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On Husserl's Thin Combination View: Structuralism, constructivism, and what not

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Abstract

After a brief outline of his method, the paper discusses Husserl's view of mathematics by means of two theses, namely the Incompleteness Claim and the Dependence Claim, with which Øystein Linnebo (2008) has characterized non-eliminative structuralism as opposed to the more traditional Platonist view of mathematics. According to the Incompleteness Claim, mathematical objects are incomplete in the sense that they have no non-structural properties. The Dependence Claim holds that the mathematical objects are dependent on each other and/or structure to which they belong. Husserl's view is shown to be a combination view: It is generally a species of non-eliminative structuralism, of which the two claims hold. However, in addition the Incompleteness Claim motivates constructivist approach to the mathematical objects. Moreover, due to the "thinness" of his "mathematics-first" approach, he is also open to the more traditionally Platonist approaches to mathematical objects.

Keywords: Husserl, structuralism, mathematical Platonism, constructivism, mathematical naturalism

1. Introduction, Husserl's method: radikale Besinnung

Husserl's philosophy of mathematics is primarily a method with which to approach mathematics. Hence, any attempt to explain his views about mathematics has to be preceded by an account of the used method. He explained it in the most mature way in the introduction to the *Formal and Transcendental Logic* (1929), where Husserl claims that the work is a result of *Besinnung* (for more detail, see Hartimo 2018a). He defines *Besinnung* as follows:

> "*Besinnung* signifies nothing but the attempt actually to produce the sense 'itself', …, it is the attempt to convert the 'intentive sense' … the sense 'vaguely floating before us' in our unclear aiming, into the fulfilled, the clear, sense, and thus to procure for it the evidence of its clear possibility" (Husserl 1969, 9).[1]

Assuming that rational activities are goal directed, *Besinnung* means clarifying the sense of the activity by explicating the typically implicit goals that guide the activity. Husserl assessed these goals by looking at the history of formal sciences, from ancients onwards, trying to capture the "point" of these sciences and how the mathematicians' goals are situated within the tradition of formal sciences. He sorted these activities into two kinds: to formal mathematics that has *non-contradictoriness* as its primary goal. The search for this is manifested in the search for definite manifolds. In logic, that is theory of science, the primary goal is *truth*.

According to Husserl, finding out what people, here the scientists, are aiming at requires entering in "a community of empathy with the scientists" [Mit den Wissenschaftlern in Einfühlungsgemeinschaft stehend oder tretend] (Husserl 1969, 9; Hua 17, 8). Husserl thus claims that *Formal and Transcendental Logic* (1929) is based on his empathetic engagement with the goals of the mathematicians and logicians around him (see Hartimo 2018b for the list of books he had read). Indeed, *Formal and Transcendental Logic* can be read as a commentary on foundations of mathematics and logic in the 1920s, and especially of David Hilbert's aims (Hartimo 2017).

The role of transcendental phenomenology is crucial for Husserl's method. Transcendental logic is ultimately about examining the presuppositions assumed in formal sciences and clarifying the evidence striven at in them. In other words, it is examination of mathematicians' aims, i.e., reflection of what exactly mathematicians are after when they seek non-contradictoriness or truth. Husserl's transcendental reflections showed that pure mathematics is guided by what he calls evidence of distinctness, that is, *Deutlichkeit*. In contrast, logic, striving at critically verified judgments, aims at having the objects themselves in the evidence of clarity, *Klarheit*. Husserl claims that the difference between the kinds of evidence made him realize that pure mathematics (what Husserl calls formal mathematics) has to be separated from logic that has a (different) notion of truth as its goal. In a way then, transcendental logic served to Husserl as a heuristic device for foundational research. However, its primary aim is to sort out conceptual confusions and making sure that the activities have a "point" that its practitioners have a clear awareness of. With the help of transcendental logic, the norms guiding formal mathematics and logic could be clarified, and if needed, revised.

In starting from examining mathematicians' activities and in the attempt of making sense of these activities, Husserl's approach is "mathematics first", and reminiscent of Penelope Maddy's naturalistic method, summarized to be to: "identify the goals and evaluate the methods by their relations to those goals" (Maddy 1997, 194). To be sure, Husserl incorporates into it transcendental phenomenological reflection, which is not of interest to Maddy. Nevertheless, his method is similarly "mathematics-first" and in it activities are criticized in so far as they do not serve the purposes they were supposed to, or their goals are unclear, conflated, or confused. The clarification of these goals leads to amelioration of the used concepts so that the renewed norms will be adopted habitually into the practices. Thus Husserl's "mathematics-first" approach is also revisionist: transcendental and historical study aims at finding out what the used concepts, norms, and values should be. However, this is not philosophy-first revisionism, in which, in words of Shapiro, "[t]he criticism does come from outside, from pre-conceived first principles" (Shapiro 2012, 13). It is criticism that arises from the consideration of the goals and values within the activities themselves. This may lead to embracing a plurality of normative goals, as I believe Husserl was led to. Despite of this, Husserl's picture is not relativist either: it aims at one unified picture within which all genuine practices have their proper roles. In it the confused goals and aims of the mathematicians are clarified and sorted out to form one sensible whole.

In what follows I will try to draw a picture of this whole as it seems to have looked to Husserl, when approached with the method characterized above. I will argue that Husserl sees mathematics mainly as a structuralist enterprise. I will argue that his structuralism differs from the more traditional Platonism and is Platonist in a "Lotzean" sense. Husserl also finds a need for more "material determination," which shows in his occasional constructivism. Finally, Husserl is open to a possibility of there being Platonistic, independent abstract objects, if mathematics develops in the way that commits mathematicians' to their existence. Thus, his view can be characterized to be a combination view, a combination of structuralism, constructivism, and even Platonism – all considered "thinly," as views to which mathematicians are committed, rather than as philosophical postulations about what there is.

2. Husserl's non-eliminative structuralism vs. Platonism: Dependence and Incompleteness

The role of *Besinnung* in Husserl's methodology makes his views contextual and "mathematics first". It led Husserl to a belief that mathematics is ultimately about striving for "definite manifolds", domains of categorical theories that should also be syntactically complete – something to which he still refers to in FTL (Hua 17, §31). Husserl's structuralism is particularly clear in the following passage from the *Prolegomena* (1900):

> "The *objective correlate* of the concept of a possible theory, definite only in respect of form, is *the concept of a possible field of knowledge over which a theory of this form will preside.* Such a field is, however, known in mathematical circles as a *manifold*. It is accordingly a field which is uniquely and solely determined by falling under a theory of such a form, whose objects are such as to permit of *certain* associations which fall under certain basic laws of this or that *determinate* form (here the only determining feature). The objects remain quite indefinite as regards their matter, to indicate which the mathematician prefers to speak of them as 'thought-objects'. They are not determined directly as individual or specific singulars, nor indirectly by way of their material species or genera, but solely by the form of the connections attributed to them. These laws then, as they determine a *field* and its *form*, likewise determine the theory to be

> constructed, or more correctly, the theory's form. In the theory of manifolds, e.g. '+' is not the sign for numerical addition, but for any connection for which laws of the form $a + b = b + a$ etc., hold. The manifold is determined by the fact that its thought-objects permit of these 'operations' (and of others whose compatibility with these can be shown *a priori*)." (Husserl 1970, 156; Hua 18, §70)

A formally definite manifold has a form. In terms defined by Stewart Shapiro this form is a *structure*: "A structure is the abstract form of a system, highlighting the interrelationships among the objects, and ignoring any features of them that do not affect how they relate to other objects in the system" (1997, 74). The objects, the pure positions in the structure, are abstract "thought-objects." They are determined "solely by the form of the connections attributed to them". They comprise what Husserl calls a '*manifold'*, and formal mathematics is about such manifolds and their relationships to each other. Husserl seems to suggest that the formally definite manifolds are domains of categorical theories, i.e., theories for which any two realizations are isomorphic with each other. For Husserl, the "thought-objects" are bona fide objects, even though they are only "formally determined." (To anticipate what is to come later, Husserl seems to have two notions of definiteness in mind: one merely formal, and the other more "material.") Husserl's discussion of mathematical objects by means of structures does not aim at eliminating them, but at demarcating a legitimate domain of formal objects. Husserl's structuralism is thus a species of non-eliminative structuralism (cf. Parsons 2008, 52).

On Husserl's view, the mathematicians are thus committed to the existence of abstract objects in so far as they are guided by the notion of definite manifold (HUA 17 §31). To determine how his view relates to Platonism about mathematics, it is very useful to consider two claims, termed Incompleteness Claim and Dependence Claim, with which Øystein Linnebo (2008) has characterized mathematical (non-eliminative) structuralism. The Incompleteness Claim holds that mathematical objects are incomplete in the sense that they have no non-structural properties. According to the Dependence Claim the mathematical objects are dependent on each other and/or structure to which they belong. The more traditional

Platonists differ from structuralists in ascribing richer and more independent nature to the mathematical objects.

On Husserl's *Prolegomena* formulation given above, both of these claims hold. The mathematical objects are "quite indefinite as regards to their matter" and "They are not determined directly as individual or specific singulars, nor indirectly by way of their material species or genera, but solely by the form of the connections attributed to them" (Incompleteness), and they are determined "solely by the form of the connections attributed to them" (Dependence). Husserl further explains that this approach banishes "all metaphysical fog and all mysticism from the mathematical investigations in question" (Huserl 1970, 157 [§70]). Husserl wrote this remark when he was in Halle with Georg Cantor as his colleague, so the suspicion is that the remark is directed at Cantor. In his *Grundlagen*, Cantor explicitly suggested that his definition of a manifold or a set captures something akin to the Platonic idea (Cantor 1996, 916). Whether Husserl thinks of Cantor or not, consistently with his remark about "metaphysical fog" (*Prolegomen*a, §70), in the [*Logical*] *Investigations* (II, § 7)*,* Husserl is critical of Platonic realism, i.e., "the metaphysical hypostatization of the universal, the assumption that the Species really exists externally to thought" (Husserl 1970, 248). On this formulation universals are given richer nature and independence, which is ruled out with the two structuralist claims.

However, to Brentano Husserl has conceded that already the *Prolegomena* had been influenced by Lotze's interpretation of Plato (BW 1, 39). In his attempt to rewrite the introduction to the 1913 edition of the *Logical Investigations* Husserl credits Lotze's discussion of Plato for his development towards anti-psychological idea of logic, calling Lotze's interpretation genial:

> "The fully conscious and radical turn and the related „Platonism" I owe to the study of Lotze's Logik. As little as Lotze himself could overcome contradictions and psychologism, as much his genial interpretation of Platonic ideas helped me and my further studies. Lotze's discussion of truths in themselves suggested to me the thought to place all mathematics and a good part of traditional logic into the realm of ideality." (Hua 20/1, 297)

For Lotze's Plato, the ideas do not exist as things do, but they possess validity in virtue of the relations between them.[2] On Lotze's view the Platonic ideas are thus dependent on other ideas and the structure they are a part of. Husserl's view could thus be said to be Platonist in Lotze's sense, that is, within the limits of the two structuralist theses.

In sum, for Husserl the structures, i.e., the unique formal domains, form a clearly circumscribed idea of the "essential content of logic" (Hua 18, §3). In so doing, they banish "the metaphysical fog" out of his Platonism in substituting modern mathematics in place of the doctrine of ideas in Lotze's Plato. In mathematics, the structures, but nothing else, exist "in themselves". Stewart Shapiro calls this kind of structuralism *ante rem* structuralism in accordance to the traditional distinction between *ante rem* and *in re* theories of universals.[3]

3. Structuralism and its thinness in Husserl's later works

Husserl's *ante rem* structuralism can be found more or less unchanged in Husserl's *Ideen I* (1913). In it, the notion of definite manifolds is presented as an ideal norm for scientific rationality (Hua I, §72). Husserl writes, for example, that

> "the closer an experiential science comes to the 'rational' level, the level of 'exact,' of nomological science - ….- the greater will become the scope and power of its cognitive-practical performance." (Husserl 1982, 19; Hua 3/1, §9)

The definite manifolds provide the empty forms of any region whatever. Husserl defines this as formal ontology that thus contains the forms of all ontologies "and prescribes for material ontologies *a formal structure common to them all*" (Hua 3/1, §10). Husserl establishes the term "essence" to refer to mathematical objects "themselves":

> "One occasionally reads in a treatise that the series of cardinal numbers is a series of concepts and then, a little further on, that concepts are products of thinking. At first cardinal numbers themselves, the essences, were thus designated as concepts. But are not cardinal numbers, we ask, what they are regardless of whether we 'form' or do not form them?" (Husserl 1982, 42)

These essences still conform to categorical axiomatic theories, so that, considered purely formally, there are mere essence-forms (i.e., the thought-objects of the *Prolegomena*) that fit all possible essences. Husserl also points out that these exact essences should be regarded as Kantian ideas (Husserl 1982, 97; Hua 3/1, §83). As such they have a normative, guiding role for our perception and also for concept formation.

In the *Formal and Transcendental Logic* (1929) Husserl explains that the concept of the definite manifold "has continually guided mathematics from within" (Hua 17, §31; Husserl 1969, 95); it is thus a typical example of goal-senses he thinks guides mathematics and is revealed to him by *Besinnung*. Husserl cites his *Prolegomena* discussion of the definite manifolds and terms pure positions, i.e., thought-objects (*Prolegomena*), essence-forms (*Ideas I*), as 'pure modes of anything-whatever' (Hua 17, §24; Husserl 1969, 78). In his transcendental examination of the givenness of the objects (whether abstract or real), Husserl describes them as somethings-themselves that are transcendent (§61). In other words, even though the mathematical objects are dependent, they nevertheless are given as *bona fide* objects, transcendent even though ideal. He also refers to the axiomatic ideal as a regulative ideal norm "beneath actually experienced Nature" (Husserl 1969, 292; Hua 17, 257).

But then in the *Crisis*, written in the 1930s Husserl suddenly renounces such realism as a misleading view:

> "Mathematics and mathematical science, as a garb of ideas, or the garb of symbols of the symbolic mathematical theories, encompasses everything which, for scientists and the educated generally, represents the life-world, dresses it up as "objectively actual and true" nature. It is through the garb of ideas that we take for true being what is actually a method—a method which is designed for the purpose of progressively improving, in infinitum, through "scientific" predictions, those rough predictions which are the only ones originally possible within the sphere of what is actually experienced and experienceable in the lifeworld. It is because of the disguise of ideas that the true meaning of the method, the formulae, the "theories," remained unintelligible and, in the naive formation of the method, was never understood." (Husserl 1976, §9h).

Until the *Crisis*, Husserl's view of mathematics is a species of *non-eliminative structuralism*, in particular *ante rem* structuralism, which however explicitly turns into a normative ideal, or an ideal which reason places into nature. It is the "garb of ideas that we take for true being" as he puts it in the above quote. But this is an illusion, in fact, the "substructed" structure is only a method. Husserl is now instrumentalist about the structures.

This turn of the events can be explained in many ways, for example, psychologically as a result of a general crisis Husserl went through in the early 1930s. A philosophically more satisfactory explanation highlights the importance of Husserl's newly acquired awareness of the Löwenheim-Skolem Theorem. Husserl's primary source to developments in the foundations of mathematics in the 1930s, around the time he was writing the *Crisis* was Friedrich Waismann's (1896-1959) *Einführung in das mathematische Denken: die Begriffsbildung der modernen Mathematik* (1936).[4] In this work Waismann states about unique structures of natural (and later similarly also of real numbers) that:

> "It is now extremely significant that Skolem has thwarted every hope of this kind. That is, he proved a general proposition which says that it is impossible to characterize the number series by finitely many axioms. For, every statement which is valid in the arithmetic of natural numbers is also valid for structures of another kind, so that it is impossible to distinguish the number series by any inner properties from sequences of another kind." (Waismann 1936, 84; 1966, 105)

On Waismann's understanding Löwenheim-Skolem Theorems show that there are no unique structures such as the structure of natural numbers or the structure of reals. It makes Husserl's belief in formal structures underlying his view of formal ontology a wild goose chase. Either Husserl should have shown the well-known results false, or else he had to admit that his own earlier beliefs were illusions. Husserl chose the latter alternative. This development of Husserl's views from *ante re*m structuralism to instrumentalism about the structures show that his metaphysical commitments cannot be discussed independently of his *Besinnung*, that is, his understanding of the mathematicians' view about the nature of mathematics. Instead of taking it as a philosophy-first defense of a certain metaphysical position, Husserl's view is about mathematicians' view of the mathematical reality.

4. The incompleteness of mathematical objects and Husserl's constructivist leanings

In the above analyses, I have mainly assessed Husserl's view of mathematical objects in terms of the Dependence Claim, i.e., the claim that mathematical objects are dependent on each other or the structure to which they belong to. I will now move on to consider the Incompleteness of mathematical objects. According to this claim, the structuralist objects are pure positions of structures, and thus they have no identity or features outside of a structure. Husserl puts this kind of claim forward in the *Prolegomena* in the passage cited already once*:*

> "The objects remain quite indefinite as regards their matter, to indicate which the mathematician prefers to speak of them as 'thought-objects'. They are not determined directly as individual or specific singulars, nor indirectly by way of their material species or genera, but solely by the *form* of the connections attributed to them" (Husserl 1970, 156; Hua 18, §70).

The incompleteness of structuralist objects has been criticized in the literature. Probably the best-known criticism is due to Paul Benacerraf (Benacerraf 1964, 291). The argument is that it must be possible to individuate the abstract objects independently of the role they play in the structure. The structuralist objects are "incomplete," because they can only be ascribed certain properties defined by the structure. This indeterminateness poses problems for example for the applications of mathematics (Parsons 2008, 106, 151).

Husserl's view with regard to the Incompleteness Claim is extremely interesting. He acknowledges that the thought-objects are incomplete, and have no more nature than what the structure ascribes to them. But to him, this motivates using constructive means to further determine these otherwise incomplete objects. Linnebo would call this a "compromise view". He interestingly takes Kant to represent such a view with his [Kant's] distinction between *totum analyticum* (totality prior to its constituents) and *totum syntheticum* (totality synthesized from its parts) (Linnebo 2008).

Indeed, the incompleteness or inauthenticity of the structuralist objects occupied mathematicians already in the 19th century. In the 19th century the problem was typically construed in terms of existence; it seemed that the structuralist "thought-objects" could not be thought as properly existing as such but their existence had to be established by some other means. Dedekind, too, established the existence of the simply infinite systems by correlating them with the realm of things that can be objects of his own thought (Dedekind 1996, 806-807).

Sometimes the worry about the actual existence of the structural objects was related to the worry about their applicability. Citing Frege's remark that "It is applicability alone that raises arithmetic from the rank of a game to that of a science. Applicability therefore belongs to it of necessity" (*Grundgesetze* II §91, cited by Parsons 2008, 73), Parsons (2008) points out that Frege and Russell seemed to have regarded this as an objection towards structuralists.

Husserl shared such concerns. Accordingly, in his Double Lecture he first discusses manifolds as structures, holding that such a domain is "a determinate, but formally defined, manifold" [in German "eine bestimmte, aber formal definierte Mannigfaltigkeit"]. The expression invites the thought that there could be also materially determined manifolds; and as we go further, it becomes clear that this indeed seems to have been Husserl's idea, and that the sought for "material determination" is related to computable constructibility. This is because the purely formal mathematics is difficult to apply. In the Double Lecture, Husserl writes for example that: "But the difficulties lie precisely in the relationship between formal mathematics and its employment in substantive mathematics or in the particular domains of knowledge" (Husserl 2003, 411 {92}). He then starts developing a more constructive method to determine the objects, with a view to determine the objects more individually (see Hartimo 2018c):

> "The essential point is the following: In the axiom system I define not only sentences which hold true for all members of the manifold in general. I therefore operate not only with general, indeterminate concepts of objects, but rather I also introduce individually designating concepts of objects – as it were proper names for objects (or species of objects) – and I axiomatically establish their existence" (Husserl 2003, 445 {116}).

He seems to have thought that for the sake of application the formal objects should have more "material" definiteness. To enable the use of the formal objects he correlated them, or named them, with determinate numbers, and then established the "term-re-writing" reductions to equalities. Thus he came to give the criteria of identity for different kinds of expressions of natural numbers (see Hartimo 2018c). Similarly motivated, in *Ideas I* Husserl adds to structuralist formal ontology material ontologies by means of eidetics, and in *Formal and Transcendental Logic* Husserl regarded the structuralist objects too abstract to have something to do with truth related to experience of objects 'themselves':

> "Each multiplicity defined by a system of axiom-forms presented them with the task of explicitly constructing the form of the corresponding deductive science itself; and the execution of the task involved precisely the same work of constructive deduction that is done in a concrete deductive science with concepts having material contents" (Husserl 1969, 98; Hua 17, §32).

In the end, for him, the proper objects of formal ontology have to be intuited, whether immediately or mediately. They must draw 'fullness' from the evidence of clarity (e.g., Husserl 1969, 203). For Husserl, this takes place in judgments about individuals. The evidence can be 'transferred' by the rules of the judgment theory to more complex judgments (I discuss this in more detail in Hartimo, forthcoming). These evident judgments determine the objects suitably for the needs of sciences and truth. Husserl thus thinks that formal mathematics should be thus constructively proven, and thus it should be ideally, not only formally, but also materially definite.

But in line of his "mathematics first" attitude, Husserl also writes that this kind of evidence is of no particular interest to mathematicians (Husserl 1969, 203), to whom *distinctness*, instead of *clarity* is of interest. (ibid., §52-53). The existence of abstract objects in Husserl's view can thus be either thin or thick, either as indeterminate thought-objects (in pure mathematics) or else as immediately or mediately evident objects (in logic). While the structuralist mathematical objects are distinctly given, Husserl clearly thinks that it would be desirable to bring them into evidence of clarity. For Husserl then, the Incompleteness Claim and the Dependence Claims do not hold of all mathematical objects; they do not hold of the constructed formal objects that are given in the evidence of clarity. Husserl often seems to imply that all structuralist formal objects could be constructively given. This is a belief that the development of mathematics showed false in the 1930s. Given his methodological, "mathematics-first" views, it should not be taken as a thesis about the nature of mathematics but rather as a belief about what he thought mathematicians of his time were thinking about mathematics, hence dependent on the stage of mathematical research.

5. Husserl and the iterative conception of sets

Linnebo (2008) discusses sets on the iterative conception as a counter-example to the structuralist Dependence Claim. Since sets are formed from their elements, the elements of the sets have to be "available" before they can be comprehended into a set. The elements of the sets are thus not dependent of the sets they are members of. The sets they will be members of may not even yet exist on iterative conception. In it sets are formed in stages so that on each stage the sets will be formed from the objects of the previous stage. This results in an open-ended hierarchy that can always be further extended (Linnebo 2008, 2013). The cumulative hierarchy motivates most of the axioms of set theory, rendering them in some sense evident (Boolos's phrase is that "there is a thought behind" it) (Linnebo 2017, 140), and I will argue next that this kind of evidence could have well be of interest to Husserl, too.

This brings us to a consideration about Husserl and the Dependence Claim, but this time possibly in favor of the more traditional Platonism. While in the case of constructive judgment theory the fullness is added to the background structure, set theory might suggest a Platonist departure from structuralism. Husserl may have known about iterative conception of sets developed by Zermelo.[5] Zermelo's iterative view of sets postulates a dynamic, open-ended sequence of bigger and bigger domains (models), uniquely characterized by the cardinality of their basis and a "boundary number" that is the least ordinal not in the model. In the hierarchy of models, the sets of one layer are 'grounded' [würzelnd] in the preceding layers, so that their elements are in the previous layers and serve as material for the following layers (Zermelo 1996, 1219).

There are some indications in Husserl's texts that might indicate awareness of Zermelo's cumulative hierarchy, or at least some similar phenomenon. In FTL, for example, in the context of discussing idealizations involved in analytics and, in particular, the fundamental form 'and so forth', Husserl discusses the reiterational 'infinity' that is presupposed in mathematics that "is the realm of infinite constructions, a realm of ideal existences, not only of 'infinite' senses but also of constructional infinities" (§74). And then he continues:

> "Obviously we have here a repetition of the problem concerning subjective constitutive origins: as the hidden method of constructions which is to be uncovered and reshaped as a norm, the method by which 'and so forth', in various senses, and infinities as categorial formations of a new sort become evident… Precisely this evidence, in all its particular formations, must now become a theme" (Husserl 1969, 189; Hua 17, §74)

Husserl refers to a new kind of evidence related to "constructional infinities." The new task of transcendental phenomenology would then be to examine the notion of evidence involved.

A possible reference to the cumulative hierarchy can be found in Husserl's correspondence with Dietrich Mahnke. In a letter to Mahnke in 1933 Husserl discusses his view of the infiniteness of transcendental subjectivity and infinity or endlessness of phenomenology, which studies the infinity of being within the totality [Alleinheit]. He calls the transcendental subjectivity as 'constitutive infinity' and then compares it to the infiniteness of the structural system of the world:

> "The infiniteness of the world, the infiniteness of teleology, that, as the world that prevails for the infinity of monads, recedes and becomes, in evermore new and changing ways, but yet remains as one identical world, - that is not a one-dimensional or multi-dimensional infinity, it is an infinite system of rays of infinity, I think with an infinity of levels, that each has its axiomatics." (BW3, 498)[6]

In fact, Husserl's use of the word "Strahlensystem von Unendlichkeiten" is curious. The reference to an infinity of levels, Stufen, could be informed by Zermelo's cumulative hierarchy introduced in (Zermelo 1996) or it could refer to

Gödel's incompleteness results and the ensuing infinity of axiomatizations where the Gödel sentence is decided by adopting a hierarchy of ever stronger theories in which the Gödel sentence of the previous level can be decided. The topological wording is curious and suggests something related to Riemann. In any case, it indicates Husserl's general attitude towards mathematics at the time to be one that endorses a kind of inexhaustibility or incompleteability: one cannot exhaust the reality with any finite set of axioms. In Zermelo's hierarchy individual domains may be uniquely characterized, which thus suggest structuralist existence for the objects defined by them and relative to them. But the characterization of the infinite sequence of them makes existence claims about, e.g., boundary numbers. So, the question arises: "does this make Husserl more traditionally Platonist, after all?" Indeed, for Husserl, this seems to be a question about the evidence related to "and so forth," to which he refers to already in the FTL. In any case, whether the cumulative hierarchy can be understood structurally or not, Husserl's methodological considerations imply that he should think that the task of transcendental logic is to examine any kind of evidence that surfaces in mathematics, hence certainly the one given by cumulative hierarchy.

6. Conclusion

In sum, while Husserl's (to be more specific, Husserl's view of mathematicians') view of mathematics is mostly *ante rem* structuralist, a closer examination of the Dependence and Incompleteness Claims shows that his *ante rem* structuralism does not hold of all mathematical objects. It holds of algebraic structures, hence much of what is studied in mathematics. Husserl would also like to bring as much of it as possible into clarity by means of computable constructions. Thus Husserl's structuralism has preferably "thick constructive patches," but it is also open to there being Platonist objects beyond it.

These different approaches ultimately differ in their normative goals, in particular, in the kinds of evidence they are after. Depending on the kind of evidence, different kinds of methods of proofs and definitions are adopted in different areas.

Husserl's transcendental logic demands consideration of these various evidences, purifying them [his terms], and then, if found worthy, adopting them as new norms guiding mathematical practices. In the *Formal and Transcendental Logic* these goals were divided into two main kinds of evidences: clarity and distinctness. In the early 20th century the distinction between these two kinds of evidences provided an important new insight to the developing modern structural mathematics as opposed to the more constructive or applied approaches. Nothing in Husserl's approach precludes new evidences and new goals to surface in mathematics. Consideration of all of them and how they relate to each other gives a unified picture of pluralistically given mathematics and should help in understanding its place in our conception of the world and our lives.

NOTES

[1] "*Besinnung* besagt nichts anderes als Versuch der wirklichen Herstellung des Sinnes ‚selbst', der in der bloßen Meinung gemeinter, vorausgesetzter ist; oder den Versuch, den ‚intendierenden Sinn', ... den im unklaren Abzielen ‚vage vorschwebenden' in den erfüllten Sinn, den klare überzuführen, ihm also die Evidenz der klaren Möglichkeit zu verschaffen" (Hua 17, 8).

[2] Lotze concludes his discussion of Plato's *Ideenlehre* as follows "Thus we readily understand the significance of Plato's endeavour to bind together the predicates which are found in the things of the external world in continual change, into a determinate and articulated whole, and how he saw in this world of Ideas the true beginnings of certain knowledge; for the eternal relations which subsist between different Ideas, and through which some are capable of association with each other and others exclude each other, form at all events the limits within which what is to be *possible* in experience falls; the further question what is real in it, and how things manage to have Ideas for *their* predicates, appeared to Plato not to be the primary, and was for the time reserved."(Lotze 1884, §315). After having established the unchangeable validity of the world of Ideas, the next task for Plato "was to investigate the universal laws which govern its structure, through which alone, in an Ideal world as elsewhere, the individual elements can be bound together into a whole" (Lotze 1884, §321). Thus the Dependence Claim is true of Lotze's Plato.

[3] Shapiro's ante rem structuralism is a species of Parsons' non-eliminative structuralism, but not *vice versa*. Parsons' structures are not defined by Dedekind abstraction but by taking the language of mathematics as the background structure. On Parsons' view, the most elementary way of describing a mathematical structure is by introducing a one-place predicate

true of an object, with other predicates and functors true of this same object. The uniqueness is not central to him but the intended interpretations of the language of mathematics, which quantify over formal objects that are then, in a Quinean manner, thought to exist (Parsons 2004; Parsons 2008, esp. 111-115). In contrast to Shapiro, in the dilemma between first order logic and determinate ontology, Quine and Parsons opt for the first option on the expense of the latter.

[4] In the end of *Mathematische Existenz*, Oskar Becker discusses Löwenheim-Skolem theorem. However, Husserl probably did not read it until in Mars 1937. According to Husserl-Chronik, this is when "H. hat grössere Abschnitte gelesen (insbesondere zum ersten Mal auch [?] die zweite Hälfte) von Oskar Becker, Mathematische Existenz, 1927." (Schuhmann 1977, 484)

[5] The theory was developed by Zermelo in Freiburg, where Husserl, too, lived at the time (Zermelo 1996). It seems likely that Zermelo's theory is at least indirectly influenced by Husserl. Husserl's assistant of the time, Oskar Becker, apparently had lectured in Zermelo's seminar on problems in the theory of transfinite ordinals that same year (Mancosu 2010, 281, 539). Becker in turn, in his discussion of transfinite "Strukturkomplikationen" of the consciousness refers to Husserl's *Ideas I* (§100), where Husserl discusses hierarchical structures of intentionalities, such as remembering in remembering and so forth. Husserl thinks that they build up a hierarchy. According to him, "[a]ll the types of objectivation-modifications previously dealt with are always accessible for always newer hierarchical formations of such a kind that the intentionalities in the noesis and noema are hierarchically built up on one another or, rather, in a unique way, encased in one another". The intentional acts and the objectifications of them allow for a hierarchy of levels of them. Husserl even assigns indices for these levels. "To every noematic level there belongs a characteristic appropriate to that level as a kind of index with which each thing characterized manifests itself as belonging to its level… For indeed to every level belong possible reflections at that level, so that, e.g., with respect to remembered things at the second level of remembering, [there are] reflections on perceivings of just these things belonging to the same level (thus presentiated at the second level). Furthermore: each noematic level is an 'objectivation' 'of' the data of the following [level]." (Hua 3/1, §101). Acknowledging that Husserl is not motivated to iterate intentional acts infinitely many times, Becker suggests using such hierarchy to clarify Cantor's view of transfinite numbers. Using intuitionist terminology, he then characterizes Cantor's transfinite numbers as a "werdende Folge, deren 'zukunft' nicht voraussehbar ißt" (Becker 1973, 112); as a becoming succession that has a future that is not foreseeable. Becker is explicit about the potential character of the hierarchy. (Similar hierarchies were at the time also proposed by many, e.g., by Russell in his type theory and Weyl's construction in The Continuum (1917) to combat paradoxes of set theory).

[6] „Die Unendlichkeit der Welt, die Unendlichkeit der Teleologie, die in der Unendlichkeit von Monaden waltend Welt werden ließ und fortwerden, immerfort neu und anders werden lässt, und doch als identische Welt – das ist nicht eine einlinige oder mehrlinige Unendlichkeit, es ist ein unendliches

Strahlensystem von Unendlichkeiten, ich denken mit einer Unendlichkeit von Stufen, deren jede ihre Axiomatik hat“ (BW3, 498).

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