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TOWARDS BETTER COMPREHENSION OF THE THEORY BY ENHANCING LANGUAGING EXERCISES IN ENGINEERING MATHEMATICS COURSE DIFFERENTIAL CALCULUS

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ABSTRACT

This paper introduces the use of languaging exercises in the engineering mathematics course ´Differential Calculus´. Mathematical exercises are typically seen as expressible by symbols and expressions. Languaging exercises are expressed by natural language, by pictures, or by a combination of these. In this paper it is demonstrated how the languaging exercises were utilized to clarify and deepen the learning of the theory. The study aimed at enhancing the understanding of theory by exploiting languaging exercises. Studies conducted among the university students have shown that using languaging exercises develops students' mathematical understanding. The findings indicate that among the students who studied in the group where the languaging exercises were used got better exam results in the exercises related to understanding the theory.

INTRODUCTION

Essentials skills in the knowledge society are numeracy, mathematical and digital competences and an understanding of science (COM, 2008). The fundamental aim of mathematics in engineering education is mathematics competencies which means the ability to apply mathematical concepts and procedures in relevant contexts (SEFI, 2013). In higher education, mathematics has an important role in engineering courses (OECD, 1996).

Whilst in mathematics, if small changes are made to the exercises, for example the name of variables are changed, the environment of the problem, or the problem statement, students struggle with the exercises. Students seem to be able to mechanically repeat the known procedures to solve problems or patch different parts of previous solutions together to match the new problem (Woods, Hrymak, Marshall, Wood, Crowe, Hoffman, & Bouchard, 1997). To be able to apply the mathematics in engineering studies students need to be able to enact mathematics in different sorts of contexts. Boudon (2016) pointed out in his study that writing mathematics does not only strengthen students' conceptual understanding but can also develop their ability to communicate the meaning of such concepts.

This research examines the effects of expressing mathematics using speech, writing and drawings. The research was designed with the goal of requiring students to think about what they are doing, not just mechanically calculate with the symbols, which fosters the depth of understanding.

Languaging method

When the important mathematical skills are considered, rarely spoken language/writing comes first in the mind. Mathematical exercises are typically seen as expressible by symbols and expressions to obtain answers. In this paper the use of the languaging method is presented. By languaging is meant expressing mathematics with speech, writing and drawings (Joutsenlahti, 2010).

Mathematics lessons are typically predominated by direct instruction which easily encourage students to imitation without ensuring conceptual understanding of the subject. As a result of this, although a student may be able to find the right solution to an exercise, s/he may not be able to explain the meaning of the solution. The purpose of the study is to find out how to develop conceptual thinking with the help of the languaging exercises, languaging exercises are expressed by natural language (i.e. spoken language, mother tongue), by pictures (graphs, charts, geometric patterns), or by a combination of these (Joutsenlahti, 2010; Joutsenlahti, Sarikka, Kangas, & Harjulehto, 2013). Boudon (2016) highlighted that writing prompts encouraged students to build their conceptual knowledge and their ability to express mathematical ideas. The theoretical framework consisted of conceptual thinking and mathematical languaging.

The use of the languaging method aims at achieving a better theoretical knowledge of mathematics, and to deepen the learning of mathematics. With the languaging exercises the aim is a successful internalization of concepts, which has been demonstrated to be a prerequisite of deep learning (Joutsenlahti, Ali-Löytty, & Pohjolainen, 2016).

The purpose of the languaging exercises is to structure and clarify the student's own thinking processes by providing additional tools and ways to express mathematics (Joutsenlahti, 2010). In the languaging exercises, the student is guided towards structuring and explaining the solution and the result of the exercise in their own words (orally, in writing) or by drawings. The key challenge of mathematics teaching is how to describe mathematical thinking and how to make it visible. The languaging exercises enable the making visible students' mathematical thinking processes and also support the development of these processes (Joutsenlahti, & Kulju, 2017).

THE METHODS

This section beginnings with a quote from Benjamin Franklin "Tell me and I forget, teaching me and I understand, involve me and I learn".

This paper introduces the use of languaging exercises in the engineering mathematics course ´Differential Calculus´ during the spring semester 2018, at Tampere University of Applied Sciences (TAMK). The engineering mathematics course ´Differential Calculus´ was taught at the beginning of the millennium by using a book, formula book, blackboard, overhead projector, symbolic calculator and some of the materials were shared online. During the spring semester 2018 on the same course the resources used included a book, formula book, symbolic calculator, smart screen, document camera, Moodle learning platform, short videos, computer aided assessment, google sheets, STACK (System for Teaching and Assessment using a Computer algebra Kernel) exercises and languaging exercises. Digitalization has brought a lot of tools to mathematics teaching and in this era of digitalization the question arises as to whether students gain a conceptual understanding of mathematics.

In this paper it is demonstrated how the languaging exercises were utilized to clarify and deepen the learning of the theory - the aim of using languaging exercises is to involve students to study the theory by doing languaging exercises.

In this study there were two electrical engineering student groups, one ICT engineering and one construction engineering. The languaging exercises were utilized in two student groups, and in the two other groups these were not used. Three groups had the same teacher and one had another teacher. Table 1 illustrates the deviation of the languaging exercises between the groups.

Table 1. Division of languaging exercises by groups.

Teacher	Group	Languaging exercises
Teacher 1	Electrical engineering A	yes
Teacher 1	ICT	yes
Teacher 1	Civil engineering	no
Teacher 2	Electrical engineering B	no

Traditionally teaching mathematics classes are based on a model whereby the teacher presents the theory and gives examples of the exercises based on the theory, and then the students do the exercises which are mainly symbolic expressions. One way to enrich mathematical learning experiences is through the use of different types of activities and exercises. In this study greater emphasis was placed on the theory by utilizing languaging exercises and in this way involving the students to a greater extent in the study of theory. The aim of using languaging exercises was to give the students a better understanding of the mathematics with the aim that with this better knowledge the students would show greater competence in engineering subjects. The aim of all of this is to promote higher-order thinking, i.e. analysis and critical thinking.

In Finland languaging exercises have been used and studied both at primary school level and at higher levels of education, but the use of mathematics languaging at an applied science university among engineering students is quite new, likewise the use of online languaging exercises. Rinneheimo and Joutsenlahti (2019) conducted the first study of the use of languaging exercises among the engineering students at an applied science university in the years 2016 and 2017. The digitalization aspect was taken into account in 2017, when the languaging exercises were online whereas in 2016 the languaging exercises were of the 'pen and paper' type. In this present study all the languaging exercises were online exercises.

The previous study of using the languaging method was conducted in the engineering mathematics course 'Orientation for Engineering Mathematics' (OEM) during the autumn semesters of 2016 and 2017, at TAMK. OEM is an optional course offered to first year students before their engineering studies. The content of the OEM course is as follows: mathematical notations, unit conversions, mathematical expressions, solving linear, quadratic and simultaneous equations, solving right triangle, simple areas and volumes.

The feedback from that study was that the languaging exercises were seen as helpful in mathematics studies, students said that the languaging exercises brought good variation to the studies and the use of natural language was useful and improved learning. The feedback also included statements recommending the use of the languaging exercises in the forthcoming courses. There were, for example, comments such as: "Preferably I would like to have these languaging exercises in the compulsory mathematics courses.", "The course's languaging exercises dealt with rather basic issues. I believe that I would benefit from more of the languaging exercises on the forthcoming courses: Differential Calculus, Integral Calculus etc." (Rinneheimo, & Joutsenlahti, 2019)

The previous study also promotes in a similar way to this new study the students' desire for the creation of languaging exercises for the other engineering mathematics courses.

This following study was conducted on the Differential Calculus course to ascertain whether the languaging exercises improve the learning process and increase mathematical understanding.

The research questions are: 1) Do the students gain a better knowledge of the theory with the help of the languaging exercises? 2) How effectively do the languaging exercises work online?

There are different types of languaging exercises and in this study the following were used ´Argumentation of the solution´, ´Explaining in your own words´ and ´Seeking errors´. In the task ´Argumentation of the solution´, the student writes or selects a natural language explanation for the solution in place of using symbolic language (or vice versa), also pictorial language could be used. ´Explaining in your own words´ means that the student provides an explanation by using natural language. In the ´Seeking errors´ task the student has to find errors or missing items in the given solution and to correct the errors. (Joutsenlahti et al. 2013; Joutsenlahti, Sarikka, & Pohjolainen 2014; Sarikka 2014; Joutsenlahti 2010)

On the courses where the languaging exercises were used there were three online sets of languaging exercises, and these sets included 26 languaging exercises all together. The topic of the first set was regression and limits (seven exercises), the second set dealt with graphical, numerical and symbolic differentiation (thirteen exercises) and the third set consisted of applied exercises (six exercises). The online languaging exercises were done using different question types on the Moodle learning platform. The question types used Moodle were 'Essay', 'Multiple Choice', 'Matching' and 'STACK'. 'Essay' allows students to write a response of a few sentences or paragraphs and this must be manually graded. When this question type was used students were asked to explain in their own words or asked to seek errors and explain in own words the correction to the error. With the 'Multiple Choice' question type single-answer and multiple-answer questions were included. Pictures were also included in the question and/or answer options. 'Matching' enabled the creation of questions with a list of sub-questions together with a list of answers and the respondent must each time match the correct answer with the question. 'Multiple Choice' and 'Matching' were used to select a natural language explanation for a symbolic presentation or graph. 'STACK' provides mathematical questions, these questions use a computer algebra system to establish the mathematical properties of the student's responses. Where the question type 'STACK' was used the solution to the problem was explained with natural language and the student was asked to complete the missing calculations. 'STACK' was used also in the exercises where the students interpret a graph.

The advantages of the online exercises were the possibilities to provide hints, feedback and/or a model solution of the exercise to the student while s/he is doing the exercise. Also, depending on the type of Moodle question, the online exercises were assessed automatically.

Exercises where the students were asked to explain in their own words were for example: Explain in own words what does $\lim_{x \to a} f(x) = b$ means. Another example related to the function and the graph given in figure 1. Then it was asked from the students: Explain in your own words a) what is the difference whit the markings h(1) and h´(1), b) how would you define the derivate for the function at the point t = 3 graphically, c) how would you define the derivate for the function at the point t = 3 numerically, d) how would you define the derivate for the function at the point t = 3 symbolically.

In figure 2 is an example of the exercise of interpreting the graph. In figure 3 an example of the ´Seeking errors´ exercise is presented. This exercise included exam answers taken from the previous years' courses.

$$h(t) = 18\frac{\mathrm{m}}{\mathrm{s}}\cdot t - 4{,}9\frac{\mathrm{m}}{\mathrm{s}^2}\cdot t^2$$

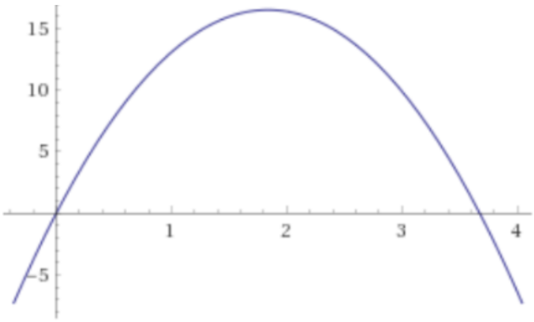
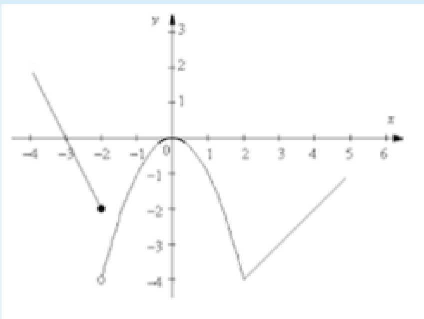


Figure 1. Explain in your own words.



a) $\lim_{x\to -2+} f(x) =$

b) $\lim_{x\to -2-} f(x) =$

c) $\lim_{x\to -2} f(x) =$

d) $\lim_{x\to 2+} f(x) =$

e) $\lim_{x\to 2-} f(x) =$

f) $\lim_{x\to 2} f(x) =$

g) $\lim_{x\to \infty} f(x) =$

h) $\lim_{x\to -\infty} f(x) =$

Figure 2. Interpret the graph.

Student

$$D\left(\frac{x^5}{15} + 3\sqrt[3]{x} - \frac{2}{x^3}\right)$$
$$1. = 15\cdot\frac{1}{x^{-5}} + 3x^{\frac{1}{3}} - 2x^{-3}$$
$$2. = 15x^5 + 3x^{\frac{1}{3}} - 2x^{-3}$$

Student

$$D\left(\frac{x^5}{15} + 3\sqrt[3]{x} - \frac{2}{x^3}\right)$$
$$1. = D\left(\frac{1}{15}x^5 + 3x^{\frac{1}{3}} - 2x^{-3}\right)$$
$$2. = \frac{5}{15}x^4 + x^{-\frac{1}{3}} + 6x^{-2}$$

Student

$$D\left(\frac{x^5}{15} + 3\sqrt[3]{x} - \frac{2}{x^3}\right)$$
$$1. = \frac{5x^4\cdot 15 - x^5\cdot 0}{(15)^2} + 3\cdot\frac{1}{3}x^{-\frac{2}{3}} - \frac{0\cdot x^3 - 2\cdot 3x^2}{(x^3)^2}$$
$$2. = \frac{75x^4}{225} + \frac{x}{\sqrt[3]{x^2}} + \frac{6x^2}{(x^3)^2}$$

Student

$$D\left(\frac{x^5}{15} + 3\sqrt[3]{x} - \frac{2}{x^3}\right)$$
$$1. = \frac{5x^4}{15} + 3\cdot\frac{1}{3\sqrt[3]{x^2}} - \frac{2}{3x^2}$$

Figure 3. Seek errors and correct them.

As we can perceive from these languaging exercises most of the exercises are expressed using a of the combination of natural language, pictures and symbols. Mathematical exercises and solutions are typically seen as expressible by symbols and expressions and with these languaging exercises natural language (i.e. spoken language, mother tongue) and pictures (i.e. graphs, charts, geometric patterns) are used alongside the symbolic language of mathematics.

Usually students are able to differentiate to get a derivative of a function. Some of the students are very good at using derivative rules, even the tricky ones, but on the other hand the meaning of the derivative of the function is a more demanding task for the students. The purpose of using these three languages is to create meanings for the concepts and processes used, and to make the student understand what s/he is doing and not just do mechanical calculations.

Lee (2006) has pointed out that when students express their thoughts out loud and by writing, they remember things better and they are able to apply them later. Students will also be able to transform mathematical concepts into new situations and they are capable of building meanings of concepts and expressions (Lee 2006). In this study with the languaging exercises students were guided to express their thoughts through writing and drawings. The aim is that when the students understand the meaning of the concept, they are able to apply it later on in his/her engineering studies.

Kilpatrick, Swafford and Findell have pointed out that conceptual understanding is the ability to present mathematical solutions in different ways and the ability to evaluate how to utilize different presentations for different purposes.

Lemke (2003) has studied how natural language, mathematical symbols and visual representations create a unified system for meaning-making. Lemke shows how mathematics can be seen not only as a system of signs, but as making mathematical meaning in real contexts. Visual illustration makes it easier to understand what is represented by symbolic language, and natural language is capable of explaining and describing various visual patterns. Thus, the presentation no longer defines the mathematics of the expression, but defines the meaning of the expression. Lemke highlights ways to make explicit for students how mathematical expressions and mathematized visual representations can be translated into natural language and vice versa. (Lemke 2003)

With the help of natural language meanings for concepts can be produced, symbolic language enables the viewing of quantitative changes in concepts, and with pictures it is, for example, possible to describe relationships between concepts. (Joutsenlahti, & Tossavainen, 2018)

In the final exam all four groups had two languaging exercises of exactly the same kind. The first exercise was multiple-choice. The students selected from three options the right formula for the given task. The second exercise included tasks where the students interpreted the graph and explained mathematical notations in their own words. This second exercise placed greater emphasis on knowledge of the theory.

RESULTS AND CONCLUSIONS

One way to enrich mathematical learning experiences is through the use of different types of activities. With the languaging exercises it was possible to provide different kinds of exercises and involve the student in learning the theory.

Before the Differential Calculus course, the students have two compulsory courses (called Geometry and Vector Algebra and Functions and Matrices). The grades given on these previous mathematics courses are used as a pre-test for the study and the key figures of the grades are presented in Table 2.

Table 2. Key figures of the previous mathematics courses.

Group	Average	StDev	Median
Geometry and Vector Algebra			
Electrical eng. A	2,8	1,5	3,0
ICT eng.	2,1	1,6	2,0
Electrical eng. B	2,5	1,5	3,0
Construction eng.	1,9	1,7	1,0
Functions and Matrices			
Electrical eng. A	3,1	1,5	3,0
ICT eng.	2,1	1,7	2,0
Electrical eng. B	2,8	1,6	3,0
Construction eng.	2,0	1,6	2,0

In the final exam all four groups had two languaging exercises of exactly the same kind. The key figures of the languaging exercises in the exam are presented in Table 3. The findings indicate that the students in the groups with languaging exercises achieved better results. This strengthens the understanding that languaging exercises really are useful in learning mathematics.

The electrical engineering group A and the ICT group had languaging exercises on their Differential Calculus course and the electrical engineering group B and civil engineering group did not. As we can perceive from the pre-test (Table 2) the electrical engineering groups gained better marks from the previous mathematics courses, so these groups starting level was better for the study course.

Table 3. Key figures of the languaging exercises in the exam.

Group	Exercise	n	Average	StDev	Median
With languaging exercises on the course					
Electrical eng. A	1 (max. 3,5 p)	31	1,46	0,79	1,50
ICT eng.	1	27	1,42	1,00	1,50
Without languaging exercises on the course					
Electrical eng. B	1	36	1,35	0,71	1,50
Construction eng.	1	31	0,74	0,62	0,75
With languaging exercises on the course					
Electrical eng. A	2 (max. 7 p)	31	4,50	1,49	4,50
ICT eng.	2	27	4,40	1,29	4,80
Without languaging exercises on the course					
Electrical eng. B	2	36	4,30	1,65	4,25
Construction eng.	2	31	3,50	1,98	3,50

From Table 3 we can perceive that the electrical engineering group A, which had languaging exercises during the course, did gain the best results of all the groups. The lowest scores were with the construction engineering group, which did not have the languaging exercises during the course and also the pre-test scores were the lowest. As can be seen from the key figures in Table 3 the groups who had languaging exercises during the course got the best results in exercises 1 and 2. Exercise 2 was an exam question that contained languaging exercises such as explaining in your own words and interpreting the graph, revealing enhanced knowledge of the theory. By comparing results between the ICT group and the electrical engineering group B we can perceive that the ICT group did get better scores in both exercises - even though the pre-test shows that the electrical engineering group B gained better scores from their previous mathematics courses. The findings indicate that among the students who studied in the group where the languaging exercises were used, these students got better results in the exam in the exercises related to understanding the theory.

An independent-samples t-test was conducted to compare do the students gain a better knowledge of the theory with the help of the languaging exercises. In the exercise 1 there was a significant difference in the scores for using the languaging exercises during the course (M= 1,44, SD= 0,89) and not using the languaging exercises during the course (M= 1,07, SD= 0,73) conditions; t(123)= 2,57, p = 0,011. Also, in the exercise 2 there was a significant difference in the scores for using the languaging exercises during the course (M= 4,49, SD= 1,39) and not using the languaging exercises during the course (M= 3,89, SD= 1,84) conditions; t(123)= 2,04, p = 0,043. These results suggest that languaging exercises do have an effect on knowledge of the theory. The findings indicate that the languaging exercises clarified and deepened the learning of the theory.

In the three online languaging tasks during the course students wrote down the key statements from the theory discussed during the classes. Phrases were sometimes merely repetition of what was said in class, but some students tried to put the ideas into their own words. The implementation of regular writing in the form a languaging exercises appeared to have a positive influence on the learning in terms of learning the theory. Online languaging exercises gave the teacher formative assessment which was used to inform the instruction and improve the students' abilities to express their thoughts and problems in mathematics, leading to better conceptual understanding. Also, a benefit of the online languaging exercises is that the possibility to provide hints, feedback and/or a model solution of the exercise to the student while s/he is doing the exercise. Also, depending on the type of question, the online exercises are able to be assessed automatically. Another benefit is instant feedback. Students are able to get instant feedback and grading with the help of online exercises, a feature which was seen by students as a very helpful. Students told that if they had problems solving the exercise the hints and the model solution helped them to find their own errors and mistakes immediately, at a time when they had exercise in their mind.

Studies conducted with university students have shown that using languaging exercises develops the students' mathematical understanding (Joutsenlahti et al. 2016; Joutsenlahti et al. 2013; Joutsenlahti et al. 2014; Sarikka 2014). The findings of this study indicate that among the students who studied in a group where languaging exercises were used, these students got better exam results in the questions related to understanding the theory.

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