TAMPEREEN TEKNILLINEN YLIOPISTO
TAMPERE UNIVERSITY OF TECHNOLOGY

NIKO HOSIAISLUOMA

PATH DEPENDENT ELECTRICITY OPTION PRICING

Master of Science Thesis

Examiner: prof. Juho Kanniainen
Examiner and topic approved by the
Faculty Council of the Business and
Built Environment on
17th December 2018

ABSTRACT

NIKO HOSIAISLUOMA: Path Dependent Electricity Option Pricing
Tampere University of Technology
Master of Science Thesis, 76 pages, 4 Appendix pages
December 2018
Master's Degree Programme in Industrial Engineering and Management
Major: Industrial and Business Economics
Examiner: Professor Juho Kanniainen

Keywords: Electricity spot price modeling, Stochastic models, Option pricing, Asian option, Electricity derivatives, Nordic power market

This thesis studied hourly electricity spot models and their application in path dependent option pricing. The first goal was to find a suitable hourly spot model that captures the price dynamics and enables risk-neutral pricing of the path dependent electricity spot options. The second goal was to study the pricing consistency of different models.

In order to price path dependent options on electricity spot price, spot models obtaining the price dynamics, most importantly the mean-reversion and jumps, and the risk factors are needed. In the first part of this thesis, the literature was studied regarding the main background theory, the stochastic electricity spot models being the main focus area. It was found that there are multiple alternative ways to describe the underlying stochastic process, which is usually assumed to be mean-reverting process with jumps. The most challenging part of the modeling is regarded to be the jumps. In addition, it can be even assumed that the underlying stochastics of the electricity spot price could be presented by more than one stochastic factor. In this thesis, there were four different two-factor models, two jump-diffusion models and two regime-switching models, two of which were actually daily spot models to which the hourly prices were generated by historical profile sampling technique.

The second part of the thesis focused on estimating the different models and applying them on pricing Asian options by using Monte Carlo simulation. To obtain the risk-neutrality, all of the models were calibrated with quoted monthly future contracts after estimating the model parameters with the spot data. The models were then used to price quarterly Asian options with different strikes and maturities, and Black-Scholes implied volatilities were calculated. In line with the prior research, it was found that the implied volatilities were affected by the definitions of the model and their parameters: the mean-reversion, jump size, jump volatility and jump intensity have clear implications on the implied volatility. Additionally, this thesis showed that the daily spot models generated consistently lower prices for Asian spot options than the hourly spot models. To validate the resulting implied volatilities of all of the models, they were compared with the prices of corresponding European options on future contracts observed from the markets. It was found that the resulting implied volatilities are in line with the market prices.

TIIVISTELMÄ

NIKO HOSIAISLUOMA: Polkuriippuvaisen sähköoption hinnoittelu
Tampereen teknillinen yliopisto
Diplomityö, 76 sivua, 4 liitesivua
Joulukuu 2018
Tuotantotalouden diplomi-insinöörin tutkinto-ohjelma
Pääaine: Talouden ja liiketoiminnan hallinta
Tarkastaja: professori Juho Kanniainen

Avainsanat: sähkön spot-hintojen mallintaminen, stokastiset mallit, option hinnoittelu, aasialainen optio, sähköjohdannaiset, pohjoismainen sähkömarkkina

Tässä diplomityössä tutkitaan stokastisia sähkön tunti-spot-hinnan malleja, ja niiden käyttämistä polkuriippuvaisten optioiden hinnoittelussa. Tämän diplomityön tavoitteena oli löytää sopiva stokastinen malli kuvaamaan sähkön spot-hintaa tunnin aikafrekvenssillä. Toiseksi tavoitteeksi asetettiin eri mallien tuottamien hintojen johdonmukaisuuden tutkiminen.

Polkuriippuvaisten sähköoptioiden hinnoittelemiseen tarvitaan stokastisia malleja sähkön spot-hinnalle. Mallin tulee ottaa huomioon sähkön spot-hinnan kausivaihtelut, keskiarvoon hakeutuvuus (engl. mean-reversion), hypyt (engl. jump) sekä sähkön futuureista havaittava riskikomponentti, jotta voidaan saavuttaa markkinahintojen kanssa linjassa olevia optioiden hintoja. Sähkön spot-hinnalle on olemassa useita vaihtoehtoisia stokastisia malleja, ja osa malleista sisältää useamman kuin yhden stokastisen komponentin. Hypyt ovat tunnistettu yhdeksi haasteellisimmista osa-alueista spot-hinnan mallinnuksessa. Tämän työn puitteissa kehitettiin kirjallisuuteen nojaten neljä eri kahden stokastisen komponentin tunti-spot-hinnan mallia. Tarkemmin määriteltynä kaksi malleista oli oikeastaan päivä-spot-hinnan malleja, joihin tunti-spot-hinnat generoitiin jälkikäteen.

Mallien parametrit estimoitiin spot-hinnan historiadatan avulla, ja kaikki mallit lopuksi kalibroitiin listattujen futuurien kanssa riskineutraalien hintojen saavuttamiseksi. Tämän jälkeen malleja käytettiin hinnoittelemaan aasialaisia optioita neljälle seuraavalle kvartaalille eri strike-hinnoilla Monte Carlo -menetelmällä. Tuloksena todettiin, että optioiden hinnoissa oli eroja. Tulokset ovat linjassa aiemman tutkimuksen kanssa, joka osoittaa, että mallien keskiarvoon hakeutuvuudella sekä hypyn suuruudella, volatiliteetilla ja intensiteetillä on vaikutus option hintaan. Lisäksi havaittiin, että päivä-spot-hinnan mallit tuottivat johdonmukaisesti alempia optioiden hintoja kuin tunti-spot-hinnan mallit. Black-Scholes implisiittisen volatiliteetin perusteella mallien tuottamia hintoja voitiin validoida listattujen eurooppalaisten optioiden hintoihin ja tuloksena todeta, että kaikkien mallien tuottamat hinnat ovat linjassa markkinahintojen kanssa.

PREFACE

This thesis was written for Fortum Power and Heat Oy's financial trading desk. In addition to providing the financial trading desk with insights under a relevant topic, this thesis project was a rewarding learning trip for me. During the project, I deepened my technical skills in quantitative analysis and methods and gained a lot of understanding in power markets and commodities markets in general.

I would not have been able to carry out the thesis this successfully without the help and support of other people. First, I would like to thank Lead Quantitative Analyst John Pettersson for both guiding my work and sparring me during the whole project. I am also grateful to Professor Juho Kanniainen for providing me with valuable comments and guidance in the main milestones of the project. Big thanks also go to Manager of Hedging & Structuring team Kalle Kuokka for providing me with this opportunity and the whole team for letting me to focus on the thesis with minimal distractions. Finally, I would also like to thank my family, friends and all the other people who have supported me in different ways not only during this thesis project but also during my whole study time in Tampere University of Technology.

In Espoo, Finland, on 27 December 2018

Niko Hosiaisluoma

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APPENDIX D: Time-dependent transition matrices of HPS RS model

1. INTRODUCTION

This thesis is about stochastic models on hourly electricity spot price and their application to path dependent option pricing. Motivated by the prior research, two different types of regime-switching models and two different types of jump-diffusion models having two stochastic factors are developed and their consistency in option pricing will be studied. Moreover, two of the models will have time-dependent properties which can be regarded as a quite natural choice in the case of electricity, as it will be shown. What will be found is that the models provide prices for Asian spot options that are in line with the quoted European future options. However, inconsistency in pricing can be detected between the models, which is due to the definition of the models and their parameters. Interestingly, the models estimated with daily spot prices generated consistently lower prices than the models estimated with hourly spot prices. Before presenting the research questions and the goals of this thesis, some motivation for the subject is first presented.

Electricity derivatives can be used for risk management purposes: an electricity producer or a consumer can protect itself from exposing to the movements of electricity spot price. Due to the non-storability of electricity, limited transportability and uncertain and inelastic demand, the electricity prices are extremely volatile, and there is a large need for electricity derivatives by market participants who want to hedge their exposure on the electricity spot price (Deng and Oren, 2006; Pineda and Conejo, 2012). Hourly inelastic demand and rigidity of supply expose energy retailers' net profits to both hourly volumetric and price risk (Boroumand et al., 2015). Different market participants can mitigate their risks by hedging, which reduces the financial distress and the variance of their profits. Thus, electricity derivatives are needed for hedging purposes, and future and forward contracts are the most common instruments for that purpose. (Deng and Oren, 2006) Producers want to hedge their exposure for low spot prices, and therefore sell future contracts to mitigate the spot price risk for the electricity that will be delivered in the future. On the contrary, consumers and load serving entities (LSE) hedge their exposure of high spot prices and buy future contracts. (Branger et al., 2010; Deng and Oren, 2006) In other words, usually producers act as a short-side and consumers or LSEs as a long-side of a trade (Deng and Oren, 2006). Electricity call and put options can be seen as efficient tools of power plants and power marketers to hedge the price risk, since the electricity generation capacities can be viewed as call options on electricity when the production costs are fixed (Deng and Oren, 2006). Compared to future and forward contracts, Asian options e.g. offer a protection for the electricity consumer from high average spot prices, and still enable the consumer to profit from lower spot prices (Vehvilainen, 2002).

The most famous option pricing model in the financial industry is the Black-Scholes formula, which offers an analytical closed-form solution for standard European option (Joshi, 2008, p. 161). However, due to the special characteristics that was mentioned above, the models developed for financial markets cannot be fully relied on when pricing derivatives on electricity. Moreover, when pricing path dependent options, whose price depends on its historical prices, there exist no closed-form solution for any security. Although approximal solutions exists, a common approach to price path dependent options is to use numerical computational techniques, such as Monte Carlo simulation. Thus, when pricing path dependent options on electricity, a model for electricity spot price is needed. Moreover, as Burger et al. (2004) underlined in their study, the need for a simulation model to replicate the market is much higher in the power markets, since the prices of complex and not-standardized electricity spot options are not directly observable from the market, and thus, the inaccuracies of the models cannot be dealt just by calibrating the models with market data on options. This is basically the practice in financial markets, where the liquid options can be used to calibrate the option pricing models. This is not the case in power markets, and in order to obtain a sufficient model, historical market data has to be analyzed extensively. (Burger et al. 2004) This leads to a conclusion that a so-called model risk is much higher in the electricity option pricing models (Branger et al. 2010), meaning that the resulting price depends highly on the structure and obtained dynamics of the model.

Pricing of path dependent options on electricity is problematic due to the absence of observable market prices on options. Moreover, already the special characteristics of the electricity spot price makes the spot price modeling challenging, and among academics and practitioners, there does not seem exist a consensus of the ideal way to model the electricity spot price, when pricing derivatives (Gürtler and Paulsen, 2018; Weron, 2014). Moreover, while a large variety of different types of models exist for daily frequency, models on hourly prices are scarcer. The daily average price, called also as the base load price, is the main reference price for financial contracts, and thus has been the main focus of academics and practitioners. However, there is a need for models with hourly frequency to price options that depend on hourly behavior of electricity spot price. In addition, there has been found evidence, that even when aiming to model daily average prices, by utilizing hourly prices and taking the hour-specific information into account leads to better results (Maciejowska and Weron, 2015; Raviv et al., 2015).

Even if decreasing the time increment of a daily spot price model to hourly frequency might sound intuitive, the outcome would be more or less clumsy due to the electricity price dynamics and patterns on an hourly level. In addition, as it will be later seen in this thesis, the spot price seems to have time-dependent behavior. The most common type of stochastic models that are used for modeling electricity spot price and option pricing are jump-diffusion and regime-switching models (Branger et al., 2010; Gürtler and Paulsen, 2018; Weron, 2014; Weron et al., 2004). In addition, models having more than one stochastic factor have provided promising results (Benth et al., 2012; Burger et al., 2004), and thus they will be in focus in this thesis as well.

The main goal of this thesis is to find a way to price path dependent options that depend on hourly behavior of electricity spot price. The main arguments behind the hourly spot model is that there is a need to study hourly spot models more in-depth. In addition, by using hourly spot models any information in the data would not be lost, compared to the daily spot models that are based on aggregated data. In order to price these path dependent options, an hourly electricity spot price model need to be developed. The studies regarding hourly spot models being rare, the models themselves and their analysis will be already a clear contribution of this thesis. To my knowledge, there are only a few studies about hourly electricity spot models and their application on option pricing (see e.g. Branger et al., 2010; Burger et al., 2004; Culot et al., 2014; Hirsch, 2009). In order to develop a proper hourly spot model for option pricing, the model should capture all the seasonality, price dynamics and enable pricing in line with the quoted electricity derivatives. The research questions are formulated as follows:

1. *What are suitable hourly electricity spot price models to price path dependent options?*
2. *How consistently do the different models price path dependent options?*

The questions above are the main research questions of this thesis. In order to provide a solid answer for the question 1, at least the following additional questions need to be also covered: what kind of seasonality and dynamics does the hourly electricity spot price have? How the data should be filtered in order to obtain sufficient estimation of the stochastic process? What kind of parameters should the spot model have, and can they be assumed to be constant? What is a proper number of stochastic factors in the model? The question 2 will be supported with a brief comparison with the only quoted options on electricity, European options on futures, to get some idea of the validity of the results. In order to achieve a solid outcome in the end, this thesis project has the following limitations regarding the methods and focus of the study:

- Technical approach is used, and thus the models used are based only on historical market data and statistical analysis
- Only stochastic modeling is considered, and thus the models including fundamental variables are excluded
- The spot model and option pricing are based on numerical computational technique, Monte Carlo Simulation
- The market under focus is the Nordic power market, and the electricity spot price the Nord Pool's system price

The structure of this thesis is the following. First, the underlying theory and concepts are briefly discussed in the chapter 2. After that in chapter 3, the prior research regarding the spot models is presented. The data that is used in this study is presented and analyzed in the chapter 4. Rest of the chapters focus on estimating and calibrating the candidate models and comparing them in the terms of statistical properties and pricing of path dependent options. Finally, in the chapter 9, the results are summed up and discussed.

The data that is used in this study consists of the historical data (1.1.2011-30.6.2018) of the system price of the Nord Pool market. The data is downloaded from Bloomberg database, and the analysis and model development are performed in Matlab.

2. BACKGROUND

The aim of this chapter is to provide basis for the reader of the subjects that this thesis is dealing with, before continuing to the chapter 3 dealing with the main background theory of this thesis, stochastic modeling of the electricity spot price. In this chapter, the following are discussed: electricity as a commodity, physical and financial power markets, derivatives on electricity and their pricing, electricity spot price dynamics and Monte Carlo simulation.

2.1 Electricity

Commodities form a large and heterogenous group of assets, and electricity is one of them. However, even if electricity is labelled as a commodity, it differs from other commodities in many ways. Koekebakker (2002) sums up the literature and lists the most important differing features of electricity: 1. non-storability, 2. limited transportability, 3. no lower bound, 4. correlation between short- and long-term pricing and 5. seasonality.

The storability and transportability of electricity are extremely limited. Due to these special characteristics, electricity is called as a flow commodity. (Lucia and Schwartz, 2002) The non-storability of electricity means that electricity cannot be stored for later use. In other words, the amount of electricity delivered matches always the actual need in a right place at a right time. The demand and supply of electricity is continuously balanced in the transmission network (Koekebakker, 2002). The limited transportability of electricity derives from the capacity limits of transmission lines and transportation losses. This means that it may not be efficient in economic sense to transport electricity across certain regions or long distances. Thus, electricity prices vary across different geographic locations and are highly dependent on the local supply and demand. (Lucia and Schwartz, 2002) In addition, as demonstrated by Ergemen et al. (2016), the production cost depends on the way the electricity is generated. For example, using hydro rather than nuclear for generating power is more cost efficient.

Since electricity cannot be sold short, there is no lower bound for electricity spot price (Koekebakker, 2002). Negative prices have occurred in different markets during periods when demand is low, and the producers have to get rid of the excess electricity and stopping the production would cost more. The correlation between short- and long-term pricing means that the short-term prices are much more demand driven, whereas forward prices are driven by the expectations of the market's production capacity, improved technology and long run cost (Koekebakker, 2002).

The demand of electricity is driven by factors such as economic activity and weather. (Lucia and Schwartz, 2002) Due to these factors, there are yearly, weekly and intraday seasonality or patterns in the electricity prices (Kiesel et al., 2018). Yearly seasonality is mainly caused by natural phenomenon, such as temperature differences between summer and winter (Kiesel et al., 2018), and in addition, in hydro-dominated Nordic market, the level of water in reservoirs has also a large impact on the seasonality of the prices (Weron and Zator, 2014). The yearly seasonality is basically so that the prices are higher during the winter and lower during the summer due to the temperature differences. The yearly seasonality in the Nordic market is demonstrated in Figure 1 by monthly average prices.

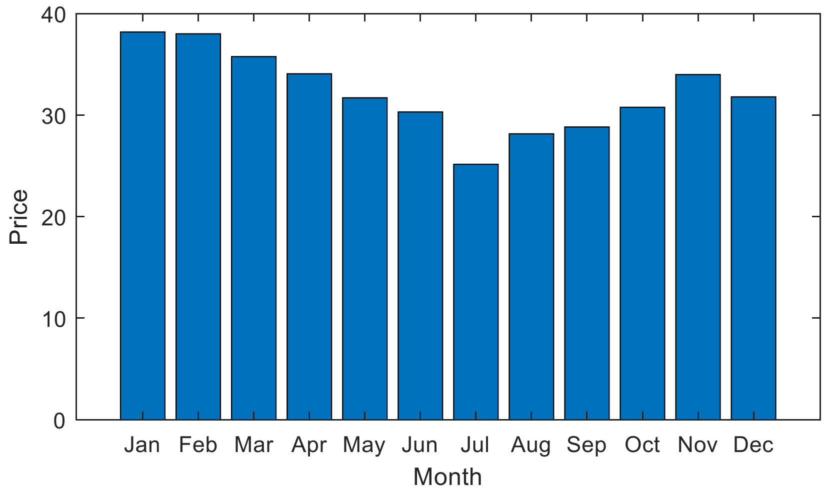

Figure 1. *Average hourly system prices per month during 1.1.2011-30.6.2018.*

In addition to the monthly pattern, the electricity prices can be characterized by weekly and intraday patterns. The weekly pattern refers to the price differences between working and non-working days, whereas the daily pattern refers to the price differences within a day. The average hourly prices of electricity by day in the Nordic market are demonstrated in Figure 2. As it can be observed, the electricity price clearly varies depending on the day and hour. The weekly pattern is roughly so that during the business days the prices are higher than during the weekend days. However, it can be observed that there are slight differences between the business days and weekend days, especially on Fridays, when the average price level is lower than during the other business days. Going deeper from the daily level it can be observed that there are clear patterns on the hourly level as well, and there exist so called intraday pattern: during the night hours the prices are lower and start to increase during the morning reaching a peak around 09:00 on business days and around 11:00 during the weekends. After that, the prices decrease a bit and reach a second peak around 18:00 and then start to decrease again for the night. Compared to the morning peak, the evening peak is lower during the business days but higher during the weekends.

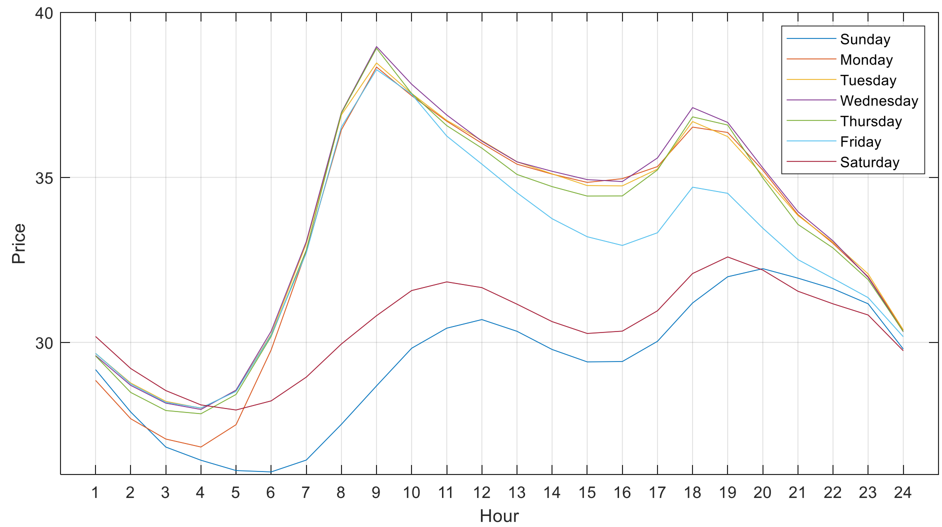

Figure 2. *Average hourly system prices by day during 1.1.2011-30.6.2018.*

These weekly and intraday patterns can be explained quite intuitively by the economic activity: when the economic activity is intense, the prices are higher (Kiesel et al., 2018). The morning peak occurs when people wake up, start their mornings and factories are running, whereas the evening peak occurs when people get away from jobs and start to use electricity at home (Kiesel et al., 2018). The lower average evening peak on Fridays might be due to people leaving earlier from their jobs. During the weekends people may tend to wake up later and have more different routines and daily rhythms, and thus the peaks are flatter and occur slightly later. Overall the price level during the weekends is lower due to the economic inactivity.

2.2 Nordic power markets

Nord Pool is the largest power market in Europe and offers the physical day-ahead and intraday markets across the Nordics, Baltic countries, UK and Germany. In addition, Nord Pool provides the intraday markets also for the Netherlands, Belgium, Luxembourg, France and Austria. (Nord Pool, 2018) Nasdaq Commodities offers the financial market, in which the financially settled derivative contracts can be traded (Nasdaq Commodities, 2018).

In the Nord Pool's day-ahead market, market members trade power that will be delivered physically on the next day. The market members submit their purchase and/or sell volumes for each next day's hour (€/MWh) each day before the noon (12.00 CET). After the bids are submitted, the hourly prices are calculated by Nord Pool's pricing algorithm and announced to the market participants afterwards. As previously discussed, the electricity prices depend on the local supply and demand due to the limited transportability. For this reason, there are different prices for different areas, called as area prices. For example, Finland is one price area (FI) and Sweden is divided into four price areas (SE1, SE2, SE3 and SE4). However, besides area prices, an unconstrained market clearing price is calculated, called as the system price. The system price is calculated based on the bids from Nordic and Baltic countries, and it is the most commonly used as the reference price for the financial instruments, which will be discussed later. (Nord Pool, 2018) The system price is also the price of which historical data is used in this thesis.

In addition to the day-ahead market, physical trading of electricity is possible in the intraday markets as well. In the intraday markets, the balance between the supply and the demand is secured in case of e.g. capacity outages, and buyers and sellers can trade power close to the physical delivery. The intraday market is open 24/7 365 days a year, and market members have access into larger liquidity pool across 12 European countries. (Nord Pool, 2018)

Where Nord Pool provides the market for physical delivery, Nasdaq Commodities provides the market for financially settled derivative contracts. As already mentioned, the system price, and more precisely the base load system price, acts as the main reference price of the financial contracts. Base load means the average price of the hourly values of a day. Nasdaq Commodities offers a market for other commodities as well across the Europe. (Nasdaq Commodities, 2018) In the next chapter, the different kind of derivatives on electricity are discussed more thoroughly. After going through the standard listed derivatives traded in Nasdaq Commodities, the more exotic derivatives traded in the over-the-counter (OTC) markets are also discussed.

2.3 Derivatives on electricity

Derivatives offering in Nasdaq Commodities include futures contracts and European options on futures. (Nasdaq Commodities, 2018) Options are traded also in the OTC-markets, which include additionally a large variety of different and more exotic derivatives (Deng and Oren, 2006). Derivatives on electricity have a few differences compared to other financial derivatives. Contracts on electricity always refer to the delivery over a certain time period, whereas derivatives on stocks, for instance, are sold at a specific point of time. Thus, derivatives on electricity refer to delivery periods, such as a week, a month, a quarter or a year. (Hepperger, 2012)

A future contract on electricity is an agreement between two counterparties to buy or sell a fixed amount of electricity with predetermined price throughout a future time period (Fanelli et al., 2016). A similar agreement is traded in OTC-markets as well, but just called as a forward contract. (Deng and Oren, 2006) Nasdaq Commodities provide futures for base load system price with different maturities on weekly, monthly, quarterly and yearly level. For instance, futures on base load system price are provided with maturities of the next 10 years. The base load futures for area prices, or more precisely Electricity Price Area Differentials (EPADs), are provided with shorter maturities. (Nasdaq

Commodities, 2018) In addition, EPAD futures are much less liquid contracts than system price futures, and some price areas do not even have quoted futures contracts at all. To conclude, in terms of the open interest, the most liquid future contracts are written on the system price with the nearest expiration dates.

An option is a type of derivative, which gives its holder a right, but not the obligation, to buy (call option) or to sell (put option) the underlying asset at a fixed price (strike price) at a specific time or time period (Joshi, 2008, p. 9). Options can be roughly divided into standard and non-standard ones. The standard options are also called as plain vanilla options, which include only the strike price and expiration date, whereas the exotic options can include different additional features (Deng and Oren, 2006). European option is an example of a standard option, and it can be traded both in the Nasdaq Commodities market and in the OTC-market, whereas an Asian option is the most common path dependent or exotic option. Next, the payoffs of European and Asian options are presented.

The payoff of a European option depends on the price of the underlying asset at the expiration date. It is the difference between the strike and the spot price at the expiration date. The payoff of European call and put options are defined as follows:

$$Payoff_{EuropeanCall} = \max(S_T - K, 0)$$

$$Payoff_{EuropeanPut} = \max(K - S_T, 0)$$

where K is the strike and S_T the spot price at the expiration. The value of a standard European option can be solved analytically by the Black-Scholes formula, when strike, time to maturity, interest rates and current price of the asset are known (Joshi, 2008, p. 161). The payoff of an Asian option depends on the average (either arithmetic or geometric) price of the underlying asset through a certain time interval (Koekebakker, 2002). Since the payoff depends also on the past values of the underlying asset at the expiration, Asian option can be called as a path dependent option. The payoff of an arithmetic average Asian call and put options are defined as follows:

$$Payoff_{AsianCall} = \max\left(\frac{1}{N}\sum_{i=1}^{N} S_i - K, 0\right)$$

$$Payoff_{AsianPut} = \max\left(K - \frac{1}{N}\sum_{i=1}^{N} S_i\, , 0\right)$$

The underlying asset of Asian and exchange traded European options are the electricity spot and a future contract on electricity, respectively. To be precise, we are talking about European future option and Asian spot option (Koekebakker, 2002) Since the relation of European and Asian option is rather interesting and might be even confusing when dealing with electricity, a few points should be brought up. A future contract refers always to a delivery over a certain period, and the future price can be regarded as the expected average spot price of the time period. This means that the value of a European future option depends on the expected average spot price, which is the same as for Asian spot options. Thus, a European future option on arithmetic based future contract with delivery period $[T_1, T_2]$ and settlement at the maturity T_2 is identical to an Asian option of which payoff depends on the arithmetic average of the realized spot price during the time period $[T_1, T_2]$ (Koekebakker, 2002). However, the exercise date of a European future option is usually the day before the beginning of the delivery period of the underlying future contract. This means that the main difference is that the payoff of a European future option depends on the expected average spot price, whereas the payoff of an Asian option depends on the realized spot prices.

In addition to European options, Asian options and future contracts, a large variety of different kind of more exotic derivatives exist also for electricity as for any other commodity or asset class. Deng & Oren (2006) list a few common derivatives on electricity: electricity swap, spark spread option and swing option. Electricity swap is contract in which it is agreed to pay a fixed price for electricity instead of the floating spot price. Spark spread option is a cross-commodity options of which payoff is determined by the price difference of the electricity spot price and the fuel price that is used to generate the electricity. Swing option's holder has a right to exercise the option at certain amount of times during certain time periods with a strike which can be either fixed or set at the beginning of each time period by a pre-specified formula. (Deng and Oren, 2006)

2.4 Pricing of electricity derivatives

According to Benth et al. (2007), models on electricity can be divided roughly into two categories: spot price models and future price models. The advantage of the models for future prices is that the market can be assumed to be complete and in addition, futures are less volatile than the spot price. (Benth et al., 2007) As already mentioned, the underlying asset for European options is a future contract, so the Black-Scholes formula can be applied for pricing that kind of options. In order to price path dependent options on electricity, spot models are needed. In practice, derivatives are priced so that the prices are risk neutral. In risk-neutral pricing, the aim is to avoid arbitrage by constructing probabilities for the asset value movements such that at any trading time and state of the world the expectation of the asset's future value is equal to its current value. This property is also called as the martingale property (Joshi, 2008, p. 129). However, since the electricity spot cannot be traded, using spot models to price derivatives on electricity raises up some problems that need to be covered. Before going into that problematics and the practices dealing with them, the normal approach of derivatives pricing will be briefly covered.

The standard approach of pricing derivatives is to construct another portfolio, which replicates precisely the payoff of the derivative contract. This technique is based on the no-arbitrage argument, which implies that the payoffs of both the derivative and the replicating portfolio must be exactly the same, since otherwise there would exist an arbitrage and one could get a risk-free profit. (Weron, 2008) For instance, when pricing a forward contract on a commodity, a replicating portfolio would be to buy the underlying commodity now and store it. In the end, no-arbitrage argument states that both of the ways should be exactly the same by value. Among commodities in general, the relationship between the spot and forward prices is described as a convenience yield. The convenience yield is defined as the premium or the price for the commodity holder to hold the physical asset. (Weron, 2008) The relationship is defined as follows:

$$F_{t,T} = P_t^{(r_t - y_t)(T-t)},$$

where P_t is the spot price, $F_{t,T}$ the forward price, r_t the risk-free interest rate, y_t the convenience yield, T the delivery time and t the current time. However, when dealing with electricity as the underlying, the no-arbitrage argument fails, due to the non-storability and transportability of electricity. Thus, the convenience yield can be questioned in the case of electricity, since one cannot define the benefit of holding the asset or the storage cost (Benth et al., 2007; Lucia and Schwartz, 2002; Weron, 2008). This also implies that information about spot prices cannot be derived from the analysis of forward price models (Benth et al., 2007). However, if the underlying asset is a future contract, as for European options on electricity, the case is different, and the no-arbitrage argument holds. Spot models are needed when valuing options that depend on the behavior of hourly electricity spot price. (Benth et al., 2007) Since the standard assumptions from the financial industry do not work properly, additional assumptions need to be made when using spot models to price derivatives on electricity. (Burger et al., 2004; Weron, 2008)

In order to price derivatives on electricity according to the market prices, the market price of a risk need to be calculated. The market price of a risk is the difference between the drift in the real-world risky probability measure and the drift in the risk-neutral probability measure (Weron, 2008) Risk-neutral pricing of electricity derivatives means that the spot models are usually calibrated with the future contracts, since they are the only quoted contracts on electricity spot (Seifert and Uhrig-Homburg, 2007). According to Benth et al. (2007), different risk factors would have to be considered when identifying the market price of a risk. It is proposed to include at least two risk factors: one for short-term hourly behavior with strong volatility, including jumps, and one for more long-term behavior, which is observable from the future contracts (Benth et al., 2007). Since there are none, at least liquid enough, traded contracts available on thinner time granularity, such as an hour or even a day, the short-term risk factor is difficult to calculate (Burger et al., 2004). Thus, the usual way seems to be to exclude the short-term risk and calculate only the long-term risk (Burger et al., 2004; Seifert and Uhrig-Homburg, 2007). In other words, when pricing path dependent options, a common approach is to assume zero market price of risk for short-term risk factors, or first calibrate the model on spot prices, after which calibrate the model on futures (Burger et al., 2004). How this is done in technical terms, is explained next.

When using spot models to price derivatives, there are a couple of common approaches to calibrate the models to be risk-neutral. The first is to assume the following expectation hypothesis (Burger et al., 2004; Seifert and Uhrig-Homburg, 2007):

$$F_{t,T} = \mathbb{E}_t(S_T|\mathcal{F}_t),$$

which basically says that the forward price $F_{t,T}$ for time T at time t is the expected spot price S_T for time T at time t with filtration of information $\mathcal{F}_t$. Another approach is to calibrate the market price of risk for the risk factors and then change to an equivalent martingale measure $P^{\boldsymbol{Q}}$, so that the following relation holds (Burger et al., 2004):

$$F_{t,T} = \mathbb{E}_t^{\boldsymbol{Q}}(S_T|\mathcal{F}_t)$$

The above formula can be further derived (Seifert and Uhrig-Homburg, 2007), so that the long-term risk factor λ_s is extracted as follows:

$$F_{t,T} = e^{\int_t^T \lambda_s \mathrm{d}s} * \mathbb{E}_t^{\boldsymbol{P}}(S_T|\mathcal{F}_t),$$

where $\mathbb{E}_t^{\boldsymbol{P}}$ is the expected spot price under real-world probability measure. As a result, a relation has been obtained which allows to calculate the long-term risk factor from the observable future prices and the expected spot price of the model. (Burger et al., 2004; Seifert and Uhrig-Homburg, 2007) At this point, it is clear that the electricity derivative pricing is a whole other world if compared to other financial derivatives, but that is just one part of the problem. Even if the risk-neutral pricing of electricity derivatives is possible using the previous relations and exotic path dependent options can be priced so that they are in line with the market prices (futures), additional problem can be brought up in hedging the positions. Since the spot price is the underlying of the structured products, complete hedging is at least complicated, if not impossible. The only financially traded contracts are the futures, which can be used to hedge the long-term risk factor, but the short-term risk factor cannot be hedged, since the spot price is not tradable. What this means in practice is that an additional risk premium will be charged in order to cover the part of the risk that cannot be hedged. (Burger et al., 2004)

Lastly, when talking about prices of options, they are usually regarded as volatilities rather than as euros or dollars. To be more precise, the prices are actually implied volatilities, meaning that the volatilities of the underlying asset are implied by the option prices. What this means is that the option prices are observed from the market and the implied volatility is derived from them by using Black-Scholes formula with the option price, strike, time to maturity, interest rates and current asset price given. (Joshi, 2008, p. 157-161) When observing market prices for options with the same maturity but different strikes and calculating the implied volatility, a volatility smile is obtained, which is the implied volatility as a function of strike (Joshi, 2008, p. 74). It is called a volatility smile, since the plot of the function is smile-shaped, meaning that the implied volatilities increase when going deeper in-the-money (ITM) or out-the-money (OTM). An option is said to be ITM (OTM), when the payoff would have a positive (zero) value at the expiry provided the price of the underlying did not change. If the assumptions under Black-Scholes would hold, such as the constant volatility, the shape of the implied volatility as a function of strike would be obviously flat. Thus, the smile shape can be said to be expressing the market's view about the imperfections of the Black-Scholes model (Joshi, 2008, p. 75). However, when dealing with mean-reverting models with jumps, the shape of the Black-Scholes implied volatility plot can be affected by the model parameters. Nomikos & Soldatos (2010) studied the model implied volatility of European spot options on electricity, and found that the mean-reversion, the jump size and the jump intensity have an effect on the implied volatility. This means that even if analyzing the model implied volatilities and not the market prices, the shape of the implied volatility plot is not necessarily flat when dealing with mean-reverting models with jumps. More precisely, Nomikos & Soldatos (2010) explain that the existence of jumps increases the probability of OTM call options to ending up ITM, and thus have a clear impact on option prices. This will be elaborated in the chapter 8 in this thesis as well, when the implied volatilities by the prices generated by the models are plotted as a function of strike.

To conclude, the pricing of derivatives on electricity and the hedging practices are extremely interesting issues to cover. It should be now clear that the models developed for financial markets cannot be fully relied on when pricing derivatives on the electricity spot price, and additional assumptions need to be made. In addition, it is already a complicated issue to develop a spot model that successfully captures the seasonality, mean reversion, varying volatility and most importantly the jumps, as it will be also demonstrated later in this thesis. (Weron, 2008) The risk-neutral calibration is just another issue on top of developing a sufficient model to replicate the electricity spot price behavior. The price dynamics is discussed more thoroughly in the next subchapter.

2.5 Electricity spot price dynamics

In addition to the seasonality that was covered earlier, electricity is regarded to be mean reverting and exhibit large jumps or spikes (Weron et al., 2004), and the volatility of the electricity spot is not constant (Koekebakker, 2002; Simonsen, 2005). Mean reversion implies that the spot price tends to fluctuate around its mean, whereas the jumps are usually defined as extreme price differences between single hours, and are usually quite short lived. (Weron et al., 2004) The mean-reverting nature and spike occurrence can be detected from the Figure 3.

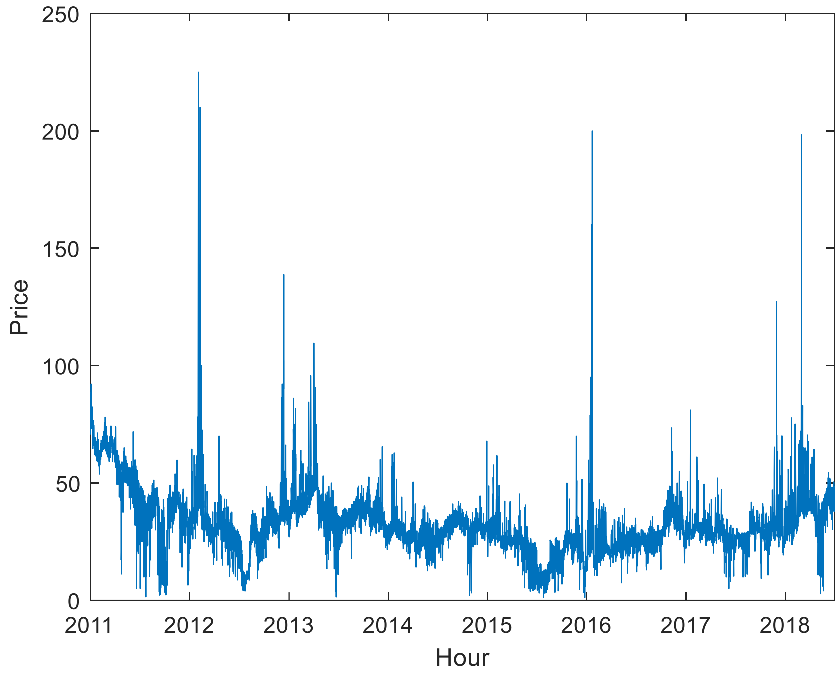

Figure 3. *Hourly system price during 1.1.2011-30.6.2018.*

A mean-reverting process that is commonly used to describe the mean reversion of electricity spot price is called Ornstein-Uhlenbeck (OU), also known as the Vasicek-model (Vasicek, 1977). Schwartz (1997) was one of the earliest pioneers to apply it for describing the dynamics of commodity prices. The OU-process can be regarded as a modified Wiener process and defined as follows:

$$\mathrm{d}X_t = (\alpha - \beta X_t)\mathrm{d}t + \sigma \mathrm{d}B_t = \beta(L - X_t)\mathrm{d}t + \sigma \mathrm{d}B_t,$$

where β is the magnitude of adjustment towards to mean $L = \alpha/\beta$ and $\mathrm{d}B_t$ the increment of the standard Brownian motion. (Weron et al., 2004) OU-process can be regarded as the basis of the electricity spot price dynamics from which different extensions are developed, as it will be seen in chapter 3. What should be noted from the OU-process is that it allows negative prices to occur as well. However, this can be regarded reasonable since negative prices have been witnessed, at least in the German market. However, in case the negative prices would like to be avoided, a squared root process could be used. As showed by Heston, if the volatility follows an OU-process, the squared volatility h_t can be defined as (Kanniainen and Piché, 2013):

$$\mathrm{d}h_t = \kappa(\theta - h_t)\mathrm{d}t + \sigma_t\sqrt{h_t}\mathrm{d}B_t^x,$$

B_t^x presenting the Brownian motion and h_t the squared volatility x_t^2. However, the standard way to describe the electricity spot price dynamics is the OU-process presented earlier.

The price jumps are usually included in the model as an additional component. One common way is to model the jump size as a random variable with an intensity that follows a Poisson process. (Benth et al., 2007; Cartea and Figueroa, 2005; Geman and Roncoroni, 2006; Weron et al., 2004) The technical definition of a jump or a spike will be discussed more in-depth in the chapter 4, in which the data will be presented and analyzed. At this point, let us just say that in technical terms, a jump or a spike is a large price difference between the time steps (Janczura et al., 2013). However, before even providing any technical definition of a spike or a jump, the fundamental reasons behind them should be dealt.

As discussed earlier, the electricity prices are driven by the basic economics: supply and demand. Jumps usually occur due to fluctuations of demand, which can be caused by weather, and/or in combination with supply capacity outages or transmission failures (Weron et al., 2004). In their study, Hellström et al. (2012) found that whether the demand or supply shocks translate into price jumps depends largely on how far the market is functioning from the capacity constraints (Hellström et al., 2012). Without going any deeper, already the result by Hellström et al. (2012) combined with the basic economics of the supply and demand of electricity justifies the following simple reasoning: upward jumps occur most likely during the time when the expected demand is higher, whereas downward jumps occur most likely during the time when the expected demand is lower. This would imply, that in order to capture the true nature of electricity, an electricity spot model should take this time-dependency into account.

As one might have already figured out, the significance of the previous conclusion regarding the time-dependency would be expected to be different if an aggregated, e.g. daily, or hourly data is used. When dealing with daily data, a downward or an upward spike could basically occur during any time step throughout a year, whereas such an event in an hourly spot data would sound highly unlike, given that there are clear demand patterns in a day, as demonstrated earlier. In addition, as it can be detected from the Figure 4 below, electricity tends to have strong autocorrelation, especially on the hourly level. The autocorrelation can be also visually observed from the Figure 9 and Figure 15.

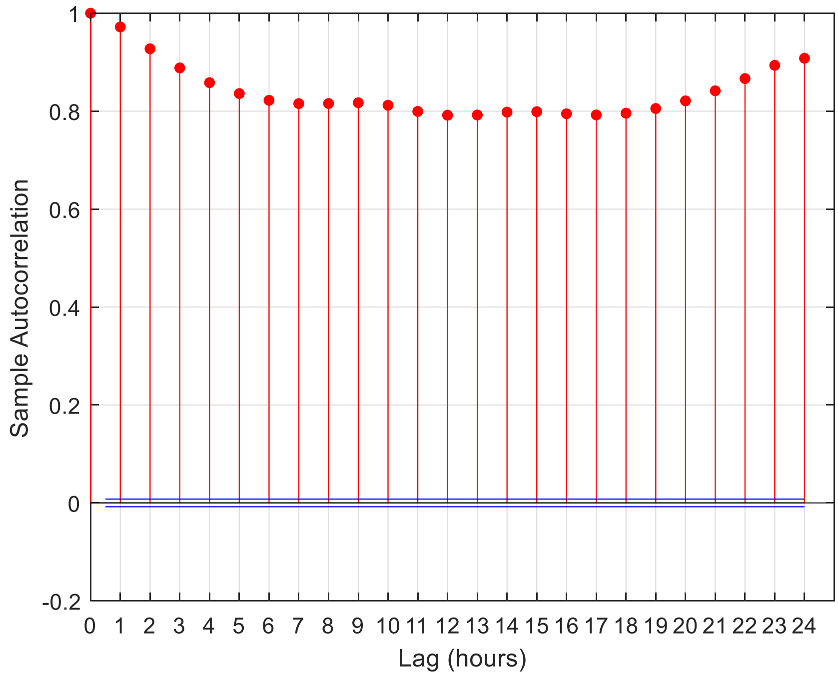

Figure 4. *Autocorrelation in the hourly system price data during 1.1.2011-30.6.2018.*

All in all, the time-dependency of the jump occurrence seem to be not that important issue when dealing with aggregated data. This might be one reason why most of the studies performed on daily prices do not take the time-dependency of the jump parameters into account in their models, as will be seen in chapter 3. However, since we are dealing with unaggregated hourly data, the model should take this time-dependency into account, in order to obtain reasonable results. For example, thinking intuitively, it is highly unlike that a downward spike would occur during the business hours or that an upward spike would occur in the middle of the night.

In addition to mean reversion and jumps, the volatility of electricity spot price tends to have different features. Volatility clustering, log-normal distribution, and long-range correlations are features that describe the power markets. In addition to these features, it has been also found that the volatility of Nordic power market shows time-dependent cyclic behavior and also dependence on the price level. More precisely, the volatility dependence of the price level occurs mainly when the spot price is low. (Simonsen, 2005; Lucia & Schwartz 2002)

2.6 Monte Carlo simulation

Monte Carlo simulation is a common and widely used numerical computational technique. (Boyle et al., 1997; Fu et al., 1998) It is based on the law of large numbers theorem, which can be regarded important in mathematical finance (Joshi, 2008, p. 191). Monte Carlo simulation can be considered, when the stochastic nature of a process is known but the outcomes cannot be easily predicted due to the random variables. The basic idea behind the Monte Carlo simulation is the following: knowing the stochastic nature of the underlying asset, simulate a large number of price scenarios or price paths, average the outcomes and end up with the expected outcome (Boyle et al., 1997). Monte Carlo simulation enables one to come up with expected end result with distributional properties.

As already mentioned earlier, numerical techniques are needed when pricing exotic or path dependent options that do not have a closed-form solution for the value. Numerical computational methods are widely used for variety of purposes in finance, such as risk analysis and stress testing of portfolios. Path dependent option, such as an arithmetic Asian style option, is often priced by using Monte Carlo simulation (Fu et al., 1998). Monte Carlo simulation is utilized in this thesis as well.

3. STOCHASTIC ELECTRICITY SPOT PRICE MODELS

This chapter covers the prior research regarding the stochastic models on electricity spot. Before going through the models used in different studies, the basic structure of the models and their estimation is briefly elaborated. In this thesis, the focus is on stochastic models that are based purely on statistical analysis of the historical market data. The models that are out of the scope of this study, and also not usually used for derivatives pricing and financial risk management, include e.g. fundamental models, multi-agent models and computational intelligence models (for a more thorough review see e.g. Weron, 2014). Traditionally, a stochastic model on electricity spot price include a deterministic seasonal component and a stochastic component. In addition, the deterministic seasonal component is usually divided into a short-term seasonal component (STSC) and a long-term seasonal component (LTSC). (Janczura et al., 2013; Lucia and Schwartz, 2002) The relationship of the components might vary in different models (Janczura et al., 2013). P_t, T_t, s_t and X_t representing the spot price, LTSC, STSC and stochastic component respectively, the relationship between the components can be formulated e.g. in the following ways:

- Additive: $P_t = T_t + s_t + X_t$
- Multiplicative: $P_t = T_t * s_t * X_t$

Based on the prior research (see e.g. Janczura et al. 2013), the model calibration procedure is roughly so that first the raw data is deseasonalized with a deterministic seasonality function, after which the stochastic parameters can be estimated. As covered in more detail by Janczura et al. (2013), some might even focus first more on detecting the outlier values (jumps) and exclude them when estimating the deterministic component. However, there seems to not exist a consensus regarding this procedure, and in the end, as it will be seen in this thesis as well, the filtering and estimation procedures relate much to the model that is used.

As discussed by Janczura et al. (2013), there are open issues in the ideal way of calibrating a spot model on electricity and in addition, there exist a large variety of different types of models that could be used (Gürtler and Paulsen, 2018; Weron, 2014). On top of that, it becomes even more interesting when we are dealing with hourly data, since a major part of the studies has been focusing on daily average prices. In the next chapters, a few common stochastic models are presented.

3.1 Stochastic process

There are different types of stochastic models which obtain the spot price dynamics, mean-reversion and jumps in different manner. The models that are presented next are categorized as in the study by Benth et al. (2012): factor models, jump-diffusion models, threshold models and regime-switching models.

After the deregulation of the power markets started to become more popular around the world, academics started to apply different models for electricity spot price. The model application by Lucia & Schwartz (2002) can be regarded as one of the pioneers. Lucia & Schwartz present two types of models on spot price and log-spot price: one factor model and two factor model. Lucia & Schwartz compare different models by making variations with the following: number of stochastic factors and the way the deterministic component is incorporated.

One-factor model on spot price can be regarded as a simple way to model electricity spot price. In one factor model, the spot price P_t is a sum of two components as follows:

$$P_t = f_t + X_t,$$

where f_t is a deterministic function and X_t a diffusion stochastic process over time. X_t is assumed to follow an OU-process

$$\mathrm{d}X_t = -\kappa X_t \mathrm{d}t + \sigma \mathrm{d}B_t,$$

where $-\kappa > 0, X_0 = x_0$, $\mathrm{d}B_t$ represents and increment to a standard Brownian motion B and κ is reversion rate. The higher the value of κ, the faster the mean-reversion. The process can be risk-adjusted so that the risk-neutral process for the state variable X_t is defined as follows:

$$\mathrm{d}X_t = \kappa(\alpha^* - X_t)\mathrm{d}t + \sigma \mathrm{d}B_t^*,$$

where

$$\alpha^* \equiv -\lambda\sigma/\kappa,$$

α^* denoting the market price per unit risk linked to the state variable X_t. (Lucia & Schwartz, 2002)

Lucia & Schwartz (2002) extended the one factor models by adding a second stochastic factor and presented the two-factor model. The two-factor spot price model by Lucia & Schwartz is defined as follows:

$$\mathrm{d}X_t = -\kappa X_t \mathrm{d}t + \sigma_X \mathrm{d}Z_X + \mathrm{d}\varepsilon_t,$$

where

$$\mathrm{d}\varepsilon_t = \mu_\varepsilon \mathrm{d}t + \sigma_\varepsilon \mathrm{d}Z_\varepsilon,$$

ε_t following an arithmetic Brownian motion, and the two Wiener processes $\mathrm{d}Z_X$ and $\mathrm{d}Z_\varepsilon$ are correlated. The two-factor model by Lucia & Schwartz can be categorized as a **factor model**. The name of the category refers to the study by Benth et al. (2012). The main idea behind that kind of models is that there are multiple stochastic factors representing the price dynamics. In the factor model by Benth et al. (2007), stochastic part X_t is a weighted sum of three independent non-Gaussian Ornstein-Uhlenbeck process

$$X_t = \sum_{i=1}^{n} w_{i,t} Y_{i,t},$$

where

$$\mathrm{d}Y_{i,t} = -\lambda_i Y_{i,t}\mathrm{d}t + \mathrm{d}L_{i,t}$$

Each of the stochastic factors captures the mean-reversion at different scales (Benth et al. 2012). In the model by Benth at al. (2007), the first OU-process is responsible for daily fluctuations, the second OU-process captures the larger price movements with faster mean-reversion rate and the third OU-process is responsible for the spikes. The factor model by Benth et al. (2007) is an additive multifactor model that separates the base and spike signals (Benth et al. 2012). Branger et al. (2010) present a modification of the factor model by Benth et al. (2007), and define the spot price as a sum of deterministic seasonal component, a jump-diffusion component X, and a spike component Y as follows:

$$P_t = f_t + X_t + Y_t,$$

where X_t and Y_t are additive non-Gaussian OU-processes:

$$\mathrm{d}X_t = -\kappa X_t \mathrm{d}t + \sigma \mathrm{d}W_t^{\mathbb{P}} + J_t^{\mathrm{X}} \mathrm{d}N_t^{\mathrm{X}}$$

$$\mathrm{d}Y_t = -\gamma Y_t \mathrm{d}t + J_t^{\mathrm{Y}} \mathrm{d}N_t^{\mathrm{Y}}$$

The **jump-diffusion model** can be regarded as a one factor model that is extended with a stochastic parameter obtaining the spikes or jumps. Merton (2001) made a pioneer study about jump-diffusion models. However, the original jump-diffusion models cannot be applied to electricity spot price modeling as such, since the mean-reversion and jumps are not captured simultaneously. Academics have presented different extensions to the Merton's jump diffusion models (see e.g. Weron et al. 2004; Cartea & Figueroa 2005) Cartea & Figueroa (2005) present a jump-diffusion model that captures the mean-reversion, jumps and seasonality, in which the stochastic process X_t is defined as follows:

$$\mathrm{d}X_t = -\alpha X_t \mathrm{d}t + \sigma_t \mathrm{d}Z_t + \ln J \mathrm{d}q_t,$$

where α is the mean-reversion speed, $\mathrm{d}Z_t$ the increment of the standard Brownian motion, σ_t the time-dependent volatility, J the proportional random jump size and $\mathrm{d}q_t$ is a Poisson process with l as the frequency of the process as follows:

$$\mathrm{d}q_t = \begin{cases} 1 \text{ with probability } l\, \mathrm{d}t \\ 0 \text{ with probability } (1-l)\mathrm{d}t \end{cases}$$

The jump size in jump diffusion models is usually represented by a normal random variable, as in the studies e.g. by Weron et al. (2004) or Cartea & Figureoa (2005). Assuming the jump size to be a random variable from normal distribution makes things a lot easier when estimating the model, since the sum of the normally distributed OU-process and the normally distributed jump size is a normally distributed random variable as well (Seifert and Uhrig-Homburg, 2007).

The **threshold model**, as labelled by Benth et al. (2012), can be regarded as a modification of a jump-diffusion model. Geman & Roncoroni (2006) propose a threshold model, which basically differs from standard jump-diffusion model only on how the jump direction is defined. In their model, the jump direction is defined by a function h, whose value depend on a threshold τ_t. h representing the jump direction, it takes values plus one and minus one as follows:

$$h(X_t) = \begin{cases} +1 \text{ if } X_t < \tau_t \\ -1 \text{ if } X_t \geq \tau_t \end{cases}$$

Therefore, the jump direction is defined by the spot price X_t and the threshold τ_t. If the spot price is below the threshold the direction is negative and positive otherwise. It can be noted, that the threshold value plays a central role: the greater the threshold value, the higher the prices can reach during the spike periods and the smaller the value is, the sooner the downward jump effect reverts the price level towards the mean level (Geman & Roncoroni 2006). Geman & Roncoroni (2006) proposed additionally an interesting way to model the time-dependent intensity s_t of the jumps, defined as follows:

$$s_t = \left(\frac{2}{1+|sin[\pi(t-\tau)/k]|} - 1\right)^d,$$

where k is the positive constant multiple of the peaking levels, beginning at time τ. The exponent d adjusts the dispersion of the jumps around the peaking times (Benth et al. 2012), and Geman & Roncoroni (2006) used the value of 2 for the coefficient.

The main idea behind **regime-switching models** is that there are two or more independent processes that are followed at each time step, and each of the processes represent a certain state (or regime), and these states are switched by an unobservable variable (Janczura and Weron, 2012; Weron et al., 2004). Weron et al. (2004) present a model that has two regimes: a base mean reverting regime and a spike regime. This means that in their model, the spot price follows either a mean reverting or a jump process at each time step. (Weron et al., 2004) Models that have more than two regimes, are called “multi-regime models” (Weron et al., 2004). The switching between the state processes is controlled by the transition matrix $\boldsymbol{P}$ that includes the probabilities of the process to stay in the same regime or to switch to another. In a two-regime model, the transition matrix $\boldsymbol{P}$ holds the information about the probabilities p_{ij} of switching from the regime i at time t to the regime j at time $t+1$, as follows (Weron et al., 2004):

$$\boldsymbol{P} = (p_{ij}) = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{pmatrix}$$

To be more precise, these models can be called as Markov regime-switching models. It is defined under Markov chain, that the current state of R_t depends on the past only and through the most recent value R_{t-1} (Janczura and Weron, 2012; Weron et al., 2004):

$$P\{R_t = j | R_{t-1} = i, R_{t-2} = k, \dots\} = P\{R_t = j | R_{t-1} = i\} = p_{ij}$$

There are obviously different alternatives to define the number of regimes and the dynamics of each regime. For instance, Weron et al. (2004) define the base regime ($R_t = 1$) as an OU-process and the spike regime ($R_t = 1$) as a lognormal distribution. Janczura & Weron (2012) on the contrary define three possible regimes: base regime, spike regime and drop regime. The drop regime is defined as an inverse log-normal distribution. Nomikos & Soldatos (2008) show another example of defining the regimes. In their study, they modify a jump-diffusion model so that the value of the real probability measure in the OU-process has two possible values which are each used based on the regime, and the switching mechanism depending on the water levels in reservoirs. Janczura & Weron (2012) refer to other studies and conclude that regime-switching models are very common in other application areas and totally different science fields as well, such as population dynamics, speech recognition, river flow analysis and traffic modeling.

Most of the studies, as the ones discussed above, are dealing with daily electricity spot prices, whereas clearly fewer studies are on hourly spot price. Hirsch (2009) performed a study regarding pricing of hourly exercisable swing options, and used three different spot models, one using regime-switching AR-processes (base regime, positive spike regime and negative spike regime), the second one using a jump-diffusion process and the third one using a normal inverse Gaussian process. The models were basically daily spot models, to which the hourly profiles were sampled afterwards and a 24-dimensional autoregressive moving average (ARMA) was used for that purpose. Hirsch found that the regime-switching model replicated the hourly characteristics best, but regarding the swing option pricing it was identified, that there exists a so-called model risk. What this model risk means in this case, is that if a swing option with only a few exercise rights is priced, a bet is basically taken on the used underlying price process, since it has a remarkable effect on the end result. Naturally, when the number of exercise rights grow, the model risk decreased. It could be concluded that the study by Hirsch underlined the fact, that using hourly spot models to price exotic options is much challenging than using daily spot models. The study by Branger et al. (2010) showed, that a hourly spot model obtaining the mean-reversion and jumps, is able to price options on futures, but a difference was identified between the model prices and market prices, when pricing options on spot. Branger et al. (2010) argue that this difference regarding spot option prices might be either due to mis-specification of the model or the inefficiency of the observed German market. Burger et al. (2004) develop a spot market for derivatives pricing, that includes two independent stochastic processes, one for describing the short-term market fluctuation presented as a seasonal ARIMA process and one for describing the variation of the futures prices, following random walk. Without performing that thorough analysis of the pricing, they demonstrate that the model is able to price swing options written on hourly electricity spot. Culot et al. (2014) develop an interesting and rather simple technique to generate hourly spot prices by generating them afterwards to daily spot series using "hourly profile sampling". As a quite recent study, Gyamerah and Ngare (2018) estimate a regime-switching model on Nord Pool's hourly system prices, but however, do not use the model in option pricing.

It is clear, that there are different alternatives to model the stochastic dynamics of electricity spot price. And moreover, when talking about option pricing models, the construction of a sufficient spot model can be regarded as a rather challenging task. Even if the stochastic component could be regarded as the core of the option pricing model, the deterministic part has its implications on the end result as well. After all, it is the deterministic part that is used to filter the raw data, in order to obtain solid estimates for the stochastic parameters. Different alternatives for the deterministic component will be covered in the next subchapter.

3.2 Deterministic process

As it should be clear, the stochastic part obtains the dynamics of the spot price process and represents the randomness of electricity spot price process after all the cyclical patterns and seasonality are filtered out. The time increment of the data sets its requirements to the deterministic seasonal component, since the job of that component is to take care that all the time-dependent patterns are captured. As it was demonstrated in the chapter 2, electricity spot prices include monthly, weekly and daily patterns. In this subchapter, it is showed how this filtering is performed in the field of research, and more precisely discussed, how the filtering is performed on an hourly level.

Lucia & Schwartz (2002) state that there are several alternatives for specifying the deterministic component, but the choice should be made so that it is based on the nature and characteristics of the time-series properties of the price. In their study, Lucia & Schwartz use both monthly dummy variables and a trigonometric function to capture the long-term seasonality, whereas the short-term seasonality is captured with dummy variables for days. Lucia & Schwartz propose two versions for the deterministic part f_t:

$$f_{1,t} = \alpha + \beta D_t + \sum_{i=2}^{12} \beta_i M_{it}$$

$$f_{2,t} = \alpha + \beta D_t + \gamma \cos((t+\tau)\frac{2\pi}{365}),$$

where D is a dummy variable for holidays or weekends, M a dummy variable for months and α, β and γ are constant parameters. The difference between the models is that how they obtain the monthly seasonality (third term). In the first version, the monthly seasonality is obtained by dummy variables, whereas the second version uses a cosine function. In both of the models, the daily seasonality (second term) is presented by a dummy variable. A trigonometric function seems to be the most common way to capture the long-term seasonality. Geman & Roncoroni (2006) study the U.S. power markets and use the following deterministic function μ_t in their threshold model:

$$\mu_t = \alpha + \beta t + \gamma \cos(\varepsilon + 2\pi t) + \delta \cos(\zeta + 4\pi t),$$

where the first term can be seen as a fixed cost linked to the power production, the second term as the long-run linear trend in the total production cost, whereas the third and fourth terms together takes the yearly seasonality into account and displays two maxima per year. Meyer-Brandis & Tankov (2008) study multiple European power markets, including Nord Pool, and follow Geman & Roncoroni (2006) and define quite similar seasonal component with 12- and 6-month periods:

$$f_t = a + bt + c_1 \sin(2\pi t) + c_2 \cos(2\pi t) + d_1 \sin(4\pi t) + d_2 \cos(4\pi t)$$

The ones presented above are just a couple of examples of the trigonometric seasonality function, but there can be found many more variations as well (see e.g. Benth and Detering, 2013; Hayfavi and Talasli, 2014; Klüppelberg et al., 2010). The underlying idea is still the same and it can be argued that what is the best fitting trigonometric function for the long-term seasonality depends on the market that is observed and moreover on the data that is used for the calibration. After all, the definition of the long-term seasonality can be regarded rather straightforward, but things become more interesting when trying to capture the hourly pattern.

Branger et al. (2010) examined the German market and used dummies for the long-term seasonality, but a trigonometric function for the hourly pattern. The yearly seasonality $f_{year,t}$ is defined by twelve dummies, each representing a month and in addition, the two phases of the EU-Emissions Trading Scheme have been taken into account by dummy variables as well. The intraday pattern $f_{day,t}$ is modelled by a trigonometric function with a seasonality of 24, 12, 8 and 6 hours. The seasonal component f_t is defined as follows:

$$f_t = f_{year,t} + f_{day,t},$$

where

$$f_{year,t} = \sum_{i=1}^{12} \mathbf{1}_{\{Month(i)\},t} m_i + j_1 * t + \sum_{i=1}^{2} \mathbf{1}_{\{CO2(i)\},t} m_t * j_{2,i}$$

$$\begin{aligned} f_{day_{i,t}} = {} & k_{1,i} \sin(k_{0,i,t}) + k_{2,i} \cos(k_{0,i,t}) + k_{3,i} \sin(2k_{0,i,y}) + k_{4,i} \cos(2k_{0,i,t}) \\ & + k_{5,i} \sin(3k_{0,i,t}) + k_{6,i} \cos(3k_{0,i,t}) + k_{7,i} \sin(4k_{0,i,t}) \\ & + k_{8,i} \cos(4k_{0,i,t}) + k_{10,i} \end{aligned}$$

Kiesel et al. (2018) discuss in more detail about the pros and cons of dummy variables and trigonometric functions, when modeling the seasonality. Kiesel et al. (2018) argue that, in general, using dummy variables for modeling the seasonality causes a problem that relates to the price levels of each season/period that a dummy variable present. This means that the transition is not smooth when moving e.g. from a month to another, while in reality the price changes smoothly. According to Kiesel et al. (2018), sums of trigonometric functions are more commonly used to model cycles. Their advantage is that they have natural periodicity and thus are continuous and make it possible for the price to change smoothly. However, Kiesel et al. (2018) demonstrate that a trigonometric function is not that suitable for modeling the hourly pattern, or at least for all hours. Lucia & Schwartz (2002) add that dummy variables are sensitive to anomalies (outliers) in the sample. This means that when estimating the values of the dummies, the effect of the jump or spike values in the data should be taken into account. This would imply that these extreme values should be filtered out before estimating the deterministic seasonality function (Janczura et al., 2013). Kiesel et al. (2018) compared the performance of dummy variables and a trigonometric function in modeling the hourly pattern and ended up into a conclusion that either of them is uniformly better for all hours, but some sort of combination might work the best.

It can be concluded that there are various alternatives for defining the deterministic component and none of them can be regarded as the correct one. As Lucia & Schwartz (2002) noted, it seems that the deterministic part should be so that it fits to the seasonal characteristics of the data from a specific market, and additionally to the model that is used. However, it seems that a common approach to model the long-term seasonality is a sinusoidal function, whereas it is hard to make any conclusions based on the literature what would be the best way to model the hourly pattern.

3.3 Identified pros and cons of the models

The models can be analyzed based on their capability to present the actual spot price and its dynamics. When we are dealing with electricity spot, it can be analyzed how a model obtains the jumps and the mean-reverting nature. Based on the prior research and their observations, one cannot name a best model on electricity spot, but one can recognize the benefits and pitfalls of specific models. Before bringing them up it should be noted that if a some type of model works well in some market, it might not be that good when estimated on data from different market. (Bennedsen, 2017) This implies, that electricity spot price has different kind of characteristic in different markets, and the goodness of any model depends largely on the data that is used for estimating it.

In their study, Weron et al. (2004) presented two types of models: a regime-switching model and jump-diffusion model. They did not compare the models by thorough testing, but in the end concluded that both of the models achieved to capture the seasonality, mean reversion and jumps, and thus they believe that both of them can be used to price derivatives in the Nordic power market. Regarding jump-diffusion models, the usual way seems to be to model the jump component as random variable that follows a Poisson process. Interestingly however, Klüpperberg et al. (2010) argue that the “spike risk” is underestimated by using Poisson process or exponentially distributed shocks.

Benth et al. (2012) compared more thoroughly three of the models that were presented earlier: jump-diffusion model by Cartea and Figueroa (2005), threshold model by Geman and Roncoroni (2006) and the factor model by Benth et al. (2007), and the models were estimated with EEX data. Benth et al. (2012) argue that that the structure of the factor model allows for more flexible capturing the faster mean-reversion of the spikes and a slower mean-reversion of the base signal, whereas the mean-reversion rates in the threshold and the jump-diffusion models are the average of the reversion of spikes and intra-spike behavior. Benth et al. (2012) also note that the threshold model parameter estimates are very sensitive to the changes in the spike sizes. They conclude that the mean-reversion parameter in jump-diffusion and the threshold models is not able to capture the nature of the spikes and base signal, which means that the mean-reversion is too slow for spikes and too fast for the base signal. On the contrary, the factor model is able to capture these two different mean-reversion speeds, but Benth et al. (2012) also note that the factor model underestimates the noise in the base signal. In the end they conclude that all of the models require careful specification and estimation in order to replicate the spot price dynamics.

It can be concluded, that there are various different alternatives for both the stochastic and the deterministic part, but in the end, they should be defined so that the model succeeds to capture the identified characteristics of the electricity spot price. However, most of the existing models in the literature are defined for modeling daily spot models. Therefore, it can be questioned whether the presented models can or to what extent they can be applied when aiming to model hourly prices. Interestingly, the models having more than one stochastic factor seem to be common and provide better results than the one-factor models. The assumption stating that there are so called short- and long-term processes driving the electricity spot prices with different underlying dynamics, could be regarded rather intuitive as well. But in the end, when developing a model for hourly spot prices, a careful study on the data should be performed to get an idea of the characteristics of the spot price data that is under focus. In the next chapter, the data is analyzed in more detail in order to get the overall view of the price dynamics on an hourly level.

4. DATA

This chapter is about presenting and analyzing the data that was used in this thesis. The data will be analyzed in the extent that enables conclusions to be made regarding the candidate models that will be presented later in this thesis. First, an overall look on the data is taken, after which the jumps are studied in more detail.

4.1 Whole sample

The data that is used in this study is hourly Nordic system spot price data during 1.1.2011-30.6.2018, the total amount of observations being 65709. The descriptive statistics of the hourly data set is presented in Table 1 with different statistical units: spot price, log-spot price and log-return. The spot price represents the actual monetary value (EUR), log-spot price the logarithmic spot price and log-return is defined as the difference between the log-prices. The same descriptive statistics of the daily data set is presented in the Table 2. The daily observations are daily averages of the hourly observations.

Table 1. *Descriptive statistics of the hourly system prices during 1.1.2011-30.6.2018.*

Data	Mean	Standard deviation	Skewness	Kurtosis
Spot	32,349	12,495	1,667	14,323
Log-Spot	3,400	0,414	-1,162	7,462
Log-Return	-8,307e-06	0,075	1,690	79,791

Table 2. *Descriptive statistics of the daily system prices during 1.1.2011-30.6.2018.*

Data	Mean	Standard deviation	Skewness	Kurtosis
Spot	32,349	11,559	1,024	5,440
Log-Spot	3,411	0,378	-0,848	5,591
Log-Return	-1,863e-04	0,120	0,362	12,868

It can be detected that all of the statistical units have quite large standard deviation and kurtosis, and all other units except log-spot are positively skewed. It is good to remind that all of the statistics are largely driven by the fundamentals, such as the seasonality, demand and supply, as discussed in the chapter 2. The high values of kurtosis are mainly explained by the jumps or spikes, which result as extreme values in the data. And as Lucia & Schwartz (2002) noted, the positive sign of the skewness for the spot price indicates that the large extreme values are more probable than low extreme values. When comparing the statistics between the Table 1 and Table 2, it should be noted that the daily prices are averages of the hourly prices and that the time increment for hourly prices is one hour. Log-return is a common statistical unit in the financial industry, and it is also the main focus in this thesis as well. Therefore next, a bit more thorough look is taken into the log-returns.

The log-return series is plotted in Figure 5, which illustrates this extreme behavior of the electricity spot price quite well. It can be seen from the figure that from time to time, the prices change strongly, and these changes are soon followed by another about the similar size change to the opposite direction. In addition, the Figure 5 shows pretty clearly that the standard deviation of the electricity spot price is not even close constant and seems to be rather stochastic. As presented in the table above, the standard deviation of log-returns is 0.075, which of course represents the average standard deviation through the whole sample. However, having a constant standard deviation in spot models is a common assumption in the prior research, and makes things a lot simpler. The standard deviation of log-returns, or volatility, is assumed to be constant in the models of this thesis as well. However, obtaining stochastic volatility in the spot model would be an interesting study to cover.

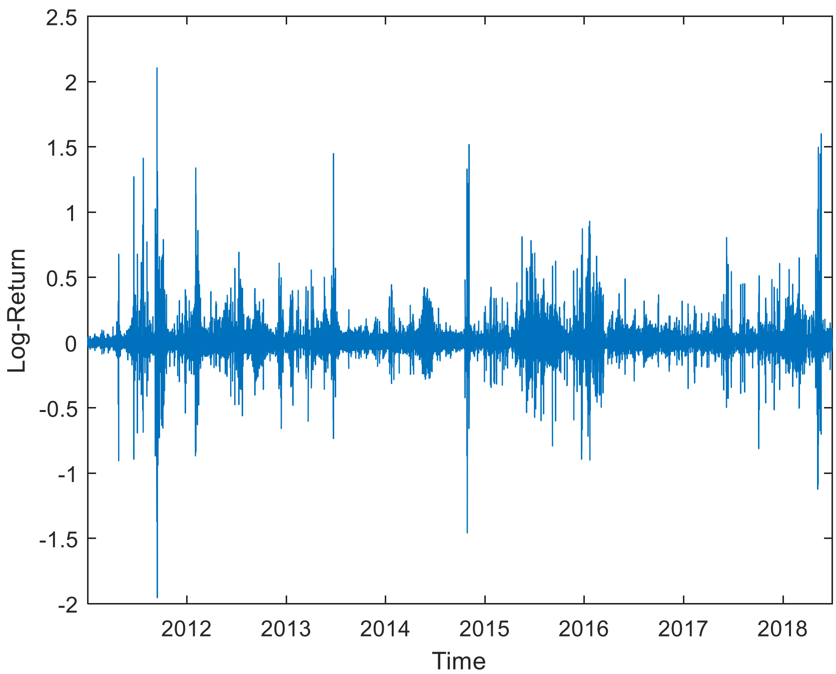

Figure 5. *Log-returns of the hourly system prices during 1.1.2011-30.6.2018.*

Just to illustrate more the high value of kurtosis, log-returns are presented as a histogram in Figure 6. High kurtosis indicates that most of the time the values are close to the mean value, which in this case is about 0, but then there are clear outliers in the data as well. These outliers or the extreme values are in the tails, far away from the mean, and thus, the histogram can be regarded as heavy-tailed. Just by looking at the histogram it is clear that the log-returns are not normally distributed, and it can be speculated that e.g. a t-distribution could provide a better fit. The skewness is hard to visually observe from the histogram, but the skewness being 1.690 log-returns are positively skewed, meaning that the tail on the right-hand side is more far away from the mean than the tail on the left-hand side.

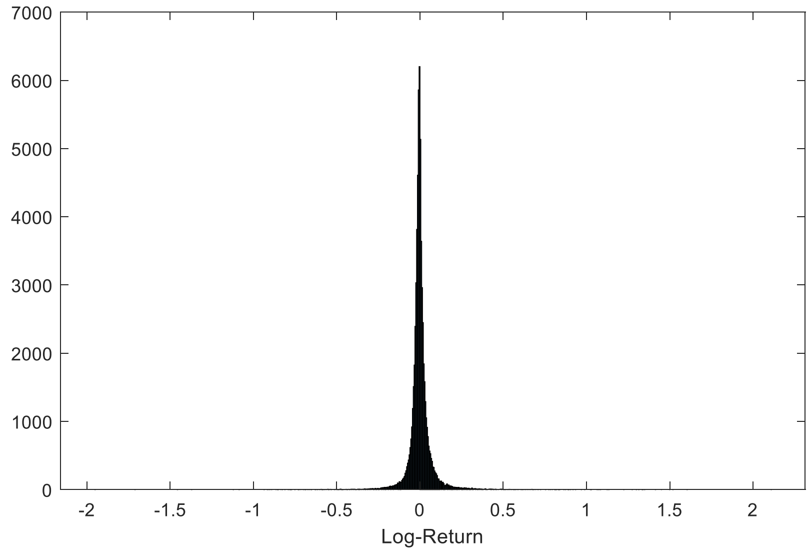

Figure 6. *Histogram of log-returns of the hourly system prices during 1.1.2011-30.6.2018.*

Next, a bit deeper look is taken into the spot prices as well to get clearer picture of the spot price behavior depending on the hour and day. The descriptive statistics of the spot price data is presented in Table 3, divided into observations in business days and holidays, the three highest values in each column being bolded. This division into business days and holidays let us to see more clearly the deterministically changing nature of the spot price in the function of time. If just a general look is taken into the statistics in business days and holidays, a couple of simple conclusions can be made: the values of standard deviation, skewness and kurtosis in business days seem to be higher during times when the demand is higher (during the peak-hours), whereas on holidays, when the demand is generally lower, the highest values of standard deviation seem to be higher when the demand is expected to be the lowest (night hours). These conclusions are also quite intuitive: the positive price jumps are usually driven by the fluctuating demand, whereas negative jumps usually occur when the demand is very low, as discussed in chapter 2. And it is most likely precisely the jumps that explain the increased standard deviation during the high and low demand hours, as will be demonstrated later.

Even if the focus of this thesis is not studying whether the volatility of electricity spot price is constant, stochastic or partly stochastic and deterministic, a couple of interesting thoughts can be highlighted. By looking at the Table 3, it could be hypothesized that the volatility is not necessarily fully stochastics, even if that would have been the initial conclusion after looking at the log-returns in Figure 5. In a conclusion, it could be argued that the volatility is obviously not constant, but rather could be partly dependent on time or the balance of demand and supply, and partly stochastics. Additionally, it could be concluded that it is precisely the observations during the high-demand hours on business days that explain largely the varying standard deviation of the spot price. Simonsen (2005) studied the volatility of Nordic power market more in-depth, but obtaining the non-constant volatility in the models could be an interesting issue to cover in more detail.

Table 3. *Descriptive statistics of hourly system prices during 1.1.2011-30.6.2018 (SD=standard deviation; Skew=skewness; Kurt=kurtosis).*

	Business days (n = 1880)				Holidays (n = 858)			
Hour	Mean	SD	Skew	Kurt	Mean	SD	Skew	Kurt
1	28,15	11,20	0,92	5,67	27,74	11,36	0,98	5,53
2	27,84	11,22	0,93	5,63	27,13	11,42	0,96	5,46
3	28,14	11,20	0,93	5,63	26,79	**11,53**	0,95	5,35
4	29,69	11,03	0,97	5,63	26,91	**11,59**	0,92	5,27
5	32,05	10,87	0,98	5,38	27,56	**11,54**	0,91	5,22
6	34,72	11,29	0,94	5,24	28,58	11,25	0,96	5,26
7	**37,94**	**14,70**	2,80	**23,84**	29,54	11,09	0,98	5,25
8	**38,66**	**16,50**	**3,87**	**34,73**	30,40	10,93	1,00	5,26
9	**37,59**	14,20	**2,53**	20,05	30,94	10,96	0,99	5,21
10	36,53	12,55	1,30	6,98	31,13	10,99	1,01	5,29
11	35,71	12,13	1,21	6,58	30,85	11,01	1,00	5,33
12	35,07	11,75	1,01	5,30	30,34	11,02	0,99	5,37
13	34,64	11,66	0,98	5,18	29,86	10,97	0,99	5,46
14	34,35	11,64	0,95	5,14	29,69	10,93	1,00	5,54
15	34,45	11,91	0,98	5,37	30,04	10,92	1,00	5,60
16	35,39	13,15	1,62	10,11	31,01	11,01	0,99	5,53
17	36,85	**16,11**	**3,51**	**30,65**	**32,09**	11,22	0,96	5,31
18	36,34	14,36	2,49	20,59	**32,51**	11,44	**1,17**	**6,53**
19	34,60	12,50	1,71	13,01	**32,11**	11,19	1,05	5,48
20	33,30	11,43	1,22	8,11	31,58	10,99	1,06	5,57
21	32,35	11,07	0,94	5,24	31,14	11,00	**1,09**	5,68
22	30,90	11,07	0,93	5,49	30,10	11,01	**1,09**	**5,87**
23	29,88	11,06	0,91	5,48	29,16	11,08	1,07	5,74
24	29,08	11,17	0,88	5,49	28,10	11,24	1,04	**5,77**

Since the aim of this thesis is to develop a spot model that can replicate the hourly electricity spot price dynamics, it is important to understand the behavior of the hourly pattern during a day. As already illustrated in the Figure 4 in the chapter 2, the electricity spot price includes clear autocorrelation, especially in the hourly frequency, meaning that the values in the time series, price data, are dependent on the prior observations, and there are a lot of studies regarding different kind of autoregressive models on hourly spot price (Branger et al., 2010; Maciejowska and Weron, 2015). The intraday pattern is mainly driven by the economic activity (demand), as discussed in the chapter 2. This seasonality means that the shape or the profile of the intraday pattern (as in the Figure 2 in chapter 2) is similar looking through the whole series. However, observing this average pattern actually tells quite little in this case. This is illustrated in the Figure 7, in which the daily spread, or the difference, between the average log-prices of peak-hours (08:00-20:00) and off-peak-hours (20:00-08:00) are plotted.

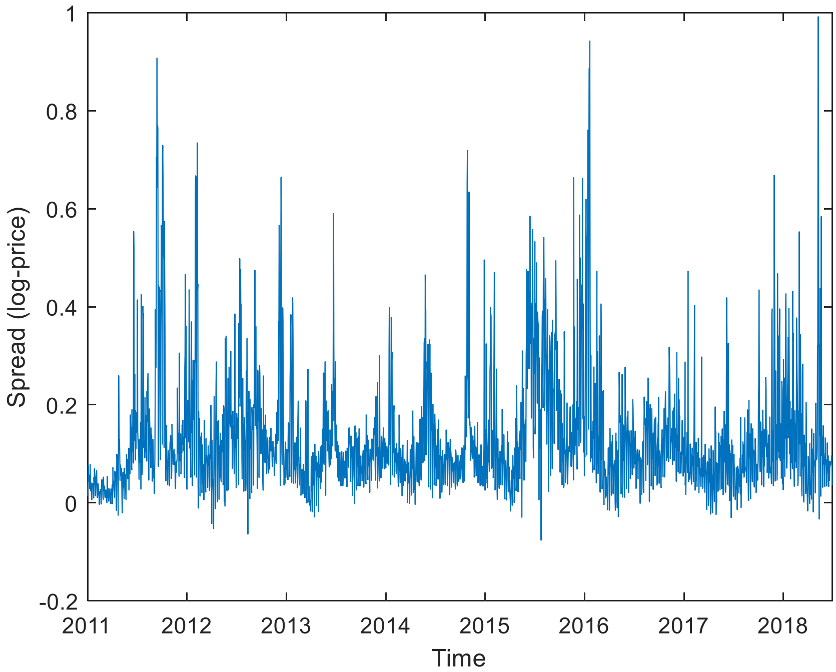

Figure 7. *Daily spread of the average values of peak-hours (08:00-20:00) and off-peak-hours (20:00-08:00).*

Figure 7 illustrates that the spread is naturally usually positive, meaning that the prices during the peak-hours are higher than during the off-peak-hours, but this price difference varies strongly. This varying spread is naturally partly driven by the jumps in the prices, but that is not necessarily always the case. Since the spread fluctuates that much, it seems that the spread between the price levels in a day can vary without any single observation being defined as jump or spike value in that day. The conclusion that can be made is that even if there would be a strong autocorrelation in the hourly data and a clear deterministic pattern, the filtering procedure and later the modeling of this pattern is rather difficult by using deterministic functions only.

In order to get even clearer picture of the time-dependency of the electricity spot price, the deseasonalized data was analyzed between different groups. The data was divided into 32 different groups based on the month, day and hour, so that each group represent different properties regarding the spot price statistics in the Table 3 and jump occurrence, which will be elaborated more in the next subchapter. Months were divided into four groups based on seasons, and days were divided into two groups based on whether it is a business day or a holiday and hours were divided into four groups (different for holidays and business days). The equality of the variance between these different groups was tested by Bartlett's test. The test is about testing the null hypothesis that the variance is equal between the groups, which in this case was rejected in all the tests. So, even if the data is deseasonalized, there exist statistically significant variation in the data, which seem to be dependent on the time (hour, day and month). This unequal variance between the groups might be due to the occurrence of jumps, which clearly depend on time, as will be illustrated in the next subchapter. The test results of the Bartlett's tests are presented in the Appendix A.

4.2 Jumps

Among the studies dealing with the electricity spot price models, modeling the jumps or the spikes are usually considered as the most difficult part. In addition, as the last clause already brought up, it can be sometimes misleading when the same extreme behavior of the electricity spot price is either referred as a jump or a spike. However, when looking at this from the technical perspective, it could be concluded that a jump usually refers to the large price change, whereas a single value can be regarded as a spike value if it is above some threshold. Then, what is considered as large or a right threshold is a whole another question, and there seem to exist any definite answer (Janczura et al., 2013). It can be also concluded, that whether the extreme behavior of the electricity spot price is referred to as a jump or a spike depends on the model that is used. For example, going back to the chapter 3, we are talking about spikes when regime-switching models are used, and jumps when jump-diffusion models are used. Also, later in this thesis, it will be elaborated more clearly how these jumps or spikes are modelled.

In this chapter, when analyzing the data regarding the extreme behavior of the electricity spot price, it is chosen to deal with jumps rather than spikes. In order to get some idea of the characteristics of the jumps, the jumps need to be detected from the data in some manner. There exist different alternatives to detect the jump values, such as classifying log-returns or log-prices as jumps or spikes if they exceed a certain threshold, as presented in the study by Janczura (2013), but in this case a nonparametric test by Lee & Mykland (2008) is used. Even if this technique is originally presented to detect jumps in the spot prices of stocks or stock indices, it suits quite well for the electricity spot price data as well, as it will be soon shown.

The underlying idea in the technique is to assume that each observation in the log-return series follow either a base process (standard Brownian motion) or a base process extended with an independent jump component (random variable following a Poisson process). The intuition behind the technique is that the realized return is much greater than usual, when a jump occurs. This means that the volatility increases after an occurrence of a jump. Since the volatility is not constant, the technique considers "instantaneous volatility", which is basically a rolling volatility with a given window. The test statistic of the technique is the ratio of the realized return and the instantaneous volatility. To conclude, an observation is defined as a jump if the ratio to the instantaneous volatility exceeds the threshold with a given significance level. In other words, if the realized return is much greater than the instantaneous volatility, it is defined as a jump. (Lee and Mykland, 2008)

The technique by Lee & Mykland was applied with a window of 10 observations and rejection region was set to 1% on hourly log-return data, which was deseasonalized down to the daily level. The intraday pattern was left to the data, since the filter consisting of dummy variables can be regarded as damaging the data in some extent. The filtering of the intraday pattern could even increase the volatility in some parts, where the average pattern (hour-dummies) is more extreme than the actual. Thus, leaving the intraday pattern to the data is assumed to provide more reliable results when aiming to analyze the jump occurrence. However, when analyzing the jump size, it is true that the sizes are now partly explained by the predictable pattern, since it is still included in the data. To conclude, whether to filter the intraday pattern out or not is basically a trade-off between damaging the input data or output data, both of which are indeed harmful. However, since the detection of the jumps is not critical in that sense that these results are not used for estimating the models, but just to describe the characteristic of the data, the choice regarding the filtering of the intraday pattern is not crucial. As a result of the Lee & Mykland technique, 702 jumps were detected in total, which are presented in the Figure 8. A closer look of the detected jumps is presented in the Figure 9 with the log-price observations. For the daily log-return data 43 jumps were detected, the rejection region being 1 % and the window being 5. The values of the window and rejection region were defined so that the end result was reasonable, taking into account both the high volatility of the log-returns of the hourly electricity spot price and jump occurrence in clusters. The jump occurrence in clusters means that if a jump occurs, it is more likely that during the same hour on the subsequent days jumps occur as well. This can be observed just by closer visual inspection of the data. So, in order to detect all the jumps in a so-called cluster, the window cannot be too large. If the window would be too large, the last jumps in a cluster would not be defined as jumps, since the instantaneous volatility would be increased due to the first jumps. In addition, when setting the value of the window the hourly pattern regarding the high-demand hours and low demand hours with the highest and lower values of kurtosis, respectively, was taken into account. The window is reasonable to set so that instantaneous volatility is not calculated through all those high-kurtosis hours. To conclude, the window was set partly arbitrarily and partly based on the statistics presented earlier and the visual inspection of the detected jumps.

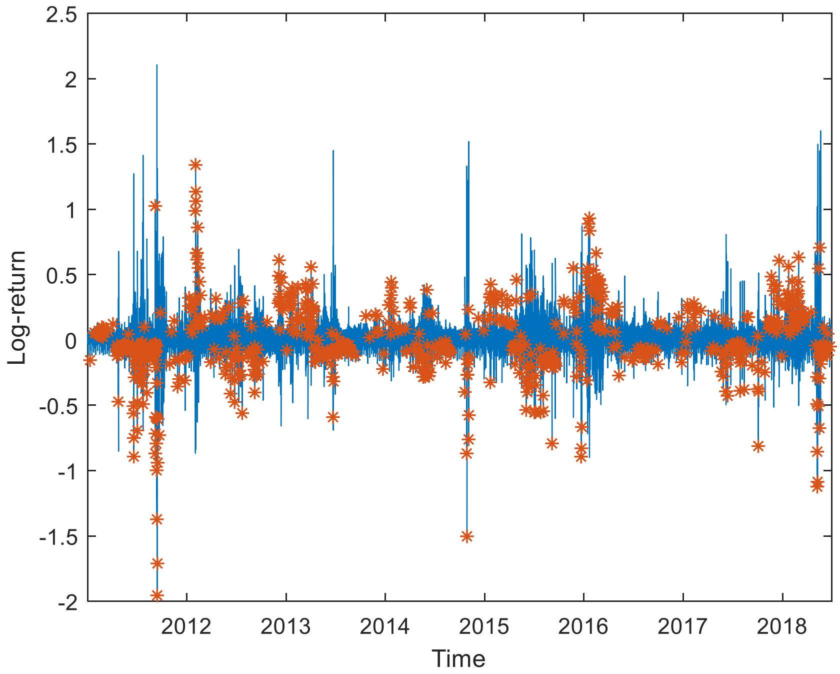

Figure 8. *Detected jumps from the log-price data.*

One reason why the technique by Lee & Mykland applies quite well on electricity spot price, is precisely due to the instantaneous volatility that is taken into account. Since it is typical that when a jump occurs, the spot price reverts if not immediately but soon back to the "normal" level. So, when detecting jumps with this technique, these so-called correction jumps are not defined as jumps. This is convenient especially when the so-called correction move is supposed to obtain by the mean-reversion rate of the model. A demonstration of this correction effect is presented in the Figure 9. Note, that only the initial jump is marked as a jump, and none of the so-called correction moves, which are basically also jumps (or drops) to the opposite direction, are not defined as jumps.

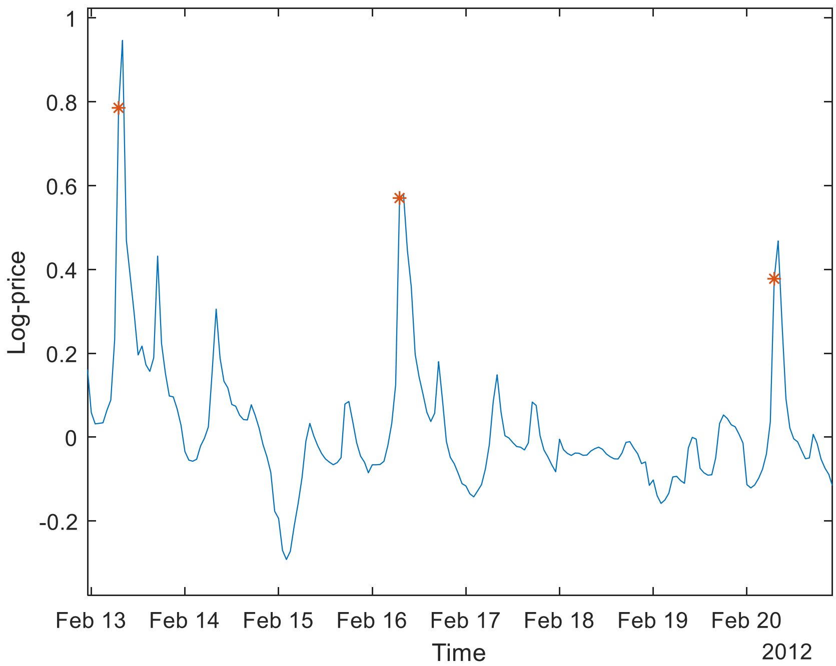

Figure 9. *A closer look of the detected jumps in the log-price data.*

The occurrence of the detected jumps per hour is presented in the Figure 10 below. As mentioned, the total amount of detected jumps is 702, which can be further divided into 415 negative jumps and 287 positive jumps. What can be observed from the Figure 10 is that negative jumps occur mostly during the night hours when the prices are lower, whereas positive jump occur mostly when the prices are higher. This is quite intuitive since the fundamental reasons for the jump (negative or positive) occurrence roots back to the balance of demand and supply, as discussed in the chapter 2. Clearly the most intensive hours for negative jumps to occur are in the night, especially during 23:00-01:00. Whereas for positive jumps occur most intensively during the morning 07:00-08:00. Additionally, positive jumps occur also during the evening hours 17:00-19:00.

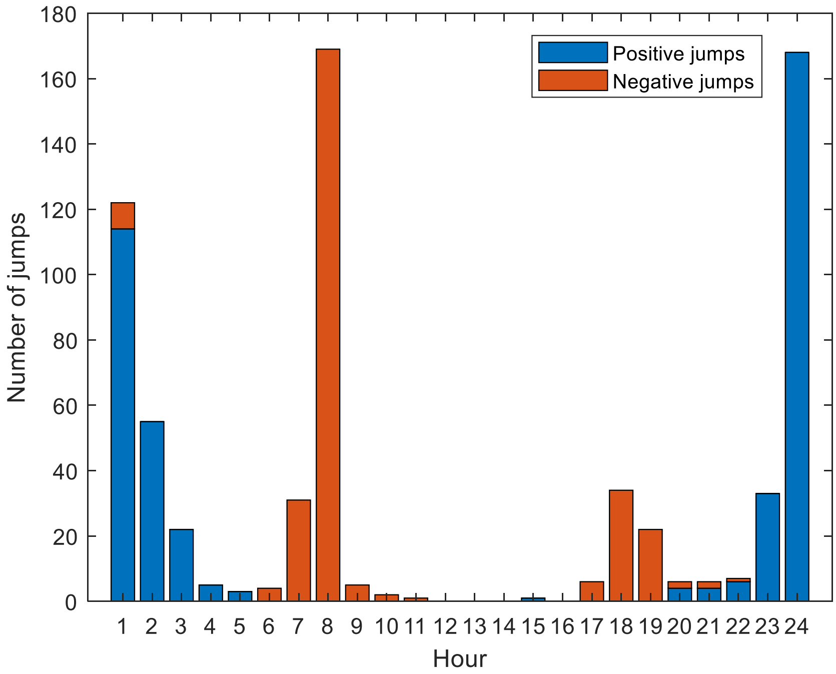

Figure 10. *Number of detected jumps per hour.*

After the jumps are detected with the technique by Lee & Mykland (2007), a suitable probability distribution for the jump observations can be studied. The descriptive statistics of the detected jumps are presented in the Table 4. As it was discussed in the chapter 2, the jump size in the jump-diffusion model is usually represented by normal random variable. Next, the empirical cumulative distribution function (CDF) of the whole sample of jump observations is plotted with a theoretical normal CDF and positive and negative jump observations with gamma CDFs, in Figures 11, 12 and 13, respectively.

Table 4. *Descriptive statistics of the detected jumps.*

Model	Mean	Standard deviation	Skewness	Kurtosis
Positive Jumps	0,258	0,201	1,950	8,556
Negative Jumps	-0,201	0,240	3,284	17,417
Whole sample	-0,013	0,319	-0,676	8,210

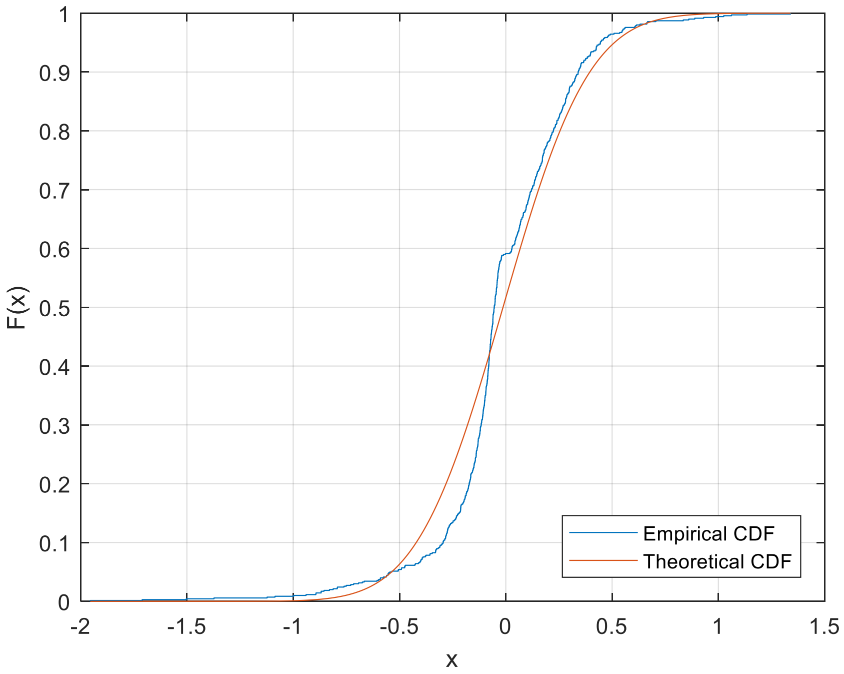

Figure 11. *Empirical CDF of jump observations and theoretical normal CDF.*

Observing the Figure 11 it can be detected that the empirical CDF seems not to be normally distributed. This is also confirmed by $\chi 2$ test, in which the null hypothesis, that the jump observations are from a normal distribution, was rejected with a p-value of 1.3364e-17. However, it should be noted that the detected jump observations with the defined window and significance level, resulted into jump observations that are rather extreme. Thus, there are not really observations near zero, as it can be detected from the Figure 11.

The empirical CDFs and theoretical gamma CDFs are presented below in Figure 12 and Figure 13 for both negative and positive jump observations in the hourly spot data, respectively. What can be observed from the CDFs is that gamma distribution seems to provide a quite good fit for the jump observations. However, in statistical terms the good fit is only confirmed for the positive jumps, when testing the goodness-of-fit of the fitted gamma distributions by the Kolmogorov-Smirnov test.

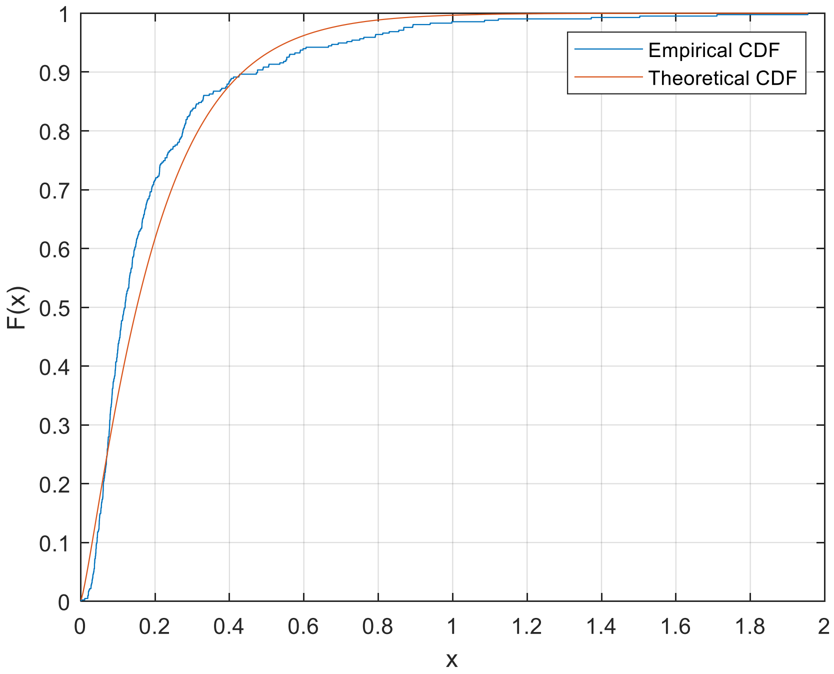

Figure 12. *Empirical CDF of negative jump observations and theoretical gamma CDF.*

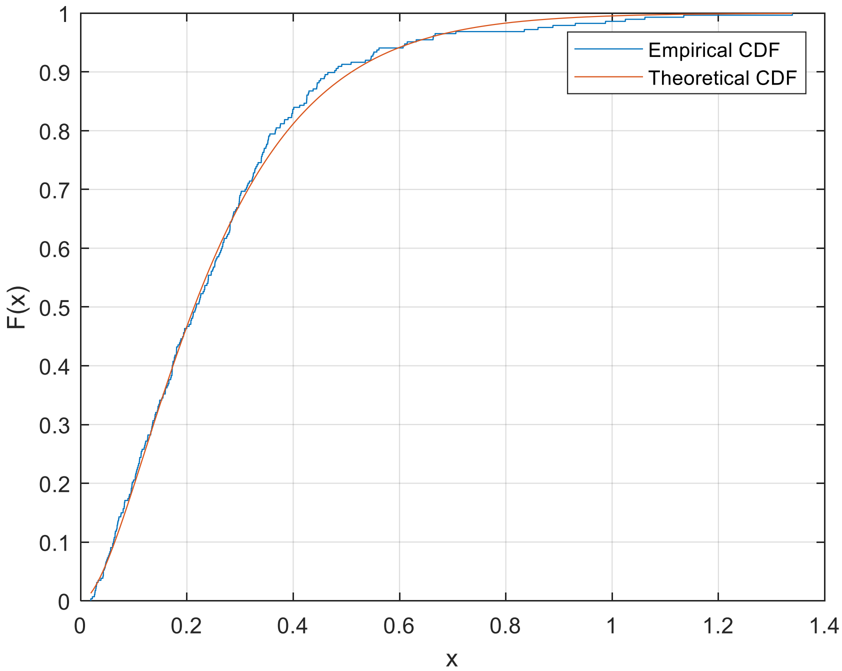

Figure 13. *Empirical CDF of positive jump observations and theoretical gamma CDF.*

The Kolmogorov-Smirnov test was performed as follows: first, 1 million random variables were generated from the fitted gamma distributions. Then, the test was performed with a null hypothesis that the actual jump observations and the generated ones are from the same distribution. The null hypothesis is not rejected for the positive jumps with a large p-value of 0.89, but rejected for the negative jumps. This rather bad fit of the negative jumps could indicate that the sample of the observed negative jumps is too small, even if there were more negative jumps than positive jumps. However, it can be observed from the figures, that there is more variation in the negative jump observation values, and the tail values in the negative jumps are much larger than in the positive jumps. As briefly discussed earlier, modeling the jumps is considered to be the most difficult part in the electricity spot price modeling. This would be a rather interesting to topic to cover, but maybe a whole another study itself. This could be also regarded as the next step of this thesis, when improving the models and developing more sophisticated ones.

After having analyzed the jumps and their occurrence in the electricity spot price, a couple of clear conclusions can be made. First, it is clear that the jump occurrence and the direction depends on time. When modeling the jumps, it would be reasonable to take this time-dependency into account to get realistic simulated price paths. The results regarding the detected jumps and their occurrence time (hour) are well in line with the electricity spot price intraday pattern, presented in the chapter 2. The results being reasonable, the second conclusion is that the detection technique by Lee & Mykland (2007) seem to obtain good results and fit well for the purpose detecting the jumps from electricity spot price data. Thirdly, gamma distribution seems to provide a rather good fit for the jump observations, as also used by Benth et al. (2012). However, having a jump-diffusion model with the jump size defined as a gamma random variable the estimation procedure would become challenging. The model candidates of this thesis are presented in the next chapter.

5. MODEL CANDIDATES

Regarding the models, the main conclusion from the data-analysis concerns the time-dependency of the jumps. In order the models to provide realistic simulated price paths, two of the models will have time-dependent properties, at least in some extent. This means that the models that will be shortly presented, will generate jumps during the times (hour of the day, month) they should occur based on the historical observations.

All of the candidate models will have the same deterministic seasonal component f_t, whereas the stochastic component X_t will be defined differently. In the most general level, all the spot price models take the following form:

$$P_t = f_t + X_t,$$

where the stochastic component X_t will be actually a sum of two independent stochastic processes. Thus, all of the models will be so-called two-factor models. All of the models will have one same stochastic factor, whereas the differences between the models are based on the other factor. Next, the candidate models will be elaborated more.

5.1 Deterministic seasonal component

The deterministic component is a combination of a sinusoidal function (long-term seasonality), dummies (for days and hours) and a linear trend, motivated by the prior studies. The long-term seasonality is in the same form as in the study by Meyer-Brandis & Tankov (2008), but it is a rather similar in most of the studies. The choice of day- and hour-dummies were mainly motivated by the prior research as well (Kiesel et al., 2018; Seifert and Uhrig-Homburg, 2007). The deterministic component is defined as follows:

$$f_t = a + bt + c_1 \sin(2\pi t) + c_2 \cos(2\pi t) + d_1 \sin(4\pi t) + d_2 \cos(4\pi t) + \sum_{i=1}^{6} e_i \, D_i + \sum_{j=1}^{4} \sum_{i=1}^{24} f_{i,j} H_{i,j} \tag{5.1}$$

where D's are dummy variables for different types of days and H's are hour-specific dummy variables. There are six different types of day dummies: 1. Mondays that are business days, 2. Tuesdays, Wednesdays and Thursdays that are business days, 3. Fridays that are business days, 4. Weekdays (Mon-Fri) that are holidays, 5. Saturdays and 6. Sundays. The hour specific dummy variables depend on the day type and season: 1. hours during business days in winter season, 2. hours during business days in summer season, 3. hours during holidays in winter season and 4. hours during holidays in summer season. The winter and summer seasons are defined accordingly with the daylight-saving time, which means that the summer season starts from the last Sunday of March and end in the last Sunday of October.

As it can be detected, the number of parameters is quite large in the seasonality function. One reason why there are hour-dummies for different types of days is that there is at least some clear variation between the price levels of the night and day hours during a year, as presented in the chapter 4. An intuitive assumption would be that the more there are hour-dummies, the better the fit. But on the other hand, when classifying hours into different groups, the number of parameters increases quite rapidly, since there are 24 hours in a day. In addition, a large number of parameters could also have some sort of implications for the estimates. Without going any deeper in the analysis of the right number of deterministic parameters, some extent reasonable number of parameters was chosen in the models in this thesis. After all, the main complexity of the models refers to the stochastic part.

5.2 Stochastic component

Based on the data-analysis and prior research, two regime-switching models and a two jump-diffusion models were chosen as candidates. The regime-switching models are defined as in the study by Janczura & Weron (2012) and the jump-diffusion models as in the study by Benth et al. (Benth et al., 2012). There are two versions of the jump-diffusion model and the regime-switching model, since the other versions will be calibrated to daily spot prices, whereas hourly spot prices are used to calibrate the other two. The intraday pattern will be generated differently for the daily spot models by using the technique called “historical profile sampling”, which will be explained in more detail later. Additionally, none of the models will not be exactly as such as they were defined in the studies, since each of them can be classified as two-factor models, which will have two stochastic factors, the other describing the short-term and the other the long-term stochastic variation around the mean. This will be elaborated later clearer. Naturally, the main difference to the original models is also that the models in this thesis are hourly spot models.

The main argument behind the regime-switching model is that they are able to consider both single jumps and as well multiple spike values in a row, since the transition matrix allows the process stay in the same regime with higher probability (Weron, 2008). This is highlighted especially when dealing with hourly data when there are clearly more jumps or spike values than in a data set containing daily observations. Moreover, by regime-switching models, it could be expected that the time-varying jump intensity, size and direction could be captured in a straightforward manner through the transition matrices. Jump diffusion models are also considered as comparison. The jump-diffusion models used in this thesis will have a normally distributed jump size without any time-dependent properties. This way the calibration can be performed in a sophisticated manner, since the process can be assumed to be normally distributed (Seifert and Uhrig-Homburg, 2007). The jump-diffusion models also sometimes allow the process stay in the high (low) level a bit longer when a jump has occurred, since the mean-reversion does not necessarily take the process back to the mean level immediately.

Two of the models will be calibrated with daily spot prices, and thus totally different technique for simulating hourly spot prices is to be studied, following the study by Culot et al. (2014). Similar idea was obtained also by Hirsch (2009). Following the technique by Culot et al., the counterparties of the regime-switching model and the jump-diffusion model will be calibrated first on daily spot prices. Then, these models will be used to simulate daily prices, after which the hourly patterns of each day will be sampled from corresponding day profiles from historical observations. This technique is called "historical profile sampling". The main advantage of this technique is to obtain stochastic hourly pattern within a day, whereas the other two of the models use deterministic dummy variables to obtain the hourly profile within a day. Naturally, this way simulated price path will have more variating daily spread between the peak and off-peak-hours, similar looking than the actual observations, as seen in the Figure 7 in chapter 4.

Before presenting the models, a one common feature of all the models should be highlighted and argued. Motivated by (Benth et al., 2012), all of the models will have a stochastic component which has two factors, which means that the stochastic component is basically a sum of two independent stochastic processes. As argued by Culot et al. (2014), single-factor models are not able to capture the long-term price movements, as they will be calibrated on high-frequency spot data only. The basic idea to use two-factor models in this thesis roots also to the deseasonalization of the spot price data, which will be presented in more detail in the next chapter. The core message will be that the so-called deseasonalized data does not look a mean-reverting process that has only one mean-reversion component. Moreover, the deseasonalized spot price data seem have a two different mean-reversion rates, and thus two mean-reverting processes: the first being responsible for the hour-by-hour variation with faster mean-reversion, whereas the other process being responsible for more long-term fluctuation around the mean-level with slower mean reversion. For all the models this long-term process is assumed to be a mean-reverting Ornstein-Uhlenbeck (OU) process $X_{1,t}$, defined as follows:

$$\mathrm{d}X_{1,t} = \left(\alpha_1 - \kappa_1 X_{1,t}\right)\mathrm{d}t + \sigma_1 \mathrm{d}W_t \tag{5.2}$$

Next, the different models or more precisely the different short-term processes will be presented.

5.2.1 Jump-diffusion model

Motivated by Weron et al. (2004) and Benth et al. (2012), the first model considered is a mean-reverting jump-diffusion model (JD model), where the short-term stochastic process $X_{2,t}$ is a mean-reverting jump-diffusion process (OU with a jump component):

$$\mathrm{d}X_{2,t} = \left(\alpha_2 - \kappa_2 X_{2,t}\right)\mathrm{d}t + \sigma_2 \mathrm{d}W_t + J\mathrm{d}q \tag{5.3}$$

The jump component is assumed to be a normally distributed random variable with mean μ and variance σ^2 that follows a Poisson process with a time-dependent intensity $\mathrm{d}q$:

$$J \sim \mathrm{N}(\mu, \sigma^2) \tag{5.4}$$

And $\mathrm{d}q$ is a Poisson process with the intensity defined as follows:

$$\mathrm{d}q = \begin{cases} 1 \text{ with probability } l \,\mathrm{d}t \\ 0 \text{ with probability } (1-l)\mathrm{d}t \end{cases},$$

where l is the jump intensity.

5.2.2 Regime-switching model

The regime-switching model (RS model) presented here is defined as in the study by Janczura & Weron (2012). The proposed RS model includes three possible regimes that the price path follows at each time step: a base regime, a spike regime and a drop regime. However, the RS model presented here has a couple of differences compared to the model by Janczura & Weron. The first difference is obviously that the model is estimated with hourly data. The second is that the transition matrix of the model will be time-dependent, which allows the model to control the switching between the regimes so that is reasonable. Reasonable in this case means that it would be e.g. highly unlike that the process would switch to the spike regime in the middle of the night.

In the base regime, the process follows a mean-reverting heteroskedastic process, defined as follows:

$$X_{t,2} = \alpha_2 + (1-\beta_2)\, X_{t-1,2} + \sigma_2 |X_{t-1,2}|^{\gamma} \varepsilon, \tag{5.6}$$

where ε is Gaussian noise. In the spike regime, the values are assumed to be i.i.d. random variables from shifted log-normal distribution, defined as follows:

$$\log\big(X_{t,3} - X(q_2)\big) \sim \mathrm{N}(\alpha_4, \sigma_4^2) \tag{5.7}$$

The drop regime represents the negative jumps. In this regime, the values are assumed to be i.i.d. random variables from shifted log-normal distribution, defined as follows:

$$\log\big(-X_{t,4} + X(q_2)\big) \sim \mathrm{N}(\alpha_4, \sigma_4^2) \tag{5.8}$$

The lower indexes in the functions above indicate the different regimes and keeping in mind that the long-term OU-process was defined with the lower index 1, as defined earlier. The transition matrices $\boldsymbol{P_t}$'s will be time-dependent, so that there are 32 different transition matrices including probabilities of switching the regime or staying in the same regime. The 32 different transition matrices depend on the season, day and hour, grouped accordingly with the Appendix A. The transition matrices of the hourly spot model and daily spot model are presented in Appendices C and D, respectively, each of them having the following form:

$$\boldsymbol{P_t} = (p_{ij}) = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix},$$

where the numbers 1, 2 and 3 in the lower indexes represent the base, spike and drop regime, respectively. p_{ij} is a probability of switching from regime i to regime j.

5.2.3 Historical profile sampling models

Both of the previously presented JD and RS models have similar counterparties that are used in the historical profile sampling technique. The models used are otherwise similar, except that the models are estimated with the daily spot data. The deterministic component is otherwise the same as defined earlier, but the hour-dummies are naturally excluded from these models. What comes to the time-dependency of the stochastic parameters, there are now eight (8) instead of 32 different groups of observations, depending on the season and day type. The two additional models are named as HPS JD and HPS RS, HPS standing for “Historical profile sampling” as presented by Culot et al. (2014).

In order to perform historical profile sampling, the different day profiles have to be defined, which are as follows:

1. Normal business day
2. Positive jump business day
3. Negative jump business day
4. Normal holiday
5. Positive jump holiday
6. Negative jump holiday

The profiles differ from the ones used by Culot et al. (2014). In this thesis it is hypothesized, that the during the so-called jump days the hourly pattern is different. After the daily spot prices are simulated, each observation is identified as one of the six profiles presented above. Then, the hourly profiles are sampled from the hour specific probability density functions (PDF) of the corresponding day profile. The hourly profiles are estimated from the actual observations. When sampling hour values for a same day from the hour specific PDFs, a common random number is used to get smooth pattern. The hour specific PDFs in each profile are assumed to be normally distributed random variables.

6. MODEL CALIBRATION

As discussed in the chapter 3, the model calibration starts with filtering out the predictable seasonality from the data, and that is explained in the first subchapter. After the predictable part of the data is filtered out, the stochastic parameters can be estimated. The estimation of the stochastic parameters will be covered in the subchapter 6.2. Note, that in this chapter more focus is put on the hourly data filtering and hourly spot models. However, when calibrating the deterministic and stochastic components, identical steps are followed regardless of which data is used, hourly or daily.

6.1 Deterministic component

The deterministic seasonal function, the equation 5.1, presented in the subchapter 5.1 is fitted to the log-price data by using the least squares method. The fitted function can be seen in the Figure 14 below and a closer look of the function is presented in Figure 15. As it can be observed, the fit is clearly not perfect, and the function does not succeed to filter out all the seasonality. Since it is just a function of time, the function represents an average seasonality through the sample, and it is obvious that the function does not manage to filter out the extreme seasonality e.g. in the summers of 2012 and 2015. However, in order to replicate this seasonality to the simulated stochastic price paths, a deterministic seasonality function is needed. In addition, if the seasonality function is found too regular, it would be then reasonable to assume that it is the stochastic process that is driving the fluctuation around the "regular" seasonal pattern.

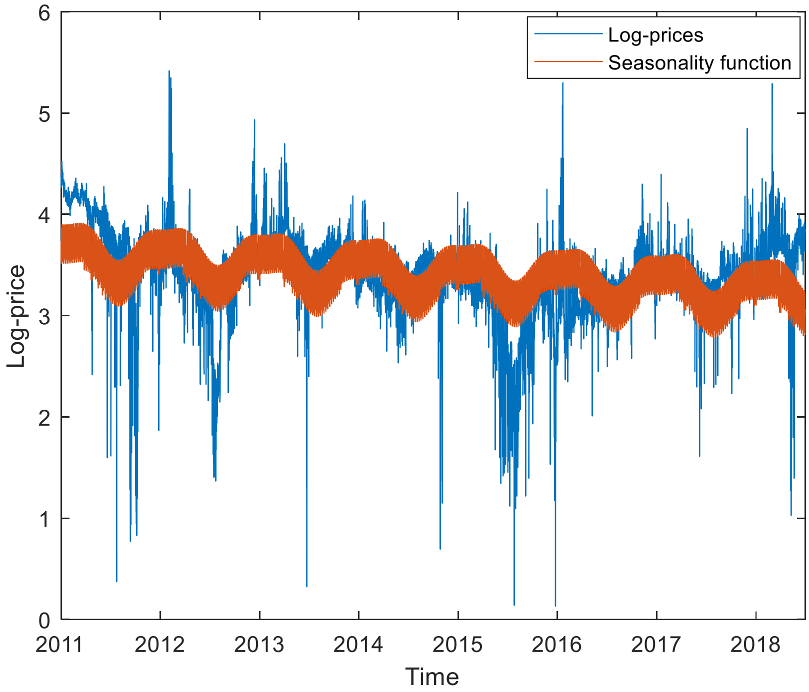

Figure 14. *Fitted seasonality function.*

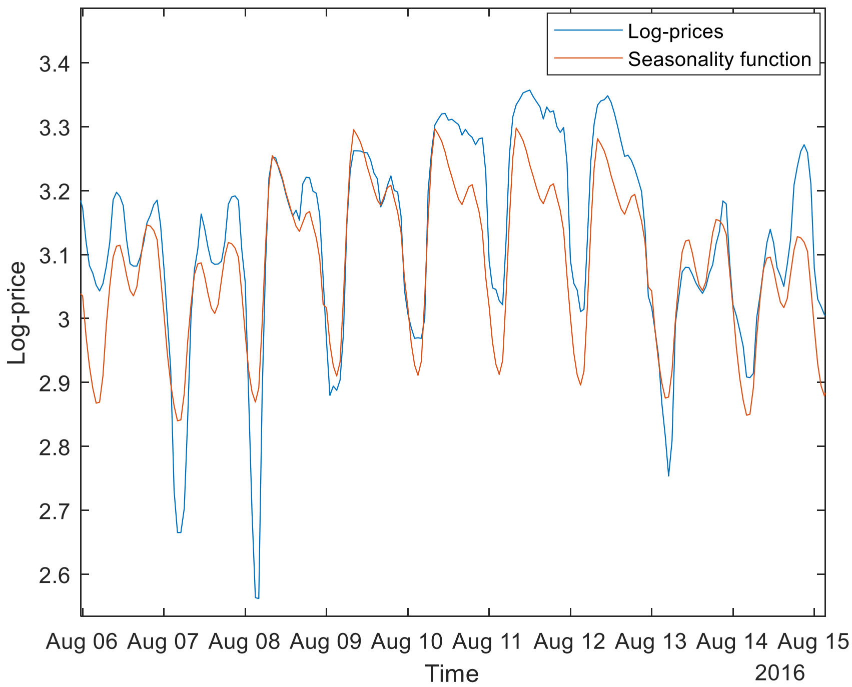

Figure 15. *A closer look of the fitted seasonality function.*

The intraday pattern can be observed clearer from the Figure 15. Just by visual inspection, it can be seen that the hour-dummies perform quite well representing the intraday pattern.

On the other hand, it can be also detected that hour-dummies that are estimated throughout the sample, do not of course provide a perfect fit when looking at single days, since the dummies represent the average values throughout the sample of the corresponding day type. By deseasonalizing the data, the jumps or spikes can be more easily detected, as it can be seen from the figure above, which includes a couple of clear negative jumps.

As it was brought up in the chapter 3, there are plenty of different alternative to present the deterministic seasonal pattern. Different seasonality functions could have been used and tested, and then the one providing the best fit, e.g. based on the Mean Squared Error, could have been chosen. However, using and comparing different seasonality functions was left out from the scope of this thesis, since after all, the main focus of this thesis relates mainly to the stochastic process and path dependent option pricing. In addition, in order to analyze the pricing consistency of the different spot models, it is reasonable to use same deterministic seasonality function for all of the models.

6.2 Stochastic component

The stochastic component is estimated with the deseasonalized data, which is obtained by subtracting the log-prices by the previously estimated deterministic component. The deseasonalized log-prices are presented in Figure 16 below.

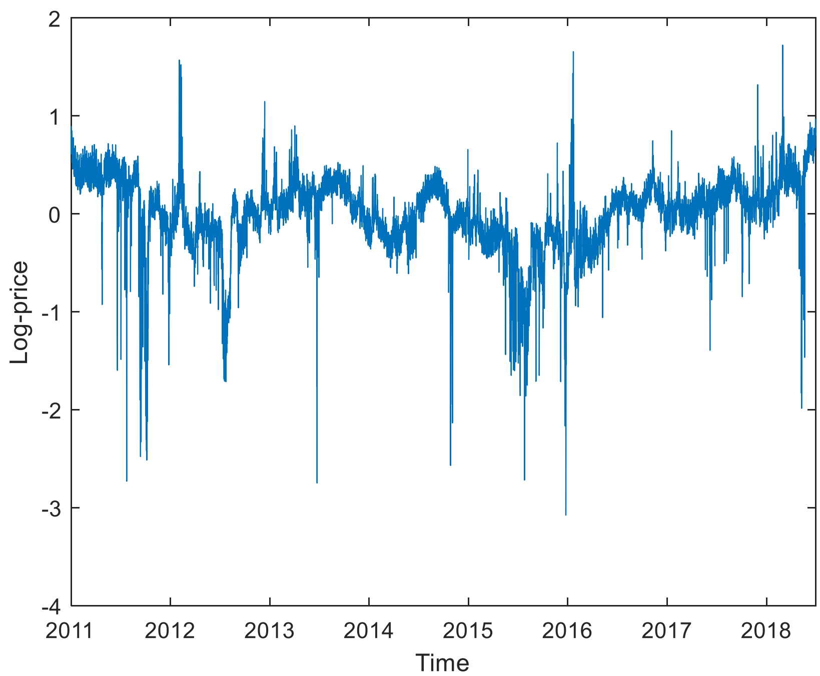

Figure 16. *Deseasonalized log-prices.*

The deseasonalized data is used to estimate the stochastic parameters. However, the stochastic process between the RS models and the JD models will be estimated differently. The RS models will be estimated by using an algorithm called Expectation Maximization, whereas the calibration of the JD models will be calibrated with the Maximum Likelihood method (MLE).

As described in the chapter 5, all of the models include a stochastic component, which is a sum of two independent stochastic processes, the other describing the short-term and the other the long-term stochastic variation around the mean. The long-term mean-reverting Ornstein-Uhlenbeck (OU) process, the equation 5.2, is the same for all of the models, but for the hourly spot models (JD model and RS model) it is estimated with the 24-hour moving average of the deseasonalized log-price data, whereas for the daily spot models (HPS JD and HPS RS models) it is estimated with the 7-day moving average of the deseasonalized log-price data. The estimation is performed with MLE by using Matlab's MLE-function. The estimated values for the parameters of the long-term OU-process are presented in Table 5 below. As it can be detected from the Table 5, the mean-reversion parameter κ is rather small number for both of the long-term OU-processes. The values are annualized, meaning that the long-term OU-process of the RS and JD models crosses the mean level 1.5441 times per year on average. For the HPS models the reversion is a bit slower.

Table 5. *Estimated parameter values of the long-term OU-process.*

Model	Alpha, α	Kappa, κ	Sigma, σ
RS and JD	0,0162	1,5441	0,6076
HPS RS and HPS JD	0,0083	1,3737	0,5424

The different models vary in the definition of the short-term stochastic process, and their estimation procedure will be explained in the next subchapters. Next, the estimation of the regime switching model will be explained first and the jump-diffusion model after that. In the end of the chapter, the estimation of the probability density functions of the historical profile sampling models will be explained.

6.2.1 Regime-switching models

In the RS models used in this thesis, there are three different stochastic processes or regimes, equations 5.6-5.7, and the transition matrices to be calibrated. In addition, the long-term OU-process is calibrated separately, as explained earlier. Usually in the regime-switching models, the parameter estimation for all of the stochastic parameters is performed simultaneously in a two-step iterative procedure with an algorithm called Expectation Maximization (EM). This form of EM-algorithm was originally introduced by Hamilton (1990). The EM-algorithm by Janczura & Weron (2012) is used in this thesis to estimate the parameters for the RS models. According to Janczura & Weron (2012), their EM-algorithm is about 100-1000 times faster than the competing ones. The main reason for the efficiency is that the algorithm utilizes only the last conditional probability instead of storing the conditional probabilities through all the prior time steps. Next, the EM-algorithm will be briefly explained, as presented by Janczura & Weron (2012). In the Matlab implementation, the code by Janczura & Weron (2018) is used with a few modifications.

In the beginning of the algorithm, a set of initial parameters are chosen arbitrarily. Then, in the "E-step", inferences of the state process are performed with a set of observations given. These inferences are expectations of the state process, which result into conditional probabilities for the process being in specific regime at a specific time step. Next, in the "M-step", new estimates of the parameters of the state process are calculated with the Maximum likelihood method based on the inferences derived in the E-step. These steps are iteratively repeated until the local maximum of the likelihood function is achieved. To get more detail description of the algorithm, see e.g. the study by Janczura & Weron (2012).

With the EM-algorithm, the underlying regime for each time step can be estimated. The identified underlying regimes from the deseasonalized hourly log-price data are presented in the Figure 17 below. A closer look of the log-prices is presented in the Figure 18 with the intraday pattern to get better demonstration of the time-dependency of the regimes. The corresponding figures for the daily data are presented in the Appendix B. As a result of the EM-algorithm, each observation has a conditional probability for being in each of the regimes. It can be observed, that the EM-algorithm seems to perform rather well in identifying the underlying regime. Since the figure highlights the identified spike and drop regimes, it should be noted that the most probable regime is still naturally the base regime.

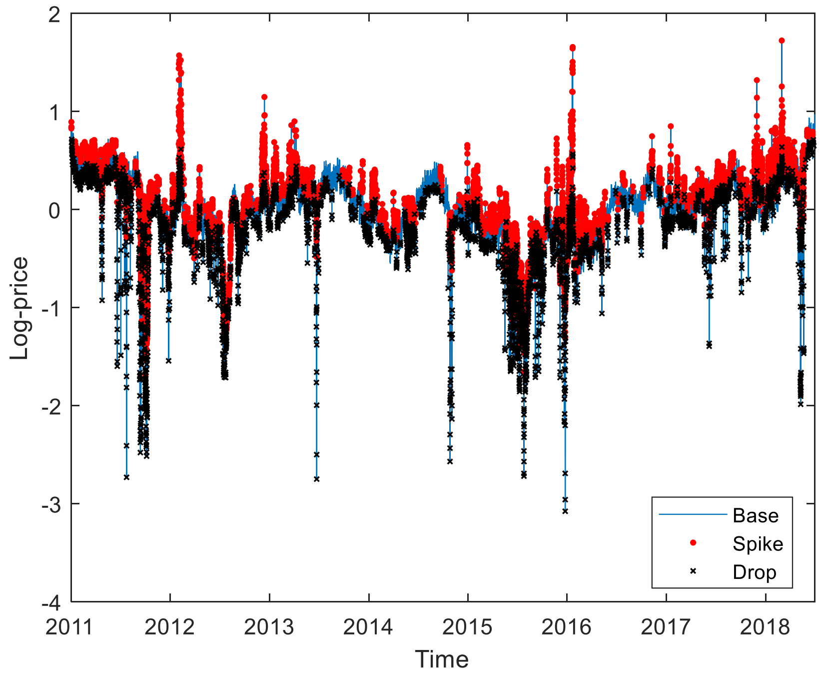

Figure 17. *Identified regimes in the hourly data by the EM-algorithm.*

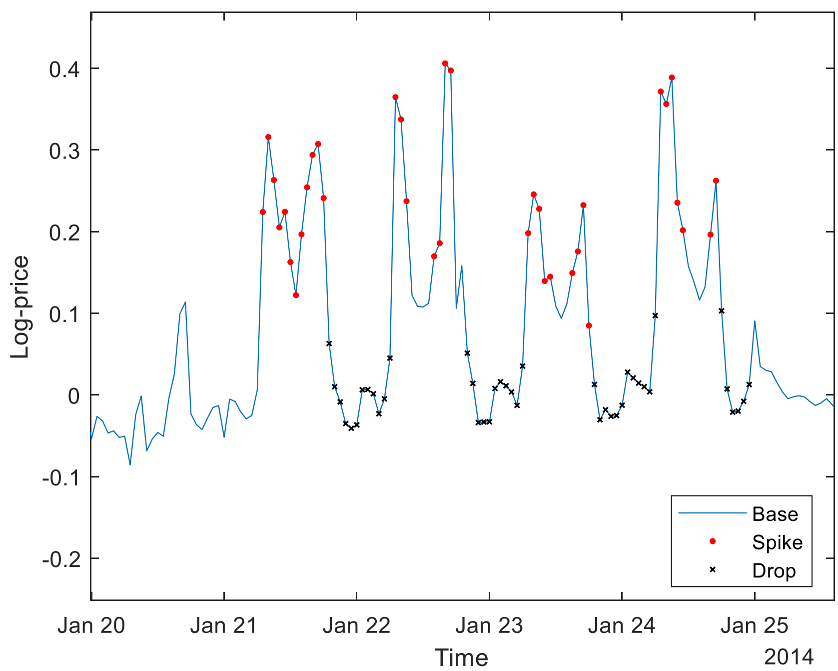

Figure 18. *A closer look of the identified regimes in the hourly data.*

The estimated parameter values for the base regime of the RS models are presented in Table 6 below, whereas the Table 7 contains the estimated parameter values for the spike and drop regimes.

Table 6. *Estimated parameter values of the base regimes of the regime-switching models.*

Model, regime	Alpha, α	Phi, ϕ	Sigma, σ
RS model, base regime	0,0002	0,8897	0,0332
HPS RS model, base regime	-0,0002	0,2487	0,0514

Table 7. *Estimated parameter values of the spike and drop regimes of the regime-switching models.*

Model, regime	mean, (log-norm distr.)	variance, (log-norm distr.)
RS model, spike regime	0,1500	0,0159
RS model, drop regime	-0,1468	0,0253
HPS RS model, spike regime	0,1148	0,0062
HPS RS model, drop regime	-0,1260	0,0132

In this thesis, the additional modification to the regime-switching model estimation by Janczura & Weron (2012) is to obtain time-dependent transition matrices. This is obtained in a straightforward manner as follows: first, the data set is divided into 32 different samples based on the hour, day and month, as defined in the last chapter. Then, the probabilities in each transition matrix are calculated for each sample, given that the underlying regime is known for each observation as a result of the EM-algorithm. The resulting 32 transition matrices of the RS model are presented in the Appendix C, and the transition matrices of the HPS RS model are presented in the Appendix D.

6.2.2 Jump-diffusion models

The short-term OU-process with jumps, the equation 5.3, in the JD models is estimated with the deseasonalized log-price data subtracted by the moving average that was used for estimating the long-term OU-process. The short-term OU-process with jumps is estimated similarly with MLE, as performed in the study by Escribano et al. (2002), which further refers to the studies (Lucia and Schwartz, 2002; Seifert and Uhrig-Homburg, 2007; Villaplana, 2003). In the Matlab implementation, it is referred to the code provided by MathWorks (MathWorks, 2018). The parameter values of the short-term OU-process is presented in the Table 8 below for both JD and HPS JD models. It can be seen that there is a clear difference between the estimated parameter values of the two OU-processes, when the other is calibrated with hourly data and the other with aggregated daily data. It is obvious that the standard deviation (sigma) is much lower with the aggregated data, but the difference in the mean-reversion (kappa) and the jump intensity (lambda) is extremely clear as well: the mean-reversion is almost three times as fast and the jump intensity nearly 14 times larger in the OU-process calibrated with hourly data (JD model).

Recall that there were 702 detected jumps with the Lee & Mykland technique, meaning annualized lambda of 93.6, which is much smaller than the one estimated here. However, the technique by Lee & Mykland detected jumps that are more extreme, whereas in this case the jumps are assumed to be normally distributed with mean nearly zero. This obviously results to jump observations that are not that extreme. Additionally, the use of the instantaneous volatility in the technique by Lee & Mykland resulted the so-called correction jumps not to be classified as jumps.

Table 8. *Estimated parameter values of the short-term OU-process with jumps.*

Model	**Alpha, α**	**Kappa, κ**	**Sigma, σ**	**Mu, μ (Jump)**	**Sigma, σ (Jump)**	**Lambda, λ**
JD model	0,2815	739,3213	2,5413	-0,0004	0,1854	781,6859
HPS JD model	0,4993	273,0696	0,9218	-0,0090	0,1507	56,7753

What is notable in this calibration procedure is that the OU parameters are estimated simultaneously with the jump parameters. Other alternative would be to estimate the OU and jump parameters separately. However, this would require a removal or replacement of the jumps, in other words the data should be manipulated. This kind of manipulation could be considered questionable for the following reasons: firstly, given the complex characteristics of the hourly electricity spot price, it is hard to define a replacement value that would be well-justified. And secondly, no matter what the argument behind the replacement value, this kind of manipulation can always be regarded as damaging the data and also intentionally losing information. To conclude, in order not to lose any information nor damage the data, it seems to be better alternative to use the data including the jump values. However, if the jump size would be modelled e.g. as a gamma-distributed random variable, the estimation would not be possible to perform in a similar manner that was used here. In that kind of scenario, the estimation procedure would might require to separate the jump observations or damage the data somehow. Now, the OU-process with the jumps can be assumed to have a normal probability density function, but in the case of gamma-distributed jump sizes, the convolution distribution of the normally distributed OU-process and gamma-distributed jumps would not be that straightforward to solve analytically.

6.3 Hour profile distributions in HPS models

The classification of the daily observations in the HPS JD model is based on the jump detection technique by Lee & Mykland, whereas for the HPS RS it is performed by the EM-algorithm. After the jumps are detected by using either of the manners, each daily observation can be categorized based on the day type (business day/holiday) and whether it was a normal day, a positive jump day or a negative jump day, resulting in six alternative labels for each daily observation: 1. Normal business day, 2. Positive spike business day, 3. Negative jump business day, 4. Normal holiday, 5. Positive jump holiday and 6. Negative jump holiday.

The procedure of estimating the hour specific probability distributions for each day profile is based on the EM-algorithm for both HPS RS and HPS JD models. Since the end result is reasonable regarding the identified "spike" and "drop" days and the sample sizes of each profile, these hour PDFs are used with the HPS JD model as well. The technique by Lee & Mykland would also be an alternative for identifying the day profiles. However, this technique would result into quite small samples for the jump profiles, and the hour PDFs would not be that representative. For this reason, the profile samples were based on the EM-algorithm.

After identifying the day profiles of each daily observation, each observation in the raw hourly price series is divided by the corresponding daily value in the raw daily price data. As a result, we have a series of hourly ratios with respect to the corresponding daily average prices. And as the day profiles are also known, the entire sample of hourly ratios can be divided into six different samples, each representing a specific profile, respectively. From the day profile samples, a probability density function (PDF) for each hour is estimated in each profile. Each PDF is assumed to be a normal distribution, which will be truncated so that negative ratios cannot occur, since that would result into unreasonable end result. There are 144 hour PDFs (6 profiles, 24 hours), of which only 15 the null hypothesis was rejected, when testing the normality with $\chi 2$-test. This obviously implies that for all the hours a normal distribution might not be the best representation. This might mean that either the sample size is too little for some or then the day profile classification is not that successful. However, the mean values of the ratios of each profile are presented in the Figure 19 below, and it can be observed that at least the averages seem good, since clear difference can be detected between the profiles.

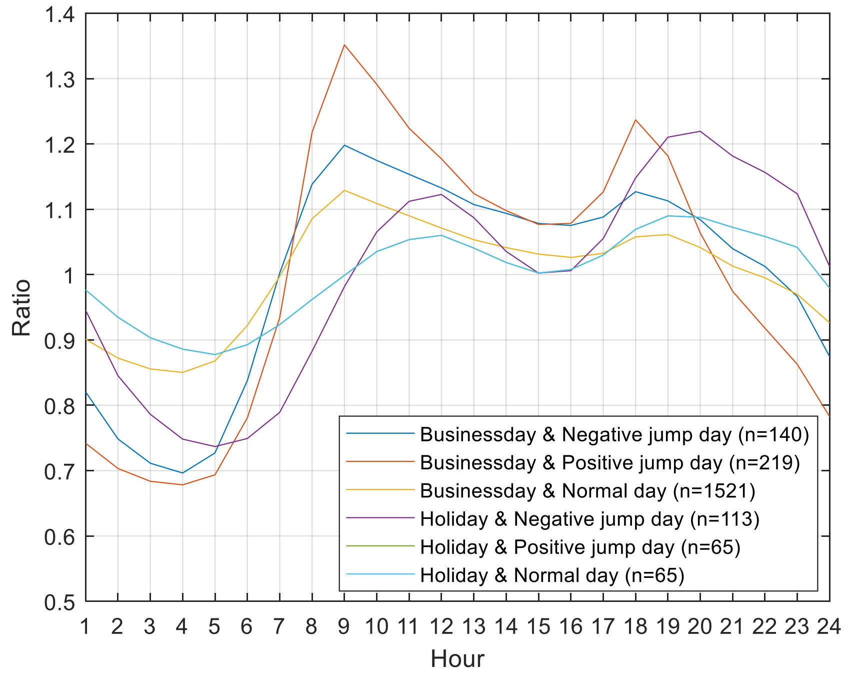

Figure 19. *Mean ratios of the hour PDFs of each profile.*

From the Figure 19 it can be observed, that there is clear difference between the hourly mean ratios within each profile. This will result into different shapes of hourly patterns for each different day profiles. The reason why the prices are used instead of log-prices relates to the risk-neutral calibration, which will be elaborated more clearly in the next subchapter. The hourly patterns will be generated after the risk-neutral simulation of daily prices, so in order to obtain the risk-neutrality, the daily average prices cannot be changed when generating the hourly patterns. This will be obtained by scaling the generated hourly prices with the corresponding daily average. Since the use of log-prices in this step would result into slightly different expected spot price of the simulation, the prices are used in order to retain the simulated expected spot price.

6.4 Risk-neutral calibration

After the models are calibrated with the spot data, the models are calibrated with the forward prices in order to obtain risk-neutral pricing. As discussed in the chapter 2, at least two different risk-factors can be identified for electricity spot prices: short- and long-term risk factors. As the common way seems to be, and used in this thesis as well, the short-term risk factor is assumed to be zero. Thus, only the long-term risk factor is calculated. In the risk-neutral calibration, the steps by Lucia & Schwartz (2002) and Seifert & Uhrig-Homburg (2007) are followed. Following Lucia & Schwartz (2002), the risk-neutral price in the mean-reverting Ornstein-Uhlenbeck process is defined as follows:

$$\mathrm{d}X_t = \kappa(\alpha^* - X_t)\mathrm{d}t + \sigma \mathrm{d}Z_t^*,$$

where

$$\alpha^* \equiv -\lambda\sigma/\kappa,$$

λ denoting the market price per unit risk linked to the state variable X_t.

As showed in the chapter 2, the long-term risk factor can be abstracted from the relation of the forward and spot prices. Seifert & Uhrig-Homburg (2007) derive the discretized formula of the market price of risk as follows:

$$\lambda_{T-1} = \frac{\ln\left(\frac{F(t,T)}{E_t^P\left(e^{D_T+S_T+L_T}|\mathrm{F}_t\right)}\right)}{\Delta t} - \sum_{s=t}^{T-2}\lambda_s \tag{6.1}$$

The discretized formula above can be used to calculate the long-term risk factor. In practice, the models are first calibrated with the spot data, after which they are used to simulate large enough sample (500000) of price paths. Then, the expected spot price is calculated by taking the average of the simulated price paths. This expected spot price is then used to calculate the market price of a risk with the formula presented above. In the Matlab implementation, it is referred to the code provided by MathWorks (MathWorks, 2018).

In order to perform the calibration in a sophisticated manner and take the seasonality into account, the used forward prices need to be seasonalized as well. When calibrating the models, monthly future contracts are used. These monthly prices are then seasonalized with respect to the average values of the hourly and daily spot prices and used in calibrating the hourly and daily spot models, respectively. The seasonalization is naturally performed in way that the average of the forward prices stays the same. The seasonalized hourly forward prices and the monthly forward prices are presented in the Figure 20 below.

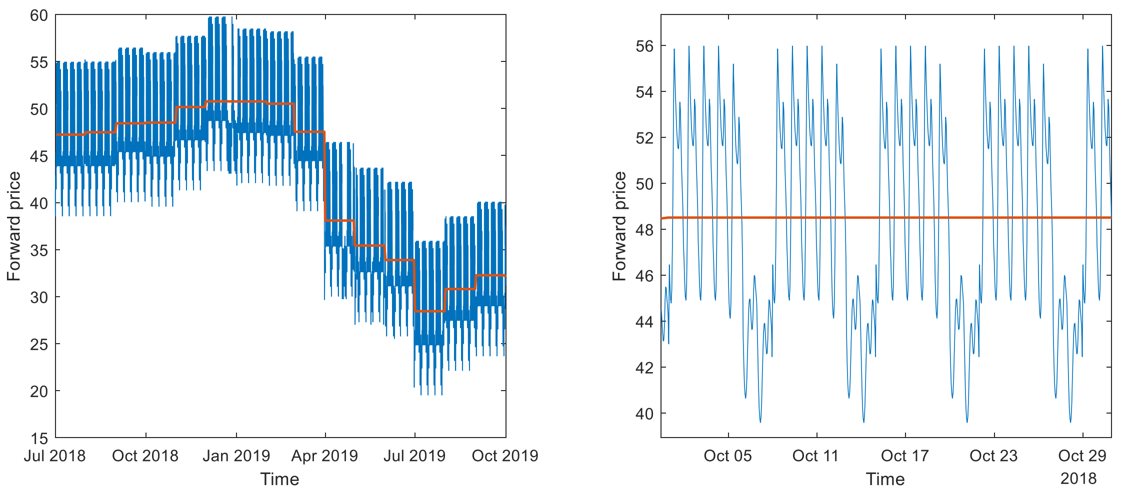

Figure 20. *Seasonalized and actual monthly future contract values.*

After the risk-neutral calibration, risk-neutral price paths can now be generated by the models. What this means in practice, is that the expected spot price generated by the models matches the prices of the future contracts. In the next chapter, the simulated price paths generated by the models are analyzed more thoroughly.

7. ANALYSIS OF THE SIMULATED PATHS

In this chapter, the simulated samples generated by the models are analyzed statistically and compared with the actual observations. What is done is that first, the models are estimated with the spot data during 1.1.2011-30.6.2018 and after that 100000 paths are simulated by each model for the period of 1.7.2018-30.9.2019. These paths are then compared with the actual historical observations in the terms of the descriptive statistics of the samples. Since only the characteristics of the log-returns are observed, it is reasonable to compare the simulated samples to the actual observations that were used to estimate the models. This way it can be seen how well the estimation succeeded the capture the characteristics of the log-returns. The descriptive statistics are compared both before the risk-neutral calibration and after the risk-neutral calibration. However, in the first subchapter, the simulated paths are first visually examined.

7.1 Simulation

The simulation was performed in two steps with all of the models. First, 100000 paths were simulated with the models estimated on the spot data. After that the long-term risk factor was calculated by the equation 6.1 using the expected spot price of the models and the forward prices observed from the market. When the risk factor was known, new 100000 paths were simulated so that the risk factor was now taken into account in the drift term of the model, as presented earlier. As an example, ten simulated risk-neutral paths and a closer look of one path are provided in Figures 21-24 for each of the models.

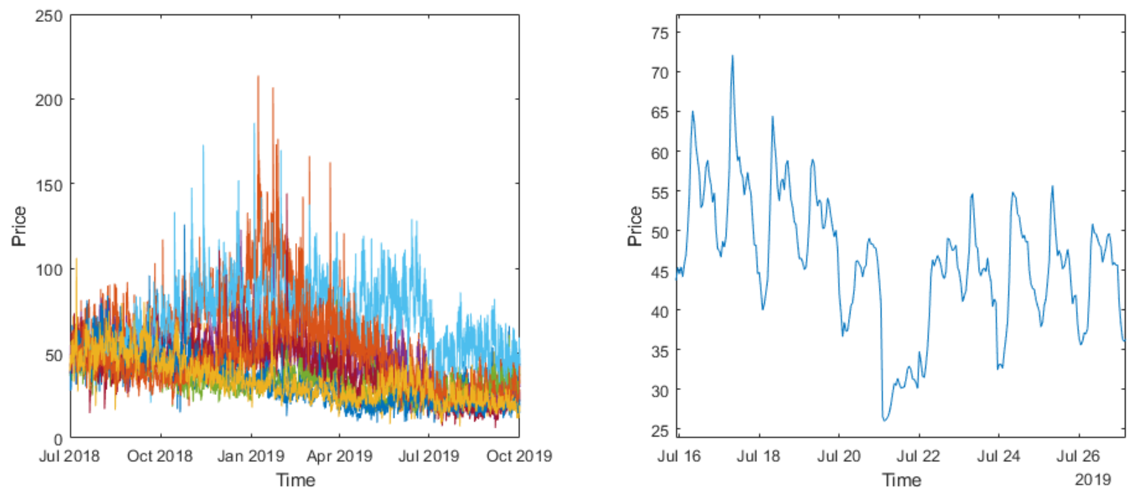

Figure 21. *A sample of simulated electricity spot price paths by the RS model.*

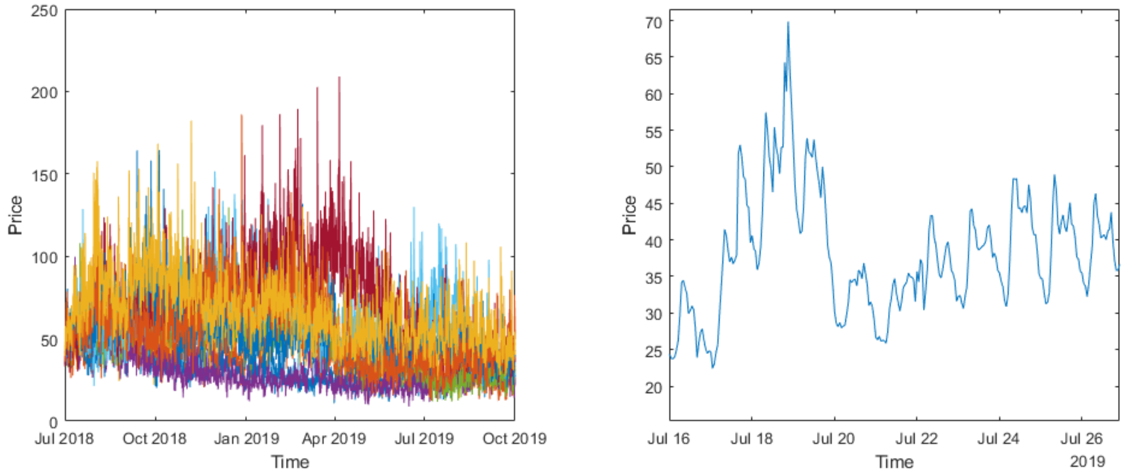

Figure 22. *A sample of simulated electricity spot price paths by the JD model.*

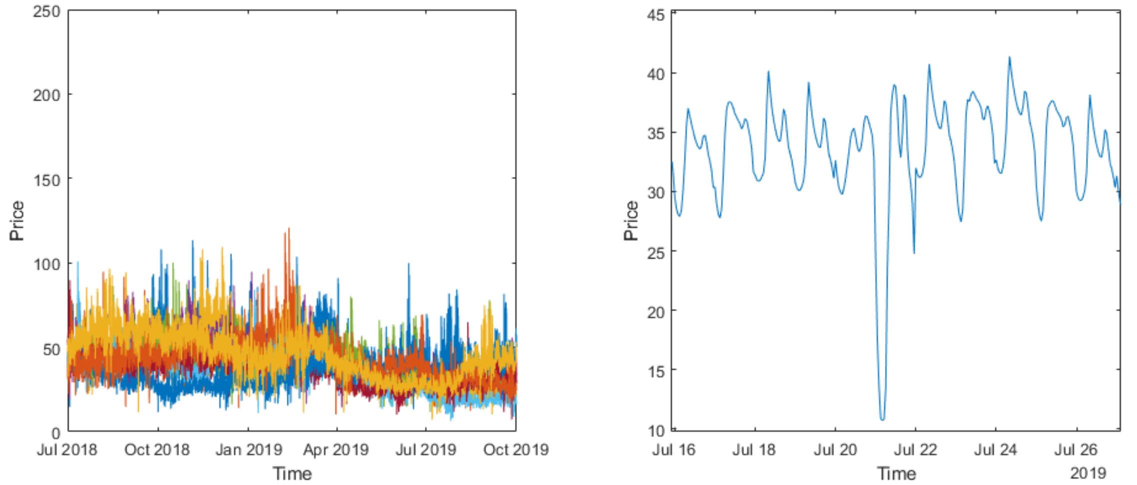

Figure 23. *A sample of simulated electricity spot price paths by the HPS RS model.*

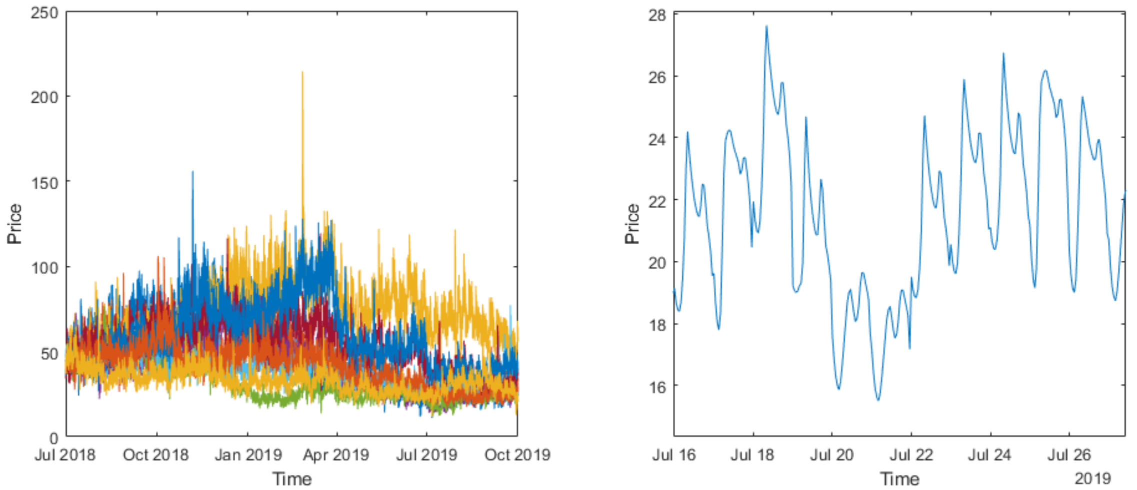

Figure 24. *A sample of simulated electricity spot price paths by the HPS JD model.*

What can be observed from the figures providing the closer look of the price paths, is that the hourly pattern looks pretty good for all of the models. However, it can be also detected that the pattern is somewhat rough for the RS and JD models, whereas for HPS models the pattern is smoother. Recall that the hourly pattern is generated by the short-term OU-process in the RS and JD models, whereas in the HPS models it is generated by the sampling technique. Thus, it is natural that the pattern is smoother and less volatile in HPS models. The roughness or smoothness of the patterns are also affected by the assumption of a constant volatility and the filtering procedure. However, any thorough analysis or conclusions should not be performed just by looking at the figures. It can be noted that when comparing the complete price paths on the left-hand side of the figures, the RS and JD models have more frequent extreme jumps, whereas for the HPS models they seem to be more controlled. Recall that hourly data was used to estimate the RS and JD models, and hourly data is not naturally so smooth as aggregated data. Even if the hourly data was used as well for deriving the hour-specific distributions for the HPS models, they were assumed to be normally distributed which obviously means values that are not that extreme. Additionally, it should be noted that even if the Figure 22 does not reveal it, the jump occurrence does not have any kind of time-dependency in the JD model, meaning that positive or negative jumps can occur just as likely during the day hours as during the night hours. After getting some visual taste of the simulated paths, the descriptive statistics of each model are presented and analyzed in the next subchapter.

7.2 Descriptive statistics

The descriptive statistics of the models, mean, standard deviation, skewness and kurtosis, are presented in the Table 9 and Table 10 below. Table 9 contain the statistics before the risk-neutral calibration. Since the HPS models are extended to hourly level after the risk-neutral calibration, hourly statistics before risk-neutral calibration is not available for those models. Table 10 contains the statistics after the risk-neutral calibration for all of the models.

From the Table 9 it can be observed that the obtained kurtosis of the RS model and the standard deviation of the JD model are quite close compared to the actual observations. The standard deviation of the RD model is a bit higher than the actual, which might imply that the underlying processes of the spike and drop regimes obtain too volatile end result. However, the value of kurtosis of the RD model can be considered pretty good, meaning that most of the log-returns lie close to the mean level, but time to time more extreme values are obtained, as in the actual observations. The standard deviation of the JD model can be considered extremely good, whereas the value of kurtosis is rather lower than it should be. Recall that the jump-diffusion process is assumed to be normally distributed, and this assumption naturally leads to a smaller value of kurtosis.

Table 9. *Descriptive statistics before the risk-neutral calibration of the simulated samples of JD model and RS model and actual observations during 1.1.2011-30.6.2018 (100000 simulations).*

Model	Standard deviation	Skewness	Kurtosis
RS model	0,113	0,111	64,731
JD model	0,077	0,155	11,399
Actual	0,075	1,690	79,791

After the risk-neutral calibration, all the models can be compared. First, what can be observed from the results of RS and JD model in the Table 10 is that they slightly differ from the statistics before the risk-neutral calibration. It should be reminded, that what was done in the risk-neutral calibration, was that the short-term risk factor was assumed to be 0 and the long-term risk factor was only calculated. This long-term risk factor, which was calculated based on the expected spot price generated by the model, was then subtracted from the drift term of the long-term OU-process in the risk-neutral simulations. As it can be detected, both the values of the standard deviations of the RS and JD models are lower than they were before the risk-neutral calibration. Interestingly, the values of kurtosis are now closer to the kurtosis of the actual observations for both models. This could imply that the calculated long-term risk-factor now makes the so-called regular log-returns to be more in line and thus closer to the mean-level. And since the short-term risk factor was assumed to be 0, the generated jumps are still there, and cause these extreme tail values. The statistics are also naturally affected by the Monte Carlo technique as well.

Table 10. *Descriptive statistics after the risk-neutral calibration of the simulated samples of all the four models and actual observations during 1.1.2011-30.6.2018 (100000 simulations).*

Model	Standard deviation	Skewness	Kurtosis
RS model	0,108	0,071	73,541
JD model	0,071	0,083	14,554
HPS RS model	0,054	1,136	43,535
HPS JD model	0,042	0,332	17,593
Actual	0,075	1,690	79,791

The results of the Table 10 indicate, that the HPS models generate less volatile price paths than the JD and RS models, and that the JD model provide the best fit in terms of the standard deviation. In addition, the RS and HPS RS models have higher values of kurtosis, and the RS model being quite close the actual. Recall that the major difference between the HPS models and RS and JD models was how the hourly profile for each day was derived. For the RS and JD models, the profile was largely constant and derived by the seasonality function. On the contrary, the hourly profile of the HPS models has stochastic characteristic, since the profiles are sampled from probability distributions based on the historical observations. However, the low values of standard deviation of the HPS models implies, that the log-returns between the hour observations varies quite little, even if the intraday pattern would have more dynamic shape. In addition, the extreme values in HPS models are rarer since the patterns are generated by the hourly ratios from normal distributions. Even if the intraday pattern is represented by the deterministic hour-dummies in the RS and JD models, the stochastic processes generate some variation to the regular pattern. The stochastic processes in the RS model can be even regarded to be too extreme, since the volatility is that high. In a conclusion, it seems that the HPS models are able to generate a quite regular intraday pattern, whereas for the RS model it might be too rough due to the state processes of the spike and the drop regimes. It should be reminded, that the HPS models are estimated on daily spot data, which is obviously less volatile, and the hourly ratios in this case are assumed to be normally distributed. This means that the normal distributions with fat and short tails are not able to generate extreme values, as for instance a long-tailed t-distribution would do. However, having a t-distribution as a representation of the hourly values might result into too extreme values, meaning unrealistically large jumps or drops. The skewness of the RS and JD models are quite close to zero, whereas the HPS models are more positively skewed as are the actual observations.

In a conclusion, it seems that none of the models cannot be labelled as extremely poor but nor extremely good either. However, the visual inspection also confirms, that the models are clearly able to generate good looking hourly patterns and obtain the jumps. The RS and HPS RS models are even able to generate jumps in a realistic manner regarding their time-dependency. Since the statistical properties are not that close to the actual observations, it might mean that the estimation of the models was not completely successful. However, it is good to also remind that having those two-factor stochastic components can have implications on the statistical properties of the generated paths. In order to obtain even better statistical properties, the estimation of a two-factor model should be studied further. In addition, both of the independent stochastic processes were assumed to follow mean-reverting processes. Alternative assumption for the long-term process would be a e.g. random walk, as in the study by Burger et al. (2004). But in the end, when looking at the descriptive statistics of the log-returns of the models, they can be said to provide decent results. In the next chapter, their consistency regarding option pricing is to be analyzed.

8. ASIAN OPTION PRICING

In this chapter, the pricing consistency of the spot models is analyzed. Asian spot options are priced with different strikes and maturities based on 500000 simulated price paths generated by the models. The payoffs of arithmetic Asian options are calculated over four different time intervals, which are the following four quarters: Q4-2018, Q1-2019, Q2-2019 and Q3-2019. The prices are considered here as implied volatilities, which are calculated by the Black-Scholes formula. This way the results can be validated with the implied volatilities of the quoted European options on futures, observed from the market. When deriving conclusions from the implied volatilities, Nomikos & Soldatos (2010) are referred. Their article was about analyzing the model implied volatilities of European spot options on electricity.

The implied volatilities of Asian spot options are calculated by the Black-Scholes vanilla option formula. Alternatively, the implied volatilities of Asian options could be calculated based on the pricing mechanism as well, but there is no closed-form solution for that purpose. Thus, the advantage of the Black-Scholes vanilla option formula is the analytical outcome and the ease of obtaining it. In addition, the Asian spot options will be compared and validated against the quoted European future options. Since the Black-Scholes formula is basically just used as a conversion formula to convert the EUR prices to implied volatilities, the underlying assumptions of the Black-Scholes formula are not critical. In other words, the payoffs are determined based on the simulated paths, which are generated by the models assuming mean-reverting behavior of the underlying, and then the implied volatility by the price is calculated with the Black-Scholes formula.

What is done in practice is that the average values during the defined time intervals are calculated based on the 500000 simulated paths generated by the models. After that, the payoffs are calculated with different strikes ranging from +/- 20 % around the at-the-money (ATM) strike which is the current (as of 30.6.2018) forward price for the time interval. Then, the volatility implied by the price, with the other parameters known (risk-free interest rate assumed to be zero), can be calculated. The payoffs of the Asian spot options are based on the arithmetic averages over the following four quarters, Q3-2018, Q1-2019, Q2-2019 and Q3-2019, and expiration days of the beginnings of the quarters are used. This obviously does not make much sense in reality, but in order to compare the prices against the quoted European future options, that are exercised before the beginning of the delivery period of the underlying future contract, the expiration is set accordingly with the Asian spot options. The Asian spot option implied volatilities with the different maturities are plotted in the Figure 25, Figure 26, Figure 27 and Figure 28, respectively, as a function of moneyness, which is defined as a the ratio between the strike (K) and the forward price (F). The implied volatility of call options is located on the left-hand side and the implied volatility of put options on the right-hand side of the figures. The implied volatilities generated by the different models are color marked.

As it will be soon seen in the following figures, the Black-Scholes implied volatilities of the models, when presented as a function of moneyness (K/F), are not flat as they would be if the model would assume the underlying price dynamics to be geometric Brownian motion. As mentioned briefly in the chapter 2, the mean-reversion, the jump size and the jump intensity have an effect on the shapes of the implied volatility plots of the models (Nomikos and Soldatos, 2010). In general, the results of this thesis are in line with the study by Nomikos & Soldatos (2010). The shapes of the implied volatility plots are clearly impacted by the used models and their parameters. In addition, in line with their results, the implied volatility of call options tends to be higher than the implied volatility of put options. Even if the differences between the implied volatilities of the calls and puts are quite minor in general, this indicates that the put-call parity does not hold in this case, which can be regarded as a rather interesting result. The higher implied volatility for call option can be partly explained by the more extreme positive jumps than negative jumps, which would imply that the more extreme positive jumps increase the likelihood of a call option to end up ITM. In addition, as it was shown in the descriptive statistics in the chapter 7, all of the models generated positively skewed log-returns. Also, in line with the results of Nomikos & Soldatos (2010), the implied volatilities decrease when the maturity of the option increases. According to Nomikos & Soldatos (2010), this phenomena of decreasing implied volatilities when time to the maturity increases can be explained by Samuelson effect (Samuelson, 1965), which states that for non-storable commodities any new market information has a more remarkable effect on the derivative prices that are closer to the maturity. Nomikos & Soldatos (2010) add that due to the mean-reversion, electricity prices have a tendency to revert to the “equilibrium level”, which in the long-term has a decreasing effect on the volatility term structure. In addition, it can be observed that the JD and HPS JD models, that did not have any time-dependent parameters, obtain implied volatilities that are almost flat but still slightly increase for call options when moving deeper ITM. This shape of an implied volatility implies that that the volatility is mostly explained by the so-called long-term stochastic factor of the models and not the short-term stochastic factor. As showed by Nomikos & Soldatos (2010), the shape of the implied volatility plot would be more pronounced if the jump size volatility or intensity would have higher values. Due to the time-dependent parameters in the RS and HPS RS models and the different equilibrium levels (forward price levels) of the quarters, the underlying time period seems to have clear implications on the implied volatilities as well. Thus, the implied volatilities of the options with the different expiries are discussed separately, starting from the Figure 25 with the Q4-18 Asian spot option. The analysis is performed mainly for implied volatilities of the call options, since the same conclusions apply for the puts as well, the implications being basically just the opposite.

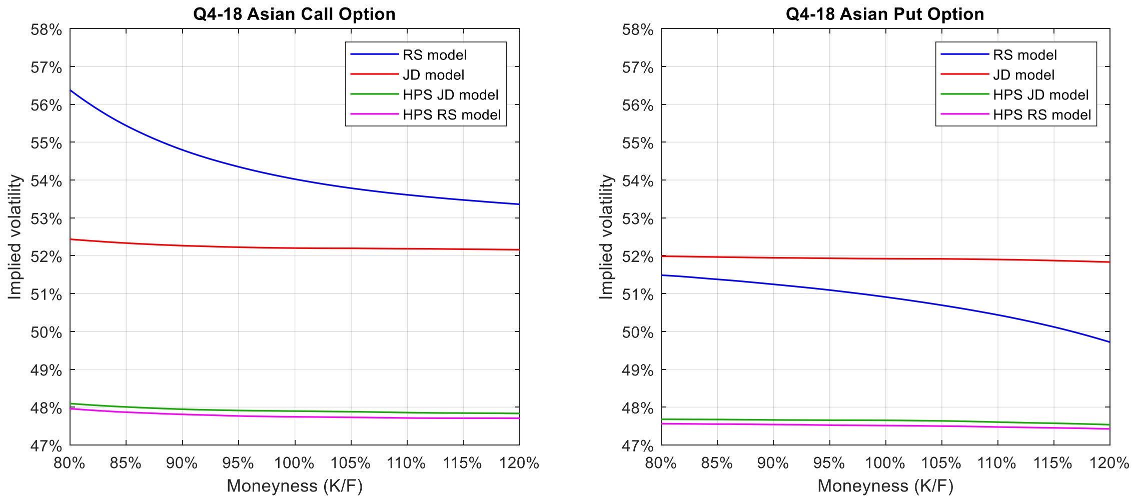

Figure 25. *Implied volatilities of Q4-2018 Asian spot call and put options expiring in the beginning of the quarter.*

The ATM Q4-2018 Asian spot option implied volatilities of the RS model, JD model, HPS JD model and HPS RS model are 54.0%, 52.2%, 47.9% and 47.7% for call options, and 50.9%, 52.0%, 47.7% and 47.5% for put options, respectively. Whereas the shape of the implied volatility plot is almost flat for the other models, it is curvier for the RS model. Positive jumps have an increasing effect on the implied volatilities of ITM call options. Since the mean-reversion pulls prices towards the mean-level, positive jumps increase the likelihood of call options ending up ITM. (Nomikos and Soldatos, 2010) It can be noted that this effect caused by positive jumps is more extreme for RS model than for the other models. Recall that the RS and HPS RS models had time-dependent transition matrices, meaning that the occurrence of positive and negative jumps depends on the time. This indicates that the more likely occurring positive jumps than negative jumps in the winter time explains the shape of the implied volatility plot. The reason why the implication is so extreme for the RS model is most likely due to the fact that the RS model generates the most volatile price paths, as it was shown in the descriptive statistics in the chapter 7. In addition, the intensity of jumps (or spikes) in RS model is quite large when compared to the other models. All in all, this means that the RS model generates more volatile jumps more intensively, and both of those factors have an increasing effect on the ITM call option implied volatility, as shown by Nomikos & Soldatos (2010). As it was already mentioned, the shape of the implied volatility plots of the other models are similar to the RS model, even if being not that extreme. Since the JD and HPS JD models did not have any time-dependent parameters in the stochastic process, the shape can be partly also explained by the difference of the spot price before the start of Q4-2018 and the "equilibrium" level (forward price) during the Q4-2018. Nomikos & Soldatos (2010) show that when the current spot price is equal or below the "equilibrium level", the mean-reversion increases the likelihood of ITM call options ending up ITM, since the prices are pulled above the strike. Since the models are calibrated with the forward prices showed in the Figure 20 in the chapter 6, the forward prices basically define the so-called "equilibrium level" in this thesis. It can be observed from the Figure 20, that the forward prices prior to Q4-2018 are below the level during the quarter.

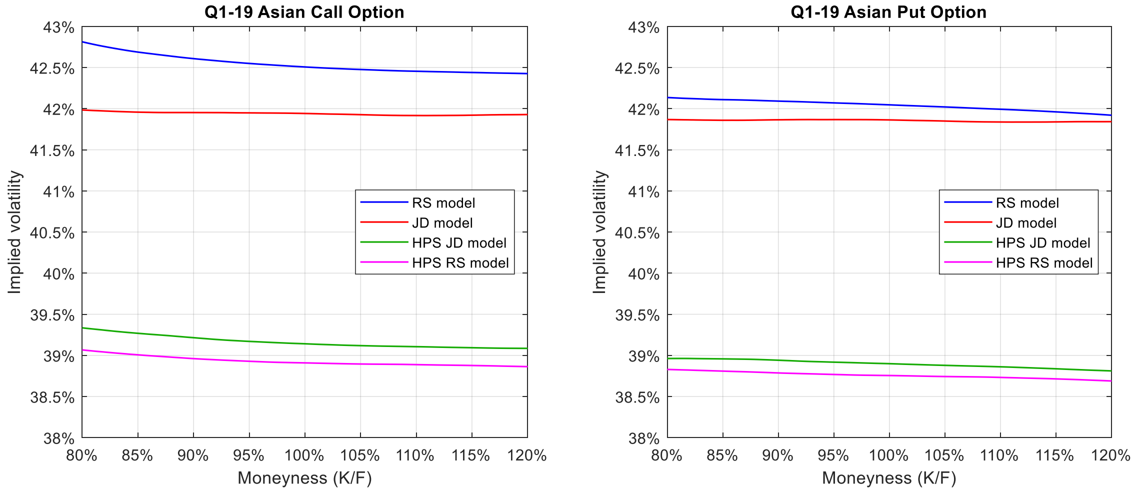

Figure 26. *Implied volatilities of Q1-2019 Asian spot call and put options expiring in the beginning of the quarter.*

The ATM Q1-2019 Asian spot option implied volatilities of the RS model, JD model, HPS JD model and HPS RS model are 42.5%, 41.9%, 39.1% and 38.9% for call options, and 42.1%, 41.9%, 38.9% and 38.8% for put options, respectively. Q1-2019 belongs also to the winter season, so the same implications of the positive jumps that were previously discussed can be detected from the Figure 26 as well. It can be seen from the Figure 20, that the forward price level is somewhat equal between Q4-2018 and Q1-2019. Thus, the results of the increased implied volatility for ITM call options are in line with the study by Nomikos & Soldatos (2010). By looking at the Figure 25 and Figure 26, the Samuelson effect can be detected quite clearly, since the ATM implied volatilities drop about 9-12 % depending on the model and the option type (call/put).

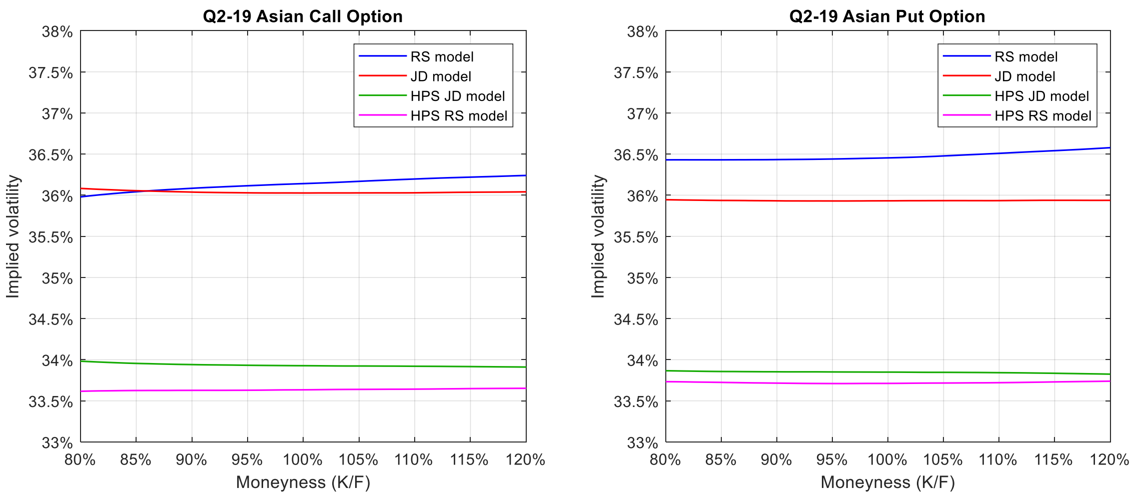

Figure 27. *Implied volatilities of Q2-2019 Asian spot call and put options expiring in the beginning of the quarter.*

The ATM Q2-2019 Asian spot option implied volatilities of the RS model, JD model, HPS JD model and HPS RS model are 36.1%, 36.0%, 33.9% and 33.6% for call options, and 36.5%, 35.9%, 33.9% and 33.7% for put options, respectively. Differing from the previous implied volatility plots, now the shape of the plots differs between the models. It can be observed that the implied volatility of the call option decreases when moving ITM for RS and HPS RS models and increases for JD and HPS models. The decreasing implied volatility of RS and HPS models can be explained by the time-dependent transition matrices, which now make the negative jumps to be more likely since it is the warm season. This means that it is now more likely that the prices achieve lower levels and thus, decrease the likelihood of ITM call option ending up ITM. In other words, this is just the opposite phenomena compared to the previous two quarters, where the positive jumps where more probable. Since the shape of the implied volatility plot is similar looking than in the previous plots for the JD and HPS models, the reason for this shape seems to be most likely the rather low values of jump size volatility and intensity, as discussed earlier. As argued by Nomikos & Soldatos (2010), in this case the volatility would be only slightly explained by the jump process.

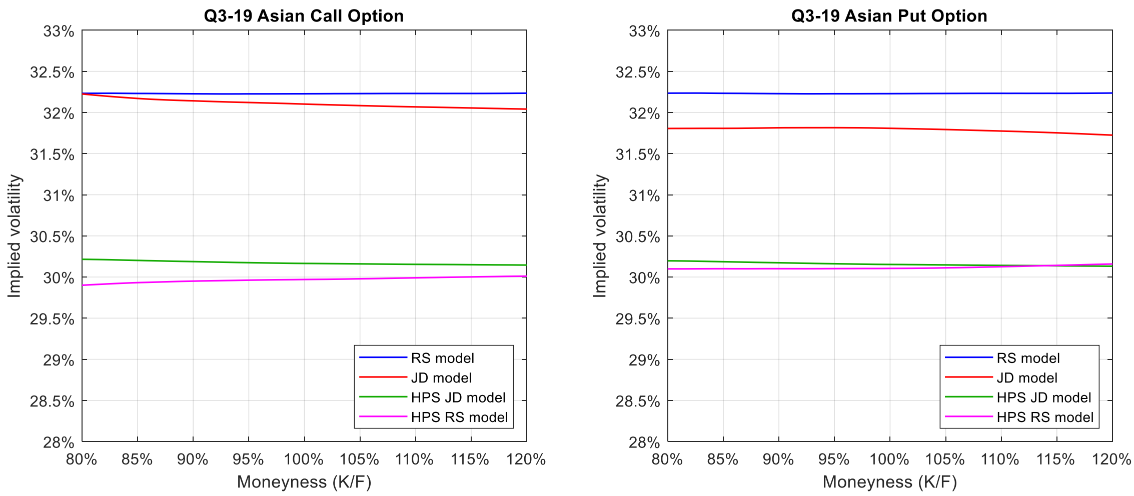

Figure 28. *Implied volatilities of Q3-2019 Asian spot call and put options expiring in the beginning of the quarter.*

The ATM Q3-2019 Asian spot option implied volatilities of the RS model, JD model, HPS JD model and HPS RS model are 32.2%, 32.1%, 30.2% and 30.0% for call options, and 32.2%, 31.8%, 30.2% and 30.1% for put options, respectively. The shapes of the implied volatility plot of the models are similar looking than earlier in the Figure 27, except that the implied volatility of the RS model is flat. Negative jumps cause most likely the decreasing implied volatility of the ITM call options for the HPS RS model, but for the RS model the balance between the positive and negative jumps seems to be somewhat steady. Otherwise the conclusions here are in line with the previous ones.

After observing the implied volatilities of the Asian spot options with different maturities, both general and specific conclusions can be made regarding the pricing consistency of the models. First of all, the results of this thesis indicate that the put-call parity can be questioned in the case of electricity as the underlying and mean-reverting models with jumps. Meaning that higher prices were obtained for call options than for put options, which is most likely due to the models obtaining more extreme positive jumps than negative jumps. The difference of the prices of calls and puts was the clearest with the RS model, that generated the most volatile price paths and obtained quite extreme spike regime. In general, it can be also concluded that the different models in this thesis do not generate consistent prices for the options, the difference being larger with shorter maturities. However, a clear difference can be detected between the RS and JD models versus the HPS models. The implied volatilities of the HPS models were consistently lower than the implied volatilities of the RS and JD models.

Recall that the short- and long-term mean-reverting processes of the HPS models were estimated both with aggregated data, daily average prices and moving weekly average prices, respectively. Meaning that that the aggregated data used for estimation is less volatile than the hourly spot price data, that was used to estimate the RS and JD models. Thus, an intuitive conclusion is that the HPS models generate less volatile price paths, even if the hourly patterns are stochastic. But as we recall, the hourly pattern is performed so that it will not change the corresponding daily average price in order to retain the risk-neutrality obtained by the simulated daily spot, so the hourly pattern generation does not have an effect on the option prices. All in all, based on these results, a conclusion is that daily and hourly spot models generate different prices for path dependent spot options. This can be regarded as a rather important result, since the daily spot models seem to be more popular at least among the academics, as discussed earlier. This implies, that even if this type of an Asian style option is possible to price by using either a daily or hourly spot model, the choice between the models has already an impact on the option price. These results, however, do not reveal which type of a model generates a price that is closer to the “right price”. However, it could be argued that the use of non-aggregated data leads to better results, since the data is less manipulated. In addition, as briefly mentioned in the introduction chapter, there has been also other studies indicating that the use of aggregated data might result into information losses in electricity spot modeling (Maciejowska and Weron, 2015; Raviv et al., 2015). It should be reminded, that in order to derive generalized conclusions, a more thorough comparison between the daily and hourly spot models should be conducted. In addition, this should be performed in way that the estimation procedure is exactly the same for both type of models, so that the possible sources of this pricing difference can be closed out. For instance, in this thesis there were differences in the data filtering, which might slightly damage the hourly data and also the use of moving averages might have implications as well. However, these differences between the estimation procedures are quite in line so to say, and the outcome should not make that big of a difference. And it is a fact, that due to the seasonality of the electricity spot price the hourly and daily data must always to be filtered in a different manner.

In line with the results by Nomikos & Soldatos (2010), the results of this thesis indicate that the model type and its parameters have clear implications on the option price, especially when dealing with mean-reverting models with jumps (or spikes). To conclude, differently defined models generate inconsistent prices, which seem to be most affected by the definition of the jump (or spike) component. It is precisely the jump size and both its volatility and intensity that have a clear effect on whether an option end up ITM or OTM. The most inconsistent with other models was the RS model. This is also quite intuitive, since as its was shown in Table 10 in chapter 7, the RS model generated the most volatile price paths and differed the most from the other models. In addition, the effect of the time-dependent parameters on the implied volatilities was also shown with the RS and HPS RS models, RS model providing more clear implications. As it was shown by Nomikos & Soldatos (2010) as well, the positive jumps tend to increase the implied volatility of ITM call options, whereas negative jumps tend to have the opposite implication.

Finally, it can be questioned whether the obtained implied volatilities are in line with the market prices or not. Some kind of validation can be performed when looking at the implied volatilities of the quoted European options. Observed from the Nasdaq Commodities market, the implied volatility of the Q4-2018 European option with ATM strike (as of 30.6.2019) lies around 37.5% (Nasdaq Commodities 2018). This means, that the ATM Q4-2018 Asian spot option implied volatilities presented above are in line with the quoted European future options in that sense, that the prices are higher, as they should be. Recalling the discussion about the relation of European and Asian spot options from the chapter 2, the price of an Asian spot option should be higher than the corresponding European option, since the underlying of the Asian spot option is the spot price, whereas for the European option it is a future contract, which is also an average of the expected spot price. Since the Asian spot option implied volatilities are not observable from the market, the obtained prices are hard to validate in a sophisticated manner. But as said, the prices are at least higher than for the European option. However, based on this analysis it is hard to say whether the price difference is valid or not.

Even if the implied volatility results seem to be in line with the market prices, based on these results only, it cannot be argued whether these models could be used as such to price path dependent options on electricity spot. Let us now recall, that the aim of this thesis was to study whether the different kind of models provide consistent prices for path dependent options, and not to develop a model that obtains the "right" price. As it was observed, the consistency of the models seems to be depending on the definition of the models and the way they are estimated, on aggregated or non-aggregated data. However, another interesting topic to cover would be to study the consistency of pricing more exotic options, since after all, the Asian spot options regarded here were not that dependent on exact hourly prices nor the time-dependency of the prices and moreover, the jumps.

9. DISCUSSION

The main goal of this thesis was to study how to price path dependent options on hourly electricity spot price. The goal required two concrete actions, developing an hourly spot model and applying it to price path dependent options, both of which can be regarded to be achieved. This thesis also provided contribution to a research area that is almost untouched or at least not extensively covered in the prior research. What this means is that prior research has not yet focused extensively on hourly spot models, time-dependent model parameters and path dependent option pricing and moreover, not covered the aforementioned simultaneously. Additionally, this thesis is one of the rare studies analyzing the option pricing consistency between the daily spot models and the hourly spot models. In addition to achieving the goals, this thesis included many more interesting results, which will be covered in more detail next by going back to the research questions that were initially set. Lastly, this thesis has also its limitations, which will be discussed in the end.

The first research question was defined as follows: *What are suitable hourly electricity spot price models to price path dependent options?* A suitable hourly spot model to price path dependent options consist of a deterministic seasonal component, that obtains the seasonality pattern from yearly to hourly level, and a stochastic component, that obtains the mean-reversion and jumps in a reasonable manner. As it was found, the deterministic component can be defined in many different ways, and in this case a trigonometric function with dummy variables and a linear trend was used. It was also found that the deterministic component provides only quite regular seasonal pattern, which hardly never provides a perfect fit. As it was also observed, the hourly pattern seems to be predictable and clearly deterministic in large part, but still the shape of the pattern is rather dynamic in terms of the highly volatile daily spread between the peak- and off-peak-hours. Thus, it would be interesting to study in more detail, if the seasonality component of the model would include some stochastic factors. A one solution for this is of course the type of "historical profile sampling" (HPS) models that were used in this thesis. This problematics regarding the hourly pattern has not yet been brought up in the prior research, which has mainly been focusing on daily models. However, it is obvious that filtering the data by using deterministic hour-dummy values might damage the data and have some implications on the stochastic parameter estimates. Due to these reasons, it might be reasonable to generate the hourly prices as in the HPS models or by using some kind of an autoregressive function approach, as performed by Burger et al. (2004).

In this thesis, the stochastic component in the models was defined as a sum of two independent stochastic processes, one obtaining the short- and the other the long-term fluctuation around the mean level. As Benth et al. (2012) also found, the multiple-factor models provide promising results. This thesis is in line with those results and in addition, provides contribution to the use of two-factor models in option pricing. Branger et al. (2010) developed a two-factor spot model as well but used that to price real options on spot price. In this thesis, the estimation of the long-term stochastic factor with the moving average of the deseasonalized log-prices is also something that has not been done before. The main argument behind this was the deseasonalization itself, which obviously did not result to that stationary process in short-term. By using moving averages, the short-term process could be smoothed in way that a faster mean-reversion was obtained, and none information was lost in the estimation procedure since the moving average was then used to calibrate the long-term process. In addition, this thesis also shed some light to the use of time-dependent parameters in the regime-switching models, since the usual approach seems to be to assume the stochastic parameters to be constant. However, it still remains for further study to obtain a jump-diffusion process with time-dependent properties. All in all, based on the results of this thesis, it can be argued that the two-factor models provide promising results. In addition, by having this kind of two-factor models, the risk-neutral calibration can be also performed in a reasonable manner, since two different risk factors were also assumed, a short- and a long-term risk factor. However, the short-term risk factor was assumed to be zero and only the long-term risk was calculated. It can be concluded that there are different risk-factors that need to be taken into account in the electricity spot models, but only the long-term risk factor observed from the quoted future contracts can be calculated in a sophisticated manner. The jumps are linked to the short-term risk, and as discussed before, obtaining the jumps in a solid way is the most challenging part of the spot models. In this thesis, jumps were obtained by Poisson distributed normal random variables (jump-diffusion models) or log-normal random variables controlled by regime-switching probabilities (regime-switching models). In addition, since there were two stochastic factors in the models calibrated on hourly spot prices (RS and JD models), the jumps were able to model in the short-term process with faster mean-reversion than in the long-term process, which can be regarded as reasonable practice when dealing with hourly data. Based on the statistical properties and the pricing results, all the models can be said to provide results that are in line with the market prices. It can be concluded, that in order to obtain both the short-lived jumps and the long-term fluctuation around the mean-level, the model need to have at least two different mean-reversion rates. As an additional note, it can be said that a suitable hourly spot model can be either a jump-diffusion, a regime-switching or other type of a model, as long as it is estimated in a reasonable way. The models should be estimated both with the spot and the forward prices, in order to obtain realistic results in terms of the spot price characteristics and option pricing.

The second research question was defined as follows: *How consistently do the different models price path dependent options?* In a conclusion, the models used in this thesis generated inconsistent option prices, which were mainly affected by the definition of the models and their parameters. As found by Nomikos & Soldatos (2010), the mean-reversion, the jump size and the jump intensity have an effect on the implied volatility. Whereas their study was about European spot options, this thesis showed the implications on the Asian spot options. In addition to pricing Asian spot options, the results of this thesis contributed to this filed of research especially regarding the comparison of the daily and hourly spot models. The HPS models that were estimated with daily spot data obtained lower option prices than the RS and JD models that were estimated with hourly spot data. Since the hourly pattern generation in the HPS models do not change the average daily price, the pricing inconsistency roots to the use of aggregated and non-aggregated spot data in HPS models and RS and JD models, respectively. Based on these results, the implied volatility of an Asian spot option is different whether aggregated or non-aggregated data is used. This also implies that the use of aggregated data might result into information losses in the data, which in this case is indeed harmful for option pricing. Regarding the use of aggregated data and information losses, similar results were found in the studies by Maciejowska & Weron (2015) and Raviv et al. (2015). However, the results of this thesis additionally provide implications to option pricing, which can be regarded as one of the most remarkable contributions of this thesis.

In addition to pricing Asian spot options, this thesis analyzed the validity of the results by comparing the prices to the quoted European future options. So, ignoring the pricing inconsistency, when comparing the obtained implied volatility of the Q4-2018 Asian spot option to the corresponding implied volatility of the European future option, all the results were in line. All of the models were able to generate higher price for the Asian spot option. If we now imagine, that none of the models used in this thesis would not have the long-term mean-reverting process with slower mean-reversion and would have only one stochastic factor driven by the fast mean-reversion, the implied volatilities of the Asian spot options would have been probably much lower, since that kind of models would have most likely been only able to generate quite regular price paths driven mainly by the deterministic seasonality component.

With respect to the scope and defined limitations, the results of this thesis can be regarded to be interesting. The prior research has mostly been focusing on daily spot models, studies regarding hourly spot models and their application to pricing path dependent options being scarcer, this thesis can be argued to provide a clear contribution to this field of science. However, this thesis has its limitations and some of the made assumptions can be questioned. First of all, it should be noted that the definitions of the stochastic components, especially regarding the jumps, are not perfect. By using the Lee & Mykland technique, the time-dependency of the jumps was shown, and some implications for the underlying probability distribution for the extreme jumps was provided. Despite these observations, jumps were chosen to model as a normal random variable following a Poisson process with a constant intensity. However, the rarer and more extreme e.g. gamma-distributed jumps should be further studied in jump-diffusion models, as Benth et al. (2012)

analyzed their use in factor and threshold models. In addition, the estimation of the different stochastic factors using moving averages of the deseasonalized data can be questioned, since a clear argument was not provided. However, the obtained results cannot be labelled poor, so the estimation can be regarded to be successful, but to conclude, the estimation of the different stochastic factors could be further studied. Another clear criticism, which is a rather common in this field of research, can be appointed to the constant volatility assumption. It is clear that the volatility of the electricity spot price is not constant, and a spot model including stochastic volatility, at least to some extent, would most likely provide better results. This thesis does not either reveal, whether the results regarding option pricing would be different, if more exotic options would be used. In addition, even if the models including time-dependent parameters can be regarded to be one of the strengths of this thesis, the contribution of the time-dependent parameters for path dependent option pricing remains rather unclear, since only Asian spot options were priced. If the valuated option is just based on some average over a certain period, it does not really matter when the single jumps have occurred, as long as they are included in the simulated sample. This thesis however showed that the time-dependency can be obtained, and the results are in line with the market prices. In addition, the results of this thesis highlighted the implications of the model and its parameters on the implied volatility as well as on the put-call parity.

As some of the potential issues for further study was already pointed out, the following should be highlighted. An interesting result of this thesis was the inconsistent option pricing of daily spot models and hourly spot models, and this should be studied more in-depth by using different models and paying attention to the estimation procedure of the models. For future research, it would be also interesting to study the pricing consistency of different hourly spot models, when more exotic options are under focus. And what comes to the spot models, the models having two or even more independent stochastic factors should be further studied, since the two-factor models used in this thesis showed already promising results, just as it has been found in the prior research as well. Moreover, the dynamics of the different stochastic factors should be also studied. In this thesis, a mean-reverting process was assumed for the long-term stochastic process, but models obtaining e.g. geometric Brownian motion could be studied more in-depth as well. Finally, this thesis was about stochastic models only, but since the electricity spot price is clearly driven by fundamental factors, it could be interesting to apply spot models with both fundamental and stochastic parameters to price path dependent options. As brought up in chapter 3, regime-switching models with the switching probabilities depending on the water levels on reservoirs have been studied. By including fundamental factors to the stochastic models, one could obtain results with distributional properties, and more realistic expected values of the spot prices. Since the option pricing models cannot be calibrated on future contracts only and prices for options on spot are not observable from the market, including fundamental factors in the models could be reasonable.

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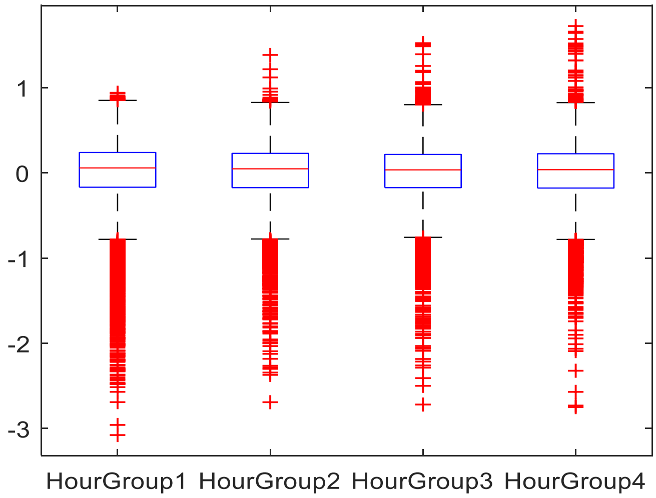
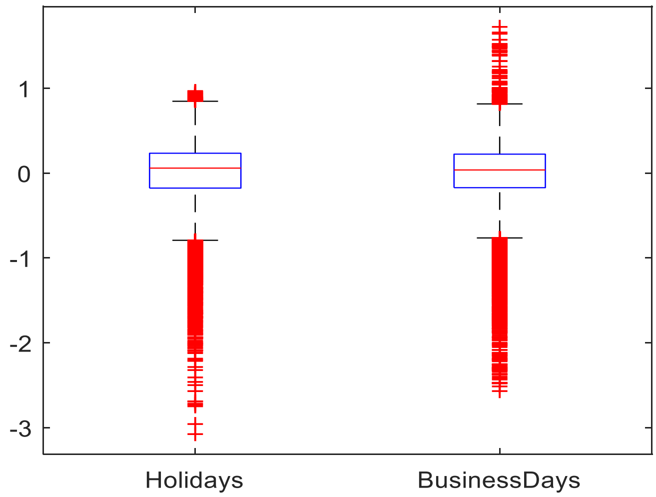
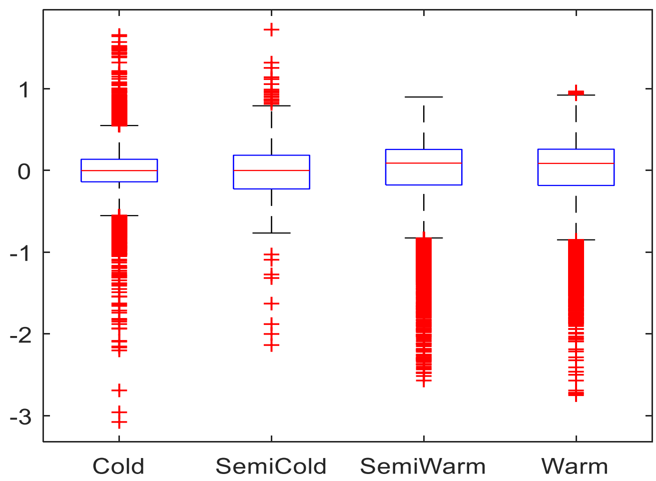
APPENDIX A: BARTLETT'S TEST RESULTS FOR THE VARIANCE EQUALITY



Group	Count	Mean	Std Dev
HourGroup1	19162	0.000	0.394
HourGroup2	8214	0.000	0.367
HourGroup3	24642	0.000	0.335
HourGroup4	13690	0.000	0.351
Pooled	65708	0.000	0.360
Bartlett's statistic	603.36		
Degrees of freedom	3		
p-value	0		

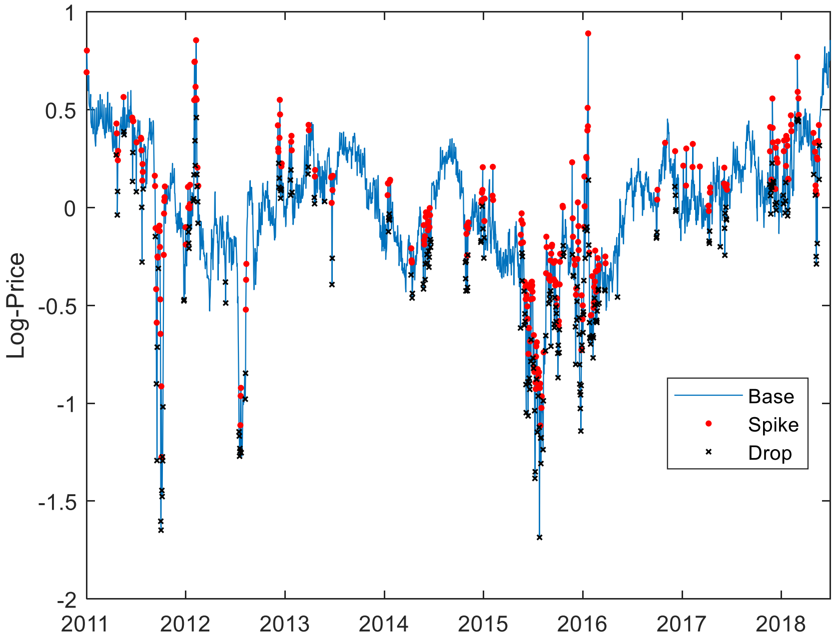


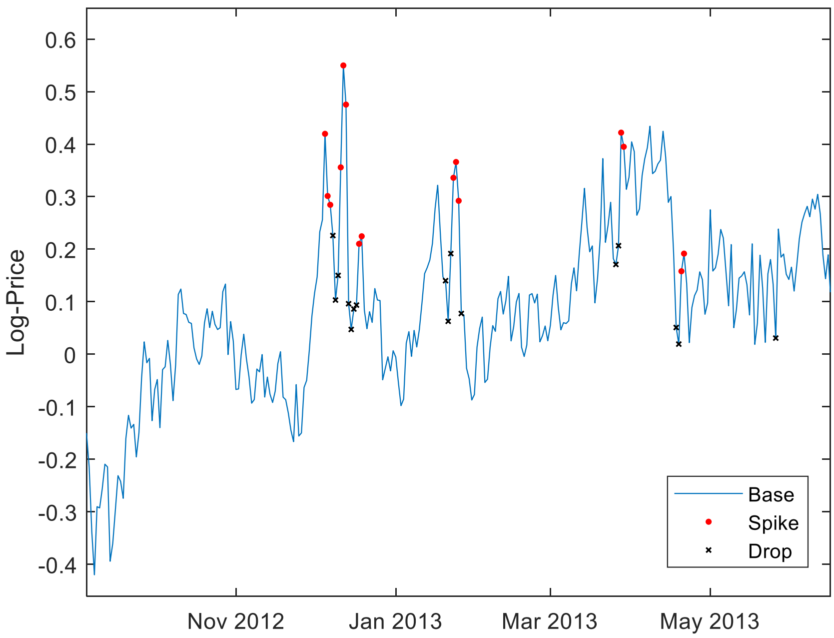
Group	Count	Mean	Std Dev
Holidays	20588	0.001	0.379
BusinessDays	45120	-0.001	0.351
Pooled	65708	0.000	0.360
Bartlett's statistic	169.19		
Degrees of freedom	1		
p-value	0		



Group	Count	Mean	Std Dev
Cold	16584	0.001	0.306
SemiCold	10984	0.001	0.278
SemiWarm	21220	0.003	0.368
Warm	16920	-0.004	0.438
Pooled	65708	0.000	0.360
Bartlett's statistic	3553.33		
Degrees of freedom	3		
p-value	0		

APPENDIX B: IDENTIFIED REGIMES IN THE DAILY SPOT DATA





APPENDIX C: TIME-DEPENDENT TRANSITION MATRICES OF RS MODEL

Base = Base regime, Pos = Spike regime, Neg = Drop regime

Group	Base to Base	Base to Pos	Base to Neg	Pos to Base	Pos to Pos	Pos to Neg	Neg to Base	Neg to Pos	Neg to Neg
'WarmBusinessdayHourblock1'	0.957	0.011	0.032	0.043	0.812	0.145	0.011	0.006	0.983
'WarmBusinessdayHourblock2'	0.998	0.000	0.002	0.146	0.843	0.011	0.059	0.007	0.933
'WarmBusinessdayHourblock3'	0.994	0.003	0.002	0.092	0.902	0.005	0.201	0.019	0.780
'WarmBusinessdayHourblock4'	0.946	0.015	0.039	0.092	0.866	0.042	0.145	0.178	0.677
'WarmHolidayHourblock1'	0.938	0.041	0.021	0.084	0.792	0.124	0.061	0.111	0.828
'WarmHolidayHourblock2'	0.953	0.000	0.047	0.051	0.836	0.113	0.036	0.024	0.939
'WarmHolidayHourblock3'	0.987	0.008	0.004	0.094	0.903	0.003	0.220	0.159	0.622
'WarmHolidayHourblock4'	0.986	0.012	0.003	0.091	0.909	0.000	0.108	0.324	0.568
'SemiWarmBusinessdayHourblock1'	0.968	0.011	0.022	0.040	0.815	0.145	0.056	0.012	0.932
'SemiWarmBusinessdayHourblock2'	0.991	0.004	0.005	0.055	0.904	0.041	0.140	0.018	0.841
'SemiWarmBusinessdayHourblock3'	0.991	0.004	0.005	0.096	0.873	0.030	0.125	0.048	0.827
'SemiWarmBusinessdayHourblock4'	0.964	0.020	0.016	0.118	0.819	0.063	0.134	0.173	0.693
'SemiWarmHolidayHourblock1'	0.964	0.023	0.013	0.053	0.805	0.142	0.118	0.106	0.776
'SemiWarmHolidayHourblock2'	0.989	0.003	0.008	0.033	0.918	0.049	0.053	0.023	0.924
'SemiWarmHolidayHourblock3'	0.991	0.003	0.006	0.233	0.733	0.034	0.189	0.111	0.700
'SemiWarmHolidayHourblock4'	0.988	0.012	0.000	0.000	1.000	0.000	0.217	0.261	0.522
'ColdBusinessdayHourblock1'	0.984	0.000	0.016	0.148	0.630	0.222	0.036	0.000	0.964
'ColdBusinessdayHourblock2'	0.990	0.001	0.009	0.234	0.396	0.370	0.100	0.006	0.894
'ColdBusinessdayHourblock3'	0.983	0.011	0.006	0.114	0.850	0.035	0.233	0.017	0.750
'ColdBusinessdayHourblock4'	0.847	0.066	0.087	0.034	0.959	0.007	0.048	0.201	0.751
'ColdHolidayHourblock1'	0.920	0.076	0.004	0.322	0.610	0.068	0.300	0.043	0.657
'ColdHolidayHourblock2'	0.976	0.000	0.024	0.210	0.776	0.014	0.018	0.000	0.982
'ColdHolidayHourblock3'	0.993	0.004	0.002	0.349	0.628	0.023	0.242	0.076	0.682
'ColdHolidayHourblock4'	0.975	0.022	0.002	0.075	0.900	0.025	0.000	0.400	0.600
'SemiColdBusinessdayHourblock1'	0.991	0.002	0.006	0.149	0.766	0.085	0.133	0.000	0.867
'SemiColdBusinessdayHourblock2'	0.996	0.002	0.001	0.238	0.613	0.150	0.355	0.016	0.629
'SemiColdBusinessdayHourblock3'	0.983	0.012	0.005	0.160	0.773	0.066	0.160	0.026	0.814
'SemiColdBusinessdayHourblock4'	0.899	0.029	0.072	0.100	0.863	0.038	0.025	0.041	0.934
'SemiColdHolidayHourblock1'	0.924	0.074	0.003	0.200	0.767	0.033	0.273	0.000	0.727
'SemiColdHolidayHourblock2'	0.979	0.005	0.016	0.164	0.826	0.009	0.119	0.000	0.881
'SemiColdHolidayHourblock3'	0.997	0.002	0.001	0.571	0.429	0.000	0.286	0.000	0.714
'SemiColdHolidayHourblock4'	0.994	0.006	0.000	0.111	0.889	0.000	1.000	0.000	0.000

APPENDIX D: TIME-DEPENDENT TRANSITION MATRICES OF HPS RS MODEL

Base = Base regime, Pos = Spike regime, Neg = Drop regime

Group	Base to Base	Base to Pos	Base to Neg	Pos to Base	Pos to Pos	Pos to Neg	Neg to Base	Neg to Pos	Neg to Neg
'WarmBusinessday'	0.989	0.011	0.000	0.101	0.667	0.232	0.149	0.468	0.383
'WarmHoliday'	0.916	0.022	0.062	0.091	0.409	0.500	0.107	0.214	0.679
'SemiWarmBusinessday'	0.975	0.011	0.013	0.208	0.583	0.208	0.083	0.389	0.528
'SemiWarmHoliday'	0.974	0.013	0.013	0.182	0.545	0.273	0.048	0.381	0.571
'ColdBusinessday'	0.939	0.037	0.024	0.210	0.617	0.173	0.066	0.377	0.557
'ColdHoliday'	0.950	0.013	0.038	0.182	0.273	0.545	0.077	0.231	0.692
'SemiColdBusinessday'	0.986	0.007	0.007	0.231	0.462	0.308	0.067	0.400	0.533
'SemiColdHoliday'	0.992	0.000	0.008	0.250	0.375	0.375	0.167	0.500	0.333