Forecasting the Diffusion of Innovation: A Stochastic Bass Model with Log-Normal and Mean-Reverting Error Process

Juho Kanniainen*, Saku Mäkinen, Robert Piché, Alok Chakrabarti

Abstract

Forecasting the diffusion of innovations plays a major role in managing technology development and in engineering management overall. In this paper, we extend the conventional Bass model stochastically by specifying the error process of sales as log-normal and mean-reverting. Our model satisfies the following reasonable properties that are generally ignored in the existing literature: sales cannot be negative, the error process can have a memory, and sales fluctuate more when they are high and less when they are low. The conventional and widely used model that assumes normally distributed error term does not have these properties. We address how to forecast properly under the log-normal and mean-reverting error process and show analytically and numerically that in our extended model sales forecasts can substantially alter conventional Bass forecasts. We also analyze the model empirically, showing that our extension can improve the accuracy of future sales forecasts.

Keywords: Innovation forecasting; diffusion models; stochastic processes

IEEE TRANSACTIONS ON ENGINEERING MANAGEMENT, VOL. 58, NO. 2, MAY 2011
Digital Object Identifier 10.1109/TEM.2010.2048912

* Corresponding author. Department of Industrial Engineering, Tampere University of Technology, P.O.Box 541, FI-33101 Tampere, Finland. Tel: +358-40-707-4532. Fax: +358-3-3115-2027 . Email: juho.kanniainen@tut.fi

I. Introduction

Changing expectations of future sales has become a common theme in the news lately during the current global financial crisis. This reflects the fact that uncertainty plays a key role in the adoption of innovations, a fact that holds in all industry sectors. Future sales are stochastic because of unexpected changes in several factors such as economic and financial conditions, technological improvements, and competition over market share.

Absolute accuracy in forecasting the future demand of current innovations even in the short term is, of course, unattainable. This, however, does not lessen the importance and relevance of forecasting, because the point is to make the best possible forecasts. Since the work of Bass [1], many models have been introduced in the literature, and widely applied, to explain and predict product diffusions (see, for example Meade and Islam [2] and Linton [3], and references therein). However, most of these studies have focused on model selection without much attention to the properties of the model's error process. For example, most studies assume that the error process of sales volume is normally distributed, leading to possible negative sales expectations. Moreover, it would be reasonable to assume that unexpected changes in the sales volume are on average greater the greater the sales volume, or, in other words, that the sales volume is a heteroscedastic variable. Yet the sales volume is typically assumed to be homoscedastic.

From the engineering management point of view, innovation forecasting presents multiple possibilities and challenges as well. For example traditional technology road mapping [4] and product characteristics' influence on portfolio management [5] heavily rely on views of expected future diffusion of innovations and market evolution. The roadmaps further define the actual technology development and engineering effort of a company. Similarly, operations and production engineering are dependent on the company's ability to forecast future demand of its offering. However, innovation forecasting as a task is subject to uncertainties both endogenous and exogenous to the company. For example, the recent economic turbulence has created shocks on the demand of innovations and also directly to organizations' operations. Therefore, an innovation forecast model should explicitly consider the error processes resulting from volatility in order to improve forecasting accuracy.

This paper shows how to forecast future sales volumes under a stochastic version of the Bass model assuming a log-normal and mean-reverting error process. To start with, we assume that the stochastic evolution of log-sales, rather than sales, is normally distributed in time and possibly mean-reverting. In particular, we assume that the sales volume is driven by the Bass model such that the logarithmic sales volume is the sum of the logarithm value of the Bass model and a normally distributed error process, which can be mean-reverting. In contrast to the assumption of normally distributed sales, we consider the log-normal assumption plausible, because sales cannot be negative and variance in the sales volume can depend on the level of the volume. Our model can thus capture heteroscedastic errors. Moreover, our assumption of mean reversion implies that the error process can be auto-correlated. The paper aims to provide better forecasts in a situation when the underlying data or theory suggest the sales are log-normal and mean-reverting. To examine the validity of the assumptions of log-normality and mean reversion with respect to empirical data, one can estimate the diffusion model in both the conventional way and as suggested in this study. In the empirical part of this paper, we find that our log-normal and mean-reverting model gives a better out-of-sample forecasting accuracy with telecommunication data. Though we focus here on the Bass model, which has dominated the forecasting literature (see [3]), the lessons learned are likely to apply to non-Bass diffusion models as well, such as any deterministic extension of the Bass model, see, for example, [6], or other non-Bass deterministic diffusion models, such as the Gompertz model (see, e.g., [2], [7], [8] for the survey of various models).

Some stochastic diffusion models have been developed to capture the "econometrics" of diffusion dynamics. Skiadas and Giovanis [9] introduced a stochastic version of the Bass model but with a very different starting point and without mean reversion. Recently, Boskwijk and Franses [10] have presented a model that satisfies non-negativeness, heteroscedasticity, and mean reversion. However, their model is characterized differently from ours: they assume that the sales volume follows a square root process, such as that of Cox, Ingresoll, and Ross [11] for interest rates; however, ours is a log-normal characterization. Our approach has the advantage of an analytical solution to the error process that can be subsequently used in future studies. Moreover, in comparison to [10], our paper focuses on forecasting with an extensive out-of-sample analysis. Furthermore, also the Gompertz model embraces some stochastic generalizations (see, for example, [12], [13]). The log-normal error process of the sales volume has previously been assumed in the context of some models (see, for example, [14], [15], and [16]). However, there is little research focused on forecasting under the log-normal error process.

This paper shows that our settings impact forecasting in two important ways: first, Bass's expectations must be adjusted because of log-normality and mean reversion; second, as time goes by, expectations of future sales must be further adjusted when observed sales do not equal the value of the Bass model. The adjustments depend on the speed of the mean reversion and the volatility of the sales, meaning that it is not enough to estimate the Bass parameters m, p, and q, but that to forecast future sales, also the speed of the mean reversion and volatility must be estimated. As we show, these two parameters may, indeed, play a key role in sales forecasting. Our stochastic specification yields analytical expressions for conditional expectations and variances of future sales, thus constituting an important property in forecasting and risk-management. The contribution of this paper is twofold. The primary contribution is methodological; we show how the conventional expectations must be adjusted under log-normal and mean-reverting error process. The secondary contribution is empirical. We examine six time series representing analog mobile telephone adoption in different countries and carry out 540 estimations to have an extensive out-of-sample analysis. Third, we discuss the role and meaning of mean reversion in innovation forecasting.

The paper is organized as follows. The following section introduces our model in continuous time. Section 3 discusses and illustrates the theory of forecasting in terms of our settings and concretizes the roles of the speed of mean reversion and volatility in forecasting. Section 4, starting with discretization of the continuous-time process, gives an empirical demonstration. We compare our model with two alternative error models, using data for analog mobile phones in six countries to test our improvements in underlying forecasting methodology. Finally, we present our conclusions and discuss further research.

II. Model

Our model can be seen as a stochastic extension of the Bass [1] model. Basically, we add mean-reverting noise to the logarithmic forecast of the Bass model. Specifically, we assume that future sales, S_t, can be characterized by a continuous time process represented logarithmically as the sum of two components – the logarithm of deterministic function of time, g_t, and a stochastic diffusion, X_t; that is,

$$\ln S_t = \ln g_t + X_t. \tag{1}$$

Here g_t is the sales volume forecast of the Bass model. It is important to note that g_t could be any deterministic extension of the Bass model, such as [6], or other non-Bass deterministic diffusion models. In this characterization, we require that for $T > t$

$$\mathbb{E}_t[\ln S_T | X_t = 0] = \ln g_t, \tag{2}$$

which means that if the stochastic term X_t is zero, then the best forecast at time t of logarithmic sales at time T is the logarithmic forecast of a deterministic model (in this case, the Bass model). Later, we consider forecasts of non-logarithmic sales with non-zero stochastic terms in our model, but (2) serves as our basic assumption for further analysis.

First, let us look at the formulation of the Bass [1] model. It assumes that, $F(t)$, the fraction of individuals adopting the product by time t, evolves as

$$\frac{dF}{dt} = (p + qF)(1 - F),$$

where p and q are constant real numbers, called the coefficients of innovation and imitation, respectively. Bass [1] gives the closed form solution of F as

$$F(s;p,q) = \frac{1 - e^{-(p+q)s}}{(q/p)e^{-(p+q)s} + 1},$$

where time $s > 0$. By denoting $f = dF/dt$, the sales at time $s > 0$, $g_s = g(s;m,p,q)$ is given by

$$\begin{aligned} g(s;m,p,q) &= mf(s;p,q) \\ &= \frac{m(p+q)^2 e^{-(p+q)s} p}{\left(p + qe^{-(p+q)s}\right)^2}, \end{aligned} \tag{3}$$

where m is the total number of adopters.[1] As mentioned before, in our model, $\ln g_s$ provides the best logarithmic sales forecast for time s when the stochastic term is zero.

Second, let us characterize the stochastic component of our model. We assume that $\{X_t; t \geq 0\}$ follows the Ornstein-Uhlenbeck process:

$$dX_t = -\kappa X_t dt + \sigma dW_t, \quad X_0 = x, \tag{4}$$

where $\kappa \geq 0$ is the speed of the mean reversion, and dW_t represents an increment to a Wiener process. By applying Ito's lemma with (1) and (4), we can write the sales process as

$$dS_t = \kappa(Y_t - \ln S_t)S_t dt + \sigma S_t dW_t, \quad S_0 > 0, \tag{5}$$

where

$$Y_t = \frac{1}{\kappa}\left(\frac{\sigma^2}{2} + \frac{d\ln g}{dt}(t;m,p,q)\right) + \ln g_t.$$

Notice that, under our model, $S_t > 0$ for all $t > 0$, satisfying then the condition of positive sales. Such a mean-reversion concept was earlier used to model interest rates and commodity prices (see, for example, [17], [18]). Our sales model can also be seen as an extension of Schwartz's model of commodity prices [18]. In particular, we assume that the equilibrium level of log-sales, Y_t, is time-varying, whereas in [18] it is constant. Moreover, our model is quite analogous to that proposed by Lucia and Schwartz [19] for modeling electricity prices.[2]

To understand the model given in Eq. (5), notice that when $\ln S_t > Y_t$, the drift term of the sales process is negative, resulting in a pull down toward the instantaneous equilibrium level or "instantaneous normal level," and that if $\ln S_t < Y_t$, the drift term is positive, pulling up to the higher equilibrium value. This feature causes the sales process to oscillate around the time-varying instantaneous equilibrium levels driven by the Bass model. The Wiener process continually pushes the sales volume away from the instantaneous equilibrium level, and the mean-reverting term pushes the sales volume back to the equilibrium level. The greater the speed of the mean reversion, κ, the greater the pull back to the instantaneous equilibrium level.

To calibrate our model, in addition to estimating the Bass parameters m, p, and q for forecasts of future sales, we must also estimate the speed of the reversion, κ, and volatility, σ. To see how these parameters can be estimated, let us write our continuous time model (model A) in discrete form. Equation 4 takes the form

$$X_t = \psi X_{t-1} + u_t^A,$$

where $\psi := (1 - \kappa\Delta t)$, and u^A is i.i.d. normal random variable with mean zero and variance $\sigma^2\Delta t$. Thus

$$\begin{aligned} \ln S_t &= \ln g(t;m,p,q) + X_t, \\ X_t &= \psi X_{t-1} + u_t^A, \end{aligned}$$

[1]We assume, following most of existing literature, that each adopter buys only one unit.

[2]Lucia and Schwartz [19] incorporate periodic behavior in the deterministic function via trigonometric functions to reflect the general seasonal pattern of electricity price.

where $t = 1, 2, \ldots, n$, which can also be expressed as

$$S_t^* = \psi S_{t-1}^* + g_t^* - \psi g_{t-1}^* + u_t^A, \tag{6}$$

where $S_t^* := \ln S_t$ and $g_t^* := \ln g(t; m, p, q)$. Parameters ψ, m, p, and q can be estimated simultaneously using a non-linear least squares (NLS) approach. We take $\tilde{\kappa} = (1 - \tilde{\psi})/\Delta t$ as the estimate of the speed of reversion. In addition, the above expression implies that the standard error of regression can be taken as the estimate of $\sigma\sqrt{\Delta t}$.

In addition to our log-normal and mean-reverting error model we consider two alternative error models. We call our error model (5) model A and the others B and C. The alternative error models are based on the error model proposed by Srinivasan and Mason [20] that with $\Delta t = 1$ has a normally distributed error process without mean reversion:

$$S_t = m(F(t) - F(t-1)) + u_t, \tag{7}$$

where u_t is normally distributed with a positive variance. As the authors note, this can lead to negative sales. Moreover, this characterization ignores heteroscedasticity and under this model, the forecasts of future sales will not converge to the Bass curve. In order to carry out comparisons, we use *continuous time* versions of their model. Specifically, instead of (7), we write model B as

$$S_t = g_t + u_t^B, \tag{8}$$

where u_t^B is a zero-mean error term with variance σ_B^2. Clearly, the expected sales under model B is

$$\mathbb{E}_t S_T = g_T.$$

Thus, according to model B, the expected observable sales level equals the Bass curve g_T regardless of the current observed sales, which means that the observed sales are expected *immediately and discontinuously* to jump to the Bass curve. Assuming a fixed set of parameters, the prediction under error term B is the same whether the current observed sales is, for example, 100 or 200. That is because the sales observations affect the prediction only via the estimation procedure. Another drawback is that the variance of future sales,

$$\mathrm{Var} S_T = \sigma_B^2,$$

is independent of the length of the forecasting period, $T - t$. This means that the future sales level for the next minute is as uncertain as for the next decade.

Model C has the form

$$S_t = g_t + Z_t, \tag{9}$$

where

$$dZ_t = \sigma_C dW_t.$$

In discrete time, model C can be written as

$$S_t - S_{t-1} = g_t - g_{t-1} + u_t^C, \tag{10}$$

where u_t^C is a zero-mean error term with variance $\sigma_C^2 \Delta t$. The expected sales volume under model C is

$$\mathbb{E}_t S_T = g_T + S_t - g_t.$$

Error model C adjusts the expected sales by the difference of the current observed sales and Bass's prediction, $S_t - g_t$, and hence the current sales observation has a direct effect on the prediction. In addition, the variance of future sales,

$$\mathrm{Var}_t S_T = (T - t)\sigma_C^2,$$

is directly proportional to the length of the forecasting period constituting the "cone of uncertainty", as is true also for model A.

Clearly, both conventional error models, B and C, can give negative simulated values. Specifically, model B assumes that the sales level is normally distributed with the mean of g_t and model C assumes that the absolute change of the sales is normally distributed with the mean of g_t. In the case of model C, the expected sales can become negative if the observed sales volume is less than originally predicted by the Bass model, in which case $g_t - S_t$ can be greater than g_T.

In our model the mean reversion term makes the error process autoregressive, that is, roughly speaking, the next realization is dependent on the current one, or in other words, that the sequential changes are correlated. Why could the error structure of the sales process be autoregressive? First, we can say that empirically the error process can be autoregressive (see [21] and [10] and the empirical part of this study). The current literature of the diffusion of innovations has little or no discussion of the mean reversion in the error process, so we now proceed to a more detailed elucidation of it.

Most importantly, as we will demonstrate, if we make conventional assumptions about normally distributed changes in sales without mean reversion (model C), then the forecasts of the future sales will not converge to the Bass curve. This means that in the long term, future sales are expected to be negative if the current realized sales volume is less than forecasted by the Bass model for the current time. Similarly, under these assumptions about normally distributed changes in sales, the future sales volume is expected to converge to a strictly positive constant if the current realized sales volume is greater than forecasted by the Bass model. Obviously, it is unreasonable to expect that, in the long term, the sales volume will not converge to zero, or at least we can expect the sales to be non-negative. On the other hand, as we will see, if we employ the assumption of normally distributed sales (model B), rather than normally distributed changes in sales, we (implicitly) assume that the observed sales volume is expected immediately and discontinuously to converge to the Bass curve. This too is somewhat unrealistic because under these assumptions the expected sales volumes are not directly dependent on the current observed sales level. If the error process is mean-reverting, future sales are expected to converge to the Bass curve in the long term, and the above problems are avoided. The mean reversion plays an important role because if we ignore it by assuming that the changes in sales are i.i.d. normally distributed (model C), the expected sales level never converges to the Bass curve. Alternatively, if we suppose that the sales are normally distributed with i.i.d. error term (model B) without allowing an arbitrary mean reversion, the expected sales jumps immediately to the Bass curve, an unrealistic situation as well.

It must be noted here that the assumption of log-normal error process ensures that sales volumes remain non-negative, and if we assume both mean reversion and log-normality, forecasts have plausible properties regarding the convergence of sales to the Bass curve. One consequence of the mean reverting log-normal error process is that the greater κ, the easier it is to forecast the future sales. This is illustrated in Figure 1, which shows ten possible sales trajectories generated with our model using Monte Carlo simulation methods. The figure also plots the Bass curve, and we can see that the possible sales trajectories oscillate around it. In plot (a), the speed of the mean reversion $\kappa = 0$, which means that the stochastic variable x does not tend toward zero; in other words, the drift term of the sales process does not act against stochastic shocks, and thus the sales volume will not return to the Bass curve when the sales volume is pushed randomly away from the equilibrium. In plot (b), the stochastic variable x is strongly mean-reverting; consequently, the sales volume oscillates nearer the Bass curve.

Notice from Figure 1 how our model A captures the heteroscedasticity, too: the greater the sales volume, the greater its variance. Why should the instantaneous variance of the sales volume (expressed in units) be dependent on the sales volume? Consider a situation in which the "long-term mean" of the sales volume, i.e. the value of g_t, is very low, say close to zero (remember that according to the Bass model, sales eventually approach zero). Then the variance of the sales (in units) must be infinitesimal because sales are non-negative. That is, if the sales level is close to zero, it cannot fluctuate much around its mean. Otherwise the sales would have to be able to be negative. The greater the mean of the sales is, the more the sales can vary. This phenomenon can well be observed from our data. Figure 2 plots the monthly telecommunication data from the USA that we use in the empirical part of this study (along with data from other countries). Note that this property can also be captured with multiplicative error structures.

Fig. 1. Sample paths of sales volume with parameters $x_0 = 0; p = 0.01; q = 0.8; m = 100000; \sigma = 0.35; \Delta t = 1/52$. In plot (a) the stochastic term is non-mean-reverting with $\kappa = 0$, whereas in plot (b) mean reversion is strong with $\kappa = 1$.

Fig. 2. The heteroscedasticity of the sales. Time series represents monthly analog mobile telephone adoption in the USA.

III. Forecasting

What may be surprising is that according to our error process characterization, the Bass model does not provide forecast of future sales, because, on one hand, the sales process is log-normally distributed, and, on the other hand, the error process is mean-reverting. Figure 3 illustrates this fact by showing probability distributions and expected values of future sales volumes together with the Bass curve for different forecasting periods with $x_0 = 0$. Note how the expected sales volumes do not lie on the Bass curve, especially if mean reversion is low ($\kappa = 0$). With the figure, we can postulate that Bass forecasts must be adjusted more when the mean reversion is smaller and the volatility is greater. This section examines mainly how to adjust the Bass model to obtain non-skewed forecasts.

To put this formally, let us address the expected level and variance of future sales according to our model. The logarithmic sales volume, $\ln S_T$, $T > t$, is normally distributed with conditional mean and variance

$$\begin{aligned}
\mathbb{E}_t \ln S_T &= \ln g_T + X_t e^{-\kappa(T-t)} \\
&= \ln g_T + (\ln S_t - \ln g_t) e^{-\kappa(T-t)} \\
\mathrm{Var}_t \ln S_T &= \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa(T-t)}\right).
\end{aligned}$$

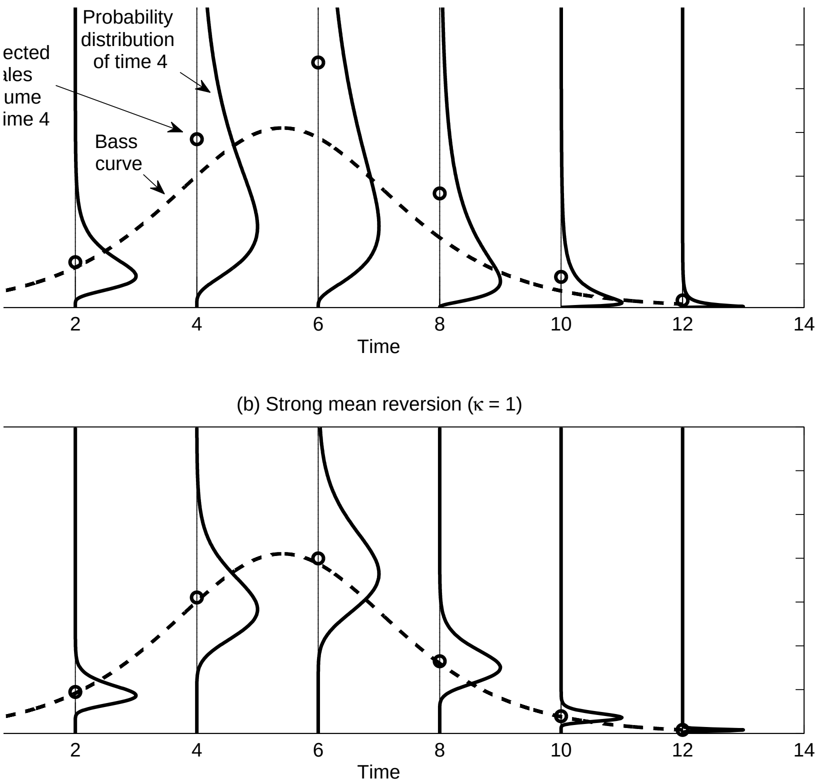

Fig. 3. Probability distributions of future sales volumes, expected values of sales volumes, and the Bass curve. Parameter values are $x_0 = 0; p = 0.01; q = 0.8; m = 100000; \sigma = 0.35$. The Bass curve is a dashed line and expected values of future sales volume by the symbol 'o'. Note that for graphical purposes the distributions are scaled to have equal amplitudes.

Because log sales are normally distributed, sales are log-normally distributed with a conditional mean

$$\begin{aligned} \mathbb{E}_t S_T &= \exp\left(\mathbb{E}_t \ln S_T + \frac{1}{2}\mathrm{Var}_t \ln S_T\right) \\ &= \exp\left(\ln g_T + (\ln S_t - \ln g_t)e^{-\kappa(T-t)} + \frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa(T-t)}\right)\right) \\ &= g_T b_1(t,T) b_2(t,T), \end{aligned} \tag{11}$$

where

$$b_1(t,T) = \exp\left((\ln S_t - \ln g_t)e^{-\kappa(T-t)}\right)$$

and

$$b_2(t,T) = \exp\left(\frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa(T-t)}\right)\right).$$

The above implies that if the actual sales volume differs from the Bass value at the current time, then also expected future sales volumes differ from the forecasts of the Bass model, which is intuitively reasonable. What is more interesting is that even if $S_t = g_t$, that is, even if the error process were zero at the current time t, the expected value of future sales volume S_T would still be affected by the parameters of the stochastic diffusions, σ and κ. Thus there are two reasons why the pure Bass model, or any other deterministic model, cannot be used to predict future sales if the sales process is assumed log-normal with a possible mean reversion component. First, as time goes by, and as the Bass model does not fully explain current observed sales, the Bass forecast at time t of the sales volume at a future time T, $T > t$, must be adjusted by the factor

$$b_1 = \exp\left((\ln S_t - \ln g_t)e^{-\kappa(T-t)}\right).$$

If the current sales volume, S_t, is greater (resp. less) than the Bass value, g_t, then b_1 is greater (resp. less) than one and decreases (resp. increases) with respect to T. Note that if $\kappa > 0$, then $b_1 \to 1$ as $T \to \infty$.

Second, the log-normal property of the sales volume together with mean reversion imply that the Bass model, or any other deterministic model, does not fully predict future sales, even when the current stochastic term is zero, that is, $x_t = 0$. In particular, Bass forecasts of S_T must be further adjusted by the factor

$$b_2 = \exp\left(\frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa(T-t)}\right)\right).$$

This adjustment takes place even if the current sales volume equals the value of the Bass model, which is the case if, for example, the product has not yet been launched. Note that for $\kappa > 0$, $b_2 > 1$; consequently, the adjustment makes the expectations greater than the values implied by the Bass model. However, for a strongly mean-reverting error process, $b_2 \approx 1$. To see this, note that $b_2 \to 1$ as $\kappa \to \infty$. This is also intuitively clear, because if the stochastic variable is strongly mean-reverting, the stochastic term is negligible. Note also that the greater the time interval, $T - t$, the greater the adjustment factor b_2. As $T \to \infty$, $b_2 \to \exp\left(\sigma^2/(4\kappa)\right)$. Moreover, the diffusion coefficient, σ, increases b_2 and thus also the expected future sales. Therefore, the greater the uncertainty in future sales, the greater the expected level of future sales at any future time instant.

For $\kappa > 0$, $\mathbb{E}_t \ln S_T = \ln g_T = -\infty$ and $\mathbb{E}_t S_T = g_T = 0$ as $T \to \infty$. This means that if the error-term is *both* log-normal and mean-reverting, the expected future sales volume converges to the Bass curve in the long term.

However, the case of $\kappa = 0$ (no mean reversion) is more complex. It is true that also in this case $\mathbb{E}_t \ln S_T = \ln g_T = -\infty$; however, this does not necessarily imply that for $\kappa = 0$, $\mathbb{E}_t S_T = 0$ as $T \to \infty$. In particular, for $\kappa = 0$ and $T \to \infty$, $\mathbb{E}_t S_T = \infty$ if $p + q < \frac{1}{2}\sigma^2$ and $\mathbb{E}_t S_T = 0$ if $p + q \geq \frac{1}{2}\sigma^2$. To see this, note that if $\kappa = 0$, then $b_1 = S_t/g_t$ and $b_2 = \exp\left(\frac{1}{2}\sigma^2(T-t)\right)$, and the expectation takes the form

$$\mathbb{E}_t S_T = \frac{m(p+q)^2 \exp\left(-(p+q-\frac{1}{2}\sigma^2)T - \frac{1}{2}\sigma^2 t\right) p}{\left(p + qe^{-(p+q)T}\right)^2} \frac{S_t}{g_t}.$$

Violating the condition of $\lim_{T\to\infty} \mathbb{E}_t S_T = 0$, however, suggests a somewhat unrealistic situation for two reasons. First, no matter how small the speed of the mean reversion, κ, if it is strictly positive, the expected value of future sales eventually converges to the Bass curve. Second, many empirical studies have shown that in annual terms, $p + q$ is typically over 0.5. This would mean an extremely high annual volatility.

Conditional variance can be obtained from

$$\begin{aligned}
\mathrm{Var}_t S_T &= \{\exp(\mathrm{Var}_t \ln S_T) - 1\} \exp\left(2\mathbb{E}_t \ln S_T + \mathrm{Var}_t \ln S_T\right) \\
&= \{\exp(\mathrm{Var}_t \ln S_T) - 1\} \left(\mathbb{E}_t S_T\right)^2 \\
&= \left(b_2^2 - 1\right) b_1^2 b_2^2 g_T^2.
\end{aligned}$$

Note that for $\kappa > 0$, $\mathrm{Var}_t S_T = 0$ as $T \to \infty$, because $b_1 = S_t/g_t$ and $b_2 = \exp\left(\sigma^2/(4\kappa)\right)$ as $T \to \infty$. Moreover, it is quite straightforward to show that for $\kappa = 0$, $\lim_{T\to\infty} \mathrm{Var}_t S_T = 0$ if $p + q \geq \sigma^2$. Thus even though $\mathrm{Var}_t \ln S_T$ is increasing for all T, $\mathrm{Var}_t S_T$ can be decreasing, especially for large T.

The above arguments can be seen to hold true in Figure 1 (a) and Figure 3 (a), where both the mean and variance visibly decrease after seven years. They both eventually decline to zero, even if $\kappa = 0$. If, however, the volatility σ were high enough (that is, very large), the mean and the variance would increase without bound.

Forecasting with our model is illustrated in Figure 4. The stochastic path of the sales volume is simulated with Eq. (5) up to the fourth year. Then by using the true parameter values (as used in the simulation), future sales volumes are forecast by taking the conditional means using Eq. (11). In plot (a), the actual sales volume at time 4 is above the Bass curve; consequently, the expected future sales volumes must be adjusted upward from the Bass curve. The forecast of future sales under model A converges to the Bass curve quite fast (note that $\kappa = 1$), whereas the error model C implies that the forecast of the sales never meet the Bass curve, and hence, model C implies that in this case, the future sales are expected to remain substantially positive forever. Note that according to the other conventional error model B, the expected sales lie on the Bass curve and the sales are expected to *jump* to the Bass curve immediately from the current observed level. That is, as the observed sales level at time 4 is about 20320, the expected sales level after one day (at time 4.0027) is the Bass value at that time, 15070. This would mean -25% expected change in *one day*.

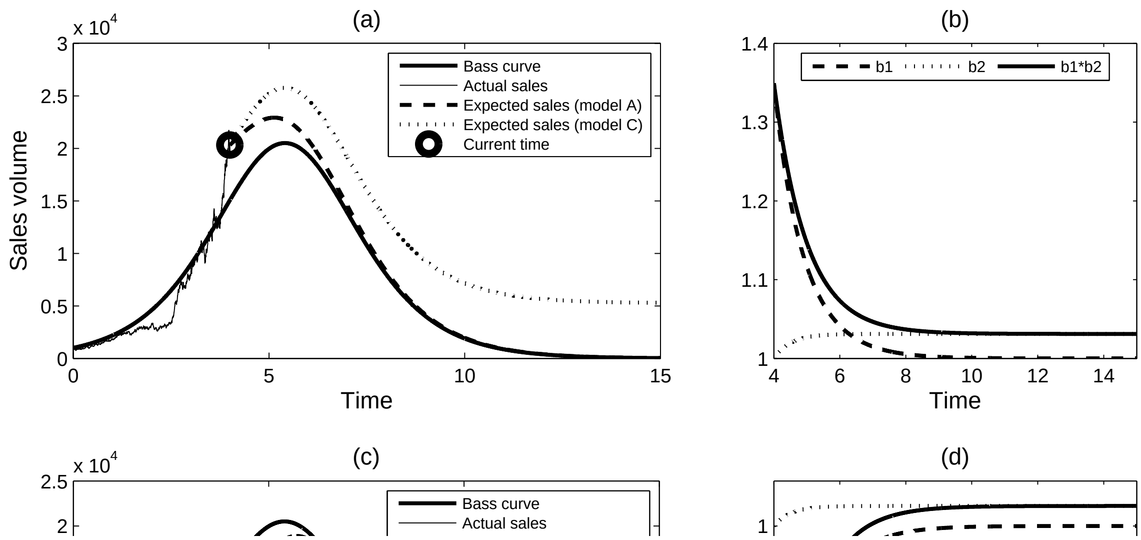

Fig. 4. Illustration of the adjustment of expectation of future sales volume. The product was launched four years ago, and the sales volume has evolved stochastically over the past four years. The figure shows how the adjusted expected values differ from the Bass curve. Plot (b) shows the adjustment factors b_1 and b_2 of plot (a), and plot (c) shows the factors of plot (c). Parameter values are $x_0 = 0; p = 0.01; q = 0.8; m = 100000; \kappa = 1; \sigma = 0.35; \Delta t = 1/365$.

Plot (b) shows the adjustment factors b_1 and b_2 related to plot (a). Both are now greater than one, which is intuitive. Moreover, b_1 is decreasing and b_2 increasing, which is in line with our above analysis. Their product, $b_1 b_2$ is decreasing. Plot (c) shows a situation in which the observed sales volume at time 4 is less than originally predicted with the Bass model. In this case, expected sales volumes lie below the Bass curve.[3] A great problem with the conventional error-term (model C) here is that the forecasts of future sales become negative at time 8.4 and finally the forecast is substantially negative. This happens due to

[3]To be precise, expected sales volumes lie below the Bass curve for a low T, but, as plot (d) shows, $b_1 b_2$ is greater than one for large T. This, however, has no great absolute effect on expected sales for large T, because the Bass model eventually drives the sales down.

TABLE I
THE MAIN PROPERTIES OF THE DATA SET. THE LENGTH OF THE TIME INTERVAL BETWEEN THE OBSERVATIONS IS THREE MONTHS.

	Japan	**Finland**	**UK**	**US**	**Australia**	**Korea**
First time point	Dec 1981	Mar 1982	Mar 1985	Sep 1986	Mar 1987	Mar 1984
Last time point	Jun 1996	Dec 1995	Dec 1995	Mar 1998	Dec 1996	Sep 1996
Num of obs.	59	56	44	47	40	51

the conventional assumption of normally distributed sales. Note that in the case of (c), both b_1 and b_2 are increasing (see plot d). If κ were greater (resp. lesser), expected sales would more quickly (resp. more slowly) converge to the Bass curve.

IV. Empirical Analysis

This section aims to demonstrate empirical estimation procedures and forecasting and it examines the usefulness of our error-term characterization. We estimate models A, B, and C using time series data for analogue mobile phones in six countries using NLS approach.[4] To present an extensive out-of-sample test, we estimate the three models 30 times for country. In particular, the models are estimated at the 30 last time series points using the data up to these times, i.e. the estimates were obtained using the data from the first observation to the observation of $n-29$, $n-28$,...,n, where n is the number of the observation. Thus, we estimate the three models 180 times, and altogether calculate 540 model estimations. In fact, this empirical analysis is among the most extensive out-of-sample analysis we have encountered in the literature.

Our data consisted of six countries selected to represents different cultural, economic, social and legal environments. The sources for our data were International Telecommunication Union (ITU), UN (United Nations), and OECD (Organisation for Economic Co-operation and Development), equipment vendors, telecommunication operators and trade journals and magazines like Telecommunications, Communications International, Cellular Business etc. Multiple data sources were used to validate our original data set of quarterly data and to additionally ensure its reliability. We further calculated the sales figures from the original adoption time series in order to have at least 40 consecutive positive sales time series points. We have attempted to validate our approach with empirical analysis that considers the merits of our approach as thoroughly as possible in one technology area, rather than only select one or two time series from different technological areas in order to increase generalizability of our results. Further, for practical reasons we had to limit our empirical testing to six countries due to the sheer number of estimations. Therefore, we settled on two countries from EU, namely UK and Finland, USA, and three countries from APAC-region, namely, Australia, Japan, and Korea. Table I summarizes our data set.

We estimate model A according to Eq. (6), and we estimate also the initial error, X_1, which can be positive or negative. Thus, the parameter set of model A is $\{p, q, m, \psi, \sigma, X_1\}$. Models B and C can be estimated using equations (8) and (9). We estimate the initial error, X_1, also for model C.

Tables V – XIV (Appendix) summarize NLS estimates for models A, B, and C for all the six countries. The first row presents the estimates at the first time point and the second row presents the standard error of the coefficients, and so on. The parameters are presented in terms of three-month periods, but the parameters can be easily converted to annual levels.[5] We observe that the data is strongly mean-reverting: On average (and in terms of three-month periods), ψ is between 0.147 (Korea) and 0.352 (Australia), depending on the country. This means that the annual value of κ is, on average, between 2.59 and 3.43,

[4]Schmittlein and Mahajan [22] proposed a maximum likelihood estimation (MLE) approach that would yield better forecasting accuracy and more stable parameter estimates than the ordinary least squares. Srinivasan and Mason [20] proposed the NLS approach, because the MLE approach underestimates standard errors of the estimated parameters.

[5]The conversion formulas are: $p_a = p_q/\Delta t; q_a = q_q/\Delta t; m_a = m_q\Delta t; \sigma_a = \sigma_q/\sqrt{\Delta t}; \kappa_a = \kappa_q/\Delta t = (1-\psi_q)/\Delta t$, where $\Delta t = 1/4$, and where x_a represents parameter annual value and x_q parameter value in the terms of three months (quarter). Note that for model B, $\sigma_a^B = \sigma_q^B$.

which represents a very high level of mean reversion. It is important to note here that these estimation results support the assumption made about the mean-reverting error process. If we ignored the property of mean reversion in the error process (that is, if we assumed that $\kappa = 0$), our forecasts would be worse. Naturally, κ and σ are not shown for model B and C because these parameters do not appear in these models.

We perform our analysis as follows (let us take UK data as an example): First, we estimate models A, B, and C using the in-sample-data $(\hat{S}_1, \hat{S}_2, \ldots, \tilde{S}_{n-i})$, at time points $n-i$, $i = 0, 1, \ldots, n-1$, where $\hat{S}_i$ denotes the observed adoption at ith time point. That is, for UK (see Table VII) we estimate the three models 30 times using the sets of observations of $\{S_1, S_2, \ldots, S_{14}\}$, $\{S_1, S_2, \ldots, S_{15}\}, \ldots,$ $\{S_1, S_2, \ldots, S_{44}\}$. Then we calculate the forecasting errors as squared errors for each model:

$$\begin{aligned}
\mathbf{SE}^A_{n-i,j} &= \left(\hat{S}_{n-i+j} - \mathbb{E}^A_{n-i}\left(S_{n-i+j}\right)\right)^2 = \left(\hat{S}_{n-i+j} - g^A_{n-i+j} b_1(n-i, n-i+j) b_2(n-i, n-i+j)\right)^2, \\
\mathbf{SE}^B_{n-i,j} &= \left(\hat{S}_{n-i+j} - \mathbb{E}^B_{n-i}\left(S_{n-i+j}\right)\right)^2 = \left(\hat{S}_{n-i+j} - g^B_{n-i+j}\right)^2, \\
\mathbf{SE}^C_{n-i,j} &= \left(\hat{S}_{n-i+j} - \mathbb{E}^C_{n-i}\left(S_{n-i+j}\right)\right)^2 = \left(\hat{S}_{n-i+j} - \left(g^C_{n-i+j} + \hat{S}_{n-i} - g^C_{n-i}\right)\right)^2,
\end{aligned}$$

where for $n = 30$, $i = 1, 2, \ldots, n-1$ and $j = 1, 2, \ldots, i$, $\hat{S}_{n-i+j}$ is the observation at time $n-i+j$, and $\mathbb{E}_{n-i}\left(S_{n-i+j}\right)$ is the forecast of sales level at time $n-i+j$ based on the information up to time $n-i$.[6] We emphasize here that only past data is used such that the end of fitting equals the start of forecasting. Moreover, g^A is the Bass model with paramaters estimated with error model A, and similarly g^B and g^C are the Bass models whose parameters are estimated with error models B and C, respectively. The forecasting error matrices are too large to be presented in the paper, but they are available upon request.[7] After that, we take the mean squared errors for i step predictions, $i = 1, 2, \ldots, n-1$:

$$\mathbf{MSE}_i = \sum_{j=1}^{i} \mathbf{SE}_{n-i,j}$$

Thus, $\mathbf{MSE}_i$ is the average forecasting error when predicting i periods ahead. These average forecasting error vectors (1×29) are calculated for each model and each country, making altogether 18 vectors. The results are summarized in Table II, which shows the ratios of $\mathbf{MSE}^B_i / \mathbf{MSE}^A_i$ and $\mathbf{MSE}^C_i / \mathbf{MSE}^A_i$ for $i = 1, 2, \ldots, n-1$ period forecasts. The summary of the results comparing MSEs are shown in Table II.

We observe that model A outperforms models B and C in the most of the cases. Specifically, for Japanese data, in 22 of the 29 cases model A yields more accurate forecasts than model B and in 29 of the 29 cases (i.e. in every case) model A outperforms model C. The corresponding numbers for the other countries are (first A/B and then A/C): Finnish 26/29, 19/29; UK 26/29, 20/29; US 26/29, 18/29; Australian 25/29, 15/29; and Korean 16/29, 29/29. Thus, with respect to the number of ratios that are more than one, for all the countries model A outperforms the alternative models. Japanese data seems to favor model A the most. The worst for model A results were obtained with Australian data in comparison to model C and Korean data in comparison to model B. When we take averages over the ratios of MSEs according to the countries, i.e., we calculate

$$\sum_{i=1}^{n-1} \frac{\mathbf{MSE}^B_i}{\mathbf{MSE}^A_i} \quad \text{and} \quad \sum_{i=1}^{n-1} \frac{\mathbf{MSE}^C_i}{\mathbf{MSE}^A_i}$$

for each country, we see that the average ratios are clearly greater than one for each country (see the second last row of Table II). On the other hand, if we take the average over countries for given values of i, $i = 1, 2, \ldots, n-1$, we again see that for almost each forecasting period length, i, model A yields

[6]To be more specific, the expectation $\mathbb{E}_{n-i}(.)$ is based on parameters values that are estimated using the observations $1, 2, \ldots, n-i$.

[7]We have 18 30-by-30 matrices, that is three for each six country.

more accurate forecasts than the other models. Specifically, the average ratio of models C to model A (with respect to countries) is always greater than one, and the ratio of the models B to A is less than one only for $i = 2$. Overall, our results confirm that the assumptions of log-normality and mean reversion are plausible and they can make the forecasting more accurate.

V. Conclusions

This paper dealt with a new stochastic extension to the Bass model for the diffusion of innovations in order to provide more realistic forecasts of future sales. Conventional assumptions about a normally distributed error process without mean reversion can yield unreasonable forecasts, such as negative sales. Our model assumes a log-normal and mean-reverting error process and under our characterization sales volumes are strictly positive and heteroscedastic with autoregressive error structure. The heteroscedasticity here means that sales fluctuate more when they are high and less when they are low. Mean reversion drives the stochastic sales process towards the Bass curve, and this can make especially the long-term forecasts more accurate. We showed that the forecasts of future sales must substantially be adjusted upward from the Bass curve due to the log-normality, especially when the error process is not mean-reverting and volatility is high. Therefore, even if we rely on the Bass model as a good deterministic part of our model, we need to adjust our forecast. We analyzed the model empirically by using a large set of telecommunication data and found that the error process displayed substantially mean revision, implying a fast pull-back to the Bass curve. We compared our error model with conventional ones and found that heteroscedastic sales and mean-reverting error process can make forecasts more accurate. The greater forecasting accuracy is reasonable because the error-term was estimated to be quite strongly mean-reverting, hence driving the stochastic sales back to the Bass curve.

Our results present several promising avenues in relation to engineering management and its utilization of forecasting results. For example, as volatility increases in the diffusion of innovations, future development projects become increasingly risky and this further highlights the need to pay special attention to their management. This insight might direct the engineering effort and further guide road mapping of a company. Naturally, improved forecasting accuracy is important as such, but the explicit consideration of error terms in innovation diffusion modelling may guide the organization's engineering efforts.

The existing literature on the diffusion of innovations has traditionally dealt with the problem of selecting a model mostly by comparing deterministic models or deterministic parts of models to produce favorable fit statistics or, at best, good forecasting capability. However, this represents only one side of the forecasting problem. As this study has shown, characterizing the error process can greatly impact the performance of forecasting. Hence, when selecting a model, one should consider not only the model's deterministic term but also its stochastic term. We anticipate future work to address this point and we view our characterization as one possibility among others. A natural extension of our study is to consider also other diffusion models with log-normal and mean-reverting error process, such as the Gompertz model. Moreover, one may also want to consider other possible error structures than presented in this paper. For example, Poison processes could be used to model extreme changes in the sales. Furthermore, higher-order autoregressive error processes should be studied in discrete time for larger data sets. Finally, this paper also serves as a preliminary starting point for real option valuation with stochastic product life cycles in continuous time.

TABLE II

OUT-OF-THE-SAMPLE FORECASTING ERRORS FOR 1-29 PERIOD FORECASTS (THE LENGTH OF ONE PERIOD IS THREE MONTH) FOR SIX COUNTRIES. HERE **B/A** MEANS $\mathbf{MSE}_i^B/\mathbf{MSE}_i^A$ AND **C/A** MEANS $\mathbf{MSE}_i^C/\mathbf{MSE}_i^A$. THE STANDARD DEVIATIONS (OVER THE PERIODS AND OVER THE COUNTRIES) ARE NOT CALCULATED BECAUSE THE RATIOS CANNOT BE ASSUMED TO BE NORMALLY DISTRIBUTED.

Periods	**Japanise Data**		**Finnish Data**		**UK Data**		**US Data**		**Australian Data**		**Korean Data**		**Average B/A**	**Average C/A**
ahead	**B/A**	**C/A**	**B/A**	**C/A**	**B/A**	**C/A**	**B/A**	**C/A**	**B/A**	**C/A**	**B/A**	**C/A**	**over countries**	**over countries**
1	**1.455**	**1.019**	**1.107**	**1.444**	**1.296**	**1.197**	0.870	**1.539**	**1.091**	**1.216**	**1.101**	**1.484**	**1.153**	**1.316**
2	0.972	**1.118**	**1.012**	**1.723**	**1.164**	**2.781**	0.932	**2.448**	**1.046**	**1.224**	0.927	**1.500**	**1.009**	**1.799**
3	0.916	**1.523**	0.998	**1.231**	**1.116**	**3.636**	0.907	**2.461**	0.941	**1.162**	0.661	**1.272**	0.923	**1.881**
4	**1.024**	**1.275**	**1.075**	**2.283**	**1.049**	**4.165**	**1.079**	**4.477**	**1.026**	0.912	**1.075**	**1.897**	**1.055**	**2.501**
5	**1.111**	**1.311**	**1.267**	**4.423**	**1.182**	**3.451**	**1.426**	**5.107**	0.626	0.646	**1.159**	**2.819**	**1.129**	**2.959**
6	**1.212**	**1.339**	**1.361**	**7.016**	**1.174**	**2.439**	**1.442**	**4.662**	0.834	0.706	**1.007**	**3.062**	**1.172**	**3.204**
7	**1.150**	**1.315**	**1.441**	**10.269**	**1.181**	**1.782**	**1.584**	**2.167**	**1.089**	0.731	**1.177**	**2.181**	**1.270**	**3.074**
8	**1.181**	**1.443**	**1.681**	**17.931**	**1.099**	**1.492**	**2.003**	**1.861**	**1.272**	0.538	**1.497**	**3.434**	**1.456**	**4.450**
9	**1.241**	**1.501**	**1.965**	**25.847**	**1.036**	**1.149**	**1.940**	**2.137**	**1.512**	0.598	**1.843**	**3.732**	**1.589**	**5.827**
10	**1.359**	**1.843**	**2.253**	**32.661**	**1.014**	0.832	**1.674**	**2.066**	**1.376**	0.715	**2.120**	**4.001**	**1.633**	**7.020**
11	**1.646**	**2.259**	**2.631**	**35.195**	0.975	0.756	**1.625**	**1.357**	**1.304**	0.775	**2.186**	**4.190**	**1.728**	**7.422**
12	**1.989**	**3.280**	**3.073**	**33.280**	0.964	0.593	**1.647**	**1.068**	**1.215**	0.751	**1.948**	**3.912**	**1.806**	**7.147**
13	**3.932**	**6.196**	**3.598**	**25.430**	0.961	0.599	**1.345**	**1.287**	**1.157**	0.783	**2.304**	**4.247**	**2.216**	**6.424**
14	**3.684**	**5.896**	**4.358**	**16.655**	**1.029**	0.606	**1.475**	**1.233**	**1.177**	0.897	**1.957**	**3.236**	**2.280**	**4.754**
15	**2.676**	**3.813**	**5.182**	**9.987**	**1.095**	0.579	**1.310**	**1.312**	**1.120**	0.925	**1.490**	**2.726**	**2.145**	**3.224**
16	**1.774**	**2.375**	**5.295**	**6.164**	**1.145**	0.628	**1.226**	**1.057**	**1.118**	**1.010**	**2.097**	**4.258**	**2.109**	**2.582**
17	**1.716**	**2.069**	**5.937**	**4.797**	**1.292**	0.709	**1.189**	**1.048**	**1.081**	0.988	**1.724**	**4.340**	**2.156**	**2.325**
18	**1.764**	**2.263**	**5.865**	**1.428**	**1.548**	0.868	**1.126**	0.938	**1.147**	**1.072**	**1.011**	**4.122**	**2.077**	**1.782**
19	**1.794**	**2.139**	**6.097**	0.963	**2.107**	**1.317**	**1.076**	0.998	**1.124**	**1.115**	0.898	**4.302**	**2.182**	**1.806**
20	**1.498**	**2.012**	**6.296**	0.750	**2.383**	**1.874**	**1.146**	0.875	**1.156**	**1.045**	0.993	**3.294**	**2.245**	**1.642**
21	**1.324**	**1.734**	**7.476**	0.683	**4.443**	**3.509**	**1.100**	0.665	**1.210**	**1.156**	0.834	**3.041**	**2.731**	**1.798**
22	**1.129**	**1.645**	**6.382**	0.698	**6.497**	**5.980**	**1.028**	0.879	**1.228**	**1.171**	0.754	**3.429**	**2.836**	**2.300**
23	**1.082**	**1.400**	**4.995**	0.730	**6.852**	**5.961**	**1.036**	0.833	**1.322**	**1.314**	0.801	**3.526**	**2.681**	**2.294**
24	0.964	**1.252**	**4.288**	0.412	**5.986**	**6.124**	**1.045**	0.879	**1.211**	**1.246**	0.785	**3.133**	**2.380**	**2.174**
25	0.930	**1.034**	**4.316**	0.449	**7.528**	**7.438**	**1.056**	0.792	**1.192**	0.710	0.744	**2.499**	**2.628**	**2.153**
26	0.886	**1.241**	**3.467**	0.606	**4.981**	**8.527**	**1.060**	**1.038**	**1.552**	**1.053**	0.760	**2.479**	**2.118**	**2.490**
27	0.914	**1.588**	**1.756**	0.283	**5.471**	**6.012**	**1.068**	0.916	**5.373**	**5.956**	0.653	**2.590**	**2.539**	**2.891**
28	0.799	**4.320**	0.318	0.586	**4.205**	**3.723**	**1.072**	0.917	0.762	**2.120**	0.665	**1.974**	**1.304**	**2.273**
29	**1.495**	**1.781**	0.183	**1.257**	**2.732**	**1.880**	**1.085**	0.897	**1.661**	**1.907**	0.713	**1.807**	**1.312**	**1.588**
Average over periods	**1.504**	**2.137**	**3.299**	**8.454**	**2.535**	**2.780**	**1.261**	**1.652**	**1.308**	**1.188**	**1.237**	**3.051**		

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Appendix

The following tables summarize NLS estimates for models A, B, and C for all the six countries. The first row presents the estimates at the first time point and the second row presents the standard error of the coefficients, and so on.

TABLE III

NLS ESTIMATES FOR JAPANESE DATA, MODEL A.

Data used up to (yyyymmdd)	Num of obs.	Model A p	q	m	ψ	σ	X_1
19881201	29	0.000158	0.113826	5367081	0.186452	0.137898	13.385648
		(0.000708)	(0.011956)	(24450123)	(0.063975)	(0.004994)	(4.343982)
19890301	30	0.000156	0.114969	5367081	0.193031	0.136926	12.998911
		(0.000585)	(0.011084)	(20581702)	(0.063155)	(0.004841)	(4.020694)
19890601	31	0.000148	0.118011	5459749	0.209859	0.145233	12.132390
		(0.000492)	(0.011074)	(18702669)	(0.066391)	(0.005358)	(3.612113)
19890901	32	0.000138	0.120095	5646678	0.226993	0.147288	11.337574
		(0.000415)	(0.010811)	(17432945)	(0.067180)	(0.005423)	(3.137939)
19891201	33	0.000135	0.125857	5320947	0.267847	0.173365	9.892849
		(0.000366)	(0.012590)	(14990711)	(0.078494)	(0.007399)	(2.663520)
19900301	34	0.000134	0.128632	5297960	0.328113	0.203449	7.244156
		(0.000374)	(0.015548)	(15565332)	(0.091973)	(0.010039)	(1.820171)
19900601	35	0.000113	0.135586	5297940	0.496025	0.210337	5.618356
		(0.000343)	(0.020946)	(17275862)	(0.094754)	(0.010576)	(0.875487)
19900901	36	0.000114	0.135586	5297940	0.496025	0.208058	5.618356
		(0.000293)	(0.019598)	(14841300)	(0.093006)	(0.010203)	(0.867218)
19901201	37	0.000115	0.135586	5297960	0.496024	0.209394	5.618357
		(0.000247)	(0.018208)	(12490546)	(0.091291)	(0.010194)	(0.872875)
19910301	38	0.000114	0.135586	5297960	0.496024	0.209044	5.618358
		(0.000208)	(0.016891)	(10707674)	(0.089282)	(0.010025)	(0.867286)
19910601	39	0.000115	0.135586	5297960	0.496024	0.207151	5.618358
		(0.000164)	(0.014929)	(8502291)	(0.083191)	(0.009718)	(0.846121)
19910901	40	0.000115	0.135586	5297940	0.496024	0.206211	5.618357
		(0.000134)	(0.013638)	(6991853)	(0.080065)	(0.009508)	(0.837999)
19911201	41	0.000061	0.137261	9791364	0.371134	0.189913	7.631591
		(0.000083)	(0.009304)	(14491901)	(0.071860)	(0.007966)	(1.374865)
19920301	42	0.000056	0.135015	10991454	0.355999	0.190815	7.854021
		(0.000077)	(0.008574)	(16112480)	(0.071680)	(0.007945)	(1.492206)
19920601	43	0.000063	0.134635	9871671	0.370088	0.189683	7.447495
		(0.000065)	(0.008051)	(11041615)	(0.068984)	(0.007760)	(1.333255)
19920901	44	0.000222	0.152143	2192764	0.437775	0.316571	6.884820
		(0.000058)	(0.016235)	(732276)	(0.115052)	(0.021366)	(1.635294)
19921201	45	0.000199	0.144182	2801125	0.364674	0.317906	7.896236
		(0.000053)	(0.011494)	(867923)	(0.091827)	(0.021306)	(2.014037)
19930301	46	0.000211	0.146413	2563483	0.374956	0.316142	7.779784
		(0.000049)	(0.011145)	(614262)	(0.090124)	(0.020840)	(1.902082)
19930601	47	0.000212	0.146733	2535276	0.376414	0.312797	7.761305
		(0.000046)	(0.010530)	(514994)	(0.087908)	(0.020183)	(1.859692)
19930901	48	0.000216	0.147385	2469177	0.377760	0.309623	7.704636
		(0.000045)	(0.010055)	(423985)	(0.086496)	(0.019569)	(1.818120)
19931201	49	0.000191	0.141150	3059799	0.359391	0.322308	7.912872
		(0.000042)	(0.009507)	(627930)	(0.089446)	(0.020987)	(2.063625)
19940301	50	0.000198	0.142406	2902857	0.353020	0.320247	8.099929
		(0.000040)	(0.009190)	(489913)	(0.088248)	(0.020512)	(2.102232)
19940601	51	0.000176	0.136885	3531759	0.342581	0.332874	8.096854
		(0.000038)	(0.008894)	(710504)	(0.091622)	(0.021943)	(2.281117)
19940901	52	0.000155	0.133338	4205476	0.364344	0.337918	7.450183
		(0.000037)	(0.009004)	(993267)	(0.092082)	(0.022394)	(2.022227)
19941201	53	0.000126	0.129464	5395881	0.388919	0.344343	6.893205
		(0.000036)	(0.009197)	(1647232)	(0.092726)	(0.023034)	(1.803713)
19950301	54	0.000109	0.135574	5297960	0.496016	0.356164	5.618369
		(0.000034)	(0.011433)	(1542429)	(0.094177)	(0.024413)	(1.210251)
19950601	55	0.000108	0.135572	5297960	0.496022	0.359195	5.618361
		(0.000032)	(0.011388)	(1412117)	(0.092980)	(0.024603)	(1.193958)
19950901	56	0.000109	0.133998	5379844	0.496166	0.373926	5.618331
		(0.000032)	(0.011617)	(1441682)	(0.095733)	(0.026424)	(1.231455)
19951201	57	0.000074	0.123099	9885429	0.422607	0.361592	6.246105
		(0.000028)	(0.009162)	(4376830)	(0.091920)	(0.024491)	(1.539155)
19960301	58	0.000109	0.133749	5461761	0.490775	0.371624	5.661810
		(0.000030)	(0.010961)	(1222513)	(0.093002)	(0.025645)	(1.224327)
19960601	59	0.000111	0.135586	5297940	0.496024	0.390608	5.618414
		(0.000032)	(0.011297)	(1057019)	(0.095770)	(0.028091)	(1.258663)

TABLE IV
NLS ESTIMATES FOR JAPANESE DATA, MODELS B AND C.

Data used up to (yyyymmdd)	Num of obs.	Model B			Model C			
		p	q	m	p	q	m	X_1
19881201	29	0.000143909	0.115993	5827921	4.187E-05	0.314293	188153	11548.29
		(0.001085717)	(0.032427)	(46391342)	(0.000239)	(0.259322)	(253864)	(2331.93)
19890301	30	0.000129484	0.120297	5888590	4.624E-06	0.226477	6211129	11522.94
		(0.00074014)	(0.028641)	(36011163)	(0.000324)	(0.492250)	(504695768)	(2460.43)
19890601	31	9.70051E-05	0.138224	5156829	2.196E-05	0.171280	6725480	11383.70
		(0.000375041)	(0.030127)	(22441037)	(0.000921)	(0.342919)	(334131910)	(2811.84)
19890901	32	7.74888E-05	0.147020	5156842	9.888E-08	0.334634	9741835	11381.78
		(0.000214695)	(0.027391)	(16763808)	(6.11E-06)	(0.417638)	(707794463)	(2470.62)
19891201	33	2.1498E-05	0.195777	4802222	1.175E-07	0.451762	321422	11558.02
		(4.78371E-05)	(0.042771)	(14960868)	(7.36E-07)	(0.232568)	(297121)	(2547.01)
19900301	34	4.86085E-05	0.171589	5113805	7.714E-10	0.500951	5143063	10262.63
		(8.99912E-05)	(0.044832)	(14060296)	(6.16E-09)	(0.356223)	(90372890)	(3118.36)
19900601	35	3.85294E-05	0.187691	4138035	0.0002811	0.139096	5134110	0.00
		(2.61782E-05)	(0.037509)	(5821981)	(0.000422)	(0.113272)	(11655602)	(7743.18)
19900901	36	2.73987E-05	0.198812	3941623	8.03E-05	0.167942	5133939	0.47
		(1.12206E-05)	(0.032784)	(3812359)	(0.000152)	(0.137286)	(14994014)	(5996.02)
19901201	37	9.91382E-06	0.257557	1898392	5.238E-10	0.594449	595734	11600.66
		(6.32647E-06)	(0.028196)	(452479)	(1.2E-09)	(0.070263)	(129630)	(4001.10)
19910301	38	1.30416E-06	0.341618	1146458	1.861E-06	0.319502	1346296	11555.56
		(1.04273E-06)	(0.026879)	(90822)	(5.54E-06)	(0.096223)	(662399)	(5736.17)
19910601	39	3.2529E-06	0.308121	1303141	2.826E-07	0.391507	1095690	11536.09
		(2.19667E-06)	(0.022628)	(94798)	(6.7E-07)	(0.072299)	(393718)	(6668.11)
19910901	40	1.80186E-05	0.242223	1791627	9.129E-06	0.274251	1515072	11541.49
		(1.06971E-05)	(0.021376)	(192710)	(2.56E-05)	(0.087224)	(865462)	(8172.27)
19911201	41	6.545E-05	0.169228	4104230	0.0001676	0.144110	4478809	7026.41
		(2.54577E-05)	(0.022810)	(1568218)	(0.000484)	(0.124816)	(7878110)	(9553.17)
19920301	42	0.000151242	0.138133	5133951	0.0001364	0.140036	5134120	0.01
		(4.32706E-05)	(0.020603)	(2374496)	(0.000461)	(0.153995)	(12965820)	(11059.67)
19920601	43	4.63932E-05	0.194952	2769879	2.375E-05	0.221749	2394077	11925.34
		(2.22716E-05)	(0.017558)	(335893)	(8.29E-05)	(0.103433)	(1910929)	(12264.51)
19920901	44	8.303E-06	0.261681	1853944	7.114E-05	0.181285	3093912	11295.65
		(7.22242E-06)	(0.026285)	(154422)	(0.00024)	(0.102668)	(2890831)	(12374.98)
19921201	45	8.73061E-06	0.260029	1864459	1.058E-09	0.481469	1389042	11530.56
		(6.93758E-06)	(0.023536)	(134878)	(5.56E-09)	(0.135523)	(804551)	(18647.25)
19930301	46	1.02361E-05	0.255018	1893924	4.832E-06	0.272949	1762416	11542.07
		(7.45431E-06)	(0.021283)	(124081)	(2.83E-05)	(0.165223)	(2203113)	(21964.41)
19930601	47	1.32997E-05	0.246281	1954824	1.995E-06	0.296761	1468094	11554.44
		(9.04225E-06)	(0.019692)	(121865)	(1.35E-05)	(0.187701)	(1865902)	(21734.56)
19930901	48	1.92492E-05	0.234312	2034667	1.008E-06	0.316790	1142357	11594.57
		(1.22692E-05)	(0.018378)	(124059)	(8.11E-06)	(0.222020)	(1532743)	(21655.95)
19931201	49	9.56798E-05	0.179290	2548214	8.084E-05	0.194633	2554961	9496.38
		(7.0157E-05)	(0.022299)	(258425)	(0.000359)	(0.132330)	(3536487)	(21596.67)
19940301	50	0.000131954	0.167239	2716574	3.173E-05	0.132904	5137121	9.91
		(8.87154E-05)	(0.020529)	(272914)	(0.000473)	(0.730786)	(106221348)	(26054.81)
19940601	51	0.000367752	0.117952	4167677	6.444E-06	0.273654	1055719	11567.64
		(0.000219492)	(0.022773)	(947763)	(6.6E-05)	(0.291438)	(2064946)	(27446.82)
19940901	52	0.000331373	0.094548	7917779	3.038E-05	0.132916	5136332	9.95
		(0.000138753)	(0.025149)	(5145751)	(0.000554)	(0.718917)	(86907080)	(31604.94)
19941201	53	9.33087E-05	0.092480	23484398	2.608E-05	0.132957	5134110	0.03
		(0.000154703)	(0.029094)	(57816253)	(0.0005)	(0.736657)	(90296507)	(31710.49)
19950301	54	1.75214E-05	0.168859	5134138	2.032E-05	0.132933	5134110	0.02
		(3.14969E-05)	(0.050840)	(2237851)	(0.000441)	(0.863310)	(116613006)	(33023.99)
19950601	55	4.85185E-05	0.148877	5134133	1.57E-05	0.132899	5134110	0.02
		(8.61249E-05)	(0.051061)	(2225721)	(0.000401)	(1.066352)	(160379129)	(35945.42)
19950901	56	8.06642E-05	0.133326	5134110	1.814E-05	0.132913	5134110	0.02
		(0.000203146)	(0.076282)	(3968104)	(0.000427)	(0.803126)	(95912619)	(35489.21)
19951201	57	8.06642E-05	0.133326	5134110	1.244E-05	0.132788	5137799	9.87
		(0.000193855)	(0.069601)	(3329657)	(0.000418)	(1.284850)	(189065605)	(44384.33)
19960301	58	8.06642E-05	0.133326	5134110	4.757E-05	0.139943	5134100	0.00
		(0.000184087)	(0.063509)	(2811456)	(0.00083)	(0.397765)	(24219446)	(71924.62)
19960601	59	8.06642E-05	0.133326	5134110	7.449E-05	0.134360	5134100	0.00
		(0.000176726)	(0.058835)	(2426420)	(0.001117)	(0.338619)	(22729659)	(72694.22)

TABLE V

NLS ESTIMATES FOR FINNISH DATA, MODEL A.

Data used up to (yyyymmdd)	**Num of obs.**	**Model A** p	q	m	ψ	σ	X_1
19880601	26	0.002878	0.134259	202021	0.000157	0.201455	-3874.412
		(0.000619)	(0.015610)	(57990)	(0.188434)	(0.011256)	(4635448)
19880901	27	0.002796	0.132926	209443	0.000258	0.199343	-2384.863
		(0.000510)	(0.013786)	(50515)	(0.174656)	(0.010815)	(1611882)
19881201	28	0.002509	0.128302	239317	0.000625	0.200517	-1018.654
		(0.000493)	(0.013066)	(61456)	(0.172482)	(0.010746)	(281021.6)
19890301	29	0.001972	0.119672	320658	0.030609	0.208131	-22.24901
		(0.000596)	(0.013547)	(120643)	(0.173651)	(0.011376)	(125.2218)
19890601	30	0.001165	0.108249	586146	0.125943	0.213926	-5.930361
		(0.000881)	(0.015213)	(500975)	(0.168276)	(0.011816)	(7.801703)
19890901	31	0.001112	0.107564	617077	0.131379	0.210482	-5.714994
		(0.000736)	(0.013628)	(462423)	(0.156662)	(0.011253)	(6.793448)
19891201	32	0.001006	0.106243	687762	0.135449	0.207385	-5.599067
		(0.000655)	(0.012654)	(505698)	(0.153537)	(0.010752)	(6.338110)
19900301	33	0.000862	0.104399	812605	0.141257	0.204712	-5.445232
		(0.000604)	(0.011881)	(636289)	(0.150839)	(0.010317)	(5.812777)
19900601	34	0.000663	0.103128	1056977	0.156545	0.207554	-4.666754
		(0.000576)	(0.011516)	(1006056)	(0.151862)	(0.010448)	(4.577451)
19900901	35	0.000636	0.101994	1116997	0.143538	0.204704	-5.439798
		(0.000497)	(0.010427)	(957503)	(0.145083)	(0.010017)	(5.548084)
19901201	36	0.000814	0.104363	858864	0.144262	0.204868	-5.319091
		(0.000383)	(0.009685)	(462631)	(0.145141)	(0.009893)	(5.424780)
19910301	37	0.000961	0.106735	715446	0.156910	0.203823	-4.798733
		(0.000299)	(0.009149)	(265554)	(0.143487)	(0.009659)	(4.483176)
19910601	38	0.000821	0.104273	853081	0.144853	0.203263	-5.306700
		(0.000288)	(0.008735)	(353328)	(0.142539)	(0.009479)	(5.266427)
19910901	39	0.001011	0.108406	668432	0.113923	0.206375	-6.465999
		(0.000204)	(0.007811)	(165388)	(0.143045)	(0.009645)	(8.206378)
19911201	40	0.001191	0.112769	548654	0.156520	0.213499	-4.520241
		(0.000172)	(0.007928)	(94704)	(0.146126)	(0.010192)	(4.387373)
19920301	41	0.001220	0.113569	532559	0.170791	0.211264	-4.112717
		(0.000153)	(0.007635)	(77297)	(0.139742)	(0.009858)	(3.530323)
19920601	42	0.001122	0.110287	595702	0.155819	0.214906	-4.671145
		(0.000146)	(0.007493)	(94509)	(0.141890)	(0.010078)	(4.361734)
19920901	43	0.001182	0.113015	549454	0.117619	0.215874	-5.968117
		(0.000127)	(0.006669)	(63179)	(0.138702)	(0.010050)	(7.190385)
19921201	44	0.001169	0.112481	558219	0.113161	0.213641	-6.240801
		(0.000118)	(0.006335)	(58060)	(0.136517)	(0.009731)	(7.652795)
19930301	45	0.001124	0.110168	594620	0.122583	0.214989	-5.935779
		(0.000114)	(0.006272)	(63445)	(0.137197)	(0.009744)	(6.747127)
19930601	46	0.001031	0.105193	685026	0.185758	0.224907	-4.184962
		(0.000125)	(0.007010)	(96403)	(0.140721)	(0.010547)	(3.292957)
19930901	47	0.001028	0.105033	688246	0.188464	0.222509	-4.136407
		(0.000117)	(0.006528)	(84070)	(0.130702)	(0.010213)	(3.043578)
19931201	48	0.001004	0.103784	714484	0.196494	0.221145	-4.031699
		(0.000112)	(0.006333)	(84117)	(0.129314)	(0.009983)	(2.824267)
19940301	49	0.000958	0.101303	770347	0.217097	0.222555	-3.769056
		(0.000111)	(0.006386)	(96953)	(0.129134)	(0.010007)	(2.405489)
19940601	50	0.000890	0.097582	868034	0.263001	0.226698	-3.269783
		(0.000117)	(0.006801)	(132332)	(0.128655)	(0.010278)	(1.763940)
19940901	51	0.000896	0.097916	858329	0.257525	0.224517	-3.323627
		(0.000108)	(0.006308)	(111123)	(0.122449)	(0.009982)	(1.773738)
19941201	52	0.000895	0.097877	859546	0.257743	0.222349	-3.322585
		(0.000102)	(0.006010)	(99673)	(0.120912)	(0.009696)	(1.753906)
19950301	53	0.000826	0.094213	973464	0.269586	0.229878	-3.326151
		(0.000104)	(0.006291)	(137153)	(0.124895)	(0.010265)	(1.711900)
19950601	54	0.000823	0.094017	980033	0.273070	0.227762	-3.292854
		(0.000098)	(0.005950)	(123459)	(0.118712)	(0.009983)	(1.628020)
19950901	55	0.000837	0.094941	950932	0.268401	0.225960	-3.310468
		(0.000094)	(0.005603)	(101064)	(0.117028)	(0.009736)	(1.653057)
19951201	56	0.000859	0.096466	909745	0.266959	0.227332	-3.261934
		(0.000094)	(0.005390)	(81597)	(0.117673)	(0.009767)	(1.662741)

TABLE VI
NLS ESTIMATES FOR FINNISH DATA, MODELS B AND C.

Data used up to	Num of	Model B			Model C			
(yyyymmdd)	obs.	p	q	m	p	q	m	X_1
19880601	26	0.0013937	0.093474	617589	0.0003021	0.065483	5507948	-1415.93
		(0.00124)	(0.015891)	(629566)	(0.023916)	(0.277000)	(465303101)	(9673.20)
19880901	27	0.0014761	0.094170	580708	1.052E-08	0.468330	2111152	359.96
		(0.00094)	(0.013870)	(438492)	(1.97E-06)	(0.737008)	(430379086)	(487.06)
19881201	28	0.0007855	0.088044	1137864	0.0015924	0.064335	1286421	-1821.24
		(0.00102)	(0.012887)	(1619605)	(0.011633)	(0.206639)	(14916239)	(10009.94)
19890301	29	0.0001816	0.088870	4594021	0.0002999	0.133234	738435	223.79
		(0.00113)	(0.014853)	(29223767)	(0.001608)	(0.205521)	(6797189)	(1275.01)
19890601	30	0.0001502	0.098404	4580531	1.155E-07	0.372032	1382910	359.85
		(0.00089)	(0.017268)	(28172492)	(3.76E-06)	(0.433232)	(59496333)	(602.40)
19890901	31	0.0001558	0.095557	4690573	6.387E-07	0.317534	1275860	358.89
		(0.00079)	(0.015462)	(24689198)	(6.74E-06)	(0.268760)	(21731368)	(595.25)
19891201	32	0.0001559	0.093292	4920974	8.654E-10	0.677180	55082	376.36
		(0.00069)	(0.013847)	(22683152)	(3.26E-09)	(0.133600)	(19679)	(731.41)
19900301	33	0.0001531	0.092596	5087179	1.185E-06	0.407090	95383	360.27
		(0.00057)	(0.012270)	(19749749)	(4.12E-06)	(0.126827)	(51681)	(851.83)
19900601	34	0.0001293	0.100676	4962919	0.0002214	0.180874	339794	270.27
		(0.00044)	(0.014081)	(18130873)	(0.000544)	(0.117753)	(397598)	(937.21)
19900901	35	0.0001404	0.094965	5263828	3.925E-05	0.197587	738376	223.80
		(0.00048)	(0.013657)	(19001596)	(0.000103)	(0.179078)	(2479628)	(1119.45)
19901201	36	0.0007348	0.112628	784080	6.883E-06	0.311103	176610	376.11
		(0.00015)	(0.014583)	(298598)	(2.85E-05)	(0.142206)	(137698)	(1461.19)
19910301	37	0.0008184	0.127604	538760	4.903E-08	0.470503	101666	359.99
		(0.00015)	(0.013927)	(100076)	(2.31E-07)	(0.149907)	(64369)	(1442.14)
19910601	38	0.0007834	0.113487	731677	2.482E-06	0.346463	143543	359.50
		(0.00012)	(0.013394)	(193159)	(9.26E-06)	(0.122156)	(102607)	(1459.05)
19910901	39	0.0007719	0.131574	520767	8.215E-05	0.091016	8324852	-388.94
		(0.00017)	(0.013286)	(70982)	(0.003425)	(0.233932)	(399039783)	(5469.92)
19911201	40	0.0006392	0.149016	429455	4.104E-05	0.245243	251701	347.06
		(0.00018)	(0.013576)	(39233)	(0.000153)	(0.122794)	(251967)	(2136.78)
19920301	41	0.0005872	0.153510	416918	2.837E-05	0.257656	256898	350.88
		(0.00017)	(0.012459)	(30920)	(9.79E-05)	(0.113032)	(232410)	(2124.24)
19920601	42	0.0008316	0.130399	507661	4.481E-05	0.244627	211351	347.92
		(0.00022)	(0.013112)	(52879)	(0.000195)	(0.144873)	(257344)	(2187.34)
19920901	43	0.0007739	0.135939	481153	5.734E-06	0.191961	747234	222.45
		(0.00021)	(0.012233)	(39903)	(5.87E-05)	(0.700707)	(16512891)	(2506.49)
19921201	44	0.00082	0.131960	498313	0.0002982	0.172965	368324	237.41
		(0.00021)	(0.011327)	(38519)	(0.001129)	(0.128544)	(554510)	(2882.99)
19930301	45	0.0009771	0.119113	562846	0.0009728	0.124053	509055	-193.54
		(0.00024)	(0.011211)	(50738)	(0.003343)	(0.123708)	(927886)	(3946.67)
19930601	46	0.0011565	0.094409	800883	0.0001156	0.073535	5418666	-159.32
		(0.00024)	(0.013144)	(152150)	(0.006947)	(0.433759)	(399022807)	(10637.81)
19930901	47	0.0011564	0.093908	808820	6.249E-05	0.131673	746917	222.83
		(0.00024)	(0.012168)	(135517)	(0.000628)	(0.481428)	(8464813)	(3204.03)
19931201	48	0.0011573	0.091637	845714	0.0017519	0.076210	1088205	-1692.84
		(0.00023)	(0.011340)	(134260)	(0.005979)	(0.145025)	(3113087)	(12544.44)
19940301	49	0.0011353	0.084539	991373	0.001898	0.063395	1460824	-2585.83
		(0.0002)	(0.010958)	(189322)	(0.005961)	(0.155775)	(5423660)	(19043.66)
19940601	50	0.0010026	0.072686	1426715	8.779E-05	0.121654	746342	234.37
		(0.00019)	(0.011233)	(479667)	(0.00091)	(0.397786)	(5926483)	(3528.19)
19940901	51	0.0010295	0.076479	1270814	3.965E-05	0.135070	730257	227.03
		(0.00016)	(0.010475)	(309864)	(0.000474)	(0.431241)	(6134896)	(3380.43)
19941201	52	0.0010719	0.079791	1145260	0.00123	0.079212	1161284	-1168.26
		(0.00018)	(0.009784)	(206159)	(0.004502)	(0.130745)	(2944355)	(9700.91)
19950301	53	0.0009476	0.069883	1590743	0.0010546	0.088097	1006813	-798.24
		(0.00016)	(0.010160)	(482326)	(0.003774)	(0.115296)	(2132727)	(7133.65)
19950601	54	0.0009499	0.071416	1522001	3.647E-05	0.129811	746668	229.21
		(0.00015)	(0.009496)	(378064)	(0.000523)	(0.464843)	(6630994)	(3945.55)
19950901	55	0.0010084	0.076123	1300566	0.0002123	0.115700	700004	212.55
		(0.00016)	(0.009004)	(222425)	(0.001508)	(0.186896)	(1848218)	(4341.50)
19951201	56	0.001015	0.083254	1098795	3.621E-08	0.308631	201411	359.99
		(0.0002)	(0.008962)	(131437)	(4.37E-07)	(0.253765)	(337724)	(4040.88)

TABLE VII
NLS ESTIMATES FOR UK DATA, MODEL A.

Data used up to (yyyymmdd)	Num of obs.	Model A					
		p	q	m	ψ	σ	X_1
19880601	14	0.000760	0.136630	10434171	-0.408617	0.191934	-0.332387
		(0.007150)	(0.039177)	(99308176)	(0.290717)	(0.013924)	(0.598257)
19880901	15	0.000772	0.133909	10428199	-0.491986	0.187394	-0.249009
		(0.005480)	(0.031960)	(74956631)	(0.249337)	(0.012823)	(0.457769)
19881201	16	0.000759	0.136922	10432897	-0.561741	0.186058	-0.243647
		(0.003695)	(0.024809)	(51617668)	(0.224761)	(0.012239)	(0.381106)
19890301	17	0.000762	0.136313	10431126	-0.577624	0.180713	-0.231201
		(0.002881)	(0.021165)	(40191767)	(0.208239)	(0.011201)	(0.357632)
19890601	18	0.000763	0.136044	10429186	-0.572774	0.175680	-0.230377
		(0.002173)	(0.017725)	(30360181)	(0.192509)	(0.010288)	(0.344212)
19890901	19	0.000764	0.135866	10429192	-0.574684	0.171024	-0.227932
		(0.001694)	(0.015270)	(23731051)	(0.186822)	(0.009490)	(0.332585)
19891201	20	0.000753	0.137814	10431165	-0.588202	0.171296	-0.243250
		(0.001291)	(0.013311)	(18438425)	(0.185842)	(0.009279)	(0.324410)
19900301	21	0.000755	0.137713	10424310	-0.590849	0.167185	-0.240985
		(0.001028)	(0.011731)	(14735716)	(0.178580)	(0.008626)	(0.314654)
19900601	22	0.001855	0.146428	4114793	-0.499534	0.194113	-0.330508
		(0.000779)	(0.012659)	(1952506)	(0.202769)	(0.011361)	(0.444475)
19900901	23	0.003826	0.184083	1651215	-0.087374	0.287902	-3.644377
		(0.000599)	(0.023157)	(317810)	(0.277901)	(0.024442)	(12.761904)
19901201	24	0.003974	0.196259	1473515	0.076818	0.286851	4.985643
		(0.000679)	(0.022597)	(181732)	(0.203570)	(0.023753)	(13.477714)
19910301	25	0.003996	0.203269	1402095	0.109864	0.284635	3.824838
		(0.000719)	(0.021246)	(133595)	(0.197555)	(0.022915)	(7.211447)
19910601	26	0.003646	0.237351	1210256	0.246815	0.352342	2.532503
		(0.001116)	(0.031163)	(126266)	(0.243886)	(0.034432)	(2.644185)
19910901	27	0.003142	0.258187	1163434	0.404552	0.353697	1.956467
		(0.001265)	(0.034313)	(144548)	(0.190408)	(0.034048)	(1.297014)
19911201	28	0.003515	0.243449	1189221	0.318881	0.355830	2.108910
		(0.001121)	(0.026427)	(126641)	(0.172825)	(0.033839)	(1.703996)
19920301	29	0.003989	0.221592	1237471	0.330391	0.385463	1.599579
		(0.001339)	(0.027923)	(140801)	(0.186078)	(0.039019)	(1.708569)
19920601	30	0.003901	0.003080	9949523	0.770752	0.435222	-1.707701
		(1.075819)	(1.444568)	(2777531563)	(0.181976)	(0.048908)	(3.862282)
19920901	31	0.004451	0.202083	1281198	0.418413	0.385433	0.966054
		(0.001670)	(0.029354)	(160232)	(0.168359)	(0.037734)	(1.280864)
19921201	32	0.005693	0.159337	1412815	0.550952	0.428364	0.189238
		(0.003042)	(0.046279)	(260106)	(0.179625)	(0.045874)	(1.244948)
19930301	33	0.002428	0.039104	9259769	0.746919	0.441483	-1.081086
		(0.050207)	(0.213969)	(205907249)	(0.153484)	(0.047983)	(2.098604)
19930601	34	0.002202	0.040194	10012715	0.750445	0.435158	-1.051631
		(0.038422)	(0.174500)	(187872470)	(0.131732)	(0.045927)	(1.881739)
19930901	35	0.001945	0.045599	10452289	0.759101	0.431215	-0.940632
		(0.026031)	(0.144214)	(152069481)	(0.121077)	(0.044450)	(1.765309)
19931201	36	0.001585	0.058995	10462755	0.781424	0.436693	-0.670109
		(0.013529)	(0.120260)	(100779318)	(0.115317)	(0.044949)	(1.768120)
19940301	37	0.001478	0.063381	10462755	0.792473	0.431779	-0.578490
		(0.009882)	(0.109621)	(80901049)	(0.106720)	(0.043345)	(1.751014)
19940601	38	0.001406	0.066595	10430459	0.799777	0.426654	-0.510726
		(0.007643)	(0.102053)	(67235930)	(0.101748)	(0.041761)	(1.732406)
19940901	39	0.001379	0.067692	10430459	0.802114	0.421226	-0.486812
		(0.006443)	(0.095659)	(58779391)	(0.098627)	(0.040180)	(1.690042)
19941201	40	0.001118	0.078829	10430459	0.821028	0.424130	-0.234134
		(0.003737)	(0.092919)	(45036483)	(0.098977)	(0.040224)	(1.809274)
19950301	41	0.001181	0.076045	10430459	0.814168	0.419416	-0.300095
		(0.003672)	(0.086734)	(41927466)	(0.096514)	(0.038852)	(1.703200)
19950601	42	0.001143	0.077680	10430459	0.817190	0.414612	-0.260900
		(0.003036)	(0.082501)	(36770229)	(0.095390)	(0.037512)	(1.677350)
19950901	43	0.001250	0.073242	10430459	0.809071	0.411656	-0.368652
		(0.003250)	(0.077150)	(35480172)	(0.094467)	(0.036547)	(1.573644)
19951201	44	0.001394	0.067838	10430459	0.804638	0.410193	-0.499170
		(0.003687)	(0.074124)	(35498266)	(0.093578)	(0.035873)	(1.511446)

TABLE VIII
NLS ESTIMATES FOR UK DATA, MODELS B AND C.

Data used up to	**Num of**	**Model B**			**Model C**			
(yyyymmdd)	**obs.**	p	q	m	p	q	m	X_1
19880601	14	0.0004265	0.158829	15408518	5.98596E-05	0.846778	112543	10381.57
		(0.00889)	(0.085044)	(327126201)	(0.00017374)	(0.296257)	(79544)	(4211.89)
19880901	15	0.0048692	0.176194	1393424	2.21799E-07	0.776775	5081724	8989.06
		(0.00365)	(0.074198)	(1504552)	(1.2925E-05)	(1.407389)	(380630208)	(8202.65)
19881201	16	0.0004325	0.162551	13934315	1.06121E-05	0.787388	215595	10401.58
		(0.00543)	(0.076446)	(181323638)	(7.5152E-05)	(0.564850)	(283540)	(13341.80)
19890301	17	0.0033675	0.174415	1934017	4.32028E-05	0.369844	5064287	1337.52
		(0.00217)	(0.063021)	(1908285)	(0.00113013)	(1.126110)	(200869818)	(17060.86)
19890601	18	0.0017927	0.155653	3933918	6.17894E-11	1.470003	144741	10400.00
		(0.00273)	(0.054941)	(7371585)	(7.004E-10)	(0.731586)	(140333)	(16891.48)
19890901	19	0.0026348	0.172555	2393508	3.20442E-05	0.333251	5064541	1297.48
		(0.00104)	(0.044213)	(1587367)	(0.0014588)	(1.528106)	(335837585)	(19820.81)
19891201	20	0.0005672	0.150691	11658787	0.001207672	0.270806	1137444	8601.89
		(0.002)	(0.044563)	(45314994)	(0.00604632)	(0.418591)	(2794104)	(22485.93)
19900301	21	0.0016002	0.154727	4404039	1.56693E-06	0.462869	5867958	8366.15
		(0.00105)	(0.037927)	(4193980)	(3.8836E-05)	(1.294311)	(265343236)	(19322.35)
19900601	22	0.0023239	0.203997	1937630	1.06683E-06	0.656271	434818	10415.87
		(0.00069)	(0.035554)	(424569)	(9.6714E-06)	(0.500393)	(591225)	(20704.03)
19900901	23	0.0015498	0.267022	1373051	3.93961E-12	1.312660	202395	10400.33
		(0.00078)	(0.039976)	(166576)	(4.6731E-11)	(0.613664)	(169500)	(19225.14)
19901201	24	0.001366	0.279085	1319686	0.000272695	0.352781	123276	9777.46
		(0.00067)	(0.036025)	(129634)	(0.01029521)	(2.255274)	(1582100)	(23298.02)
19910301	25	0.0012724	0.285148	1297573	0.000276523	0.365327	123457	9775.80
		(0.0006)	(0.032789)	(111257)	(0.00963082)	(2.145523)	(1464250)	(23010.55)
19910601	26	0.000982	0.305361	1235802	0.000331017	0.357614	123359	9762.53
		(0.00048)	(0.032646)	(99390)	(0.0111591)	(2.091902)	(1403017)	(22630.45)
19910901	27	0.0008593	0.315055	1210463	0.000275449	0.351208	116595	9488.69
		(0.00042)	(0.031719)	(92952)	(0.01050311)	(2.256528)	(1454911)	(22912.68)
19911201	28	0.000868	0.314314	1212384	0.000375307	0.346602	115059	9490.25
		(0.00041)	(0.030131)	(89331)	(0.01295002)	(2.094366)	(1286550)	(22534.49)
19920301	29	0.0010437	0.301458	1243388	0.000300929	0.348481	123678	9149.97
		(0.00048)	(0.029266)	(92578)	(0.00960717)	(1.863133)	(1214887)	(22140.64)
19920601	30	0.001147	0.294826	1259704	6.62434E-08	0.373450	5000918	1397.67
		(0.0005)	(0.028165)	(92386)	(6.9525E-05)	(7.435187)	(6161152076)	(22274.51)
19920901	31	0.001283	0.286918	1279343	0.000558095	0.329848	124894	8840.82
		(0.00055)	(0.027418)	(93724)	(0.01560875)	(1.710596)	(1161393)	(22016.22)
19921201	32	0.0017588	0.264078	1338925	0.000308113	0.359156	129628	8850.39
		(0.00081)	(0.029920)	(114265)	(0.00786619)	(1.499078)	(974770)	(21437.97)
19930301	33	0.0032257	0.215820	1485617	1.77635E-07	0.349647	5060151	1417.46
		(0.00162)	(0.034266)	(166317)	(6.172E-06)	(1.809756)	(400175839)	(21975.32)
19930601	34	0.0063472	0.143805	1815448	1.31218E-07	0.367877	5065839	1927.68
		(0.00274)	(0.033638)	(254111)	(1.5098E-06)	(0.963710)	(114146517)	(21703.49)
19930901	35	0.0050857	0.055552	4531755	1.96157E-07	0.341575	4413352	2501.11
		(0.00436)	(0.043399)	(5266848)	(2.7547E-06)	(1.218872)	(141500235)	(22121.48)
19931201	36	9.04E-06	0.248316	4055030	1.67577E-07	0.338826	4978008	1574.66
		(2.7E-05)	(0.183915)	(11443857)	(2.1155E-06)	(0.971939)	(117035104)	(21831.75)
19940301	37	2.954E-07	0.351525	3397868	7.93482E-10	0.496699	5068613	7432.30
		(1.4E-06)	(0.197850)	(5423810)	(1.1464E-08)	(0.645981)	(42390532)	(22342.45)
19940601	38	1.925E-06	0.298027	3568961	4.86306E-10	0.553452	1614273	10401.45
		(6.3E-06)	(0.128173)	(3272963)	(4.7065E-09)	(0.290784)	(1384420)	(23112.63)
19940901	39	2.518E-06	0.289957	3604568	5.7802E-10	0.547999	1639534	10220.82
		(6.9E-06)	(0.100005)	(2137558)	(3.7537E-09)	(0.187726)	(939568)	(22813.17)
19941201	40	3.755E-07	0.342587	3438788	1.59784E-09	0.516784	1772347	9853.20
		(1.1E-06)	(0.092906)	(1256691)	(7.9554E-09)	(0.141327)	(899314)	(22592.95)
19950301	41	2.868E-07	0.349660	3438788	7.80878E-09	0.401750	5065244	7521.15
		(7E-07)	(0.074262)	(802590)	(1.0513E-07)	(0.446951)	(19274920)	(28733.31)
19950601	42	2.595E-07	0.351202	3479748	3.85975E-07	0.339221	5056014	1337.39
		(5.4E-07)	(0.061076)	(576599)	(1.4583E-06)	(0.106278)	(2728942)	(33609.13)
19950901	43	3.419E-07	0.344722	3479748	3.70184E-07	0.336781	4938866	2536.26
		(6E-07)	(0.050412)	(432717)	(1.3079E-06)	(0.097067)	(2575721)	(33341.09)
19951201	44	2.694E-07	0.352972	3397828	2.92355E-08	0.408157	3023079	10387.03
		(4.2E-07)	(0.044052)	(335874)	(1.314E-07)	(0.120820)	(1833798)	(33284.26)

TABLE IX
NLS ESTIMATES FOR US DATA, MODEL A.

Data used up to (yyyymmdd)	**Num of obs.**	**Model A**					
		p	q	m	Ψ	σ	X_1
19900901	17	0.004654	0.138121	16350008	0.148518	0.114885	0.022913
		(0.002483)	(0.024778)	(10162649)	(0.244562)	(0.004527)	(1.047152)
19901201	18	0.005566	0.145576	13317066	0.177099	0.112893	0.132083
		(0.001563)	(0.021498)	(4764180)	(0.235931)	(0.004248)	(0.868992)
19910301	19	0.005294	0.142955	14141084	0.165907	0.110099	0.094700
		(0.001238)	(0.018353)	(4250052)	(0.226238)	(0.003933)	(0.882264)
19910601	20	0.004834	0.137610	15862230	0.170112	0.108344	-0.013013
		(0.001109)	(0.016576)	(4642415)	(0.222516)	(0.003712)	(0.828247)
19910901	21	0.004730	0.137164	16199985	0.171941	0.105733	-0.015450
		(0.000866)	(0.014538)	(3863588)	(0.215154)	(0.003450)	(0.788891)
19911201	22	0.004668	0.136435	16469514	0.172984	0.103339	-0.029995
		(0.000689)	(0.012884)	(3226166)	(0.210105)	(0.003220)	(0.757323)
19920301	23	0.004443	0.133572	17531357	0.176631	0.101725	-0.088830
		(0.000597)	(0.011719)	(3132557)	(0.206704)	(0.003051)	(0.727045)
19920601	24	0.004154	0.129670	19113992	0.195156	0.100796	-0.159915
		(0.000550)	(0.011047)	(3362375)	(0.203366)	(0.002933)	(0.660491)
19920901	25	0.003102	0.116704	27354018	0.281871	0.110014	-0.304454
		(0.000876)	(0.013811)	(9617805)	(0.218362)	(0.003423)	(0.529973)
19921201	26	0.001567	0.104066	57562220	0.472164	0.122304	-0.288384
		(0.001747)	(0.020817)	(71350582)	(0.213838)	(0.004149)	(0.402231)
19930301	27	0.001606	0.102981	56764932	0.423482	0.120907	-0.344028
		(0.001250)	(0.016233)	(49623467)	(0.173113)	(0.003979)	(0.420735)
19930601	28	0.001753	0.103421	52101708	0.413418	0.119725	-0.356845
		(0.000995)	(0.014458)	(34041349)	(0.170039)	(0.003831)	(0.420018)
19930901	29	0.001377	0.101223	66682564	0.392394	0.121404	-0.384327
		(0.000937)	(0.013358)	(50954471)	(0.172221)	(0.003871)	(0.436919)
19931201	30	0.001480	0.106653	58894164	0.451603	0.128459	-0.230960
		(0.000747)	(0.013900)	(34880183)	(0.177944)	(0.004261)	(0.414245)
19940301	31	0.001488	0.106145	58894164	0.430623	0.126568	-0.254346
		(0.000626)	(0.012457)	(29376381)	(0.164286)	(0.004069)	(0.420153)
19940601	32	0.001436	0.109035	58894164	0.465698	0.132546	-0.165271
		(0.000533)	(0.012990)	(26754631)	(0.172144)	(0.004392)	(0.413869)
19940901	33	0.001395	0.112847	57826753	0.573443	0.137301	-0.057808
		(0.000494)	(0.016414)	(26860987)	(0.174273)	(0.004641)	(0.400860)
19941201	34	0.001268	0.118972	57750759	0.713725	0.144540	0.079587
		(0.000459)	(0.026293)	(31418107)	(0.178904)	(0.005067)	(0.504167)
19950301	35	0.001317	0.116802	57750759	0.661925	0.143425	0.031929
		(0.000421)	(0.021615)	(27571615)	(0.169233)	(0.004917)	(0.447054)
19950601	36	0.001337	0.115906	57750829	0.645651	0.141929	0.011102
		(0.000375)	(0.019160)	(23873206)	(0.166540)	(0.004748)	(0.422913)
19950901	37	0.001269	0.117389	59061469	0.653107	0.144498	0.054196
		(0.000307)	(0.018308)	(20945033)	(0.162901)	(0.004854)	(0.421591)
19951201	38	0.001264	0.117789	59061497	0.286755	0.170498	0.229984
		(0.000191)	(0.010052)	(12496905)	(0.191814)	(0.006669)	(0.701004)
19960301	39	0.001149	0.113337	68236554	0.566184	0.147907	-0.010645
		(0.000249)	(0.013253)	(20484951)	(0.165322)	(0.004954)	(0.403061)
19960601	40	0.001193	0.111233	68236527	0.557217	0.155185	-0.073607
		(0.000247)	(0.012183)	(18342323)	(0.164835)	(0.005385)	(0.411743)
19960901	41	0.001262	0.118745	58406183	0.286326	0.171388	0.235656
		(0.000135)	(0.007842)	(7438114)	(0.157078)	(0.006488)	(0.694015)
19961201	42	0.001044	0.108906	78722366	0.450372	0.161738	-0.107225
		(0.000163)	(0.008509)	(14996101)	(0.144716)	(0.005708)	(0.457830)
19970301	43	0.001185	0.111603	68236524	0.294602	0.190562	-0.111392
		(0.000146)	(0.007648)	(9601595)	(0.167759)	(0.007832)	(0.757863)
19970601	44	0.001186	0.111600	68236557	0.313333	0.188281	-0.117672
		(0.000135)	(0.007199)	(8043154)	(0.145425)	(0.007558)	(0.700991)
19970901	45	0.001201	0.112657	66729546	0.353681	0.188492	-0.078532
		(0.000138)	(0.007307)	(7034925)	(0.143309)	(0.007490)	(0.628221)
19971201	46	0.001267	0.118063	58406131	0.273592	0.195876	0.230748
		(0.000131)	(0.006523)	(4453665)	(0.146018)	(0.008000)	(0.813654)
19980301	47	0.001252	0.118102	60262195	0.340174	0.209888	0.081920
		(0.000153)	(0.007410)	(4826438)	(0.156448)	(0.009087)	(0.714924)

TABLE X
NLS ESTIMATES FOR US DATA, MODELS B AND C.

Data used up to	Num of	Model B			Model C			
(yyyymmdd)	**obs.**	p	q	m	p	q	m	X_1
19900901	17	0.005956468	0.162048	11254428	0.000896	0.401104	3128212	83019.24
		(0.0006666)	(0.016655)	(2044071)	(0.001066)	(0.104807)	(1411945)	(24085.59)
19901201	18	0.006205648	0.167509	10487452	0.0032519	0.256994	5790965	62957.04
		(0.0004523)	(0.013893)	(1227842)	(0.002953)	(0.092013)	(3242125)	(40211.32)
19910301	19	0.005806906	0.156689	11936524	0.0038801	0.231749	6681941	54665.13
		(0.0004181)	(0.012603)	(1402806)	(0.003091)	(0.078719)	(3559442)	(45614.92)
19910601	20	0.005091663	0.141168	14917966	0.0057511	0.109929	21534850	-50142.58
		(0.0004703)	(0.012486)	(2260481)	(0.00351)	(0.123849)	(40105096)	(275585.42)
19910901	21	0.004861326	0.136777	16044758	0.0015059	0.062522	152765944	-157330.79
		(0.0003914)	(0.010890)	(2134059)	(0.023854)	(0.229705)	(3050468530)	(1013591.53)
19911201	22	0.00474627	0.134395	16685353	0.0051961	0.106034	24390820	-52943.31
		(0.0003169)	(0.009467)	(1843696)	(0.002994)	(0.098107)	(36392046)	(244905.52)
19920301	23	0.004463927	0.128272	18482746	0.0052314	0.112780	21864768	-40091.91
		(0.0002914)	(0.008659)	(2000020)	(0.003069)	(0.079253)	(23367819)	(188349.00)
19920601	24	0.004089019	0.120722	21255548	0.0045227	0.076662	45209873	-132490.29
		(0.0002917)	(0.008241)	(2502254)	(0.003012)	(0.099435)	(105800918)	(446456.92)
19920901	25	0.002197143	0.097644	45773718	0.0024802	0.052755	126783652	-243440.87
		(0.0009282)	(0.012963)	(23825143)	(0.011313)	(0.137645)	(966778666)	(1041625.93)
19921201	26	0.000860887	0.097953	105691571	5.05E-07	0.400685	45330886	87557.81
		(0.0015067)	(0.020859)	(204466216)	(6.12E-06)	(0.408249)	(922036306)	(35825.93)
19930301	27	0.000817305	0.097312	112037309	3.303E-09	0.595182	45357289	87432.53
		(0.0012035)	(0.018171)	(184081418)	(1.5E-08)	(0.337278)	(521392581)	(33748.14)
19930601	28	0.001562531	0.107557	53347039	9.6E-06	0.387842	7187838	87605.80
		(0.0006147)	(0.016786)	(29537680)	(4.31E-05)	(0.202835)	(6127037)	(75905.54)
19930901	29	0.001026127	0.100596	85671158	1.67E-05	0.367376	8337999	89145.13
		(0.000692)	(0.015275)	(71472213)	(5.16E-05)	(0.134458)	(5327242)	(75632.18)
19931201	30	0.000665058	0.103763	119297287	0.0001931	0.152099	80374001	75577.25
		(0.0006373)	(0.016624)	(137532090)	(0.000812)	(0.244100)	(704573906)	(137158.95)
19940301	31	0.000672418	0.100857	125059000	3.43E-07	0.374298	45329198	87337.30
		(0.0005525)	(0.014735)	(124990991)	(1.58E-06)	(0.444359)	(449176196)	(93100.09)
19940601	32	0.000574278	0.104869	132941229	2.049E-06	0.395778	9797991	87191.97
		(0.000408)	(0.014103)	(118951945)	(1E-05)	(0.185940)	(7708194)	(113094.72)
19940901	33	0.000504416	0.112617	126514357	6.07E-05	0.208069	45343744	89989.97
		(0.0002672)	(0.014447)	(92878897)	(0.000238)	(0.234830)	(145700095)	(126996.53)
19941201	34	0.000397152	0.160900	54735983	0.0001248	0.186920	45328174	89999.76
		(0.0001211)	(0.023117)	(15952249)	(0.000427)	(0.181077)	(98063436)	(131570.28)
19950301	35	0.000404426	0.117236	138622595	7.885E-05	0.185136	45328171	89999.76
		(0.0001727)	(0.015265)	(91989621)	(0.000375)	(0.277788)	(191023859)	(146283.18)
19950601	36	0.000582137	0.113167	112407830	2.706E-07	0.415940	17347784	79572.99
		(0.0001652)	(0.015929)	(56955493)	(1.09E-06)	(0.132510)	(9683163)	(166525.80)
19950901	37	0.000650801	0.121608	86267157	0.0001736	0.202839	45306833	90009.37
		(0.0001067)	(0.014203)	(23982078)	(0.000376)	(0.080444)	(31034214)	(181584.33)
19951201	38	0.000511829	0.106848	143870004	8.667E-05	0.213602	36729117	83326.26
		(0.0001442)	(0.013267)	(69777755)	(0.000281)	(0.114000)	(32968317)	(194382.19)
19960301	39	0.000645663	0.128572	75707422	2.631E-05	0.208388	45337098	89847.33
		(0.0001298)	(0.013400)	(13050009)	(0.000175)	(0.259586)	(120363252)	(207731.83)
19960601	40	0.000460075	0.156186	55445277	2.841E-05	0.250114	34850389	85752.38
		(0.0001483)	(0.014858)	(5586178)	(9.35E-05)	(0.105969)	(28430621)	(245435.96)
19960901	41	0.000444607	0.158212	54550036	0.0001347	0.204714	45328158	89999.77
		(0.0001385)	(0.013478)	(4491939)	(0.000353)	(0.088668)	(39083434)	(254552.42)
19961201	42	0.000607055	0.137909	65221662	3.767E-05	0.243405	29443774	85911.17
		(0.0001817)	(0.014074)	(7197963)	(0.00015)	(0.128473)	(32052510)	(269179.62)
19970301	43	0.000396506	0.160856	54735940	1.52E-05	0.186175	46013021	89502.30
		(0.0001474)	(0.014645)	(4320485)	(0.000241)	(0.679468)	(522577054)	(313305.94)
19970601	44	0.000398067	0.160862	54735940	7.557E-05	0.214036	45328160	89999.72
		(0.0001406)	(0.013455)	(3793505)	(0.000311)	(0.131666)	(57461955)	(399096.39)
19970901	45	0.000401301	0.160877	54735940	0.0001153	0.202385	39158308	81788.05
		(0.0001349)	(0.012443)	(3386089)	(0.000561)	(0.158558)	(62839448)	(404521.38)
19971201	46	0.000392095	0.160834	54735940	0.0001115	0.203147	39859364	81133.45
		(0.0001295)	(0.011899)	(3173968)	(0.000528)	(0.153542)	(60857146)	(399972.45)
19980301	47	0.000402879	0.160919	54735980	1.73E-05	0.270601	20068128	86912.77
		(0.0001296)	(0.011363)	(2967884)	(0.000127)	(0.231692)	(31777951)	(399783.25)

TABLE XI
NLS ESTIMATES FOR AUSTRALIAN DATA, MODEL A.

Data used up to (yyyymmdd)	**Num of obs.**	**Model A** p	q	m	ψ	σ	X_1
19890601	10	0.001093	0.184698	2865479	0.022619	0.121853	-53.4704922
		(0.013064)	(0.081625)	(34884964)	(0.135431)	(0.006640)	(314.4152327)
19890901	11	0.007184	0.198636	440772	0.000096	0.158925	-12860.19188
		(0.007302)	(0.076918)	(543945)	(0.173572)	(0.010770)	(23358212.74)
19891201	12	0.001178	0.173747	2779716	0.016981	0.183448	-73.16356156
		(0.012724)	(0.092741)	(30871804)	(0.199762)	(0.013739)	(849.2858729)
19900301	13	0.001916	0.170091	1754459	0.000178	0.183346	-7122.934901
		(0.007694)	(0.063897)	(7414453)	(0.166440)	(0.013185)	(6664382.82)
19900601	14	0.003773	0.177987	882756	0.000163	0.180297	-7738.427317
		(0.004511)	(0.052357)	(1220137)	(0.163488)	(0.012286)	(7750523.269)
19900901	15	0.004687	0.188256	684958	0.000155	0.171935	-7996.20309
		(0.002455)	(0.041002)	(462035)	(0.154837)	(0.010794)	(8010806.67)
19901201	16	0.005405	0.194574	584047	0.000241	0.168752	-5069.652846
		(0.001421)	(0.032940)	(218160)	(0.148093)	(0.010068)	(3108701.939)
19910301	17	0.006595	0.217466	440010	0.000120	0.169061	-9687.577233
		(0.000846)	(0.026071)	(68214)	(0.147481)	(0.009803)	(11918633.64)
19910601	18	0.006169	0.206266	491107	0.000366	0.172377	-3264.519259
		(0.000752)	(0.025704)	(80987)	(0.150497)	(0.009905)	(1343331.428)
19910901	19	0.005260	0.185330	626652	0.000315	0.191487	-3990.444767
		(0.000834)	(0.027139)	(151683)	(0.158742)	(0.011896)	(2008653.099)
19911201	20	0.004931	0.178336	688909	0.000759	0.189290	-1689.086069
		(0.000738)	(0.022957)	(146027)	(0.142241)	(0.011331)	(316488.4046)
19920301	21	0.004762	0.174478	726025	0.001734	0.185633	-747.3593412
		(0.000636)	(0.020238)	(131491)	(0.136086)	(0.010634)	(58618.1322)
19920601	22	0.004458	0.167124	803972	0.008889	0.184653	-149.0555204
		(0.000586)	(0.018880)	(144984)	(0.133414)	(0.010281)	(2230.397757)
19920901	23	0.003874	0.152875	1001176	0.051465	0.189842	-27.017648
		(0.000664)	(0.019672)	(245872)	(0.133930)	(0.010628)	(69.072597)
19921201	24	0.002589	0.130098	1712644	0.132842	0.201890	-11.316830
		(0.001224)	(0.023439)	(1027406)	(0.136243)	(0.011766)	(11.066267)
19930301	25	0.002542	0.129383	1752481	0.135723	0.197824	-11.103841
		(0.000951)	(0.019935)	(836169)	(0.124454)	(0.011069)	(9.798371)
19930601	26	0.001626	0.117904	2925092	0.159208	0.200219	-9.813652
		(0.001254)	(0.020509)	(2615021)	(0.124636)	(0.011118)	(7.328055)
19930901	27	0.001080	0.112948	4527172	0.183035	0.201324	-8.635890
		(0.001296)	(0.019608)	(5994304)	(0.120979)	(0.011031)	(5.447848)
19931201	28	0.000621	0.111836	7823043	0.200905	0.211925	-7.854464
		(0.001249)	(0.018940)	(16723546)	(0.122973)	(0.012003)	(4.633432)
19940301	29	0.000524	0.111049	9314659	0.207143	0.207844	-7.636467
		(0.001045)	(0.016774)	(19625264)	(0.114467)	(0.011345)	(4.123995)
19940601	30	0.000553	0.116113	8249902	0.261813	0.212973	-5.190946
		(0.000821)	(0.016374)	(13167273)	(0.116538)	(0.011711)	(2.348245)
19940901	31	0.000469	0.115853	9853359	0.222585	0.216080	-6.887062
		(0.000687)	(0.014558)	(15408717)	(0.117056)	(0.011859)	(3.618319)
19941201	32	0.000352	0.115513	13092485	0.236670	0.213583	-6.467758
		(0.000601)	(0.013536)	(23582747)	(0.113565)	(0.011404)	(3.138671)
19950301	33	0.000358	0.116944	12683928	0.246751	0.212776	-6.148578
		(0.000494)	(0.012628)	(18618150)	(0.112663)	(0.011146)	(2.867950)
19950601	34	0.000328	0.116934	13837626	0.247061	0.209219	-6.133272
		(0.000418)	(0.011624)	(18802040)	(0.110706)	(0.010616)	(2.811658)
19950901	35	0.000350	0.117086	12977964	0.248660	0.206987	-6.093977
		(0.000351)	(0.010832)	(14055991)	(0.109492)	(0.010242)	(2.742943)
19951201	36	0.000255	0.118377	17271803	0.222587	0.216451	-6.682130
		(0.000296)	(0.010216)	(21442655)	(0.114109)	(0.011043)	(3.486147)
19960301	37	0.000300	0.114880	15304753	0.143046	0.264569	-10.626210
		(0.000324)	(0.010885)	(17749736)	(0.137562)	(0.016274)	(10.204138)
19960601	38	0.001429	0.141316	2852908	0.545639	0.394176	-2.612670
		(0.000619)	(0.028591)	(890774)	(0.184400)	(0.035645)	(1.317629)
19960901	39	0.001429	0.141310	2852908	0.545640	0.389090	-2.612668
		(0.000567)	(0.025943)	(656234)	(0.133108)	(0.034283)	(1.121837)
19961201	40	0.001427	0.141910	2852948	0.545588	0.384529	-2.612714
		(0.000553)	(0.024581)	(565612)	(0.123907)	(0.033063)	(1.070665)

TABLE XII
NLS ESTIMATES FOR AUSTRALIAN DATA, MODELS B AND C.

Data used up to	**Num of**	**Model B**			**Model C**			
(yyyymmdd)	**obs.**	p	q	m	p	q	m	X_1
19890601	10	0.007437881	0.287187	275853	0.0064115	0.049426	5987721	-38943.75
		(0.001869363)	(0.055584)	(114098)	(0.229455)	(1.399425)	(384652329)	(1102423.54)
19890901	11	0.008750358	0.379149	163730	0.0024363	0.051499	12711618	-31399.56
		(0.001900459)	(0.050779)	(19464)	(0.249242)	(1.282976)	(1582089407)	(693589.59)
19891201	12	0.000829277	0.190918	3395241	0.001842	0.609528	97355	791.17
		(0.009697562)	(0.102061)	(41102397)	(0.002682)	(0.196541)	(61125)	(2110.42)
19900301	13	0.005846989	0.240622	418473	6.017E-08	1.028401	1617920	1695.25
		(0.001981915)	(0.076199)	(245726)	(5.84E-06)	(2.454641)	(198029482)	(3723.74)
19900601	14	0.006248546	0.256317	363940	8.699E-06	0.931855	76021	1121.28
		(0.001539829)	(0.058782)	(106547)	(5.96E-05)	(0.625434)	(90221)	(5361.04)
19900901	15	0.005789591	0.227918	454768	0.0065074	0.264043	361864	-1920.72
		(0.001230377)	(0.049283)	(139042)	(0.016766)	(0.356414)	(742489)	(13620.61)
19901201	16	0.00590308	0.237541	422803	0.0057775	0.061871	4615062	-27068.10
		(0.001231467)	(0.039786)	(77223)	(0.084245)	(0.905047)	(144268309)	(466329.21)
19910301	17	0.005775589	0.258542	377832	0.0073034	0.219490	492841	-3322.53
		(0.001343739)	(0.033937)	(41487)	(0.015276)	(0.265144)	(912430)	(16499.49)
19910601	18	0.006183799	0.220099	457126	0.0037669	0.304970	333522	-574.39
		(0.001305613)	(0.033123)	(62484)	(0.008178)	(0.218563)	(443252)	(7413.54)
19910901	19	0.004977685	0.153439	845510	3.141E-07	0.552481	1617405	2124.69
		(0.001356417)	(0.039866)	(383546)	(9.22E-05)	(3.993649)	(572136123)	(5709.12)
19911201	20	0.004670384	0.147666	929996	2.947E-07	0.570768	1616290	2120.95
		(0.001123987)	(0.034192)	(373952)	(1.19E-05)	(1.733979)	(108872375)	(5573.17)
19920301	21	0.004746648	0.149304	906320	0.0056405	0.039215	8505213	-48479.73
		(0.00085215)	(0.029070)	(262479)	(0.057619)	(0.625713)	(245993249)	(929837.40)
19920601	22	0.004388608	0.140770	1033795	0.0083977	0.077130	2478121	-21138.65
		(0.000759878)	(0.025689)	(295736)	(0.011422)	(0.298753)	(14538838)	(139527.17)
19920901	23	0.00304568	0.117942	1720953	0.003111	0.023173	26276547	-82275.49
		(0.001204841)	(0.025411)	(979015)	(0.127219)	(0.835528)	(2047295952)	(3066774.58)
19921201	24	0.000358414	0.099251	15505647	5.491E-08	0.539831	1618015	2115.61
		(0.002671427)	(0.029429)	(119234594)	(2.52E-06)	(1.916063)	(133077899)	(5785.15)
19930301	25	0.001156281	0.104984	4690146	4.577E-08	0.555057	1622935	2055.78
		(0.0016176)	(0.025258)	(7572293)	(3.12E-07)	(0.929760)	(31966428)	(5688.04)
19930601	26	0.000330132	0.101660	16099800	6.564E-11	0.966816	154560	1236.81
		(0.001647482)	(0.023746)	(83953208)	(4.68E-10)	(0.311244)	(93802)	(6080.72)
19930901	27	0.000276893	0.110256	16407388	2.655E-08	0.529364	1618035	2095.61
		(0.001125506)	(0.022945)	(70751124)	(2.79E-07)	(1.309306)	(49325298)	(7484.58)
19931201	28	0.000135152	0.161978	10591824	5.245E-06	0.338373	1756160	1108.93
		(0.00034289)	(0.038610)	(32944141)	(3.23E-05)	(0.448268)	(9806510)	(7294.32)
19940301	29	0.000194248	0.130747	15492135	1.899E-08	0.543845	1618036	2095.61
		(0.000545527)	(0.026754)	(49216477)	(1.56E-07)	(0.432086)	(5615607)	(7514.67)
19940601	30	0.000170994	0.134749	16023661	1.807E-12	0.953598	397198	1123.37
		(0.000362913)	(0.023519)	(39593268)	(8.73E-12)	(0.179381)	(137251)	(8633.30)
19940901	31	0.000124179	0.157573	12681866	3.19E-05	0.270232	2264999	1033.28
		(0.000148377)	(0.026157)	(20707833)	(0.000148)	(0.221771)	(3706082)	(11178.82)
19941201	32	0.000115182	0.189053	6365169	2.283E-09	0.567897	1618122	1381.15
		(3.1734E-05)	(0.026630)	(3705000)	(2.93E-08)	(0.546074)	(5647344)	(11900.84)
19950301	33	9.38145E-05	0.213365	4405486	1.191E-07	0.465277	1373911	1437.52
		(3.6526E-05)	(0.024820)	(1176808)	(6.03E-07)	(0.182530)	(876164)	(12464.09)
19950601	34	0.000110197	0.208026	4410514	3.165E-06	0.344751	2099365	1107.80
		(3.88989E-05)	(0.020183)	(825257)	(1.1E-05)	(0.126273)	(1254512)	(12247.85)
19950901	35	0.000187902	0.185464	4883589	2.526E-06	0.352886	2025063	1116.30
		(5.29549E-05)	(0.016595)	(802424)	(7.1E-06)	(0.097722)	(950924)	(12066.99)
19951201	36	0.000101624	0.142953	21453195	1.997E-06	0.361500	1962188	1114.48
		(9.45593E-05)	(0.023352)	(29757712)	(4.73E-06)	(0.079848)	(807886)	(11898.27)
19960301	37	8.10099E-05	0.225123	3922129	7.832E-10	0.538389	1606691	2097.19
		(6.8656E-05)	(0.034953)	(658449)	(1.43E-08)	(0.586086)	(3579926)	(26582.11)
19960601	38	2.6068E-07	0.430117	2178950	1.096E-09	0.569086	1618003	1765.18
		(4.62035E-07)	(0.058097)	(228662)	(1.05E-08)	(0.290301)	(1664930)	(49551.73)
19960901	39	2.42829E-07	0.433221	2176470	9.115E-12	0.703641	1595033	1134.29
		(4.01765E-07)	(0.053786)	(211926)	(7.95E-11)	(0.257956)	(1168902)	(48074.77)
19961201	40	3.50948E-06	0.348466	2565510	3.347E-09	0.539549	1617985	2124.89
		(4.1123E-06)	(0.039149)	(225391)	(3.11E-08)	(0.283665)	(1629749)	(49620.20)

TABLE XIII
NLS ESTIMATES FOR KOREAN DATA, MODEL A.

Data used up to (yyyymmdd)	Num of obs.	Model A					
		p	q	m	Ψ	σ	X_1
19890301	21	0.000035	0.151887	3886295	0.635799	0.266215	2.888273
		(0.006984)	(0.165570)	(776021468)	(0.247207)	(0.021871)	(1.386678)
19890601	22	0.000035	0.149750	4074213	0.631475	0.260126	2.857124
		(0.006248)	(0.150518)	(740590526)	(0.238952)	(0.020402)	(1.261744)
19890901	23	0.000037	0.146655	3965151	0.621817	0.254733	2.850168
		(0.005040)	(0.121947)	(544361975)	(0.215363)	(0.019135)	(1.057268)
19891201	24	0.000039	0.142521	3965151	0.612389	0.250152	2.814481
		(0.003952)	(0.094279)	(404932985)	(0.186854)	(0.018064)	(0.897882)
19900301	25	0.000033	0.156009	3965147	0.634598	0.256140	2.962896
		(0.002152)	(0.081141)	(262354112)	(0.165297)	(0.018557)	(0.898408)
19900601	26	0.000026	0.172787	3886285	0.678234	0.259283	3.115575
		(0.001251)	(0.085442)	(189443090)	(0.159990)	(0.018646)	(1.032581)
19900901	27	0.000028	0.168037	3886285	0.664540	0.254984	3.072678
		(0.001165)	(0.075774)	(163714062)	(0.156730)	(0.017695)	(0.917633)
19901201	28	0.000034	0.154456	3954452	0.626557	0.260687	2.952238
		(0.001294)	(0.061108)	(152789062)	(0.154542)	(0.018162)	(0.789908)
19910301	29	0.000035	0.153381	3945239	0.625814	0.256309	2.935222
		(0.000941)	(0.048403)	(109202147)	(0.132636)	(0.017252)	(0.742821)
19910601	30	0.000027	0.172607	3886297	0.641021	0.298935	3.239600
		(0.000485)	(0.051482)	(71671223)	(0.140709)	(0.023073)	(0.849220)
19910901	31	0.000031	0.164415	3886297	0.604069	0.297148	3.220223
		(0.000512)	(0.044739)	(65644595)	(0.140770)	(0.022427)	(0.809446)
19911201	32	0.000034	0.156650	3965150	0.577473	0.300198	3.174048
		(0.000511)	(0.038123)	(60444914)	(0.139002)	(0.022530)	(0.824794)
19920301	33	0.000034	0.160105	3886297	0.577949	0.298542	3.245568
		(0.000346)	(0.032868)	(41374221)	(0.126591)	(0.021942)	(0.814185)
19920601	34	0.000032	0.164211	3886297	0.589181	0.297342	3.284739
		(0.000249)	(0.031129)	(32021807)	(0.122956)	(0.021443)	(0.792540)
19920901	35	0.000118	0.159695	1152622	0.553627	0.322263	3.318161
		(0.000237)	(0.029427)	(2671169)	(0.133597)	(0.024826)	(0.901808)
19921201	36	0.000038	0.153966	3886297	0.522943	0.320128	3.381206
		(0.000217)	(0.023256)	(23363468)	(0.119981)	(0.024155)	(0.955981)
19930301	37	0.000078	0.156140	1829267	0.527860	0.317344	3.385256
		(0.000161)	(0.021886)	(4157044)	(0.117238)	(0.023414)	(0.934808)
19930601	38	0.000034	0.153289	4366124	0.517787	0.315292	3.397143
		(0.000147)	(0.019538)	(19795387)	(0.114092)	(0.022806)	(0.952192)
19930901	39	0.000031	0.154786	4687734	0.522571	0.312581	3.410934
		(0.000116)	(0.018572)	(18622827)	(0.112860)	(0.022126)	(0.929180)
19931201	40	0.000031	0.154676	4700449	0.522069	0.308651	3.410637
		(0.000097)	(0.017497)	(15701180)	(0.111433)	(0.021302)	(0.913748)
19940301	41	0.000034	0.160366	3886337	0.463264	0.311637	3.651776
		(0.000060)	(0.015204)	(7380203)	(0.112055)	(0.021450)	(1.031698)
19940601	42	0.000036	0.158934	3847689	0.461164	0.314062	3.663328
		(0.000054)	(0.014615)	(6276841)	(0.112976)	(0.021524)	(1.053796)
19940901	43	0.000035	0.160788	3847089	0.459166	0.311731	3.664407
		(0.000042)	(0.014124)	(5187332)	(0.111963)	(0.020958)	(1.034547)
19941201	44	0.000035	0.160802	3847089	0.458096	0.311035	3.667464
		(0.000035)	(0.013493)	(4388660)	(0.111492)	(0.020626)	(1.035706)
19950301	45	0.000035	0.160796	3847089	0.458459	0.307693	3.666545
		(0.000030)	(0.012832)	(3742317)	(0.110082)	(0.019959)	(1.022306)
19950601	46	0.000036	0.160842	3827535	0.457330	0.313192	3.670467
		(0.000025)	(0.012320)	(3123455)	(0.110270)	(0.020453)	(1.045520)
19950901	47	0.000036	0.160843	3827535	0.457330	0.309936	3.670467
		(0.000021)	(0.011771)	(2685863)	(0.109031)	(0.019816)	(1.033573)
19951201	48	0.000036	0.160843	3827535	0.457330	0.306699	3.670467
		(0.000018)	(0.011093)	(2261138)	(0.106719)	(0.019201)	(1.022655)
19960301	49	0.000036	0.160844	3827515	0.457327	0.328401	3.670468
		(0.000016)	(0.011261)	(2027929)	(0.112226)	(0.021788)	(1.095881)
19960601	50	0.000035	0.162538	3828135	0.457720	0.325773	3.670480
		(0.000013)	(0.011064)	(1746576)	(0.111094)	(0.021226)	(1.070093)
19960901	51	0.000035	0.162102	3830055	0.457948	0.337707	3.670621
		(0.000012)	(0.011028)	(1605830)	(0.114683)	(0.022584)	(1.111304)

TABLE XIV
NLS ESTIMATES FOR KOREAN DATA, MODELS B AND C.

Data used up to (yyyymmdd)	Num of obs.	Model B p	Model B q	Model B m	Model C p	Model C q	Model C m	Model C X_1
19890301	21	2.0549E-05	0.189185	3657237	3.304E-05	0.147933	3476839	867.01
		(0.001381635)	(0.075227)	(248857482)	(0.018728)	(0.704037)	(2005024307)	(1429.35)
19890601	22	2.14766E-05	0.186575	3657250	1.845E-07	0.415935	3583057	749.51
		(0.000965418)	(0.060111)	(166927946)	(5.85E-05)	(0.676150)	(1178535658)	(462.86)
19890901	23	2.99246E-05	0.167852	3657174	1.312E-05	0.452188	37784	999.24
		(0.001016656)	(0.049283)	(126320906)	(7.49E-05)	(0.328734)	(46138)	(510.06)
19891201	24	4.16457E-05	0.149282	3657006	2.507E-07	0.660143	21480	1000.02
		(0.001189777)	(0.043381)	(106276491)	(1.26E-06)	(0.248596)	(14213)	(495.36)
19900301	25	2.77919E-05	0.170711	3657140	1.843E-05	0.436601	38297	998.91
		(0.000584904)	(0.043096)	(79067374)	(6E-05)	(0.165081)	(25929)	(491.98)
19900601	26	9.24654E-06	0.222449	3656725	3.896E-07	0.340492	3625422	672.04
		(0.000161995)	(0.056875)	(67478180)	(5.19E-05)	(0.506221)	(520667797)	(675.63)
19900901	27	1.28595E-05	0.207891	3657197	1.406E-07	0.386047	3757603	640.71
		(0.000148947)	(0.044270)	(45072765)	(6.83E-06)	(0.321826)	(209020985)	(681.14)
19901201	28	3.55901E-05	0.162603	3657029	2.934E-10	0.814510	49053	1011.14
		(0.000489188)	(0.048994)	(53030135)	(1.23E-09)	(0.170082)	(16950)	(756.79)
19910301	29	5.33562E-05	0.144207	3656919	1.879E-09	0.743800	58036	939.89
		(0.000641105)	(0.043946)	(46365236)	(5.5E-09)	(0.117519)	(18451)	(815.07)
19910601	30	4.20265E-06	0.245569	3676284	1.524E-11	0.942929	28467	990.35
		(5.24451E-05)	(0.099606)	(53444130)	(9.5E-11)	(0.248199)	(13622)	(1006.06)
19910901	31	1.11443E-05	0.209556	3657140	2.898E-08	0.424353	3775003	459.60
		(9.28325E-05)	(0.073605)	(35911659)	(4.39E-07)	(0.460156)	(100203880)	(2798.67)
19911201	32	4.12639E-05	0.162587	3422489	9.81E-06	0.316762	321308	987.35
		(0.000308602)	(0.064785)	(29740030)	(9.02E-05)	(0.378311)	(657447)	(4069.14)
19920301	33	9.12337E-05	0.211140	544575	1.219E-11	0.815417	124860	1019.77
		(7.13183E-05)	(0.052338)	(334599)	(8.22E-11)	(0.232608)	(72259)	(3637.54)
19920601	34	2.69363E-05	0.175276	3657141	2.482E-06	0.232633	3863637	47.44
		(0.000100555)	(0.049530)	(17450087)	(0.000171)	(0.911854)	(359545932)	(4488.79)
19920901	35	2.42227E-05	0.276532	370962	3.264E-08	0.366513	3775773	815.00
		(2.97413E-05)	(0.049924)	(74239)	(9.72E-07)	(0.777324)	(194434873)	(4483.68)
19921201	36	4.38577E-05	0.246315	446557	3.353E-10	0.634964	170818	995.22
		(4.47003E-05)	(0.042252)	(90375)	(3.29E-09)	(0.310211)	(164650)	(5757.61)
19930301	37	6.36183E-05	0.228832	492907	0.0002535	0.163903	827657	770.54
		(5.51165E-05)	(0.035692)	(88700)	(0.001852)	(0.312418)	(2678179)	(6774.54)
19930601	38	0.000132884	0.145798	1770386	0.0001285	0.201038	587762	913.08
		(7.09681E-05)	(0.035990)	(2019707)	(0.00085)	(0.237367)	(1163332)	(6270.00)
19930901	39	4.00485E-05	0.155730	3714632	3.976E-07	0.268013	3780297	309.97
		(5.66602E-05)	(0.038883)	(8554403)	(7.05E-06)	(0.742106)	(147872502)	(6470.72)
19931201	40	3.27206E-05	0.160657	3830674	6.591E-08	0.316904	3806643	503.73
		(3.11142E-05)	(0.034094)	(6673564)	(4.73E-07)	(0.550435)	(76721576)	(6388.90)
19940301	41	1.65483E-05	0.176850	4265447	4.332E-06	0.211067	3439830	921.65
		(1.1647E-05)	(0.034198)	(6439514)	(2.36E-05)	(0.379691)	(37353468)	(6559.30)
19940601	42	1.68244E-06	0.241693	3657450	1.496E-08	0.346818	3780197	892.43
		(1.26962E-06)	(0.047465)	(4124634)	(1.06E-07)	(0.418097)	(40315088)	(6650.15)
19940901	43	2.18079E-06	0.235129	3659690	2.513E-06	0.235235	3775705	3.33
		(1.55622E-06)	(0.037084)	(2642104)	(1.24E-05)	(0.182728)	(8967718)	(7887.73)
19941201	44	1.59606E-06	0.242720	3657140	7.512E-09	0.407011	1274314	1007.15
		(1.14254E-06)	(0.030538)	(1711898)	(4.06E-08)	(0.141811)	(725038)	(8382.24)
19950301	45	2.40312E-06	0.245544	2518321	2.951E-07	0.292476	2589380	971.18
		(2.0679E-06)	(0.028124)	(559400)	(1.53E-06)	(0.146232)	(2586763)	(8714.51)
19950601	46	2.75939E-06	0.226551	3948162	1.711E-08	0.384983	1528601	966.05
		(2.0977E-06)	(0.027621)	(1314556)	(5.89E-08)	(0.085803)	(610514)	(9579.14)
19950901	47	3.10475E-06	0.227667	3523990	1.492E-06	0.234395	3783686	9.12
		(2.2611E-06)	(0.023249)	(711520)	(1.19E-05)	(0.234477)	(8738450)	(14674.95)
19951201	48	1.21154E-06	0.259504	2696175	6.913E-08	0.329974	2017941	1000.67
		(9.07696E-07)	(0.020506)	(258989)	(3.9E-07)	(0.136243)	(1409084)	(16037.42)
19960301	49	5.8092E-07	0.261847	3704490	3.611E-08	0.346674	1908728	999.95
		(9.37072E-07)	(0.043979)	(961948)	(1.7E-07)	(0.111093)	(1145770)	(15892.18)
19960601	50	2.71115E-07	0.277722	3707685	1.087E-06	0.234223	3773780	9.95
		(4.0613E-07)	(0.038380)	(673356)	(1.55E-05)	(0.364582)	(10987140)	(32371.49)
19960901	51	1.108E-06	0.250891	3577962	1.086E-06	0.235479	3778715	29.81
		(1.59319E-06)	(0.036369)	(542689)	(1.37E-05)	(0.307426)	(8362409)	(35438.08)