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Implications for Aggregate Inflation of Sectoral Asymmetries: Generalizing Woodford

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Abstract

This paper develops and simulates a simple two sector DSGE model for studying aggregate inflation and output dynamics under sectoral adjustment asymmetries. The CES aggregate consumption bundle consists of two different groups of goods with imperfect substitutability between as well as within the groups. Allowing for different within group CES aggregators implies that the degree of substitutability between goods in a group is group-specific. To generate sector-specific price rigidities the model assumes sector-specific Calvo pricing. The paper focuses on potential post-shock divergences across sectors as well as on the implications for aggregate inflation and output of the sectoral asymmetries and identifies an important role for the sectoral relative price for aggregate dynamics. More specifically, the paper generalizes Woodford (2003), which only allows for the price rigidity to differ across sectors. Incorporating sector-specific price elasticities is important and well in line with the micro-level evidence on individual as well as sectoral prices. From the point of view of allocational efficiency and welfare, relative price movements occupy a central role in models incorporating Calvo pricing. This particular feature underscores the perceived macroeconomic benefits of low and stable inflation. This paper takes this logic a step further by incorporating movements both in individual and sectoral relative prices.

Keywords:

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The views expressed here are those of the authors.

1 Introduction

Relative price movements lie at the core of the market mechanism and the associated resource allocation. While conditions under which full price flexibility is efficient and welfare maximizing are well known, price rigidities of various forms are equally well known to have the potential of generating inefficiencies. Take for example the famous Calvo pricing mechanism that is incorporated in so many dynamic macroeconomic models of business cycle fluctuations to allow for pricing frictions. Calvo pricing generates relative price movements between firms currently optimizing their output price and those not optimizing. Hence, inefficient resource allocation due e.g. pure inflationary shocks can emerge in models incorporating the Calvo pricing mechanism. This potential for inefficiencies underscores the perceived macroeconomic benefits of low and stable inflation and justifies the appropriate policy intervention.

Microeconomic data on individual prices clearly indicate that the distribution of the frequency of price changes is non-trivial. The range of frequencies is impressive: some of the prices, e.g. price of gasoline, behave like diffusions, while others, e.g. some of the services, change via infrequent, discrete jumps. Then again, price elasticities seem to differ across goods as well as across sectors. For example Anderson *et al* (1997)¹ document that in the US estimated own price elasticity of demand ranges from 0.1 to 4.6. Examples of inelastic goods include salt, matches and physician services, while fresh tomatoes, fresh green peas and restaurant meals represent elastic goods.

These observations on sectoral asymmetries are the starting point of this study. We seek to analyze the aggregate dynamics of an economy subject to different degree of sectoral price rigidity as well as subject to sector specific price elasticities. We will model these asymmetries by extending Woodford's (2003)² model of sectoral asymmetries to allow for the price elasticity of two groups of goods to differ from each other. As a byproduct we will derive a more general condition under which fluctuations in the sectoral relative price (around the frictionless benchmark) is of no consequence to the aggregate economy.

After introducing the model and solving it, we run simulations in order to assess the quantitative importance to the aggregate economy of the fluctuations in the sectoral relative prices, when the economy is bombarded by various shocks. Of the various shocks we can think of, we will in particular derive the impulse responses to a cost push and interest rate shock. Given the New Keynesian structure of the model, a cost push shock – shock to inflation – is a standard one to consider, whereas a shock to the interest rate is particularly relevant from the point of view of monetary policy.

As a byproduct of our analysis we are able to generalize the condition under which variations in the sectoral relative price do not affect aggregate inflation dynamics. Needless to say, identical frequency of price adjustment across sectors is not any more sufficient for aggregate inflation to be independent of the changes in the sectoral relative price. Sectoral price elasticity, or sector specific mark-up

¹ Anderson P, McLellan R, Overton J and Wolfram G (1997), "Price Elasticity of Demand," http://scholar.harvard.edu/files/alada/files/price_elasticity_of_demand_handout.pdf

² Section 2.5 "Consequences of Sectoral Asymmetries," p. 200 in *Interest and Prices: Foundation for a Theory of Monetary Policy* (2003), OUP.

emerges as a factor that matters. More generally, the combination of the sector specific frequency of price adjustment and price elasticity basically determines how sensitive aggregate inflation is to changes in sectoral relative price and, hence, to shocks that affect sector specific price levels.

The modelling framework in this study is an extended version of a dynamic stochastic general equilibrium (DSGE) model with two sectors, nominal rigidities and imperfect competition presented in Woodford (2003). More specifically, we allow for the price elasticities to differ between the two sectors. This feature is important and well in line with the micro-level evidence on individual as well as sectoral prices. Moreover we allow for external habit formation (Campbell and Cochrane, 1999). Model simulations indicate that relative price movements³ can be important both from the policy transmission and welfare point of view.

The rest of the study is organised as follows; the model used in the analysis is derived and presented in chapter 2. Chapter 3 presents the simulation and its principles while resulting graphs are shown in Appendixes. Chapter 4 concludes.

2 A Two Sector Model

The economy is composed of two sectors within which the goods are imperfectly substitutable. Hence, there is imperfect competition in the relevant markets. A representative household in this economy derives utility from a consumption bundle that is a CES aggregate of the sector specific consumption indices. These sector specific consumption indices are CES aggregates over a continuum of individual goods. Our representative household also works, thus generating disutility in the usual manner. The (flow) budget constraint determines the feasible choices for our representative household: on top of allocating income on consumption, the household can invest in one period bonds, which generates interest income. By working, the household earn wage income.

We thus assume that the representative household seeks to maximise the following intertemporal utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t, \quad (1)$$

where β is the discount factor and where

$$U_t = \varphi_t^B \left(\frac{1}{1-\sigma} (C_t - h_t C_{t-1})^{1-\sigma} - \frac{1}{1+\psi} (L_t)^{1+\psi} \right) \quad (2)$$

³ Sectoral asymmetries could lead to dual inflation; for examples, see e.g. Arratibel, Rodriguez-Palenzuela and Thimann (2002) and Estrada and Lopez-Salido (2002) and ECB (2003) for a more general discussion.

σ is the inverse of the inter-temporal elasticity of substitution in consumption and ψ the inverse of the Frisch elasticity. C_t is now an index of the household's consumption of the goods that are supplied, while L_t is the labor supply. Eq (2) contains also a general preference shocks φ_t^B . The external habit formation is captured by the term h .

Households maximise their objective function (1) subject to the (flow) budget constraint:

$$P_t B_t + P_t C_t = (P_t W_t L_t) + R_{t-1}^b B_{t-1}, \quad (3)$$

where P_t is the price level and B_t denotes bonds.

Total nominal income consists of two components: labour income ($P_t W_t L_t$) and the gross return on the bonds ($R_{t-1}^b B_{t-1}$). As the capital stock is assumed to be fixed in the considerations we do not include it here. This is because we will focus on the dynamic effects of cost push and interest rate shocks at the business cycle frequency. The underlying assumption here is that variations in the capital stock are not the main driver for business cycles.

Consumption behavior

The Euler condition for the optimal intertemporal allocation of consumption is derived from the maximization problem of the objective function (2) subject to budget constraint (3) with respect to consumption and (nominal) return on bonds R_t . This yields

$$E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t P_t}{P_{t+1}} \right] = 1 \quad (4)$$

where $\lambda_t = \varphi_t^b (C_t - hC_{t-1})^{-\sigma}$ is the usual consumption Euler equation describing the marginal utility of consumption.

This leads to the optimal consumption dynamics of the log-linear form:

$$\widehat{c}_t = \frac{h}{1+h} \widehat{c}_{t-1} + \frac{1}{1+h} E_t \widehat{c}_{t+1} - \frac{1-h}{\sigma(1+h)} [\widehat{r}_t - E_t \widehat{\pi}_{t+1}] + \widehat{\varphi}_t^b. \quad (5)$$

The hatted variables represent log deviations from the steady state.

2.1 Deriving a model for sectoral asymmetries

The aggregate consumption index C_t is of the CES-form and consists of two sub-indexes for the commodity groups n_1 and n_2 .

$$C_t = \left[(n_1 \varphi_{1t})^{\frac{1}{\eta}} C_{1t}^{\frac{\eta-1}{\eta}} + (n_2 \varphi_{2t})^{\frac{1}{\eta}} C_{2t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad (6)$$

where $\varphi_{j,t}$ is a shock to the relative weight of the commodity group in a consumption basket and η is the elasticity of substitution between the groups and the sectoral consumption index aggregates a continuum of sector-specific goods

$$C_{jt} = \left[n_j^{-\frac{1}{\theta_j}} \int_0^{\frac{\theta_j}{\theta_j-1}} c_{jt}(i)^{\frac{\theta_j-1}{\theta_j}} di \right]^{\frac{\theta_j}{\theta_j-1}}, \quad j=1, 2 \quad N_1 = [0, n_1], N_2 = [n_1, 1] \quad (7)$$

Here θ_j is the elasticity of substitution between sector j goods defining the own price elasticity of the demand for these goods. We allow the two θ :s to differ.

Sectoral price indices, which defines the minimum cost of buying a unit of the sector j good, satisfy

$$P_{j,t} = \left[n_j^{-\frac{1}{\theta_j}} \int_{N_j} p_{j,t}(i)^{1-\theta_j} di \right]^{\frac{1}{1-\theta_j}}, \quad j = 1, 2 \quad (8)$$

whereas the aggregate price index corresponding to the aggregator in (6) is given by

$$P_t = \left[n_1 \varphi_{1,t} P_{1,t}^{1-\eta} + n_2 \varphi_{2,t} P_{2,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (9)$$

Optimal allocation for different goods in sector $j = 1, 2$ can be derived from the minimization problem

$$\min \int p_{j,t}(i) c_{j,t}(i) di \quad \text{s.t.} \quad \left[n_j^{-\frac{1}{\theta_j}} \int_{N_j} c_{j,t}(i)^{\frac{\theta_j-1}{\theta_j}} di \right]^{\frac{\theta_j}{\theta_j-1}} \geq C_{jt} \quad \text{and its first order conditions.}$$

Demand for different brands $c(i)$ within a group j is then

$$c_{jt}(i) = n_j^{-1} \left[\frac{p_{jt}(i)}{P_{jt}} \right]^{-\theta_j} C_{jt}. \quad (10)$$

The sectoral market demand functions in equilibrium are then

$$C_{jt} = n_{jt} \varphi_{jt} \left(\frac{P_{jt}}{P_t} \right)^{-\eta} C_t. \quad (11)$$

Note that as in Woodford (2003) the aggregators have been normalized so that under the common individual price in both sectors, $p_{jt}(i) = p_t \forall j, i$,

$$c_{jt}(i) = \varphi_{jt} C_t.$$

Disutility of labour is given by $v(h(i); \xi_{jt})$, where ξ is a vector of parameters of interest, so that sector-specific shocks to preferences regarding labour supply is allowed for, but not good-specific. Production of the good i is obtained via the production function

$$y_{jt}(i) = A_{jt} f(h_t(i)) \quad (12)$$

which thus implies that sector-specific shocks are allowed for. Assuming that each firm is a wage-taker in the firm specific labour market, (nominal) profits of firm i in sector j can be written as

$$\begin{aligned} \Pi_t^j(p_t(i)) &= p_t(i) y_{jt}(i) - W_t(i) h_t(i) \\ &= n_j^{-1} p_t(i)^{1-\theta_j} P_{jt}^{\theta_j} Y_{jt} - W_t(i) f^{-1}(A_{jt}^{-1} n_j^{-1} p_t(i)^{-\theta_j} P_{jt}^{\theta_j} Y_{jt}) \end{aligned} \quad (13)$$

The real total cost of supplying any good i in sector j can be represented as

$$\begin{aligned} TC(y_{jt}(i), Y_t; \tilde{\xi}_t) &= w_t(i) h_t(i) \\ &= w_t(i) f^{-1}(A_{jt}^{-1} n_j^{-1} p_t(i)^{-\theta_j} P_{jt}^{\theta_j} Y_{jt}). \end{aligned} \quad (14)$$

Hence, real marginal cost of supplying any good i in sector j can be represented as

$$\begin{aligned} s^j(y_{jt}(i), Y_t; \tilde{\xi}_t) &= \frac{w_t(i)}{mpl_t|_{h=f^{-1}(c)}} \\ &= \frac{v_h(f^{-1}(y_{jt}(i)/A_{jt}); \xi_{jt})}{u_c(Y_t; \tilde{\xi}_t)} \frac{1}{A_{jt} f' [f^{-1}(A_{jt}^{-1} n_j^{-1} p_t(i)^{-\theta_j} P_{jt}^{\theta_j} Y_{jt})]} \\ &\equiv \frac{v_h(f^{-1}(y_{jt}(i)/A_{jt}); \xi_{jt})}{u_c(Y_t; \tilde{\xi}_t)} \Psi(y_{jt}(i)/A_{jt}) \end{aligned} \quad (15)$$

Note that we have used the assumption that the households are on their labour supply schedule so that the real wage equals the marginal rate of substitution between labour and consumption. Note also that mpl denotes marginal product of labour.

The desired mark-up in sector j is now $\mu_j = \frac{\theta_j}{\theta_j - 1}$, a constant, but sector specific. The natural level of output in sector j , Y_{jt}^n , is defined as the common level of sector j output under flexible prices. It satisfies

$$\mu_j s^j(Y_{jt}^n, Y_t^n; \tilde{\xi}_t) = \frac{P_{jt}}{P_t} \left(\frac{Y_{jt}^n}{n_j \varphi_{jt} Y_t^n} \right)^{-\frac{1}{\eta}} \quad (16)$$

with the intended interpretation that the utmost right hand side indicates the relative price P_{jt} / P_t required to induce the relative demand Y_{jt}^n / Y_t^n . The natural rate of aggregate output Y_t^n aggregates sectoral natural outputs according to the CES-aggregator. If $\tilde{\xi}_t = 0$ and $\varphi = 1$ for all t and for both sectors, the flex price equilibrium involves a common output \bar{Y} for all goods (satisfying the above equilibrium pricing equation)

$$s^j(\bar{Y}, \bar{Y}; 0) = \frac{1}{\mu_j} \left(\frac{1}{n_j} \right)^{-\frac{1}{\eta}}.$$

Log-linearizing (15) around this equilibrium gives us

$$\begin{aligned} \hat{s}_t^j(i) &= \ln s^j(y_{jt}(i), Y_t; \tilde{\xi}_t) - \ln s(\bar{Y}, \bar{Y}; 0) \\ &= \ln s^j(y_{jt}(i), Y_t; \tilde{\xi}_t) + \ln \mu_j - \frac{1}{\eta} \ln n_j \\ &= \ln s^j(y_{jt}(i), Y_t; \tilde{\xi}_t) - \frac{1}{\eta} [\ln Y_{jt}^n - \ln Y_t^n - \ln n_j - \ln \varphi_{jt}] - \frac{1}{\eta} \ln n_j - \ln s(Y_{jt}^n, Y_t^n; \tilde{\xi}_t) \\ &= \ln s^j(y_{jt}(i), Y_t; \tilde{\xi}_t) - \frac{1}{\eta} [\ln Y_{jt}^n - \ln Y_t^n - \ln \varphi_{jt}] - \ln s(Y_{jt}^n, Y_t^n; \tilde{\xi}_t) \\ &\approx \omega(\hat{y}_{jt}(i) - \hat{Y}_{jt}^n) + \sigma^{-1}(\hat{Y}_t - \hat{Y}_t^n) + \eta^{-1}(\hat{\varphi}_{jt} + \hat{Y}_t^n - \hat{Y}_{jt}^n) \end{aligned} \quad (17)$$

Here ω denotes the elasticity of the real marginal cost function with respect to y_{jt} (and

Y_{jt}^n i.e. the first argument of the marginal cost function).

Assume Calvo-type (Calvo 1983) price staggering in each of the two sectors with α_j the fraction of goods prices that remain constant in any given period in sector j . A firm i in that sector that is lucky to get the change to optimize its price in period t chooses its new price $p_t(i)$ to maximize the expected present value of its profits

$$E_t \left\{ \sum_{T=t}^{\infty} \alpha_j^{T-t} Q_{t,T} \left[\prod_T^j (P_t(i)) \right] \right\} \quad (18)$$

The F.O.C for this programme is, after log-linearizing, given by

$$E_t \sum_{T=t}^{\infty} (\alpha_j \beta)^{T-t} \left\{ \hat{p}_{jt}^* - \left[\hat{s}_{t,T}^j - \hat{p}_{jT} + \sum_{\tau=t+1}^T \pi_{j\tau} \right] \right\} = 0, \quad (19)$$

where $\hat{p}_{jt}^* = \ln \left(\frac{p_{jt}^*}{P_j} \right)$ denotes the relative price (relative to others in sector j) of the firms that get to optimize their price at date t and $\hat{p}_{jt} = \ln \left(\frac{P_{jt}}{P_j} \right)$ is the real price of sector j at date t (i.e. the relative date t price of sector j relative to “cpi” or overall price level). On the other hand, $\hat{s}_{t,T}^j$ is the real marginal cost of the firms in sector j that last change their prices at date t . By (17) we have the following decomposition

$$\begin{aligned} \hat{s}_{t,T}^j &= \omega(\hat{y}_{jt,T} - \hat{Y}_{jt}^n) + \sigma^{-1}(\hat{Y}_T - \hat{Y}_T^n) + \eta^{-1}(\hat{\phi}_{jT} + \hat{Y}_T^n - \hat{Y}_{jT}^n) \\ &= \omega(\hat{Y}_{jT} - \hat{Y}_{jt}^n + \hat{y}_{jt,T} - \hat{Y}_{jT}^n) + \sigma^{-1}(\hat{Y}_T - \hat{Y}_T^n) + \eta^{-1}(\hat{\phi}_{jT} + \hat{Y}_T^n - \hat{Y}_{jT}^n) \\ &= \omega(\hat{Y}_{jT} - \hat{Y}_{jT}^n) + \sigma^{-1}(\hat{Y}_T - \hat{Y}_T^n) + \eta^{-1}(\hat{\phi}_{jT} + \hat{Y}_T^n - \hat{Y}_{jT}^n) + \omega(\hat{y}_{jt,T} - \hat{Y}_{jT}^n) \\ &= \hat{s}_T^j - \omega\theta_j \ln \left(\frac{P_{jt}^*}{P_{jT}} \right) = \hat{s}_T^j - \omega\theta_j \ln \left(\frac{P_{jt}^*}{P_{jt}} \cdot \frac{P_{jt}}{P_{j,t+1}} \cdot \dots \cdot \frac{P_{j,t+1}}{P_{jT}} \right) \\ &= \hat{s}_T^j - \omega\theta_j \left\{ \ln \left(\frac{P_{jt}^*}{P_{jT}} \right) - \ln \left(\frac{P_{jt}}{P_{j,t+1}} \cdot \dots \cdot \frac{P_{j,t+1}}{P_{jT}} \right) \right\} \\ &= \hat{s}_T^j - \omega\theta_j \left\{ \hat{p}_{jt}^* - \sum_{\tau=t+1}^T \pi_{j\tau} \right\} \end{aligned} \quad (20)$$

where \hat{s}_T^j denotes the (deviation of the) average (i.e. real marginal cost corresponding to average sectoral output Y_{jt}) real marginal cost in sector j . Sectoral price indexes are given in (8) and repeated here for convenience

$$P_{jt}^{1-\theta_j} = n_j^{-1} \int_{N_j} P_{jt}(i)^{1-\theta_j} di, \quad j=1, 2$$

$$\begin{aligned}
&= n_j^{-1} \int_{N_j} \left[(1 - \alpha_j) p_{jt}^*{}^{1-\theta_j} + \alpha_j p_{j,t-1}^{1-\theta_j} \right] di \\
&= (1 - \alpha_j) p_{jt}^*{}^{1-\theta_j} + \alpha_j P_{j,t-1}^{1-\theta_j} \Leftrightarrow \\
&1 = (1 - \alpha_j) \left(\frac{p_{jt}^*}{P_{jt}} \right)^{1-\theta_j} + \alpha_j \left(\frac{P_{j,t-1}}{P_{jt}} \right)^{1-\theta_j} \\
&= (1 - \alpha_j) \left(\frac{p_{jt}^*}{P_{jt}} \right)^{1-\theta_j} + \alpha_j \left(\frac{1}{1 + \pi_{j,t}} \right)^{1-\theta_j} \tag{21}
\end{aligned}$$

so a log-linear approximation (around the steady state) allows us to derive the following relationship

$$\begin{aligned}
0 &= \left[(1 - \alpha_j) \hat{p}_{jt}^* - \alpha_j \hat{\pi}_{jt} \right] (1 - \theta_j) \Leftrightarrow \\
\hat{p}_{jt}^* &= \left(\frac{\alpha_j}{1 - \alpha_j} \right) \hat{\pi}_{jt} \tag{22}
\end{aligned}$$

Now, insert everything into the optimal pricing equation

$$\begin{aligned}
&E_t \sum_{T=t}^{\infty} (a_j \beta)^{T-t} \left\{ \hat{p}_{jt}^* - \left[\hat{s}_{t,T}^j - \hat{p}_{jT} + \sum_{\tau=t+1}^T \pi_{j\tau} \right] \right\} = 0 \\
&E_t \sum_{T=t}^{\infty} (a_j \beta)^{T-t} \left\{ (1 + \omega \theta_j) \hat{p}_{jt}^* - \left[\hat{s}_T^j - \hat{p}_{jT} + (1 + \omega \theta_j) \sum_{\tau=t+1}^T \pi_{j\tau} \right] \right\} = 0 \\
&E_t \sum_{T=t}^{\infty} (a_j \beta)^{T-t} \left\{ \frac{\alpha_j (1 + \omega \theta_j)}{(1 - \alpha_j)} \hat{\pi}_{jt} - \left[\hat{s}_{t,T}^j - \hat{p}_{jT} + \sum_{\tau=t+1}^T \pi_{j\tau} \right] \right\} = 0
\end{aligned}$$

where from further manipulations give us

$$\begin{aligned}
\left[\frac{\alpha_j}{1 - \alpha_j} \right] \hat{\pi}_{jt} &= \left(\frac{1 - \alpha_j \beta}{1 + \omega \theta_j} \right) E_t \sum_{T=t}^{\infty} (a_j \beta)^{T-t} (\hat{s}_T^j - \hat{p}_{jT}) \\
&+ (1 - \alpha_j \beta) E_t \sum_{T=t}^{\infty} (a_j \beta)^{T-t} \sum_{\tau=t+1}^T \pi_{j\tau} \\
&= \left(\frac{1 - \alpha_j \beta}{1 + \omega \theta_j} \right) E_t \sum_{T=t}^{\infty} (a_j \beta)^{T-t} (\hat{s}_T^j - \hat{p}_{jT}) \\
&+ \alpha_j \beta E_t \sum_{T=t}^{\infty} (a_j \beta)^{T-t} \pi_{j\tau}
\end{aligned}$$

$$\begin{aligned}
&= \left(\frac{1 - \alpha_j \beta}{1 + \omega \theta_j} \right) (\hat{s}_t^j - \hat{p}_{jt}) + \alpha_j \beta E_t \hat{\pi}_{j,t+1} + \alpha_j \beta \left[\frac{\alpha_j}{1 - \alpha_j} \right] E_t \hat{\pi}_{j,t+1} \\
&= \left(\frac{1 - \alpha_j \beta}{1 + \omega \theta_j} \right) (\hat{s}_t^j - \hat{p}_{jt}) + \left[\frac{\alpha_j \beta}{1 - \alpha_j} \right] E_t \hat{\pi}_{j,t+1} \Rightarrow \\
\hat{\pi}_{jt} &= \underbrace{\left(\frac{(1 - \alpha_j \beta)(1 - \alpha_j)}{\alpha_j (1 + \omega \theta_j)} \right)}_{\zeta_j} (\hat{s}_t^j - \hat{p}_{jt}) + \beta E_t \hat{\pi}_{j,t+1} \\
&= \zeta_j (\hat{s}_t^j - \hat{p}_{jt}) + \beta E_t \hat{\pi}_{j,t+1} \tag{23}
\end{aligned}$$

We are almost there, as we still need to express the sector specific real marginal cost in terms of the relevant average sectoral output measure. From the expression for the demand for the sectoral composite good we obtain

$$\begin{aligned}
\hat{Y}_{jt} &= \hat{\phi}_{jt} + \hat{Y}_t - \eta \hat{p}_{jt} \\
\hat{Y}_j^n &= \hat{\phi}_{jt} + \hat{Y}_t^n - \eta \hat{p}_{jt}^n . \tag{24}
\end{aligned}$$

Substituting this into the average real marginal cost of sector j gives us

$$\begin{aligned}
\hat{s}_t^j &= \omega (\hat{Y}_{jt} - \hat{Y}_{jt}^n) + \sigma^{-1} (\hat{Y}_t - \hat{Y}_t^n) + \eta^{-1} (\hat{\phi}_{jt} + \hat{Y}_t^n - \hat{Y}_{jt}^n) \\
&= -\omega \eta (\hat{p}_{jt} - \hat{p}_{jt}^n) + (\sigma^{-1} + \omega) (\hat{Y}_t - \hat{Y}_t^n) + \eta^{-1} (\hat{\phi}_{jt} + \hat{Y}_t^n - \hat{Y}_{jt}^n) \\
&= -\omega \eta (\hat{p}_{jt} - \hat{p}_{jt}^n) + (\sigma^{-1} + \omega) (\hat{Y}_t - \hat{Y}_t^n) + \hat{p}_{jt}^n + \hat{p}_{jt} - \hat{p}_{jt}^n \Leftrightarrow \\
\hat{s}_t^j - \hat{p}_{jt} &= (\sigma^{-1} + \omega) (\hat{Y}_t - \hat{Y}_t^n) - (1 - \omega \eta) (\hat{p}_{jt} - \hat{p}_{jt}^n) \tag{25}
\end{aligned}$$

Inserting this into (23), we obtain

$$\hat{\pi}_{jt} = \zeta_j (\sigma^{-1} + \omega) (\hat{Y}_t - \hat{Y}_t^n) + \gamma_j (\hat{p}_{Rt} - \hat{p}_{Rt}^n) + \beta E_t \hat{\pi}_{j,t+1}, \tag{26}$$

where $\hat{p}_{Rt}^n \equiv \frac{1}{\eta} \left[(\hat{\phi}_{2t} - \hat{\phi}_{1t}) - (\hat{Y}_{2t}^n - \hat{Y}_{1t}^n) \right]$ and the sectoral relative price $\hat{p}_{Rt} = \ln \left(\frac{P_{2t}}{P_{1t}} \right)$ is obtained from the aggregate price index as follows

$$P_t = \left[n_1 \varphi_{1,t} P_{1,t}^{1-\eta} + n_{2,t} \varphi_{2,t} P_{2,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

$$\begin{aligned}
1 &= \left[n_1 \varphi_{1,t} \left(\frac{P_{1t}}{P_t} \right)^{1-\eta} + n_{2,t} \varphi_{2,t} \left(\frac{P_{2t}}{P_t} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \Rightarrow \\
0 &= \frac{1}{1-\eta} \left[n_1 (\hat{\varphi}_{1t} + (1-\eta) \hat{p}_{1t}) + n_2 (\hat{\varphi}_{2t} + (1-\eta) \hat{p}_{2t}) \right] \\
&= n_1 \hat{p}_{1t} + n_2 \hat{p}_{2t} = (1-n_2) \hat{p}_{1t} + n_2 \hat{p}_{2t} \\
&= \hat{p}_{1t} + n_2 (\hat{p}_{2t} - \hat{p}_{1t}) \equiv \hat{p}_{1t} + n_2 \hat{p}_{Rt} \Rightarrow \hat{p}_{1t} = -n_2 \hat{p}_{Rt}
\end{aligned} \tag{27}$$

and similarly for $\hat{p}_{2t} = n_1 \hat{p}_{Rt}$. Now define $\kappa_j = \zeta_j (\sigma^{-1} + \omega)$ and since $\gamma_1 = n_2 \zeta_1 (1 + \omega \eta)$ and $\gamma_2 = -n_1 \zeta_2 (1 + \omega \eta)$ we can write

$$\hat{\pi}_{jt} = \kappa_j (\hat{Y}_t - \hat{Y}_t^n) + \gamma_j (\hat{p}_{Rt} - \hat{p}_{Rt}^n) + \beta E_t \hat{\pi}_{j,t+1}.$$

We still need to derive the dynamics for aggregate inflation. Note that

$$\begin{aligned}
\hat{p}_{Rt} &= \ln \left(\frac{P_{2t}}{P_{1t}} \right) = \ln \left(\frac{P_{2t}}{P_{2,t-1}} \cdot \frac{P_{1,t-1}}{P_{1,t}} \cdot \frac{P_{2,t-1}}{P_{1,t-1}} \right) \\
&= \hat{\pi}_{2t} - \hat{\pi}_{1t} + \hat{p}_{R,t-1}.
\end{aligned} \tag{28}$$

Hence, we can represent the sectoral inflation differential in terms of the lagged relative price $\hat{p}_{R,t-1}$.

$$\begin{aligned}
(1 - \beta' F) (\hat{\pi}_{1t} - \hat{\pi}_{2t}) &= \kappa' (\hat{Y}_t - \hat{Y}_t^n) + \gamma' (\hat{p}_{Rt} - \hat{p}_{Rt}^n) \text{ or} \\
(\hat{\pi}_{1t} - \hat{\pi}_{2t}) &= \kappa' (\hat{Y}_t - \hat{Y}_t^n) + \gamma' (\hat{p}_{Rt} - \hat{p}_{Rt}^n) + \beta' E_t (\hat{\pi}_{1,t+1} - \hat{\pi}_{2,t+1})
\end{aligned} \tag{29}$$

$$F x_t = E_t x_{t+1}$$

$$\begin{aligned}
\beta' &= \frac{\beta}{1 + \gamma}, \kappa' = \frac{\kappa}{1 + \gamma}, \gamma' = \frac{\gamma}{1 + \gamma} \\
\gamma &= \gamma_1 - \gamma_2, \kappa = \kappa_1 - \kappa_2
\end{aligned}$$

Aggregate inflation rate $\hat{\pi}_t = n_1 \hat{\pi}_{1t} + n_2 \hat{\pi}_{2t}$ has a similar representation

$$\begin{aligned}
\hat{\pi}_t &= \bar{\kappa} (\hat{Y}_t - \hat{Y}_t^n) + \bar{\gamma} (\hat{p}_{Rt} - \hat{p}_{Rt}^n) + \beta E_t \hat{\pi}_{t+1} \\
\bar{\kappa} &= n_1 \kappa_1 + n_2 \kappa_2, \bar{\gamma} = n_1 \gamma_1 + n_2 \gamma_2,
\end{aligned} \tag{30}$$

where we can use equation (29) for the period t relative price to ascertain that aggregate inflation in general depends on the lagged relative price and not only on aggregate variables. Note also that

identical sectoral price adjustment frequency – $\alpha_1 = \alpha_2$ – is not anymore sufficient to eliminate the dependence of aggregate inflation on the sectoral relative price. On the other hand, if both the frequency of price adjustment and price elasticity of demand are equal across sectors, then we have

$$\bar{\gamma} = n_1\gamma_1 + n_2\gamma_2 = 0$$

so that aggregate inflation is independent of the sectoral relative price.

2.2 The model block of sector-specific aggregated model for forecasting and policy analyse

Here we briefly represent and collect the equations we utilise in the log-linearized model for simulations.

Output is given by a linear production function (output consist of consumption goods only so that $Y = C$)

$$Y_t = A_t L_t,$$

and then technical progress and productivity are catch by

$$\frac{A_t}{A_{t-1}} \equiv dA_t = \exp(\lambda + \varepsilon_{A,t}).$$

The real marginal cost, in turn, are given by

$$rmc_t = \frac{w_t}{A_t}.$$

The aggregate consumption Euler equation with external habit formation is

$$\hat{c}_t = \frac{h}{1+h} \hat{c}_{t-1} + \frac{1}{1+h} E_t \hat{c}_{t+1} - \frac{1-h}{\sigma(1+h)} [\hat{r}_t^b - E_t \hat{\pi}_{t+1}] + \hat{\varphi}_t^b,$$

and the sectoral market demand

$$\hat{c}_{j,t} = n_j \log \left(\left(\frac{P_j}{P} \right)^{-\eta} C_t \right).$$

Inflation dynamics is given by the New Keynesian Phillips curve

$$\hat{\pi}_{j,t} = \kappa_j (\hat{c}_t - \hat{c}_t^n) + \gamma_j (\hat{p}_{R,t} - \hat{p}_{R,t}^n + \varphi^b) + \beta E_t \hat{\pi}_{j,t+1} + \varphi_{j,t,p},$$

where $\hat{c}_t^n = \log\left(\left(\frac{\eta}{\eta-1}\right)^{-1/\sigma+\psi} A_t^{\frac{1+\psi}{\sigma+\psi}}\right)$.

and $\hat{\pi}_t = n_1\varphi_{1,t}\hat{\pi}_{1,t} + n_2\varphi_{2,t}\hat{\pi}_{2,t}$,

$$\bar{\kappa} = \bar{\zeta}(\sigma^{-1} + \omega), \quad \zeta = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\omega\theta)} \text{ and } \gamma = \zeta(1+\omega\eta)$$

for aggregate and sectoral inflation.

This formulation then focus on output gap role in inflation formation. If we instead would focus on the role played by the marginal costs we could utilise

$$\hat{\pi}_{j,t} = \zeta_j(\hat{mc}_{j,t} - \hat{p}_{j,t}) + \beta E_t \hat{\pi}_{j,t+1} + \lambda_{jp,t}, \quad j=1, 2$$

for sectoral and aggregate inflation and ζ is defined as earlier. The hatted variables represent log deviations from the steady state. We continue by focus on the output gap as we assume demeaning (of the data) would eliminate the natural level of output and productivity growth as

$$\tilde{c}_t = \hat{c}_t - \bar{c} \approx \hat{c}_t - (\hat{c}_t^n - dA_t), \text{ so that } E_{t-1}\tilde{c}_t = 0.$$

Moreover $\tilde{p}_{j,t} = \ln\left(\frac{P_{j,t}}{P_t}\right) - \ln\left(\frac{\bar{P}_{j,t}}{\bar{P}_t}\right)$, $\hat{p}_{R,t} = \ln\left(\frac{P_{2,t}}{P_{1,t}}\right)$ and

$$\hat{p}_{R,t}^n \equiv \frac{1}{\eta} [(\hat{\varphi}_{2,t} - \hat{\varphi}_{1,t}) - (\hat{c}_{2,t}^n - \hat{c}_{1,t}^n)] \text{ so that } \tilde{p}_{R,t} = \hat{p}_{R,t} - \hat{p}_{R,t}^n \text{ and as labour effort is given by}$$

$\hat{l}_{j,t} = \hat{c}_{j,t} - \hat{a}_{j,t}$, it follows that $\tilde{l}_{j,t} = \tilde{c}_{j,t}$ etc. It is hence assumed that the labour supply equals labour demand i.e. the labour market equilibrium holds. Then we have the following linearized system of equations for the inflation adjustment

$$\tilde{\pi}_{j,t} = \kappa_j(\tilde{c}_t) + \gamma_j(\tilde{p}_{R,t}) + \beta E_t \tilde{\pi}_{j,t+1}, \text{ where } \tilde{\pi}_{j,t} = \tilde{p}_{j,t} - \tilde{p}_{j,t-1} + \varphi_t^{jp}$$

$$\tilde{\pi}_t = n_1\tilde{\pi}_{1,t} + n_2\tilde{\pi}_{2,t}$$

$$\tilde{c}_t = \frac{h}{1+h}\tilde{c}_{t-1} + \frac{1}{1+h}E_t\tilde{c}_{t+1} - \frac{1-h}{\sigma(1+h)}[\tilde{r}_t^b - E_t\tilde{\pi}_{t+1}], \quad \tilde{c}_t = \tilde{c}_{1,t} + \tilde{c}_{2,t} \text{ and}$$

$$\tilde{c}_{j,t} = n_j\left(\left(\tilde{p}_t^{dj}\right)^{-\eta}\tilde{c}_t\right), \text{ where } \tilde{p}_t^{dj} = \left[\ln\left(\frac{P_{j,t}}{P_t}\right) - \ln\left(\frac{P_{j,t-1}}{P_{t-1}}\right)\right] - \bar{p}.$$

$$\tilde{l}_{j,t} = \tilde{c}_{j,t}$$

$$\tilde{r}_t^b = \tilde{r}_{t-1}^b + \varphi_t^r.$$

Endogenous variables are then aggregate and sectoral inflation π , π_j , aggregate and sectoral consumption c , c_j , relative prices p_R , market rate of interest r^b , aggregate and sectoral labour effort l , l_j (and the real wage as $w=l$).

Moreover we have the following exogenous shocks variables that hit the economy: $\varphi^{i,p}$ a general cost push shock and φ^r an interest rate shock to bond rate. These shock variables are assumed to follow an independent first-order autoregressive stochastic process.

$$\varphi_t^{ap} = \rho^{ap} \varphi_{t-1}^{ap} + u_t^{ap},$$

$$\varphi_t^{bp} = \rho^{bp} \varphi_{t-1}^{bp} + u_t^{bp},$$

$$\varphi_t^r = \rho^r \varphi_{t-1}^r + u_t^r.$$

The interest rate shock affects the consumption decisions. Then the aggregate output gap is affected as we postulate that the aggregate level of inflation is fixed (inflation target) in the simulations so that $dP = 0 = P_{p_a} dP_a + P_{p_b} dP_b$.

Here we do not made any assumptions about the covariance of the shocks.

3. Simulation

The linearized non-linear system of optimality conditions and structural equations has resulted in a system of linear stochastic difference equations under rational expectations, which can be written in state-space form

$$A_0 E_t Y_{t+1} = A_1 Y_t + B_0 \varepsilon_{t+1} \quad (31)$$

where A_0 , A_1 and B_0 are matrices of coefficients of the linearized model and Y_t denotes a vector of all endogenous and ε_t a vector of all exogenous variables. These linearized equations form a dynamic system that determines the path of the variables of the model. This system contains backward and forward-looking elements. Depending on the parameterization of the model, either no stable rational expectations solution exists, or the model yields determinacy, i.e., the stable solution is unique. Indeterminacy, in turn, means that multiple stable solutions exist and the parameter space has to be restricted in order to focus on the case of interest. A solution for finding the rational expectations solution of the system is a feedback rule relating the current endogenous variables to the state variables of the model. This study utilises the method of eigenvector-eigenvalue decomposition, developed by Blanchard and Kahn (1980), where the stable and unstable variables of the system are decoupled and solved. The model is first partitioned into predetermined and non-predetermined variables and then a Jordan decomposition of the coefficient matrix is computed. Next the problem is transformed into decoupled stable and unstable equations. The model under scrutiny has a unique solution only if the Blanchard-Kahn condition for determinacy is met: the number of unstable eigenvectors (roots) must be exactly equal to the number of non-predetermined (forward looking) variables. Then this system has saddle path stability.

A model simulation is performed utilising Dynare toolbox for Octave. Calibrated parameter values (as priors) from earlier empirical studies e.g. Smets and Wouters (2003), Kilponen and Ripatti (2005), Gelain and Kulikov (2009) and Koskinen (2010).

Parameters and the respective calibrated values in (sector-specific) simulation:

- β - discount factor (0.99)
- α - Calvo parameter (0.75)
- ω – elasticity of real marginal cost to y (in this case c) (1)
- (λ growth rate)
- θ_a price elasticity of demand within category a goods (-6)
- θ_b price elasticity of demand within category b goods (-15)
- η the elast. of subst. between groups a and b ,
- and price elast. of demand in consumption basket (-10)
- κ weight of the output gap in infl.
- category a : 0.00225, category b : 0.009817
- γ weight of the relative price index in infl. eq
- category a : 0.13488, category b : 0.0590
- σ intertemporal elasticity of substitution in cons (0.6)
- ψ the inverse of the elasticity of work effort w.r.t real wage (1)
- h habit formation parameter (0.47)
- $n = 0.6$ for category a
- u^{ap} stderr 0.02
- u^{bp} stderr 0.01
- u^f stderr 0.005

Sectoral price setting power and inflation adjustment

The sectoral and aggregate inflation adjustment equations reveal that expectations concerning the future prices (based on pricing power) play a crucial role in determining the rate of inflation. The exact manner, how parameter θ_j the price elasticity of demand within a category j affect the sectoral inflation dynamics under Calvo-type pricing mechanism, is shown in in equations (18) – (19) in accordance with eq. (16). These equations together with eq. (21) relates pricing cycle together with pricing power of producers and it is obvious that as parameter θ_j determines the desired mark-up in sector j , the producers in that sector will optimize their prices (i.e. set p^* as high as possible) accordingly. The magnitude (standard error) of a cost push shock in sector a and b is set according to the price mark-ups and difference in the price elasticity between these sectors. The fact that we have set an inflation target ($dP = 0$) by the central bank causes counter wise reactions of some variables to common shocks.

Cost Push and Interest Rate Shocks driving the dynamics of the Model

The motivation of this study is to analyse the reactions of the endogenous model variables to structural shocks in a case where aggregated economy consists of two different kinds of commodity groups with diverge price elasticity of demand parameter values. The structural shocks of interest are a cost push and interest rate (real rate of return on bonds) shocks as they have a central role in macroeconomic fluctuations. In order to distinguish the differences between two separate sectors with different price elasticity of demand for their products we carried out the impulse response analysis in respect for that.

The endogenous variables impulse response to (one standard deviation orthogonalized innovation to) an interest rate shock is plotted on pictures 1 to 2. These pictures plots the impulse responses (IRFs) in an model with two separate sectors (forming an aggregate economy). In a model for an aggregate economy the special features of IRFs of separate sectors are hidden (i.e. they are not identifiable) while these are clearly present in a model allowing for sectoral considerations (e.g. two sectors with asymmetries in the price setting behaviour and price elasticity of demand). More over an aggregate model could hide important counter wise reactions to shocks and the policy actions taken could treat the sub sectors of an economy unfairly. This is obvious when one look at the IRFs of consumption and labour in picture 1 and 2 to an interest rate shock as the sectors react with opposite effects but the aggregate consumption and labour in the same pictures give no information about that.

The sectors response to a positive cost push shock is depicted in pictures 3 to 6. As mentioned previously, in sector *a* this cost push shock is twice as large in magnitude as in sector *b*. An interesting feature is the sectoral reactions of consumption / output and labour to this cost push shock as they are of opposite sign to both of the shocks i.e. to u^{ap} and u^{bp} . Otherwise the reactions are as expected in this model specification.

The variance decomposition methodology measures the relative importance of each structural shocks included in the model. The relative share of total variation that a particular shock explains for each endogenous variable in the model is presented in table 2 in appendixes.

4. Conclusions

In this study we have derived and simulated a two sector DSGE model in order to highlight the potential differences of these sectors with respect to shocks they face in an imperfect competition environment. The modelling framework in this study is an extended version of a DSGE model with two sectors, nominal rigidities and imperfect competition presented by Woodford (2003) in the sense that we allow for the sectoral price elasticities to differ between the two sectors. This feature is important and well in line with the microlevel evidence on individual as well as sectoral prices.

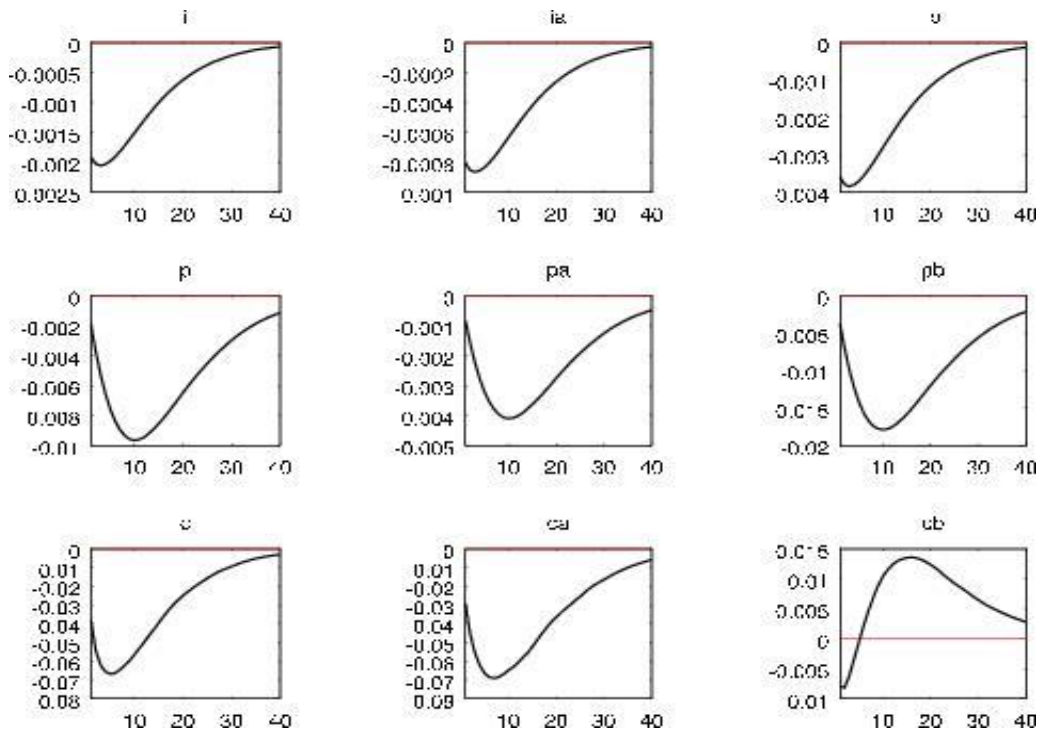
A typical economy consists of several kinds of markets for commodities (sectors) with possibly different structural features and dynamics that characterise the inter-temporal behaviour of agents and their preferences. As these differences between the sectors are due to the agent's preferences (as well as technologies applied) the price setting behaviour within a sector could diverge markedly from the rest of the economy. Then the response of output, labour and inflation or price level formation to (cost push and interest rate) shocks could also be asymmetric within the sectors of economy. In these circumstances the outcome of any economic policy is not equal across the sectors and it is crucial to recognise and react to these differences. The simulations with calibrated parameter values demonstrate, that the relative price movements are potentially important from the policy transmission and welfare point of view as the investigated response of consumption, labour and price level formation to shocks are asymmetric within the two sectors of the simulated model's economy. The consequences of the resulting tradeoff for optimal policies are left for further (empirical) studies in future. Expectations concerning the future prices are one important driving force beyond the dynamics of the model.

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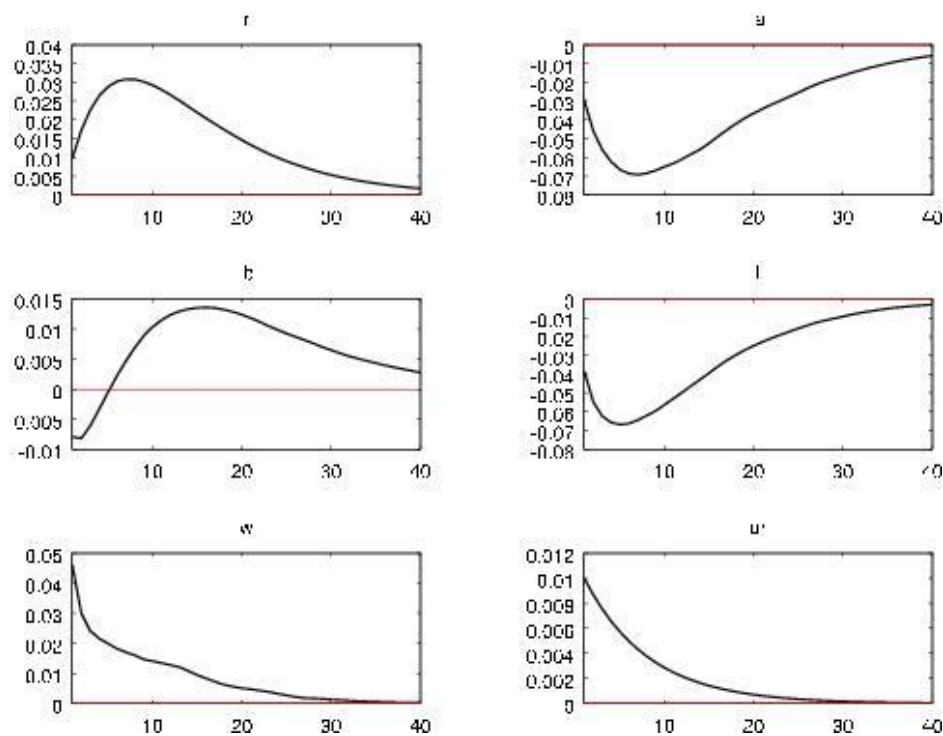
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Appendix.

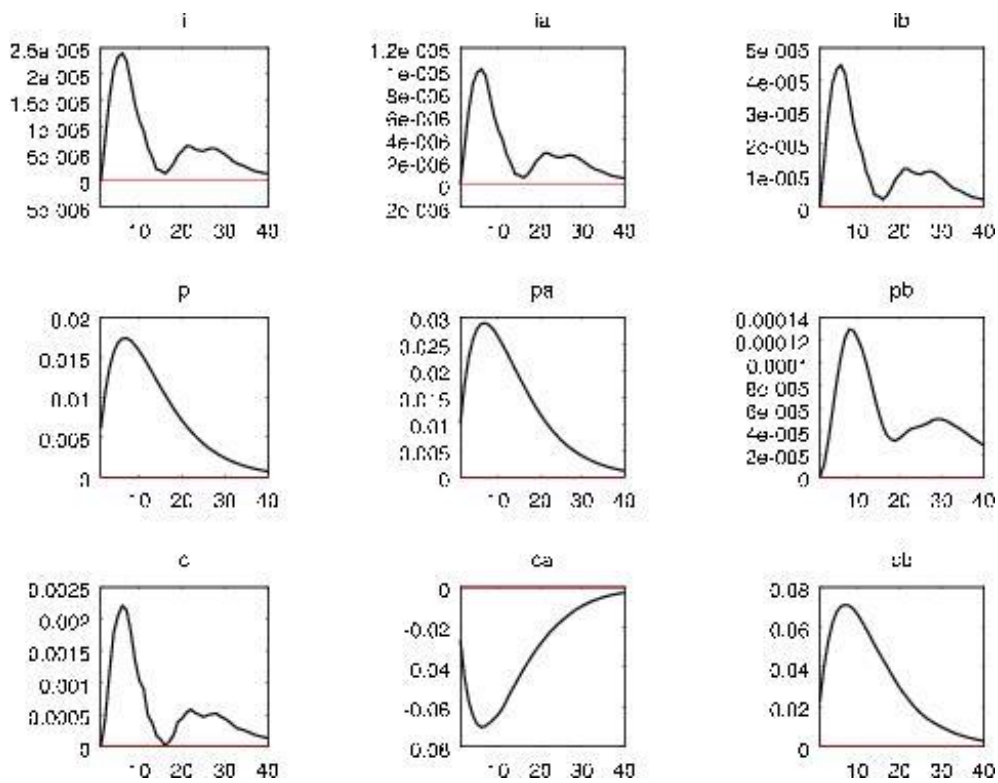
Impulse responses



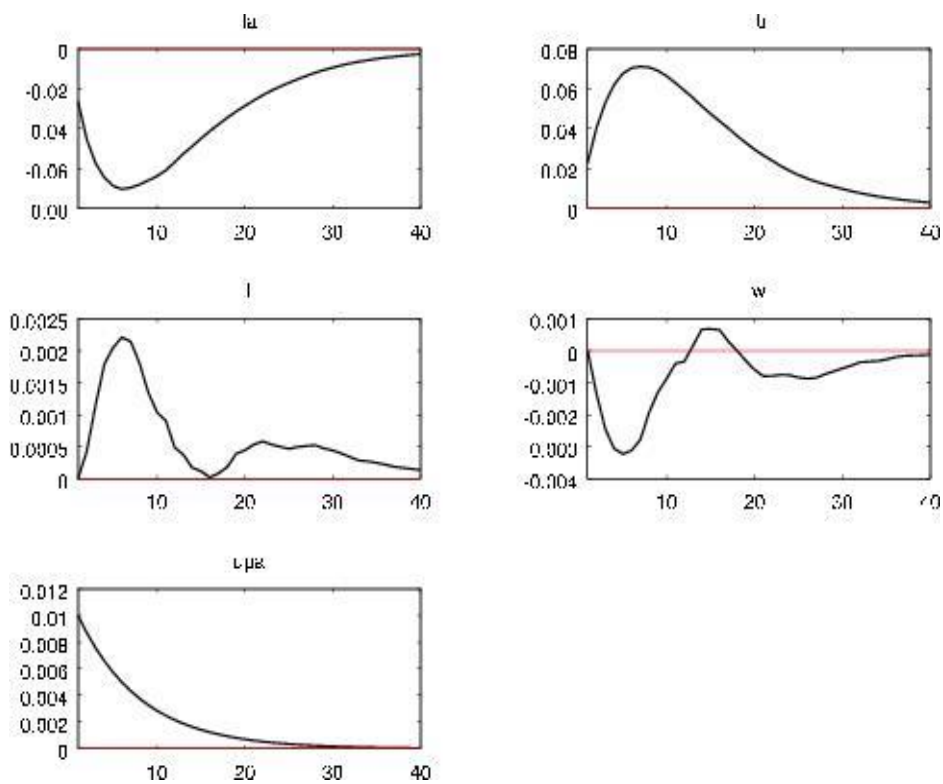
Picture 1. Interest rate shock to inflation i :s, price level p :s and consumption c :s in sectoral model.



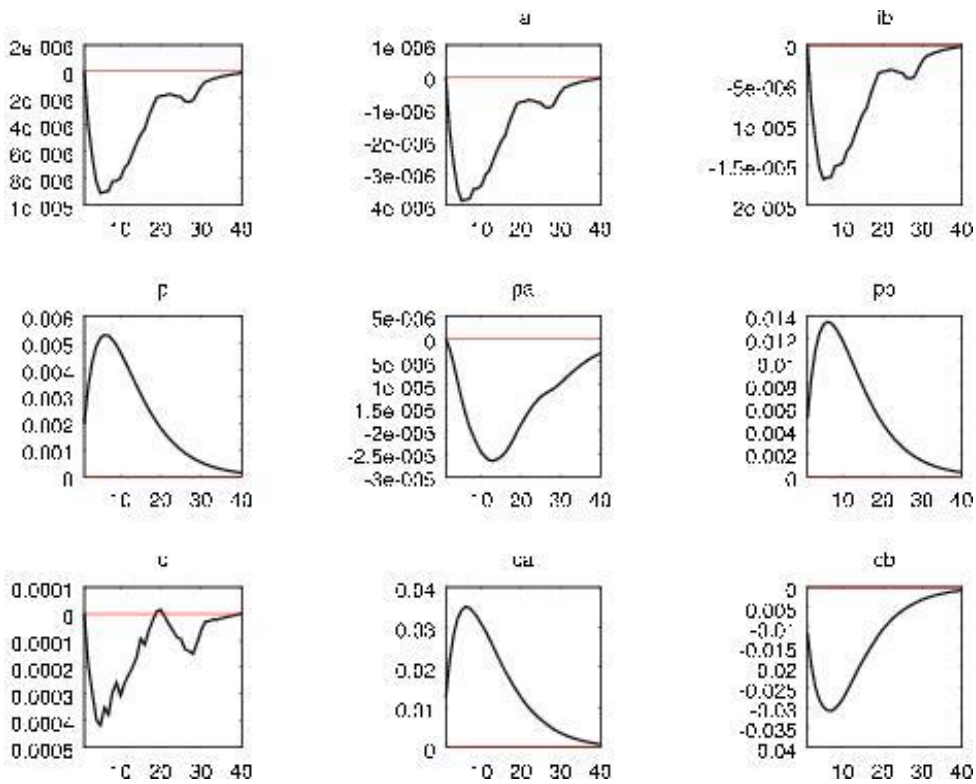
Picture 2. Interest rate shock to r , labour effort ls and wage w .



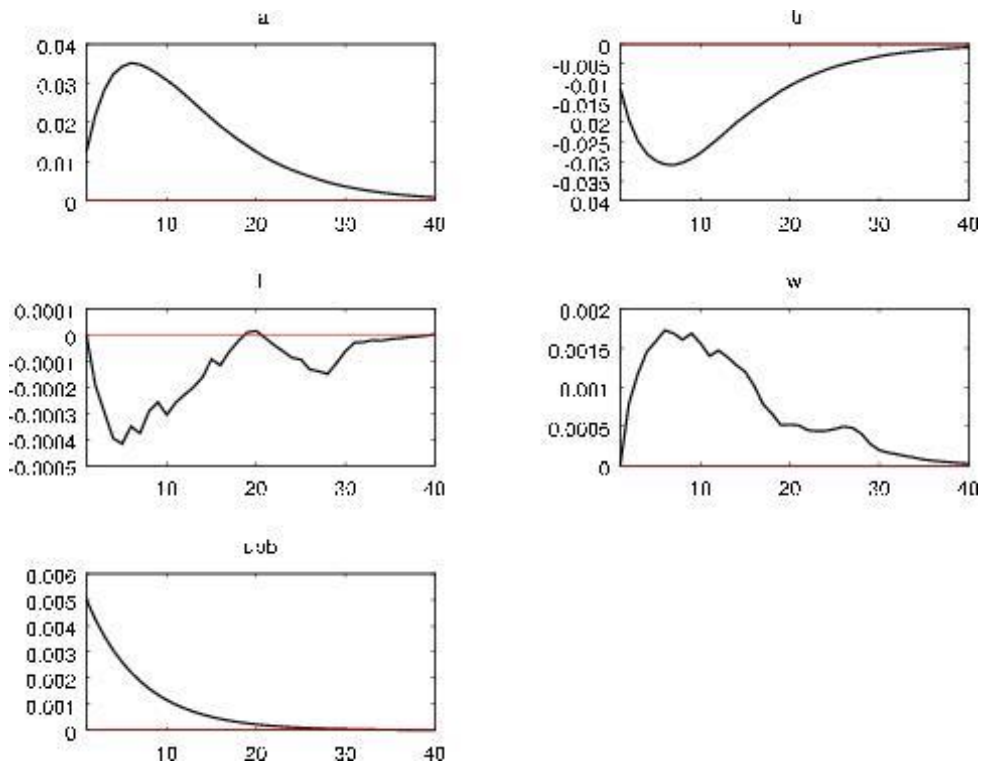
Picture 3. A cost push shock in sector *a* to *is*, *ps* and *cs*.



Picture 4. A cost push shock (*upa*) in sector *a* to *ls* and *w*.



Picture 5. A cost push shock (upb) in sector *b* to *is*, *ps* and *cs*.



Picture 6. A cost push shock in sector *b* to *ls* and *w*.