Tampere University

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# VALUE AT RISK MODELING

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# ABSTRACT

Juho Vänskä: Value at Risk modeling
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This thesis focuses on Value at Risk calculation models in the context of Nordic energy markets. Necessary background information from commodity derivatives is given along with an introduction to common Value at Risk calculation models. Additionally, financial time series analysis is studied.

Value at Risk is a common tool for risk management in financial industry. It measures the market risk of an investment portfolio. Value at Risk is defined as the largest possible loss with a given confidence level during a set period of time. Value at Risk is used as a tool in the regulation of financial institutions. It is also used by many businesses to help their own risk management.

There are several common approaches to calculating Value at Risk. This thesis studies the linear model, historical simulation and Monte Carlo simulation. The linear model assumes a distribution of portfolio returns, usually normal distribution or Student's t-distribution, and calculates the necessary distribution parameters from past price change data. The Value at Risk is then derived from the cumulative distribution function of the returns distribution. Historical simulation does not assume any distribution. Value at Risk is calculated directly from historical price data. Monte Carlo simulation does not use historical price data. Instead, a stochastic process is used to generate many different price paths for the portfolio and Value at Risk is calculated from them.

In the results section the linear model is used to calculate Value at Risk using different distributions and parameters for some common energy derivatives. The results are compared with historical price changes to study the quality of the model. For each derivative, the best distribution and parameters are calculated. The model can be refined further with the help of the results to better suit the specific needs a modeler may have in their operations.

Keywords: Value at Risk, risk management, energy markets, commodity derivatives

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# TIIVISTELMÄ

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Tämä diplomityö keskittyy Value at Risk -laskentaan Pohjoismaiden energiamarkkinoilla. Työssä tutustutaan hyödykejohdannaisiin ja yleisiin Value at Risk -laskentamalleihin. Lisäksi perehdytään aikasarja-analyysiin ja sen erityispiirteisiin rahoitusalalla.

Value at Risk on yleinen riskinhallinnan tunnusluku finanssialalla. Se kertoo sijoitusportfolion markkinariskin suuruudesta. Value at Risk määritellään suurimpana mahdollisena tappiona valitulla luottamustasolla valitun aikajakson aikana. Pankkien toiminnan säännöstelyssä käytetään apuna Value at Risk -lukua. Myös monet yksityiset toimijat käyttävät sitä oman riskinhallintansa tukena.

Value at Risk -luvun laskentaan on useita tunnettuja malleja. Tässä työssä perehdytään lineaariseen malliin, historialliseen simulaatioon ja Monte Carlo -simulaatioon. Lineaarinen malli olettaa tutkittavan portfolion tuottojakauman, yleensä normaalijakauman tai Studentin t-jakauman, ja laskee aikaisempien hintamuutosten perusteella oletetun tuottojakauman tarvittavat parametrit. Näin saadusta jakaumasta lasketaan Value at Risk analyyttisesti oletetun tuottojakauman kertymäfunktiosta. Historiallinen simulaatio ei oleta mitään jakaumaa, vaan Value at Risk lasketaan suoraan toteutuneista hintamuutoksista. Monte Carlo -simulaatiossa ei käytetä toteutuneita hintoja, vaan stokastisen prosessin avulla lasketaan paljon erilaisia mahdollisia vaihtoehtoja hintamuutoksille, joiden perusteella Value at Risk lasketaan.

Tulos-osiossa lasketaan lineaarisella mallilla Value at Risk eri jakaumaoletuksia ja parametreja käyttäen muutamille yleisille energiajohdannaisille. Tuloksia verrataan toteutuneisiin hintamuutoksiin, joita hyödyntäen tutkitaan laskentamallin tarkkuutta. Eri johdannaistuotteille selvitetään juuri niille sopivimmat jakaumaoletukset ja parametrit. Tulosten perusteella mallia voidaan kehittää vastaamaan vielä paremmin kunkin toimijan erityisiä tarpeita.

Avainsanat: Value at Risk, riskinhallinta, energiamarkkinat, hyödykejohdannaiset

Tämän julkaisun alkuperäisyys on tarkastettu Turnitin OriginalityCheck –ohjelmalla.

# PREFACE

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Helsinki, May 16th 2020

Juho Vänskä

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# LIST OF SYMBOLS AND ABBREVIATIONS

| | |
|---|---|
| ARCH | Autoregressive conditional heteroscedasticity |
| ARMA | Autoregressive moving average |
| CVaR | Conditional value at risk |
| GARCH | Generalized autoregressive conditional heteroscedasticity |
| VaR | Value at risk |
| var | Variance |
| | |
| $E$ | Expected value operator |
| $\{\varepsilon_t\}$ | Sequence of independent and identically distributed random variables |
| $\Phi$ | Standard normal distribution cumulative distribution function |
| $\varphi$ | Standard normal distribution probability density function |
| $\lambda$ | Decay factor |
| $P$ | Probability operator |
| $p$ | Price |
| $r$ | Return |
| $\rho$ | Correlation function |
| $\Sigma$ | Covariance matrix |
| $\sigma$ | Volatility |
| $t$ | Time index |
| $\mu$ | Mean |
| $v$ | Degrees of freedom of Student's t-distribution |
| $w_t$ | Gaussian white noise |
| $\{x_t\}$ | A collection of random variables |
| $\Upsilon$ | Covariance function |

# 1. INTRODUCTION

Derivatives are financial contracts which can be used for hedging risks. The role of derivatives has increased for decades in many different fields. In energy markets the prices change for many different reasons. It can be a result of natural causes like temperature or long rain period, shocks to the supply of fuels like oil or natural gas, changes in exchange rates or even political matters affecting energy markets. Sudden changes are hard to predict and from there rises the need for hedging.

Hedging is a common activity in businesses where the price, liquidity or other important aspects of an asset might change during a short period of time affecting financial stability and business operations. Hedging is usually executed in energy industry by trading derivatives, predominantly forwards, futures and options. A forward contract is a promise to buy or sell at a specific time for a given price. In financial context, buying results in a long position and selling in a short position. Futures have the same conditions but are mainly traded in exchanges, therefore being more standardized than forwards. The main focus of this thesis is on forwards and futures of different energy commodities.

Derivatives trading raises the issue of managing financial risk related those activities. Risk modeling is a vital part of financial risk management. The goal of risk modeling is to measure the uncertainty of the future. In the context of financial markets, one important component of risk to measure is the market risk. Market risk is the uncertainty in the future value of an investment portfolio due to changes in asset prices. The fundamental purpose of measuring market risk is to summarize the potential for deviations from the expected value of an investment portfolio. Thus, it is needed to know about the potential for individual asset prices to vary and about the dependency between movements of different asset prices.

Traditional approach to limiting risk has been position limits. The most simple form of a risk limit is a nominal limit. An example of such a limit is for a portfolio to have a maximum value of 10 million euros. The deficiency of such limits is that they don’t measure risk at all. A portfolio value of 10 million euros has a lot more risk related to it during volatile market conditions than during stable ones. Hedging operations might be affected by simple limits as well. Properly hedging assets reduces risks, yet simple limits might consider hedging as increasing risk. Value at risk measures the actual risk by estimating possible losses within the relevant context. It accounts for the size of exposure, hedging and market conditions.

Value at risk became a very common approach for measuring market risk in all financial institutions during 1990s due to regulators adopting value at risk as a tool for regulation and large banks developing their own risk management systems based on the value at risk approach. The appeal of value at risk approach to risk management is that it is universal, easy to understand and can be applied to all activities and to all kinds of risk, across different markets and different exposures to market at any level, from a single trade up to a company-wide portfolio. The output of a value at risk calculation is the amount of money that could be lost over a period of time with some chosen probability. As such, it is a good tool for a risk manager to assess and compare risks related to different activities, markets and investment portfolios.

In the following chapters general results from literature are introduced. Nordic energy markets are explored along with relevant properties of financial markets. The concept of Value at Risk (VaR) is introduced and its usefulness in financial risk management demonstrated. Different modeling approaches to VaR are explored with their pros and cons.

The main focus is on a linear VaR model. Its properties are defined and analytic formulas for VaR calculation are derived. The strengths and weaknesses of the approach are explored and the validity of different distribution choices and parametrization is studied for commodity derivatives including oil, natural gas and coal as well as electric power. The results give a strong basis for refining a linear VaR model suitable for specific needs in the context of energy markets.

# 2. BACKGROUND INFORMATION

The main focus of the thesis is on the Nordic Europe energy forwards and futures markets and the associated risks. Artzner et al (1999) define *risk* in the financial context as variability of the future value of an asset or portfolio. Market actors that include energy futures in their portfolios or otherwise conduct business in energy markets are exposed to many sources of risk. Local and global events can impact the delivery of commodities, weather conditions vary greatly from one year to another and power plants may experience unplanned downtime. Energy prices can fluctuate considerably both on long and short term. Futures contracts have been used for decades to hedge against risks.

Energy futures have varying risk characteristics compared with other assets, such as stocks and commodities like copper. Some energy commodities, such as oil and coal, can be stored but others, like electric power, cannot. Especially the commodities that cannot be stored have a unique market dynamic. In the case of electric power, for example, supply and demand must be balanced at every moment in time making the related markets different from most. The energy commodities that can be stored have their own unique characteristics as well. One example of that is how different commodities are stored in different geographical locations, introducing risks related to the location of stores and the transportation of commodities.

## 2.1 Characteristics of Nordic energy markets

According to Westgaard et al (2019) the Nordic energy markets have unique characteristics. The findings are similar to previous studies and to the general consensus about the characteristics of these markets. Coal and crude oil prices depend on global economy conditions, and power futures and natural gas depend more on local market conditions. The returns for all commodities studied exhibit a distribution with high peaks and fat tails. Based on Jarque-Bera normality test, the null hypothesis of normal distribution is rejected for all of them. The mean of the distribution is centered around zero, supporting using zero mean assumption for the return distribution used in the thesis. The volatility varies between commodities. For oil and coal, the historic price data shows volatility clustering around 2009 and 2016. For gas and power, the data shows spiky trends with some very extreme returns, driven by more local shocks to supply and demand.

All of the commodities studied are quite symmetrical about minimum and maximum returns. The exceptions are found in Nordic power first-month position and German power third month position. The finding justifies simplified assumptions about using a symmetric distribution to model the distribution of returns. It makes value at risk calculation models easier to build, because similar tail behavior can be expected for both long and short positions.

## 2.2 Financial returns

Most financial analyses use returns instead of asset prices. The main reasons for that are laid out by Campbell et al (1997). Returns give a scale-free summary of the investment opportunity and a series of returns is statistically easier to handle than a series of prices. The scaling issue is very important especially in the case of Nordic power markets, because power prices in the Nordics have considerable seasonal fluctuation. A price change of 1 euro has a different significance if the price is 20 euros than if the price is 50 euros. Using returns instead of prices is a good way to deal with the issue.

**Definition.** Return *r* is defined as

$$r = \frac{p_t - p_{t-1}}{p_{t-1}},$$

where $p_t$ is the market price at time t and $p_{t-1}$ the previous observed price. In this thesis, the mean of returns is assumed to be zero. The assumption is justified by focusing on a short period of one day ahead forecast. Estimating the mean returns is difficult to do accurately. Additionally, the variance of returns can be written as

$$var = E(r_t^2) - \left(E(r_t)\right)^2.$$

According to Jorion (1995) the term $E(r_t^2)$ dominates the term $\left(E(r_t)\right)^2$ by a typical factor of 700:1. Therefore, the assumption of zero mean returns is reasonable and likely more precise than an attempt to estimate the mean based on historic data.

The studies of financial return series show the distribution of returns to be non-normal. They are usually leptokurtic, meaning fat tails and high peak at the mean. It implies that there are many days when the market prices do not change much at all and on the days that they do, the changes tend to be quite significant. There is also a lot of evidence for the return distributions being skewed (Albuquerque 2012). These empirical findings pose a challenge for modeling the distribution of returns. Fat tails and skewness are the most important characteristics from risk modeling perspective. A risk model should reflect the tail behavior of returns and also account for the skewed distribution, which means the model might need to handle long and short positions in a different manner.

From historic data, it is clear that there have been periods of high returns and low returns while most of the time returns float around the historical mean. The phenomenon of high (low) returns being more likely following high (low) returns is usually referred to as volatility clustering, because high returns mean high volatility and low returns low volatility. The characteristic of returns having volatility clustering leads to the desire to have a volatility model that reflects these observations.

## 2.3 Issues with data

Historical market data time series are constructed by recording prices at a regular interval, such as daily. Settlement and closing prices are released by the exchange and are easily recorded. Closing prices are most commonly used in value at risk calculation (Holton 2014). In this thesis, closing prices are used for all calculations.

For some products, such as commodity futures, there are multiple contracts being traded simultaneously forming multiple overlapping time series corresponding to different contracts. Each one begins when the corresponding contract starts to trade and ends when that contract expires. Exhibit 1 illustrates what this may look like in practice. Therefore, it is common to construct a continual time series from the overlapping time series by sorting the contracts into different buckets based on their expiration dates (Holton 2014). For a contract with monthly delivery, for example, the continual time series can be constructed by sorting data points by time to maturity. Suppose today is July 15$^{th}$ and we are studying a contract that expires at the end of August. Then, the continual time series can be constructed such that only data points between June 16$^{th}$ and July 15$^{th}$ are considered for the contract maturing at the end of August. More data for the continual time series is added by considering data points between May 16$^{th}$ and June 15$^{th}$ for the same contract maturing at the end of July and so on.

| Time (month) | Contract (indicated by month of expiration) | | | | |
|---|---|---|---|---|---|
| | Jan-00 | Mar-00 | May-00 | Jul-00 | Sep-00 |
| Apr-99 | 241.2 | | | | |
| May-99 | 244.1 | | | | |
| Jun-99 | 243.6 | 248.8 | | | |
| Jul-99 | 239.3 | 244.0 | 246.0 | | |
| Aug-99 | 245.7 | 248.5 | 251.5 | | |
| Sep-99 | 237.3 | 242.3 | 246.5 | 252.0 | |
| Oct-99 | 230.0 | 235.0 | 239.0 | 243.0 | |
| Nov-99 | 223.5 | 225.2 | 228.9 | 232.4 | 235.5 |
| Dec-99 | 216.7 | 218.5 | 221.6 | 224.7 | 231.5 |
| Jan-00 | | 221.7 | 225.0 | 229.0 | 233.0 |
| Feb-00 | | 221.8 | 222.1 | 225.0 | 230.0 |
| Mar-00 | | | 231.0 | 232.5 | 240.5 |
| Apr-00 | | | 238.2 | 241.5 | 244.5 |
| May-00 | | | | 224.0 | 227.0 |
| Jun-00 | | | | 218.0 | 218.0 |
| Jul-00 | | | | | 214.8 |
| Aug-00 | | | | | 219.0 |

***Exhibit 1.*** Monthly prices for flaxseed futures traded on the Winnipeg Commodities Exchange. Prices are in Canadian dollars per tonne. (Holton 2014)

The continual time series captures relevant price behavior when constructed in a way that accounts for time to maturity. Time to maturity has an effect on the price movements in such a way that volatility is usually much higher close to expiry. This bucketing system also provides means to gather more data than only the available data for the specific contract because contracts are traded only for a limited time. For example, a contract might be traded only for six months which is around 125 trading days. 125 data points can be, and often is, insufficient for value at risk calculations.

## 2.4 Value at Risk

**Definition** (Artzner et al. 1999) For $\alpha \in (0,1)$, the number q is an $\alpha$ quantile of the random variable X under the probability distribution P if one of the three equivalent properties below is satisfied:

i) $P(X \leq q) \geq \alpha \geq P(X < q)$

ii) $P(X \leq q) \geq \alpha$ and $P(X \geq q) \geq 1 - \alpha$

iii) *$F_X(q) \geq \alpha$ and $F_X(q_-) \leq \alpha$ with $F_X(q_-) = lim_{X->q,x<q}F(x)$*, where $F_X$ is the cumulative distribution function of X.

**Definition** (Artzner et al. 1999) Given α ∈ (0,1), and a reference instrument r, the value at risk $VaR_\alpha$ at level α of the final net worth X with distribution P is the negative of the quantile $q_\alpha^+$ of X/r; that is,

$$VaR_\alpha(X) = -\inf\{\, x \mid P(X \leq xr) > \alpha\}.$$

Value at risk estimates how much an asset or portfolio might lose in value in a set time period. The quantile α to be used in the calculation sets the probability that the losses exceed the value at risk. It is commonly used in the financial industry to quantify risk. It is usually assumed that there is no trading during the set time period. Then the relative weights of different assets in the portfolio may change during the set time period due to fluctuations in the asset values. In contexts where that is undesirable, value at risk can be calculated assuming constant rebalancing of the portfolio weights as the asset values fluctuate.

# 3. TIME SERIES ANALYSIS

Developing mathematical models which provide plausible descriptions for sample data is the primary objective of time series analysis. It requires a statistical setting for describing the character of data that seemingly fluctuates randomly over time. The usual approach is to assume a time series can be defined as a collection of random variables which are indexed according to the order they are obtained in time. A collection of random variables, $\{x_t\}$, is called a stochastic process. In financial time series analysis, t is usually discrete and can only have integer values. The observed values of a stochastic process are referred to as a realization of the stochastic process. Below are given definitions for common time series analysis terminology.

**Definition** (Tsay 2010) A *white noise* is a time series $x_t$ where $\{x_t\}$ is a sequence of independent and identically distributed random variables with finite mean and variance.

**Definition** (Shumway et al, 2006) The *mean function* is defined as

$$\mu_{xt} = E(x_t) = \int_{-\infty}^{\infty} x f_t(x) dx,$$

where E is the expected value operator.

**Definition** (Shumway et al, 2006) The *autocovariance function* is defined as the second moment product

$$\Upsilon_x(s,t) = E[(x_s - \mu_s)(x_t - \mu_t)],$$

for all *s* and *t*.

**Definition** (Shumway et al, 2006) The *autocorrelation function* is defined as

$$\rho(s,t) = \frac{\gamma(s,t)}{\sqrt{\gamma(s,s)\gamma(t,t)}}.$$

**Definition** (Shumway et al, 2006) The *cross-covariance function* between two series $x_t$ and $y_t$ is

$$\gamma_{xy}(s,t) = E\big[(x_s - \mu_{xs})(y_t - \mu_{yt})\big].$$

**Definition** (Shumway et al, 2006) *The cross-correlation function* is a scaled cross-covariance function

$$\rho_{xy}(s,t) = \frac{\gamma_{xy}(s,t)}{\sqrt{\gamma_x(s,s)\gamma_y(t,t)}}.$$

*Stationarity* is an important concept in financial time series analysis. Usually some kind of stationarity is assumed in financial models.

**Definition** (Shumway et al, 2006) A *strictly stationary* time series is one for which the probabilistic behaviour of every collection of values

$$\{x_{t_1}, x_{t_2}, \dots, x_{t_k}\}$$

is identical to that of the time shifted set

$$\{x_{t_1+h}, x_{t_2+h}, \dots, x_{t_k+h}\}.$$

That is,

$$P\{x_{t_1} \le c_1, \dots, x_{t_k} \le c_k\} = P\{x_{t_1+h} \le c_1, \dots, x_{t_k+h} \le c_k\}$$

for all $k$ = 1, 2, …, all time points $t_1, t_2, \dots, t_k$, all numbers $c_1, c_2, \dots, c_k$, and all time shifts $h = 0, \pm 1, \pm 2, \dots$ .

If a time series is *strictly stationary*, then all of the multivariate distributions functions for subsets of variables must agree with their counterparts in the shifted set for all values of the shift parameter $h$.

Strictly stationary assumption is usually too strong for practical model development. Additionally, assessing strict stationarity from a single data set is difficult. A weakly stationary time series assumption is generally more useful for financial time series modeling.

**Definition** (Shumway et al, 2006) A *weakly stationary* time series $x_t$ is a finite variance process such that

i) the mean value function $\mu_t$ is constant and does not depend on time t

ii) the covariance function γ(s,t) depends on s and t only through their difference $|s - t|$.

The mean function of a weakly stationary time series is independent of time and thus will be noted as $\mu$,

$$\mu_t = \mu.$$

Let $s = t + h$, where h represents the time shift or lag. Then,

$$\gamma(t+h,t) = E[(x_{t+h} - \mu)(x_t - \mu)] = E[(x_h - \mu)(x_0 - \mu)] = \gamma(h,0)$$

does not depend on the time argument $t$. Thus, notation $\gamma(h) = \gamma(h,0)$ is used for weakly stationary time series. Using the notation, the autocovariance function of a weakly stationary time series is written as

$$\gamma(h) = E[(x_{t+h} - \mu)(x_t - \mu)]$$

and the autocorrelation function of a weakly stationary time series is written as

$$\rho(h) = \frac{\gamma(t+h,t)}{\sqrt{\gamma(t+h,t+h)\gamma(t,t)}} = \frac{\gamma(h)}{\gamma(0)}.$$

**Definition** (Shumway et al, 2006) A *linear process* $x_t$ is defined to be a linear combination of white noise variates $w_t$, and is given by

$$x_t = \mu + \sum_{j=-\infty}^{\infty} \psi_j w_{t-j},$$

where

$$\sum_{j=-\infty}^{\infty} |\psi_j| < \infty .$$

**Definition** (Shumway et al, 2006) A process $\{x_t\}$ is said to be a *Gaussian process* if the k-dimensional vectors **x** = $(x_{t_1}, x_{t_2}, \dots, x_{t_k})$ for every collection of time points $t_1, t_2, \dots, t_k$ and every positive integer k have a multivariate normal distribution.

## 3.1 ARMA model

ARMA model describes weakly stationary time series data in a simple polynomial form. It is a tool for understanding and predicting future values of time series data. ARMA model combines autoregressive and moving average models and requires the time series to be weakly stationary. The underlying assumption of the model is constant volatility. According to the ARMA model, future occurrences are linearly dependent on past values and white noise.

**Definition** (Shumway et al, 2006) An autoregressive model of order p, AR(p), is of the form

$$x_t - \mu = \varphi_1(x_{t-1} - \mu) + \varphi_2(x_{t-2} - \mu) + \cdots + \varphi_p(x_{t-p} - \mu) + w_t \,,$$

where $x_t$ is weakly stationary, $\varphi_1, \varphi_2, \ldots, \varphi_p$ are constants ($\varphi_p \neq 0$) and $w_t$ is a Gaussian white noise series with mean zero and variance $\sigma_\omega^2$.

**Definition** (Shumway et al, 2006) The moving average model of order q, MA(q), is defined to be

$$x_t = w_t + \theta_1 w_{t-1} + \theta_2 w_{t-2} + \cdots + \theta_q w_{t-q} \,,$$

where there are q lags in the moving average, $\theta_1, \theta_2, \ldots, \theta_q$ are parameters ($\theta_q \neq 0$) and $w_t$ is Gaussian white noise.

**Definition** (Shumway et al, 2006) A time series {$x_t$; t = 0, $\pm$1, $\pm$2, …} is ARMA(p, q) if it is weakly stationary and

$$x_t = \varphi_1 x_{t-1} + \cdots + \varphi_p x_{t-p} + w_t + \theta_1 w_{t-1} + \cdots + \theta_q w_{t-q} \,,$$

with $\varphi_p \neq 0, \theta_q \neq 0$, and $\sigma_w^2 > 0$. Parameters *p* and *q* are called the *autoregressive* and the *moving average* orders, respectively and $w_t$ is a Gaussian white noise sequence. If $x_t$ has a nonzero mean μ, an additional parameter $\alpha = \mu(1 - \varphi - \cdots - \varphi_p)$ is added to the right side of the above equation:

$$x_t = \alpha + \varphi_1 x_{t-1} + \cdots + \varphi_p x_{t-p} + w_t + \theta_1 w_{t-1} + \cdots + \theta_q w_{t-q}.$$

Calculating Value at Risk using the ARMA model involves predicting the future value of portfolio and its error distribution with the ARMA model. Value at Risk measure is found by determining the desired quantile from the error distribution.

# 4. VOLATILITY MODELING

Volatility is the statistical measure used to assess the potential for individual asset prices to vary, usually by standard deviation or variance of the return of an asset. Larger deviations around the mean imply higher risk and lower deviations imply lower risk. The characteristics of volatility enable volatility forecasting with the help of statistical theory and time series analysis.

Modeling volatility is an important part of financial literature. It is used in modern portfolio theory and option pricing models. Therefore, there is a lot of literature available on volatility and its modeling. A simple approach is to calculate the volatility from historical data, but that doesn't allow volatility to fluctuate over time.

Based on empirical observations, volatility tends to cluster (Tsay, 2010). That is, during some periods it is higher than during other periods. Additionally, volatility fluctuation tends to be small in a short time period. Volatility tends to fluctuate differently during times of increasing returns than during times of decreasing returns. Also, based on historical data, volatility is always finite. These properties are desirable to capture when developing a model for volatility.

There is a lot of research on the forecasting performance of volatility models. Unfortunately, many of them contradict each other and there is no consensus for the best model. The best performing models are restricted to specific applications and their model parameters need to be adjusted from time to time. More general models can be applied to a wider range of applications with the downside of being less accurate. An example of contradicting results is the study by Hansen and Lunde (2005). They found the simple GARCH(1, 1) model to outperform many other, more sophisticated models. Overall, the forecasting performance of a model depends on which dataset it is applied on. The difficulty of developing an accurate general model for volatility has led to financial institutions developing their own proprietary models, which are fine tuned for their particular needs and are unfortunately not publicly available for study.

## 4.1 Uniformly weighted moving average

The most simple approach to estimating volatility is to calculate the standard deviation from a sample of returns. The sample size is kept constant; every time new data is added to the sample, the oldest data is removed from it. The volatility estimate is calculated by

$$\sigma_t^2 = \frac{1}{T}\sum_{i=1}^{T} r_{t-i}^2,$$

where $\sigma_t^2$ is the volatility forecast for time t, $r_t$ is the return on time t and T is the sample size.

The method has several weaknesses. It is sensitive to the chosen sample size. A large sample leads to a very slowly changing volatility estimate, whereas a small sample leads to a quickly changing estimate. Due to equally weighted datapoints, the oldest and the most recent observations have an equal effect on the volatility estimate. It is an undesirable feature, especially under changing market conditions. It may take a long time for the volatility estimate to properly reflect the changing market conditions. Similarly, a short period of higher volatility increases the estimate as long as the period is within the sample. Then, the volatility estimate will drop for seemingly no reason as the sample doesn't include the period anymore. Exponentially weighted moving average reduces these undesirable properties from which a moving average model inherently suffers.

## 4.2 Exponentially weighted moving average

To improve the shortcomings of equally weighted moving average, each observation is weighted by a weighting factor $\lambda^i$. The constant λ is a number between zero and one, often called the decay factor. The index *i* is an integer and the value *i* = 0 is given to the most recent observation, *i* = 1 to the observation before that and so on. This way, the most recent observations are weighted much more heavily than the older ones. It leads to a volatility estimate, that better reflects current market conditions. With exponential weighting, the choice of sample size becomes less important as well. The effect of a more volatile period decreases exponentially over time and by the time it is not included in the sample anymore its significance has been weighted down.

J.P. Morgan (1996) developed RiskMetrics model during the 1990's and they concluded $\lambda = 0.94$ to be the best choice for daily returns data. Different decay factors can be considered, such as $\lambda = 0.9$ or $\lambda = 0.97$, depending on the purpose of the model. The greater the decay factor, the more the model resembles equally weighted moving average with $\lambda = 1$ reducing the model to an equally weighted moving average model. A high value for $\lambda$ is therefore undesirable. On the other hand, smaller values for $\lambda$ have their own deficiencies as well. A low value for $\lambda$ leads to a sensitive volatility estimate, because the newest observations are so heavily weighted. As a result, the volatility estimate is effectively using only few observations.

The volatility estimate is calculated by

$$\sigma_t^2 = \frac{\sum_{i=1}^{t} \lambda^{i-1} r_{t-i}^2}{\sum_{i=1}^{t} \lambda^{i-1}},$$

where $\sigma_t^2$ is the volatility forecast for time *t*, $\lambda$ is the decay factor and $r_t$ is the return on time *t*. For practical calculation purposes, it is useful to derive an equivalent formula for volatility when $t \rightarrow \infty$:

$$\sigma_t^2 = \frac{\sum_{i=1}^{t} \lambda^{i-1} r_{t-i}^2}{\sum_{i=1}^{t} \lambda^{i-1}}$$

The denominator $\sum_{i=1}^{t} \lambda^{i-1} = \frac{1}{(1-\lambda)}$ , thus

$$\sigma_t^2 = (1-\lambda) \sum_{i=1}^{t} \lambda^{i-1} r_{t-i}^2 \qquad (i)$$

Taking $r_{t-1}^2$ out of the summation gives

$$\sigma_t^2 = (1-\lambda)\left(r_{t-1}^2 + \sum_{i=2}^{t} \lambda^{i-1} r_{t-i}^2\right)$$

$$\sigma_t^2 = (1-\lambda)\left(r_{t-1}^2 + \sum_{i=1}^{t-1} \lambda^{(i-1)+1} r_{(t-1)-i}^2\right)$$

Taking $\lambda$ out of the summation gives

$$\sigma_t^2 = (1-\lambda) r_{t-1}^2 + \lambda(1-\lambda) \sum_{i=1}^{t-1} \lambda^{i-1} r_{(t-1)-i}^2$$

Finally, applying $(i)$ gives

$$\sigma_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2.$$

## 4.3 ARCH model

The ARCH (autoregressive conditional heteroskedasticity) model provides a systematic framework for volatility modeling. ARCH models introduce a shock $x_t$ to the asset return which is serially uncorrelated. However, it is dependent and the dependence of $x_t$ can be described by a simple quadratic function of its lagged values. Specifically, an ARCH(m) model assumes that (Tsay, 2010)

$$x_t = \sigma_t \varepsilon_t , \qquad \sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \cdots + \alpha_m x_{t-m}^2 ,$$

where $\{\varepsilon_t\}$ is a sequence of independent and identically distributed random variables with mean zero and variance 1, $\alpha_0 > 0$, and $\alpha_i \geq 0$ for $i > 0$. The coefficients $\alpha_i$ must satisfy some regularity conditions to ensure that the unconditional variance of $x_t$ is finite. In practice, $\varepsilon_t$ is often assumed to follow the standard normal or a standardized Student's-t or a generalized error distribution. The historical appeal of ARCH model is in its ability to include volatility clustering in the model, during a time when other models could not account for it.

Let's examine the ARCH(1) model

$$x_t = \sigma_t \varepsilon_t , \qquad \sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 ,$$

where $\alpha_0 > 0$ and $\alpha_1 \geq 0$.

The unconditional mean of $x_t$

$$E(x_t) = E[E(x_t | F_{t-1})] = E[\sigma_t E(\epsilon_t)] = 0$$

and the unconditional variance of $x_t$ can be obtained as

$$\mathrm{Var}(x_t) = \mathrm{E}(x_t^2) = \mathrm{E}[\mathrm{E}(x_t^2 | F_{t-1})] = E(\alpha_0 + \alpha_1 x_{t-1}^2) = \alpha_0 + \alpha_1 E(x_{t-1}^2).$$

Since $x_t$ is a stationary process with $E(x_t) = 0$,

$$Var(x_t) = Var(x_{t-1}) = E(x_{t-1}^2).$$

Thus,

$$Var(x_t) = \alpha_0 + \alpha_1 Var(x_t)$$

$$Var(x_t) = \frac{\alpha_0}{1 - \alpha_1}$$

The variance of $x_t$ must be positive, thus

$$0 \leq \alpha_1 < 1.$$

### 4.3.1 GARCH model

GARCH (generalized autoregressive conditional heteroskedasticity) model is an extension of ARCH model, where dependence from the variance of previous time step is added to the model in the following way:

$$x_t = \sigma_t \varepsilon_t , \qquad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i x_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2,$$

where $\{\varepsilon_t\}$ is a sequence of independent identically distributed random variables with mean zero and variance 1, $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_i) < 1$. Additionally, $\alpha_i = 0$ for $i > m$ and $\beta_j = 0$ for $j > s$.

Setting $i = 1$ and $j = 1$ results in the GARCH(1,1) model

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \beta_1 \sigma_{t-1}^2 ,$$

which can be shown to be equivalent to ARCH(∞). In other words, GARCH model generalizes ARCH model to include infinitely many past squared returns when estimating conditional volatility.

### 4.3.2 Estimation of GARCH parameters

A common way to estimate GARCH model parameters is maximum likelihood. From an identically and independently distributed random variable following a parametric distribution with density function f(·) can be drawn a sample $z = \{z_1, z_2, \ldots, z_T\}$ size T. A likelihood function

$$L(\theta, z) = \prod_{t=1}^{T} f(z_t, \theta)$$

is defined, where θ is the parameters and z is the sample. The parameter estimations are found by maximizing the likelihood function. In practice it is difficult to solve for the maximum likelihood functions. Solver algorithms and optimizers are commonly used to estimate parameters instead.

# 5. VALUE AT RISK MODELS

## 5.1 Parametric linear VaR models

Parametric linear VaR models calculate VaR using analytic formulae that are based on an assumed parametric distribution for the risk factor returns, when the portfolio value is a linear function of its underlying risk factors. The most basic assumption behind the models is that the returns of the portfolio are independent and identically distributed with a normal distribution.

In the parametric linear VaR models, all dependencies between the risk factors are assumed to be represented by correlations. The correlations are used to build a covariance matrix. It is done by using a moving average model with historical returns data.

### 5.1.1 Normal linear VaR

The portfolio's returns are assumed to be normally distributed and independent and identically distributed in the normal linear method for calculating VaR (Holton, 2014). Let *X* be the portfolio return and $x_\alpha$ be the α quantile return, such that

$$P(X < x_\alpha) = \alpha. \tag{i}$$

Then,

$$P(X < x_\alpha) = P\left(\frac{X-\mu}{\sigma} < \frac{x_\alpha - \mu}{\sigma}\right) = P\left(Z < \frac{x_\alpha - \mu}{\sigma}\right), \tag{ii}$$

where $Z \sim N(0, 1)$. Substituting i) into ii) gives

$$P\left(Z < \frac{x_\alpha - \mu}{\sigma}\right) = \alpha.$$

Since *Z* is normally distributed,

$$P\left(Z < \varphi^{-1}(\alpha)\right) = \alpha$$

$$\frac{x_\alpha - \mu}{\sigma} = \varphi^{-1}(\alpha),$$

where $\varphi$ is the standard normal cumulative distribution function. The standard normal distribution is symmetric and thus

$$\varphi^{-1}(\alpha) = -\varphi^{-1}(1 - \alpha).$$

Thus, an analytic formula for α quantile VaR is

$$VaR = \varphi^{-1}(1 - \alpha)\sigma - \mu.$$

VaR is often calculated for such a short time period that μ is assumed to equal zero.

### 5.1.2 Student's t-distributed linear VaR

Student's t-distribution can be a better assumption for the distribution of returns than normal distribution. Historically, the distribution of financial returns have had heavier tails than a normal distribution assumption would predict, especially for daily returns. The student's t-distribution can reflect this more accurately than normal distribution due to its leptokurtic properties. A *Leptokurtic* distribution has kurtosis greater than 3 (Holton, 2014). Leptokurtic distributions have simultaneously higher peaks and fatter tails than a normal distribution.

If a random variable $T$ has a student's t-distribution with $v$ degrees of freedom, its density function is

$$f_v(t) = (v\pi)^{-\frac{1}{2}}\Gamma\left(\frac{v+1}{2}\right)(1 + v^{-1}t^2)^{-\left(\frac{v+1}{2}\right)},$$

where $\Gamma$ is the gamma function. Its variance for $v > 2$ is $V(T) = v(v - 2)^{-1}$. Let's denote the α quantile of the standard Student's t-distribution by $t_v^{-1}(\alpha)$. Then the α quantile of the standardized Student's t-distribution with v degrees of freedom is $\sqrt{v^{-1}(v-2)}\ t_v^{-1}(\alpha)$. Let $X$ denote the daily return on a portfolio and suppose it has standard deviation $\sigma$ and mean $\mu$. Then $X = \mu + \sigma T$, where $T$ is a standardized Student's t random variable. Like in the case of the normal linear value at risk, Student's t VaR is calculated as

$$VaR_{\alpha,v} = \sqrt{v^{-1}(v-2)}\, t_v^{-1}(1 - \alpha)\sigma - \mu.$$

## 5.2 Historical simulation method for calculating VaR

Historical approach to calculating VaR doesn't make any assumptions about the shape of the distribution of returns or the dependencies between different assets. Instead, the observed past returns are used directly to estimate VaR. The simple approach is to arrange the returns data in ascending order and use the $\alpha$ quantile to determine VaR directly from the data. Another approach is to randomly select $n$ values with replacement from the data and use them to determine the VaR estimate. For example, for $n$ = 100, the 95% VaR is determined from the sample by arranging the sample values in ascending order and using the fifth lowest return value as the VaR estimate.

The main advantages of the historical approach to value at risk is its easy implementation. It requires no volatility or distribution modeling. It accommodates nonlinearities and all kinds of distributions. The main disadvantage of historical approach is in the philosophy itself. The method assumes that past events reflect the future perfectly and all future risks are already captured in the observation period. The observation period is a problem by itself. As the returns distribution is assumed to be exactly the one in the observation period, the choice of the observation period affects the value at risk measure directly. For example, dropping the oldest data points from the observation period or adding in new ones cause the value at risk measure to fluctuate. The effect can be significant even if nothing is happening at the market. The method has been empirically found to underestimate value at risk (Alexander 2008). Nevertheless, it can be useful in situations where a simple method is needed for a quick estimate of value at risk with the understanding of its shortcomings. It is also easy to understand and completely free of modeling and implementation risk. In fact, it is widely in use in the banking industry. Pérignon and Smith (2010) found that 73% of banks that disclosed their value at risk calculation methodology used historical simulation in 2005.

## 5.3 Monte Carlo simulation approach for VaR

Monte Carlo simulation is an extremely flexible tool that has numerous applications to finance. It is usually used when analytic solutions do not exist or when other numerical methods fail. In a Monte Carlo approach to VaR calculation, the future portfolio value is simulated with a mathematical model that is specifically built to forecast asset price changes. The simulation is run many times and from the simulation results VaR is estimated like in the case of historical simulation.

### 5.3.1 Cholesky matrix

Cholesky matrix is useful when simulating correlated normal random variables from a specified covariance matrix $\Sigma = AA^T$, which is square and symmetric. Using a transformation $\boldsymbol{y} = A\boldsymbol{x}$, a vector of normal random variables with covariance matrix $\Sigma$ can be created by simulating a vector **x** of independent normal random variables. The vector **y** accounts for the dependencies between assets in a portfolio.

A positive semidefinite matrix $A$ can be factored in the form $A = KK^T$ for some real square matrix $K$. A particularly easy factorization $A = KK^T$ is known as the *Cholesky factorization*. A positive semidefinite matrix has a factorization of the form $A = LL^T$, where $L$ is a lower triangular matrix. Solving for $L$ is straightforward. As an example, let's consider a 2×2 matrix $A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$. Its Cholesky factorization takes the form

$$A = \begin{bmatrix} l_{1,1} & 0 \\ l_{2,1} & l_{2,2} \end{bmatrix} \begin{bmatrix} l_{1,1} & l_{2,1} \\ 0 & l_{2,2} \end{bmatrix}$$

Thus,

$$a_{1,1} = l_{1,1}^2, \qquad a_{1,2} = l_{1,1} l_{2,1}, \qquad a_{2,2} = l_{2,1}^2 + l_{2,2}^2$$

$$l_{1,1} = \sqrt{a_{1,1}}, \qquad l_{2,1} = \frac{a_{1,2}}{l_{1,1}}, \qquad l_{2,2} = \sqrt{a_{2,2} - l_{2,1}^2}$$

The algorithm generalizes for higher dimensions (Holton 2014). It entails two kinds of calculations. The diagonal elements $l_{i,i}$ entail taking a square root and off-diagonal elements $l_{i,j}$ $i > j$ entail dividing by the last calculated diagonal element. For a positive definite matrix A, all diagonal elements will be nonzero and its Cholesky matrix is unique, if only the positive square roots are considered.

# 6. CONDITIONAL VALUE AT RISK

Value at risk has been a controversial subject due to its shortcomings. One such issue is that value at risk does not help quantify risk beyond the quantile that is used for calculating it. A 95 % 1 day VaR of 10 000 € for a portfolio tells nothing about risk in the worst 5 % of occurances, except that it is greater or equal than 10 000 €. The risk could be 3*VaR, 10*VaR, or even much more than that in the worst 5 % of the return distribution. Therefore, it is difficult for a risk manager to be prepared for such a risk.

Another problem with value at risk is that it is not a coherent risk measure. Artzner et al. (1999) defined four axioms for a coherent risk measure:

1) Translation invariance

   for $\alpha \in \mathbb{R}$, $\varphi(X + \alpha r) = \varphi(X) - \alpha$

2) Subadditivity

   $\varphi(X_1 + X_2) \leq \varphi(X_1) + \varphi(X_2)$

3) Positive homogeneity

   for $\beta \geq 0$, $\varphi(\beta X) = \beta\varphi(X)$

4) Monotonicity

   for $X \leq Y$, $\varphi(Y) \leq \varphi(X)$

In the four axioms, $\varphi$ is an arbitrary risk measure and X and Y are arbitrary investments. Artzner et al. show that value at risk does not always satisfy the subadditivity axiom and thus is not a coherent risk measure. This can lead to a situation where, according to VaR, it would be less risky to split a portfolio into sub portfolios than to have all assets in one portfolio.

Another risk measure, that is coherent (Alexander 2008), conditional value at risk (CVaR) is closely related to value at risk and can be used to amend the shortcomings of value at risk. Let X denote the returns over the set time period. Then

$$VaR_\alpha = -x_\alpha,$$

where $x_\alpha$ is the $\alpha$ quantile of the distribution of $X$.

**Definition** (Alexander 2008)

$$CVaR_\alpha(X) = -E(X|X < x_\alpha).$$

Since $P(X < x_\alpha) = \alpha$, if X has a probability density function f(x), then

$$CVaR_\alpha(X) = -\frac{1}{\alpha}\int_{-\infty}^{x_\alpha} xf(x)dx.$$

Conditional value at risk is the expected loss given that the loss exceeds value at risk. In that regard, it is more informative than value at risk. Because it is also coherent, the aggregation of risks can be done accurately under all circumstances.

## 6.1 Conditional value at risk in the normal linear VaR model

Let *X* be a random variable to denote the returns of a portfolio over the set time period. If $X \sim N(\mu, \sigma^2)$, then

$$CVaR_\alpha(X) = \frac{1}{\alpha}\varphi\left(\Phi^{-1}(\alpha)\right)\sigma - \mu,$$

where $\varphi$ and $\Phi$ are the standard normal density and cumulative distribution functions.

**Proof**. (Alexander, 2008)

Let Z ~ *N(0, 1)*, then

$$\varphi(z) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}z^2}$$

Thus,

$$CVaR_\alpha(Z) = -\frac{1}{\alpha}\int_{-\infty}^{\Phi^{-1}(\alpha)} z\varphi(z)dz = -\frac{1}{\left(\sqrt{2\pi}\,\alpha\right)}\int_{-\infty}^{\Phi^{-1}(\alpha)} ze^{-\frac{1}{2}z^2}dz = \frac{1}{\alpha}\varphi\left(\Phi^{-1}(\alpha)\right).$$

Since Z ~ N(0, 1), X = Zσ + μ and

$$CVaR_\alpha(X) = CVaR_\alpha(Z)\sigma - \mu \qquad Q.E.D$$

# 7. MODEL VALIDATION

Financial models can be tested by using historical data to study the performance of the model. It is called backtesting. In backtesting, historical results are compared against the results predicted by the model. A model is not valid if it does not perform well in a backtest. In the context of value at risk modeling, the test is performed by using one or more candidate portfolios. The candidate portfolio represents a typical exposure to the relevant risk factors.

The primary concerns when backtesting a VaR model are how frequently the model overestimates or underestimates risks along with the time distribution of such events. If the model overestimates VaR, it restricts risk taking too much. On the other hand, underestimating VaR leads to taking higher risks. Therefore, a good result from a backtest has the VaR figure exceeded just often enough. For example, for 95% confidence level it would be ideal to have the VaR calculated with the model to be exceeded 5% of the time by the losses in historical data. These events should be randomly distributed in time. Let's consider a model that has the underestimated events clustered in time. That would signal a problem with the model under certain market conditions. A good model performs in all market conditions and thus it does not have periods where it consistently overestimates or underestimates VaR.

## 7.1 Bernoulli trial

A *Bernoulli trial* is an experiment with only two possible outcomes (Holton, 2014). Let p be the probability of success and 1 – p the probability of failure. Given a Bernoulli trial, define a random variable

$$X = \begin{cases} 1, for\ succes \\ 0, for\ failure \end{cases}$$

It has a *Bernoulli distribution* with probability function

$$\begin{cases} p \quad\ \ , for\ x = 1 \\ 1 - p, for\ x = 0 \end{cases}$$

Bernoulli trial is useful for testing the coverage of a value at risk model. First, the backtest is run through all historical sample data. Then, the times when value at risk estimate is too low compared to the historical results are counted. The result is compared against an identically and independently distributed Bernoulli distribution. Let's define an indicator function $I_{\alpha,t}$ on the time series of daily returns relative to the value at risk measure as (Alexander 2008)

$$I_{\alpha,t+1} = \begin{cases} 1, & if\ Y_{t+1} < -VaR_{\alpha,t}, \\ 0, & otherwise. \end{cases}$$

$Y_{t+1}$ is the realized return on the portfolio from time $t$ to time $t + 1$, $\alpha$ is the quantile used to calculate value at risk and $VaR_{\alpha,t}$ is the value at risk measure at time $t$. If the value at risk model is accurate and $\{I_{\alpha,t}\}$ follows an independent and identically distributed Bernoulli distribution, the probability of realized return being lower than VaR at any time is t is α. Hence, for $n$ observations

$$E\left(X_{n,\alpha}\right) = n\alpha$$

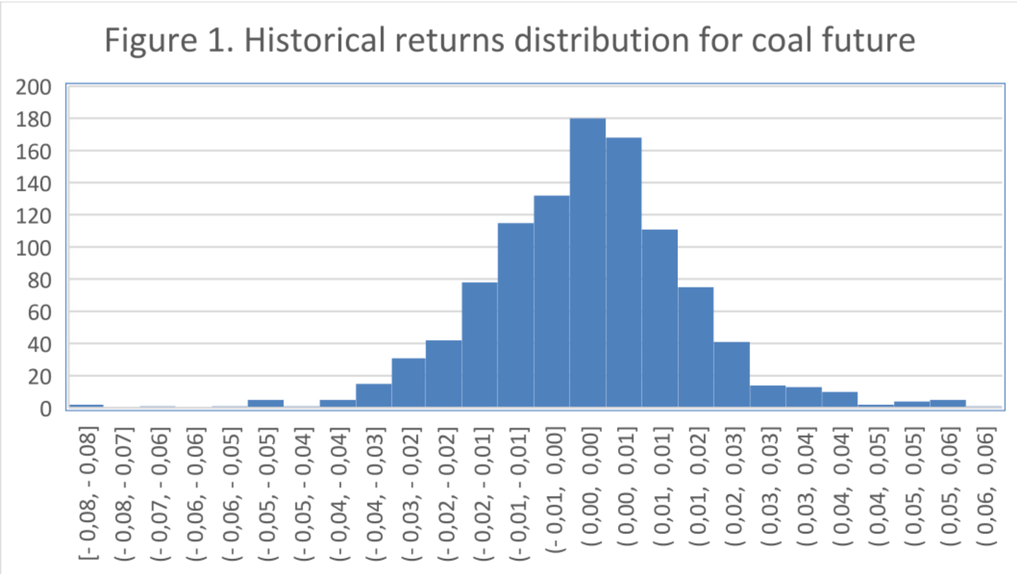
$$Var\left(X_{n,\alpha}\right) = n\alpha(1 - \alpha).$$

The standard error of the estimate, $\sqrt{n\alpha(1 - \alpha)}$, provides a measure of uncertainty around the expected value. Due to sampling error, it is unlikely to obtain exactly the expected number of exceedances in a backtest. Instead, the confidence interval around the expected number of exceedances within which it is very likely that the observed number of exceedances will fall should be considered. When n is very large the distribution of $X_{n,\alpha}$ is approximately normal, so a two-sided 95% confidence interval for $X_{n,\alpha}$ under the null hypothesis that the value at risk model is accurate is approximately

$$\left(n\alpha - 1.96\sqrt{n\alpha(1 - \alpha)}, n\alpha + 1.96\sqrt{n\alpha(1 - \alpha)}\right).$$

# 8. TEST RESULTS

The linear VaR model was tested with coal, natural gas, oil and electric power futures. Historical closing price data between April 1st 2016 and March 31st 2020 was used in the test. Financial contracts used in the test are coal (API-2), natural gas (TTF) and oil (BRENT) futures for the front month product. Additionally, Nordic power system price for front month, quarter and year and German power front month and year were used as well. Figures 1-8 have returns distributions for each product in a histogram for the test period. The python code used for calculations is on appendix A.



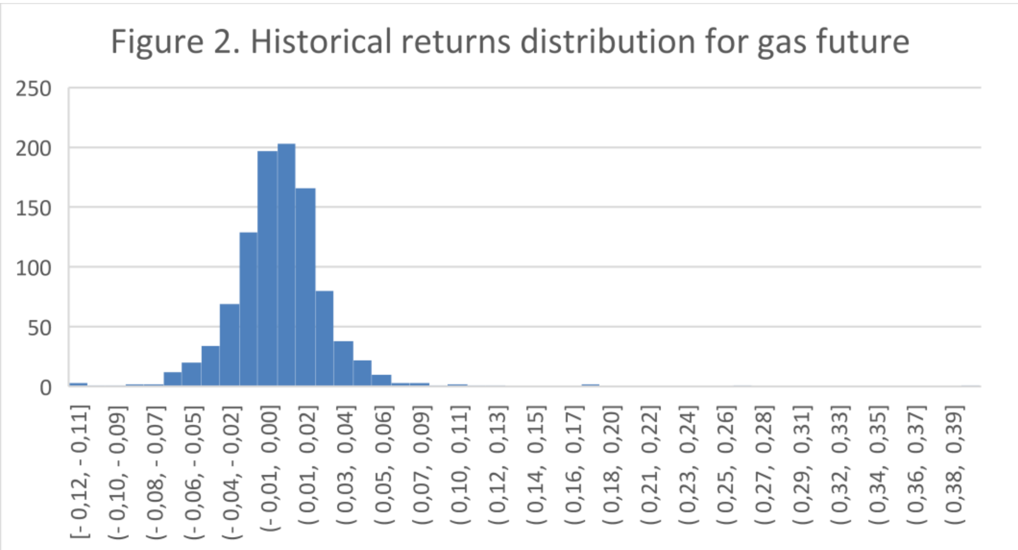



Figure 3. Historical returns distribution for oil future

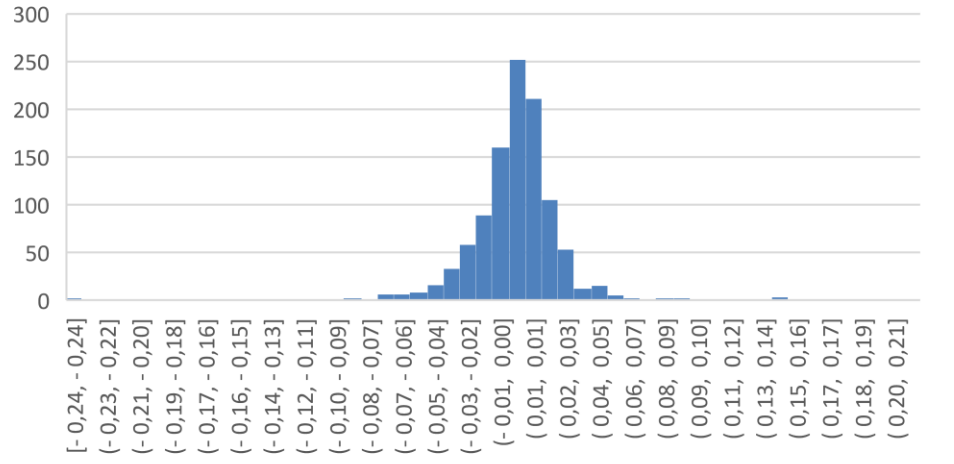



Figure 4. Historical returns distribution for Nordic power system price (Q) future

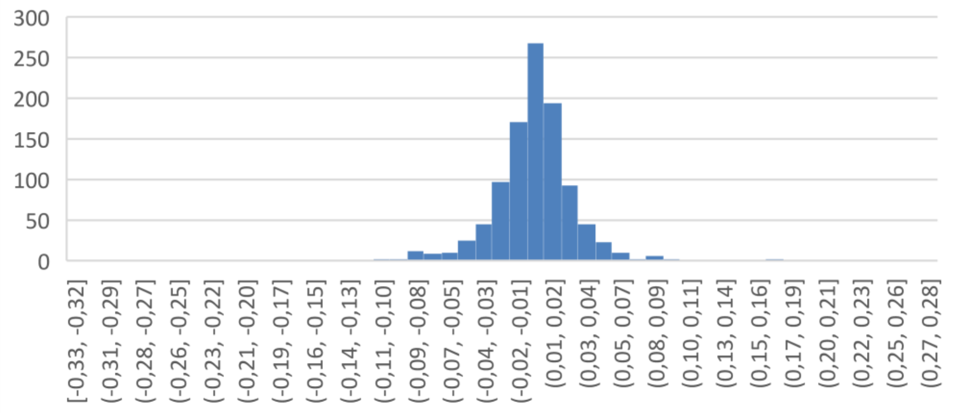



Figure 5. Historical returns distribution for Nordic power system price (M) future

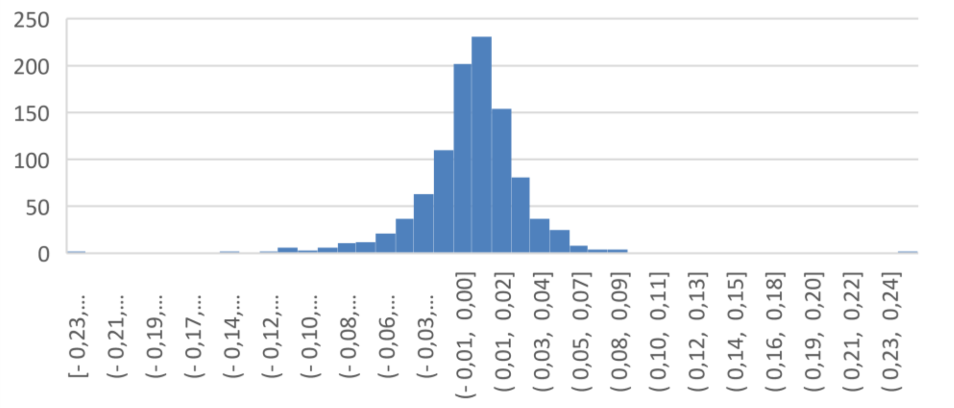


Figure 6. Historical returns distribution for Nordic power system price (Y) future

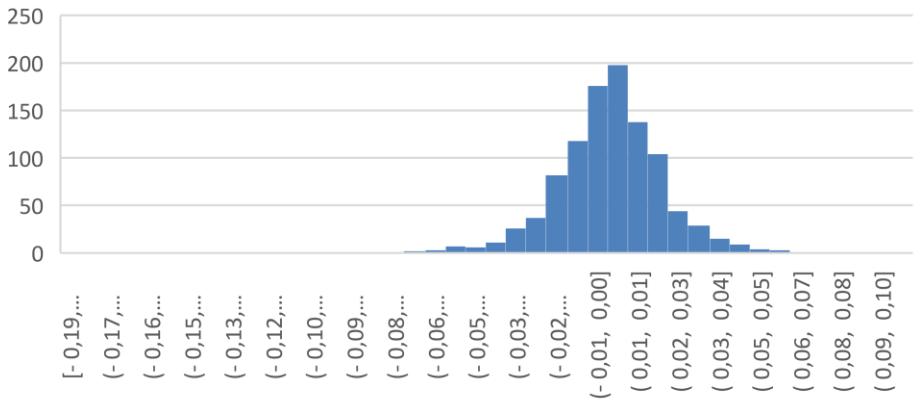



Figure 7. Historical returns distribution for German power (M) future

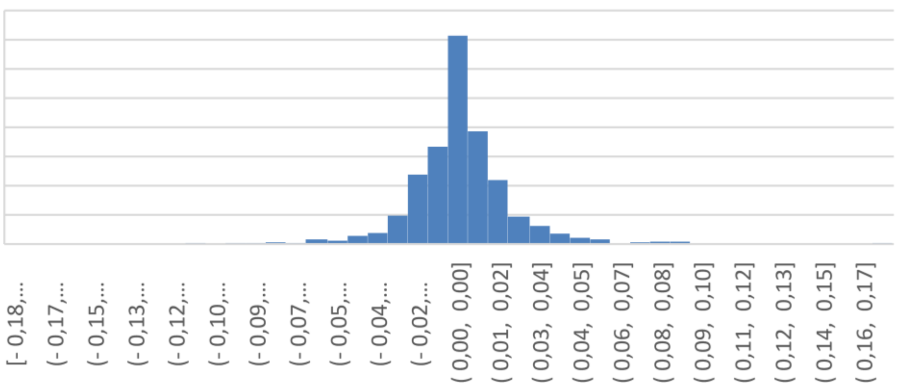
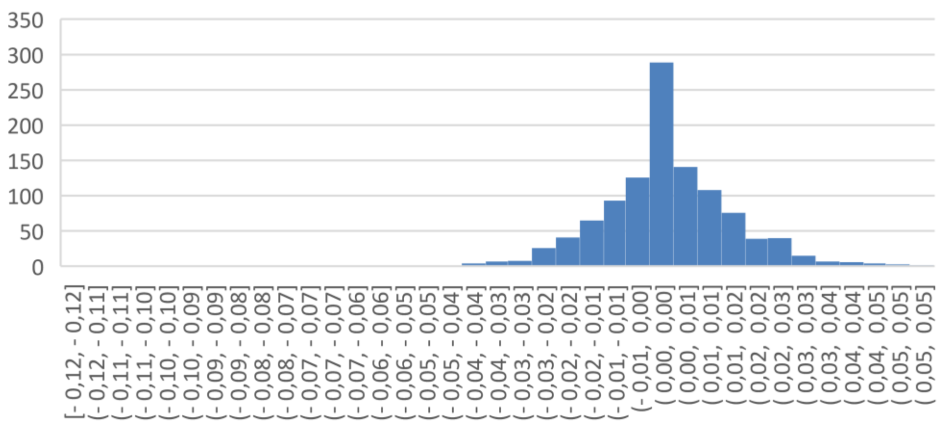



Figure 8. Historical returns distribution for German power (Y) future



The distributions shown in figures 1-8 are certainly not normally distributed. Some of them have too high peaks and most are clearly skewed. However, the distributions resemble a normal distribution and the linear VaR model can perform well enough. It is especially true when normal distribution is replaced by a t-distribution due to t-distribution having fatter tails, which is important in financial risk models.

## 8.1 Bernoulli trial backtest results

Backtesting was performed with linear VaR model using Bernoulli trial. One day 95% VaR was calculated with different decay factors and return distributions. VaR was calculated for every trading day between April 1st 2017 and March 31st 2020. For each VaR calculation, closing prices of previous 365 days were used to calculate the return distribution parameters. The test period includes 748 trading days meaning the expected number of VaR exceedances for the test period is 37.4 and the 95% confidence interval for exceedances is (26, 49). The results for coal future front month product are listed in tables 1, 2 and 3.

***Table 1*** *Backtest results for coal. Normal distribution*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 35 | 4.68 | 50 | 6.68 |
| 0,98 | 32 | 4.28 | 53 | 7.09 |
| 0,96 | 35 | 4.68 | 54 | 7.22 |
| 0,94 | 41 | 5.48 | 55 | 7.35 |

***Table 2*** *Backtest results for coal. T-distribution with 4 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 17 | 2.27 | 28 | 3.74 |
| 0,98 | 10 | 1.34 | 26 | 3.48 |
| 0,96 | 16 | 2.14 | 31 | 4.14 |
| 0,94 | 19 | 2.54 | 32 | 4.28 |

***Table 3*** *Backtest results for coal. T-distribution with 7 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 25 | 3.34 | 35 | 4.68 |
| 0,98 | 24 | 3.21 | 39 | 5.21 |
| 0,96 | 23 | 3.07 | 40 | 5.35 |
| 0,94 | 25 | 3.34 | 39 | 5.21 |

Tables 1, 2, and 3 show that for coal future front month product, linear VaR model with a t-distribution and 7 degrees of freedom performs the best. Normal distribution underestimates and t-distribution with 4 degrees of freedom overestimates VaR. Even using t-distribution with 7 degrees of freedom is not good enough for the short position, though. T-distribution with higher degrees of freedom than 7 would be an even better fit for coal. The choice of decay factor has little effect on the results. Tables 4, 5 and 6 contain the results for natural gas future front month product.

***Table 4*** *Backtest results for natural gas. Normal distribution*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 35 | 4.68 | 44 | 5.88 |
| 0,98 | 43 | 5.75 | 34 | 4.55 |
| 0,96 | 41 | 5.48 | 34 | 4.55 |
| 0,94 | 43 | 5.75 | 34 | 4.55 |

***Table 5*** *Backtest results for natural gas. T-distribution with 4 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 22 | 2.94 | 19 | 2.54 |
| 0,98 | 18 | 2.41 | 13 | 1.74 |
| 0,96 | 19 | 2.54 | 15 | 2.01 |
| 0,94 | 24 | 3.21 | 15 | 2.01 |

***Table 6*** *Backtest results for natural gas. T-distribution with 7 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 30 | 4.01 | 29 | 3.88 |
| 0,98 | 27 | 3.61 | 19 | 2.54 |
| 0,96 | 29 | 3.88 | 16 | 2.14 |
| 0,94 | 28 | 3.74 | 18 | 2.41 |

Tables 4, 5, and 6 show that for natural gas future front month product the normal distribution has the best performance in the backtest. T-distribution with both 4 and 7 degrees of freedom overestimates VaR. Decay factor has some significance to the results. The results are different with decay factor and without one. The value of decay factor has little effect on the results. Tables 7, 8 and 9 contain the results for oil future front month product.

***Table 7*** *Backtest results for oil. Normal distribution*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 35 | 4.68 | 50 | 6.68 |
| 0,98 | 36 | 4.81 | 47 | 6.28 |
| 0,96 | 35 | 4.68 | 46 | 6.15 |
| 0,94 | 37 | 4.95 | 47 | 6.28 |

***Table 8*** *Backtest results for oil. T-distribution with 4 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 16 | 2.14 | 25 | 3.34 |
| 0,98 | 15 | 2.01 | 27 | 3.61 |
| 0,96 | 16 | 2.14 | 25 | 3.34 |
| 0,94 | 16 | 2.14 | 26 | 3.48 |

***Table 9*** *Backtest results for oil. T-distribution with 7 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 22 | 2.94 | 34 | 4.55 |
| 0,98 | 25 | 3.34 | 36 | 4.81 |
| 0,96 | 26 | 3.48 | 35 | 4.68 |
| 0,94 | 28 | 3.74 | 35 | 4.68 |

Tables 7, 8 and 9 show that for oil future front month product t-distribution with 4 degrees of freedom is overestimating VaR. T-distribution with 7 degrees of freedom is also overestimating VaR a little. However, with decay factors of 0.94 and 0.96 the results are within the confidence interval of (26, 49). The best result comes from the normal distribution by a small margin, however, without a decay factor the result for long position is out of the confidence interval of (26, 49). A t-distribution with higher degrees of freedom than 7 would have better results, though. The choice of decay factor has little effect on the results. Tables 10, 11 and 12 contain the results for Nordic power system price future front quarter product.

***Table 10*** *Backtest results for Nordic power system price quarter product. Normal distribution*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 42 | 5.62 | 51 | 6.83 |
| 0,98 | 38 | 5.09 | 40 | 5.35 |
| 0,96 | 40 | 5.35 | 36 | 4.82 |
| 0,94 | 39 | 5.22 | 36 | 4.82 |

***Table 11*** *Backtest results for Nordic power system price quarter product. T-distribution with 4 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 20 | 2.68 | 26 | 3.48 |
| 0,98 | 18 | 2.41 | 23 | 3.08 |
| 0,96 | 20 | 2.68 | 22 | 2.95 |
| 0,94 | 21 | 2.81 | 24 | 3.21 |

***Table 12*** *Backtest results for Nordic power system price quarter product. T-distribution with 7 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 24 | 3.21 | 34 | 4.55 |
| 0,98 | 27 | 3.61 | 27 | 3.61 |
| 0,96 | 31 | 4.15 | 29 | 3.88 |
| 0,94 | 31 | 4.15 | 30 | 4.02 |

Tables 10, 11 and 12 show that normal distribution with a decay factor of 0.94, 0.96 or 0.98 have good results for Nordic power system price future front quarter product. Normal distribution without a decay factor underestimates VaR so much that for long position the number of exceedances fall out of the confidence interval of (26, 49). T-distribution with 4 degrees of freedom overestimates VaR too much, with 7 degrees of freedom only slightly. In fact, the only result falling out of the confidence interval of (26, 49) is for the short position without a decay factor. Overall, the results are better with a decay factor than without one, and the value of decay factor has little effect on the results. Tables 13, 14 and 15 contain results for Nordic power system price future front month product.

***Table 13*** *Backtest results for Nordic power system price month product. Normal distribution*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 46 | 6.15 | 66 | 8.82 |
| 0,98 | 43 | 5.75 | 69 | 9.22 |
| 0,96 | 45 | 6.02 | 70 | 9.36 |
| 0,94 | 49 | 6.55 | 71 | 9.49 |

***Table 14*** *Backtest results for Nordic power system price month product. T-distribution with 4 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 24 | 3.21 | 41 | 5.48 |
| 0,98 | 21 | 2.81 | 30 | 4.01 |
| 0,96 | 24 | 3.21 | 33 | 4.41 |
| 0,94 | 28 | 3.74 | 36 | 4.81 |

***Table 15*** *Backtest results for Nordic power system price month product. T-distribution with 7 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 30 | 4.01 | 50 | 6.68 |
| 0,98 | 29 | 3.88 | 44 | 5.88 |
| 0,96 | 32 | 4.28 | 50 | 6.68 |
| 0,94 | 37 | 4.95 | 55 | 7.35 |

Tables 13, 14 and 15 show that normal distribution underestimates VaR a lot, namely for the long position for Nordic power system price future front month product. T-distribution with 7 degrees of freedom has the same problem, however, with a decay factor of 0.98 the results are within the confidence interval of (26, 49). T-distribution with 4 degrees of freedom slightly overestimates VaR and with a decay factor of 0.94 the results are within the confidence interval of (26, 49). A better fit would be a t-distribution with degrees of freedom between 4 and 7. The decay factor has little effect on the results, only for t-distribution with 4 degrees of freedom for the long position is there a noticeable difference between having one or not. Tables 16, 17 and 18 contain results for Nordic power system price future front year product.

***Table 16*** *Backtest results for Nordic power system price year product. Normal distribution*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 41 | 5,48 | 38 | 5,08 |
| 0,98 | 39 | 5,21 | 31 | 4,14 |
| 0,96 | 43 | 5,75 | 35 | 4,68 |
| 0,94 | 38 | 5,08 | 35 | 4,68 |

***Table 17*** *Backtest results for Nordic power system price year product. T-distribution with 4 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 14 | 1,87 | 20 | 2,67 |
| 0,98 | 17 | 2,27 | 18 | 2,41 |
| 0,96 | 19 | 2,54 | 18 | 2,41 |
| 0,94 | 22 | 2,94 | 15 | 2,01 |

***Table 18*** *Backtest results for Nordic power system price year product. T-distribution with 7 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 23 | 3,07 | 25 | 3,34 |
| 0,98 | 24 | 3,21 | 23 | 3,07 |
| 0,96 | 26 | 3,48 | 21 | 2,81 |
| 0,94 | 28 | 3,74 | 22 | 2,94 |

Tables 16, 17 and 18 show that normal distribution performs well for the Nordic power system price future front year product. T distribution with 4 and 7 degrees of freedom overestimate VaR. Decay factor has little effect on the results. Tables 19, 20 and 21 contain results for German power future front month product.

***Table 19*** *Backtest results for German power month product. Normal distribution*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 27 | 3,79 | 38 | 5,34 |
| 0,98 | 29 | 4,07 | 38 | 5,34 |
| 0,96 | 28 | 3,93 | 38 | 5,34 |
| 0,94 | 31 | 4,35 | 38 | 5,34 |

***Table 20*** *Backtest results for German power month product. T-distribution with 4 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 10 | 1,40 | 15 | 2,11 |
| 0,98 | 12 | 1,69 | 18 | 2,53 |
| 0,96 | 15 | 2,11 | 21 | 2,95 |
| 0,94 | 14 | 1,97 | 19 | 2,67 |

***Table 21** Backtest results for German power month product. T-distribution with 7 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 20 | 2,81 | 23 | 3,23 |
| 0,98 | 16 | 2,25 | 25 | 3,51 |
| 0,96 | 19 | 2,67 | 25 | 3,51 |
| 0,94 | 19 | 2,67 | 27 | 3,79 |

Tables 19, 20 and 21 show that normal distribution performs best with results in the confidence interval of (26, 49). T-distribution with 4 and 7 degrees of freedom overestimate VaR. Decay factor has little effect on the results with normal distribution and t-distribution with 7 degrees of freedom. The choice of decay factor has an effect on the results with t-distribution with 4 degrees of freedom. Tables 22, 23 and 24 contain results for German power future front year product.

***Table 22** Backtest results for German power year product. Normal distribution*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 40 | 5,40 | 23 | 3,10 |
| 0,98 | 38 | 5,13 | 21 | 2,83 |
| 0,96 | 46 | 6,21 | 23 | 3,10 |
| 0,94 | 50 | 6,75 | 27 | 3,64 |

***Table 23** Backtest results for German power year product. T-distribution with 4 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 16 | 2,16 | 10 | 1,35 |
| 0,98 | 18 | 2,43 | 10 | 1,35 |
| 0,96 | 18 | 2,43 | 8 | 1,08 |
| 0,94 | 19 | 2,56 | 9 | 1,21 |

***Table 24*** *Backtest results for German power year product. T-distribution with 7 degrees of freedom*

| decay factor | number of exceedances short position | exceedance % short position | number of exceedances long position | exceedance % long position |
|---|---|---|---|---|
| no decay factor | 26 | 3,51 | 18 | 2,43 |
| 0,98 | 27 | 3,64 | 12 | 1,62 |
| 0,96 | 31 | 4,18 | 12 | 1,62 |
| 0,94 | 32 | 4,32 | 15 | 2,02 |

Tables 22, 23 and 24 show that short position has many more exceedances than long position. One way to resolve the issue would be to use one set of model parameters for short position and another set for long position. The ideal choice of distribution for the short position is a t-distribution with greater degrees of freedom than 7, because normal distribution slightly underestimates VaR and t-distribution with 7 degrees of freedom slightly overestimates VaR. For long position even normal distribution overestimates VaR as well as t-distribution with 4 and 7 degrees of freedom. A possible solution for the problem could be to scale down the VaR given by normal distribution. Decay factor has little effect on the results, aside from normal distribution with a decay factor of 0.94.

Overall, based on the backtesting results, the choice of the decay factor is not very important. The only major difference in the results is that it is better to have one. It means that a modeler is justified to use any decay factor between 0.94 and 0.98. The result is a little surprising, because VaR results can be quite different with a decay factor of 0.94 and a decay factor of 0.98. A decay factor of 0.94 weighs recent observations heavily and it makes VaR change fast with market price changes while a decay factor of 0.98 makes VaR change slower with market price changes. This finding is good news for modelers who would prefer their models to behave in a certain way. Choosing a decay factor between 0.94 and 0.98 does not affect the validity of the model much while giving the freedom to tweak the model to fit specific practical needs better.

## 8.2 Qualitative analysis of backtest results

The VaR model should be robust and perform well in different market conditions. The performance of the model can be analysed qualitatively by listing all days in the backtesting where a VaR exceedance occurred. The exceedances should not cluster around certain time periods. Instead, they should be spread quite evenly over time with perhaps a small random cluster here or there. Tables 25, 26, 27 and 28 contain VaR exceedance dates for the Bernoulli trial results in chapter 8.1 with normal distribution and no decay factor.

***Table 25*** *VaR exceedance dates for coal and gas*

| COAL short | COAL long | GAS short | GAS long |
|---|---|---|---|
| 16.5.2017 | 31.1.2018 | 22.9.2017 | 28.9.2017 |
| 12.2.2018 | 5.2.2018 | 3.11.2017 | 13.11.2017 |
| 16.2.2018 | 7.2.2018 | 17.11.2017 | 12.12.2017 |
| 2.3.2018 | 8.2.2018 | 11.12.2017 | 12.1.2018 |
| 19.3.2018 | 23.2.2018 | 13.12.2017 | 16.1.2018 |
| 28.3.2018 | 28.2.2018 | 27.12.2017 | 2.2.2018 |
| 9.5.2018 | 16.4.2018 | 30.1.2018 | 23.2.2018 |
| 11.5.2018 | 19.6.2018 | 16.2.2018 | 13.3.2018 |
| 6.7.2018 | 20.6.2018 | 19.2.2018 | 30.4.2018 |
| 17.7.2018 | 19.7.2018 | 22.2.2018 | 24.5.2018 |
| 3.8.2018 | 21.8.2018 | 8.3.2018 | 31.5.2018 |
| 18.9.2018 | 29.8.2018 | 9.3.2018 | 14.6.2018 |
| 18.10.2018 | 31.10.2018 | 18.5.2018 | 11.9.2018 |
| 15.1.2019 | 2.11.2018 | 28.5.2018 | 25.9.2018 |
| 12.2.2019 | 7.11.2018 | 7.9.2018 | 16.11.2018 |
| 14.2.2019 | 12.11.2018 | 14.9.2018 | 17.12.2018 |
| 3.4.2019 | 13.11.2018 | 18.9.2018 | 21.12.2018 |
| 4.4.2019 | 15.11.2018 | 11.10.2018 | 4.1.2019 |
| 5.4.2019 | 16.11.2018 | 9.11.2018 | 18.1.2019 |
| 8.4.2019 | 21.12.2018 | 16.1.2019 | 23.1.2019 |
| 10.4.2019 | 2.1.2019 | 26.3.2019 | 15.2.2019 |
| 16.4.2019 | 4.1.2019 | 2.4.2019 | 20.3.2019 |
| 2.5.2019 | 18.1.2019 | 3.4.2019 | 27.3.2019 |
| 2.7.2019 | 23.1.2019 | 4.4.2019 | 5.4.2019 |
| 3.7.2019 | 31.1.2019 | 8.4.2019 | 9.4.2019 |
| 9.7.2019 | 6.2.2019 | 3.6.2019 | 12.4.2019 |
| 18.7.2019 | 7.2.2019 | 4.7.2019 | 17.4.2019 |

| | | | |
|---|---|---|---|
| 28.8.2019 | 5.3.2019 | 5.7.2019 | 29.5.2019 |
| 9.9.2019 | 7.3.2019 | 10.7.2019 | 5.6.2019 |
| 13.9.2019 | 20.3.2019 | 22.7.2019 | 17.6.2019 |
| 18.11.2019 | 26.3.2019 | 31.7.2019 | 18.6.2019 |
| 3.2.2020 | 29.3.2019 | 7.8.2019 | 15.7.2019 |
| 12.2.2020 | 1.4.2019 | 28.8.2019 | 19.7.2019 |
| 23.3.2020 | 2.4.2019 | 9.9.2019 | 23.7.2019 |
| 24.3.2020 | 12.4.2019 | 2.1.2020 | 9.8.2019 |
| | 17.4.2019 | | 13.8.2019 |
| | 23.4.2019 | | |
| | 15.5.2019 | | |
| | 16.5.2019 | | |
| | 8.7.2019 | | |
| | 25.7.2019 | | |
| | 26.7.2019 | | |
| | 26.8.2019 | | |
| | 23.9.2019 | | |
| | 24.9.2019 | | |
| | 27.12.2019 | | |
| | 13.1.2020 | | |
| | 15.1.2020 | | |
| | 28.1.2020 | | |
| | 17.2.2020 | | |

***Table 26*** *VaR exceedance dates for oil and Nordic system power quarter product*

| OIL short | OIL long | Nordic power (Q) short | Nordic power (Q) long |
|---|---|---|---|
| 24.7.2017 | 18.4.2017 | 8.1.2018 | 15.5.2017 |
| 17.8.2017 | 3.5.2017 | 11.1.2018 | 2.6.2017 |
| 30.8.2017 | 24.5.2017 | 30.1.2018 | 2.2.2018 |
| 22.9.2017 | 6.6.2017 | 31.1.2018 | 1.3.2018 |
| 3.11.2017 | 13.6.2017 | 12.2.2018 | 23.3.2018 |
| 13.2.2018 | 4.7.2017 | 16.2.2018 | 4.6.2018 |
| 8.3.2018 | 5.12.2017 | 8.3.2018 | 8.6.2018 |
| 20.3.2018 | 8.2.2018 | 9.3.2018 | 25.7.2018 |
| 9.4.2018 | 27.2.2018 | 11.5.2018 | 26.7.2018 |
| 17.4.2018 | 24.5.2018 | 14.5.2018 | 31.7.2018 |
| 8.5.2018 | 14.6.2018 | 18.5.2018 | 7.8.2018 |
| 29.5.2018 | 29.6.2018 | 1.6.2018 | 31.8.2018 |

| | | | |
|---|---|---|---|
| 6.6.2018 | 10.7.2018 | 11.6.2018 | 7.9.2018 |
| 15.6.2018 | 13.7.2018 | 15.6.2018 | 13.9.2018 |
| 21.6.2018 | 31.7.2018 | 18.6.2018 | 17.9.2018 |
| 21.8.2018 | 7.8.2018 | 27.6.2018 | 19.9.2018 |
| 21.9.2018 | 10.10.2018 | 24.8.2018 | 8.10.2018 |
| 28.9.2018 | 22.10.2018 | 29.8.2018 | 16.11.2018 |
| 23.11.2018 | 31.10.2018 | 24.9.2018 | 1.2.2019 |
| 30.11.2018 | 12.11.2018 | 9.11.2018 | 8.2.2019 |
| 28.12.2018 | 19.11.2018 | 12.12.2018 | 20.2.2019 |
| 8.1.2019 | 22.11.2018 | 14.12.2018 | 25.4.2019 |
| 31.1.2019 | 7.12.2018 | 16.1.2019 | 13.5.2019 |
| 17.4.2019 | 17.12.2018 | 26.2.2019 | 16.5.2019 |
| 19.6.2019 | 19.12.2018 | 27.2.2019 | 31.5.2019 |
| 9.7.2019 | 21.12.2018 | 4.4.2019 | 27.6.2019 |
| 12.8.2019 | 22.2.2019 | 5.4.2019 | 2.8.2019 |
| 3.9.2019 | 25.4.2019 | 9.4.2019 | 30.8.2019 |
| 13.9.2019 | 22.5.2019 | 5.7.2019 | 23.12.2019 |
| 3.12.2019 | 29.5.2019 | 29.8.2019 | 27.12.2019 |
| 2.1.2020 | 11.6.2019 | 9.9.2019 | 3.1.2020 |
| 11.2.2020 | 1.7.2019 | 27.9.2019 | 13.1.2020 |
| 28.2.2020 | 15.7.2019 | 10.10.2019 | 17.1.2020 |
| 9.3.2020 | 31.7.2019 | 1.11.2019 | 20.1.2020 |
| 18.3.2020 | 2.8.2019 | 22.1.2020 | 27.1.2020 |
| | 6.8.2019 | 24.1.2020 | 28.1.2020 |
| | 16.9.2019 | 10.2.2020 | 29.1.2020 |
| | 7.1.2020 | 20.2.2020 | 4.2.2020 |
| | 31.1.2020 | 21.2.2020 | 5.2.2020 |
| | 21.2.2020 | 24.2.2020 | 14.2.2020 |
| | 27.2.2020 | 4.3.2020 | 17.2.2020 |
| | 5.3.2020 | 18.3.2020 | 18.2.2020 |
| | 6.3.2020 | | 19.2.2020 |
| | 11.3.2020 | | 27.2.2020 |
| | 13.3.2020 | | 28.2.2020 |
| | 17.3.2020 | | 13.3.2020 |
| | 19.3.2020 | | 16.3.2020 |
| | 26.3.2020 | | 17.3.2020 |
| | 27.3.2020 | | 20.3.2020 |
| | 31.3.2020 | | 25.3.2020 |
| | | | 30.3.2020 |

***Table 27*** *VaR exceedance dates for Nordic system power month and year product*

| Nordic power (M) short | Nordic power (M) long | Nordic power (Y) short | Nordic power (Y) long |
|---|---|---|---|
| 30.6.2017 | 15.5.2017 | 12.9.2017 | 28.9.2017 |
| 17.8.2017 | 2.6.2017 | 13.10.2017 | 15.12.2017 |
| 25.8.2017 | 6.10.2017 | 3.11.2017 | 23.2.2018 |
| 13.10.2017 | 11.10.2017 | 12.2.2018 | 16.4.2018 |
| 8.1.2018 | 23.2.2018 | 16.2.2018 | 24.5.2018 |
| 11.1.2018 | 1.3.2018 | 19.3.2018 | 29.5.2018 |
| 30.1.2018 | 23.3.2018 | 20.3.2018 | 4.6.2018 |
| 31.1.2018 | 8.6.2018 | 28.3.2018 | 5.6.2018 |
| 12.2.2018 | 25.7.2018 | 18.4.2018 | 8.6.2018 |
| 16.2.2018 | 26.7.2018 | 25.4.2018 | 14.6.2018 |
| 22.2.2018 | 31.7.2018 | 11.5.2018 | 19.6.2018 |
| 27.2.2018 | 7.8.2018 | 18.5.2018 | 31.8.2018 |
| 8.3.2018 | 7.9.2018 | 1.6.2018 | 7.9.2018 |
| 9.3.2018 | 13.9.2018 | 11.6.2018 | 12.9.2018 |
| 11.5.2018 | 14.9.2018 | 12.6.2018 | 13.9.2018 |
| 18.5.2018 | 17.9.2018 | 15.6.2018 | 25.9.2018 |
| 1.6.2018 | 19.9.2018 | 26.6.2018 | 9.10.2018 |
| 11.6.2018 | 5.10.2018 | 27.6.2018 | 12.10.2018 |
| 15.6.2018 | 16.11.2018 | 29.6.2018 | 2.1.2019 |
| 27.6.2018 | 11.1.2019 | 10.8.2018 | 4.1.2019 |
| 10.7.2018 | 1.2.2019 | 24.8.2018 | 18.1.2019 |
| 24.8.2018 | 8.2.2019 | 29.8.2018 | 8.2.2019 |
| 24.9.2018 | 12.2.2019 | 28.9.2018 | 8.3.2019 |
| 28.9.2018 | 20.2.2019 | 18.10.2018 | 2.8.2019 |
| 16.10.2018 | 17.4.2019 | 6.11.2018 | 30.8.2019 |
| 9.11.2018 | 16.5.2019 | 9.11.2018 | 3.1.2020 |
| 12.11.2018 | 24.5.2019 | 12.12.2018 | 13.1.2020 |
| 12.12.2018 | 31.5.2019 | 21.1.2019 | 17.1.2020 |
| 14.12.2018 | 27.6.2019 | 26.2.2019 | 28.1.2020 |
| 16.1.2019 | 2.8.2019 | 27.2.2019 | 29.1.2020 |
| 26.2.2019 | 4.10.2019 | 3.4.2019 | 5.2.2020 |
| 4.4.2019 | 25.10.2019 | 4.4.2019 | 6.3.2020 |
| 5.4.2019 | 23.12.2019 | 9.4.2019 | 11.3.2020 |
| 1.7.2019 | 27.12.2019 | 29.8.2019 | 13.3.2020 |
| 5.7.2019 | 30.12.2019 | 9.9.2019 | 16.3.2020 |
| 9.7.2019 | 3.1.2020 | 10.10.2019 | 17.3.2020 |
| 10.7.2019 | 7.1.2020 | 1.11.2019 | 20.3.2020 |

| | | | |
|---|---|---|---|
| 27.9.2019 | 13.1.2020 | 6.2.2020 | 26.3.2020 |
| 22.1.2020 | 16.1.2020 | 10.2.2020 | |
| 24.1.2020 | 17.1.2020 | 11.2.2020 | |
| 10.2.2020 | 20.1.2020 | 21.2.2020 | |
| 20.2.2020 | 27.1.2020 | | |
| 21.2.2020 | 28.1.2020 | | |
| 24.2.2020 | 29.1.2020 | | |
| 4.3.2020 | 30.1.2020 | | |
| 18.3.2020 | 31.1.2020 | | |
| | 4.2.2020 | | |
| | 5.2.2020 | | |
| | 14.2.2020 | | |
| | 17.2.2020 | | |
| | 18.2.2020 | | |
| | 19.2.2020 | | |
| | 25.2.2020 | | |
| | 27.2.2020 | | |
| | 28.2.2020 | | |
| | 6.3.2020 | | |
| | 13.3.2020 | | |
| | 16.3.2020 | | |
| | 17.3.2020 | | |
| | 20.3.2020 | | |
| | 24.3.2020 | | |
| | 25.3.2020 | | |
| | 26.3.2020 | | |
| | 27.3.2020 | | |
| | 30.3.2020 | | |
| | 31.3.2020 | | |

***Table 28*** *VaR exceedance dates for German power month and year product*

| **German power (M) short** | **German power (M) long** | **German power (Y) short** | **German power (Y) long** |
|---|---|---|---|
| 30.10.2017 | 28.9.2017 | 12.9.2017 | 28.9.2017 |
| 30.1.2018 | 7.2.2018 | 11.12.2017 | 29.1.2018 |
| 6.2.2018 | 23.2.2018 | 12.2.2018 | 23.2.2018 |
| 12.2.2018 | 13.3.2018 | 16.2.2018 | 16.4.2018 |
| 16.2.2018 | 4.6.2018 | 19.3.2018 | 19.4.2018 |

| | | | |
|---|---|---|---|
| 22.2.2018 | 5.6.2018 | 20.3.2018 | 24.5.2018 |
| 7.3.2018 | 8.6.2018 | 6.4.2018 | 31.5.2018 |
| 8.3.2018 | 25.7.2018 | 18.4.2018 | 19.6.2018 |
| 20.3.2018 | 26.7.2018 | 25.4.2018 | 11.7.2018 |
| 18.5.2018 | 12.9.2018 | 9.5.2018 | 19.7.2018 |
| 25.5.2018 | 25.9.2018 | 18.5.2018 | 12.9.2018 |
| 28.5.2018 | 27.9.2018 | 1.6.2018 | 25.9.2018 |
| 1.6.2018 | 16.11.2018 | 11.6.2018 | 9.10.2018 |
| 13.7.2018 | 23.11.2018 | 26.6.2018 | 10.10.2018 |
| 24.7.2018 | 21.12.2018 | 6.7.2018 | 19.10.2018 |
| 19.9.2018 | 11.1.2019 | 18.7.2018 | 25.10.2018 |
| 21.9.2018 | 18.1.2019 | 24.8.2018 | 31.10.2018 |
| 6.11.2018 | 22.2.2019 | 29.8.2018 | 16.11.2018 |
| 9.11.2018 | 12.3.2019 | 3.9.2018 | 4.1.2019 |
| 20.11.2018 | 17.4.2019 | 5.9.2018 | 12.3.2019 |
| 16.1.2019 | 16.5.2019 | 6.9.2018 | 29.5.2019 |
| 3.4.2019 | 27.6.2019 | 7.9.2018 | 30.8.2019 |
| 4.4.2019 | 15.7.2019 | 14.9.2018 | |
| 5.7.2019 | 25.10.2019 | 19.9.2018 | |
| 12.7.2019 | 25.11.2019 | 18.10.2018 | |
| 9.9.2019 | 20.12.2019 | 6.11.2018 | |
| 27.9.2019 | 27.12.2019 | 9.11.2018 | |
| | 13.1.2020 | 26.11.2018 | |
| | 17.1.2020 | 12.12.2018 | |
| | 28.1.2020 | 16.1.2019 | |
| | 29.1.2020 | 3.4.2019 | |
| | 30.1.2020 | 9.4.2019 | |
| | 7.2.2020 | 21.6.2019 | |
| | 13.3.2020 | 29.8.2019 | |
| | 16.3.2020 | 9.9.2019 | |
| | 17.3.2020 | 10.10.2019 | |
| | 20.3.2020 | | |
| | 25.3.2020 | | |

Tables 25, 26, 27 and 28 show that the Bernoulli trial results are good from qualitative point of view. There is little clustering of VaR exceedances and the exceedances are quite evenly spread out over time. The exception are the Nordic power futures and German power future front month contract. They have significant clustering in January, February and March 2020. Nordic power markets were volatile during the period due to mild and wet winter and in March 2020 the effects of Coronavirus Covid-19 had an impact in Nordic power markets as well. German power prices were affected in a similar way.

The results demonstrate the basic properties of VaR. During normal times it is possible to estimate the upper bound for losses quite accurately. On the other hand, when things are changing quickly, it is very difficult to estimate volatility accurately and therefore VaR models almost certainly underestimate risk during uncertain times.

# 9. SUMMARY

Managing risk is an important part of any operation. Financial derivatives are commonly used to hedge against price changes. The focus of the thesis is on managing market risk related to energy markets derivatives contracts in the Nordics. Value at Risk is a widely used tool to measure market risk. It gives a risk manager a powerful tool to estimate how much market risk is carried by holding derivatives contracts.

Value at Risk became mainstream in the 1990s when market regulators started using Value at Risk as a tool for regulation. Large financial institutions started developing their own models for Value at risk as well. Since then, Value at risk has spread into wide use in all organizations involved in financial markets.

Value at Risk is defined as a predefined quantile of the loss distribution of investment portfolio. In practise, instead of calculating Value at Risk from the asset price changes directly, financial returns are commonly used. Returns have desirable properties that make the calculations easier and more reliable over time with changing asset price levels.

Value at Risk has its critics as well. It is not necessarily a coherent risk measure. It can break the subadditivity requirement in the wrong circumstances leading to crazy conclusions. In order to amend the problem, another related concept has been developed. It is called conditional Value at Risk. It is also a quantile based risk measure and calculated similarly to Value at Risk. It is a coherent risk measure measuring the expected losses given a loss higher than a predefined level of loss.

Because of its wide use in the financial industry, Value at Risk has also been researched by many different scholars and institutions. Several different approaches have been proposed and they each have their pros and cons. The most common ones are a linear VaR model, historical simulation and Monte Carlo simulation.

Linear VaR model assumes a theoretical return distribution for an asset, such as normal distribution or Student’s t-distribution from which a corresponding quantile is calculated. The distribution parameters are calculated from historical asset prices. It is a very simple approach and relatively easy to implement, yet still reliable with linear futures and forwards. The model is easy to understand and explain to stakeholders who might not be familiar with risk models.

Historical simulation is an even easier method to implement and understand than linear VaR model. In the historical simulation method the historical asset prices are used directly to infer a return distribution from which a corresponding quantile is calculated. It has the advantage of capturing even the most complicated relationships between different assets, which is very difficult to do accurately with any mathematical model. It has a major drawback as well. The historical return distribution does not necessarily reflect the future in any way. Thus the approach assumes that the future will be like the past, which can lead to inaccurate Value at Risk estimates.

Monte Carlo simulation uses the general idea of historical simulation with the important distinction of generating asset price paths with a model instead of using historical asset prices. It remedies the drawback of historical simulation. However, generating different price paths that accurately reflect reality is difficult. Different stochastic processes and proprietary models are used to generate asset prices for Monte Carlo simulation and the model is only as good as the model used for asset price generation. The development of such models requires a lot of time and expertise. The approach is the most powerful one and also the most complicated one. It can be very difficult to explain to other stakeholders. Also understanding the results can be challenging since the model is necessarily complicated.

Linear VaR model was studied more thoroughly with different parametrizations and return distributions. Results were calculated for coal, natural gas, oil, Nordic power system price and German power futures.

The results show that normal distribution works best for natural gas, Nordic power system price quarter product, Nordic power system price year product and German power month products. T-distribution with 7 degrees of freedom is the best choice for coal, t-distribution with degrees of freedom between 4 and 7 suits best for Nordic Power system price month product and t-distribution with more than 7 degrees of freedom fits the best for oil. German power year product proved to be more difficult to model. The results do not support any one particular distribution to be a good choice for it. It would require different choices for short and long position to have an accurate Value at Risk model for the product.

The choice of decay factor was not found to have a significant effect on the results. It is good news for everyone trying to build a working Value at Risk calculation model that suits their specific needs. Based on the results, one is justified to use any decay factor greater or equal than 0.94. It means a model more sensitive to recent price changes can be used when appropriate, and conversely, a more stable one is equally justified.

Qualitative analysis of the results shows that the linear model works well in different market conditions as well. The times VaR estimates were exceeded by actual losses were reasonably randomly distributed over time with some small clusters scattered around the data. It indicates a model that performs well over time and not only during a specific period of time. An exception was found during January, February and March 2020. The winter 2020 in the Nordics was mild and wet. It resulted in a difficult situation for electric power producers causing unstable market conditions. Additionally, the effects of the coronavirus Covid-19 were felt in the Nordic energy markets as well. These unusual circumstances proved to be very challenging for the linear VaR model. It highlights an issue Value at risk models inherently possess. Forecasting future in the financial markets is as difficult as in any other field. Surprising events causing uncertainty and unrest are practically impossible to model accurately. Value at Risk models perform well only under normal market circumstances.

Overall, a model that accurately fits the needs a modeler may have for their Value at Risk calculation model can be refined based on the results. The linear VaR approach is shown to perform well in the context of Nordic energy markets. Further research could develop the framework used in this thesis further and produce more accurate results.

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# APPENDIX A: PYTHON FUNCTION FOR VAR CALCULATION

```
# -*- coding: utf-8 -*-
"""
Function to calculate Value at Risk for given prices.

@author: Juho Vänskä
"""

import numpy
from scipy import stats
import math


def calculate_VaR(prices, quantile=0.05, distribution='t',
   decay=0.96, df=7):

    """

    :param prices: List of asset prices used for calculation.

    :param quantile: Quantile used for calculation. Default
    value 0.05

    :param distribution: Distribution used for calculation. Op-
    tions: normal and Student's t. Default value 't'

    :param decay: Decay factor used for calculation. Must be
    between 0 and 1. If no decay factor is used for
    calculation, choose 'no'. Default value 0.96

    :param df: Degrees of freedom for Student's t-distribution.
    Can be ignored if using normal distribution.
    Default value 7

    :return: Returns Value at Risk.
    """

    # If no decay factor is used, volatility is calculated as
    standard deviation of returns.

    if decay == 'no':
        returns = []
        for i in range(len(prices) - 1):
               returns.append((prices[i] - prices[i + 1]) /
   prices[i + 1])
        volatility = numpy.std(returns)

    # If decay factor is used, volatility is calculated by
   exponentially weighted moving average of returns.

    elif 0 < decay < 1:
```

```
    volatility = 0
    for i in range(len(prices) - 1):
        volatility += (((prices[i] - prices[i + 1]) /
      prices[i + 1]) ** 2 * decay ** i) * (1 - decay)
    volatility = math.sqrt(volatility)

else:
    raise ValueError("Invalid decay factor. Decay factor "
  "must be between 0 and 1. If no decay factor is used for"
                    " calculation, choose 'no'.")

# VaR is calculated from the chosen returns distribution.

if distribution == 'normal':
    return stats.norm.ppf(q=quantile, loc=1,
  scale=volatility) * prices[0]

elif distribution == 't':
    return stats.t.ppf(q=quantile, loc=1, scale=volatility,
  df=df) * prices[0]

else:
    raise ValueError("Invalid distribution. Distribution"
  " must be either 'normal' or 't'.")
```