \cdot EI_{eff.\theta 20} \qquad N_{cr.\theta 20} = 12938.1\,\text{kN}$$

Long term effects in fire situation are not taken into account.

Design value of the effective flexural stiffness, used to determine the sectional forces.

$$EI_{effII.\theta 20} := 0.9 \cdot \left(E_{a.\theta 20} \cdot I_a + 0.5 \cdot E_{csec.\theta 20} \cdot I_c + E_{s.\theta 20} \cdot I_s\right) \qquad EI_{effII.\theta 20} = 14165.0\,\text{kN} \cdot \text{m}^2$$

$$N_{cr.eff\theta 20} := \frac{\pi^2}{L_{eff.\theta 20}^{\;2}} \cdot EI_{effII.\theta 20} \qquad N_{cr.eff\theta 20} = 10787.3\,\text{kN}$$

Long term effects in fire situation are not taken into account.

Stiffness to axial compression, used to determine the sectional forces.:

$$EA_{\theta 20} := E_{a.\theta 20} \cdot A_a + E_{s.\theta 20} \cdot A_s + E_{csec.\theta 20} \cdot A_c \qquad EA_{\theta 20} = 2379.9\,\text{MN}$$

Axial resistance

$$N_{Rd.\theta 20} := \chi_{\theta 20} \cdot N_{plRd.\theta 20}$$

$$\chi_{\theta 20} := \frac{1}{\Phi_{\theta 20} + \sqrt{\Phi_{\theta 20}^{\;2} - \lambda_{\theta 20}^{\;2}}}$$ but =< 1

$$\lambda_{\theta 20} := \sqrt{\frac{N_{plR.\theta 20}}{N_{cr.\theta 20}}} \qquad \lambda_{\theta 20} = 0.617$$

$$\Phi_{\theta 20} := 0.5 \cdot \left[1 + \alpha \cdot \left(\lambda_{\theta 20} - 0.2\right) + \lambda_{\theta 20}^{\;2}\right] \qquad \Phi_{\theta 20} = 0.792$$

Where:

α is an imperfection factor, equal to 0.49 (buckling curve C)

$$\chi_{\theta 20} := \min\left(\frac{1}{\Phi_{\theta 20} + \sqrt{\Phi_{\theta 20}^{\ 2} - \lambda_{\theta 20}^{\ 2}}}, 1\right) \qquad \chi_{\theta 20} = 0.775$$

$$N_{Rd.\theta 20} := \chi_{\theta 20} \cdot N_{plRd.\theta 20} \qquad N_{Rd.\theta 20} = 3816.9\,kN$$

Utilization check:

$$\frac{N_{Sd.\theta 20}}{N_{Rd.\theta 20}} = 0.473 \quad < \quad 1.0$$

Increse of the bending moment due to second order effects (EN 1994-1-1, 6.7.3.4)

Second order effects may be allowed for by multiplying the bigger of the $M_{Sd.\theta}$ by a factor k_{θ}, where k_{θ} is greater or equal to 1.0.

$$k_{\theta 20} := \frac{\beta_{\theta 20}}{1 - \frac{N_{Sd.\theta 20}}{N_{cr.eff\theta 20}}}$$

Where:

$$\beta_{\theta 20} := \max\left[\left[0.66 + 0.44 \cdot \left(\frac{M_{Sd2.\theta 20}}{M_{Sd1.\theta 20}}\right)\right], 0.44\right] \qquad \beta_{\theta 20} = 0.496$$

$$k_{\theta 20} := \frac{\beta_{\theta 20}}{1 - \frac{N_{Sd.\theta 20}}{N_{cr.eff\theta 20}}} \qquad k_{\theta 20} = 0.596$$

$$k_{\theta 20} := \max\left(k_{\theta 20}, 1\right) \qquad k_{\theta 20} = 1.000$$

Thus:

$$M_{Sd.\theta 20} := k_{\theta 20} \cdot M_{Sd1.\theta 20} \qquad M_{Sd.\theta 20} = 21.5\,kN \cdot m$$

Transverse shear

Resistance of the steel tube (EN 1993-1-1, 6.2.6)

$$V_{plaRds.\theta 20} := \left|A_v \cdot f_{a.\theta 20} \cdot \sqrt{3}\right| \qquad V_{plaRds.\theta 20} = 386.3kN$$

Resistance of the concrete (EN 1992-1-1, 6.2)

Resistance of the concrete without shear reinforcement. (6.2.2)

$$\sigma_{cp.\theta 20} := \min\left(\frac{N_{pmRd.\theta 20}}{N_{plRd.\theta 20}} \cdot \frac{N_{Sd.\theta 20}}{A_c}, 0.2 \cdot f_{c.\theta 20}\right) \qquad \sigma_{cp.\theta 20} = 8.0MPa$$

Design value of the shear resistance is given by:

$$V_{Rdc.\theta 20} := \left[C_{Rdc} \cdot \left[\left(100 \cdot \min\left(\rho_1, 0.02\right) \cdot f_{c.\theta 20}\right)^{\frac{1}{3}}\right] + k_1 \cdot \sigma_{cp.\theta 20}\right] \cdot b_w \cdot d$$

With the minimum value of:

$$V_{Rd.c.min.\theta 20} := \left[\left(v_{min.\theta 20} + k_1 \cdot \sigma_{cp.\theta 20}\right) \cdot b_w \cdot d\right]$$

Calculation of design shear resistance:

$$C_{Rdc} := 0.18$$

$$V_{Rdc.\theta 20} := \left[C_{Rdc} \cdot \left[\left(100 \cdot \min\left(\rho_1, 0.02\right) \cdot \frac{f_{c.\theta 20}}{MPa}\right)^{\frac{1}{3}}\right] + k_1 \cdot \frac{\sigma_{cp.\theta 20}}{MPa}\right] \cdot b_w \cdot d \cdot MPa$$

$$V_{Rdc.\theta 20} = 86.5kN$$

Minimum value of the shear resistance:

$$v_{min.\theta 20} := 0.035 \cdot k^{\frac{3}{2}} \cdot \left(\frac{f_{c.\theta 20}}{MPa}\right)^{0.5} \qquad v_{min.\theta 20} = 0.626$$ recommended value

$$V_{Rd.c.min.\theta 20} := \left[\left(v_{min.\theta 20} + k_1 \cdot \frac{\sigma_{cp.\theta 20}}{MPa}\right) \cdot b_w \cdot d\right] \cdot MPa \qquad V_{Rd.c.min.\theta 20} = 83.9kN$$

Design shear resistance.

$$V_{Rd.c.\theta 20} := \max\left(V_{Rdc.\theta 20}, V_{Rd.c.min.\theta 20}\right) \qquad V_{Rd.c.\theta 20} = 86.5kN$$

Resistance of concrete cross-section with shear reinforcement (6.2.3)

The shear resistance of a member with perpendicular shear reinforcement is equal to:

$$V_{Rds.\theta 20} := \frac{A_{sw}}{s} \cdot z \cdot \frac{f_{s.\theta 20}}{\tan(\theta)}$$

but not greater than:

$$V_{Rdmax.\theta 20} := \alpha_{cw.\theta 20} \cdot b_w \cdot z \cdot \nu_{1.\theta 20} \cdot \frac{f_{c.\theta 20}}{(\tan(\theta) + \cot(\theta))}$$

Where:

$$\nu_{1.\theta 20} := 0.6 \cdot \left(1 - \frac{f_{c.\theta 20}}{250MPa}\right) \qquad \nu_{1.\theta 20} = 0.504$$

$\alpha_{cw.\theta 20}$ depends on $\sigma_{cp.\theta 20}$.

$$\sigma_{cp.\theta 20} := \min\left(\frac{N_{pmRd.\theta 20}}{N_{plRd.\theta 20}} \cdot \frac{N_{Sd.\theta 20}}{A_c}\right) \qquad \sigma_{cp.\theta 20} = 14.7MPa$$

$$\alpha_{cw.\theta 20} := \begin{cases} 1 & \text{if } \sigma_{cp.\theta 20} = 0 \\ 1 + \dfrac{\sigma_{cp.\theta 20}}{f_{c.\theta 20}} & \text{if } 0 < \sigma_{cp.\theta 20} \le 0.25 \cdot f_{c.\theta 20} \\ 1.25 & \text{if } 0.25 \cdot f_{c.\theta 20} < \sigma_{cp.\theta 20} \le 0.5 \cdot f_{c.\theta 20} \\ 2.5 \cdot \left(1 - \dfrac{\sigma_{cp.\theta 20}}{f_{c.\theta 20}}\right) & \text{if } 0.5 \cdot f_{c.\theta 20} < \sigma_{cp.\theta 20} \end{cases} \qquad \alpha_{cw.\theta 20} = 1.25$$

The maximum effective cross-sectional area of the shear reinforcement, calculated for $\cot(\theta) = 1$, is given by:

$$A_{swmax.\theta 20} := (0.5 \cdot \alpha_{cw.\theta 20} \cdot \nu_{1.\theta 20} \cdot f_{c.\theta 20}) \cdot b_w \cdot \frac{s}{f_{s.\theta 20}} \qquad A_{swmax.\theta 20} = 1799.3mm^2$$

Resistance of stirrups in tension:

$$A_{sw} = 56.5mm^2$$

$$V_{Rds.\theta 20} := \frac{A_{sw}}{s} \cdot z \cdot \frac{f_{s.\theta 20}}{\tan(\theta)} \qquad V_{Rds.\theta 20} = 16.4kN$$

Resistance of concrete chords in compression:

$$V_{Rdmax.\theta20} := \alpha_{cw.\theta20} \cdot b_w \cdot z \cdot \nu_{1.\theta20} \cdot \frac{f_{c.\theta20}}{(\tan(\theta) + \cot(\theta))} \qquad V_{Rdmax.\theta20} = 520.9\,kN$$

Resistance of cross-section requiring shear reinforcement

$$V_{Rd.reinf.\theta20} := \min(V_{Rds.\theta20}, V_{Rdmax.\theta20}) \qquad V_{Rd.reinf.\theta20} = 16.4\,kN$$

Resistance of the column to transverse shear.

If number of longitudinal reinforcing bars is 0, resistance to shear is calculated from:

$$V_{Rd.\theta20} := V_{Rd.c.\theta20} + V_{plaRds.\theta20}$$

Otherwise it is equal to:

$$V_{Rd.\theta20} := \max(V_{Rd.c.\theta20}, V_{Rd.reinf.\theta20}) + V_{plaRds.\theta20} \qquad V_{Rd.\theta20} = 472.8\,kN$$

To simplify the calculation and avoid the reduction of the yield strength, limitation has been imposed that:

$$V_{a.Sd.\theta20} := \max(V_{Sd.\theta20} - \max(V_{Rd.c.\theta20}, V_{Rd.reinf.\theta20}), 0)$$

$$\frac{V_{a.Sd.\theta20}}{V_{plaRds.\theta20}} \le 0.5$$

In the considered case:

$$V_{a.Sd.\theta20} = 0.0\,N \qquad => \qquad \frac{V_{a.Sd.\theta20}}{V_{plaRds.\theta20}} = 0.0$$

Utilization of the cross-section in shear.

$$\frac{V_{Sd.\theta20}}{V_{Rd.\theta20}} = 0.017 \qquad < \qquad 0.5$$

Shear does not affect the bending resistance.

Resistance of column for bending (EN 1994-1-1, 6.7.3.3)

$$M_{Rd.\theta 20} := \alpha_M \cdot \mu_{\theta 20} \cdot M_{pl.Rd.\theta 20}$$

To determine the bending resistance of the cross-section, which is depending on the axial force, value of μ coefficient has to be callculated. A graphic method is used to determine μ. (see Figure 6). Symbols that are used are evaluated below.

$$\chi_{pm.\theta 20} := \frac{N_{pmRd.\theta 20}}{N_{plRd.\theta 20}} \qquad \chi_{pm.\theta 20} = 0.450$$

$$\chi_{\theta 20} := \frac{N_{Rd.\theta 20}}{N_{plRd.\theta 20}} \qquad \chi_{\theta 20} = 0.775$$

$$\chi_{d.\theta 20} := \frac{N_{Sd.\theta 20}}{N_{plRd.\theta 20}} \qquad \chi_{d.\theta 20} = 0.367$$

$$\chi_{n.\theta 20} := \chi_{\theta 20} \cdot \frac{\left(1 - \frac{M_{Sd2.\theta 20}}{M_{Sd1.\theta 20}}\right)}{4} \qquad \chi_{n.\theta 20} = 0.266$$

$$\frac{M_{max.Rd\theta 20}}{M_{pl.Rd.\theta 20}} = 1.127$$

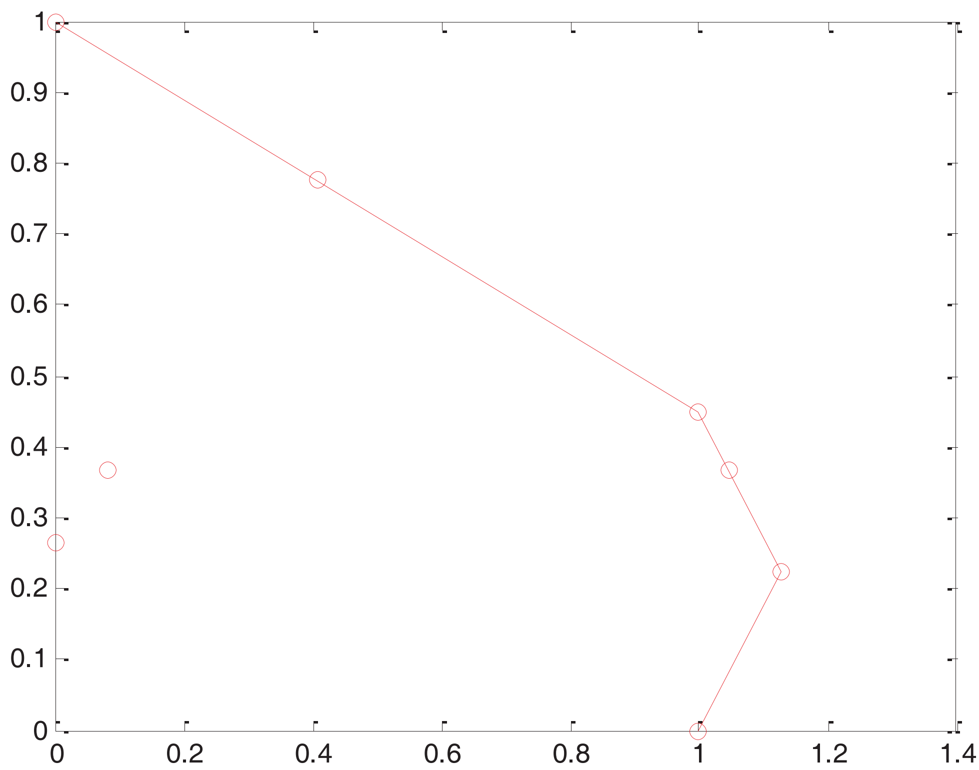


Figure 6. Calculation of μ in fire situation.

The bending resistance in fire situation is checked as follows:

$$\mu_{\theta 20} := 0.9657$$

$$\mu_{\theta 20} := \begin{cases} \mu_{\theta 20} & \text{if } \mu_{\theta 20} \le 1.0 \\ \left(\mu_{\theta 20} - 1\right) \cdot \gamma_F + 1 & \text{if } \mu_{\theta 20} > 1.0 \end{cases}$$

$$\mu_{\theta 20} = 0.9657$$

Thus:

$$\frac{M_{Sd.\theta 20}}{\mu_{\theta 20} \cdot M_{pl.Rd.\theta 20}} = 0.083 \qquad < \qquad \alpha_M := 0.9$$

Results:

$$EA_{\theta 20} = 2379.9\,MN$$

$$EI_{effII.\theta 20} = 14165.0\,kN \cdot m^2$$

$$N_{plRd.\theta 20} = 4922.7\,kN$$

$$N_{pmRd.\theta 20} = 2215.5\,kN$$

$$N_{Rd.\theta 20} = 3816.9\,kN$$

$$M_{pl.Rd.\theta 20} = 268.3\,kN \cdot m$$

$$M_{max.Rd\theta 20} = 302.3\,kN \cdot m$$

$$\mu_{\theta 20} \cdot M_{pl.Rd.\theta 20} = 259.1\,kN \cdot m$$

$$V_{Rd.\theta 20} = 472.8\,kN$$

3 CIRCULAR COLUMN

3.1 Ambient design

Data for the calculation:

$col := 355.6mm$	width and height of the steel tube
$t := 6mm$	wall thickness of the steel tube
$n := 6$	number of reinforcing bars
$fi := 25mm$	diameter of reinforcing bars
$fi_s := 8mm$	diameter of stirrups
$s := 375mm$	stirrups spacing
$u_s := 45mm$	distance from the inner surface of the tube to the surface of the bar
$L_{eff} := 2000mm$	effective length of the column

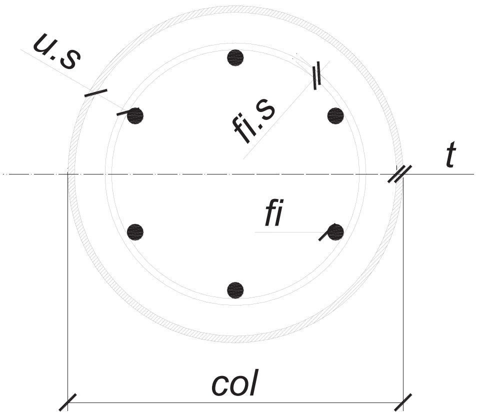


Figure 7. Symbols used for the basic design variables.

Material properties are the same is in previous example

Steel	Concrete	Reinforcement	
$f_y := 355MPa$	$f_{ck} := 40MPa$	$f_{sk} := 500MPa$	characteristic strength
$gamma_a := 1.1$	$gamma_c := 1.35$	$gamma_s := 1.15$	material safety factor
$f_{ad} = 322.7MPa$	$f_{cd} = 29.6MPa$	$f_{sd} = 434.8MPa$	design strengths
$E_a := 210000MPa$	$E_{cm} := 35000MPa$	$E_s := 210000MPa$	Young modulus

Forces in the column in normal situation:

$N_{Sd} := 542.0\text{kN}$

$M_{Sd1} := 131.5\text{kN}\cdot\text{m}$ bigger bending moment (upper or lower end)

$M_{Sd2} := -113.0\text{kN}\cdot\text{m}$ smaller bending moment (upper or lower end)

$V_{Sd} = 122.3\text{kN}$ constant shear force

Characteristics of the cross section

Steel tube

$A_a := \frac{\pi}{4}\cdot\left[(col)^2 - (col - 2\cdot t)^2\right]$ $A_a = 6590\,\text{mm}^2$ tube cross-section area

$I_a := \frac{\pi}{64}\cdot\left[(col)^4 - (col - 2\cdot t)^4\right]$ $I_a = 1.007\times 10^8\,\text{mm}^4$ tube second moment of area

$W_{pa} := \frac{1}{6}\cdot\left[(col)^3 - (col - 2\cdot t)^3\right]$ $W_{pa} = 7.334\times 10^5\,\text{mm}^3$ plastic modulus

Reinforcement

$A_{bar} := \frac{fi^2\cdot\pi}{4}$ $A_{bar} = 491\,\text{mm}^2$ cross-section area of a single bar

$A_s := n\cdot A_{bar}$ $A_s = 2945\,\text{mm}^2$ cross-section area of the reinforcement

$dist := \frac{col}{2} - t - u_s - \frac{fi}{2}$ $dist = 114.3\text{mm}$ distance between the axis of the outermost bar to the midline of the column

$W_{ps} := A_{bar}\cdot dist\cdot\left(4\cdot\sin\left(\frac{\pi}{6}\right) + 2\right)$ $W_{ps} = 224.4\text{cm}^3$ plastic modulus of the reinforcement cross-section

$I_s := A_{bar}\cdot dist^2\cdot\left[4\cdot\left(\sin\left(\frac{\pi}{6}\right)\right)^2 + 2\right]$ $I_s = 1.924\times 10^7\,\text{mm}^4$ second moment of area of the reinforcement

Concrete core

$A_{cgross} := \frac{\pi}{4}\cdot(col - 2\cdot t)^2$ $A_{cgross} = 92725\,\text{mm}^2$ gross cross-section area of the concrete

$A_c := A_{cgross} - A_s$ $A_c = 89780\,\text{mm}^2$ nett cross-section area of the concrete

$W_{pc} := \frac{1}{6}\cdot\left[(col - 2\cdot t)^3\right] - W_{ps}$ $W_{pc} = 6536.5\text{cm}^3$ plastic modulus of the concrete cross-section

$$I_{cgross} := \frac{\pi \cdot (col - 2 \cdot t)^4}{64} \qquad I_{cgross} = 6.842 \times 10^8\, mm^4$$ gross second moment of area of the concrete

$$I_c := I_{cgross} - I_s \qquad I_c = 6.650 \times 10^8\, mm^4$$ nett second moment of area of the concrete

Check if the reinforcement meets the requirement of the reinforcement level

$$\rho := \frac{A_s}{A_{cgross}} \qquad \rho_{max} := 6.0 \cdot \% \; > \; \rho = 3.2\% \; > \; \rho_{min} := 1.5 \cdot \%$$ Requirement met

Maximum design bending resistance for the cross-section is given by equation:

$$M_{max.Rd} := W_{pa} \cdot f_{ad} + W_{ps} \cdot f_{sd} + \frac{W_{pc}}{2} \cdot f_{cd} \qquad M_{max.Rd} = 431.1\, kN \cdot m$$

Calculation of the location of the neutral axis.

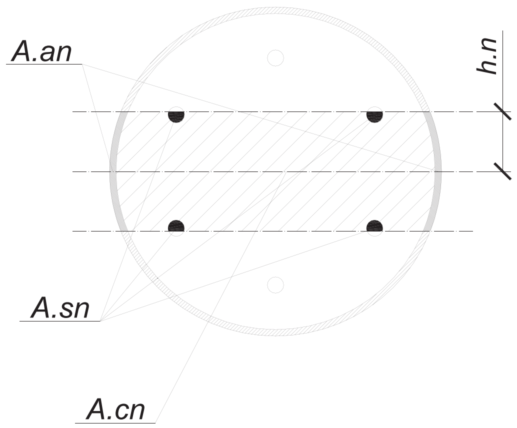


Figure 8. Symbols used in calculation of plastic bending resistance

Design axial resistance of the concrete only is equal to:

$$N_{pmRd} := A_c \cdot f_{cd} \qquad N_{pmRd} = 2660\, kN$$

Equilibrium equation:

$$0.5 \cdot N_{pmRd} - 0.5 \cdot A_{cn} \cdot f_{cd} = A_{sn} \cdot f_{sd} + A_{an} \cdot f_{ad}$$

Due to the shape of the column cross-section, the calculation of h_n is very difficult. An iterational method has been used to compute this. Concrete core has been divided into 200.000 layers, h_n has been moving away from the centerline one layer at a time and equilibrium equation has been checked at each step. The first value of h_n for which the right side of the equation got bigger than the left is the neutral axis (Appendix).

$$h_n := 55.41mm$$

Thus:

$$A_{sn} := 808mm^2$$

$$W_{psn} := 41117mm^3$$

$$W_{pan} := 37814mm^3$$

$$W_{pcn} := 985770 \cdot mm^3$$

With this data $M_{pl.Rd}$ can be now calculated.

$$M_{pl.Rd} := M_{max.Rd} - W_{pan} \cdot f_{ad} - W_{psn} \cdot f_{sd} - \frac{W_{pcn}}{2} \cdot f_{cd}$$

$$M_{pl.Rd} = 386.4kN \cdot m$$

Values of the resistance to axial force (EN 1994-1-1, 6.7.3.2)

$N_{plRd} := A_a \cdot f_{ad} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$ $\quad N_{plRd} = 6067\,kN$ $\quad$ design resistance

$N_{plR} := A_a \cdot f_y + A_c \cdot f_{ck} + A_s \cdot f_{sk}$ $\quad N_{plR} = 7403\,kN$ $\quad$ characteristic resistance

Effective flexural and axial stiffnesses (EN 1994-1-1, 6.7.3.3)

$EI_{eff} := E_a \cdot I_a + 0.6 \cdot E_{cm} \cdot I_c + E_s \cdot I_s$ $\quad EI_{eff} = 39152.5kN \cdot m^2$

$N_{cr} := \frac{\pi^2}{L_{eff}^2} \cdot EI_{eff}$ $\quad N_{cr} = 96605.0kN$

Check if the long term effect need to be taken into account

$\delta := \frac{A_a \cdot f_{ad}}{N_{plRd}}$ $\quad \delta = 0.351$ $\quad$ is the contribution ratio of the steel tube, has to be between 0.2 and 0.9 - only then column works as a composite

$\lambda := \sqrt{\frac{N_{plR}}{N_{cr}}}$ $\quad \lambda = 0.277$ $\quad$ is the relative slenderness of the column, has to be smaller than 2 (EN 1994-1-1 6.7.3.3)

$\lambda_{vert} := \frac{0.8}{1 - \delta}$ $\quad \lambda_{vert} = 1.232$

According to [1], if λ is smaller than λ_{vert}, long term effects do not need to be taken into account.

Design value of the effectibve flexural stiffness, used to determine the sectional forces.
(EN 1994-1-1 6.7.3.4)

$EI_{effII} := 0.9 \cdot \left| E_a \cdot I_a + 0.5 \cdot E_{cm} \cdot I_c + E_s \cdot I_s \right|$ $\quad EI_{effII} = 33142.6 kN \cdot m^2$

$N_{cr.eff} := \frac{\pi^2}{L_{eff}^2} \cdot EI_{effII}$ $\quad N_{cr.eff} = 81776.2 kN$

Stiffness to axial compression, used to determine the sectional forces.:

$EA := E_a \cdot A_a + E_s \cdot A_s + E_{cm} \cdot A_c$ $\quad EA = 5144.6 MN$

Axial resistance (EN 1994-1-1, 6.7.3.5)

$N_{Rd} := \chi \cdot N_{plRd}$

Where:

χ is the reduction factor for the relevant buckling mode, in terms of relative slenderness λ. (EN 1993-1-1, 6.3.1.2)
if λ =< 0.2 or N.Sd/N.plRd =< 0.1 than χ = 1.

$$\frac{N_{Sd}}{N_{plRd}} = 0.089$$

$\chi := 1$

$N_{Rd} := \chi \cdot N_{plRd}$ $\quad N_{Rd} = 6067.4 kN$

Utilization check:

$\frac{N_{Sd}}{N_{Rd}} = 0.089$ < 1.0

Transverse shear

Resistance of the steel tube (EN 1993-1-1, 6.2.6)

$A_v := A_s \cdot \frac{2}{\pi}$ $\quad A_v = 1875.0 mm^2$ area in shear for RHS

$V_{plaRds} := \left(\frac{A_v \cdot f_y \cdot \sqrt{3}}{gamma_a} \right)$ $\quad V_{plaRds} = 1048.1 kN$ plastic shear resistance of the tube.

Resistance of the concrete (EN 1992-1-1, 6.2)

As a simplification the heigth and width od the column cross-section core are taken as 0.8 of the actual core diameter [2]. Core shear resistance is calculated as for rectangular section. (Figure 9.)

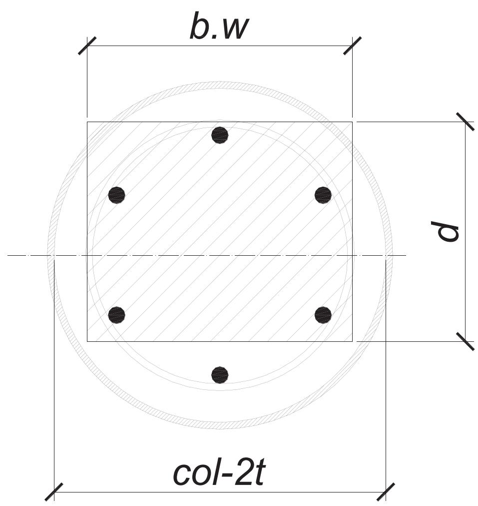


Figure 9. Effective width and depth of the cross section.

$d := 0.8\cdot(col - 2\cdot t) - u_s - \frac{fi}{2}$ $d = 217.4mm$ effective depth of the cross-section

$b_w := 0.8\cdot(col - 2\cdot t)$ $b_w = 274.9mm$ effective width of the cross section

$A_{sl} := \frac{A_s + A_{sn}}{2}$ $A_{sl} = 1876.6mm^2$ area of the tensile reinforcement, properly anchored

$\rho_1 := \frac{A_{sl}}{b_w\cdot d}$ $\rho_1 = 0.031$ ratio of the longitudinal reinforcement

Resistance of the concrete without shear reinforcement. (6.2.2)

$$\sigma_{cp} := \min\left(\frac{N_{pmRd}}{N_{plRd}}\cdot\frac{N_{Sd}}{A_c}, 0.2\cdot f_{cd}\right) \qquad \sigma_{cp} = 2.6MPa$$

Design value of the shear resistance is given by:

$$V_{Rdc} := \left[C_{Rdc}\cdot\left[\left(100\cdot\min\left(\rho_1, 0.02\right)\cdot f_{ck}\right)^{\frac{1}{3}}\right] + k_1\cdot\sigma_{cp}\right]\cdot b_w\cdot d$$

With the minimum value of:

$$V_{Rd.c.min} := \left[\left(v_{min} + k_1\cdot\sigma_{cp}\right)\cdot b_w\cdot d\right]$$

Calculation of design shear resistance:

$$C_{Rdc} := \frac{0.18}{gamma_c} \qquad C_{Rdc} = 0.133$$ recommended value

$$k_1 := 0.15$$ recommended value

$$V_{Rdc} := \left[C_{Rdc} \cdot \left[\left(100 \cdot \min\left(\rho_1, 0.02\right) \cdot \frac{f_{ck}}{MPa} \right)^{\frac{1}{3}} \right] + k_1 \cdot \frac{\sigma_{cp}}{MPa} \right] \cdot b_w \cdot d \cdot MPa \qquad V_{Rdc} = 58.1kN$$

Minimum value of the shear resistance:

$$k := \min\left[1 + \left(\frac{200mm}{d} \right)^{0.5}, 2 \right] \qquad k = 2.0$$

$$v_{min} := 0.035 \cdot k^{\frac{3}{2}} \cdot \left(\frac{f_{ck}}{MPa} \right)^{0.5} \qquad v_{min} = 0.607$$ recommended value

$$V_{Rd.c.min} := \left[\left(v_{min} + k_1 \cdot \frac{\sigma_{cp}}{MPa} \right) \cdot b_w \cdot d \right] \cdot MPa \qquad V_{Rd.c.min} = 60.0kN$$

In the end design shear resistance is the bigger of the velues V_{Rdc} and $V_{Rd.c.min}$.

$$V_{Rd.c} := \max\left(V_{Rdc}, V_{Rd.c.min} \right) \qquad V_{Rd.c} = 60.0kN$$

Resistance of concrete cross-section with shear reinforcement (6.2.3)

The shear resistance of a member with perpendicular shear reinforcement is equal to:

$$V_{Rds} := \frac{A_{sw}}{s} \cdot z \cdot \frac{f_{sd}}{\tan(\theta)}$$

but not greater than:

$$V_{Rdmax} := \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot \frac{f_{cd}}{\left(\tan(\theta) + \cot(\theta) \right)}$$

Where:

$$\theta := 45deg$$ angle between concrete compression strut and column axis [degrees]. If not proven otherwise equal to 45 degrees.

$$z := 0.9 \cdot d \qquad z = 195.6mm$$

$$\nu_1 := 0.6 \left(1 - \frac{f_{ck}}{250\text{MPa}}\right) \qquad \nu_1 = 0.504$$

ν_1 is the strength reduction factor for concrete cracked in shear

α_{cw} is the coefficient taking account of the state of the stress in the compression chord, it depends on σ_{cp}.

$$\sigma_{cp} := \min\left(\frac{N_{pmRd}}{N_{plRd}} \cdot \frac{N_{Sd}}{A_c}\right) \qquad \sigma_{cp} = 2.6\text{MPa}$$

$$\alpha_{cw} := \begin{cases} 1 & \text{if } \sigma_{cp} = 0 \\ 1 + \dfrac{\sigma_{cp}}{f_{cd}} & \text{if } 0 < \sigma_{cp} \le 0.25 \cdot f_{cd} \\ 1.25 & \text{if } 0.25 \cdot f_{cd} < \sigma_{cp} \le 0.5 \cdot f_{cd} \\ 2.5 \cdot \left(1 - \dfrac{\sigma_{cp}}{f_{cd}}\right) & \text{if } 0.5 \cdot f_{cd} < \sigma_{cp} \end{cases} \qquad \alpha_{cw} = 1.09$$

The maximum effective cross-sectional area of the shear reinforcement, calculated for $\cot(\theta) = 1$, is given by:

$$A_{swmax} := (0.5 \cdot \alpha_{cw} \cdot \nu_1 \cdot f_{cd}) \cdot b_w \cdot \frac{s}{f_{sd}} \qquad A_{swmax} = 1928.4\text{mm}^2$$

Cross sectional area of a single stirrup.

$$A_{sw} := 2 \cdot fi_s^{\,2} \cdot \frac{\pi}{4} \qquad A_{sw} = 100.5\text{mm}^2$$

Resistance of stirrups in tension:

$$V_{Rds} := \frac{A_{sw}}{s} \cdot z \cdot \frac{f_{sd}}{\tan(\theta)} \qquad V_{Rds} = 22.8\text{kN}$$

Resistance of concrete chords in compression:

$$V_{Rdmax} := \alpha_{cw} \cdot b_w \cdot z \cdot \nu_1 \cdot \frac{f_{cd}}{(\tan(\theta) + \cot(\theta))} \qquad V_{Rdmax} = 437.4\text{kN}$$

Resistance of cross-section requiring shear reinforcement

$$V_{Rd.reinf} := \min(V_{Rds}, V_{Rdmax}) \qquad V_{Rd.reinf} = 22.8\text{kN}$$

Resistance of the column to transverse shear.

If number of longitudinal reinforcing bars is 0, resistance to shear is calculated from:

$$V_{Rd} := V_{Rd.c} + V_{plaRds}$$

Otherwise it is equal to:

$$V_{Rd} := \max\left(V_{Rd.c}, V_{Rd.reinf}\right) + V_{plaRds} \qquad V_{Rd} = 1108.1\mathrm{kN}$$

Influence of shear on the resistance to bending has to be taken into account if the part of the shear force that is coming to the steel tube $V_{a.Sd}$ is greater than half it's resistance V_{plaRds}
It is done by reduction of the design resistance of the steel tube, calculated using reduced yield strength in the part of the tube that carries shear - A_v (EN 1993-1-1, 6.2.8).

$$\left(1 - \rho_{shear}\right) \cdot f_{yd}$$

Where:

$$\rho_{shear} := \left(2 \cdot \frac{V_{a.Sd}}{V_{plaRds}} - 1\right)^2$$

Where:

V_{plaRds} is the plastic design resistance of the steel tube only

$V_{a.Sd}$ is the part of the shear force that is coming to the steel tube, assuming that the resistance of the concrete core to shear is fully utilized. $V_{a.Sd}$ is calculated as follow:

$$V_{a.Sd} := \max\left(V_{Sd} - \max\left(V_{Rd.c}, V_{Rd.reinf}\right), 0\right)$$

To avoid the reduction of the yield strength, limitation has been imposed:

$$\frac{V_{a.Sd}}{V_{plaRds}} \le 0.5$$

In the considered case:

$$V_{a.Sd} = 62.3\mathrm{kN} \qquad => \qquad \frac{V_{a.Sd}}{V_{plaRds}} = 0.059$$

Utilization of the cross-section in shear.

$$\frac{V_{Sd}}{V_{Rd}} = 0.110 \qquad < \qquad 0.5$$

Rewritten in another form:

$$\frac{2 \cdot V_{Sd}}{V_{Rd}} = 0.221 \qquad < \qquad 1.0$$

Shear does not affect the bending resistance.

Resistance of column for bending (EN 1994-1-1, 6.7.3.3)

$$M_{Rd} := \alpha_M \cdot \mu \cdot M_{pl.Rd}$$

To determine the bending resistance of the cross-section, which is depending on the axial force, value of μ coefficient has to be calculated. A graphic method is used to determine μ (Figure 11.) Symbols that are used are evaluated below.

$$\chi_{pm} := \frac{N_{pmRd}}{N_{plRd}} \qquad \chi_{pm} = 0.438$$

$$\chi := \frac{N_{Rd}}{N_{plRd}} \qquad \chi = 1.000$$

$$\chi_d := \frac{N_{Sd}}{N_{plRd}} \qquad \chi_d = 0.089$$

$$\chi_n := \chi \cdot \frac{\left(1 - \frac{M_{Sd2}}{M_{Sd1}}\right)}{4} \qquad \chi_n = 0.465$$

$$\frac{M_{max.Rd}}{M_{pl.Rd}} = 1.116$$

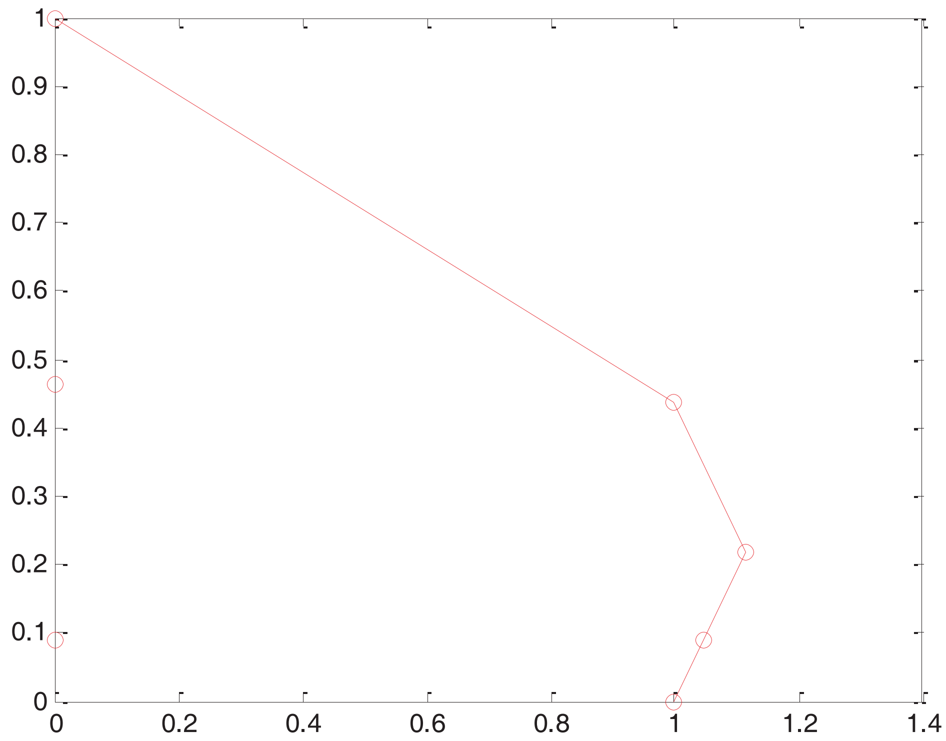


Figure 10. Graphic method for calculating μ.

$$\mu := 0.8 \cdot \left| \mu - 1 \right| + 1$$

$$\mu = 1.0377$$

Thus:

$$\frac{M_{Sd}}{\mu \cdot M_{pl.Rd}} = 0.328 \qquad < \qquad \alpha_M := 0.9$$

Results:

$$EA = 5144.6\mathrm{MN}$$

$$EI_{effII} = 33.1\mathrm{MN \cdot m^2}$$

$$N_{plRd} = 6067.4\mathrm{kN}$$

$$N_{pmRd} = 2660.1\mathrm{kN}$$

$$N_{Rd} = 6067.4\mathrm{kN}$$

$$M_{pl.Rd} = 386.4\mathrm{kN \cdot m}$$

$$M_{max.Rd} = 431.1\mathrm{kN \cdot m}$$

$$\mu \cdot M_{pl.Rd} = 401.0\mathrm{kN \cdot m}$$

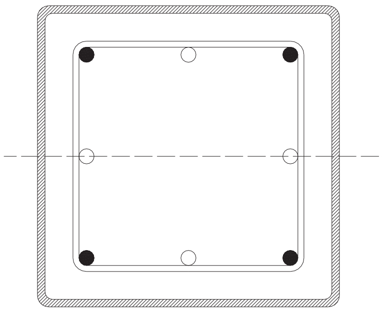
$$V_{Rd} = 1108.1\mathrm{kN}$$

4 RESULTS

Following some resistances of different column cross-section are presented.

The results are for three design situations:

- At ambient temperature
- In fire situation (R60), at elevated temperature
- In fire situation, at 20ºC



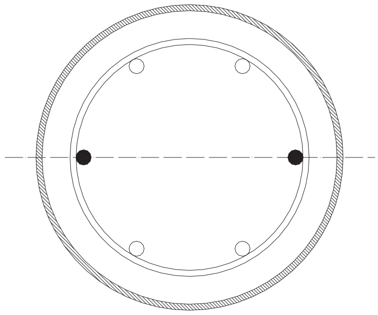
S355

C 40/50

A500 HW

Table 1. Resistances of square columns.

COLUMN		AMBIENT			FIRE - R60			FIRE at 20 C		
steel tube	reinf.	N.pl.Rd	M.pl.Rd	N.pm.Rd	N.fi.pl.Rd	M.fi.pl.Rd	N.fi.pm.Rd	N.20.pl.Rd	M.20.pl.Rd	N.20.pm.Rd
150 x 150 x 5	0T00	1517	57	581	323	4	265	1814	64	784
150 x 150 x 5	4T12	1700	63	567	450	8	258	2022	70	766
150 x 150 x 5	4T16	1842	67	557	550	11	254	2183	75	752
180 x 180 x 5	0T00	1986	84	856	634	6	564	2399	94	1156
180 x 180 x 5	4T12	2169	93	843	780	13	555	2607	104	1138
180 x 180 x 5	4T16	2312	99	832	893	18	548	2768	111	1124
180 x 180 x 5	4T20	2495	106	819	1039	24	539	2977	119	1106
180 x 180 x 5	4T25	2781	116	798	1399	37	525	3302	131	1077
200 x 200 x 5	0T00	2328	106	1070	848	7	769	2829	118	1444
200 x 200 x 5	4T16	2654	124	1046	1113	22	752	3198	139	1412
200 x 200 x 5	4T20	2837	133	1032	1262	30	742	3407	149	1394
200 x 200 x 5	4T25	3124	146	1011	1634	48	727	3732	164	1365
200 x 200 x 6	0T00	2550	124	1047	848	9	753	3067	138	1414
200 x 200 x 6	4T16	2876	141	1023	1108	23	736	3437	158	1382
200 x 200 x 6	4T20	3059	150	1010	1343	35	726	3645	169	1363
200 x 200 x 6	4T25	3345	163	989	1621	47	711	3970	183	1335
220 x 220 x 6	0T00	2939	152	1282	1130	11	1025	3554	169	1731
220 x 220 x 6	4T16	3265	173	1258	1392	28	1006	3924	193	1698
220 x 220 x 6	4T20	3449	184	1245	1630	43	996	4132	206	1680
220 x 220 x 6	4T25	3735	200	1224	1911	58	979	4457	224	1652
250 x 250 x 6	0T00	3568	199	1678	1565	14	1444	4345	222	2266
250 x 250 x 6	4T16	3894	225	1655	1761	31	1424	4715	252	2234
250 x 250 x 6	4T20	4077	239	1641	1973	47	1412	4923	268	2215
250 x 250 x 6	4T25	4364	260	1620	2203	64	1394	5248	292	2187
250 x 250 x 6	8T20	4586	272	1604	2381	84	1380	5501	308	2165
250 x 250 x 6	8T25	5159	307	1562	2841	117	1344	6151	350	2109
300 x 300 x 8	0T00	5405	377	2390	2544	28	2347	6543	421	3226
300 x 300 x 8	4T20	5915	429	2353	2933	70	2311	7121	481	3176
300 x 300 x 8	4T25	6201	457	2332	3296	105	2290	7447	512	3148
300 x 300 x 8	4T32	6709	503	2294	3776	150	2253	8023	566	3098
300 x 300 x 8	8T25	6996	518	2273	4048	187	2233	8350	587	3069
300 x 300 x 8	8T32	8012	595	2199	5007	270	2160	9503	678	2969
350 x 350 x 10	0T00	7616	636	3227	3714	52	3398	9184	709	4356
350 x 350 x 10	4T20	8125	695	3189	4259	117	3358	9762	777	4306
350 x 350 x 10	4T25	8411	726	3168	4600	155	3336	10087	813	4277
350 x 350 x 10	4T32	8919	779	3131	5207	220	3297	10664	874	4227
350 x 350 x 10	8T25	9207	797	3110	5487	268	3275	10990	900	4199
350 x 350 x 10	8T32	10223	888	3036	6700	393	3197	12144	1008	4099
400 x 400 x 10	0T00	9313	845	4279	5070	69	4707	11314	943	5776
400 x 400 x 10	4T20	9822	918	4241	5615	149	4666	11892	1027	5726
400 x 400 x 10	4T25	10109	957	4220	5954	196	4643	12217	1071	5697
400 x 400 x 10	4T32	10616	1023	4183	6560	277	4603	12794	1148	5647



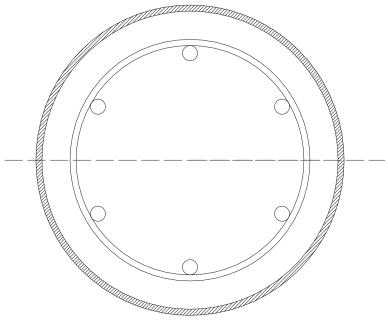
S355

C 40/50

A500 HW

Table 2. Resistances of circular columns, with some of the bars lying on the centerline

COLUMN		AMBIENT			FIRE - R60			FIRE at 20 C		
steel tube	reinf.	N.pl.Rd	M.pl.Rd	N.pm.Rd	N.fi.pl.Rd	M.fi.pl.Rd	N.fi.pm.Rd	N.20.pl.Rd	M.20.pl.Rd	N.20.pm.Rd
168,3 x 5	0T00	1411	49	583	381	4	325	1698	55	787
168,3 x 5	4T12	1594	55	570	539	10	317	1906	62	769
168,3 x 5	4T16	1737	58	559	662	14	311	2068	66	755
168,3 x 5	4T20	1920	62	546	821	16	304	2276	70	737
219,1 x 5	0T00	2103	86	1017	852	8	773	2567	97	1374
219,1 x 5	4T16	2429	103	994	1212	30	755	2937	116	1341
219,1 x 5	4T20	2612	110	980	1414	39	744	3146	126	1323
219,1 x 5	4T25	2898	119	959	1752	49	728	3471	138	1295
219,1 x 5	6T25	3296	129	930	2201	60	706	3922	149	1256
273 x 5	0T00	2968	138	1610	1538	13	1434	3667	155	2173
273 x 5	4T16	3294	162	1586	1884	45	1413	4037	183	2141
273 x 5	4T20	3477	174	1572	2079	61	1401	4246	198	2123
273 x 5	4T25	3764	191	1551	2428	84	1382	4571	218	2094
273 x 5	6T25	4162	206	1522	2873	100	1356	5022	235	2055
273 x 6	0T00	3209	162	1585	1556	15	1429	3927	181	2140
273 x 6	4T16	3535	185	1561	1907	48	1407	4297	208	2108
273 x 6	4T20	3719	196	1548	2105	63	1395	4505	222	2090
273 x 6	4T25	4005	212	1527	2448	85	1376	4830	242	2062
273 x 6	6T25	4403	227	1498	2894	101	1350	5282	259	2022
323,9 x 6	0T00	4198	233	2264	2426	22	2269	5183	261	3056
323,9 x 6	4T20	4707	279	2227	2983	87	2232	5762	315	3006
323,9 x 6	4T25	4993	301	2206	3322	120	2211	6087	343	2978
323,9 x 6	6T25	5391	320	2177	3770	142	2182	6538	365	2938
323,9 x 6	4T32	5501	337	2169	3930	167	2174	6663	386	2928
323,9 x 6	6T32	6153	367	2121	4683	202	2126	7403	421	2863
323,9 x 6	8T32	6804	438	2073	5435	284	2078	8143	503	2799
355,6 x 6	0T00	4874	284	2747	3012	28	2837	6048	318	3709
355,6 x 6	4T20	5383	334	2710	3601	103	2799	6626	378	3659
355,6 x 6	4T25	5670	360	2689	3933	139	2777	6952	409	3630
355,6 x 6	6T25	6067	380	2660	4394	163	2747	7403	432	3591
355,6 x 6	4T32	6177	400	2652	4522	194	2739	7528	458	3580
355,6 x 6	6T32	6829	432	2604	5277	231	2690	8268	495	3516
355,6 x 6	8T32	7481	507	2557	6032	318	2641	9008	582	3452
406,4 x 8	0T00	6778	485	3547	4103	53	3807	8343	543	4788
406,4 x 8	6T20	7542	559	3491	4986	125	3747	9210	631	4713
406,4 x 8	12T20	8306	647	3435	5868	279	3687	10077	733	4637
406,4 x 8	6T25	7971	597	3460	5482	161	3713	9698	676	4670
406,4 x 8	6T32	8733	661	3404	6363	302	3653	10562	753	4595
406,4 x 8	8T32	9385	753	3356	7116	408	3602	11302	858	4531
457 x 8	0T00	8168	623	4526	5379	69	5041	10116	697	6110
457 x 8	8T20	9186	755	4451	6553	256	4958	11272	853	6009
457 x 8	8T25	9759	826	4409	7213	346	4911	11922	935	5953
457 x 8	8T32	10774	945	4335	8384	472	4828	13075	1076	5852
508 x 10	0T00	10591	947	5542	6917	124	6359	13036	1060	7482
508 x 10	8T20	11609	1096	5467	8089	336	6274	14192	1235	7381



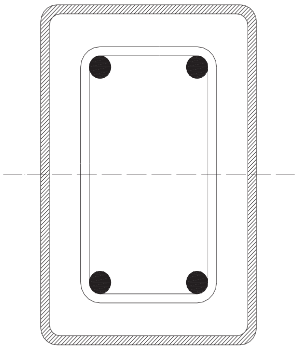
S355

C 40/50

A500 HW

Table 3. Resistances of circular columns, without any bars lying on the centerline

COLUMN		AMBIENT			FIRE - R60			FIRE at 20 C		
steel tube	reinf.	N.pl.Rd	M.pl.Rd	N.pm.Rd	N.fi.pl.Rd	M.fi.pl.Rd	N.fi.pm.Rd	N.20.pl.Rd	M.20.pl.Rd	N.20.pm.Rd
168,3 x 5	0T00	1411	49	583	381	4	325	1698	55	787
168,3 x 5	4T12	1594	54	570	539	8	317	1906	60	769
168,3 x 5	4T16	1737	57	559	662	11	311	2068	64	755
168,3 x 5	4T20	1920	62	546	821	15	304	2276	69	737
219,1 x 5	0T00	2103	86	1017	852	8	773	2567	97	1374
219,1 x 5	4T16	2429	101	994	1212	21	755	2937	112	1341
219,1 x 5	4T20	2612	108	980	1414	30	744	3146	121	1323
219,1 x 5	4T25	2898	119	959	1752	45	728	3471	134	1295
219,1 x 5	6T25	3296	137	930	2201	69	706	3922	156	1256
273 x 5	0T00	2968	138	1610	1538	13	1434	3667	155	2173
273 x 5	4T16	3294	160	1586	1884	32	1413	4037	177	2141
273 x 5	4T20	3477	170	1572	2079	41	1401	4246	190	2123
273 x 5	4T25	3764	186	1551	2428	62	1382	4571	208	2094
273 x 5	6T25	4162	212	1522	2873	106	1356	5022	242	2055
273 x 6	0T00	3209	162	1585	1556	15	1429	3927	181	2140
273 x 6	4T16	3535	183	1561	1907	33	1407	4297	205	2108
273 x 6	4T20	3719	195	1548	2105	43	1395	4505	217	2090
273 x 6	4T25	4005	211	1527	2448	65	1376	4830	235	2062
273 x 6	6T25	4403	234	1498	2894	107	1350	5282	266	2022
323,9 x 6	0T00	4198	233	2264	2426	22	2269	5183	261	3056
323,9 x 6	4T20	4707	276	2227	2983	56	2232	5762	308	3006
323,9 x 6	4T25	4993	298	2206	3322	82	2211	6087	332	2978
323,9 x 6	6T25	5391	328	2177	3770	149	2182	6538	372	2938
323,9 x 6	4T32	5501	334	2169	3930	129	2174	6663	373	2928
323,9 x 6	6T32	6153	383	2121	4683	216	2126	7403	436	2863
323,9 x 6	8T32	6804	432	2073	5435	275	2078	8143	494	2799
355,6 x 6	0T00	4874	284	2747	3012	28	2837	6048	318	3709
355,6 x 6	4T20	5383	328	2710	3601	78	2799	6626	364	3659
355,6 x 6	4T25	5670	350	2689	3933	89	2777	6952	389	3630
355,6 x 6	6T25	6067	386	2660	4394	173	2747	7403	439	3591
355,6 x 6	4T32	6177	388	2652	4522	138	2739	7528	433	3580
355,6 x 6	6T32	6829	446	2604	5277	243	2690	8268	508	3516
355,6 x 6	8T32	7481	499	2557	6032	308	2641	9008	571	3452
406,4 x 8	0T00	6778	485	3547	4103	53	3807	8343	543	4788
406,4 x 8	6T20	7542	563	3491	4986	177	3747	9210	636	4713
406,4 x 8	12T20	8306	777	3435	5868	467	3687	10077	883	4637
406,4 x 8	6T25	7971	605	3460	5482	230	3713	9698	684	4670
406,4 x 8	6T32	8733	676	3404	6363	317	3653	10562	768	4595
406,4 x 8	8T32	9385	741	3356	7116	398	3602	11302	844	4531
457 x 8	0T00	8168	623	4526	5379	69	5041	10116	697	6110
457 x 8	8T20	9186	751	4451	6553	261	4958	11272	851	6009
457 x 8	8T25	9759	817	4409	7213	353	4911	11922	929	5953
457 x 8	8T32	10774	930	4335	8384	492	4828	13075	1060	5852
508 x 10	0T00	10591	947	5542	6917	124	6359	13036	1060	7482
508 x 10	8T20	11609	1091	5467	8089	341	6274	14192	1232	7381



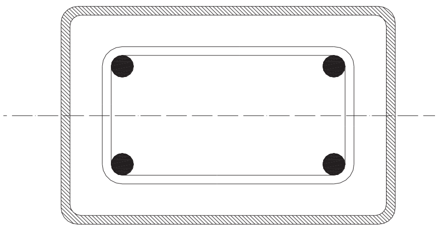
S355

C 40/50

A500 HW

Table 4. Resistances of rectangular columns – about major axis.

COLUMN		AMBIENT			FIRE - R60			FIRE at 20 C		
steel tube	reinf.	N.pl.Rd	M.pl.Rd	N.pm.Rd	N.fi.pl.Rd	M.fi.pl.Rd	N.fi.pm.Rd	N.20.pl.Rd	M.20.pl.Rd	N.20.pm.Rd
200 x 120 x 5	0T00	1620	77	619	313	6	251	1937	86	836
200 x 120 x 5	4T10	1747	84	610	372	9	247	2081	94	823
250 x 150 x 6	0T00	2476	145	973	735	11	640	2967	162	1314
250 x 150 x 6	4T12	2659	160	960	844	20	632	3175	179	1296
250 x 150 x 6	4T16	2802	171	949	930	27	625	3337	192	1282
300 x 200 x 8	0T00	4048	291	1548	1449	23	1285	4839	325	2090
300 x 200 x 8	4T16	4373	325	1524	1673	47	1266	5209	365	2058
300 x 200 x 8	4T20	4557	343	1511	1800	60	1255	5417	385	2040
300 x 200 x 8	4T25	4843	370	1490	2147	94	1237	5743	417	2012



S355

C 40/50

A500 HW

Table 5. Resistances of rectangular columns – about minor axis.

COLUMN		AMBIENT			FIRE - R60			FIRE at 20 C		
steel tube	reinf.	N.pl.Rd	M.pl.Rd	N.pm.Rd	N.fi.pl.Rd	M.fi.pl.Rd	N.fi.pm.Rd	N.20.pl.Rd	M.20.pl.Rd	N.20.pm.Rd
120 x 200 x 5	0T00	1620	51	619	313	3	251	1937	57	836
120 x 200 x 5	4T10	1747	53	610	372	4	247	2081	60	823
150 x 250 x 6	0T00	2476	97	973	735	7	640	2967	108	1314
150 x 250 x 6	4T12	2659	102	960	844	10	632	3175	114	1296
150 x 250 x 6	4T16	2802	106	949	930	12	625	3337	118	1282
200 x 300 x 8	0T00	4048	212	1548	1449	15	1285	4839	236	2090
200 x 300 x 8	4T16	4373	228	1524	1673	27	1266	5209	255	2058
200 x 300 x 8	4T20	4557	237	1511	1800	34	1255	5417	265	2040
200 x 300 x 8	4T25	4843	248	1490	2147	49	1237	5743	279	2012

5 SUMMARY

Calculation of the resistances of different concrete-filled tubes has been presented.

For square column the resistance calculation is presented in 3 situations:

- At ambient temperature
- At elevated temperature in fire situation (fire class R60)
- At 20ºC in fire situation

For circular column – only at ambient temperature

The calculation of h_n for round column takes into account reinforcing bars lying in the area between $-h_n$ and $+h_n$ from the centerline. It is also possible that only part of the bar is in this range. Matlab program has been written to iteratively find the location of the neutral axis. Concrete core of the column had been divided into 200 000 layers, the axis had been moved one layer at a time until the equilibrium of the internal forces was obtained.

Including the reinforcement A_{an} in the calculation lead to observation that in some cases the bending resistance of a column varies up to 57% (at elevated temperature), depending on the arrangement of the bars. The difference is maximum 17% in normal design situation.

Also calculation of shear resistance of columns has been presented. Shear resistance have to be calculated for columns that are rigidly connected with the beams, and thus carry the shear loads. Assumption has been made that shear stresses in the steel tube, due to shear, are less than half the yield strength and do not affect the bending resistance.

For the circular column the shear area of the concrete has been taken as square with the size of 0.8 times the core diameter.

In the end the resistances of different cross-sections of columns, in 3 design situations, have been presented.

REFERENCES:

[1] Teräsrakenneyhdistys, Betonitäytteisen teräsliittopilarin suunnitteluohje, 2004

[2] Suunnittelun sovellutusohjeet ja Betoninormien RakMK B4 suunnitteluosa, RakMK B1 ja B2, by 16, Suomen Betoniyhdistys r.y., Jyväskylä 1984. pp. 75, Figure S 34.

[3] EN 1994-1-1 – Eurocode 4: Design of composite steel and concrete structures – Part 1-1: General rules and rules for buildings

[4] EN 1992-1-1 – Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings

[5] EN 1993-1-1 – Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings

APPENDIX

Part of the MATLAB script that calculates h_n, W_{psn} and A_{sn} needed for the calculation of plastic bending resistance of a circular cross-section column. Presented example regards column with 6 reinforcing bars, with none of them lying on the centerline.

```
r = (col-2*t)/2;
dist = r-us-fi/2;
i = 100000;
layer = r/i;
area = 0;
x = 0;
j = 0;
y = 0;
Asn = 0*Abar;

while (0.5*Ac-(area-0.5*Asn))*fcd-4*(r+t/2)*asin(y/(r+t/2))*t*(fyd)-Asn*(fsd) >0
    j = j+1;
    y = (j-0.5)*layer;
    x = (r^2-y^2)^0.5;
    area = area+2*layer*x;
    hn = y;
     if  hn <= (sin(pi/6)*dist-fi/2)
         Wpsn = 0;
         Asn = 0;
     elseif   hn >= (sin(pi/6)*dist-fi/2)   &&   hn < (sin(pi/6)*dist+fi/2)
         rbar = fi/2;
         a = abs(hn-(sin(pi/6)*dist));
            if  hn >= (sin(pi/6)*dist));
                alfa = pi+2*asin(a/rbar);
            elseif  hn < (sin(pi/6)*dist));
                alfa = pi-2*asin(a/rbar);
            end
         Asingle = rbar^2/2*(alfa-sin(alfa));
           % Asingle is the area of the section of the disc
         xc = 4/3*rbar*(sin(alfa/2))^3/(alfa-sin(alfa));
           % xc is distance from the center of A to the axis of the bar
         Wpsn = 4* Asingle * (sin(pi/6)*dist-xc);
         Asn = 4* Asingle;
     elseif   hn >= (sin(pi/6)*dist+fi/2)
         Wpsn = 4* Abar*sin(pi/6)*dist;
         Asn = 4* Abar;
     end
end
```

Talonrakennustekniikan tutkimusraportit v. 1998 – 2010

147 Bzdawka, K., Composite column – calculation examples. TUT 2010. 54 p. 34 €
146 Bzdawka, K., Optimisation of a steel frame building. TUT 2009. 104 p. + 38 app. 34 €.
145 Leivo, V., Ohje uimahallien ja kylpylöiden lattioiden liukkauden ehkäisemiseen. TTY 2009. xx s. + x liites.
144 Leivo, V., Uimahallien laattalattioiden liukkaus. TTY 2009. xx s. + x liites.
143 Vinha, J., Viitanen, H., Lähdesmäki, K., Peuhkuri, R., Ojanen, T., Salminen, K., Paajanen, L., Strander, T. & Iitti, H. Rakennusmateriaalien ja rakenteiden homehtumisriskin laskennallinen arviointi. TTY 2009. xx s. + 2 liites. 42 €
142 Rauhala, J., Kylliäinen, M., Eristerapatun betoniseinän ilmaäänen eristävyys. TTY 2009. 119 s. + 83 liites. 42 €.
141 Aho, H., Korpi, M. (toim.) Ilmanpitävien rakenteiden ja liitosten toteutus asuinrakennuksissa. TTY 2009. 100 s. 42 €.
140 Vinha, J., Korpi, M., Kalamees, T., Jokisalo, J., Eskola, L., Palonen, J., Kurnitski, J., Aho, H., Salminen, M., Salminen, K., Keto, M. Asuinrakennusten ilmanpitävyys, sisäilmasto ja energiatalous. TTY 2009. 148 s. + 19 liites. 42 €.
139 Leivo, V, Rantala, J., Maanvastaisten rakenteiden mikrobiologinen toimivuus. TTY 2006. 57 s. + 55 liites. 34 €.
138 Heinisuo, M., Aalto, A., Stiffening of Steel Skeletons Using Diaphragms. TUT 2006. 31 p. 7 app. 34 €.
137 Kylliäinen, M., Talonrakentamisen akustiikka. TTY 2006. 205 s. 42 €.
136 Varjonen, S., Mattila, J., Lahdensivu, J. & Pentti, M., Conservation and Maintenance of Concrete Facades Technical Possibilities and Restrictions. TUT 2006. 29 p.
135 Heinisuo, M., Ylihärsilä, H., All metal structures at elevated temperatures. TUT 2006. 54 p. 37 app. 34 €.
134 Aho, H., Inha, T., Pentti, M., Paloturvallinen rakentaminen EPS-eristeillä. TTY 2006. 106 s. + 38 liites. 42 €.
133 Haukijärvi, M., Varjonen, S., Pentti, M., Julkisivukorjausten turvallisuus. TTY 2006. 25 s. + 111 liites.
132 Heinisuo, M., Kukkonen, J., Design of Cold-Formed Members Following New EN 1993-1-3. TUT 2005. 41 p. 34 €.
131 Vinha, J., Korpi, M., Kalamees, T., Eskola, L., Palonen, J., Kurnitski, J., Valovirta, I., Mikkilä, A., Jokisalo, J., Puurunkoisten pientalojen kosteus- ja lämpötilaolosuhteet, ilmanvaihto ja ilmatiiviys. TTY 2005. 102 s. + 10 liites. 42 €.
130 Vinha, J., Käkelä, P., Kalamees, T., Valovirta, I. Puurunkoisten ulkoseinärakenteiden lämpö- ja kosteustekninen toiminta diffuusion kannalta tarkasteltuna. 42 € (julkaistaan lähiaikoina)
129 Vinha, J., Valovirta, I., Korpi, M., Mikkilä, A., Käkelä, P. Rakennusmateriaalien rakennusfysikaaliset ominaisuudet lämpötilan ja suhteellisen kosteuden funktiona. TTY 2005. 101 s. + 211 liites. 42 €.
128 Leivo, V., Rantala, J., Lattialämmitetyn alapohjarakenteen rakennusfysikaalinen toiminta. TTY 2005. 140 s. 34 €.
127 Lahdensivu, J., Luonnonkiviverhottujen massiivitiiliseinien vaurioituminen ja korjausperiaatteet. TTY 2003. 156 s. + 9 liites. 34 €.
126 Leivo, V., Hirsirakennuksen yläpohjan tiiviys – vaikutus lämpöenergiankulutukseen. TTY 2003. 63 s.
125 Kylliäinen, M., Uncertainty of impact sound insulation measurements in field. TUT 2003. 63 p. + 50 app. 34 €.
124 Myllylä, P., Lod, T. (toim.), Pitkäikäinen puurakenteinen halli, toimiva kosteustekniikka ja edullinen elinkaari. TTY 2003. 143 s. + 6 liittes. 34 €.

123 Mattila, J., Pentti, M., Suojaustoimien tehokkuus suomalaisissa betonijulkisivuissa ja parvekkeissa. TTY 2004. 69 s. 42 €.
122 Leivo, V., Rantala, J., Moisture Behavior of Slab-on-Ground Structures. TUT 2003. 100 p. + 12 app. 34 €.
121 Leivo, V., Rantala, J., Maanvastaiset alapohjarakenteet – kosteustekninen mitoittaminen ja korjaaminen. TTKK 2002. 33 s. + 11 liites.
120 Leivo, V., Rantala, J., Maanvastaisten alapohjarakenteiden kosteustekninen toimivuus. TTKK 2003. 106 s. + 13 liites. 34 €.
119 Lindberg, R., Wahlman, J., Suonketo, J., Paukku, E., Kosteusvirtatutkimus. TTKK 2002. 92 s. + 3 liites. 34 €.
118 Hietala, J., Kelluvan betonilattian kaareutuminen, osa II. TTY 2003. 58 s. + 12 liites. 30 €.
117 Vinha, J., Käkelä, P., Kalamees, T., Comparison of the Moisture Behaviour of Timber-Framed Wall Structures in a One-Family House. 34 € (julkaistaan lähiaikoina)
116 Vinha, J., Käkelä, P., Kalamees, T., Puurunkoisten seinärakenteiden kosteusteknisen toiminnan vertailu omakotitalossa. TTKK 2002. 54 s. + 11 liites. 34 €.
115 Junttila, T., Venäjän rakennusalan säädöstö ja viranomaishallinto, osa I ja II TTKK 2001. 97 s. 34 €
114 Junttila, T., (toim.) Venäjän rakennusalan tuotekortit. TTKK 2001. 63 s. 34 €
113 Junttila, T., Lod, T., Aro, J., Rakennusinvestointihankkeen toteuttaminen Moskovassa. TTKK 2001. 112 s. + 11 liites. 34 €
112 Junttila, T., (toim.), Venäjän rakentamisen oppikirja.Osa B: Talonrakennustekniikka. TTKK 2001. 174 s. 34 €
111 Junttila, T., (toim.) Venäjän rakentamisen oppikirja. Osa A: Liiketoimintaympäristö ja rakennushankkeen johtaminen. TTKK 2001. 173 s. + 21 liites. 34 €
110 Юнттила, Т. (под ред.), Управление недвижимостью в России. Теория и практические примеры. Технический университет Тампере 2001. 356 стр. + приложения на 33 стр. 34 €
109 Junttila,T., (toim.) Kiinteistöjohtaminen Suomessa ja Venäjällä. Edellytykset kiinteistöalan yhteistyölle. TTKK 2001. 293 s. + 54 liites. 34 €
108 Hietala, J., Kelluvan betonilattian kaareutuminen. TTKK 2001. 80 s. + 7 liites. 34 €
107 Binamu, A., Lindberg, R., The Impact of Air Tightness of The Building Envelope on The Efficiency of Ventilation Systems with Heat Recovery. TTKK 2001. 62 p. + 7 app., 25 €
106 Leivo, V., Rantala, J., Maanvaraisten alapohjarakenteiden kosteuskäyttäytyminen. TTKK 2000. 124 s. 34 €
105 Junttila, T. (toim.), Venäjän federaation kaavoitus- ja rakennuslaki. TTKK 2000. 49 s. 34 €
104 Niemelä, T., Vinha, J., Lindberg, R., Carbon Dioxide Permeability of Cellulose-Insulated Wall Structures. TUT 2000. 46 p. + 9 app. 25 €
103 Vinha, J., Käkelä, P., Water Vapour Transmission in Wall Structures Due to Diffusion and Convection. TUT 1999. 110 s. 34 €
102 Suonketo, J., Pessi, A-M., Pentti, M.,
101 Pessi, A-M., Suonketo, J., Pentti, M., Raunio-Lehtimäki, A. Betonielementtijulkisivujen mikrobiologinen toimivuus. TTKK. 1999. 88 s. + 6 liites. 42 €
100 Pentti, M., Haukijärvi, M., Betonijulkisivujen saumausten suunnittelu ja laadunvarmistus. TTKK 2000. 2. täydennetty painos. 78 s. + 3 liites. 42 €
99 Torikka, K., Hyypöläinen, T., Mattila, J., Lindberg, R., Kosteusvauriokorjausten laadunvarmistus. TTKK 1999. 106 s. + 37 liites. 34 €
98 Mattila, J., Peuhkurinen, T., Lähiökerrostalon lisärakentamishankkeen tekninen esiselvitysmenettely. Korjaus- ja LVIS-tekninen osuus. TTKK 1999. 48 s.
97 Kylliäinen, M., Keronen, A., Lisärakentamisen rakennetekniset mahdollisuudet lähiöiden asuinkerrostaloissa. TTKK 1999. 59 s. + 37 liites. 34 €

96 Vinha, J., Käkelä, P., Vesihöyryn siirtyminen seinärakenteissa diffuusion ja konvektion vaikutuksesta. TTKK 2001. 3 painos. 81 s. + 29 liites. 34 €

95 Leivo, V. (toim.), Opas kosteusongelmiin – rakennustekninen, mikrobiologinen ja lääketieteellinen näkökulma. TTKK 1998. 157 s. 25 €

94 Pentti, M., Hyypöläinen, T., Ulkoseinärakenteiden kosteustekninen suunnittelu. TTKK 1999. 150 s. + 40 liites. 42 €

93 Lepo, K., Laatujärjestelmän kelpoisuus. TTKK 1998. 101 s. + 50 liites.

92 Berg, Malinen, P., Leivo, V., Internal Monitoring of The Technology Programme for Improving Product Development Efficiency in Manufacturing Industries – Rapid Programme. TUT 1998. 81 s. + 93 liites.

91 Berg, P., Salminen, K., Leivo, V., Nopeat tuotantojärjestelmät teknologiaohjelman painoalueet vuosille 1998-2000 sekä ohjelman arviointi- ja ohjaussuunnitelma. TTKK 1998. 55 s. + 37 liites.

90 Lindberg, R., Keränen, H., Teikari, M., Ulkoseinärakenteen vaikutus rakennuksen energiankulutukseen. TTKK 1998. 34 s. + 26 liites.

89 Pentti, M., Huttunen, I., Vepsäläinen, K., Olenius, K., Betonijulkisivujen ja parvekkeiden korjaus. Osa III Korjaushanke. TTKK 1998. 124 s. + 23 liites. 42 €

Tutkimusraportin hinta:
20 € , ellei toisin ole mainittu. Oikeus hinnanmuutoksiin pidätetään.
Hintoihin lisätään alv 8 %.
Myynti:
Juvenes-Yhtiöt Oy, Tampereen teknillisen yliopiston kirjakauppa, Rakennustalo
Korkeakoulunkatu 5, 33720 Tampere
Puh.(03) 3115 2351, faksi (03) 3115 2191, TTY.kirjakauppa@juvenes.fi
tai Tampereen teknillinen yliopisto,Terttu Mäkipää puh. (03) 3115 4804,
terttu.makipaa@tut.fi

Tampereen teknillinen yliopisto
PL 527
33101 Tampere

Tampere University of Technology
P.O.B. 527
FIN-33101 Tampere, Finland