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# A Note on the Two-Sector Model of Regional Factor Allocation

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## **Abstract**

*The standard neoclassical Two-Sector Model of inter-regional factor allocation says that a positive demand shock in one region starts immigration of both labour and capital into the region, but that the market adjustment eventually ends in re-equalisation of the factor rewards thus reproducing the inter-regional equilibrium. The issue is here examined in a Neo-Keynesian macroeconomic framework. In the present model, the demand shock does not trigger factor movements into the same direction like the Two-Sector Model postulates. Under Perfect Foresight or under Rational Expectations the factors do not move at all. Under Adaptive Expectations the factors are induced to move in the short term. However, the factors move in opposite directions: a positive demand shock causes capital to immigrate, but labour is caused rather to emigrate than immigrate. Whether the factors migrate or not depends on the relative sensitivity of the alternative adjustment mechanism, namely the within-region adaptation of the factor suppliers' expectations.*

Key words: Adaptive Expectations Hypothesis, factor migration

JEL classification: 931

## 1 Introduction

Migration and regional allocation of production factors is a major concern of the neoclassical approach to regional growth. Conventional textbook analyses of this issue are based on two analytical frameworks known as the One-Sector Model and Two-Sector Model of factor allocation (McCann, 2001, pp. 208-221). Both models are constructed in the classical general equilibrium framework and they say that factor migration leads to inter-regional equalisation of factor rewards and to an interior solution of regional factor allocation.

In the One-Sector Model, the regional production functions are assumed identical, while the Two-Sector Model allows for regional differences in production and production technologies. This means that the production possibilities frontier is linear in the One-Sector Model and concave in the Two-Sector Model. In the long run, the production pattern of the Two-Sector Model is postulated to converge to that of the One-Sector Model.

The One-Sector Model rests on the law of variable factor proportions. With different capital/labour ratios between regions, the marginal products of capital and labour will also differ. Supposing that the factors are everywhere rewarded according to their marginal products, the due differences in factor rewards make the factors migrate in opposite directions. Migration continues until both factor rewards and capital/labour ratios become equalised between regions.

In the Two-Sector Model the regions differ in the factor intensity of their output production. The model postulates that, between capital-intensively and labour-intensively producing regions, both factors may occasionally be drawn to flow in the same direction. For example an exogenous rise in commodity prices in one region improves the marginal revenue product of both factors thus generating immigration of both factors. The key insight is that the market process still ends to an interior solution, where the factor rewards are re-equalised across the regions.

The market adjustment in the Two-Sector Model rests on the effect of commodity prices on factor rewards. Referring to normal goods market model with a downward sloping demand curve and an upward sloping supply curve, the adjustment can be divided to two stages. First, a positive demand shock causes the prices to rise, which improves the marginal revenue product of both factors. Both factors start to immigrate. Second, as the immigration of factors consequently increases production and thus supply in the goods market, the market prices start to adjust downwards. Since this occurs reversely in the rest of the economy, the market process will eventually eliminate the original demand shock, and the economy will settle to a stable equilibrium with equal factor rewards everywhere. (McCann, 2001, pp. 214-217.)

This paper reconsiders the neoclassical theory of regional factor allocation in a Neo-Keynesian macroeconomic framework. The Neo-Keynesian Model facilitates a thorough investigation of the simultaneous adjustment in the goods and factor markets and provides further insight especially concerning the message of the Two-Sector Model. The present framework is set up in Section 2. The classical long-term adjustment under Perfect Foresight (or Rational Expectations) and the short-term adjustment under Adaptive Expectations are studied in Section 3. Section 4 concludes.

## **2 The framework**

The framework is an elaboration of the seminal Neo-Keynesian macroeconomic framework (Laurila, 2004). The supply side in the regional goods market is determined by the production function and by the factor market conditions. The production function for one particular region is

$$(1) q = f(L, K),$$

where  $L$  denotes regional labour, measured in terms of labour time, and  $K$  denotes the regional capital stock. The standard assumptions concerning the production function are made, namely constant returns to scale and  $f_1, f_2 > 0, f_{11}, f_{22} < 0, f_{12} = f_{21} > 0$ , where the subscripts refer to first and second partial derivatives of the function (1).

Labour demand in the regional labour market derives from profit maximisation of the regional firm sector. The labour demand function is

$$(2) \quad w = p f_1,$$

written in terms of nominal wages. Function (2) says that the demand for labour equals the marginal revenue product of labour. The labour demand curve has a negative slope in  $L$ - $w$  space. Labour supply, derived from people's optimal time use decisions, reads

$$(3) \quad w = p g(L),$$

where the right-hand-side tells the marginal opportunity cost of labour time. Since  $g' > 0$  by assumption, the labour supply curve is upward sloping in  $L$ - $w$  space.

In the regional capital market, the demand for capital derived from profit maximisation reads

$$(4) \quad p = r/f_2$$

written in terms of prices. Function (4) is manipulated from the regular demand function, which says that the market rent for capital equals its marginal revenue product. By (4), the capital demand

curve has a positive slope in the  $K$ - $p$  space. Assuming that the supply of the capital inputs is positively correlated to the real factor rewards, the supply function can be written as

$$(5) p = r/\varphi(K),$$

where  $\varphi(K)$  denotes the opportunity cost of capital and  $\varphi' > 0$  by assumption. Thus, the capital supply curve is downward sloping in  $K$ - $p$  space.

The functions (1) – (5) determine the aggregate supply (the AS curve) in the regional goods market. The AS curve is vertical unless the price level is anticipated asymmetrically by the demanders and the suppliers of the two factors. Possible differences in the anticipations are treated below in terms of graphics and a formal description of different expectation hypotheses is omitted for simplicity.

The aggregate demand conditions of the regional economy are determined by the IS-LM model. The equilibrium of the real side of the economy, the IS curve, is given by

$$(6) q = c(q) + i(r),$$

where  $c$  denotes consumption and  $i$  denotes investment. Since  $c' > 0$  and  $i' < 0$  by assumption, the IS curve is declining in  $q$ - $r$  space. The LM curve is given by the money market equation

$$(7) m/p = l(q, r),$$

where the supply of real money equals the demand of liquid money, reasoned by transaction use depending on production  $q$  and inter-temporal use depending on the interest rate  $r$ . It is assumed that  $l_1 > 0$  and  $l_2 < 0$  implying that the LM curve slopes upwards in  $q$ - $r$  space.

Equations (6) and (7) give the aggregate demand (the AD curve) in the goods market. The AD curve has a negative slope in  $q$ - $p$  space. Exogenous demand shocks shift the IS curve and thus the AD curve outwards if the shock is positive and vice versa.

### 3 Price adjustment and factor movements in the neo-Keynesian model

#### 3.1 The classical long-term effect

Reconsider the price adjustment argument of the Two-Sector Model in the present framework. For the sake of simplicity, exclude the trade of goods and concentrate only on factor mobility. Second, assume that the region considered is of atomistic size in the whole economy, meaning that the factor rewards outside the region are taken as given. Third, assume that the mobile factors are not only rewarded in their present location, but that the rewards are also exchanged to consumption in the same location. Fourth, apply the Perfect Foresight Hypothesis (or the Rational Expectations Hypothesis, which is equivalent in its effects in terms of graphics) to describe the factor market responses to the changes in commodity prices.

Figure 1 presents the analysis of the market adjustment. The figure consists of two sets of quadrants. The first set in Panel (i) includes the labour market, the production function, the goods market and the real wage, listed clockwise from northeast. The second set in Panel (ii) includes the capital market, the production function, the IS-LM framework, and the real interest rate, listed clockwise from northeast.

(Figure 1 here)

In Figure 1, the region considered is initially assumed to be in the inter-regional equilibrium given by the market price set  $(p_0, w_0, r_0)$ . Thus, the real wage is  $\omega^* = (w/p)^* = f_1^*$  and the real interest rate is  $\rho = (r/p)^* = f_2^*$  everywhere. The regional employment is  $L_0$  and the capital stock is  $K_0$  implying that production is  $q_0$ . Suppose that there emerges an exogenous demand shock, shown by an outward shift of the IS curve from  $IS_0$  to  $IS_1$  in Panel (ii) and the consequent shift of the AD curve from  $AD_0$  to  $AD_1$  in Panel (i) in Figure 1.

As is argued by the Two-Sector Model, the increase in local demand causes the prices to rise from  $p_0$  to  $p_1$ . In the classic regime of the model, however, this not only causes labour demand to increase from  $D^L_0$  to  $D^L_1$  but makes also labour supply shift back from  $S^L_0$  to  $S^L_1$  in Panel (i) of Figure 1. Nominal wages rise from  $w_0$  to  $w_1$ , the labour market equilibrium shifts vertically from  $e_0$  to  $e_1$ . Thus, the real wage, employment and production remain unchanged at  $\omega^*$ ,  $L_0$  and  $q_0$ , respectively.

In Panel (ii) of Figure 1, the shift of the IS curve from  $IS_0$  to  $IS_1$  is accompanied by an inward shift of the LM curve from  $LM_0$  to  $LM_1$ , which is due to the rise in the price level from  $p_0$  to  $p_1$ . The nominal interest rate rises from  $r_0$  to  $r_1$  so that the real interest rate remains unchanged at  $\rho^*$ . Therefore, the demand for capital shifts backwards from  $D^K_0$  to  $D^K_1$ , and the supply of capital shifts outwards from  $S^K_0$  to  $S^K_1$ . The capital market equilibrium shifts vertically from  $\varepsilon_0$  to  $\varepsilon_1$ . Capital use and production thus remain unchanged at  $K_0$  and  $q_0$ , respectively.

The conclusion from the classical case is that the demand shock has no real effects in the regional economy in the long term. Employment and production as well as real wages and real interest rates are not affected. Therefore, regional demand shocks also have no effects on factor migration. This result seems to contradict the Two-Sector Model argument, which postulates that factor migration should be induced. Whether there is a contradiction or not is a matter of opinion: what actually happens in both models is that factor prices are re-equalised in the long term.



### 3.2 The short-term adjustment path

In the classical regime of the model, with Perfect Foresight or Rational Expectations, exogenous demand shocks do not trigger migration. However, the framework is also apt to explore if such effects arise in the shorter term. Consider the short-term adjustment to the classical long-term equilibrium of the Neo-Keynesian Model inspired by the Adaptive Expectations Hypothesis. In Figure 2, this is illustrated by the assumption that, in the short term, the demand and supply of the two factors respond asymmetrically to the exogenous price shock. In particular, the firm sector is assumed to be perfectly foresighted, but the household sector is assumed to lack such rationality.

(Figure 2 here)

Start again from the positive demand shock implying that the IS curve shifts from  $IS_0$  to  $IS_1$  in Panel (i) and the AD curve shifts from  $AD_0$  to  $AD_1$  in Panel (ii) of Figure 2. In Panel (i), the firms have perfect foresight and anticipate that the price level rises according to the long-term aggregate supply curve  $AS^{LT}$  from  $p_0$  to  $p_1$ . Thus, the demand for labour shifts from  $D^L_0$  to  $D^L_1$ . Likewise, in Panel (ii), since the perfectly foresighted firms anticipate that the interest rate rises from  $r_0$  to  $r_1$  according to the long-term LM curve  $LM^{LT}$ , the demand for capital shifts backwards from  $D^K_0$  to  $D^K_1$ .

In the very short term, since people are assumed to lack perfect foresight as the suppliers of the factor inputs, the price effects are supposed to surprise them. Therefore, the supply conditions remain unchanged in the factor markets. The momentary equilibrium in the labour market shifts from  $e_0$  to  $e'$  along  $S^L_0$  in Panel (i) and that in the capital market shifts from  $\varepsilon_0$  to  $\varepsilon'$  in Panel (ii) of Figure 2. Momentarily, employment increases from  $L_0$  to  $L'$  and capital use decreases from  $K_0$  to  $K'$ . On impact, the production function shifts inwards in Panel (i) and outwards in Panel (ii). The momentary equilibrium in the labour market in Panel (i) implies that the price level is  $p'$ , the

nominal wage is  $w'$  and the real wage is  $\omega'$ , which is lower than  $\omega^*$ . The respective equilibrium in the capital market in Panel (ii) means that the nominal interest rate is  $r'$  and that the real interest rate is  $\rho'$ , which is higher than  $\rho^*$ . Production has increased from  $q_0$  to  $q'$  in both Panels.

The Adaptive Expectations Hypothesis says that, once the people face the induced changes in market prices, they begin to correct their market actions towards the long-term equilibrium. In the labour market this means that people start to respond to the decreased real wage by reducing their labour supply. The momentary equilibrium  $e'$  thus starts to move along  $D^L_I$  towards  $e_I$  in Panel (i) of Figure 2. Likewise, in the capital market, people respond to higher real interest rates by increasing their supply of capital so that the momentary equilibrium moves from  $\varepsilon'$  towards  $\varepsilon_I$  along  $D^K_I$  in Panel (ii). The momentary changes in employment and capital use also remove towards their initial stages, and so do the graphs of the production functions in the Panels (i) and (ii). The long term classical equilibrium described in the above section is thus eventually reached after the adjustment path has been gone through.

The possibility of free factor mobility, however, offers an alternative pattern of adjustment. Return to the momentary equilibrium, where people for the first time become aware of the price implications of the demand shock. In the labour market, at point  $e'$  in Panel (i) of Figure 2, people see that the local real wage  $\omega'$  has fallen below  $\omega^*$ . Since the real wage in the rest of the economy remains unaffected at  $\omega^*$  by assumption, and supposing that the people are perfectly aware of that, there clearly arises a motive for emigration. Observing at  $p_I$ , which is the price level that makes local labour demand and labour supply commensurable, the induced amount of emigration is  $L_I - L_0$ , measured in terms of labour time units. Recall that the Two-Sector Model postulates that a positive demand shock should induce immigration, not emigration.

In the capital market, in Panel (ii) of Figure 2, the short-term equilibrium at  $\varepsilon'$  reveals that the local real interest rate has improved from that prevailing in the rest of the economy,  $\rho' > \rho^*$ . Taken that capital is perfectly mobile, the difference clearly attracts capital to immigrate from the rest of the economy. The consequent inflow of capital amounts to  $K_0 - K_1$ , observed at the relevant nominal interest rate  $r_1$ .

The induced emigration of labour  $L_1 - L_0$  and the induced inflow of capital  $K_0 - K_1$  immediately reproduce the long-term classical equilibrium, where employment is  $L_0$ , capital use is  $K_0$  and production is  $q_0$ . The real wages and real interest rates are again equalised everywhere to  $\omega^*$  and  $\rho^*$ , respectively.

The analysis suggests that there are two alternative adjustment mechanisms in operation: a within-region mechanism, which concerns the gradual adaptation of the regional factor suppliers to the actual price effects, and an inter-regional mechanism, which concerns the factor migration induced by differences in the factor rewards. Both mechanisms may take place at the same time thus replacing one another. The final shares of within-region and inter-regional adjustment and thus the proper amounts of factor migration depend on the relative sensitivities of the two mechanisms. This remains an empirical issue. In any case, the real effects are those of the classical long-term equilibrium, namely that they do not exist.

## 4 Conclusions

The analysis of factor migration in the Neo-Keynesian Model makes two main contributions. First, the analysis shows that factor mobility complements the Adaptive Expectations Hypothesis by introducing an alternative migration explanation to the normal error-correction argument in describing the adjustment path towards the long-term equilibrium. In this sense a fresh

interpretation or even a rehabilitation of the Adaptive Expectations Hypothesis is provided. It might be argued that the presence of free mobility in effect explains the asymmetry of the demand and supply responses in labour and capital markets: the factor suppliers need not to be concerned about the price effects because they expect that factor mobility shall clear the market in any case.

Second, concerning the Two-Sector Model of regional factor allocation, the analysis yields both supporting and controversial results. The results are supporting in the sense that the adjustment to an exogenous demand shock ends to equalisation of the factor rewards in the longer term. The controversial aspect arises from the finding that, starting from an initial inter-regional equilibrium, an exogenous demand shock does not cause the two factors to migrate into the same direction as the Two-Sector Model indicates. The factors rather start to move in opposite directions thus substituting each other in the adjustment mechanism. In particular, a positive demand shock motivates emigration rather than immigration as is postulated by the Two-Sector Model.

The differences of the results in the present model to the results of the Two-Sector Model are due to the fact that the short-term effects arise in this model only because of the asymmetric anticipation of the price effects. Due to the asymmetry, regional real wages are anticipated to fall and real interest rates are anticipated to rise in the short term. This induces emigration of the mobile labour factor and immigration of the mobile capital factor. Relaxing the rather strong assumption that the capital rewards are used within the region, which is to say that capital movements are determined by the real interest rate, does not change the conclusion concerning the direction capital movements. The result would be the same if the nominal interest rate should determine capital migration.

The main message of the neoclassical theory of regional factor allocation is appealing: any exogenously originated inter-regional differences in factor rewards with their consequent effects on factor movements are cleared by the market mechanism, and the differences are eventually

equalised between regions. The market adjustment always ends to an interior solution where no benefits can be derived from relocation, and where social welfare is maximised. The additional insight yielded by the Neo-Keynesian Model does not change this basic message, but it certainly points out that the path of adjustment must be critically assessed.

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Figure 1: Long-term effects of a demand shock

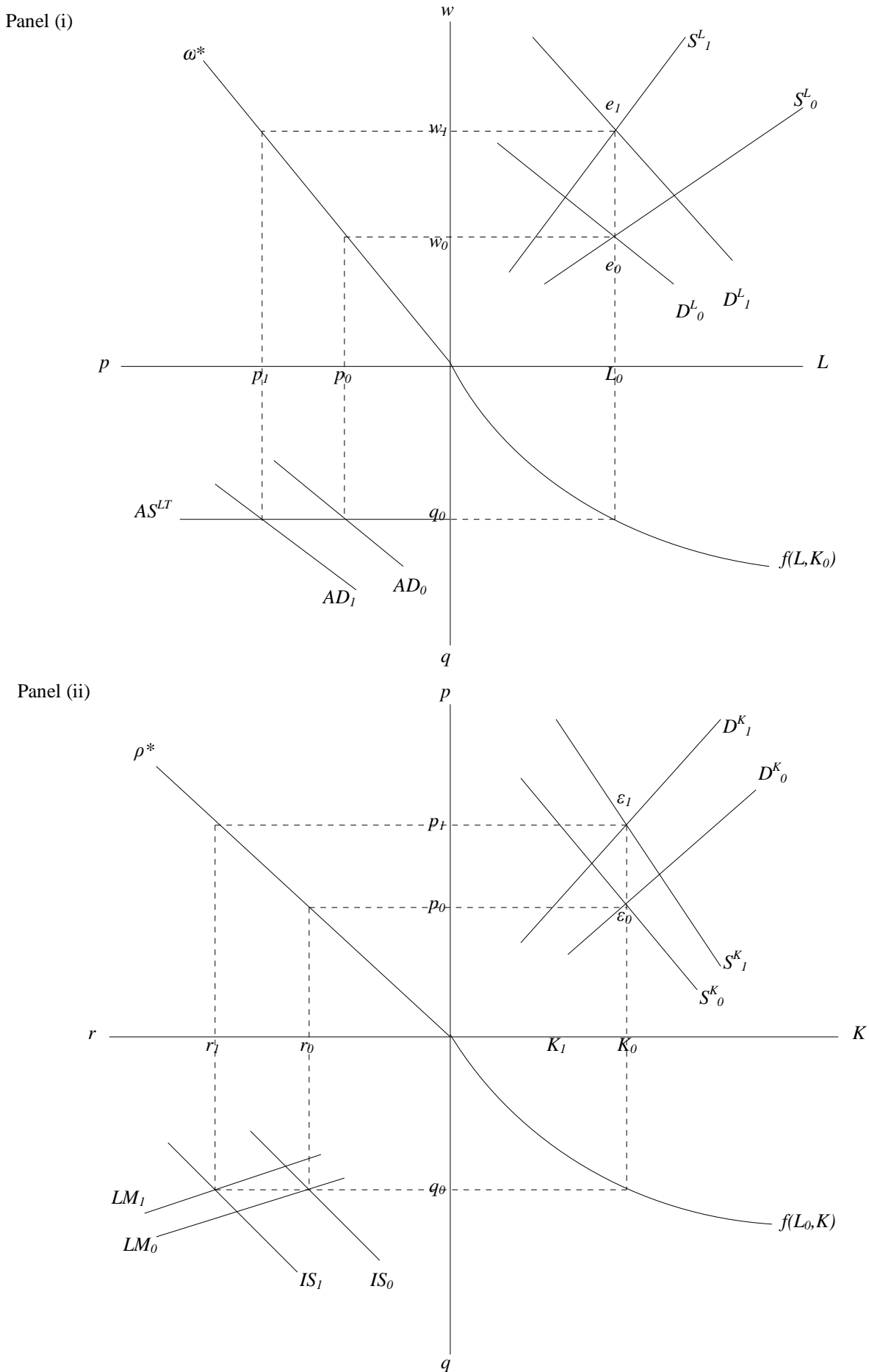
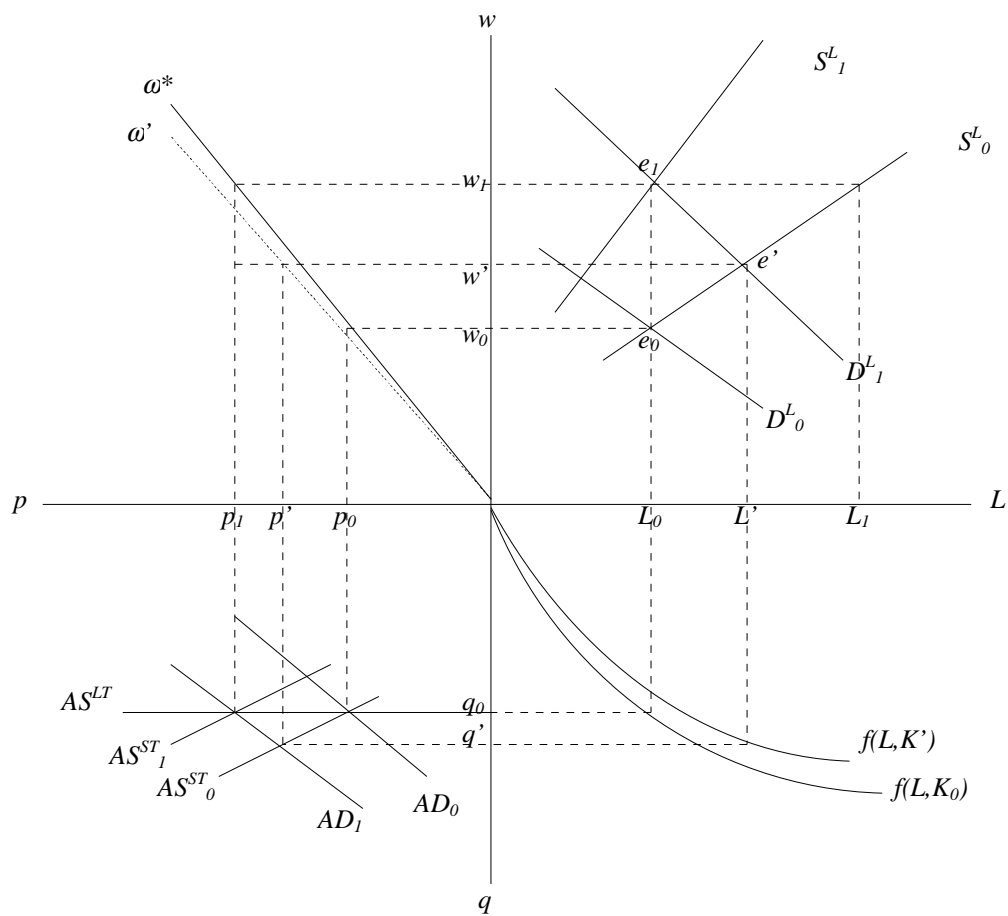


Figure 2: Short-term effects of a demand shock on factor movements

Panel (i)



Panel (ii)

