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VOLUNTARY OPT-IN PROVISION AND INSTRUMENT CHOICE IN ENVIRONMENTAL REGULATION

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Voluntary opt-in provision and instrument choice in environmental regulation*

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Abstract

We study market-based instruments under incomplete participation. Incomplete participation means that the regulation does not cover all emitters that contribute to harmful damages. Our results show that a voluntary opt-in provision should always be incorporated into regulation under incomplete participation as the provision unambiguously increases expected social welfare. Incomplete participation also affects the choice between market-based instruments, tradable permits and environmental taxes, under uncertainty. The impact will depend on whether the voluntary provision is used or not. The voluntary participation does not unambiguously favor one of the instruments, but the advantage is case-specific.

Keywords: Emission taxation, tradable emission permits, uncertainty, voluntary opt-in

JEL Codes: D62, D81, H23, Q58

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1 Introduction

In our theoretical model of market-based environmental regulation, a strictly positive but less-than-full participation[1] is a starting point. In these situations, incorporating a voluntary provision into market-based implementation has potential to increase the participation rate (Montero [10]). The general idea is to offer carrots to firms to get them to participate. Specifically, with tradable permits, the provision offers a free permit handout to a firm that agrees to cover its emissions through licenses. With environmental taxes instead, the provision should offer a fixed subsidy to a participating firm. Consequently, both implementations try to achieve two policy goals at the same time, namely, to regulate the excessive emissions by the price of emissions and to raise participation by subsidization. Overall, we have two questions in mind. First, we ask whether the inclusion of a voluntary provision raises societal welfare. Second, if it indeed represents sensible policy, does it affect the choice between market-based instruments, namely, between tradable permits and pollution taxes?

Based on the analysis of our model, the answers to both questions are definitely positive. Voluntary participation provision unambiguously raises expected social welfare. It raises expected social welfare, even if the provision does not attract all non-affected firms to participate. The provision also affects the traditional instrument choice formula that Weitzman [20] introduced in 1974. It will have a special effect on instrument choice as long as voluntary participation is incomplete, so that not every non-affected firm participates. However, we cannot say that incomplete voluntary participation favors a particular instrument as the advantage is both policy- and industry-specific.

We summarize the influence of voluntary participation on instrument choice into three concepts, namely, into a scope effect, a cost effect, and a volume effect. Scope effect is a special consequence of incomplete participation while cost and volume effects are commonly found in studies of second-best instrument choice (Nikula [12]). Scope effect reflects the fact that expanding regulation generates new socially prof-

[1] This feature has been an issue in the sulfur dioxide emissions trading program in the US (Ellerman *et. al.* [3]), in the EU Emissions Trading System (Ellerman *et. al.* [4]; European Commission [5]), or more generally, in the various flexibility mechanisms applied for greenhouse gas reductions (Newell, Pizer, and Raimi [11]).

itable abatement projects that will reduce the aggregate abatement costs. Cost effect in turn takes into account the fact that real-life extension can only be achieved by inefficient means and, consequently, by an increase in the aggregate abatement costs. We show that these effects together flattens the slope of the marginal cost function in our framework. Following the basic principles of instrument choice (Weitzman [20]), this means that the combined scope-cost effect invariably favors quantity instrument.

Volume effect arises as the quantity policy does not fix the aggregate level of emissions. More specifically, it arises under incomplete voluntary participation. The participation rate fluctuates, which further makes the aggregate number of permits to fluctuate. The non-binding permit quota is an unusual phenomenon in static models of environmental regulation. In the present context, it evolves as the policy uses free allocation of permits in the implementation of the voluntary provision. The implementation generates endogenous private permit supply, and this part of the total supply is sensitive to the changes in the business environment. We show that the volume effect does not ambiguously favor one of the instruments, but the advantage depends on the industry-specific factors.

Overall, the voluntary opt-in provision plays an important role. We show that the provision succeeds in attracting socially profitable green investments from the pool of non-affected firms. Before us, Montero [10] has studied the opt-in provision. He specifically focuses on the design of a phase-in emissions trading program. He draws experience from the sulfur dioxide emissions trading program that includes the so-called substitution provision, which allows producers unaffected in the first phase of the program to participate voluntarily (Ellerman *et. al.* [3]). Our framework applies a modified version of the Montero [10] model. Our model emphasizes the same issues that Montero does, namely, imperfect information, distributional concerns, and cost uncertainty. Our model reproduces Montero's key finding that a binding upper bound in subsidization completely changes the nature of regulation. The binding constraint turns participation incomplete and the emission becomes inefficiently distributed.[2]

Our work complements the study of instrument choice under uncertainty (Weitz-

[2]We explicitly show that the environmental agency strictly prefers incomplete voluntary participation over zero-voluntary participation. This means that an upper limit on subsidies is always binding in the policy design.

man [20]). A recent contribution by Meunier [9] considers instrument choice in a setting where participation can be interpreted as incomplete. In his setting, an unregulated good pollutes alongside a regulated polluting good.[3] However, the unregulated good remains unregulated throughout the work, so Meunier does not consider the voluntary participation at all. Other recent contributions that share our topics include Krysiak and Oberauner [6] and Mandell [7]. They develop the design of a hybrid instrument (Roberts and Spence [14]). The hybrid nature of these policies is indirectly present in our study, as the endogeneity of permit supply has the potential to soften the extreme nature of the quantity instrument. Furthermore, D'Amato and Dijkstra [2] and Krysiak [8] study technical change toward green investments, while Shinkuma and Sugeta [15] study market-based instruments and subsidization in a long-run framework.

We start by setting up the model. We define the two sectors and the technologies available for the firms in there. Next, we analyze three different markets structures one at a time, namely, complete participation, zero voluntary participation, and strictly positive voluntary participation. In each case, we study both the instrument design and instrument choice. After that, we will provide a summary of the main results in the concluding section.

2 The Polluting Industry

We will study market-based policies under three different market structures. We study markets under complete participation, zero voluntary participation, and positive voluntary participation, respectively. In complete participation, regulation mandates every firm in the polluting industry to participate. As far as we have two sectors, A and N, the policy regulates both sectors. With zero voluntary participation, only one of the sectors is regulated and there is no voluntary program in progress. The policy regulates firms in sector A while sector N stays completely out of regulation. With positive voluntary participation instead, non-affected firms become regulated

[3]Both studies belong to a subgroup label as second-best instrument choice. In these studies, there exists an additional constraint on the top of the commonly assumed informational constraint. In our case, the constraint is the upper limit in subsidization. See Meunier [9] for a review of the second-best instrument choice.

on a voluntary basis. The policy regulates every firm in sector A, while the policy regulates some firms in sector N. In the last two cases, we talk about incomplete participation. In the context of voluntary provision, we further use phrase incomplete voluntary participation when some but not every non-affected firm participates.

The division into sectors A and N is exogenously given. In both sectors, there are numerous polluters and the number of polluters remains unchanged throughout the analyses. However, the sectors have one thing in common as firms in both sectors produce same type of pollution. Pollution is homogeneous as the damages of pollution are not directly related to sector-specific emissions, but they depend on the sum of all emissions. We further assume that emissions from sectors A and N cover all emissions, so together these sectors form a polluting industry.

Regarding the pollution reductions, every firm in the polluting industry may choose between two production technologies to produce the commodity unit. The private benefit for a firm η of producing one commodity unit in sector A after choosing technology j is

$$B_{Aj}(\eta) = b_{Aj} + \theta_{Aj} - c_{Aj}\eta, \tag{1}$$

while the private benefit for a firm λ of producing one commodity unit in sector N after choosing technology j is

$$B_{Nj}(\lambda) = b_{Nj} + \theta_{Nj} - c_{Nj}\lambda, \tag{2}$$

where b_{ij} and c_{ij} are positive constants and θ_{ij} is a random variable with $i = A, N$ and $j = 0, 1$. Actually, it holds that $\theta_{A0} = \theta_{N0} = 0$, so we write $\theta_{A1} = \theta_A$ and $\theta_{N1} = \theta_N$. The random variables are identically and independently distributed with $E(\theta_A) = E(\theta_N) = 0$ and $Var(\theta_A) = Var(\theta_N) = \sigma^2$. We further assume (without any loss of generality) that $\Delta b_i \equiv b_{i1} - b_{i0} > 0$ and $\Delta c_i \equiv c_{i1} - c_{i0} > 0$ with $i = A, N$.

Our model depicts transitions towards green technology. We assume that green technology (technology 1) produces the same level of output as brown incumbent technology (technology 0) but with lower pollution content. We denote by α_{ij} the level of emissions that a firm produces by technology j in sector i. We further define $\Delta\alpha_i = \alpha_{i0} - \alpha_{i1} > 0$, so the firm-specific pollution content is lower using technology one than using technology zero in both sectors. Thus, by the assumptions above, the uncertainties in the model are related to the profitability of green technology. We assume that green technology is emerging technology with plenty of uncertainties around it.

A public agency regulates polluting industry because pollution creates harm, and no voluntary bargain between the emitters and the sufferers has thus far been successful. The instrument in regulation is either that of environmental taxation or a system of tradable permits. Both the regulatory agency and the firms face the same uncertainty. The crux of the matter is that the agency sets the regulatory parameters before everyone learns of the uncertainty, and the agency is unable to re-optimize subsequently. Firms make all their choices only after the regulation is fixed, and after everyone has learned of the uncertainty.

We denote the unit price of emissions by s. After incorporating the environmental policy into the analysis, the profit for the firm η is

$$\Pi_{Aj}(\eta) = B_{Aj}(\eta) - s(\alpha_{Aj} - l_A),$$

while the profit for the firm λ is

$$\Pi_{Nj}(\lambda) = B_{Nj}(\lambda) - s(\alpha_{Nj} - l_N), \tag{3}$$

where $j = 0, 1$. We denote by l_A and l_N the sector-specific threshold levels and set $s = p, \tau$ for permits and taxes, respectively. In the case of permits, the threshold l_i is the initial allocation of permits to a firm, while with taxes, l_i gives the tax-free level of emissions.

2.1 Complete Participation

We study the standard model of regulation, namely, the case where participation is complete. Within the sectors, there exist cut-off firms that are indifferent between the technologies. The cut-off firm η_1 satisfies

$$\begin{gathered} \Pi_{A0}(\eta_1) = \Pi_{A1}(\eta_1) \iff \\ b_{A0} - c_{A0}\eta_1 - s(\alpha_{A0} - l_A) = b_{A1} + \theta_N - c_{A1}\eta_1 - s(\alpha_{A1} - l_A), \end{gathered} \tag{4}$$

so

$$\eta_1 = \frac{\Delta b_A + \theta_A + s\Delta\alpha_A}{\Delta c_A}. \tag{5}$$

Similarly, the cut-off firm in sector N is

$$\lambda_1 = \frac{\Delta b_N + \theta_N + s\Delta\alpha_N}{\Delta c_N}. \tag{6}$$

Figure 1 illustrates. The lines B_{ij} corresponds to private benefits of technology j in sector i, while the line $\Pi_{ij}(\lambda)$ involves the influence of the regulation. In drawing the sectors, we use the solid lines to describe the benefits and the dashed lines to describe the profits. Our assumptions about Δb_i and Δc_i determine the structures of the sectors, so that firms apply green technologies at the low end of the distributions. As we denote the number of firms in sector A by η_0, then the firms $\eta \in [0, \eta_1]$ use green technology while the firms $\eta \in [\eta_1, \eta_0]$ utilize polluting brown technology in Figure 1(a). As compared to choices without regulation, we write

$$\eta_1^0 = \frac{\Delta b_A - \theta_A}{\Delta c_A},$$

so the firms $\eta \in [\eta_1^0, \eta_1]$ switch from brown to green technology because of the regulation.[4] Similarly, in sector N, the firms $\lambda \in [0, \lambda_1]$ use green and the firms $\lambda \in [\lambda_1, \lambda_0]$ use brown technology, while the firms $\lambda \in [\lambda_1^0, \lambda_1]$ switch from brown to green technology.

We denote the emissions in sector A and N by e_A and e_N, respectively, so

$$e_A = \int_0^{\eta_1} \alpha_{A1} d\eta + \int_{\eta_1}^{\eta_0} \alpha_{A0} d\eta = \eta_0\alpha_{A0} - \eta_1\Delta\alpha_A \tag{7}$$

and

$$e_N = \int_0^{\lambda_1} \alpha_{N1} d\lambda + \int_{\lambda_1}^{\lambda_0} \alpha_{N0} d\lambda = \lambda_0\alpha_{N0} - \lambda_1\Delta\alpha_N. \tag{8}$$

Pollution is homogeneous in nature, so the total level of pollution

[4] In principle, it may also be the case that $\eta_1^0 = 0$. This means that green technology becomes economically viable only because of the environmental regulation.

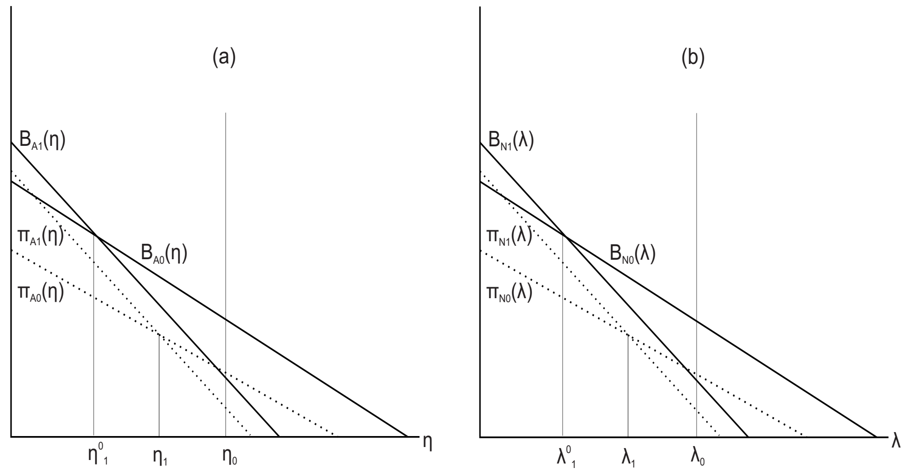

Figure 1: The Polluting Industry under Complete Participation: Sector A (a) and Sector N (b).

$$e = e_A + e_N \tag{9}$$

will be of interest. We further assume quadratic damage function

$$D(e) = \frac{d}{2}e^2, \tag{10}$$

where $d > 0$.[5] The aggregate benefits, in turn, are

$$B = \int_0^{\eta_1} B_{A1}(\eta)d\eta + \int_{\eta_1}^{\eta_0} B_{A0}(\eta)d\eta + \int_0^{\lambda_1} B_{N1}(\lambda)d\lambda + \int_{\lambda_1}^{\lambda_0} B_{N0}(\lambda)d\lambda. \tag{11}$$

It can be shown that minimum cost function $B(e)$ is quadratic and

$$\gamma = \frac{\Delta c_A \Delta c_N}{\Delta c_A \Delta \alpha_N^2 + \Delta c_N \Delta \alpha_A^2} \tag{12}$$

[5] We assume the damage function is known with certainty. It can be shown that our model is similar to Weitzman [20] as linear uncertainty in damage function does not matter as long as the random variable is uncorrelated with benefit uncertainties.

is the slope of the marginal function. Finally, we write

$$B_U = B_{AU} + B_{NU}, \tag{13}$$

where

$$B_{NU} = \int_0^{\lambda_1^0} B_{N1}(\lambda)d\lambda + \int_{\lambda_1^0}^{\lambda_0} B_{N0}(\lambda)d\lambda. \tag{14}$$

Similarly,

$$U = U_{AU} + U_{NU}, \tag{15}$$

where

$$U_{NU} = \int_0^{\lambda_1^0} \alpha_{N1}d\lambda + \int_{\lambda_1^0}^{\lambda_0} \alpha_{N0}d\lambda. \tag{16}$$

We call B_U the counterfactual benefits and U the counterfactual emissions. They are benefits and emissions in the absence of regulation.

The entire polluting industry participates in the regulation. The regulator chooses the strictness of the policy by setting a unit price on emissions. By Equations (48) and (49) in Appendix A, we write benefits

$$B(s) = B_U - \frac{1}{2\gamma}s^2 \tag{17}$$

and emissions

$$e(s) = U - \frac{1}{\gamma}s, \tag{18}$$

as functions of the unit price s. The factors B_U and U are the counterfactual variables defined in Equations (13) and (15), respectively. Denote the optimal rate by τ, so the necessary first-order condition requires that

$$E\left[\frac{dB}{d\tau} - \frac{dD}{de}\frac{de}{d\tau}\right] = 0,$$

or, that

$$\tau = dE\left[e\right]. \tag{19}$$

The derived policy rule is a standard one. It equates the price of the emissions to the expected level of marginal damage.

If the agency uses permit policy, it fixes first the number of pollution permits at the level ι and then auctions them off. At market equilibrium, the demand and supply of permits are equal, so

$$\begin{aligned}\iota &= \int_0^{\eta_1} (\alpha_{A1} - l_A)\, d\eta + \int_{\eta_1}^{\eta_0} (\alpha_{A0} - l_A)\, d\eta \\ &\quad + \int_0^{\lambda_1} (\alpha_{N1} - l_N)\, d\lambda + \int_{\lambda_1}^{\lambda_0} (\alpha_{N0} - l_N)\, d\lambda.\end{aligned} \tag{20}$$

We denote the number permits under complete participation by l. By Equation (18), the equilibrium permit price satisfies

$$p = Ep - \gamma\left(\frac{\theta_A}{\Delta c_A}\Delta\alpha_A + \frac{\theta_N}{\Delta c_N}\Delta\alpha_N\right). \tag{21}$$

The expected price Ep can be taken as the choice variable. Thus, by Equation (19), the agency should set $\iota = l$ in such a way that the condition

$$Ep = \tau.$$

is met.

In addition to the individual instrument designs, there is a choice to be made between the instruments. Different instruments will induce different responses and different expected levels of societal welfare. The choice between the instruments is based on a comparative advantage. A comparative advantage between the tax instrument τ and the permit instruments p is

$$\Delta(\tau, p) = E\left[B(\tau,\theta) - D(e(\tau,\theta) - (B(p(\theta),\theta) - D(e(p(\theta)),\theta))\right], \tag{22}$$

where $\theta = (\theta_A, \theta_N)$. We have

$$EB(s) = EB_U - \frac{1}{2\gamma}E\left[s^2\right],$$

$$ED(e(\tau)) = \frac{d}{2}l^2 + \frac{d}{2}E\left(\frac{\alpha_A}{c_A}\theta_A + \frac{\Delta\alpha}{\Delta c}\theta_N\right)^2,$$

and

$$e(p(\theta), \theta) = \frac{d}{2}l^2.$$

Based on these, we have

$$\Delta(\tau, p) = \frac{1}{2}\frac{Var(p)}{\gamma^2}(\gamma - d) \tag{23}$$

as the comparative advantage under complete participation.

The comparison $\Delta(\tau, p)$ follows the original Weitzman principle. The choice does not depend on the size of the uncertainty ($Var(p)$), so the slopes of the marginal benefit (γ) and damage (d) functions alone determine the choice between the price (environmental tax) and the quantity (tradable permits) instruments. It holds that the tax and permit policies are able to implement the policy in an efficient manner, even though the regulated industry is not standard.[6] Furthermore, the presence of subsidization neither destroy the efficiency nor affect the instrument choice. Basically, this happens as the industry responses η_1 (Equation (5)) and λ_1 (Equation (6)) are independent of the thresholds l_A and l_N. In practice, every firm gets the same compensation no matter what they choose, so the thresholds have no effect on the allocation of technology, and consequently, no effect on the policy choices.

2.2 Zero-Voluntary Participation

We start our study of incomplete participation by studying a policy without voluntary provision. The policy regulates only sector A, so firms in sector N stay completely outside the regulation. The benefits are

$$B_a = \int_0^{\eta_1} B_{A1}(\eta)d\eta + \int_{\eta_1}^{\eta_0} B_{A0}(\eta)d\eta + B_{NU}$$

while the emission equals

[6] The technology transition model (as portrayed by the piece-wise linear functions in Figure 1) is a novel feature in this study.

$$e_a = \int_0^{\eta_1} \alpha_{A1} d\eta + \int_{\eta_1}^{\eta_0} \alpha_{A0} d\eta + U_{NU},$$

where B_{NU} and U_{NU} are the sector N counterfactual variables defined in Equations (14) and (16), respectively. Consequently, in the absence of voluntary participation provision, the implemented policy do not affect the choices in sector N in any way.[7] Specifically, if we write

$$B_a = B_{aA}(e_{aA}) + B_{NU},$$

it then holds that

$$\frac{d^2 B_a}{de_{aA}^2} = \gamma_a,$$

where

$$\gamma_a = \frac{\Delta c_A}{\Delta \alpha_A^2}. \tag{24}$$

The details of the regulation resemble those of the complete participation. Consequently, the regulator either fixes the tax rate (τ_a) or the number of the auctioned permits (l_a). According to Equations (54) and (55) in Appendix A, benefits and emissions can be written as

$$B_a(s) = B_U - \frac{s^2}{2\gamma_a}$$

and

$$e_a(s) = U - \frac{1}{\gamma_a} s, \tag{25}$$

where B_U and U are defined in Equations (13) and (15), respectively. At the optimum, the tax rate (resp. the expected price level, Ep_a) satisfies

$$\tau_a = dE\left[e_a(\tau_a)\right], \tag{26}$$

or, in terms of the parameters of the model,

$$\tau_a = \frac{\gamma_a d}{\gamma_a + d} EU \tag{27}$$

(see Equation (57) in Appendix A).

[7]Meunier [9] studies the various links between regulated and unregulated goods.

Let us consider the policy choices more closely for a moment. Note first that (by assumption) the restricted regulator takes the scope of an environmental policy as given. Had she the opportunity to choose the scope, the regulator would prefer complete participation (Appendix A, Equation (63)). Second, the incomplete participation alters the nature of the abatement cost function. The function is only partial in nature as some profitable abatement projects remain outside the set of plausible projects. Third, as the agency can only influence the emissions of sector A, so the sector N emissions merely enter the policy calculations as an exogenously given entity. Regarding the instrument designs, we show in Appendix A (see Equations (61) and (62)) that

$$\tau_a > \tau$$

and

$$Ee_a > Ee,$$

where τ and Ee are the policy choices with complete participation. A shift in the marginal abatement cost function explains these differences.[8]

In the permit markets, the supply of permits equals the demand or, equivalently,

$$\iota = \int_0^{\eta_1} (\alpha_{A1} - l_A)\, d\eta + \int_{\eta_1}^{\eta_0} (\alpha_{A0} - l_A)\, d\eta, \tag{28}$$

where ι is the number of auctioned permits. The permit implementation sets $\iota = l_a$ in such a manner that the equilibrium price p_a will satisfy

$$p_a = \tau_a - \gamma_a \frac{\Delta\alpha_A}{\Delta c_A}\theta_A. \tag{29}$$

As sector N counterfactual emissions are denoted by U_{NU}, the aggregate emissions are

$$e_a(p_a) = l_a + U_{NU},$$

[8]In terms of emission reductions, the exclusion of sector N effectively means that the marginal abatement curve shifts upward at every level of emission reduction (see Fig. 1 in Montero [10]). As marginal damages are increasing, the price and the level of regulated emissions are increasing as well.

so the emissions are uncertain under tradable permits. Under tax policy instead,

$$e_a(\tau_a) = e_a(p_a) - \frac{\Delta \alpha_A}{\Delta c_A} \theta_A.$$

The comparative advantage is

$$\Delta_a(\tau_a, p_a) = E\left[B_a(\tau_a, \theta) - D(e_a(\tau_a, \theta) - (B_a(p_a(\theta), \theta) - D(e_a(p_a(\theta), \theta))\right].$$

We enter the various prices and quantities from above into this formula. It holds that

$$\Delta_a(\tau_a, p_a) = \frac{1}{2} \frac{Var(p_a)}{\gamma_a^2} (\gamma_a - d). \tag{30}$$

In principle, less-than-full participation in the environmental program does not change the basic rule of instrument choice.[9] If the slope of the marginal benefit function (γ_a) exceeds the slope of the marginal damage function (d), then the price instrument should be chosen. We can also show that the rule in Equation (30) remains intact if the sector N instead is the sole regulated industry.

The statistic $\Delta_a(\tau_a, p_a)$, however, does not provide any information about the changes that incomplete participation induces on instrument choice. In principle, the participation in regulation may turn from partial towards complete, or *vice versa.* We can show that the influence to instrument choice depends on the direction of the change: The quantity instrument will be favored as the regulatory regime somehow shifts from incomplete to complete participation. Conversely, a shift to incomplete participation favors the price instrument. The reason for these changes lies in the benefit side. As we are mainly interested in increasing participation in regulation, we briefly discuss this case here.

If voluntary provision succeeds in attracting new cost-saving projects into regulation, it will result in lower total and marginal cost curves. Following the basic principles of instrument choice, the less steep marginal cost curve means that more weight will be given to the slope of marginal damage d. In other words, as relative im-

[9]Our model rules out any covariance between the random variables in the benefit function by assumption. The presence of covariance will certainly induce differences between complete and incomplete participation. See Weitzman [20], Williams [21], Stavins [17], and Meunier [9].

portance of pollution damages increases, the relative importance of quantity control increases. More formally, it can be shown that a relation

$$\gamma_a = \gamma n \tag{31}$$

holds between γ_a and γ. We derive in Appendix B (Equation (71)) that $n > 1$, so $\gamma_a > \gamma$. Consequently, we may rewrite the comparative measure under complete participation (Equation (23)) as

$$\Delta(\tau, p) = \frac{Z}{2} \frac{Var(p_a)}{\gamma_a^2} \left[\gamma_a - nd \right], \tag{32}$$

where $Z > 0$. The multiplier $n > 1$ is the additional weight given to the slope of the marginal damage.[10]

For example, let $\gamma_a = d$, so that the regulator is indifferent between prices and quantities under zero-voluntary participation. If participation turns complete (and the policy choices turn optimal alongside it), then (by Equation (32)) $\Delta(\tau, p) < 0$ and the quantities becomes the preferred choice. We call the multiplier n the scope effect in instrument choice.

3 Regulation with Voluntary Participation

3.1 Voluntary participation

Above, we studied policy that regulates only a fraction of polluting firms. In this section, a mandatory regulation still remains infeasible in sector N but the agency will pursue a voluntary approach in there.[11] In the implementation of the voluntary provision, the agency applies subsidies in sector N.

Depending on the size of subsidy threshold l_N, the voluntary participation in itself can be complete or incomplete. In the first case, we state

[10] The size of factor Z is irrelevant for instrument choice, as it only magnifies the size of the measure Δ.

[11] This view naturally presupposes that the society benefits from increasing participation. We will return soon to this issue.

Proposition 1 *Assume that $l_N > \alpha_{N0}$. Voluntary participation is complete as every firm in sector N opts in. The environmental policy implements the same expected prices and quantities as with the complete participation. Furthermore, the instrument choice between prices and quantities follow the standard rule as written in Equation (23).*

The findings in instrument design are the same as in Montero [10]. Accordingly, if transfers are unlimited, the less-than-full participation may have no deleterious welfare effects. In the present context, as $l_N > \alpha_{N0}$, then $l_N > \alpha_{N1}$ as well, so

$$\Pi_{Nj}(\lambda) > B_{Nj}(\lambda)$$

for every λ, and $j = 0, 1$. In practice, this condition means that every firm in sector N finds it profitable to participate. To understand Proposition 1, we remind the reader of our earlier analysis under complete participation. Efficiency required that only the price of emissions, not the subsidization should affect the response λ_1 in sector N. In the present context, every firm participates (and earns the same subsidy), so the threshold l_N affects neither the participation rate nor the technology choices. These insights chiefly explain the similarities between complete mandatory and voluntary participation regimes. As the planner finds no restrictions in the implementation, it pursues the first-best policy familiar with the case of complete participation. Proposition 1 also expands the analysis of unlimited transfers (Montero [10]) toward the instrument choice. The first-best implementation explains why the voluntary participation provision does not affect the instrument choice at all.[12]

We will further follow Montero [10] as we not only model complete but also model incomplete voluntary participation. In our case, incomplete voluntary participation boils down to a condition

$$\alpha_{N0} > l_N > \alpha_{N1}. \tag{33}$$

Under this condition, transfers become limited and some firms at the high end of the type distribution refuses to participate. Specifically, the cut-off firm in sector N

[12] We will prove this result more formally in the closing section of this paper.

(Equation (6)) is replaced by the firm

$$\lambda_{v1} = \frac{\Delta b_N + \theta_N + s(l_N - \alpha_{N1})}{\Delta c_N}. \tag{34}$$

The market response λ_{v1} is seen to depend on the threshold l_N, so subsidization affects technological transition under incomplete voluntary participation.

This change does not happen without consequences. Specifically, the aggregate benefits (Equation (64) in Appendix A) become

$$B_v = B_U - \frac{1}{2}\frac{1}{\gamma_l}s^2 \tag{35}$$

while the aggregate emissions (Equation (65) in Appendix A)) are

$$e_v = U - \frac{1}{\gamma_L}s, \tag{36}$$

where

$$\gamma_l = \frac{\Delta c_N \Delta c_A}{(l_N - \alpha_{N1})^2 \Delta c_A + \Delta\alpha_A^2 \Delta c_N} \tag{37}$$

and

$$\gamma_L = \frac{\Delta c_N \Delta c_A}{(l_N - \alpha_{N1})\Delta\alpha_N \Delta c_A + \Delta\alpha_A^2 \Delta c_N} \tag{38}$$

In comparison to γ (Equation (12)), it holds that $\gamma_l \neq \gamma$ and $\gamma_L \neq \gamma$ for as long as $l_N > 0$.

We find it convenient to work in a framework, where damages and benefits are written in terms of emission price. However, if we transform our analysis into quantity framework, an important insight about the benefits under incomplete voluntary participation emerges. To see it, we rewrite first Equation (36) as

$$s_v(e) = \gamma_L \left(U - e\right).$$

We insert this price into benefits (Equation (35)), so

$$B_v = B_U - \frac{1}{2}\frac{\gamma_L^2}{\gamma_l}\left(U - e\right)^2.$$

Define a term

$$\rho = \frac{\gamma_L^2}{\gamma \gamma_l}, \tag{39}$$

so that

$$B_v = B_U - \frac{1}{2}\rho\gamma \left(U - e\right)^2.$$

We call term ρ the cost effect in instrument choice. It records the fact that subsidization under iincomplete voluntary participation turns the emission allocation inefficient. We will show that $\rho > 1$ as long as participation remains incomplete. Conversely, complete participation means that $\rho = 1$.

3.2 Implementation

Under incomplete voluntary participation, the planner (the regulator) will operate under the constraint $\alpha_{N0} > l_N$. It is shown in Appendix A (Equation (67)) that the second-best tax rate satisfies

$$\tau_v = \frac{d\gamma_l\gamma_L}{\left(\gamma_L\right)^2 + d\gamma_l} EU,$$

where EU are the expected counterfactual emissions. By Equation (36), we further write

$$E\left[e_v(\tau_v)\right] = EU \left[\frac{\left(\gamma_L\right)^2}{\left(\gamma_L\right)^2 + d\gamma_l}\right], \tag{40}$$

so that

$$\tau_v = \frac{\gamma_l}{\gamma_L} dE\left[e_v(\tau_v)\right]. \tag{41}$$

By Equations (37) and (38), it holds that $\gamma_L \neq \gamma_l$, so the standard condition $s = dE\left[e_s(s)\right]$ does not hold under incomplete voluntary participation.

Alternatively, the policy can use tradable permits. The implementation auctions off ι permits, so that the market equilibrium satisfies

$$\iota = \int_0^{\eta_1} \left(\alpha_{A1} - l_A\right) d\eta + \int_{\eta_1}^{\eta_0} \left(\alpha_{A0} - l_A\right) d\eta + \int_0^{\lambda_{v1}} \left(\alpha_{N1} - l_N\right) d\lambda. \tag{42}$$

After incorporating values of η_1 (Equation (5)) and λ_{v1} (Equation (34)) into the equilibrium condition, we can write the equilibrium price as

$$p_v = \overline{p}_v - \gamma_l \left(\frac{\theta_A}{\Delta c_A} \Delta\alpha_A + \frac{\theta_N}{\Delta c_N} (l_N - \alpha_{N1}) \right), \tag{43}$$

where γ_l is defined in Equation (37). By Equation (41), the agency should auction off permits (l_v) to the extent that the condition

$$\tau_v = \overline{p}_v.$$

will be met.

Finally, we present an important question about the overall desirability of the opt-in provision. To answer this question, we write first

Lemma 1

$$E[e_v(\tau_v)] - E[e(\tau)] = \gamma d \left(\frac{\rho - 1}{(\gamma + d)(\gamma\rho + d)} \right) EU$$

and

$$E[e_v(\tau_v)] - E[e_a(\tau_a)] = \gamma d \left(\frac{\rho - n}{(n\gamma + d)(\gamma\rho + d)} \right) EU.$$

The proofs are in Appendix A (see Equations (69) and (70) in there). We further write

Proposition 2 *If the environmental agency has an opportunity to choose between incomplete voluntary participation and complete participation, it prefers complete participation. Furthermore, the agency prefers incomplete voluntary participation to zero voluntary participation.*

Proof. We use our calculations in Appendix A. By the welfare calculations in Equations (53), (58), and (68) (together with the tax rates in Equations (19), (26), and (41)), we can write

$$EW_v(\tau_v) - EW(\tau) = d\frac{EU}{2} \left(E[e(\tau)] - E[e_v(\tau_v)] \right)$$

and

$$EW_v(\tau_v) - EW_a(\tau_a) = d\frac{EU}{2} \left(E[e_a(\tau_a)] - E[e_v(\tau_v)] \right),$$

where $EW(\tau)$, $EW_a(\tau_a)$, and $EW_v(\tau_v)$ are the maximal levels of social welfare under complete participation, zero voluntary participation, and strictly positive voluntary participation, respectively. Consequently, the differences between levels of social welfare depend on the results of Lemma 1. In the next section (in Lemma 2), we will show that conditions $\rho > 1$ and $n > \rho$ hold under incomplete participation. Thus, by Lemma 1, we have $E[e(\tau)] - E[e(\tau_v)] < 0$ and $E[e(\tau_a)] - E[e(\tau_v)] > 0$. As $d > 0$ and $EU > 0$, it holds that

$$EW(\tau) > EW_v(\tau_v)$$

and

$$EW_v(\tau_v) > EW_a(\tau_a).$$

■ By Equation (63) in Appendix A, we may also write

$$EW(\tau) > EW_v(\tau_v) > EW_a(\tau_a).$$

We briefly comment the condition $n > \rho$. It was shown above that this condition must be met in order for the incomplete voluntary participation to increase welfare. We discussed earlier that expanding the pool of regulated firms will shift the marginal abatement function downwards ("the $n-$effect"). However, this insight is based on the efficient implementations. Accounting for the inefficiency that voluntary participation provision produces, the marginal cost function will shift upwards ("the $\rho-$effect"). Overall, the condition $n > \rho$ says that regulator should promote incomplete participation only if the policy shifts marginal abatement costs (and the level of expected regulated emissions) downwards.

3.3 Instrument Choice

We are interested in the choice between environmental taxes and tradable permits in the implementation of the voluntary provision. We ask whether the size of the subsidy should be fixed (taxes) or should be let to fluctuate (permits) in the implementation. We expand the concept of comparative advantage (Weitzman [20]) toward payment flows that market-based policies inevitably generate.

We already analysed one particular case of instrument choice, namely, the case of complete voluntary participation. It was shown that the choice is similar to the choice under complete participation, as the agency is able to implement both policies in an efficient manner. However, in the current case, the presence of unlimited transfers turns implementations inefficient, and the instrument choice is fundamentally affected.

The formula of comparative advantage is

$$\Delta_v(\tau_v, p_v) = E\left[B_v(\tau_v, \theta) - D(e_v(\tau_v, \theta) - (B_v(p_v(\theta), \theta) - D(e_v(p_v(\theta), \theta))\right].$$

We state

Proposition 3 *The comparative advantage between prices and quantities under incomplete voluntary participation is*

$$\Delta_v(\tau_v, p_v) = \frac{Z^*}{2}\frac{Var(p_a)}{\gamma_a^2}(\gamma_a - \Theta\frac{n}{\rho}d),$$

where $Z^* > 0$. *The influence of voluntary participation on the instrument choice is given by* $\Theta\frac{n}{\rho}$, *where* Θ *is the volume effect,* n *is the scope effect, and* ρ *is the cost effect.*

We already met the scope effect (in Equation (31)) and the cost effect (in Equation (39)) but the volume effect is a new feature in our framework. Volume effect isolates the influence that non-fixed emission quota has on instrument choice. The non-binding quota follows as the voluntary provision applies free permits in the implementation. According to Equation (42), there emerges private permit supply into the permit markets that equals to

$$\int_0^{\lambda_{v1}} (l_N - \alpha_{N1})\, d\lambda > 0.$$

As the participation λ_{v1} fluctuates, then the private permit supply and, consequently, the aggregate permit supply fluctuates as well. This is in contrast to our models above, where aggregate permit allocation remained truly fixed. Specifically, in the traditional analysis with complete participation, the quantity instrument fixes both the level of emissions and the number of permits to a predetermined level. This assumption lies behind the traditional Weitzman [20] analysis (as calculated in Equation (23)).

The representation of comparative statistic is derived in Appendix B.[13] In the derivations, we find it convenient to introduce some new auxiliary notation. We write.

$$u = \frac{\Delta c_A}{\Delta c_N} > 0,\ a = \frac{\Delta \alpha_N}{\Delta \alpha_A} > 0,\ k = \frac{l_N - \alpha_{N1}}{\Delta \alpha_A} > 0. \tag{44}$$

We have

Lemma 2 *Under incomplete voluntary participation, $n > 1$, $\rho > 1$, and $\frac{n}{\rho} > 1$.*

Proof. See calculations in Appendix B. Using the definitions in (44), we calculate that

$$n = 1 + ua^2 \tag{45}$$

(see Equation (71)), and that

$$\rho = 1 + \frac{u\,(k-a)^2}{(1+uak)^2} \tag{46}$$

(see Equation (73)). These figures proves that $n > 1$ and $\rho \geq 1$. Furthermore, by definitions of a and k,

$$a - k = \frac{\alpha_{N0} - l_N}{\Delta \alpha_A},$$

and by the definition of incomplete voluntary participation (Equation (33)),

$$\alpha_{N0} > l_N,$$

so

$$a > k.$$

Thus, under incomplete voluntary participation, $\rho > 1$, and, as

$$\frac{n}{\rho} = \frac{(1+uak)^2}{1+uk^2}$$

[13]In specific, our representation shows that cost effect is associated with benefits alone while the volume effect is associated only with damages in the instrument choice (see Equations (80) and (82)).

(see Equation (74)), we may conclude that $n > \rho$. ■

The result $n > \rho$ has two immediate consequences. First, referring to Proposition 2, incomplete voluntary participation invariably increases social welfare. Second, referring to Proposition 3, the combined cost-scale effect $\frac{n}{p}$ invariably favors the quantity instrument, tradable permits

Regarding the volume effect, we can write it as

$$\Theta = 2q - 1,$$

where

$$q = \frac{1 + uk^2}{1 + uak} \frac{1 + ku^2 a}{1 + k^2 u^2}, \tag{47}$$

see Equations (81) and (83) in Appendix B. It holds that

$$\Theta \gtreqless 1 \Leftrightarrow ku(u-1)(a-k) \gtreqless 0.$$

Thus, as long as the voluntary participation remains incomplete ($a > k$), the factor $u = \frac{\Delta c_A}{\Delta c_N}$ is pivotal. The volume effect Θ vanishes if cost parameters are equal ($u = 1$) or if they take extreme values ($u = 0$ or $u \longrightarrow \infty$).[14] Otherwise, if $\Delta c_A > \Delta c_N$ ($\Delta c_A < \Delta c_N$) the volume effect invariably favors the quantity instrument (the price instrument). Accordingly, we write

Lemma 3 *Under incomplete voluntary participation, if* $\Delta c_A > \Delta c_N$ *then* $\Theta > 1$.

Altogether, Proposition 3 tells us that the influence of incomplete participation on instrument choice is given by $\frac{n}{\rho}\Theta$, so the influence is explained by the combined cost-scope effect and by the volume effect. We learned above that the combined cost-scope effect invariably favors the quantity instrument (Lemma 2). We also learned (Lemma 3) that the volume favors the quantity instrument as long as $\Delta c_A > \Delta c_N$. Combining these results, we may write

[14]It holds that

$$q = \left[\frac{\frac{1}{u} + k^2}{\frac{1}{u} + ka} \frac{\frac{1}{u^2} + ka}{\frac{1}{u^2} + k^2} \right] \underset{u \longrightarrow \infty}{=} \left[\frac{k^2}{ka} \frac{ka}{k^2} \right] = 1$$

so that $\Theta = 1$.

Proposition 4 *The condition $\Delta c_A > \Delta c_N$ is sufficient for the incomplete participation to favor the quantity instrument.*

More generally, referring to our calculations in Appendix B, the effect of incomplete participation on instrument choice can be written as

$$\Theta - \frac{\rho}{n} = uk\frac{(a-k)}{(1+u^2k^2)}\frac{1+uk^2}{1+uka}\left[\left(\frac{u-1}{(1+uk^2)}\right)+\left(\frac{u+1+\frac{k\left(1+ku^2a^2\right)}{(a-k)}}{(1+uka)}\right)\right],$$

see Equation (84).

Finally, let $l_N > \alpha_{N0}$, so that the participation is complete. Then, $k = a$, so $\Theta = \rho = 1$. If $l_N < \alpha_{N1}$ instead, then no firm voluntarily participates. We have $k = 0$, $\Theta = 1$ and $n = \rho$. In either case, we are back in a standard Weitzman representation of comparative advantage, as documented in Equations (23) and (30) above.

4 Conclusions

Piqouvian taxes and tradable permits provides similar means to regulate entire industries especially in the cases, where the pollutant is perfectly mixing (e.g., carbon in atmosphere). We study how the instruments do as they implement voluntary provision within industries, where green technology is emerging (but uncertain) alternative. We show that the voluntary provision is socially desirable as it contributes to the transition toward green production. Voluntary provision also affects the traditional instrument choice formula as originally represented by Weitzman [20]. We show that it affects the instrument choice formula as long as the voluntary participation is incomplete, so that not every non-affected firm opts-in.

As in Montero [10], we assume limitations to subsidize voluntary participation. Consequently, the limitations create a policy regime of incomplete participation. Under this regime, the permit implementation in particular undergo significant changes. The implementation uses free permit allocation, so it will generate endogenous private permit supply into the permit markets. As Roberts and Spence [14] show in their design of a hybrid instrument, non-binding quota is not necessarily a problem for the permit implementation. In a similar manner, our analysis finds a set of parameter values that supports endogenous private permit supply. Under these values, the quantity instrument becomes the preferred choice.

Our framework pay attention to limited transfers, but it ignores the policy challenges that excessive allocation (Montero [10]) presents. Excessive allocation accumulates as voluntary provision provides permit allocations greater than the counterfactual emissions. This means that the excessive allocation will cover reductions that would have occurred in the absence of the voluntary provision. Hopefully, we can offer an updated version of this work in the future, where we can explain the effects of excessive allocation on instrument choice in an intuitive fashion. It may be challenging as the present framework already operates under two constraints. There exists the usual Weitzman [20] constraint on information and the additional constraint on subsidization.

However, the current study has notable merits, too. It helps in specifying the separate effects that incomplete voluntary participation has on instrument choice. If we incorporate more elements into the framework, the scope, the cost, and the volume effects will exist even though their mutual relations will be affected.

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Appendix

A Social Welfare

In this section, we derive formulas for social welfare under various market structures. Using these formulas, we may derive optimal policies as well. In what follows, we will operate in terms of the price variable s. We will repeatedly apply concepts of counterfactual benefits and counterfactual emissions. These are the aggregate benefits and emissions in the absence of regulation. In our framework, counterfactual values arise as we set $s = 0$.

We study first complete participation. We insert the values of η_1 (Equation (5)) and λ_1 (Equation (6)) into the definition of aggregate benefits (Equation (11)). We have

$$B(s) = B_U - \frac{\Delta\alpha_A^2 \Delta c_N + \Delta\alpha_N^2 \Delta c_A}{\Delta c_N \Delta c_A} \frac{s^2}{2} = B_U - \frac{1}{2\gamma} s^2, \tag{48}$$

where B_U are the counterfactual benefits. Next, we insert the same cut-offs into Equations (7) and (8), so, after applying the definition of counterfactual emissions U, we have total emissions

$$e(s) = U - \frac{\Delta\alpha_A^2 \Delta c_N + \Delta\alpha_N^2 \Delta c_A}{\Delta c_N \Delta c_A} \tau = U - \frac{1}{\gamma} s. \tag{49}$$

Thus, societal welfare is

$$W(s) = B(s) - D(e(s)) = B_U - \frac{1}{2\gamma} s^2 - \frac{d}{2} \left(U - \frac{1}{\gamma} s \right)^2. \tag{50}$$

We further decompose counterfactual emissions into deterministic and stochastic parts. We have

$$U = \overline{U} + \Phi, \tag{51}$$

where $\overline{U} = EU$ and $\Phi = \frac{\theta_A}{\Delta c_A} \Delta\alpha_A - \frac{\theta_N}{\Delta c_N} \Delta\alpha_N$.

We denote the optimal tax by $s = \tau$. It will satisfy the first order condition

$$E\left[\frac{dB}{d\tau} - \frac{dD}{de} \frac{de}{d\tau} \right] = 0,$$

so, by Equation (50), it holds that

$$\tau = \frac{\gamma d}{\gamma + d} \overline{U}. \tag{52}$$

In specific, if we insert the optimal tax (Equation (52)) together with the decomposition of U (Equation (51)) into welfare, we have

$$W = B_U - \frac{1}{2\gamma}\left(\frac{\gamma d}{\gamma + d}\overline{U}\right)^2 - \frac{d}{2}\left(\left(\left(1 - \frac{1}{\gamma}\frac{\gamma d}{\gamma + d}\right)\overline{U}\right)^2 + \Phi^2 + 2\Phi\left(1 - \frac{1}{\gamma}\frac{\gamma d}{\gamma + d}\right)\overline{U}\right).$$

As $E\Phi = 0$,

$$EW = EB_U - \left[\frac{1}{\gamma}\left(\frac{\gamma d}{\gamma + d}\right)^2 + d\left(1 - \frac{1}{\gamma}\frac{\gamma d}{\gamma + d}\right)^2\right]\frac{\overline{U}^2}{2} - \frac{d}{2}E\Phi^2,$$

or after a few lines of manipulations,

$$EW = EB_U - \frac{\gamma d}{\gamma + d}\frac{\overline{U}^2}{2} - \frac{d}{2}E\Phi^2.$$

By Equation (52), we may also write

$$EW(\tau) = EB_U - \tau\frac{\overline{U}}{2} - \frac{d}{2}E\Phi^2. \tag{53}$$

Consider next incomplete participation without voluntary provision. After inserting the value of η_1 (Equation (5)) into aggregate benefits (Equation (11)) and into emission formulas (Equations (7) and (8)), we write benefits

$$B_a(s) = B_U - \frac{\Delta\alpha_A^2}{\Delta c_A}\frac{s^2}{2} = B_U - \frac{s^2}{2\gamma_a} \tag{54}$$

and emissions

$$e_a(s) = U - \frac{1}{\gamma_a}s \tag{55}$$

in terms of the unit price s. Benefits and damages together will yield (a counterpart of Equation (50))

$$W_a(s) = B_U - \frac{1}{2\gamma_a}s^2 - \frac{d}{2}\left(\overline{U} + \Phi - \frac{1}{\gamma_a}s\right)^2 \tag{56}$$

We set

$$\frac{dEW_a(\tau_a)}{d\tau_a} = -\frac{\tau_a}{\gamma_a} + \frac{d}{\gamma_a}E(e(\tau_a)) = 0,$$

so, together with Equation (55), it holds that

$$\tau_a = \frac{\gamma_a d}{\gamma_a + d}EU. \tag{57}$$

After inserting the tax rate into Equation (56), and taking expectation, we have

$$EW_a(\tau_a) = EB_U - \left[\frac{1}{\gamma_a}\left(\frac{\gamma_a d}{\gamma_a + d}\right)^2 + d\left(1 - \frac{1}{\gamma_a}\frac{\gamma_a d}{\gamma_a + d}\right)^2\right]\frac{E\overline{U}^2}{2} - \frac{d}{2}E\Phi^2.$$

As the term inside the parenthesis equals $\frac{\tau_a}{EU}$, it further holds that

$$EW_a(\tau_a) = EB_U - \tau_a\frac{\overline{U}}{2} - \frac{d}{2}E\Phi^2. \tag{58}$$

In evaluating the differences between the two market structures, we will apply the factor

$$n = \frac{\gamma_a}{\gamma}$$

(see Equation (31) in the main text) in the calculations. Then, by Equations (52) and (57), it holds that

$$\tau_a - \tau = \frac{n\gamma d}{n\gamma + d}EU - \frac{\gamma d}{\gamma + d}EU = \frac{\gamma d^2}{(\gamma + d)(n\gamma + d)}(n-1)EU. \tag{59}$$

Furthermore, by Equations (49), (52), (55), and (57), we have

$$E[e_a(\tau_a)] - E[e(\tau)] = E\left[U - \frac{\tau_a}{n\gamma} - (U - \frac{\tau}{\gamma})\right] = \gamma d\left(\frac{n-1}{(\gamma + d)(n\gamma + d)}\right)EU. \tag{60}$$

We will show below (see Equation (71)) that $n > 1$, so

$$\tau_a > \tau \tag{61}$$

and

$$E[e(\tau_a)] > E[e(\tau)]. \tag{62}$$

Finally, by Equations (53) and (58), it holds that

$$EW_a(\tau_a) - EW(\tau) = (\tau - \tau_a)\frac{EU}{2},$$

so, by Equation (61),

$$EW(\tau) > EW_a(\tau_a). \tag{63}$$

We consider voluntary participation as the third feasible market structure. We incorporate the values of η_1 (Equation (5)) and λ_{s1} (Equation (34)) into the definition of aggregate benefits (Equation (11)), so

$$B_v(s) \quad = \quad B_U - \frac{(l_N - \alpha_{N1})^2 \Delta c_A + \Delta\alpha_A^2 \Delta c_N}{2\Delta c_N \Delta c_A} s^2 = B_U - \frac{1}{2}\frac{1}{\gamma^l} s^2 \tag{64}$$

After incorporating these firms into emission formulas (Equations (7) and (8)), aggregate emissions are

$$e_v(s) = U - \left(\frac{\Delta\alpha_A^2 \Delta c_N + (l_N - \alpha_{N1})\Delta\alpha_N \Delta c_A}{\Delta c_N \Delta c_A}\right) s_s = U - \frac{1}{\gamma^L} s, \tag{65}$$

so the aggregate social welfare is

$$W_v(s) = B_v(s) - D(e_v(s)) = B_U - \frac{1}{2}\frac{1}{\gamma^l} s^2 - \frac{d}{2}\left(U - \frac{1}{\gamma^L} s\right)^2. \tag{66}$$

We calculate the (second-best) optimal tax rate, τ_v. First order condition states that

$$\frac{dE\left[W_v(\tau_v)\right]}{d\tau_v} = E\left[-\frac{1}{\gamma^l}\tau_v + \frac{d}{\gamma^L}\left(U - \frac{1}{\gamma^L}\tau_v\right)\right] = 0,$$

or that

$$-\frac{1}{\gamma^l}\tau_v + \frac{d}{\gamma^L}\left(\overline{U} - \frac{1}{\gamma^L}\tau_v\right) = 0.$$

After arrangement,

$$\tau_v = \frac{d\gamma^l \gamma^L}{\left(\gamma^L\right)^2 + d\gamma^l}\overline{U}. \tag{67}$$

Incorporate τ_v into the social welfare in Equation (66), so

$$W_v(\tau_v) = B_U - \frac{1}{2\gamma^l}\left(\frac{d\gamma^l \gamma^L}{(\gamma^L)^2 + d\gamma^l}\overline{U}\right)^2 - \frac{d}{2}\left(U - \frac{1}{\gamma^L}\left(\frac{d\gamma^l \gamma^L}{(\gamma^L)^2 + d\gamma^l}\overline{U}\right)\right)^2.$$

Next, we apply the decomposition in Equation (51) and take the expected value. It holds that

$$EW_v(\tau_v) = EB_U - \frac{1}{2\gamma^l}\left(\frac{d\gamma^l\gamma^L}{(\gamma^L)^2 + d\gamma^l}\right)^2 \overline{U}^2 - \frac{d}{2}\left[E\Phi^2 - \left(1 - \frac{d\gamma^l}{(\gamma^L)^2 + d\gamma^l}\right)^2 \overline{U}^2\right]$$

or, that

$$EW_v(\tau_v) = EB_U - \frac{d}{2}E\Phi^2 - d\left(\gamma^L\right)^2 \frac{\overline{U}^2}{2\left((\gamma^L)^2 + d\gamma^l\right)}.$$

We incorporate the second-best tax rate from Equation (67) into this equation, so we can write

$$EW_v(\tau_v) = EB_U - \frac{d}{2}E\Phi^2 - \frac{\gamma^L}{\gamma^l}\frac{\overline{U}}{2}\tau_v. \tag{68}$$

Finally, we compare optimal emission level under incomplete voluntary participation to earlier optimal emission levels. By Equations (49), (57) (65), and (67), it holds that

$$\begin{aligned} E\left[e_v\left(\tau_v\right)\right] - E\left[e\left(\tau\right)\right] &= \frac{1}{\gamma}\frac{\gamma d}{\gamma + d}EU - \frac{1}{\gamma_L}\frac{d\gamma_l\gamma_L}{\gamma_L^2 + d\gamma_l}EU \\ &= \left(\frac{1}{\gamma + d} - \frac{\gamma_l}{\gamma_L^2 + d\gamma_l}\right)dEU. \end{aligned}$$

This can be also written as

$$E\left[e_v\left(\tau_v\right)\right] - E\left[e\left(\tau\right)\right] = d\left(\frac{\gamma\gamma_l}{\gamma_l}\frac{\frac{(\gamma_L)^2}{\gamma\gamma_l} - 1}{(\gamma + d)\left(\frac{\gamma}{\gamma\gamma_l}\gamma_L^2 + d\right)}\right)dEU,$$

or, after using the definition of ρ (Equation (39) in the main text), as

$$E\left[e_v\left(\tau_v\right)\right] - E\left[e\left(\tau\right)\right] = \gamma d\left(\frac{\rho - 1}{(\gamma + d)(\gamma\rho + d)}\right)EU. \tag{69}$$

Similarly, By Equations (55), (57) (65), and (67), it holds that

$$\begin{aligned} E\left[e_v\left(\tau_v\right)\right] - E\left[e_a\left(\tau_a\right)\right] &= \frac{1}{\gamma_a}\frac{\gamma_a d}{\gamma_a + d}EU - \frac{1}{\gamma_L}\frac{d\gamma_l\gamma_L}{\gamma_L^2 + d\gamma_l}EU \\ &= \frac{d}{\gamma_a + d}EU - \frac{d\gamma_l}{\gamma_L^2 + d\gamma_l}EU \end{aligned}$$

or, alternatively written,

$$E\left[e_v\left(\tau_v\right)\right]-E\left[e_a\left(\tau_a\right)\right]=\left(\frac{\left(\gamma_L\right)^2-\gamma_a\gamma_l}{\left(\gamma_a+d\right)\left(\gamma_L^2+d\gamma_l\right)}\right)dEU.$$

It holds that $\gamma_a = n\gamma$ (see Equation (31) in the main text), so (using also the definition of ρ (Equation (39))

$$E\left[e_v\left(\tau_v\right)\right]-E\left[e_a\left(\tau_a\right)\right]=\gamma d\left(\frac{\rho-n}{\left(n\gamma+d\right)\left(\gamma\rho+d\right)}\right)EU. \tag{70}$$

B Instrument Choice

We derive various results of instruments choice under incomplete voluntary participation. In what follows, we repeatedly apply definitions.

$$u = \frac{\Delta c_A}{\Delta c_N} > 0,\ a = \frac{\Delta \alpha_N}{\Delta \alpha_A} > 0,\ k = \frac{l_N - \alpha_{N1}}{\Delta \alpha_A} > 0.$$

We have

$$\begin{aligned} n &= \frac{\gamma_a}{\gamma} = \frac{\frac{\Delta c_A}{\Delta \alpha_A^2}}{\frac{\Delta c_A \Delta c_N}{\Delta c_A \Delta \alpha_N^2 + \Delta c_N \Delta \alpha_A^2}} = \frac{\Delta c_A \Delta \alpha_N^2 + \Delta c_N \Delta \alpha_A^2}{\Delta c_N \Delta \alpha_A^2} = 1 + \frac{\Delta c_A \Delta \alpha_N^2}{\Delta c_N \Delta \alpha_A^2} \\ &= 1 + ua^2 \end{aligned} \tag{71}$$

and

$$\begin{aligned} \rho &= \frac{\gamma_L^2}{\gamma \gamma_l} = \frac{\left(\frac{\Delta c_N \Delta c_A}{\Delta \alpha_A^2 \Delta c_N + (l_N - \alpha_{N1}) \Delta \alpha_N \Delta c_A}\right)^2}{\frac{\Delta c_N \Delta c_A}{(l_N - \alpha_{N1})^2 \Delta c_A + \Delta \alpha_A^2 \Delta c_N} \frac{\Delta c_A \Delta c_N}{\Delta c_A \Delta \alpha_N^2 + \Delta c_N \Delta \alpha_A^2}} \\ &= \frac{\left((l_N - \alpha_{N1})^2 \Delta c_A + \Delta \alpha_A^2 \Delta c_N\right) \left(\Delta c_A \Delta \alpha_N^2 + \Delta c_N \Delta \alpha_A^2\right)}{\left(\Delta \alpha_A^2 \Delta c_N + (l_N - \alpha_{N1}) \Delta \alpha_N \Delta c_A\right)^2} \\ &= \frac{\left(1 + \frac{(l_N - \alpha_{N1})^2 \Delta c_A}{\Delta \alpha_A^2 \Delta c_N}\right) \left(1 + \frac{\Delta c_A \Delta \alpha_N^2}{\Delta \alpha_A^2 \Delta c_N}\right)}{\left(1 + \frac{(l_N - \alpha_{N1}) \Delta \alpha_N \Delta c_A}{\Delta \alpha_A^2 \Delta c_N}\right)^2} = \frac{(1 + uk^2)(1 + ua^2)}{(1 + uak)^2}, \end{aligned} \tag{72}$$

so that

$$\begin{aligned} \rho &= \frac{1 + 2uak + (uak)^2 + u(k^2 - 2ak + a^2)}{(1 + uak)^2} = \frac{(1 + uak)^2 + u(k - a)^2}{(1 + uak)^2} \\ &= 1 + \frac{u(k - a)^2}{(1 + uak)^2}. \end{aligned} \tag{73}$$

In particular,

$$\frac{n}{\rho} = \left(1 + ua^2\right) \frac{(1 + uak)^2}{(1 + uk^2)(1 + ua^2)} = \frac{(1 + uak)^2}{1 + uk^2}. \tag{74}$$

Next, we write prices as

$$s_v = Es_v - \gamma_L \left(R_0(s_v) \frac{\Delta \alpha_A}{\Delta c_A} \theta_A + R_1(s_v) \frac{\Delta \alpha_N}{\Delta c_N} \theta_N \right), \tag{75}$$

so the quantities can be written as

$$e_v(s_v) = E\left[e_v(s_v)\right] - \left((1 - R_0(s_v))\frac{\Delta\alpha_A}{\Delta c_A}\theta_A + (1 - R_1(s_v))\frac{\Delta\alpha_N}{\Delta c_N}\theta_N\right), \tag{76}$$

where

$$R_0(p_v) = \frac{\gamma_l}{\gamma_L} \equiv R_0, \tag{77}$$

$$R_1(p_v) = \frac{\gamma_l}{\gamma_L}\frac{(l_N - \alpha_{N1})}{\Delta\alpha_N} \equiv R_1, \tag{78}$$

and

$$R_0(\tau_v) = R_1(\tau_v) = 0$$

(see Equations (36) and (43) in the main text). With the help of the formula in Equation (75), we write difference in benefits (Equation (64)) as

$$E\left[B_v(\tau_v) - B_v(p_v)\right] = E\left[\frac{1}{2}\frac{1}{\gamma_l}\left(\gamma_L\left(R_0\frac{\Delta\alpha_A}{\Delta c_A}\theta_A - R_1\frac{\Delta\alpha_N}{\Delta c_N}\theta_N\right)\right)^2\right]$$

or, as θ_A and θ_N are independently distributed, as

$$E\left[B_v(\tau_v) - B_v(p_v)\right] = \frac{1}{2}\frac{\gamma_L^2}{\gamma_l}\left(E\left(R_0\frac{\Delta\alpha_A}{\Delta c_A}\theta_A\right)^2 + E\left(R_1\frac{\Delta\alpha_N}{\Delta c_N}\theta_N\right)^2\right).$$

We further apply the definition of ρ (Equation (72)), and the fact that

$$Var(p_v) = \gamma_L^2 E\left(R_0\frac{\Delta\alpha_A}{\Delta c_A}\theta_A\right)^2 + E\left(R_1\frac{\Delta\alpha_N}{\Delta c_N}\theta_N\right)^2, \tag{79}$$

as we write

$$E\left[B_v(\tau_v) - B_v(p_v)\right] = \frac{1}{2}\frac{Var(p_v)}{\gamma_L^2}\gamma\rho. \tag{80}$$

The difference between damages is

$$E\left[D(e_v(\tau_v)) - D(e_v(p_v))\right] = \frac{d}{2}E\left[(e_v(\tau_v))^2 - (e_v(p_v))^2\right].$$

By the formula in Equation (76), and by the fact that θ_A and θ_N are independently distributed, we may further write

$$\begin{aligned} & E\left[D(e_v(\tau_v)) - D(e_v(p_v))\right] \\ = \; & \frac{d}{2} 2 \left(R_0 \left(\frac{\Delta \alpha_A}{\Delta c_A} \right)^2 E(\theta_A)^2 + R_1 \left(\frac{\Delta \alpha_N}{\Delta c_N} \right)^2 E(\theta_N)^2 \right) \\ & - \frac{d}{2} \left(R_0^2 \left(\frac{\Delta \alpha_A}{\Delta c_A} \right)^2 E(\theta_A)^2 + R_1^2 \left(\frac{\Delta \alpha_N}{\Delta c_N} \right)^2 E(\theta_N)^2 \right). \end{aligned}$$

By Equation (79), and by the fact $E(\theta_A)^2 = E(\theta_N)^2$, it further holds that

$$\begin{aligned} & E\left[D(e_v(\tau_v)) - D(e_v(p_v))\right] \\ = \; & \frac{1}{2} \frac{Var(p_v)}{\gamma_L^2} d \left(2 \frac{R_0 \left(\frac{\Delta \alpha_A}{\Delta c_A} \right)^2 + R_1 \left(\frac{\Delta \alpha_N}{\Delta c_N} \right)^2}{R_0^2 \left(\frac{\Delta \alpha_A}{\Delta c_A} \right)^2 + R_1^2 \left(\frac{\Delta \alpha_N}{\Delta c_N} \right)^2} - 1 \right). \end{aligned}$$

We define volume effect Θ as

$$\Theta = 2 \frac{R_0 \left(\frac{\Delta \alpha_A}{\Delta c_A} \right)^2 + R_1 \left(\frac{\Delta \alpha_N}{\Delta c_N} \right)^2}{R_0^2 \left(\frac{\Delta \alpha_A}{\Delta c_A} \right)^2 + R_1^2 \left(\frac{\Delta \alpha_N}{\Delta c_N} \right)^2} - 1, \tag{81}$$

so

$$E\left[D(\tau_v) - D(p_v)\right] = \frac{1}{2} \frac{Var(p_v)}{\gamma_L^2} d\Theta. \tag{82}$$

Regarding the size of the volume effect, we write

$$q = \frac{R_0 \left(\frac{\Delta \alpha_A}{\Delta c_A} \right)^2 + R_1 \left(\frac{\Delta \alpha_N}{\Delta c_N} \right)^2}{R_0^2 \left(\frac{\Delta \alpha_A}{\Delta c_A} \right)^2 + R_1^2 \left(\frac{\Delta \alpha_N}{\Delta c_N} \right)^2} = \frac{1}{R_0} \frac{1 + \frac{R_1 \left(\frac{\Delta \alpha_N}{\Delta c_N} \right)^2}{R_0 \left(\frac{\Delta \alpha_A}{\Delta c_A} \right)^2}}{1 + \frac{R_1^2 \left(\frac{\Delta \alpha_N}{\Delta c_N} \right)^2}{R_0^2 \left(\frac{\Delta \alpha_A}{\Delta c_A} \right)^2}}.$$

By Equations (77) and (78), we have

$$R_0 = \frac{\gamma_l}{\gamma_L} = \frac{\frac{\Delta c_N \Delta c_A}{(\alpha_{N1} - l_N)^2 \Delta c_A + \Delta \alpha_A^2 \Delta c_N}}{\frac{\Delta c_N \Delta c_A}{\Delta \alpha_A^2 \Delta c_N + (l_N - \alpha_{N1}) \Delta \alpha_N \Delta c_A}} = \frac{1 + \frac{(l_N - \alpha_{N1}) \Delta \alpha_N \Delta c_A}{\Delta \alpha_A^2 \Delta c_N}}{1 + \frac{(l_N - \alpha_{N1})^2 \Delta c_A}{\Delta \alpha_A^2 \Delta c_N}} = \frac{1 + uak}{1 + uk^2}$$

and

$$R_1 \quad = \quad R_0 \frac{(l_N - \alpha_{N1})}{\Delta\alpha_N} = R_0 \frac{k}{a}.$$

Then,

$$\begin{aligned} q \quad &= \quad \frac{1}{R_0} \frac{1 + \frac{k}{a} \left(\frac{\Delta c_A}{\Delta c_N}\right)^2 \left(\frac{\Delta\alpha_N}{\Delta\alpha_A}\right)^2}{1 + \left(\frac{k}{a}\right)^2 \left(\frac{\Delta c_A}{\Delta c_N}\right)^2 \left(\frac{\Delta\alpha_N}{\Delta\alpha_A}\right)^2} \qquad (83) \\ &= \quad \frac{1 + uk^2}{1 + uak} \frac{1 + \frac{k}{a} u^2 a^2}{1 + \left(\frac{k}{a}\right)^2 u^2 a^2} = \frac{1 + uk^2}{1 + uak} \frac{1 + ku^2 a}{1 + k^2 u^2}. \end{aligned}$$

Finally, we calculate the total effect of incomplete voluntary participation on instrument choice. This is equivalent to calculating a difference

$$\Theta - \frac{\rho}{n}.$$

In terms of u, k, and a,

$$\begin{aligned} \Theta - \frac{\rho}{n} &= 2 \left[\frac{1 + uk^2}{1 + uka} \frac{1 + u^2 ka}{1 + u^2 k^2}\right] - 1 - \frac{(1 + uk^2)}{(1 + uak)^2} \\ &= \frac{1 + uk^2}{1 + uka} \left[2 \frac{1 + u^2 ka}{1 + u^2 k^2} - \frac{1 + uka}{1 + uk^2} - \frac{1}{1 + uka}\right] \end{aligned}$$

(see Equations (74), (81) and (83)). The difference can also be presented in a more illustrative fashion by straightforward calculations. In brief, it can be written as

$$\Theta - \frac{\rho}{n} = \frac{1 + uk^2}{1 + uka} \left[\left(\frac{1 + u^2 ka}{1 + u^2 k^2} - \frac{1 + uka}{1 + uk^2}\right) + \left(\frac{1 + u^2 ka}{1 + u^2 k^2} - \frac{1}{1 + uka}\right)\right]$$

or, as

$$\begin{aligned} \Theta - \frac{\rho}{n} &= uk \frac{1 + uk^2}{1 + uka} \left(\frac{u\,(a - k) - (a - k)}{(1 + u^2 k^2)\,(1 + uk^2)}\right) \\ &\quad + uk \frac{1 + uk^2}{1 + uka} \left(\frac{u\,(a - k) + a + k - k + u^2 k^2 a^2}{(1 + u^2 k^2)\,(1 + uka)}\right), \end{aligned}$$

or, finally, as

$$\Theta - \frac{\rho}{n} \tag{84}$$

$$= uk\frac{(a-k)}{(1+u^2k^2)}\frac{1+uk^2}{1+uka}\left[\left(\frac{u-1}{(1+uk^2)}\right)+\left(\frac{u+1+\frac{k\left(1+ku^2a^2\right)}{(a-k)}}{(1+uka)}\right)\right].$$