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CREDIT VALUATION ADJUSTMENT WRONG-WAY RISK MODELLING OF FOREIGN EXCHANGE SENSITIVE DERIVATIVES

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ABSTRACT

Tommi Salonen: Credit valuation adjustment wrong-way risk modelling of foreign exchange sensitive derivatives
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In derivatives pricing credit valuation adjustment (CVA) is used to quantify the counterparty credit risk. Wrong-way risk refers to possibility that the counterparty's insolvency probability is increasing at the same time as the value of the contract increases. The methods used to model CVA often assume that probability of default and exposure are independent of each other, and thus the wrong-way risk is not considered. However, market crises, for example the eurozone debt crisis of 2010, have shown that the assumption of market-credit independence is often violated. The observation is supported by empirical studies, both on historical data and market-implied data. The market is pricing wrong-way risk, which can be seen, for example, in the spreads of credit default swaps quoted in different currencies referring to same entity. The regulator also recognizes the existence of wrong-way risk and requires measures to monitor and manage it.

In this thesis CVA wrong-way risk modelling is studied in the case of a cross-currency basis swap and a systemically significant counterparty. By nature, wrong-way risk is portfolio-specific and difficult to model, and as the topic of WWR is relatively new, there are no established practices. The goals of this thesis are:

(i) Identify methods of modelling credit value adjustment wrong-way risk of derivatives contract having foreign exchange risk factor.
(ii) Model CVA WWR in realistic market setting with an example contract where bilateral collateral is posted and compare results with simple CVA model where WWR is ignored.

Two methods were selected for wrong-way risk modeling, one of which is based on a constant correlation of the error components of the models used for stochastic modeling of the exchange rate and default probability, and the other on the relative, instant jump of the exchange rate at the time of default of the counterparty. A joint model of these methods is derived, which is examined in the empirical part of the work from the point of view of a cross-currency basis swap. A sensitivity analysis is performed with key parameters of the methods.

Based on the results of the empirical part, the constant correlation method is not producing a significant wrong-way risk effect in the case of the modelled contract, when collateral is used. Instead, the assumed relative jump in the exchange rate at the time of default of the counterparty causes a significant relative change in the value of the CVA compared to the model without wrong-way risk. The effect is particularly large in the case of a collateralized contract, as the lagged collateral is not able to reduce the jump-at-default effect. The results are in line with previous literature: linear correlation alone does not cause a significant wrong-way risk to the collateralized portfolio but jump-at-default can be a significant source of additional risk. In managerial decisions the possibility of jump-at-default should not be ignored.

Keywords: CVA, WWR, jump-at-default, FX

TIIVISTELMÄ

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Mahdollisuutta vastapuolen maksukyvyttömyystodennäköisyyden kasvamiseen yhtäaikaisesti sopimuksen arvon kasvaessa kutsutaan luottoarvokorjauksen (engl. credit valuation adjustment, CVA) vääräsuuntaisuus (engl. wrong-way) riskiksi. Luottoarvokorjauksen mallintamiseen käytettävät menetelmän usein olettavat maksukyvyn ja altistuman (engl. exposure) olevan toisistaan riippumattomia, eli wrong-way riskiä ei huomioida. Markkinakriisit, esimerkiksi euroalueen velkakriisi 2010, osoittavat riippumattomuusoletuksen olevan väärä. Väitettä tukevat empiiriset tutkimukset paitsi historiadatasta, myös markkinadatasta. Markkinat siis hinnoittelevat wrong-way riskiä, mikä ilmenee esimerkiksi eri valuutoissa noteerattujen, samaan kohteeseen viittaavien luottotappioriskien vaihtosopimusten hintojen spreadeissa. Myös sääntelijä tunnistaa wrong-way riskin olemassaolon ja edellyttää toimenpiteitä sen monitoroimiseen sekä hallintaan.

Edellä mainitut syyt motivoivat mittaamaan wrong-way riskiä, jonka mallintamista ja suuruutta tutkitaan tässä työssä valuuttakurssijohdannaisen ja systeemisesti merkittävien vastapuolien tapauksessa. Wrong-way riski ilmenee eri syistä eri sopimustyypeissä ja sen tunnistaminen sekä mallintaminen on monimutkaista. Aiheen ollessa suhteellisen tuore, vakiintuneita menetelmiä ei ole ja menetelmäkenttä on hajanainen. Tämän työn tavoitteina onkin:

(i) Tunnistaa kirjallisuudesta menetelmiä CVA:n wrong-way riskin mallintamiseen valuuttajohdannaissopimuksessa.
(ii) Mallintaa CVA wrong-way riskiä esimerkkisopimuksella, jossa on käytössä kahdensuuntainen vakuus ja verrata tuloksia CVA-laskentaan ilman wrong-way riskiä.

Wrong-way riskin mallinnukseen valikoitui kaksi menetelmää, joista toinen perustuu valuuttakurssin ja konkurssitodennäköisyyden stokastiseen mallintamiseen käytettävien mallien virhekomponenttien vakiomääräiseen korrelaation ja toinen valuuttakurssin suhteelliseen, välittömään hyppyyn vastapuolen konkurssihetkellä. Työssä käytettävä malli johdetaan näiden menetelmien yhteismalliksi, jota tarkastellaan empiirisesti *cross-currency basis swap* -sopimustyypin näkökulmasta. Tuloksille suoritetaan herkkyystarkastelu mallien keskeisten parametrien suhteen.

Empiirisen osan tulosten perusteella vakiomääräinen korrelaatio ei aiheuta merkittävää wrong-way riskiä mallinnetun sopimuksen tapauksessa, kun sopimus on kollateralisoitu. Sen sijaan oletettu suhteellinen hyppy valuuttakurssissa vastapuolen konkurssihetkellä aiheuttaa merkittävän suhteellisen muutoksen CVA:n arvossa verrattuna malliin ilman wrong-way riskiä. Vaikutus on erityisen suuri kollateralisoidun sopimuksen tapauksessa, sillä edellisen päivän altistuman perusteella vaihdettu kollateraali ei vähennä konkurssihetkellä tapahtuvan hypyn vaikutusta. Tulokset ovat linjassa aikaisemmassa kirjallisuudessa saatujen tulosten kanssa, joiden perusteella pelkkä lineaarinen korrelaatio ei aiheuta kollateralisoituun portfolioon merkittävää wrong-way riskiä, mutta konkurssihetkellä markkinariskifaktorissa tapahtuva hyppy voi olla merkittävä lisäriskin lähde, jonka mahdollisuutta ei tulisi jättää huomioimatta.

Avainsanat: CVA, WWR, jump-at-default, FX

ALKUSANAT

Työ tehty on

Esteri Salonen, 1985

Voin nyt samaistua edeltävään ajatelmaan kiitos nykyisen työnantajani ja opinahjoni. Erityisesti haluan kiittää Jaakko Juntusta työn aiheen ideasta ja ohjaajaani Ville Veinoa erittäin arvokkaista kommenteista työn luonnosversioista sekä jatkuvasta tuesta työn kirjoittamisen aikana. Lisäksi haluan kiittää Juho Kanniaista työn ohjaamisesta ja Henri Hansenia sen tarkastamisesta.

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Tampereella, 18. syyskuuta 2023

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GLOSSARY

BIS	the Bank for International Settlements
BS	the Black-Scholes (model)
CCBS	cross-currency basis swap
CCP	central counterparty
CCS	cross-currency swap
CDS	credit default swap
CIP	covered interest parity
CIR	the Cox-Ingersoll-Ross (process)
CSA	credit support annex
CVA	credit valuation adjustment
OTC	over-the-counter
DVA	debt value adjustment
EAD	exposure at default
ECB	the European Central Bank
EPE	expected positive exposure
ETD	early termination date
EUR	euro
FX	foreign exchange
G-SIB	global systemically important bank
GBM	geometric Brownian motion
IBOR	interbank offered rate
IM	intial margin
ISDA	the International Swap and Derivatives Association
LGD	loss given default
LIBOR	the London Interbank Offered Rate
MPoR	margin period of risk
MtM	mark-to-market

OU	the Ornstein-Uhlenbeck (process)
PD	probability of default
PED	potential event of default
RFR	overnight risk-free rate
RWR	right-way risk
SIFI	systemically important financial institution
SOFR	the Secured Overnight Financing Rate
USD	United States dollar
VaR	value-at-risk
VM	variation margin
WWR	wrong-way risk
€STR	the Euro short-term rate

1. INTRODUCTION

In financial risk management field counterparty credit risk has been one of the hot topics for years. Failures of large derivatives dealers during the financial crisis, namely Lehman Brothers in 2008, increased attention to derivatives counterparty risk. Derivatives counterparty risk is characterized by introducing a component of market risk in addition to credit risk (Glasserman and Yang 2018). This feature makes the derivatives counterparty risk difficult to measure, since uncertainties both in market variables and credit must be considered. In derivatives pricing *credit valuation adjustment* (CVA) is commonly used tool for quantifying this risk (Glasserman and Yang 2018). According to the Bank for International Settlements (BIS) (2011a) during the financial crisis only one-third of counterparty credit risk related losses were due to actual defaults and rest was attributed to mark-to-market (MtM) valuation changes due to CVA losses.

Over-the-counter (OTC) derivatives are commonly used instruments by banks to transfer risks. OTC derivatives have huge global market value, the gross value estimated at the end of 2022 to be 2.7 trillion United States dollars (USD) (Basel Committee on Banking Supervision 2023). OTC derivatives are traded between two counterparties, without a central clearing house which exposes trades to apparent counterparty risk. According to Atkeson et al. (2015) banks participate OTC market to hedge their underlying risk exposures. The authors also list second incentive for banks to participate OTC markets: trading profits gains from intermediation services, which are possible due to price dispersion in the market. They note that some banks act endogenously as dealers, being large enough to do so, and others as customers.

This thesis is done from a viewpoint of a European bank, which is using OTC derivatives only for hedging purposes and thus acting as a customer in the market. Some counterparties of the bank's derivatives transactions are assumed to be dealers, acting as intermediates in the market with high systemic importance. Systemically important financial institutions (SIFIs) are not only large but have also high interconnectedness and many cross-border activities (Castro and Ferrari 2014). Need for measuring and assessing systemic importance of financial institutions became clear to regulators after the financial crisis, and since 2011 the Financial Stability Board has published a list of global systemically important banks (G-SIBs) using the methodology established by the Basel Committee on Banking Supervision (Bongini, Nieri, and Pelagatti 2015). By definition, a failure of some of these institutions could potentially have adverse consequences for global economy (Basel Committee on Banking Supervision 2018).

Since a failure of G-SIB might affect global economy, there is a possibility that default of this kind of counterparty might affect market parameters. For example, a failure of a systemically important eurozone bank could affect foreign exchange (FX) rate of euro (EUR) against USD. This in turn would decrease or increase value of a derivative contract, where the underlying is given exchange rate. If the counterparty of this contract is the bank which failed and it defaults the contract, it implies that there's a dependency structure between value of the contract and default.

In simple CVA calculation settings it is usually assumed that an exposure to counterparty via a portfolio of contracts with it and the counterparty's probability of default (PD) are independent. However, in the case described above it would no longer be reasonable assumption, if there is correlation or even a causal relation between default and exchange rate. The lack of independence can pose wrong-way risk (WWR) if exposure and PD increase at the same time, or right-way risk (RWR) if the relationship is negative. According to Ruiz (2015, p. 169) correct modelling of RWR and WWR is essential for financial institution since without them the institution cannot understand what is the true amount of carried risk, and the pricing of derivatives is erroneous, when CVA is used to adjust the prices for counterparty risk. For practitioner an important question is how high is the potential error made by ignoring the dependency structure. To answer this question, a fitting modelling method for the dependency structure must be chosen and results of it must be compared with the model where the dependency structure is ignored. The given analysis in the context of FX risk factor is the main practical contribution of this thesis.

The field of CVA modelling and especially the field of WWR is highly dispersed and there is no standard way of modelling and measuring CVA with WWR. By nature, WWR manifests differently in contracts having different underlyings so in WWR measurement both counterparty and contract type characteristics must be taken into account. In this thesis the focus is on measuring CVA wrong-way risk of derivatives deals with foreign exchange (FX) exposure where the counterparty is in domestic currency union (eurozone) and has high systemic importance. The goals of the thesis are:

(i) Identify methods of modeling credit value adjustment wrong-way risk of derivatives contract having FX risk factor.

(ii) Model CVA WWR in realistic market setting with an example contract where bilateral collateral is posted and compare results with simple CVA model where WWR is ignored.

The choices to focus on eurozone G-SIBs and OTC contracts with FX underlyings stem mainly from the assumed viewpoint the CVA is measured from. On the other hand, WWR of FX contracts made specifically with this kind of counterparties are not yet discussed in the literature. Thus, there is also an academic motivation in addition to practical one behind the objectives of the thesis.

This thesis is organized to theoretical and empirical part. The theoretical part aims to answer a research question, which is derived from the first goal described above. The question is: *How CVA WWR should be modelled in OTC derivatives contracts made with eurozone G-SIB, when the underlying risk factor is USD/EUR FX rate and both parties post collateral?* To answer this question, in the second chapter details of CVA are discussed. In the third chapter characteristics which drive the model choice are described, a brief review of WWR models is conducted and the choice of modelling approach is made. The fourth chapter concludes the theoretical part of the work by deriving the model(s) of choice under relevant assumptions by using literature.

The empirical part of the thesis is motivated by a research question derived from the second goal: *How significant is the WWR effect in the example contract based on the selected model(s) compared to a CVA model without WWR and how it is affected by modelling assumptions?* The example contract is a plain vanilla cross-currency basis swap. In the fifth chapter the selected model is calibrated with real-world data and in the sixth chapter it is used with the example contract to calculate illustrative CVA WWR values with two different modelling approaches. The values are compared with CVA values of a model without WWR, to understand better how large the effect of WWR in the given case is. Connection with the real-world data comes from calibration of the formulated model(s), which is done with market-implied values of a single day. Historical data is used only for comparison purposes, not for the actual calibration procedure.

The empirical results of simulations are reported in the sixth chapter in table and in graphical form. The idea of the simulation part is to answer the second research question by empirical means: using computer simulation of stochastic processes, observing results of the simulations and inducing an answer to the question. Since the calibration is done with data of single day, a sensitivity analysis of results is reported in the sixth chapter, to test the robustness of the results against market parameters. Finally in the seventh chapter the thesis is concluded with a discussion about managerial implications and some ideas for future research are given.

2. CREDIT VALUATION ADJUSTMENT WRONG-WAY RISK

In this chapter general ideas around derivatives credit loss are discussed. First mathematical foundation and intuition of credit valuation adjustment is built and in following sections each basic component of credit loss are analyzed further in the context of this thesis. Finally the topic of wrong-way risk introduced.

2.1 Basic components of credit valuation adjustment

We are considering a case of a bank having a portfolio of derivative contracts with counterparty C. The portfolio's value at time t can be seen as a payoff of *defaultable claim*[1] $\Pi(t)$ and we fix the *portfolio's time horizon* $T \in \mathbb{R}^+$. The *mark-to-market value of the portfolio* at time t is $V(t)$ and exposure to C due to Π at any future time t is given by $V(t)^+ = \max\{V(t), 0\}$ (Pykhtin and Sokol 2013). We will denote the loss which bank suffers if C defaults at time t by L_t, and the fraction of the exposure which bank is able to recover after default at time t, the recovery rate, by R_t. Now, if we knew for sure that the counterparty would default at time t, we could calculate the *loss* in this case as

$$L_t = (1 - R_t)V(t)^+$$

by following Zhu's and Pykhtin's (2007) loss formulation. However, the *default time* of C is unknown and thus we introduce the random variable τ to model it (Brigo and Vrins 2018).

We fix a filtered probability space $(\Omega, \mathcal{G}, \mathcal{G}_t = (\mathcal{G})_{0 \leq t \leq T}, \mathbb{Q})$ where all information, including τ, will be defined: A *generic outcome* ω of a random experiment in the *set of all possible outcomes* Ω is a piece of information we are considering, *an event*, if it belongs to the σ-field $\mathcal{G}$. All the information available up to time t is represented by σ-field $\mathcal{G}_t$ and the family σ-fields satisfying also $t \geq 0$ is called *filtration*. (Brigo and Masetti 2005, p. 4)

In particular we are interested if the default of C occurs before the maturity T of the portfolio Π. For this purpose we define the indicator function $\mathbf{1}_{\{\cdot\}}$, which takes value in

[1] A defaultable claim is an asset, or a combination of assets, of which payoff is paid by counterparty which has a risk of default.

case the argument is true and value zero otherwise (Zhu and Pykhtin 2007). Now the *value of the loss at random default time* τ is

$$L_\tau = (1 - R_\tau)\,\mathbf{1}_{\{\tau \leq T\}}\,V(\tau)^+.$$

Since we are interested in the value of the potential loss today, and not at random time τ in the future, we must calculate the *discounted loss* L^*. The *present value of the loss* is given by

$$L^*_\tau = (1 - R_\tau)\,\mathbf{1}_{\{\tau \leq T\}}\,D(0,\tau)V(\tau)^+,$$

where $D(t_1, t_2)$ is a chosen *discount factor* from time t_2 to time t_1. However, now default time τ and also the processes underlying the exposure $V(\cdot)^+$ are stochastic so to arrive at the formula of the CVA the expected value of L^*_τ must be considered. For this purpose we set the expectation operator considering all available information up to time t as $\mathbb{E}_t[\cdot] := \mathbb{E}[\cdot|\mathcal{G}_t]$ following the notation of Brigo and Masetti (2005, p. 5), because we assume that all information we are considering is $\mathcal{G}_t$-measurable.

CVA tells how much value of the default free OTC portfolio should be adjusted in order to take the counterparty's default risk into account (Antonelli, Ramponi, and Scarlatti 2021). This definition is further clarified in the section 2.1.2. From a perspective of *risk-neutral pricing framework*, CVA can be seen as a price of hedging counterparty credit risk (Zhu and Pykhtin 2007). In risk-neutral pricing *martingale* is an essential concept. A martingale is a random process Y that at any time $t_1 < t_2$ satisfies a condition

$$\mathbb{E}\left[Y_{t_2}|\mathcal{F}_{t_1}\right] = Y_{t_1},$$

which means that given all the available current information, the expected value of a martingale process is the current value of the process (Joshi 2003, p. 155). The idea of the risk-neutral pricing is that with a change of probability measure from the physical measure[2] $\mathbb{P}$ to the risk-neutral measure $\mathbb{Q}$ we do not need knowledge of the expected rate of return under $\mathbb{P}$: The valuation can be done by taking the expected value under $\mathbb{Q}$ discounted with a corresponding *numéraire*, which with $\mathbb{Q}$ is the *risk-free rate of return* (Joshi 2003, p. 158). Thus the expected rate of an asset under $\mathbb{Q}$ is the risk-free rate used for discounting.

Since CVA is a pricing adjustment made to derivatives portfolio and derivatives are in general priced by using risk-neutral dynamics, it is natural that CVA is also priced under the domestic risk neutral-measure $\mathbb{Q}$, where the rate of return is the domestic risk-free rate.

[2]The physical probabilities, or *real-world probabilities*, are the ones estimated from the historical data.

In sequel $\mathbb{Q}$ is used to denote the domestic risk-neutral measure. Following Brigo and Vrins (2018) we assume arbitrage free and complete market and choose the numéraire corresponding to discount factor to be the money market account

$$B_t := e^{\int_0^t r_s ds},$$

where r is the *short rate process* defining the risk-free rate. The *deflator* B is defined from time $t = 0$ forward, $B := (B_t)_{t \geq 0}$, and it has dynamics

$$dB_t = r_t B_t dt.$$

The discount factor has now form $D(0,t) = 1/B_t$. It follows from no-arbitrage assumption that all tradeable, non-divident paying assets discounted with B_t are martingales under the associated probability measure $\mathbb{Q}$ (see for example Brigo and Vrins 2018). By utilizing this property the *unilateral* CVA can be expressed as risk-neutral expectation of the discounted loss

$$\mathbf{CVA} = \mathbb{E}^{\mathbb{Q}}\left[L_\tau^*\right] = \mathbb{E}^{\mathbb{Q}}\left[(1 - R_\tau)\,\mathbf{1}_{\{\tau \leq T\}}\,\frac{V(\tau)^+}{B_\tau}\right], \tag{2.1}$$

where $\mathbb{E}^{\mathbb{Q}}$ denotes $\mathbb{E}_0[\cdot]$-expectation under $\mathbb{Q}$ (Zhu and Pykhtin 2007). In the unilateral framework only counterparty's default risk in Π is considered whereas *bilateral* CVA would also consider the bank's own default risk, which would require calculation of the *debt value adjustment* (DVA) (Brigo, Capponi, and Pallavicini 2014). In this thesis only unilateral CVA[3] is discussed, and possibility of own default in the horizon T is ignored.

In practice there are three main components of credit risk embedded in this formula: *loss given default* (LGD), *probability of default* (PD) and *exposure at default* (EAD), which is discounted with risk free account. These components are further discussed in next subsections from the perspective of this thesis.

2.1.1 Loss given default

If bank's counterparty defaults it is reasonable to assume that full value of the credit is not necessarily lost but some proportion of it will be recovered (Hull and White 1995). This proportion is generally expressed as a fraction of exposure at default and it is called *recovery rate*. The loss given default at time t can be expressed in terms of recovery rate $\mathrm{LGD}_t = 1 - R_t$ and this complement format is frequently used in the literature (Bastos 2010).

[3]For a review of WWR in bilateral CVA see for example Scherer and Schulz (2016).

In practice loss given default is usually assumed to be constant over time (see for example Ruiz 2015, p.134). Under this assumption equation (2.1) can be simplified:

$$\mathbf{CVA} = (1-R)\,\mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{\tau\leq T\}}\,\frac{V(\tau)^{+}}{B_{\tau}}\right]. \tag{2.2}$$

The assumption can be questioned since recovery rates seem to follow business cycle and decrease in recessions (Bruche and González-Aguado 2010). Especially with loan securities relying on static LGD values is heavily criticized (Frye 2003). Instead of using constant LGD it could be, for example, forecasted (Bastos 2010).

However, in the case of default-risky derivatives the whole term structure of recovery rate should be modelled until the maturity of the derivative to make use of time-dependence and, in addition, using observed historical recovery rates would not be in line with risk-neutral pricing. Using market implied recovery rates, extracted for example from credit default swap[4] (CDS) spreads, is difficult due to identification problems, since probability of default is also part of the same equation (Das and Hanouna 2009). Das and Hanouna show that the identification problem can be tackled, but for purposes of this thesis other two main components of credit risk, PD and EAD, are more important and thus the topic of estimating recovery rates is not discussed further.

2.1.2 Exposure at default

Exposure is a metric which tells "how much we are owed" (Ruiz 2015, p. 21) at time t. At default time τ exposure is the *positive net present value*[5] $\mathrm{NPV}^{+}(\cdot) := \max\{\mathrm{NPV}(\cdot), 0\}$ of the residual payoff of $\Pi(t)$ remaining before maturity T measured at default time τ (Brigo and Masetti 2005, p. 7). Within $\mathrm{NPV}(\cdot)$ all netting-set level factors reducing or increasing the amount *we are owed to* are assumed to be taken in account. For example, collateralization by margining and netting are tools for counterparty risk reduction (Pallavicini, Perini, and Brigo 2011) which can affect the residual value. Usage of these tools is controlled by contractual factors like credit support annex (CSA) (Pallavicini, Perini, and Brigo 2011).

Exposure measured at default time τ, the exposure at default, is used in the *general counterparty risk pricing formula*:

[4]A single-name credit default swap is a financial instrument providing a protection for the holder of the instrument against default of the reference entity, which can be a country or company. The *protection leg* pays periodic payments for the holder of until the end of contract or default of the reference entity. In case of default, the seller pays protection leg holder a a compensation which depends on the value of a reference instrument after the default. (Blanco, Brennan, and Marsh 2005)

[5]Net present value is a sum of cash flows in the portfolio, where each cash flow is discounted with a corresponding discount factor.

$$\mathbb{E}^{\mathbb{Q}}\left[\Pi(t)\right] = \mathbb{E}^{\mathbb{Q}}\left[\Pi_{DF}(t)\right] - (1-R)\,\mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{\tau \le T\}}\,D(t,\tau)\,\mathrm{NPV}(\tau)^{+}\right], \tag{2.3}$$

where $\Pi_{DF}(\cdot)$ is similar claim as $\Pi(\cdot)$ but with a *default-free counterparty* (Brigo and Masetti 2005, pp. 6–7) having no risk of default before T. If we use the same definition of exposure $V(\tau)^{+} = \mathrm{NPV}(\tau)^{+}$ in (2.2) and plug in the general discount factor $D(t,\tau)$ we can see how CVA formula (2.2) is part of the special case of the general pricing formula:

$$\mathbb{E}^{\mathbb{Q}}\left[\Pi(0)\right] = \mathbb{E}^{\mathbb{Q}}\left[\Pi_{DF}(0)\right] - \mathbf{CVA},$$

where the time of measurement is current time $t = 0$. This explains why CVA is intuitively defined as adjustment for the current value of the default-free portfolio required to take the default risk into account (Antonelli, Ramponi, and Scarlatti 2021).

Another important exposure metric is the *expected positive exposure* (EPE), which tells "how much we can be owed on average" (Ruiz 2015, p. 21). If we assume that the discounted positive exposure and default time are independent, the definition of EPE is given by

$$\mathrm{EPE}^{\perp}(t) := \mathbb{E}^{\mathbb{Q}}\left[\frac{V(t)^{+}}{B_t}\right], \tag{2.4}$$

where $\perp$ denotes that the quantity is calculated under independence assumption of exposure and default time (Brigo and Vrins 2018), also known as *market-credit independence* (Ruiz 2015, p. 167).

2.1.3 Probability of default

The indicator term $\mathbf{1}_{\{\tau \le t\}}$ of equation (2.2) is connected to the probability of default: Following Li and Mercurio (2016) the *cumulative default probability* $P_\tau(\cdot)$ of counterparty before time t is given by risk-neutral expectation

$$P_\tau(t) := \mathbb{Q}[\tau \le t] = \mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{\tau \le t\}}\right]$$

and it satisfies $P_\tau(0) = 0$. The *risk neutral survival probability* of the counterparty is obtained as $1 - P_\tau(t)$ (Brigo and Vrins 2018).

CVA can now be written by applying the *law of iterated expectations*[6] to (2.2) as

$$\begin{aligned}\mathbf{CVA} &= (1-R)\,\mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{\tau\leq T\}}\,\frac{V(\tau)^{+}}{B_\tau}\right]\\ &= (1-R)\,\mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}}\left[P_\tau(T)\frac{V(\tau)^{+}}{B_\tau}\middle|\mathcal{H}_t\right]\right],\end{aligned} \tag{2.5}$$

where $\mathcal{H}_t = \sigma(P_\tau(u), 0 \leq u \leq t)$ is a subfiltration $\mathcal{H}_t \subseteq \mathcal{G}_t$ having enough information to determine the potential occurrence of counterparty credit event prior to t (Brigo and Vrins 2018).

According to Brigo and Vrins (2018) the outer expectation in (2.5) can be written as an integral and the argument of inner expectation can be simplified, if τ admits density. In our case the integral is with respect to the probability of default and the expression of CVA becomes

$$\mathbf{CVA} = (1-R)\int_0^T \mathbb{E}^{\mathbb{Q}}\left[\frac{V(t)^{+}}{B_t}\middle|\tau = t\right] dP_\tau(t). \tag{2.6}$$

Following the same notation as Li and Mercurio (2016) the *density function of survival probability* is

$$p_\tau(t) := \frac{d}{dt}P_\tau(t),$$

and by writing the integral with respect to time we finally obtain the general CVA formula:

$$\mathbf{CVA} = (1-R)\int_0^T \mathbb{E}^{\mathbb{Q}}\left[\frac{V(t)^{+}}{B_t}\middle|\tau = t\right] p_\tau(t)dt. \tag{2.7}$$

If we make again the same independence assumption as used in definition (2.4) we can define the *independent CVA* as

$$\mathbf{CVA}^{\perp} := (1-R)\int_0^T \mathbb{E}^{\mathbb{Q}}\left[\frac{V(t)^{+}}{B_t}\right] p_\tau(t)dt = (1-R)\int_0^T \mathrm{EPE}^{\perp}(t)p_\tau(t)dt \tag{2.8}$$

[6] The law of iterated expectations, or the *tower law*, says that for a general random variable X and time $s < t$ the expected value can be written as

$$\mathbb{E}[X|\mathcal{J}_s] = \mathbb{E}[\mathbb{E}[X|\mathcal{J}_t] \mid \mathcal{J}_s],$$

where $\mathcal{J}$ is a sub σ-algebra (Joshi 2003, p. 155) See for example Allen, Morris, and Shin 2006 for more about iterated expectations.

according to Brigo and Vrins (2018). The reason why the assumption is not applicable to the case of this thesis is discussed next.

2.2 Statistical dependency and wrong-way risk

The assumption of market-credit independence is common in CVA calculations (Hull and White 2012), even though the dependency between counterparty risk and exposure is one of the key drivers of CVA (Brigo and Vrins 2018). According to Zhu and Pykhtin (2007) banks' counterparty credit risk originates mainly from interest-rate derivatives and they claim that the dependency structure is less material in FX and interest rate contracts, so banks are comfortable to make the independence assumption. However there is empirical evidence of significant dependency between default risk and exchange rate (Ehlers and Schönbucher 2006; Pykhtin and Sokol 2013). Thus it is more likely that the complexity of integrating the dependency structure modelling into CVA framework is the actual reason why the WWR effect is not explicitly addressed in CVA calculation, as Brigo and Vrins (2018) point out. In the case of this thesis, with collateralized FX derivative transaction and systemic counterparty, the impact of the dependency to CVA is even more pronounced as will be discussed in the chapter 3.

The market-credit dependency is sometimes referred as market-credit *correlation* (Ruiz 2015, p. 383). However, correlation is a linear measure of dependency[7] and thus it might oversimplify the modelling of dependency and lead to exposure miss-calculations as discussed by Ruiz (2015) and in context of FX derivatives by Chung and Gregory (2019). Chung and Gregory mention that in specific market-credit dependency cases there might be causal linkage, in which case the relation cannot be correctly captured with an ordinary correlation based modelling. An example of causal effect affecting CVA is an assumed jump in market risk factor's value immediately at the default time. If the exposure at the default time $V(t)^+$ is affected by the arrival of default, CVA must be measured *conditional* on default as in equation (2.6). In this thesis *market-credit dependency* refers to any type of dependency structure between exposure and default time or default probability, which might driven by correlation, causal effects or both.

[7] There are also nonlinear correlation frameworks for CVA, for example using a stochastic correlation modelling instead of constant correlation (Kumar, Markus, and Hari 2021).

3. WRONG-WAY RISK IN DERIVATIVES WITH FOREIGN EXCHANGE RISK FACTOR

According to Gregory (2015, chapter 17) WWR is by its nature often specific and unavoidable consequence of financial markets. Thus, it is important to understand the context where WWR is analyzed and set underlying assumptions accordingly. Some simplifying assumptions are necessary, since all details cannot be included in a tractable framework, because exposure calculations alone are already computationally heavy. In the process of setting simplifying assumptions it is necessary to understand which features of the portfolio and its dynamics are the most essential for maintaining description of counterparty credit risk which is detailed enough. In practice the process is rather iterative and may benefit from numerical sensitivity analysis with respect to underlying assumptions in addition to literature-based qualitative method.

In sequel the assumed portfolio structure is discussed in detail. The nature of the portfolio sets some basic requirements for a modelling approach, both in sense of underlying risk factor and due to contractual reasons. The discussion of modelling approaches is built on the requirements, in a way that only those which fulfill the requirements are reviewed in more detail. Finally, the choice of modelling approach is made by comparing found modelling approaches and identifying which might fit best to the case of the portfolio and is tractable enough to be implemented. As already discussed, the choice of modelling approach is heavily dictated by assumptions made of the portfolio and thus the model may not fit universally to all WWR CVA cases, especially if other than FX risk factor is considered.

3.1 Cross-currency swap and direction of transactions

The most obvious question about portfolio structure is of what kind of instruments is it constructed. In this thesis the market-risk factor considered is the FX risk so naturally the focus is on FX derivatives. We are considering a bank using OTC FX derivatives to to hedge FX risk connected to foreign currency. For simplicity it is assumed that there is only one foreign currency hedged, USD, and thus the only underlying FX risk factor is the USD/EUR pair. More detailed analysis of FX risk is in section 3.5.

In addition it is assumed that the portfolio is constructed of only one type of FX derivatives, *cross-currency basis swaps*. The cross-currency basis swap is a contract in which the counterparties simultaneously exchange the same value, the principals, measured by FX spot rate in different currencies (Baba 2009). During the contract, the *swap term*, which begins from the initial transaction and ends at the maturity, counterparties pay each other agreed rates regularly. Finally at the maturity of the contract, the principals are exchanged back. In this construction the bank is effectively giving a loan in one currency and borrowing one in another currency with same counterparty and agreement to unwind (Baba and Sakurai 2011). A cross-currency basis swap could be used for example to hedge FX risk related to funding made in foreign currency, since principal exchanges are fixed at the interception (Baba 2009). However, the exposure to counterparty credit risk is relatively high, usually higher than interest rate swap, due to the exchange of principals (Duffie and M. Huang 1996).

As noted, the cross-currency basis swap fits well for hedging a funding made in foreign currency, which motivates one more assumption about the portfolio: all transactions are made to hedge foreign currency risk, that is, the bank is always the "lender" of USD "funds" and borrower of EUR funds in the swaps. This *direction assumption* simplifies structure of the portfolio, but more importantly, it follows that the exposure to counterparty is monotonic function of the underlying FX risk in a cross-currency basis swap portfolio, because cross-currency basis swap's cash flows are linear with respect to FX rate.[1] Further, the direction assumption has some interesting implications from the perspective of market-credit dependency, namely in some cases it guarantees that only WWR or RWR is present. This assumption is not as heavy as it may first seem, because for a firm trying to manage assets and liabilities only in domestic currency, this is the only direction where foreign currency swaps would be made.

As an illustrative example, consider a bank issuing a bond denominated in USD. The bank receives N dollars, and the coupons are linked to the *Secured Overnight Financing Rate* (SOFR)[2] with added premium. The bank is willing to hedge the USD/EUR FX risk of the liability, so it enters a cross-currency basis swap with same notional and payment schedule as the loan, with mirrored flows. In the swap the bank "loans" N USD and "borrows" X_0N EUR, where X_t is the USD/EUR spot rate. According to Baba (2009) the market convention is to quote the contract in terms of interest rates as Euro Inter-bank Offered Rate (Euribor) plus α basis points versus SOFR[3]. When the periodicity is three

[1] However, instruments having cash-flows that are linear functions of the underlying market risk factor are no longer linear, when CVA is taken into account in the valuation (Zwaard, Grzelak, and Oosterlee 2021).

[2] There is an ongoing paradigm shift from *interbank offered rates* (IBORs) to a new set of *overnight risk-free rates* (RFRs), the main one being SOFR. Older literature often refers to the *London Interbank Offered Rate* (LIBOR) when USD denominated liabilities are considered. For further discussion about the IBOR to RFR transition, see for example Schrimpf and Sushko (2019).

[3] Based on the no-arbitrage argument a so-called long-term *covered interest parity* (CIP) condition can be derived, which connects long-term interest rate differentials between currencies (Popper 1993). In fact,

months, the bank is paying floating rate Euribor plus α and receiving floating SOFR every three months. The bank can use the SOFR flow to pay coupons of the bond. At the maturity the bank is obligated to return X_0N EUR and receives N USD from the swap and pays N USD to bond holders. Thus, if the cross-currency swap notional and schedule match exactly to the bond, the bank has no FX risk exposure during the contract.

According to Baba (2009) similar economic effect is achieved with *FX swaps*, where the principal exchange at maturity is done at pre-agreed FX forward rate and during the contract no interest is paid. However, in this thesis the focus is on cross-currency swaps, since they are more liquid for maturities of one year or more (Baba, Packer, and Nagano 2008) which implies they fit better in hedging funding in foreign currency than FX swaps. In addition, cross-currency swap's cash flows typically mimic bond payment streams (Popper 1993). Among different types of cross-currency swap instruments, the cross-currency basis swap is the most liquid (Baba and Sakurai 2011), so it was chosen to be the main instrument to be analyzed.

3.2 Collateral posting

According to Brigo et al. (2011) continuously changing exposure of one counterparty to another makes collateral management difficult in the case of counterparty credit risk. They add that frequent collateral posting is essential in reducing credit exposure and that the type of collateral posted should not be correlated to the value of the transaction. The requirement of independence is rather obvious, since correlation could impose another source of wrong-way risk. In this thesis collateral is assumed to be cash in domestic currency, so risk mitigation effect of collateral is "pure" from the bank's own perspective.

Collateral posting is subject to margin agreement, which specifies thresholds and minimum transfer amounts (Pykhtin 2009). The *International Swap and Derivatives Association* (ISDA) is a central authority forming agreement standards in the field of OTC derivatives: the ISDA master agreement specifies general terms between counterparties governing transactions at counterparty level (Bliss and Kaufman 2006). The details of collateral posting are written in a legally enforceable margin agreement, the ISDA *credit support annex* (CSA), under the ISDA master agreement (Pykhtin 2009).

In the ISDA framework two types of collateral flows are recognized: *intial margin* (IM) and *variation margin* (VM) (Andersen, Pykhtin, and Sokol 2017). In regulation VM is intended to cover current exposure, while IM is reflecting the potential future exposure (Basel Committee on Banking Supervision 2011b). The common type of IM is fixed flow in beginning

the basis spread α, also known as *cross-currency basis spread*, measures the market implied deviation from the parity condition and more negative it is, the higher the demand for USD liquidity is relative to the funding currency EUR (Baba and Sakurai 2011). Detailed discussion about the cross-currency basis and historical violations of CIP condition are presented by Borio et al. (2016).

of the contract, but other types of IM are also possible. For example, IM may be adjusted to reflect the closeout risk (Andersen, Pykhtin, and Sokol 2017), which refers to additional costs arising after counterparty default (Gregory 2015, chapter 11). The closeout risk can be modelled and measured with *value-at-risk* (VaR) approach after which IM can be re-margined if necessary (Andersen, Pykhtin, and Sokol 2017). While IM is becoming mandatory in OTC markets through banking regulation (Basel Committee on Banking Supervision 2011b), historically IM has been rare in OTC markets and used mainly by *central counterparties* (CCPs) being dominated by use of VM (Gregory 2015, chapter 6). In addition, according to Andersen et al. (2017) dynamic IM without VM can only weakly reduce expected exposure in cross currency swap trades. In this thesis IM is ignored and only VM is considered in the collateralized exposure calculations. Typically VM, the regularly adjusted collateral, is designed to follow the value of the portfolio between parties quite closely (Andersen, Pykhtin, and Sokol 2017). For the value of the portfolio often used proxy is MtM value of the underlying transactions and thus VM is sometimes called *MtM margin* (Gregory 2015, chapter 6).

It is possible that both parties are not collateralized, but only one is required to post collateral with unilateral agreement or the margin thresholds are set to be highly asymmetric (Andersen, Pykhtin, and Sokol 2017). In this thesis the collateral posting is assumed to be bilateral and symmetrical, so that both parties are required to post collateral with similar terms. For simplicity 0-0 threshold is assumed, so the MtM-measured portfolio value is exactly matched at each collateral transfer, implying that the maximum allowed exposure against counterparty is 0. The CSA will typically cover also a number of other parameters such as minimum transfer amounts and rounding (Gregory 2015, chapter 6). These parameters of CSA are more relevant for non-cash collateral (Gregory 2015, chapter 6) so they are ignored in this thesis. In addition, the issue of netting rights after default coordinated by ISDA master agreement is not further discussed. This choice is motivated by the decision to focus on *directional portfolio*, in which case the netting effect would be minimal.

In practice there is delay in collateral posting after margin call, which is typically limited to one day (Brigo, Capponi, Pallavicini, and Papatheodorou 2011). The lag after the margin call before receiving collateral is effectively part of the margin period of risk (MPoR) (Pykhtin 2009). The concept of MPoR is specifically related to default event, while the lag occurs every time the collateral call is made. The assumed collateral lag value in this thesis is one day.

3.3 Margin period of risk

MPoR defines the length of time from the last successful margin call to the point of time when losses after default have crystallized (Andersen, Pykhtin, and Sokol 2017). The events unfold over MPoR can be divided into pre-default and post-default events. Besides of above discussed ordinary lag, including possible dispute of collateral call by collateral giver and settlement time of collateral transaction, there is also a contractual grace period in which the counterparty will not yet be deemed to be default. (Gregory 2015, chapter 6) According to Andersen et al. (2017) the *potential event of default* (PED) must be formally communicated to collateral giver and it marks the start of grace period. Following Andersen et al. in this thesis PED is the *true default time* τ which is not the same as *official default time* in contractual terms. An interesting factor which might affect pre-default window length of G-SIBs under the United States Bankruptcy Code is the ISDA Resolution Stay Protocol, which restricts some default rights temporary to let regulators better handle financial distress of a systemically important bank (see for example *ISDA 2015 universal resolution stay protocol* 2015).

If the collateral is not received in the grace period the counterparty is contractually in default and post-default events unfold (Gregory 2015, chapter 6): First, the counterparty will be informed of the event of default and the *early termination date* (ETD) of transactions will be designated by the bank (Andersen, Pykhtin, and Sokol 2017). ETD, also known as the *closeout date*, is the valuation date of the portfolio claim in MtM terms (Gregory 2015, chapter 6). The portfolio claim value includes unpaid trade flows and collateral. After the portfolio claim is adjusted by the held collateral amount, the residual value will be submitted as a claim to counterparty's insolvency, which will usually be challenged by the insolvency representative. It might need a lengthy bankruptcy court resolving process before the realized recovery rate will be known. (Andersen, Pykhtin, and Sokol 2017)

When defining the end of MPoR not all sources arrive in the same conclusion, since in some authors like Andersen et al. (2017) focus on ETD observation date and others on successful replacement of underlying transactions, like Gregory (2015). It can be argued that approach of Andersen et al. is somewhat more universal, since it doesn't take a stand on how the bank will proceed with their position after ETD. On the other hand, as discussed in chapter 1, many OTC market participants use derivatives mainly for hedging purposes. The hedging use is also an assumption in this thesis, so it is reasonable to include time needed to re-hedge the position in MPoR, since the bank remains exposed to unfavorable market variable evolutions until the successful replacement of transactions or macro-hedging of the defaulted portfolio has been performed. In the hedging use of derivatives re-hedging the position is necessary, because MtM-value of an asset or liability hedged will continue to evolve still after the counterparty has defaulted the derivative contract(s).

It is clear that modelling all the details of pre- and post-default periods is not necessarily feasible or even useful as there are multiple short delays and uncertainties in every step. In fact, according to Gregory (2015, chapter 6) MPoR is commonly used as a fairly simple parameter which is intended to include all these short delays, and is set conservatively in Basel regulation rather than being modelled in the most realistic way. According to him some typical choices for length of MPoR are from 10 to 20 days. While it might be tempting to use same MPoR for every counterparty it should be noted in case of certain counterparties, for example G-SIBs, it is not reasonable to assume that market conditions remain stable after the default event. Gregory (2015, chapter 11) discusses the conditionality of MtM volatility on default event and notes that MPoR length can be used to consider this effect, if it's not explicitly quantified in the exposure simulation.

All in all, MPoR generally in CVA calculations is a "catch-all" parameter, which is intended to collapse the essence of the close-out risk arising from pre- and post-default events into a single number. In doing so, it should not be interpret literally as the actual time that it may take to re-hedge the portfolio (Gregory 2015, chapter 6), but as a proxy which reflects the risk connected to the events and time unfolding in the case of default. However, in case of this thesis the more detailed treatment of MPoR events, related to potential WWR effects near default event, will be discussed further: it depends on the choice of modelling approach which part of close-out risk will be explicitly assessed, and which part will be allocated for MPoR. According to Gregory (2015, chapter 11) it is essential to find right balance between benefits and diminishing returns of more detailed MPoR modelling.

3.4 Systemic counterparties

Another important consideration in CVA calculations is the counterparty the bank has contracts with. It is obvious that credit quality of the counterparty is one of the parameters when measuring counterparty credit risk associated with derivatives. In this thesis one step is taken further since the WWR considered is the most relevant for a subset of potential counterparties, G-SIBs. One could argue that the modelling approach should be made independent from the type of counterparty, to keep number of models manageable. However, systemically important institutions are fundamentally different form other potential counterparties, due to potential adverse consequences for global economy after one's failure (Basel Committee on Banking Supervision 2018). Why this affects WWR modelling is discussed further in section 3.7.

There is no full consensus how systemic importance of bank should be measured. In the Basel framework the systemic risk classification is based on indicators of size, interconnectedness, substitutability, complexity and cross-jurisdictional activity (Foglia and Angelini 2021). Some other proposed methodologies are for example risk measures of interconnectedness, like conditional value-at-risk measuring marginal contribution of an institution to the overall systemic risk (Adrian and Brunnermeier 2011) and marginal expected shortfall (Acharya, Engle, and Richardson 2012). Foglia and Angelini (2021) present a methodology connecting both cross-sectional and temporal dimensions of systemic risk, aiming to bridge the gap between systemic importance and systemic risk measures. While the debate about triggers and measurement methodology of systemic importance is still ongoing (Foglia and Angelini 2021) according to Elliott and Litan (2011) the common concern is the potential failure of some financial institution systemically important enough to damage whole economy, for example due to losses for large amount of creditors.

Since the purpose of this thesis is not to deep-dive methodologies of measurement of systemic risk or evaluation of systemically important financial institutions, a proxy is needed to understand which counterparties should be deemed systemically important. In this thesis the list of global systemically important banks published by the Financial Stability Board which is based on the methodology of the Basel Committee is used for this purpose. While the methodology is criticized for example due to lack of transparency (Foglia and Angelini 2021) and arbitrary of weights of indicators (Benoit et al. 2017), the list is commonly available and updated regularly. The systemic risk is not constant (Elliott and Litan 2011) and the list of G-SIBs evolves from year to year. In this thesis the latest issue of the list is used, which is at the time of writing the one published in November 2022 (Financial Stability Board 2022). While the CVA values are not measured explicitly against any of these banks, the list is used as a reference to set directional levels of credit risk related parameters in the chapter 5. The sensitivity of CVA against these parameters is measured, so the assumption of systemic counterparty affects more the modelling choices in this thesis than the actual measurement results, since the measurements are made with multiple parameter levels.

3.5 Jump diffusion and FX risk factor

In previous sections some assumptions about the portfolio were set, including the main market risk factor considered, the FX risk. There is an extensive literature documenting evidence of abrupt FX rate movements, both observed from historical time series and implied from prices of financial instruments. To tackle the modelling of abrupt risk factor movements, jumps attached to the process dynamics are often presented in the literature. Some of this literature is reviewed next.

Pioneering work of market risk factor jumps is the one by Merton (1976) related to equity option pricing. In his paper he deviates from the so-called local Markov property of the stock price dynamics used in the Black and Scholes (1973) pricing formula. Merton adds a jump component modelled by Poisson-driven process, which allows stock price to change in a short time interval more than the pure *geometric Brownian motion* (GBM) used in the *Black-Scholes model* (BS) would. This idea is extended to currency options by Borensztein and Dooley (1987). They use a jump method to model prices of *out-of-the-money* FX options, which would require high value change of underlying to be of value at maturity. As Bates (1996) explains, values of currency options are systemically mispriced if the GBM assumption is made. The violation of the BS assumptions in currency options can be observed for example by plotting BS model implied volatilities against the spot rate divided by the strike of the option, which tends to exhibit a U-shaped curve, known as the *volatility smile*[4]. Whereas most of the option pricing literature consider occurrences of high-frequency jumps as noted by Farhi et al. (2009), like Bakshi et al. (2008) who find evidence of jump risk pricing in currency options, the currency crashes considered in this thesis are of far lower frequency. An empirical model aiming to incorporate both high frequency jumps and low frequency crashes in currency markets is presented by Chernov et al. (2018). Farhi and Gabaix (2016) explain *smirks*, non-symmetric implied volatility smiles, in currency options with a model of exchange rates where countries' different exposures to a possible global disaster are considered.

Another highly relevant event connected to currency crashes in case of CVA measurement is a default of financial institution having high level of systemic importance in the currency area. According to Pykhtin and Sokol (2013) a default of systemically important financial institution could rapidly affect the currency of the area and thus imply rapid FX movements. Since defaults of this scale are uncommon, it is not easy to estimate magnitude of possible movements from historical data. However, according to Ehlers and Schönbucher (2006) a specific market instruments, *quanto credit default swaps* can be used to extract market implied information about the possible jump size of FX rate in case of default of specific counterparty. The quanto credit default swap is a credit default swap instrument referring to a potential default of a given entity, with a payoff denominated in different currency than the assets of the given entity.[5] If the spreads of normal and quanto CDS instruments referring to same counterparty differ, the quanto basis spread implies that the market might be pricing jump-at-default risk (Brigo, Pede, and Petrelli 2019). Thus, both historical and market implied information indicate that a default of G-SIB could move FX rate of domestic currency rapidly, typically devaluating the domestic currency, which affects the risk of currency derivative transactions made with the given counterparty. In eurozone, if one is receiving dollars from systemically important counterparty in cross-currency transaction and paying euros, a failure of the counterparty might rapidly increase the USD/EUR FX rate, which means that the exposure in the transaction will increase rapidly at the same time when the counterparty fails. The devaluation effect in a the framework of sovereign CDS of eurozone countries is studied by Augustin et al. (2020). Due to assumptions of the previous chapters about the portfolio, namely direction of transactions and a type of counterparty considered, the euro devaluation at default is exactly the type of WWR relevant in the case of this thesis.

[4]This is not how the volatility smiles of OTC FX options are constructed in practice since the strike-price pairs are not directly observable (Reiswich and Uwe 2012). In addition, it is more common to quote currency option in terms of its first derivative with respect to the spot exchange rate, known as delta (Farhi, Fraiberger, et al. 2009).

[5]The quanto CDS is discussed more in the model calibration chapter 5. Theory of quanto CDSs is explained in detail for example by Augustin et al. (2020).

3.6 Requirements for the modelling approach

The requirements set here are a set of basic constraints which a modelling approach should satisfy to be considered. The requirements are quite vague on purpose and stemming mainly from practicalities of derivatives pricing. More detailed analysis of the methods are based on assumptions and requirements set in previous sections.

The requirements are:

(i) The model is in line with arbitrage-free pricing.

(ii) The model must combine the default or probability of default of a counterparty and FX movement.

(iii) The model must generalize to different counterparties and constructions of the portfolio without complete re-calibration. In other words, it cannot be really portfolio specific.

The first requirement is motivated by the fact that the derivatives pricing is in general performed with arbitrage free methods, so it is sensible to perform pricing adjustments in same manner. The second requirement is to make sure that the chosen method is capable of measuring WWR effect in currency derivative transactions. The last requirement is solely practical: there is little value in a WWR model if the pricing process of derivatives becomes too complicated to be calibrated and performed daily. In addition, results of complicated model might be difficult to communicate and interpret.

3.7 Review of WWR modelling approaches

According to Brigo and Vrins (2018) there are two main approach categories, one popular among practitioners and other proposed by academic researchers, to address WWR: *Static approaches* to WWR couple the credit and market risk components *after* the simulation process, while *dynamic approaches* require taking the dependency structure into account already during the dynamic simulation of the credit risk and exposure.

Brigo and Vrins (2018) note that in the industry the static approaches are popular because they are more tractable and less computationally heavy alternative than the dynamic approaches proposed by many researchers. However, static approaches in general lack the level of theoretical justification provided by dynamic ones: they are not arbitrage-free, which contradicts with widely used assumptions in derivatives pricing. In addition, they point out that the way how static approaches handle coupling of credit risk and exposure is rather artificial. Thus, approximations provided by static approaches are less in line with actual market data, but they avoid time-intensive simulations of dynamic approaches which can be infeasible if amount of market factors and exposures is huge. In particular, static copula approaches avoid joint modelling of exposure and credit since the coupling of the distributions is done *a posteriori* via a copula[6](Pykhtin and Rosen 2010; Cherubini 2013). While static approaches have some attractive practical properties they are not considered in this thesis, because they lack sound theoretical justification and does not fulfill the requirement of being in line with arbitrage-free pricing.

In the dynamic approaches the coupling of credit and market risk is done *a priori* and thus the joint dynamics must already be defined during the exposure simulations. According to Brigo and Vrins (2018) on can distinguish two distinct setups among dynamic approaches: The first class of dynamic models are the *structural models* focusing on the counterparty's balance sheet. The basic idea in these approaches deriving from the *Merton* (1974) credit model is to consider the default occur as soon as the value of the firm drops below the firm's assets, represented by a pre-defined barrier (Brigo and Vrins 2018). These models are applied to CVA for example by Brigo and Pede (2019). Calibration of the models may not be possible against traded instruments and the calibrated structural models might fail to reproduce credit quantities observed in markets (Eom, Helwege, and J.-z. Huang 2004). In addition structural models are more commonly used with equity linked instruments (Chung and Kwok 2016) than with FX derivatives. Structural models are by nature really counterparty specific and thus they do not satisfy the requirement of generality. Instead of structural models, reduced-form models are considered in this thesis.

The reduced-form modelling framework considers the default likelihood of counterparty (Brigo and Vrins 2018) instead of counterparty's balance sheet. The default probability is in these models driven by a *default intensity*, a counterparty spesific quantity expressing the rate of default time, given that the counterparty has survived up till that time (Ghamami and Goldberg 2014).[7] Thus, the default intensity is driving the probability of default, which is in reduced-form models expressed as a function of the intensity. In the dynamic framework *stochastic intensity* models are the most popular way to incorporate WWR in CVA (Brigo and Vrins 2018). The stochastic intensity can be made a function of the exposure itself (Hull and White 2012) or a market risk factor driving the exposure, like FX rate (Chung and Gregory 2019). The stochastic models are calibrated so that the default probabilities implied by the model agree with market observed default probabilities, which can be extracted for example from corporate bond spreads or credit default swap spreads of the corresponding counterparty (Ghamami and Goldberg 2014).

A problem with models having the default intensity a function of exposure is that the cal-

[6]Copula is a cumulative distribution function generating a multivariate distribution from univariate ones (Aas et al. 2009), hence coupling the distributions *a posteriori*.

[7]In credit risk literature *hazard rate* is commonly used as a synonym of default intensity. In this thesis the default intensity is used to highlight the fact that it is the quantity driving the rate of default probability, while hazard rate is used as term for a deterministic function calibrated against market information and used as a part of default intensity function. In addition, if the default intensity is deterministic, it is equal to hazard rate in the framework of this thesis.

ibration is then highly portfolio structure specific: The calibration will arrive to different results with different instruments, because payoffs of the instruments will affect the intensity model parameters. In addition, collateral agreements complicate the calibration even further (Hull and White 2012). It is unlikely that the calibration could be done with respect to risk-neutral probability measure, and with historical calibration with respect to physical measure the correct form of relationship is might not be present in time series, if the structure has changed or the portfolio has complex derivatives with optionality structures. Thus, making the intensity a function of exposure does not full fill the requirements set for the modelling approach.

More simple approach to WWR with stochastic intensity framework is to assume correlation between random components of market-credit dependency: for example, one can specify a linear correlation structure between FX rate and default intensity, which is then used to modify the error components of the stochastic simulations (Kumar, Markus, and Hari 2021). An advantage of this kind of approach that there is only one WWR parameter to be calibrated per *counterparty and market risk factor* pair and it is not portfolio specific. Also, if the stochastic intensity framework is already implemented, the WWR implementation only requires the generated random numbers to be multiplied by the correlation factors given by the Cholesky decomposition (Kumar, Markus, and Hari 2021). The linear correlation approach is popular due to its simplicity, but it is criticized because it is in producing only a weak WWR effects in general (Ehlers and Schönbucher 2006; Chung and Gregory 2019; Kumar, Markus, and Hari 2021), and is not able to reproduce prices observed in derivatives market (Brigo, Pede, and Petrelli 2019). In this thesis linear correlation model is used as a benchmark version of the WWR implementation, compared with a *jump-at-default* approach introduced next.

Finally, the WWR effect can also be present in the structure of the processes of market variables. A popular approach for WWR when FX risk factor is present is to add a jump component to the basic dynamics of the FX process, which occurs at the default time of the counterparty. This modelling approach was first proposed by Ehlers and Schönbucher (2006) for quanto CDS pricing. The approach fits well to CVA measurement of portfolios with FX risk because the jumps are theoretically sensible and empirically proven feature of FX rates, observed both from historical and market implied data, as is explained in section 3.5. Like with the constant correlation approach, in the *jump-at-default* approach there is only one parameter to be calibrated per *counterparty and FX rate* pair, a jump size. Obviously, calibrating this parameter with counterparty specific historical data is not possible, but in some cases market implied calibration is (Brigo, Pede, and Petrelli 2019). The jump-at-default approach, satisfying the requirements and being theoretically justifiable framework is the second method chosen to be considered in this thesis. Since the jump component can be added to the dynamics of FX risk factor and the constant correlation method requires only recalculation of the error term, both methods can be implemented at the same time in a framework consisting stochastic default intensity model and a FX model with a jump. In sequel, a model combining both features in the assumed portfolio structure is derived and the effect of both WWR frameworks is compared with a CVA without WWR effect.

4. DERIVATION OF WRONG-WAY RISK MODEL

In this chapter the chosen modelling approach is explained by presenting the assumed foreign exchange rate and default intensity processes. Joint modelling of given processes is necessary to capture the potential market-credit dependency, which in the chosen approach can be *constant correlation* or *jump-at-default* based. Since a cross-currency basis swap has cash flows in two different currencies, the risk neutral pricing in both domestic and foreign currency are required. For simplicity, the numéraires corresponding domestic and foreign risk neutral measures ensuring risk-neutrality are assumed to be deterministic and independent of other processes. It is a reasonable assumption as risk-free rate credit dependency has commonly rather limited numerical impact (Brigo and Alfonsi 2005).

Underlying market risk factor(s) must be modelled to price a derivative contract at future times as required by exposure calculations. Since the chosen contract type is a cross-currency basis swap, which has two legs paying floating interest rates, one could choose to model the forward rates with stochastic processes. However, as Li and Mercurio (2016) note, FX fluctuations are the main contributor of CVA in cross-currency swaps, with interest rates having limited effect. In addition, the chosen WWR approach is focused on FX effect and thus both cash flow projections and discounting in this thesis are assumed to follow deterministic term structure.

Both FX rates and floating rates are directly connected to exposure. In addition to exposure, probability of default is essential part of CVA as is explained in chapter 2. Incorporating stochastic processes in default probability estimation is not completely necessary in jump-at-default approach (Chung and Gregory 2019). However, it allows to compare effects of the jump-approach with one of the most popular WWR approaches, which correlates a measure called *default intensity* with chosen market risk factor(s). The default intensity is introduced in section 4.1.

The presented formulas in this chapter are mostly based on articles of Li and Mercurio (2016) and later Chung and Gregory (2019), whose approach is less restrictive and allows to incorporate more sophisticated exposure modelling, including lagging collateral and MPoR.

4.1 Default time and intensity process

Let $\mathbb{Q}$ be the domestic risk neutral measure as defined in the chapter 2 in probability space $(\Omega, \mathcal{G}, \mathcal{G}_t, \mathbb{Q})$. To tackle modelling of discrete default-event in continuous setting we define the *Azéma's supermartingale* S_t^τ, which can be interpreted as stochastic survival probability given default-free information:

$$S_t^\tau := \mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{\tau>t\}}\,|\mathcal{F}_t\right] = \mathbb{Q}\left[\tau > t|\mathcal{F}_t\right] \tag{4.1}$$

where $\mathcal{F}_t \subseteq \mathcal{G}_t$ is a filtration, which includes all *default free information* available at time t.[1] In other words, $\mathcal{F}_t$ is not rich enough to determine if default occurred before t. Together with subfiltration of the default indicator $\mathcal{H}_t$ it defines the total information $\mathcal{G}_t := \mathcal{F}_t \cup \mathcal{H}_t$ available at time t. (Brigo and Vrins 2018)

By applying the law of iterated expectations, we can connect the Azéma's supermartingale to risk neutral default probability $P_\tau(t)$ as

$$\mathbb{E}^{\mathbb{Q}}\left[S_t^\tau\right] = \mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{\tau>t\}}\,|\mathcal{F}_t\right]\right] = \mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{\tau>t\}}\right] = \mathbb{Q}\left[\tau > t\right] = 1 - P_\tau(t). \tag{4.2}$$

The formula $\mathbb{E}^{\mathbb{Q}}\left[S_t^\tau\right] = 1 - P_\tau(t)$ is known as the *calibration equation* since it allows to calibrate risk neutral expectation of S_t^τ if the deterministic default probability function $P_\tau(\cdot)$ is known.

We will work under a special case of Azéma's supermartingale process S_t^τ known as the Cox construction (or stochastic intensity framework): Following Brigo and Vrins (2018) the survival process is chosen in the construction to be

$$S_t^\tau := e^{-\Lambda_t}, \tag{4.3}$$

where $\Lambda_t := \int_0^t \lambda_s ds$ is the *martingale hazard process*. The positive $\mathcal{F}_t$-adapted stochastic process λ_t is known as *intensity process* (Brigo and Vrins 2018). Furthermore, $\Gamma_t := -\ln S_t^\tau$ is the *hazard process* (Coculescu and Nikeghbali 2012).

Azéma's supermartingale process $(S_t^\tau)_{t\geq 0}$ in the Cox construction is a decreasing stochastic process, because the intensity process λ_t is assumed to be positive. This property ensures that the random default time τ is a $(\mathcal{F}_t)$*-pseudo-stopping time*[2] (Nikeghbali and Yor 2005). According to Coculescu and Nikeghbali (2012) every pseudo-stopping time satisfies the condition where hazard process and martingale hazard process are equivalent:

[1] See Coculescu and Nikeghbali 2012 for extensive review of default time filtered processes.

[2] *Stopping time* is a random time of which passing can be determined from the information set $\mathcal{F}_t$ available at time t. Thus the event of passing the stopping time is in the information set: $\{\tau < t\} \in \mathcal{F}_t$. (Joshi 2003, p. 143)

$\Lambda = \Gamma$, which is satisfied by Cox construction, since

$$\Gamma_t = -\ln S_t^\tau = -\ln e^{-\Lambda_t} = \Lambda_t.$$

This is an important property, since it allows us to use default-free valuation techniques in defaultable claim valuation.

Another important property in Cox framework is that we can easily sample random default time τ from the survival process: Let ξ be a random variable uniformly distributed on $[0, 1]$ and independent of information $\mathcal{F}_\infty$. According to Brigo and Vrins (2018) the default time is

$$\tau = \sup \{s : S_s^\tau < \xi\},$$

the first passage time s when the survival process S_t^τ is below ξ, because

$$S_t^\tau = \mathbb{Q}\left[\tau > t|\mathcal{F}_t\right] = \mathbb{Q}\left[e^{-\Gamma_t} > \xi|\mathcal{F}_t\right].$$

By simulating survival process and for each simulation path sampling from uniform distribution, we could determine explicit default time, if it occurred in the time frame of the simulation path.

Instead of simulating explicit default times, we can discard the default time completely in CVA calulations by using Azéma's supermartingale: Assume that the exposure process of the portfolio V^+ and short-rate process B_t are $\mathcal{F}_t$-predictable[3] and $\mathbf{1}_{\{\tau \leq T\}} V(\tau)$ is $\mathbb{Q}$-integrable. With the given assumptions we can apply the *key lemma* (see Lemma 3.1.3 Bielecki et al. 2011) which states that for every $t \geq T$ we have

$$\mathbf{1}_{\{t<\tau\}} \mathbb{E}^{\mathbb{Q}} \left[\mathbf{1}_{\{\tau \leq T\}} \frac{V(\tau)^+}{B_\tau} \middle| \mathcal{G}_t \right] = \mathbf{1}_{\{t<\tau\}} e^{\Gamma_t} \mathbb{E}^{\mathbb{Q}} \left[\int_{]t,T]} \frac{V(u)^+}{B_u} d(1 - S_u^\tau) \middle| \mathcal{F}_t \right], \quad (4.4)$$

which means that we don't need the full information set $\mathcal{G}_t$ to measure the left side of the equation, but we can discard the explicit default time and use instead the $\mathcal{F}_t$-measurable information about default probabilities embedded in the Azéma's supermartingale. Applying the key lemma to the CVA formula (2.2) from $t = 0$ forward yields

[3] In other words, the exposure process V^+ cannot depend on the explicit value of τ. For example, it may not include credit instruments referring to the default of the counterparty, but it may well be depend on credit worthiness quantities, like the default intensity λ which is $\mathcal{F}_t$-measurable (Brigo and Vrins 2018).

$$\mathbf{CVA} = (1-R)\,\mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{\tau \leq T\}}\,\frac{V(\tau)^+}{B_\tau}\right] = -(1-R)\,\mathbb{E}^{\mathbb{Q}}\left[\int_0^T \frac{V(t)^+}{B_t} dS_t^\tau\right], \tag{4.5}$$

because

$$e^{\Gamma_0} = -\ln S_0^\tau = -\ln 1 = 0.$$

The discounted exposure values $\frac{V(\cdot)^+}{B}$ are calculated without default information, as $\mathcal{F}_t$-predictability of both processes is assumed.

Now that we have connected the survival probability to risk neutral default probabilities and constructed a default time, we have necessary tools to tackle default probability calibration, default time modelling and risk-neutral valuation of defaultable assets. In addition, complete credit model requires defining the (stochastic) intensity process λ_t.

For modelling the stochastic intensity dynamics we have alternatives: As Brigo et al. (2019) note, local volatility models or square root processes, like versions of the *Cox-Ingersoll-Ross* (CIR) processes[4] can be used. Another popular choice for modelling the intensity process is *Ornstein-Uhlenbeck* (OU) process (Brigo and Vrins 2018), of which the exponential version is often used for credit risk purposes (Brigo and Pede 2019; Brigo, Pede, and Petrelli 2019). In this thesis the scaled exponential version of the OU process is used, to avoid negative intensities (Brigo, Pede, and Petrelli 2019) and make the calibration process intuitive.

We begin by defining the ordinary OU process. The Ornstein-Uhlenbeck process Y_t is a univariate continuous Markov process obtained as a solution of a stochastic differential equation

$$dY_t = a(m - Y_t)dt + \sigma dW(t), \quad Y_0 = y \tag{4.6}$$

where a is the *mean reversion speed*, m is the *long-term mean*, $\sigma \leq 0$ is a volatility parameter and W_t is a *Wiener process* (Maller, Müller, and Szimayer 2009). The Wiener process, or *standard Brownian motion*, increments over positive time steps Δt are distributed normally with mean 0 and variance Δt

$$W_{t+\Delta t} - W_t \sim N(0, \Delta t), \quad W_0 = 0$$

and the increments are independent of previous increments (Joshi 2003, p. 100). According to Maller et al. (2009) a solution the stochastic differential equation (4.6) can be

[4] See Alfonsi (2005) for extensive review of CIR processes and their discretization schemes.

obtained by using a *scaled time-transformed Wiener process* $W_{(1-e^{-2at})/(2a)}$ with increments distributed normally

$$W_{(1-e^{-2a(t+\Delta t)})/(2a)} - W_{(e^{2at}-1)/2a} \sim N(0, (e^{2a\Delta t} - 1)/2a), \quad W_0 = 0.$$

By applying results of Doob (1942) the solution of the a stochastic differential equation (4.6) is then

$$Y_t = Y_0 e^{-at} + m(1 - e^{-at}) + \sigma W_{(1-e^{-2at})/(2a)}. \tag{4.7}$$

As explained by Maller et al. (2009) the process Y_t has a mean-reverting property, which means that when Y is over (under) the long-term mean level m it tends to move downward (upward) due to negative (positive) coefficient of dt.

In our setting we model the stochastic intensity under the risk-neutral martingale measure with an exponential OU process[5] scaled with a deterministic function

$$\lambda_t = h(t)e^{Z_t}, \tag{4.8}$$

where $h(t)$ is a scaling function known as *hazard rate*. Hazard rate is expressed by using the deterministic default probability $P_\tau(\cdot)$ as

$$1 - P_\tau(t) = e^{\int_0^t h(s)ds}. \tag{4.9}$$

The stochastic component Z_t is defined by an OU process

$$dZ_t = a(m - Z_t)dt + \sigma_\lambda dW^\lambda, \quad Z_0 = z, \tag{4.10}$$

where the corresponding Wiener process dW^λ is under the risk-neutral dynamics and σ_λ is *intensity volatility*. By applying the *Itô's lemma*[6] to the equation (4.10) we get the dynamics of the exponential OU process

$$de^{Z_t} = a\left(m + \frac{\sigma_\lambda^2}{2a} - Z_t\right)e^{Z_t}dt + \sigma e^{Z_t}dW, \quad Z_0 = z,$$

where the term $m + \frac{\sigma_\lambda^2}{2a}$ acts as an equilibrium level of the process (Mejía Vega 2018), same way as m is the equilibrium level of the normal OU process (4.10).

[5]The exponential OU process is applied to commodity prices by Schwartz (1997) and is known as *Schwartz one-factor model* (Mejía Vega 2018).

[6]see for example Joshi (2003, p. 110)

The conditional expectation of λ_T measured at time $t \geq s \geq 0$ is

$$\mathbb{E}^{\mathbb{Q}}[\lambda_t|\mathcal{F}_s] = \mathbb{E}^{\mathbb{Q}}\left[h(t)|\mathcal{F}_s\right]\mathbb{E}^{\mathbb{Q}}\left[e^{Z_t}\middle|\mathcal{F}_s\right] = h(t)\,\mathbb{E}^{\mathbb{Q}}\left[e^{Z_t}\middle|\mathcal{F}_s\right], \tag{4.11}$$

because $h(t)$ is assumed deterministic and uncorrelated with e^{Z_t}. According to Schwartz (1997) the conditional distribution of Z_t measured at time s is normally distributed with mean

$$\mathbb{E}^{\mathbb{Q}}\left[Z_t|\mathcal{F}_s\right] = e^{-a(t-s)}Z_s + m\left(1 - e^{-a(t-s)}\right),$$

and variance

$$\mathbb{V}^{\mathbb{Q}}\left[Y_t|\mathcal{F}_s\right] = \frac{\sigma_\lambda^2}{2a}\left(1 - e^{-2a(t-s)}\right),$$

which can be confirmed from the equation (4.7).

Since Z_t is normally distributed, e^{Z_t} is log-normally distributed with conditional expectation

$$\begin{aligned}\mathbb{E}^{\mathbb{Q}}\left[e^{Z_t}\middle|\mathcal{F}_s\right] &= \exp\left[\mathbb{E}^{\mathbb{Q}}\left[Z_t|\mathcal{F}_s\right] + \frac{1}{2}\mathbb{V}^{\mathbb{Q}}\left[Z_t|\mathcal{F}_s\right]\right] \\ &= \exp\left[e^{-a(t-s)}Z_s + m\left(1 - e^{-a(t-s)}\right) + \frac{\sigma_\lambda^2}{4a}\left(1 - e^{-2a(t-s)}\right)\right]\end{aligned} \tag{4.12}$$

as explained by Schwartz (1997). Even if we consider the initial level of the exponential OU process at level $Z_0 = 0$ and the same equilibrium level $\frac{\sigma_\lambda^2}{2a} = 0$ the conditional expected value (4.12) remains time-dependent.

For our purposes it is convenient, if the expected value of the intensity process conditional on information available at the measurement time $t = 0$ remains at the level of hazard rate

$$\mathbb{E}^{\mathbb{Q}}[\lambda_t|\mathcal{F}_0] = h(t),$$

because then we could calibrate the deterministic $h(t)$ directly against credit spreads and the calibrated model would agree with market information. However, from equation (4.12) we can see that the conditional expected values of the exponential OU model are time-dependent. Instead it is more practical to set the equilibrium level $m + \frac{\sigma_\lambda^2}{2a}$ to zero and obtain a value for m, which will make the exponential OU process on a long term approach value 1. The value for m is then

$$m = -\frac{\sigma_\lambda^2}{2a}.$$

The exact values of the exponential OU process e^{Z_t} can be calculated as with the normal OU process in the formula (4.7) as

$$Z_t = Z_0 e^{-at} + m(1 - e^{-at}) + \sigma_\lambda e^{-at} W_{(e^{2at}-1)/2a}, \tag{4.13}$$

where $W_{(1-e^{-2at})/(2a)}$ is the scaled time-transformed Wiener process (Mejía Vega 2018). In an iterative simulation process the next value of the process will be updated based on the previous realization and the Wiener process increments are sampled from a standard normal distribution, where the variance is scaled. By using the incremental updating formula of the exponential OU process (see for example Mejía Vega 2018), the exact solution of the default intensity over constant time-step Δt given information $\mathcal{F}_{t-1}$ is

$$\begin{aligned}
&\lambda_t = h(t) e^{Z_t}, \\
&Z_t = Z_{t-1} e^{-a\Delta t} + m(1 - e^{-a\Delta t}) + \sigma_\lambda \sqrt{\frac{1}{2a}(1 - e^{-2a\Delta t})}\varepsilon_t, Z_0 = 0, \\
&m = -\frac{\sigma_\lambda^2}{2a},
\end{aligned} \tag{4.14}$$

where Z_{t-1} is the previous realization of the process in the discrete time grid and ε_t is an error term, which in case of zero correlation is identically and independently normally distributed with mean 0 and variance 1. The intensity process is perturbed from the deterministic term structure $h(t)$ by stochastic process e^{Z_t}.

Hazard rates themselves are not observable, but prices of the credit default swap instruments referring to the default event of the counterparty are, if they exist. Thus CDS spreads are commonly used to calibrate the deterministic hazard rate function[7]. On the other hand our definition of hazard rate in the Cox framework ties hazard rate and default intensity together: by using the definition of Azéma's supermartingale in the stochastic intensity framework (4.3), the calibration equation (4.2) and our definition of the hazard rate (4.9) we have

$$e^{-\int_0^t h(s)ds} = 1 - P_\tau(t) = \mathbb{E}^{\mathbb{Q}}\left[S_t^\tau\right] = \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^t \lambda_s ds}\right]. \tag{4.15}$$

[7] A simple way to calibrate hazard rate is to assume it remain constant over the CDS maturity T. Then, if the recovery rate R of the CDS is known the risk neutral average hazard rate is $\bar{h}(T) = S(T)/(1-R)$, where $S(\cdot)$ is the observed CDS spread (Hull and White 2012). See chapter 5 for details.

In fact, according to Brigo and Vrins (2018) approximating stochastic intensity with deterministic hazard rate, that is $\lambda_t \approx h(t)$, gives usually satisfactory results in CVA pricing. However, with correlation based WWR framework using the given approximation is not possible, because it requires stochastic modelling of default intensity.

4.2 FX risk factor dynamics

In this section the dynamics of the FX risk factor model are defined. Together with the stochastic intensity model of previous section it allows to model two different types of market-credit dependency: correlated default intensity and FX-rate, and FX jump at default. In the following we work with respect to the total filtration $\mathcal{G}_t$ in which the default time τ is a stopping-time. Thus, passage of the default time can be determined from the information set available at time t.

Let X_t be the spot foreign exchange rate at time t measured as value of one unit of foreign currency in domestic currency. Following Li and Mercurio (2016) we set the *construction of the FX model* as

$$X_t := X_t^B \cdot M_t^J, \tag{4.16}$$

where X_t^B is a baseline FX-model without a jump-feature and M_t^J is a jump martingale process. We assume that the magnitude of the jump $J \in]-1, \infty[$ is constant and the jump of the process X_t happens only when counterparty defaults at time τ. The arrival of jump is then modelled with a default indicator

$$D_t := \mathbf{1}_{\{t \leq \tau\}}, \quad t \leq 0, \tag{4.17}$$

which is $\mathcal{G}_t$-measurable, and in addition a *compensator term*

$$dA_t = \mathbf{1}_{\{t \geq \tau\}} = (1 - D_t)\lambda_t dt, \tag{4.18}$$

must be attached to it for the jump process to be martingale (Brigo and Pede 2019). The jump martingale process dynamics are then given by

$$dM_t^J = J(dD_t - dA_t) = J(dD_t - (1 - D_t)\lambda_t dt), \tag{4.19}$$

Where the compensator term A_t is in the case of a positive FX jump $J > 0$ a decreasing drift term, which is pulling X_t downwards until the default appears and the process X_t jumps J percents.

For the baseline model we assume a geometric Brownian motion process

$$dX_t^B = \mu^B(t)X_t + \sigma_X X_t dW_t^X, \quad X_0 = x, \tag{4.20}$$

with deterministic drift $\mu^B(t)$ set by no-arbitrage considerations, constant volatility σ_X for FX-rate and Wiener process W_t^X. It can be shown that then the drift term is

$$\mu^B(t) = r(t) - \hat{r}(t) \tag{4.21}$$

if $r(t)$ and $\hat{r}(t)$ are deterministic functions representing domestic and foreign economy risk-free short rates (Brigo, Pede, and Petrelli 2019). The notation $\hat{\cdot}$ denotes processes and variables in foreign-currency economy. Since we assume that the short rates will follow exactly the deterministic term structures, the money market accounts are defined by differential equations

$$\begin{aligned} dB_t &= r(t)B_t dt, \quad B_0 = 1, \\ d\hat{B}_t &= \hat{r}(t)\hat{B}_t dt, \quad \hat{B}_0 = 1. \end{aligned}$$

By using (4.10), (4.16), (4.19), (4.20) and (4.21) we can express the *complete market-credit model dynamics* with two stochastic differential equations:

$$\begin{aligned} dZ_t &= a(m - Z_t)dt + \sigma_\lambda dW_t^\lambda, \quad z_0 = 0 \\ dX_t &= (r(t) - \hat{r}(t) - \lambda_t J(1 - D_t))X_t dt + \sigma_X X_t dW_t^X + JX_{t^-} dD_t, \quad X_0 = x \end{aligned} \tag{4.22}$$

where x is the initial FX-rate at time $t = 0$. The construction is same as used by Brigo and Pede (2019).

In addition to the jump-at-default component, another source of dependence between FX and credit can be introduced in (4.22) as *instantaneous constant correlation* $\rho_{X,\lambda} \in [-1, 1]$ between two Brownian motion processes

$$\mathbb{E}^{\mathbb{Q}}[dW_t^X dW_t^\lambda] = \rho_{X,\lambda} dt.$$

For simulation purposes it is more convenient to express the FX model in solved form instead of stochastic differential equations. By integrating (4.19) we get for the jump martingale component

$$M_t^J = \left(1 + J\,\mathbf{1}_{\{t \geq \tau\}}\right) e^{-\int_0^{\min\{\tau,t\}} \lambda_s J ds}, \tag{4.23}$$

as explained by Li and Mercurio (2016). From (4.23) we can see that at the default time τ the process X_t will jump proportionally to the level of the rate. In other words, M_t^J is a proportional scaling factor of the baseline model. The baseline model's differential equation has a well-known solution of a GBM model

$$X_t^B = X_0 e^{\left(r(t)-\hat{r}(t)-\frac{1}{2}\sigma_X^2\right)t+\sigma_X dW_t^X}, \tag{4.24}$$

obtained by integrating (4.20).

4.3 Collateral modelling

Collateral can increase or decrease value of the portfolio and thus affect the credit exposure positively or negatively. *Value of a collateralized portfolio* at time t is given by

$$V(t) = \mathrm{NPV}(t) - C(t), \tag{4.25}$$

where $C(t)$ is the collateral balance after the last collateral posting. Since we are considering only variable margin, the *collateral balance* can be expressed as

$$C(t) = \max\{\mathrm{NPV}(t-\delta) - H_C, 0\} - \min\{-\mathrm{NPV}(t-\delta) - H_B, 0\} \tag{4.26}$$

where $\delta > 0$ is the collateral settlement lag, H_B is the collateral threshold of the bank and H_C the collateral threshold of the counterparty. (Chung and Gregory 2019) We are considering zero thresholds for both parties, so the value of the collateralized portfolio, obtained from (4.25) and (4.26), reduces to

$$V(t) = \mathrm{NPV}(t) - \mathrm{NPV}(t-\delta), \tag{4.27}$$

which is an intuitive result, as held collateral at time t in two-way CSA agreement is the total price of trades in the portfolio measured at margin call time. In practice collateral is always lagged, and hence collateral account cannot perfectly track price of the portfolio, under and overshooting regularly, as price of derivatives in the portfolio change continuously.

4.4 CVA for European-style FX contracts with MPoR

To make the exposure expression $V(\cdot)^+$ more explicit, we must make more assumptions about the portfolio construction underlying the exposure. The objective of this thesis is to model CVA WWR with an example contract, which has FX underlying, and the chosen contract type is a cross-currency basis swap as explained in 3.1. One essential property in the valuation of this kind of contract is that the price of the contract is not path-dependent, as the contract does not include any option components with early-exercise rights. Derivative contracts without early-exercises optionalities are called *European-style* derivatives (Li and Mercurio 2016). In following we assume that the $V(\cdot)^+$ includes only European-style FX contracts, because it allows us to discard the explicit default time modelling and still include jump-at-default component in the FX-process (Li and Mercurio 2016). Thus we can model the exposure with slight modifications made to the jump martingale process (4.23) and still calculate CVA with (4.5).

As explained by Li and Mercurio (2016) the key is to consider the jump martingale process as if the counterparty defaulted at time $\tau = t$. This applied to (4.23) and (4.16) gives the *default-conditional FX-process*

$$X_{t|\tau=t} := \left[X_t^B \cdot M_t^J \middle| \tau = t\right] = X_t^B \left(1 + J\right) e^{-\int_0^t \lambda_s J ds},$$

obtained by observing that $\mathbf{1}_{\{t \geq t\}} = 1$ and $\min\{t, t\} = t$ and recalling that the baseline model X_t^B is assumed to be independent of default. More precisely, the baseline model must be scale invariant, which means that a scaling made to the baseline model at time t induces a constant scaling of same size to all values of X_t^B after t (Li and Mercurio 2016).[8] Due to the scalability assumption, we can consider the *post-default FX process* by letting the baseline process evolve Δ time units after the default:

$$(X_{t+\Delta|\tau=t})_{\Delta \geq 0} = X_{t+\Delta}^B \left(1 + J\right) e^{-\int_0^t \lambda_s J ds}. \tag{4.28}$$

The post-default FX process is an important tool, because it can be used to obtain directly the exposure value after MPoR, if we assume that the FX process continues to follow baseline process in the MPoR window. The *pre-default FX process* can be obtained similarly by using (4.23) as

$$(X_{t-\Delta|\tau=t})_{\Delta > 0} = X_{t-\Delta}^B e^{-\int_0^{t-\Delta} \lambda_s J ds}. \tag{4.29}$$

Now we have tools to represent the portfolio value in a form required by (4.5). Let $\mathrm{NPV}(t, X_t)$ denote NPV of a portfolio at time t with FX rate X_t. Following Ruiz (2015, p. 72) we assume that the MPoR window length is constant in time and denote it with $\mathrm{MPoR} \geq 0$. Then we can define the default conditional collateralized portfolio value by using (4.27), (4.28) and (4.29) as

[8] The scalability assumption is satisfied by common stochastic models, like Black-Scholes model the Merton (Merton 1976) jump-diffusion model and Heston (Heston 1993) stochastic volatility model (see for example Li and Mercurio 2016).

$$V_{\mathrm{MPoR}}(t) := \mathrm{NPV}(t + \mathrm{MPoR}, X_{t+\mathrm{MPoR}\,|\tau=t}) - \mathrm{NPV}(t-\delta, X_{t-\delta|\tau=t}), \tag{4.30}$$

where the FX rate in the first term is the *post-default FX process*

$$X_{t+\mathrm{MPoR}\,|\tau=t} = X^B_{t+\mathrm{MPoR}}\,(1+J)\,e^{-\int_0^t \lambda_s J ds}, \tag{4.31}$$

and the FX rate in the second term is the *pre-default FX process*

$$X_{t-\delta|\tau=t} = X^B_{t-\delta} e^{-\int_0^{t-\delta} \lambda_s J ds}.$$

The CVA for collateralized portfolio of European-style FX deals with MPoR is then

$$\mathbf{CVA} = -(1-R)\,\mathbb{E}^{\mathbb{Q}}\left[\int_0^T \frac{V_{\mathrm{MPoR}}(t)^+}{B_t} dS_t^\tau\right], \tag{4.32}$$

obtained by combining (4.5) and (4.30).

4.5 CVA discretization

To make the simulation of exposure values in (4.32) tractable, the formula must be discretized. In this section the discretization is formulated for the default conditional exposure $V_{\mathrm{MPoR}}(\cdot)^+$, but same discretization applies to general $\mathcal{F}_t$-measurable $V(\cdot)^+$.

Following Chung and Gregory (2019) we set the *exposure grid* with M time steps in interval $t \in [0, T]$ as

$$\{t_i; i = 0, 1, \ldots, M\}, \tag{4.33}$$

where step size $t_{i+1} - t_i$ could potentially be variable.[9] Ideally one would calculate the exposure daily (Ruiz 2015, p. 35) and in our case it is a natural choice since the margin call frequency is also daily. Even time-step frequency also simplifies notation used to express the exposure when collateral is considered.

For Monte Carlo simulations[10], the general CVA formula (2.7) must be adapted to time discretized form, which follows the time grid (4.33). We will use the rectangle rule for discretization of the integral, where each exposure V^+ is calculated at the exact step t_i.

[9] In practice it is common to use unevenly spread time buckets in which time points are close together near beginning and spread out so that the exposure calculation task remains computationally feasible, when portfolio maturity is long and amount of trades is high (Ruiz 2015, p. 50).

[10] see chapter 6 for discussion about the Monte Carlo method

Following Chung and Gregory (2019), the integral of equation (4.5) can then be approximated as sum

$$\mathbf{CVA} \simeq -(1-R)\,\mathbb{E}^{\mathbb{Q}}\left[\sum_{i=0}^{M-1} \frac{V_{\mathrm{MPoR}}(t_i)^+}{B_{t_i}}\left(S^{\tau}_{t_{i+1}} - S^{\tau}_{t_i}\right)\right]. \tag{4.34}$$

By recalling linearity of expectation and the definition (4.3) of the Azéma's supermartingale in the Cox construction $S^{\tau}_t = e^{-\int_0^t \lambda_s ds}$ yields

$$\mathbf{CVA} \simeq (1-R)\sum_{i=0}^{M-1} \mathbb{E}^{\mathbb{Q}}\left[\frac{V_{\mathrm{MPoR}}(t_i)^+}{B_{t_i}}\left(e^{-\int_0^{t_i} \lambda_s ds} - e^{-\int_0^{t_{i+1}} \lambda_s ds}\right)\right]. \tag{4.35}$$

If we assume that the instantaneous correlation between exposure $V(t)^+$ and default intensity λ_t is zero, which in our setting means that $\rho_{X,\lambda} = 0$, we can decompose the expectation as

$$\mathbf{CVA} \simeq (1-R)\sum_{i=0}^{M-1} \mathbb{E}^{\mathbb{Q}}\left[\frac{V_{\mathrm{MPoR}}(t_i)^+}{B_{t_i}}\right]\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^{t_i} \lambda_s ds} - e^{-\int_0^{t_{i+1}} \lambda_s ds}\right], \tag{4.36}$$

which simplifies further to

$$\mathbf{CVA} \simeq (1-R)\sum_{i=0}^{M-1} \mathbb{E}^{\mathbb{Q}}\left[\frac{V_{\mathrm{MPoR}}(t_i)^+}{B_{t_i}}\right]\left(e^{-\int_0^{t_i} h(s)ds} - e^{-\int_0^{t_{i+1}} h(s)ds}\right),$$

by using the deterministic default intensity obtained by setting $e^{Z_t} = 1$ and observing from (4.8) that the intensity is then

$$\lambda_t = h(t). \tag{4.37}$$

The same discretization of CVA in the deterministic intensity setup is observed directly from (2.6) by using (4.15), which connects default probability curve to hazard rate function as

$$P_{\tau}(t) = 1 - e^{-\int_0^t h(s)ds}. \tag{4.38}$$

Even though (4.35) is an approximation the only error with respect to the analytical formula (4.32) comes from the discretization of the integral. If we let number of time-steps M in the interval $t \in [0,T]$ become very large, the right side of (4.35) converges to the exact value of CVA.[11]

4.6 Exposure in cross-currency swap

In this section cross-currency swap exposure valuation components are explained. The valuation and theory behind it is explained only briefly, as the focus of this thesis is not in derivative instrument valuation. More detailed explanations can be found for example from Boenkost and Schmidt (2005), Brigo et al. (2013), and Burgess (2018).

Using the same notation as before, the immediate delivery rate of two currencies in the market is the spot exchange rate, in the modelling framework represented by the process X_t. As explained in section 3.1, the vanilla *cross-currency swap* (CCS) construction has an exchange of principals in the interception of transaction $t = 0$ and at the maturity of the contract $t = T$. Usually the principal amounts, known as domestic notional N_{dom} and foreign notional N_{for}, are set to be $N_{\text{for}} = N_{\text{dom}}/X_0$, which means that the amounts are fair (Boenkost and Schmidt 2005). The side in the contract receiving domestic currency notional at the interception is the *domestic trade leg* and the side receiving foreign currency is the *foreign trade leg*. Typically these notional amounts are rebalanced during the lifetime of the contract (Burgess 2018), but for simplicity the notional resets are not considered in this thesis.

As explained in section 3.1, both parties pay interest during the contract, which in case of a *cross-currency basis swap* (CCBS) are floating rates for both legs. Thus CCBS can be understood as an exchange of floating rate bonds, because the cash flows of CCBS mimic cash flows of bonds in contract currencies (Boenkost and Schmidt 2005). Let $\Pi_{\text{CCS,dom}}(t, X_t)$ be the value of domestic leg at time t and with FX spot rate X_t. Then from the viewpoint of the party paying the domestic leg, the *value of the foreign leg* is

$$\Pi_{\text{CCS, for}}(t, X_t) = -\Pi_{\text{CCS, dom}}(t, X_t) \tag{4.39}$$

as explained by Kumar et al. (2021).

Domestic leg pays n cash flows at dates $t_1 < t_2 < \cdots < t_n$, at year fractions Δ_{dom}. Following Burgess (2018) and (2021) we can express the *present value of coupons* at time t as

$$\Pi_{\text{Cpn,dom}}(t) = N_{\text{dom}} \sum_{j=1}^{n} \mathbf{1}_{t_j \geq t}\, r_{\text{dom}}(t_j) \Delta_{\text{dom}} D_{\text{dom}}(t, t_j), \tag{4.40}$$

[11] Approximations which are made by simplifying the analytical form of CVA are presented for example by Li and Mercurio (2016). These approximations do not necessarily converge to the exact value of CVA when M grows, but they can significantly simplify simulations and calculations.

where $r_{\text{dom}}(t_j)$ is the interest rate fixed for the coupon period ending at t_j and starting at $t_j - \Delta$, and $D_{\text{dom}}(t, t_i)$ is a discount factor for given cash flows. For CCBS[12] the interest rate is defined as

$$r_{\text{dom}}(t_j) = l_{\text{dom}}(t_j) + s_{\text{dom}}, \tag{4.41}$$

where $l_{\text{dom}}(t_j)$ is a *forward rate* for period $[t_j - \Delta, t_j]$ and s_{dom} is a spread over the floating rate. Forward rate is a projected interest rate for future time window (Boenkost and Schmidt 2005). Usually the spread s is added only over the floating rate of one leg (Burgess 2018) and if the CCBS price is set fair it is the same as the cross-currency basis spread α, as explained in section 3.1. The discount factor $D(t, t_j)$ is not necessarily same as the risk-free rate of given currency: for example, collateralization affects the discount factor (Burgess 2017).

In (4.41) we implicitly assume that the forward interest rates $l_{\text{dom}}(\cdot)$ follow a deterministic term structure over time. Since we assume deterministic interest rates also in the modelling approach, the assumption is in line with other modelling choices. To keep the valuation consistent with arbitrage-free pricing the future cash flows of foreign currency must be converted to domestic currency by using forward FX rate, which is dependent on spot FX rate: According to Burgess (2017) the *forward FX rate process* $F_t(T, \cdot)$ measured at time t for time T is implied from discount factors and the spot FX rate [13] as

$$F_t(T, X_t) = X_t \frac{D_{\text{for}}(t, T)}{D_{\text{dom}}(t, T)}. \tag{4.42}$$

Thus, the FX forward rate changes among the FX process X_t even though discount factors follow deterministic term structure in the modelling framework.

By applying (4.42) and (4.40) the *price of foreign currency leg coupons* can be expressed as

$$\begin{aligned} \Pi_{\text{Cpn,for}}(t, X_t) &= N_{\text{for}} \sum_{j=1}^{m} \mathbf{1}_{t_j \geq t}\, F_t(t_j, X_t) r_{\text{for}}(t_j) \Delta_{\text{for}} D_{\text{dom}}(t, t_j) \\ &= N_{\text{for}} \sum_{j=1}^{m} \mathbf{1}_{t_j \geq t}\, X_t r_{\text{for}}(t_j) \Delta_{\text{for}} D_{\text{for}}(t, t_j), \end{aligned} \tag{4.43}$$

where m is number of cash flows the foreign leg pays at dates $t_1 < t_2 < \cdots < t_n$, at

[12]The value of fixed-float or fixed-fixed CCS can be obtained as a special case of the float-float CCS, where the coupon rate is set to be fixed $r(t_j) = r$ for one or both legs (Burgess 2018)

[13]Due to imperfections it might be that the forward price replication argument used to imply forward rates does not exactly match observed prices (see for example Cornell and Reinganum 1981 for empirical study of observed differences).

year fractions Δ_{for}. We will use spread only for the domestic currency leg, so the interest rate of foreign notional is set to be $r_{\text{for}}(t_j) = l_{\text{for}}(t_j)$. The foreign currency coupons are discounted with domestic discount factor, because they are converted to domestic currency with forward FX rate.

By using (4.42) the *price of notional exchanges of domestic currency* is according to Burgess (2018) given by

$$\Pi_{\text{Exch,dom}}(t) = \underbrace{N_{\text{dom}}\,\mathbf{1}_{t=0}}_{\text{Upfront exchange}} - \underbrace{N_{\text{dom}} D_{\text{dom}}(t,T)}_{\text{Final exchange}}, \tag{4.44}$$

when notional resets are not considered. Similarly, the *value of foreign currency notional exchanges* is

$$\begin{aligned} \Pi_{\text{Exch,for}}(t, X_t) &= N_{\text{for}} X_0\,\mathbf{1}_{t=0} - N_{\text{for}} F_t(T, X_t) D_{\text{dom}}(t,T) \\ &= N_{\text{for}} X_0\,\mathbf{1}_{t=0} - N_{\text{for}} X_t D_{\text{for}}(t,T), \end{aligned} \tag{4.45}$$

where both exchanges are converted to domestic currency. Now the value of the CCBS at time t for the party paying the domestic leg is observed by combining (4.40), (4.43), (4.44) and (4.45) as

$$\Pi_{\text{CCS, dom}}(t, X_t) = \Pi_{\text{Exch,dom}}(t) - \Pi_{\text{Exch,for}}(t, X_t) - \Pi_{\text{Cpn,dom}}(t) + \Pi_{\text{Cpn,for}}(t, X_t) \tag{4.46}$$

by applying results of Burgess (2018) in our framework.

If the portfolio with a counterparty consist only one CCBS transaction, the counterparty is holding the foreign leg and we are calculating CVA at the interception of the transaction, the default conditional exposures can then be observed by using CCBS value function (4.46) in place of NPV in (4.30) as $NPV(t, X_t) := \Pi_{\text{CCS, dom}}(t, X_t)$, which yields

$$V_{\text{MPoR}}(t)^+ = \max\left\{\Pi_{\text{CCS, dom}}(t + \text{MPoR}, X_{t+\text{MPoR}\,|\tau=t}) - \Pi_{\text{CCS, dom}}(t-\delta, X_{t-\delta|\tau=t}), 0\right\} \tag{4.47}$$

for an *exposure of collateralized CCBS* calculated at time $t+\text{MPoR}$ with simulated default conditional FX process up to $t + \text{MPoR}$ and other market information available at the interception of the transaction.

5. DATA AND MODEL CALIBRATION

In this chapter, the model defined in chapter 4 is calibrated. After the portfolio structure is defined the components to be calibrated are:

(i) the intensity model (4.14),

(ii) the FX model's baseline component (4.24),

(iii) the FX model's jump martingale component (4.23),

(iv) the cross-currency swap valuation model (4.46), and

(v) the discretized CVA WWR model (4.35).

First three components define the *stochastic hybrid model for market credit dependency*, which is aiming to capture the wrong-way risk effect. The stochastic hybrid model requires joint calibration of the components, namely the jump martingale process and constant correlation, together modelling the WWR effects. Thus, calibration of these components is separated in this chapter into own section 5.3.

For counterparty specific parameters illustrative values are used, since the objective is to remain at general level. The example contract is a one year cross-currency basis swap with domestic currency being EUR and foreign USD. The holder of EUR leg receives semi-annual USD coupons based on USD SOFR and pays quarterly EUR coupons based on 3-month Euribor. The calibration is done against general market data observed on 31.3.2023 and 360-day calculation convention is used. The data sources and details are described in appendix A.

5.1 Intensity process calibration

In the stochastic intensity process (4.14) we have two parameters to be calibrated and the deterministic hazard rate function to be matched against observed credit default swap spreads. Intensity model parameters can be calibrated by using maximum likelihood estimation against historical CDS data (Kumar, Markus, and Hari 2021). The hazard rate function values are not directly observable from market. However, according to Hull and White (2012) the *risk neutral average hazard rate* $h(\bar{T})$ over time frame $[0, T]$ can be estimated by using the *credit triangle approximation*

$$\bar{h}(T) = \frac{S(T)}{1-R}, \tag{5.1}$$

where $S(T)$ is the observed single name credit default swap spread for maturity T.[1] For illustrative purposes, we will assume a constant hazard rate h of 3%, the intensity volatility σ_λ of 50%, and mean reversion a of 0.01%, motivated by Brigo et al. (2019) and the fact that similar hazard rate values can be extracted from CDS quotes referring to a systemically important European bank. Figure 5.1 shows a single simulation path of stochastic intensity with given parameter values and empirical mean of 100 000 simulation paths.

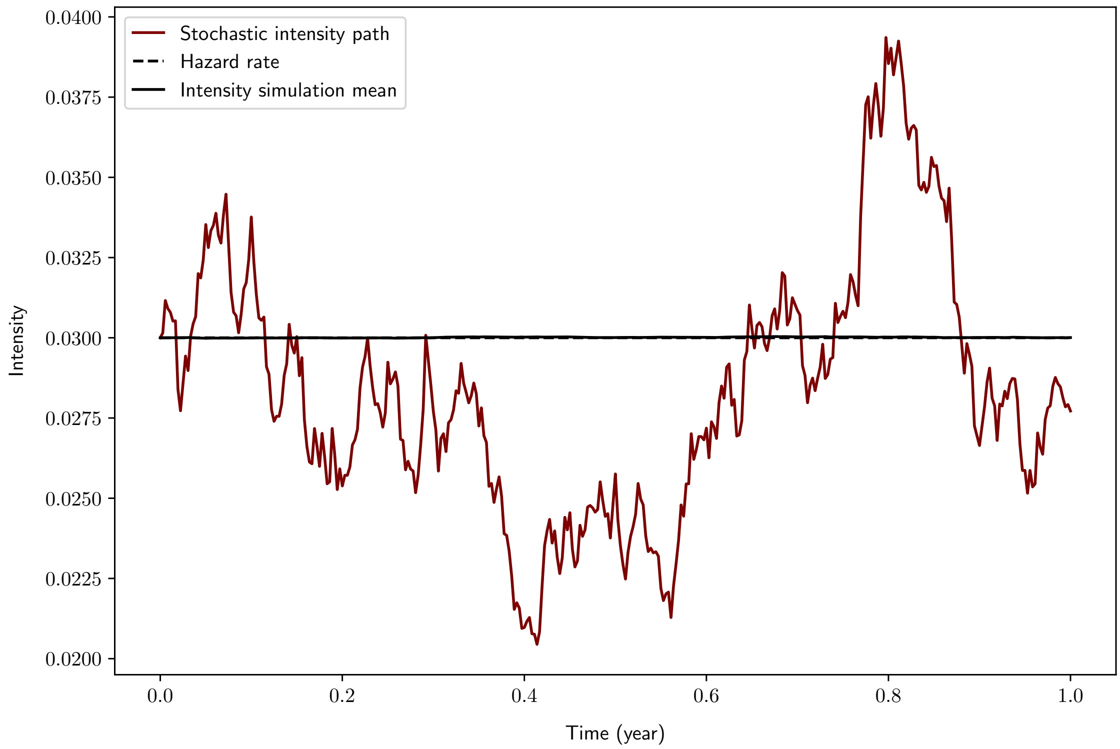

Figure 5.1. *One simulation path of stochastic default intensity and mean of 100 000 simulation paths.*

As we can see, the process fluctuates a lot due to high value of volatility parameter. However, it tends to revert toward the hazard rate function. The empirical mean of the intensity agrees well with the hazard rate, which indicates that the modelling choice of selecting the equilibrium level to be $m = -\sigma_\lambda^2/2a$ is successfully ensuring that the expected value of the intensity remains near the value of the deterministic hazard rate function $h(t)$.

The figure 5.2 shows the survival probability process derived from the same stochastic intensity path shown in the figure 5.1.

[1] A more rigorous method than assuming constant hazard rate over any time horizon T is to bootstrap the hazard rate curve from CDS spreads and assume constant hazard rate between subsequent CDS maturities (Castellacci 2008).

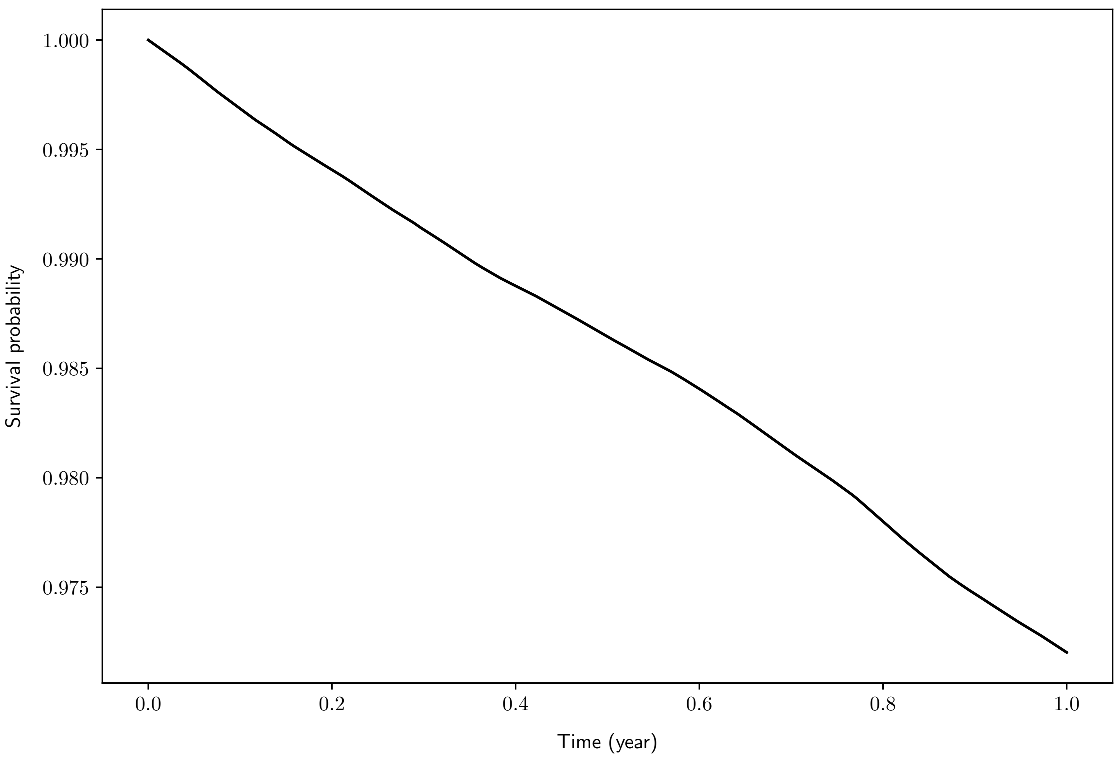

Figure 5.2. *Survival probability function derived from the stochastic intensity simulation*

As we can see, the fluctuation effect of stochastic intensity is damped when a transformation to survival probability is performed and in the survival probability decreases fairly linearly.

5.2 FX process calibration

In the FX process we have two components, as explained in chapter 4. The baseline model (4.24) has two parameters, the initial FX rate X_0 and volatility σ_X, to be calibrated, and two deterministic functions: domestic and foreign risk-free short rates, $r(t)$ and $\hat{r}(t)$ respectively. These are calibrated against market data: initial FX rate from FX quotes, FX volatility with implied volatility of USD/EUR FX option and risk-free rates against yield curves.

The initial FX rate observed on the valuation date is extracted from the *European Central Bank's* (ECB) *Euro foreign exchange rates Time series* (European Central Bank 2023). In the ECB's dataset the euro is the *currency* and the US dollar is the *valuation currency*, so the quotes are EUR/USD. We are using the euro as a valuation currency, so the reciprocal of the data is used. The observed initial FX rate X_0 at 31.3.2023 is 0.91954. USD/EUR FX history is shown in figure 5.3.

While the risk-neutral FX volatility can be calibrated by using implied volatilities of FX options, we can estimate the historical volatility under the real-world probability measure

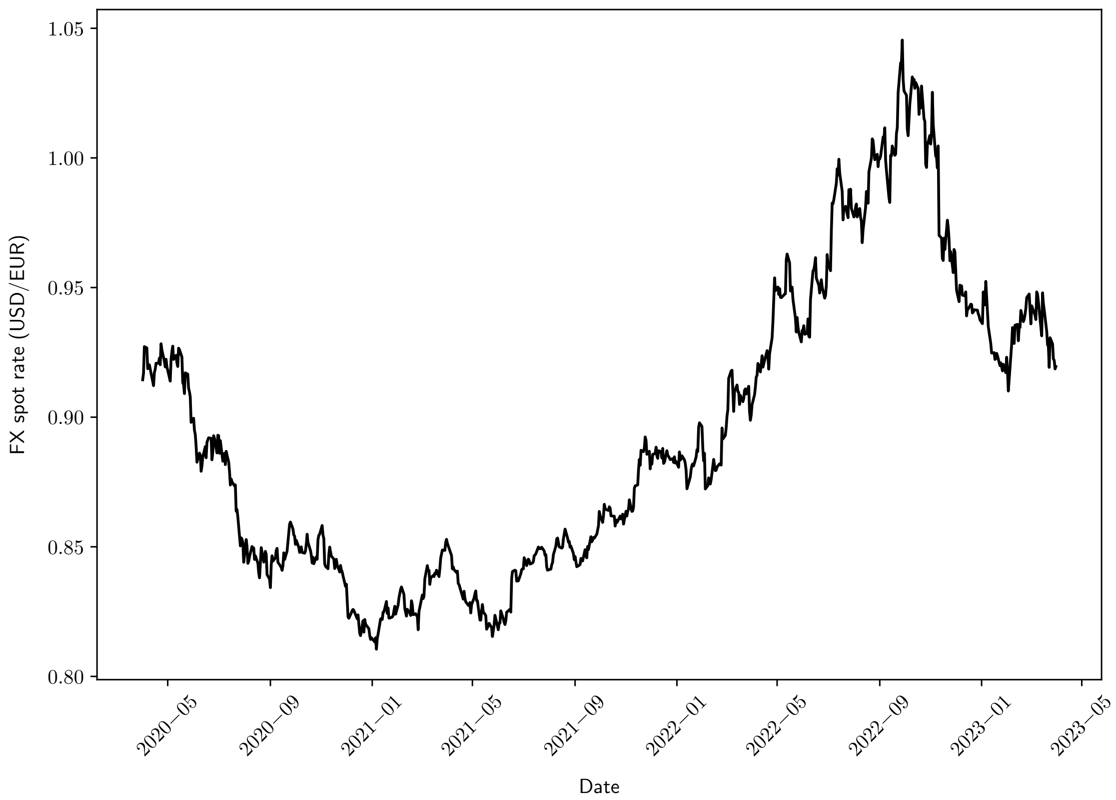

Figure 5.3. *The USD/EUR FX rate time series from 1.4.2020 till 31.3.2023*

$\mathbb{P}$ for comparison purposes. The estimation can be done from daily log returns of the FX rate. The FX rate log-return summary statistics are presented in appendix A. Figure 5.4 presents a histogram of the log returns. As we can see from the histogram and appendix, log returns are not exactly normal in the given time frame, but the shape of the distribution resembles more the Student's T-distribution, with thick tails. The minimum value of the return, -3.5% shows that large intraday changes in relative value of USD/EUR FX rate are possible, but quite rare. The annualized volatility estimated from the log returns is 0.0967. The implied volatility of at-the-money FX-option with remaining maturity of one year is approximately 0.805, which will be used as a value of parameter σ_X.

For discounting domestic currency (EUR) cash flows, the *Euro short-term rate* (€STR) is used as reference rate, as it is the standard near risk-free rate for the euro (Huerga et al. 2022). Foreign currency (USD) cash flows are discounted with the SOFR[2]. The risk-free term structures can be derived from the markets prices of derivatives linked to given risk-free rates, for example from overnight index swaps and futures (Schrimpf and Sushko 2019). Figure 5.5 shows continuously compounded risk-free rate term structure for different tenors interpolated from the derivative quotes. As we can see, the dollar

[2]The SOFR is a *secured* risk-free rate, as it reflects conditions in collateralized markets whereas €STR reflects uncollateralized conditions (Schrimpf and Sushko 2019). The rationale for using secured rate as a basis for risk-free rate in the USA markets is discussed in the Federal Reserve Bank of New York's (2018) report.

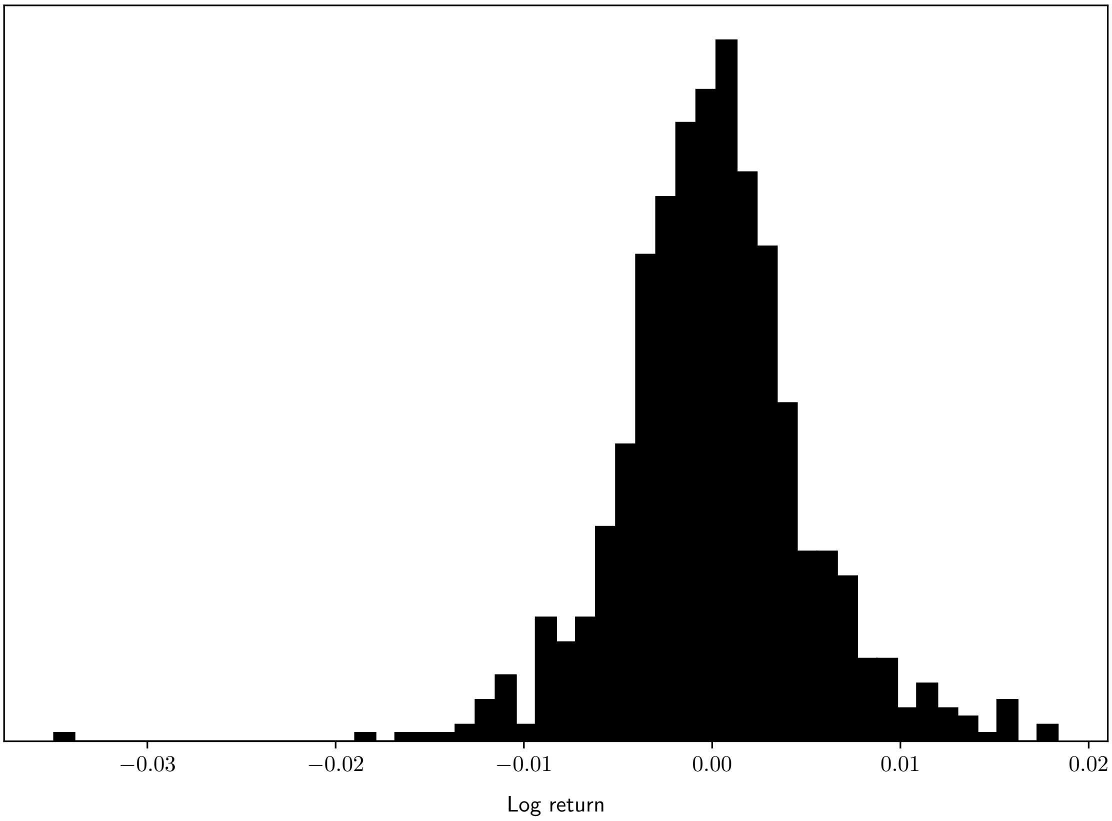

Figure 5.4. *The USD/EUR FX rate log return histogram of FX rates from 1.4.2020 till 31.3.2023*

area interest rates are higher, which will affect the drift term of the baseline model (4.24). The drift of the FX process is then downward sloping as can be seen from figure 5.6. The figure shows also how the jump compensator term A_t in (4.19) affects the drift of the USD/EUR rate pulling it downward, because the assumed jump-at-default would be positive.

5.3 Wrong-way risk effect calibration

The wrong-way risk effect of the hybrid model is summarized by two parameters: the jump parameter J of the jump martingale process (4.23) representing the expected devaluation of the domestic currency upon default, and the instantaneous correlation $\rho_{X,\lambda}$ between diffusion components of the intensity and FX processes. Obviously, both parameters are counterparty-dependent, and nature of the counterparty can heavily affect estimates of these parameters. Thus, instead of calculating the CVA value for single counterparty, multiple values of CVA are calculated in the next chapter for different values of both parameters for illustrative purposes. This gives also an idea of CVA's sensitivity against both parameters.

Ideally, one would estimate both parameters risk-neutrally from market data for each

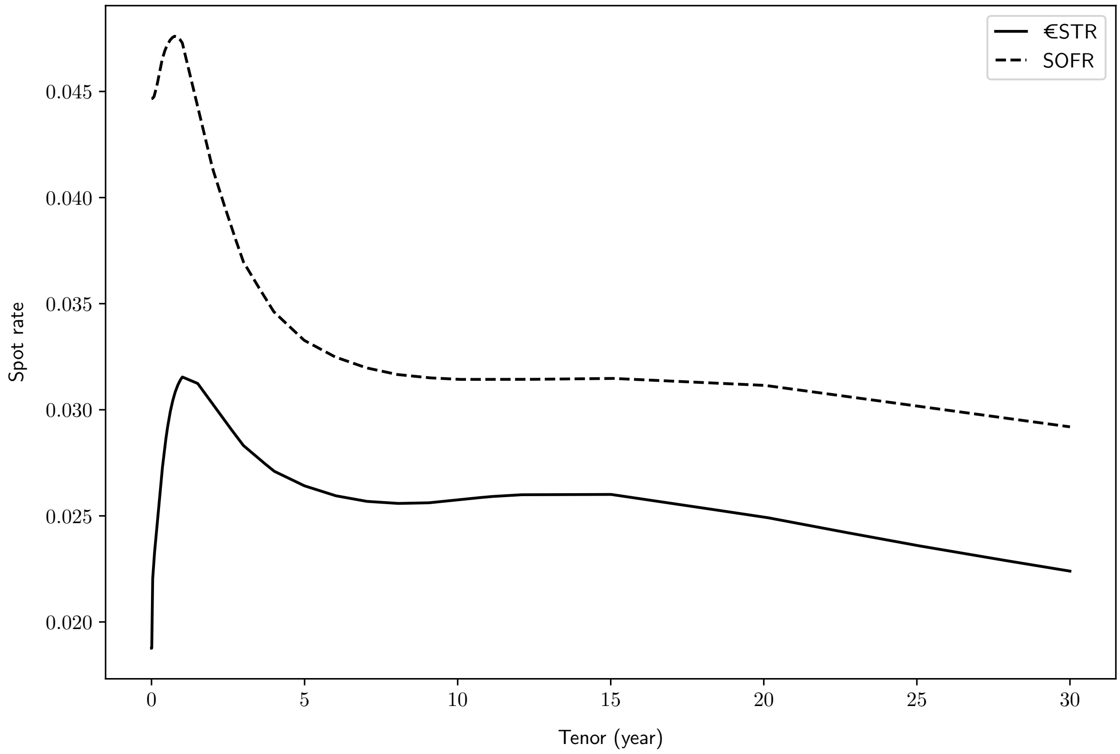

Figure 5.5. *Interpolated risk-free continuously compounding rates term structures for domestic and foreign currency on 31.3.2023*

counterparty in order to price CVA with WWR. The risk-neutral market implied calibration of the jump parameter can be done if there exists credit-default instruments denominated in domestic and foreign currency linked to same debt instrument issued by the counterparty (Du and Schreger 2016). An intuition is that if there exists a basis spread $S_{\text{for}}(T) - S_{\text{dom}}(T)$ between par spreads of CDS contracts denoted in foreign currency S_{for} and domestic currency S_{dom}, it must be explained by some driver of the default intensity (Ehlers and Schönbucher 2006). It is shown in the literature that the value of CDS denominated in different currency than the assets of the systemically important reference entity is typically higher than the CDS denominated in the same currency as the assets of the entity (Ehlers and Schönbucher 2006; Pykhtin and Sokol 2013; Du and Schreger 2016; Brigo, Pede, and Petrelli 2019; Augustin, Chernov, and Song 2020). The CDS denominated in the non-domestic currency of the reference entity is called a *quanto CDS* contract (Du and Schreger 2016).

Full derivation of the calibration formula under foreign and domestic risk-neutral measures are not provided here. However, the idea is to explain the difference in values of intensities λ and $\hat{\lambda}$ by expressing prices of CDS contracts under our modelling assumption (Brigo, Pede, and Petrelli 2019). It can be shown by using a change of measure argument that under our modelling assumptions the default intensity under the *foreign risk-neutral measure* $\hat{\mathbb{Q}}$ is given by

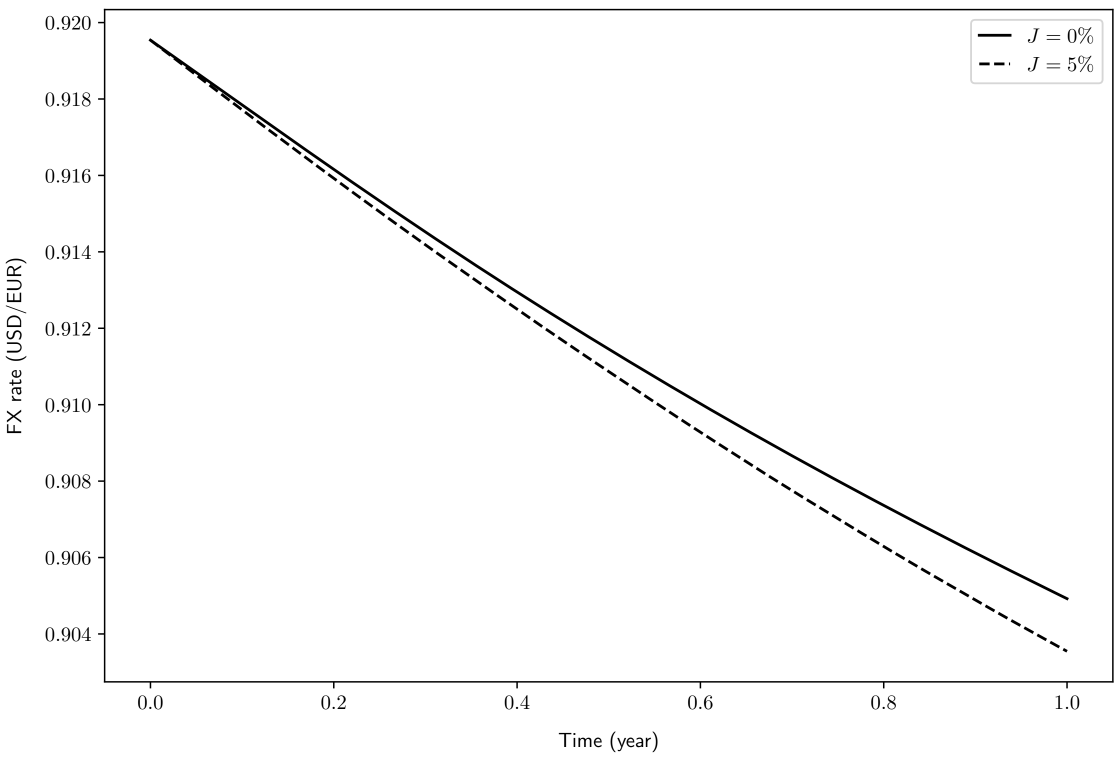

Figure 5.6. *FX process drifts obtained by setting* $\sigma_X = 0$, $h(t) = 0.03$ *and assuming that the jump does not occur in the time window. The process with jump has a higher absolute drift value due to the martingale drift component compensating the potential (upward) jump of the FX process.*

$$\hat{h} = (1 + J)h$$

for constant hazard rate $h(t) = h$ (Chung and Kwok 2016). Using the credit triangle approximation (5.1) yields then

$$J = \frac{S_{\text{for}} - S_{\text{dom}}}{S_{\text{dom}}}$$

as an easy-to-use approximation formula for the relative jump factor.

The correlation parameter can also be estimated from the credit spreads by using a heuristic formula provided by Elizalde et al. (as cited in Brigo et al. 2019):

$$\frac{S_{\text{for}}(T) - S_{\text{dom}}(T)}{S_{\text{dom}}(T)} \approx J + \sigma_X \sigma_\lambda \rho^{\text{Imp}}_{X,\lambda} A(T), \tag{5.2}$$

where $\rho^{\text{Imp}}_{X,\lambda}$ denotes implied correlation and $A(T)$ is the *risky annuity of a domestic currency denominated CDS contract* with tenor T, which can be expressed as

$$A(T) \approx \frac{1 - e^{-T(r(T)+\bar{h}(T))}}{r(T) + \bar{h}(T)}$$

by using the credit triangle approximation for the hazard rate (Chung and Gregory 2019). The correlation parameter can then be decomposed from the jump effect by subtracting (5.2) of different tenors (Brigo, Pede, and Petrelli 2019): the jump parameter cancels out and by rearranging we get

$$\rho_{X,\lambda}^{\mathsf{Imp}} \approx \frac{1}{\sigma_X \sigma_\lambda (A(T_2) - A(T_1))} \left(\frac{S_{\mathsf{for}}(T_2) - S_{\mathsf{dom}}(T_2)}{S_{\mathsf{dom}}(T_2)} - \frac{S_{\mathsf{for}}(T_1) - S_{\mathsf{dom}}(T_1)}{S_{\mathsf{dom}}(T_1)} \right), \quad (5.3)$$

where $T_2 > T_1$. Regardless of the heuristic nature of the correlation approximation (5.3) Brigo et al. (2019) find it to work acceptably and suggest that it can be used to produce rough approximations for the correlation parameter.

Another way of estimating the instantaneous correlation parameter is to measure a historical correlation between log-returns of the FX rate and the CDS spread $S_{\mathsf{dom}}(t)$. Based on studies of Brigo et al. (2019), and Chung and Gregory (2019) one will not necessary expect the historical correlation to agree with the implied one calculated with (5.3): Brigo et al. (2019) use a time window of 50 days for linear correlation and obtain that the absolute value of the historical correlation tends to be higher than the model implied. Chung and Gregory obtain similar results, and in case of neglecting the jump effect in (5.2) they report correlations over the boundaries $[-1, 1]$ which suggests that the implied correlation alone is not sufficient to explain the observed quanto basis spreads between S_{for} and S_{dom}.

5.4 Cross-currency swap valuation model inputs

As discussed in section 4.6 the CCBS valuation requires forward floating rates l_{dom} and l_{for} for coupon periods, the spead s_{dom} for the domestic leg and CSA-specific discount factors D_{for} and D_{dom}, which can be constructed as explained by Burgess (2017). We assume that the collateral is posted in EUR cash by both parties. The derived CSA discount curves are presented in figure 5.7. The figure shows clearly how higher interest rate expectations in the foreign currency area affect the discount factors.

The domestic coupons are based on three-month Euribor and foreign on the SOFR rate. The rates are fixed for each period in the beginning of the interest period[3] so the first coupon period rates are already known at the interception of the transaction. However,

[3] In practice, the interest rate is fixed for the coupon period from the floating rate observed few bank days before beginning of the period.

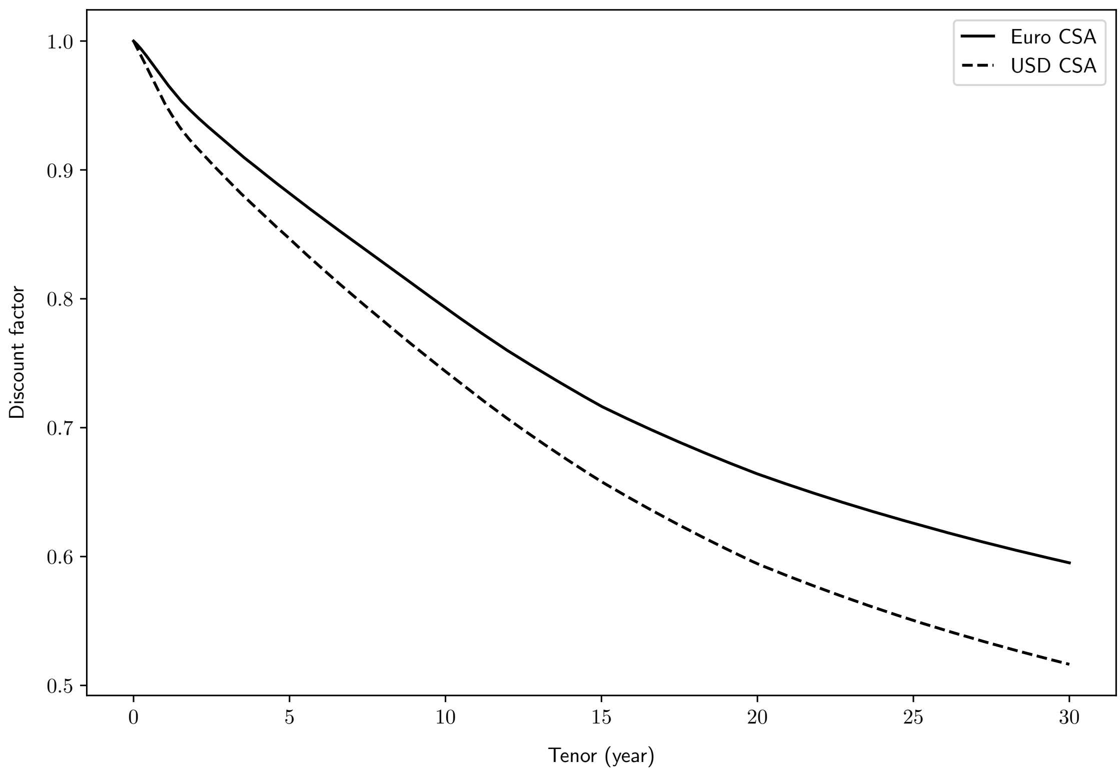

Figure 5.7. *Interpolated CSA discount curves on 31.3.2023 for domestic and foreign currency cash flows under the Euro cash collateralization*

for next coupon periods the rates must be forecasted. The Euribor forward rates can be bootstrapped for example from futures prices (Bernoth and Hagen 2004) and SOFR rates as explained earlier in this chapter. Interpolated forward rate curve of three-month Euribor and SOFR are shown in figure 5.8. The rates are simple annual rates, whereas the SOFR rate in figure 5.5 was presented in the continuously compounded form as required by the baseline FX model.

After the valuation model construction, the spread s_{dom} over the domestic leg floating rate can be set. One can simply use the CCBS valuation formula (4.46) with parameters $t = 0$ and $X_t = X_0$ and numerically set the value of s_{dom} so that the value of the swap is near 0 at the interception of the transaction. With this method $s_{\text{dom}} \approx -0.047194$ is obtained. From figure 5.8 and the CCBS coupon formula (4.40) we can see that the domestic leg coupons are negative with the given s_{dom} value.

5.5 CVA calculation inputs

Since the stochastic hybrid model is already calibrated in previous sections, the discretized CVA formula (4.35) has only the risk-free rate process in B_t to be calibrated and the recovery rate R to be chosen. In addition, the default conditional exposure of CCS (4.47) has the margin period of risk parameter MPoR to be set.

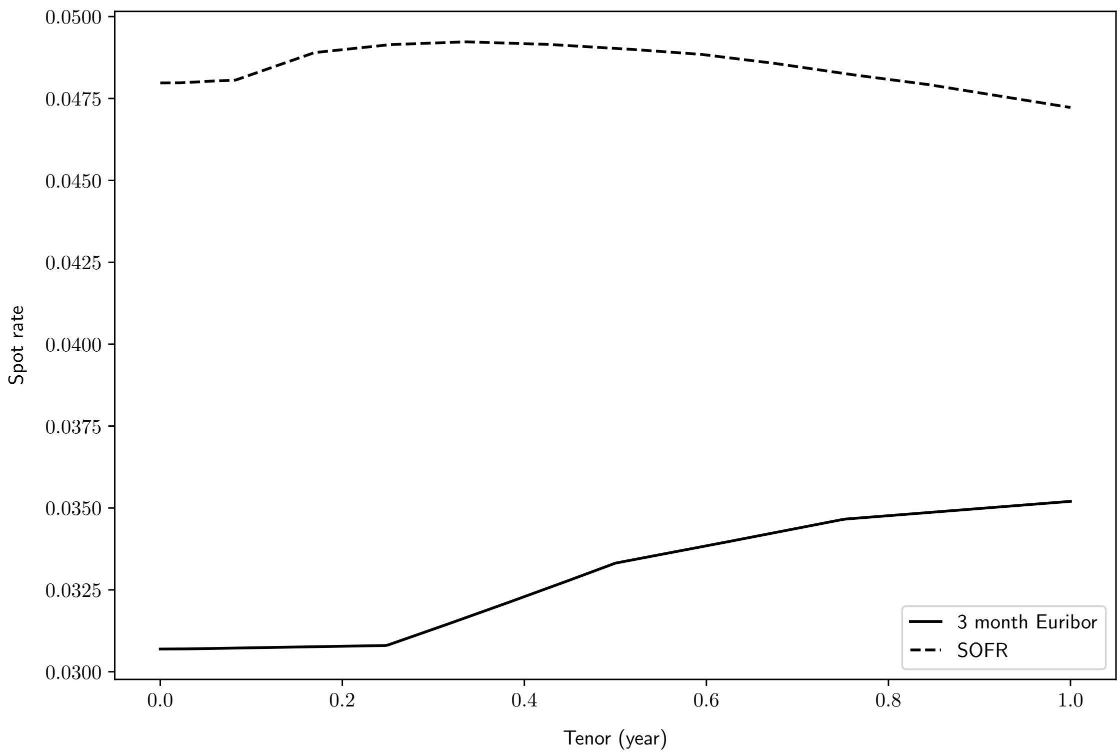

Figure 5.8. *Interpolated forward rate curves on 31.3.2023 for coupon rates*

The recovery rate is assumed constant as explained in section 2.1.2. The value is chosen to be 0.3, which can be understood as a fraction of the exposure in case of default that can be recovered until the end of MPoR window. As the CVA formula (4.35) is a linear function of R, the value of recovery rate does not affect the relative effect of WWR on CVA. However one must take into account that if the credit triangle approximation (5.1) is used to extract hazard rates, the observable recovery rate might affect CVA values through the dynamics of FX rate and default intensity (4.22).

The money market account B_t is chosen to have a initial value 1 at time $t = 0$. The corresponding short rate is assumed to follow deterministic term structure of domestic risk-free rate $r(t)$. Thus we can use same term structure of €STR as a proxy of risk-free rates as is used in section 5.2 for the FX model.

The value of MPoR can potentially affect value of CVA of a collateralized portfolio materially in case of adverse market movements after the default. During the MPoR the counterparty is not posting collateral, so the exposure can increase significantly as the value of the instruments in the portfolio deviates from the pre-default values. The value of MPoR is chosen to be 10 days. The background of MPoR assumptions is explained in section 3.3.

6. SIMULATION AND CVA MEASUREMENT

In this chapter the simulation results are presented and discussed. In first two sections some technical details of the simulations are explained briefly. In the third section the exposure simulation and results are explained and in the fourth section main results of CVA WWR measurements are presented. In the final section sensitivity analysis of results is conducted against few main parameters of the models.

6.1 Monte Carlo approach

The Monte Carlo approach, a method based on the law of large numbers and asymptotic theorems, is used for decades to address problems with unobtainable closed form solutions and chains of events with known transition probabilities (Metropolis and Ulam 1949). Nowadays Monte Carlo techniques are used in many application areas considering quantitative problems, including the fields of economics and finance[1] (Kroese et al. 2014). In the field of quantitative finance, first applications of the Monte Carlo method considered option pricing problems (Boyle 1977). In the risk management the Brownian Monte Carlo method is an essential part of exposure calculation engines (Ruiz 2015, chapter 3).

Monte Carlo method in this thesis is used to approximate the risk-neutral expectation in the discretized CVA formula (4.35). Recall that the *expectation* of an arbitrary measurable function $f(\cdot)$ of a random variable Y is given by integral

$$\mathbb{E}[f(y)] = \int f(y)g(y)dy, \tag{6.1}$$

where $g(y)$ is associated continuous density function. According to Joshi (2003, p. 191), the Monte Carlo method allows to approximate the integral (6.1) as a long run average, because the *law of large numbers* says that

$$\lim_{j\to\infty} \frac{1}{N} \sum_{j=0}^{N} f(Y_j) = \mathbb{E}[f(Y)], \tag{6.2}$$

where Y_n is a sequence of random draws of Y. By using the central limit theorem it can

[1] For an extensive review of Monte Carlo methods in applied finance see (Glasserman 2004).

be shown that the distribution of the numerical Monte Carlo error tends to the standard normal distribution with large values of N and the error measured with standard deviation is order of $\sqrt{N}$ (Boyle 1977). Thus, to decrease the standard deviation by factor of then, we have to take hundred times more samples of $f(Y)$. According to Ruiz (2015, p. 33), typical values of N in counterparty credit risk calculations range from 1 000 to 10 000.

We calculate a Monte Carlo estimate for each point t_i of the time grid with samples of two normally distributed random variables $W_{t_i}^{\lambda}$ and $W_{t_i}^{X}$ for default intensity and FX process respectively. Thus the input of scalar valued "function" within the expectation, with fixed deterministic values of other parameters, is a random vector $\left(W_{t_i}^{\lambda}, W_{t_i}^{X}\right)$. To obtain correlated Brownian motions, we can simulate a 2-dimensional random vector $W = \left(W^{(1)}, W^{(2)}\right)$ with standard normal independent draws W^j and take a linear combination

$$Y = \rho W^{(1)} + \sqrt{1-\rho^2} W^{(2)} \tag{6.3}$$

to obtain another Brownian motion Y_t, which is correlated with $W^{(1)}$ with the linear correlation coefficient ρ (Joshi 2003, p. 262). The expectation formula (6.1) extends same way to a scalar function $f(\cdot)$ when Y is a random vector, so the Monte Carlo method applies through the law of large numbers (6.2) similarly.

In this thesis 10 000 paths are simulated for each CVA calculation. As explained by Chung and Gregory (2019), the exposure calculation must be run twice to calculate the collateralized portfolio values. The exposure engine calculates two $N \times M$ matrices, which have $N = 10000$ values for each t_i. The subtraction in the CCBS exposure formula (4.47) is performed element-wise for each post- and pre-default value simulated for time step t_i and floored with zero. Now the Monte Carlo estimate for the *collateralized post-default CCBS exposure* at time t_i is obtained by averaging over all 10 000 values for $V_{\mathrm{MPoR}}(t_i)^+$. For uncollateralized portfolio same values are obtained simply by setting the pre-default term representing the collateral balance to zero.

However, unless we can assume an independence of default probabilities and exposure, the averaged post default exposures cannot be used directly to calculate the CVA, because the expectation in CVA formula (4.35) cannot be decomposed to (4.36) in general case, since the weight given by the default probability might be correlated with the exposure. In the discretized CVA formula (4.35) the average is taken of the discounted default conditional exposure at time t_i weighted with associated default probability over time step $t_{i+1} - t_i$.

6.2 Technical implementation

The numerical results are simulated with a self-written *exposure engine*, which allows to obtain both collateralized and uncollateralized exposure values for the specified contract type. As explained in the previous section the exposure calculation is ran twice: the first run is made to evaluate the post-default process in (4.30) for each t_i and the second run to evaluate pre-default values with same simulated random numbers, which are the lagged collateral account values under our assumptions. The collateralized exposure is then simulated as follows:

1. Generate $N \times M$ random numbers from the standard normal distribution and simulate the intensity process with given parameter values.
2. Generate $N \times M$ random numbers from the standard normal distribution, use the correlation formula (6.3) to obtain correlated Brownian motion and simulate the FX process with given parameter values conditional on jump value $J = j$, where j is the constant relative FX jump size.
3. Simulate MtM value of the contract for each entry of the $N \times M$ matrix: Calculate the MtM value of CCBS for each step t_i, by estimating becoming cash flows at $t_{i+1}, t_{i+2}, \ldots, t_{i=M}$ form the contract and discount them to point t_i with corresponding CSA discount factor.
4. Collateral account value (the pre-default MtM): Redo steps (2.) and (3.) but with a jump value $J = 0$.
5. Discounted exposure matrix: Subtract the collateral matrix from the post-default MtM matrix and discount with risk-free discount factor values.
6. Calculate CVA: Calculate element-wise product of the discounted exposure matrix and the intensity process matrix. Calculate mean of each entry t_i in M dimension and then sum along N dimension. Multiply with $(1 - R)$ to obtain the CVA number.

This is a lot more efficient method than explicit simulation of default times[2], since we obtain with a 360-step partition and 10 000 simulation paths 3 600 000 post-default values of collateralized portfolio, instead of single default in approximately 3 percent[3] of simulated paths. However, the used method is also time-consuming, since each time step for both runs the NPV of the portfolio must be evaluated, which requires projection and evaluation of all the future cash flows of the contract(s).

All scripts used to produce numerical results are written with *Python* by using well-known libraries *NumPy* (Harris et al. 2020) and *pandas* (McKinney 2010). The graphs are produced with the *Matplotlib* graphics environment (Hunter 2007).

[2]The default time simulation can also be performed a lot more efficiently than naïvely simulating all exposure paths (see for example Ruiz 2015, p. 171).

[3]Here we assume constant default intensity of 3 percent and maturity of one year.

6.3 Exposure simulation results

In this section few exposure graphs are presented, to understand how the CCBS's default conditional exposure component behaves in the CVA formula over time in the given market environment.

The post-default profiles for collateralized and uncollateralized portfolio when WWR effect is not considered are presented in figure 6.1. The exposure profiles are obtained by

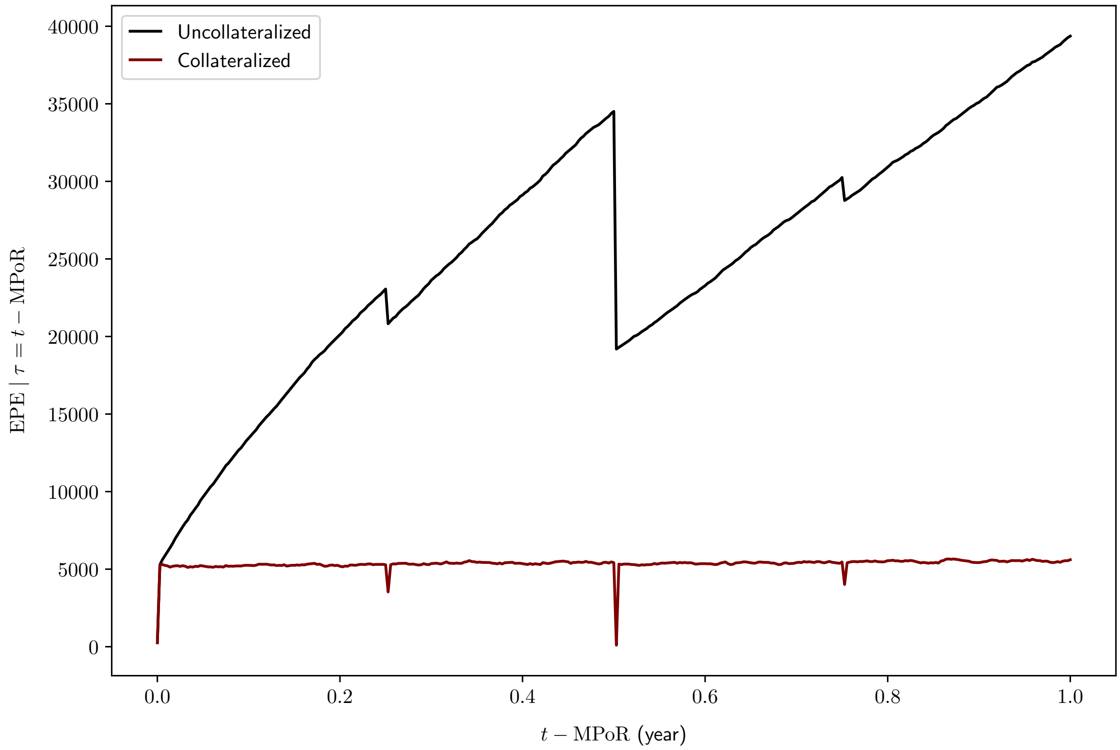

Figure 6.1. *Default conditional expected positive exposure without WWR*

simulating 10 000 paths from (4.47) with WWR parameters set to zero ($J = 0, \rho_{X,\lambda} = 0$). The x-axis starts from the first close-out date, that is $0 + \mathrm{MPoR}$. The simulated FX values are extended to cover the whole period, which means that to obtain given profile for a portfolio with maturity T one must simulate FX process from $t = 0$ up to $t = T + \mathrm{MPoR}$.

The expected exposure profiles in 6.1 jag, because interest payments affect the exposure. The jags are only downwards, because as we can see from forward rate curves 5.8 and from selected domestic currency leg spread value, the coupon rates in the domestic leg are negative. As we can see from the figure, collateral is able to decrease the expected exposure significantly when WWR is not considered.

Same graphs are presented in figure 6.2, but this time with FX jump at default $(J = 0.05)$ on. From the figure we can see how the jump at default affects the post-default expected exposure level through the post-default FX process term in (4.30) when compared to 6.2.

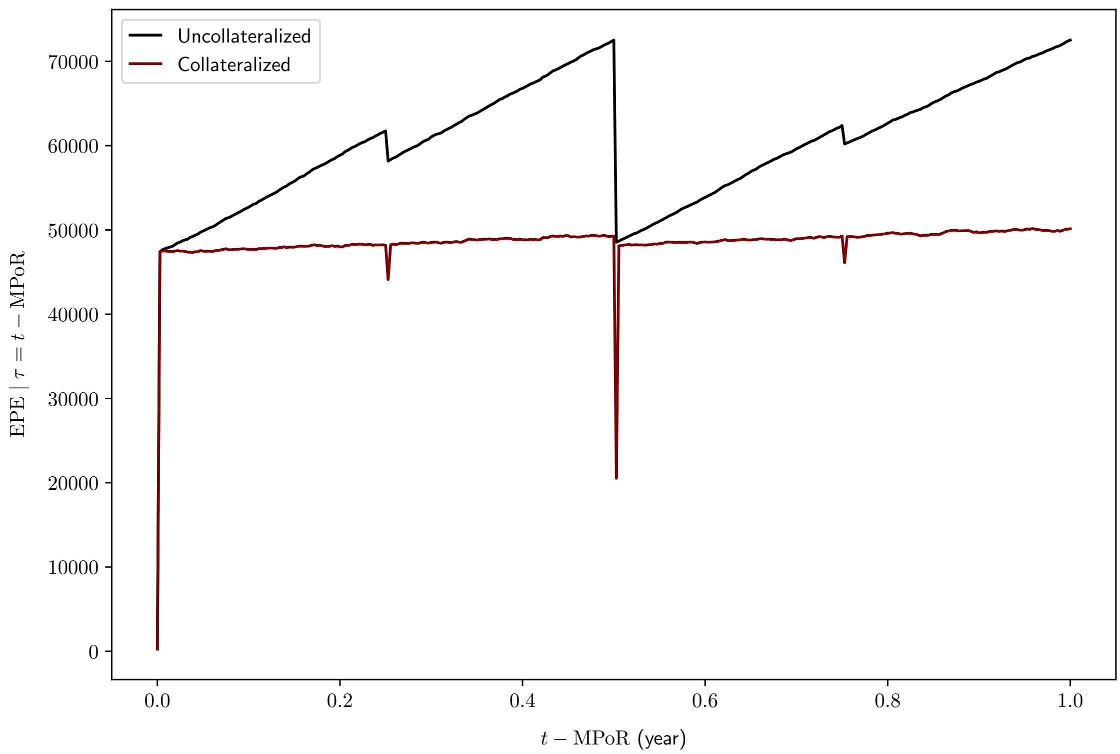

Figure 6.2. *Default conditional expected positive exposure with proportional FX jump $J = 0.05$ at default time of counterparty*

The same effect is present for both uncollateralized and collateralized case, because the lagged collateral is not able to tackle the WWR effect of jump-at-default.

In figure 6.3 the WWR effect of linear correlation $\rho_{X,\lambda} = 0.5$ on default conditional exposure is compared with an exposure values without WWR effect. The correlation effect is obtained by adjusting the FX process' Brownian motion with the intensity process' Brownian motion based on formula (6.3) as

$$W_{t_i}^X = \rho W_{t_i}^\lambda + \sqrt{1-\rho^2} W, \tag{6.4}$$

where W component is an independent and standard normal Brownian motion. As we can expect, the correlated and uncorrelated exposure profiles on are practically same *on average*, because the simulated Brownian motion for the correlated FX process from (6.4) is still normally distributed $W_{t_i}^X \sim N(0,1)$. However, on single exposure path level, presented in figure 6.4, the difference in uncorrelated and correlated paths can be seen clearly. The underlying default intensity process and FX processes of the exposure paths are shown in figure 6.5. The figure shows clearly how trending intensity (FX) process can affect the FX rate (default intensity) if correlation between random components is assumed.

As it was earlier mentioned, the default conditional expected positive exposures cannot

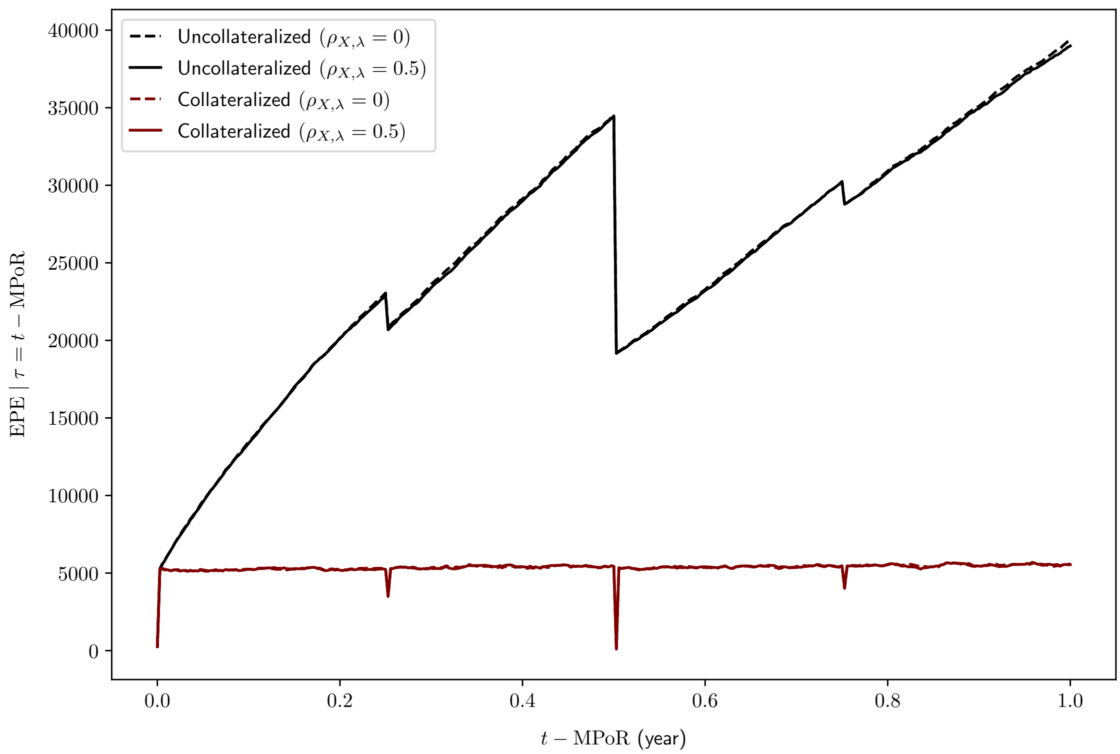

Figure 6.3. *Default conditional expected positive exposure with and without linear correlation WWR*

be directly used in the CVA formula if correlation is presented: the WWR effect of linear correlation becomes from co-variation of market and credit risk, so the correlation effect is fully present only in the final CVA number. For verification same figures as 6.3 and 6.4 were also plotted for WWR with jump and correlation present at the same time. The findings were same as with no-WWR versus the correlation WWR case: the default conditional expected positive exposure profiles are overlapping, while on a single exposure path level the values differ.

6.4 Wrong-way risk effect on CVA

The CVA calculation results for uncollateralized case are in the table 6.1 where the CVA without WWR effect, the independent collateralized CVA, $\mathbf{CVA}^{\perp}$, is used to scale the values. The value of $\mathbf{CVA}^{\perp}$ is obtained from the model by setting $J = 0$ and $\rho_{X,\lambda} = 0$. Same numbers, but for the collateralized CVA are presented in the table 6.2. Note that the scaling factors in tables are different, since value of the CVA calculated without WWR effect is in collateralized case only around fifth of the uncollateralized CVA. From the tables we can see that a constant correlation has, even with high values, quite limited effect on collateralized CVA while on uncollateralized CVA the effect is more material. On the other hand, the jump parameter has relatively material effect on collateralized CVA even with low values: A relative FX jump of 1% is already doubling the value of CVA.

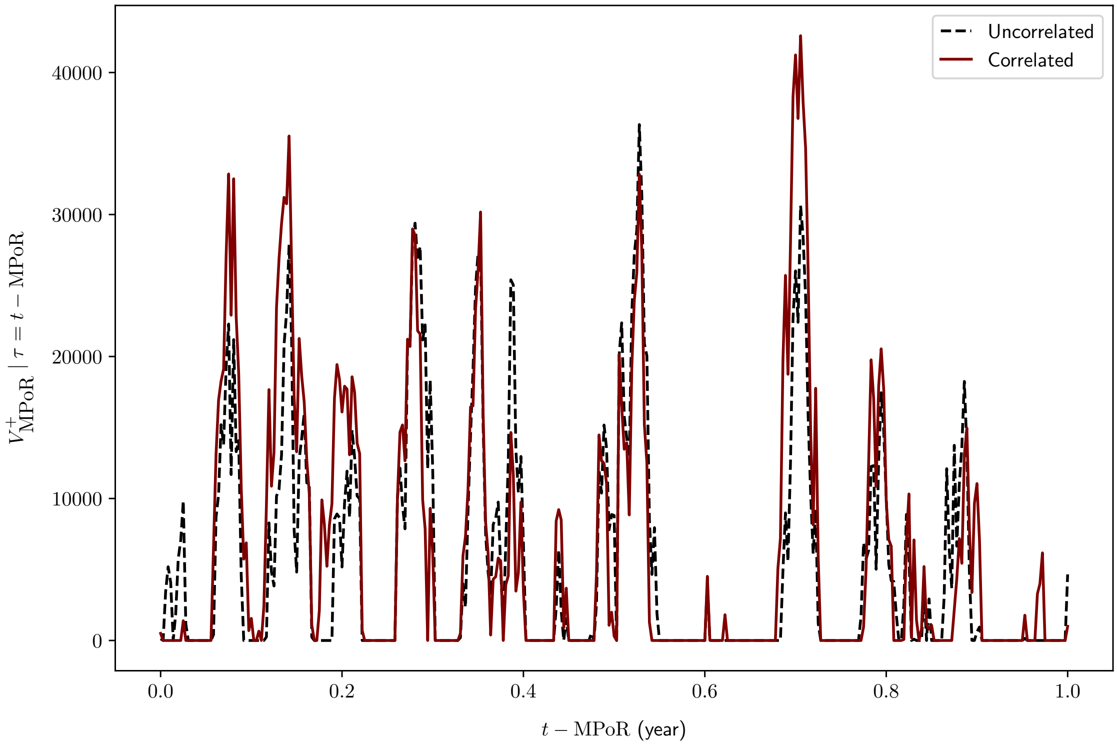

Figure 6.4. *A default conditional exposure path with and without linear correlation* $(\rho_{X,\lambda} = 0.5)$ *WWR for collateralized portfolio. The paths have same independent random components, so they would overlap if the correlation was set to 0.*

Table 6.1. *Collateralized CVA relative to the independent collateralized CVA with different FX jump and FX rate - default intensity correlation combinations. The measured independent collateralized CVA is* 108.68€ *obtained with* $J = 0$ *and* $\rho_{X,\lambda} = 0$.

$J \backslash \rho_{X,\lambda}$	0.0	0.1	0.2	0.5	0.7	1.0
0.000	1.00	1.00	1.01	1.02	1.03	1.04
0.005	1.52	1.52	1.53	1.54	1.55	1.57
0.010	2.15	2.16	2.17	2.19	2.20	2.22
0.030	5.44	5.46	5.47	5.52	5.55	5.58
0.050	9.08	9.10	9.12	9.19	9.23	9.29
0.100	18.21	18.25	18.29	18.41	18.49	18.60

Same results but for negative J and $\rho_{X,\lambda}$ values are calculated and presented in appendix B to obtain RWR effect on CVA. The effect of RWR is mirrored: USD devaluation jump-at-default decreases relative CVA in collateralized portfolio with 5% devaluation jump nearly zeroing the CVA.

For verification, WWR calculations were performed also for 10-year CCBS contract, with all other parameter values remaining same. The results are presented in appendix C. The results are similar, with constant correlation becoming slightly more important contributor

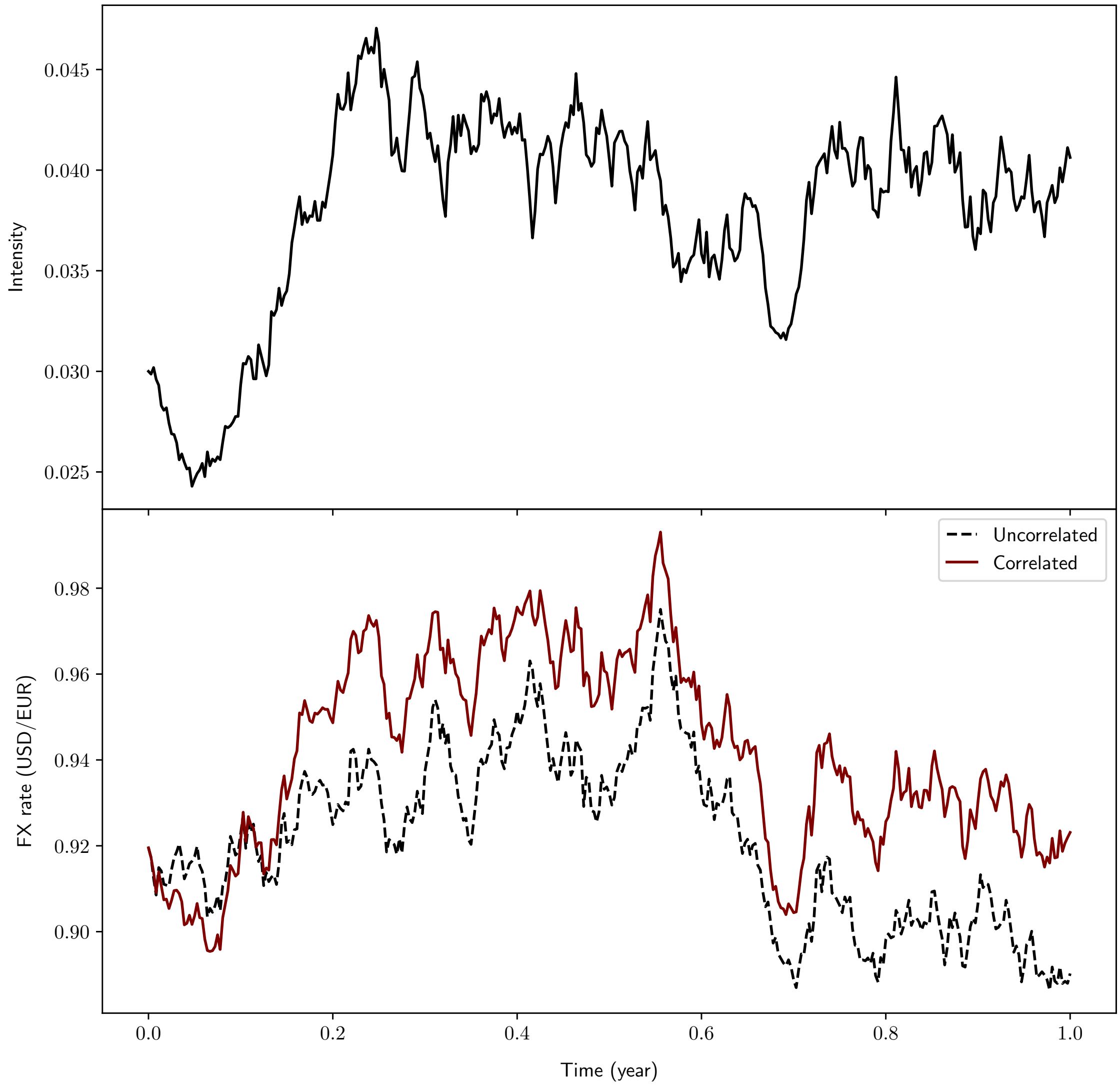

Figure 6.5. *Stochastic intensity path and correlated FX rate path with* $\rho_{X,\lambda} = 0.5$. *The uncorrelated path shows the FX rate evolution before the correlation effect.*

for WWR. With 10-year contract the jump-at-default effect is still material even with low jump values.

The differences between WWR parameter value sensitivities on collateralized portfolio are even more clear in the figure 6.6, where CVA is shown as a function of the jump parameter value with different levels of correlation. As we can see, the graphs with different correlation values are nearly overlapping. Figure 6.7 shows the same graphs but for uncollateralized case, where the correlation parameter value has clearly more significant relative effect on CVA value. Same figures but in RWR case are shown in appendix B and for 10-year CCBS in appendix C.

In both WWR figures the relative CVA increases as a function of the jump parameter, but in the collateralized case the increase is steeper. However, with all other parameters equal, the absolute value of CVA in collateralized portfolio remains still lower than

Table 6.2. *Uncollateralized CVA relative to the independent uncollateralized CVA with different FX jump and FX rate - default intensity correlation combinations. The measured independent uncollateralized CVA is* 510.60 *€ obtained with* $J = 0$ *and* $\rho_{X,\lambda} = 0$.

$J \backslash \rho_{X,\lambda}$	0.0	0.1	0.2	0.5	0.7	1.0
0.000	1.00	1.04	1.09	1.23	1.32	1.46
0.005	1.11	1.16	1.21	1.35	1.45	1.60
0.010	1.23	1.28	1.33	1.48	1.59	1.74
0.030	1.77	1.83	1.89	2.07	2.19	2.37
0.050	2.39	2.46	2.53	2.73	2.87	3.06
0.100	4.14	4.22	4.30	4.54	4.70	4.92

the absolute value of uncollateralized one. A comparison of absolute CVA values with different jump parameter values is presented in figure 6.8. The absolute difference between uncollateralized and collateralized CVA decreases as the size of assumed relative FX jump-at-default increases.

In conclusion, collateralization can reduce the effect of the constant correlation WWR, but it is incapable in reducing the jump WWR effect. Chung and Gregory (2019) report similar results for a directional portfolio with collateralization. This is an unsurprising result, because collateral can mitigate the effect of correlation by reducing the total exposure while the value of contract increases at the same time with default intensity. But with jump-at-default, the collateral is posted last time one day before the default and the increase in value of contract due to USD/EUR rate increase is in its entirety translated to the increase in exposure. This effect is clearly visible in the default conditional exposure graphs when the jump-at-default case 6.2 is compared with 6.1 having no WWR effect.

6.5 Sensitivity analysis

While the assumed relative jump-at-default J and the constant correlation $\rho_{X,\lambda}$ are the main parameters of interest when comparing jump and correlation based WWR models, other parameters may affect conclusions made of the modelling approaches. For example, higher or lower FX volatility σ_X or intensity volatility σ_λ might make difference between CVA numbers of the modelling approaches notable. To evaluate robustness of the results of previous section, the sensitivity of independent CVA and CVA with WWR is evaluated against few important model parameters. The simple sensitivity analysis is made with four different modelling choices:

(i) No WWR ($J = 0, \rho_{X,\lambda} = 0$)

(ii) Relative 5% FX jump-at-default ($J = 0.05, \rho_{X,\lambda} = 0$)

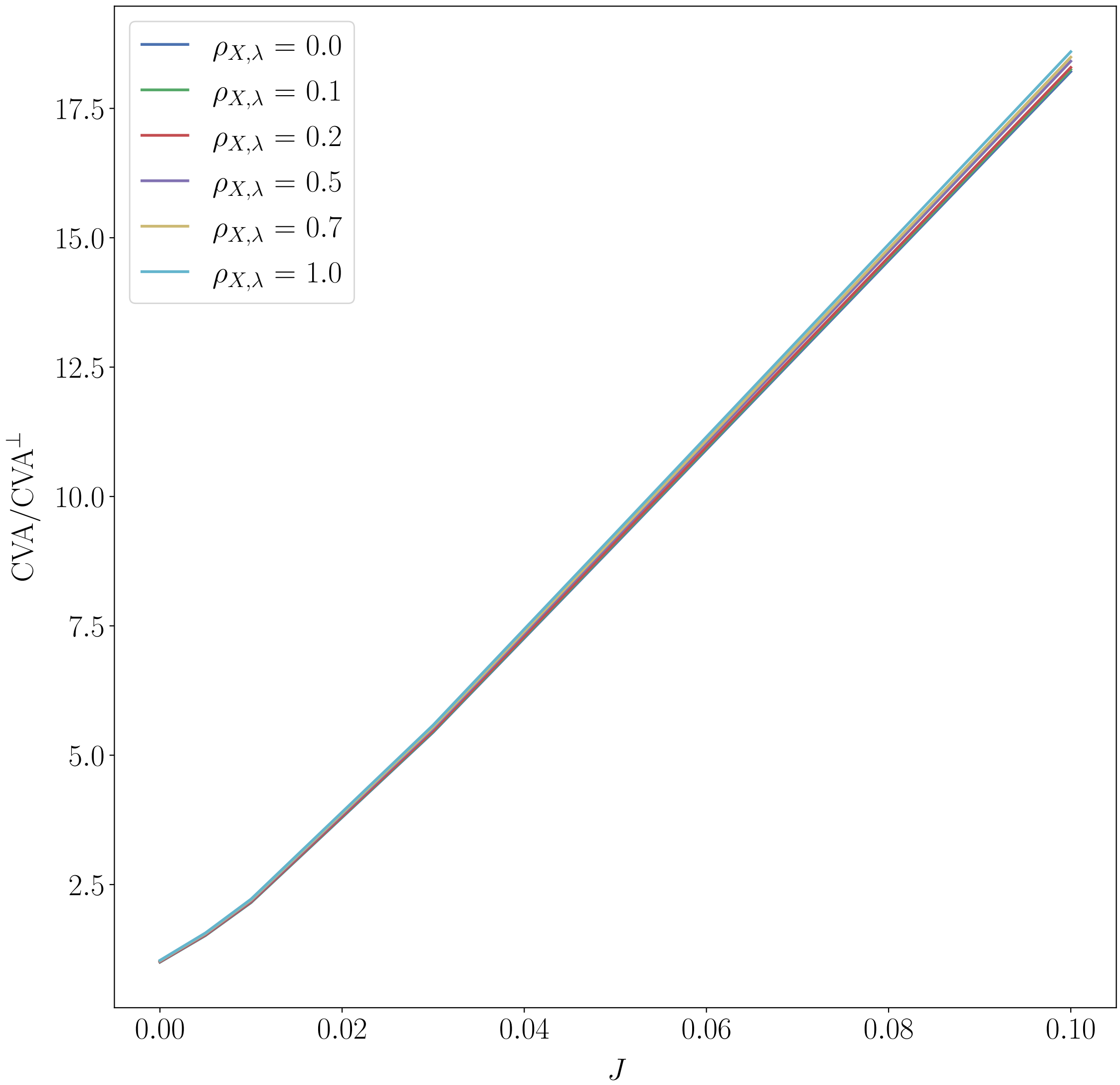

Figure 6.6. *Collateralized cross-currency basis swap CVA relative to independent CVA with different levels of FX rate and default intensity correlation*

(iii) Constant 50% correlation between FX rate error and default intensity error ($J = 0, \rho_{X,\lambda} = 0.5$),

(iv) Both jump-at-default and constant correlation WWR effects ($J = 0.05, \rho_{X,\lambda} = 0.5$)

The sensitivities are measured by adjusting the corresponding parameter value *ceteris paribus*, by keeping all the other model parameters at the same level as in the previous section. The results are presented only for the collateralized CVA which is more important in this thesis than the uncollateralized case, due to portfolio assumptions. The sensitivity analysis results may differ in an uncollateralized portfolio, because the results of previous section suggest that collateralization affects significantly on how CVA reacts to different modelling choices.

First CVA sensitivity is measured against intensity parameters constant hazard rate h and intensity volatility σ_λ. The sensitivity figure of constant hazard rate is presented in 6.9. As

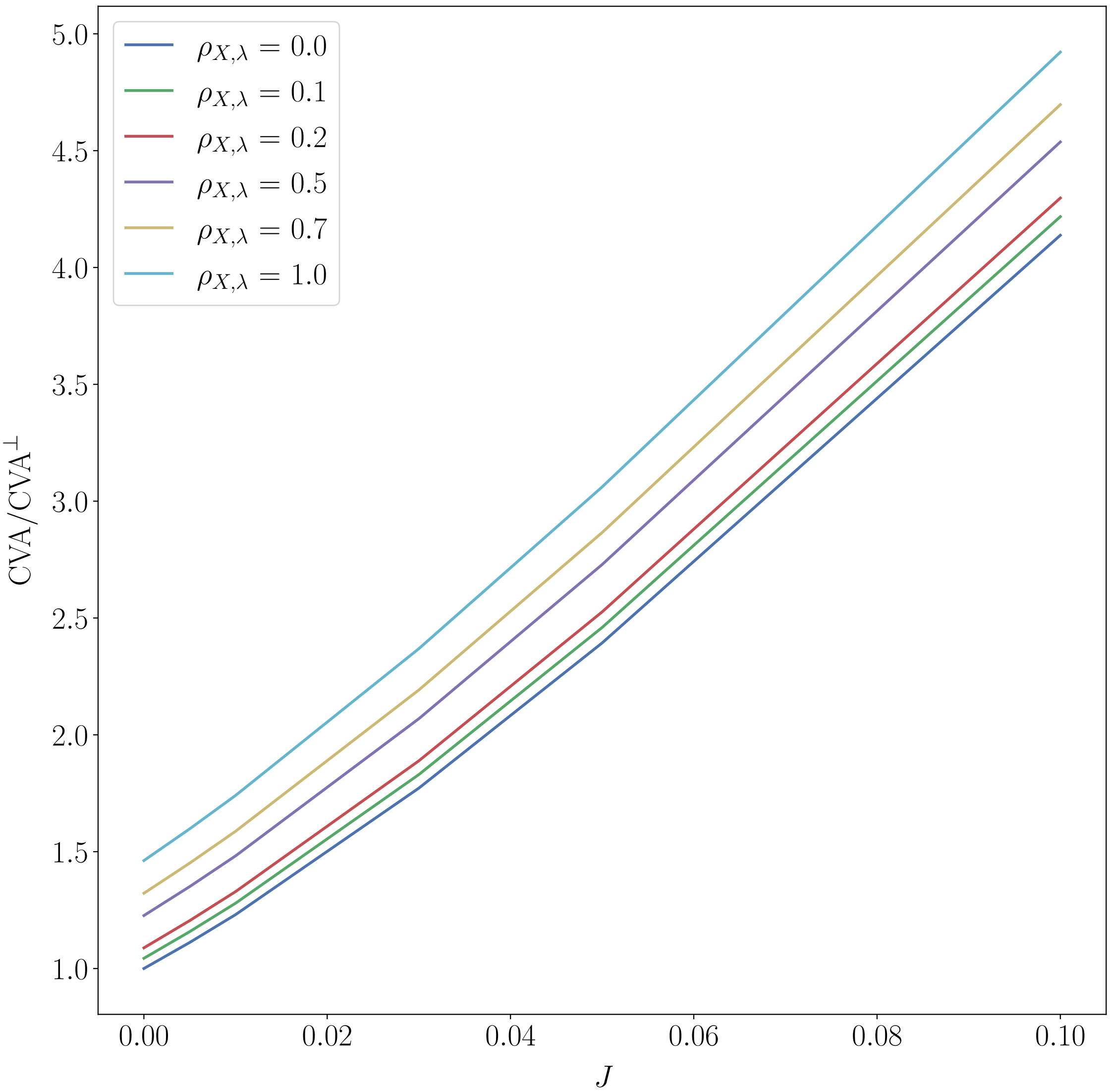

Figure 6.7. *Uncollateralized cross-currency basis swap CVA relative to independent CVA with different levels of FX rate and default intensity correlation*

we can see the the hazard rate's effect on CVA is approximately linear, with higher hazard rate yielding higher CVA values due to higher average probability of default. In the jump-at-default framework, the exposures at default are higher on average, which explains why CVA value increases more steeply when the default intensity increases.

Figure 6.10 shows default intensity volatility sensitivity of different modelling approaches. With an intensity volatility value $\sigma_\lambda = 0$ the corresponding CVA model with correlation WWR is essentially same as the one without it, because the error term in the intensity model becomes zero. Interestingly, the CVA seems to decrease as a function of the intensity volatility, when the volatility is high enough. From the equation (4.12) one would suggest that the expected value of default intensity increases as a function of default intensity volatility and ditto CVA would monotonically increase, when the value of $m = -\sigma^2/2a$ is set. The contradiction may indicate that with high volatility values the empirical mean

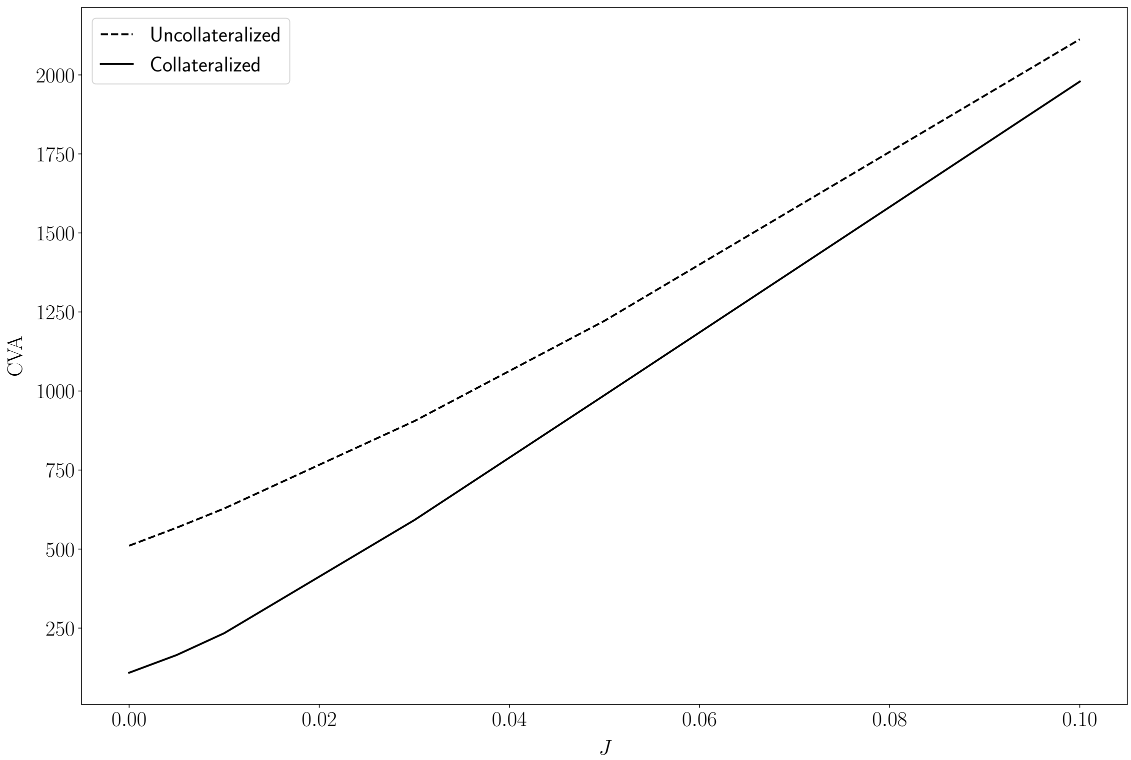

Figure 6.8. *Collateralized and uncollateralized cross-currency basis swap CVA with different jump values. The correlation parameter is set to $\rho_{X,\lambda} = 0$.*

of default intensity is not leveling to the value of the deterministic hazard rate function. However, this observation does not question the results of previous section as the level of collateralized CVA in 6.10 remains a lot higher with jump-at-default framework than with correlation framework for all tested values of default intensity.

In figure 6.11 FX volatility sensitivity of CVA is shown. With all modelling approaches the CVA increases as a function of FX volatility, but with the independent CVA and the correlation framework the increase is steeper. The difference might be due to exposure changes unfolding in the MPoR period, driven by the baseline FX model (4.24). From figure 6.12 we can see how the MPoR is a lot more significant driver of CVA in the independent CVA and the correlation framework than for the jump-at-default CVA. In fact, in the jump-at-default framework increasing MPoR decreases CVA. The effect might be explained by the tendency of FX drift to be downward and the fact that the exposure is floored to zero: With low expected positive exposure values of correlation framework and independent CVA it is more likely that with high FX volatility and longer MPoR the exposure will end up being higher than nil. With jump-at-default framework it is already more probable that the default conditional exposure will be above zero after the FX jump. In that case the decreasing drift of the FX process have higher impact on CVA in MPoR than the possibility of landing above zero.

All in all, the sensitivity analysis shows some differences on how the constant correlation

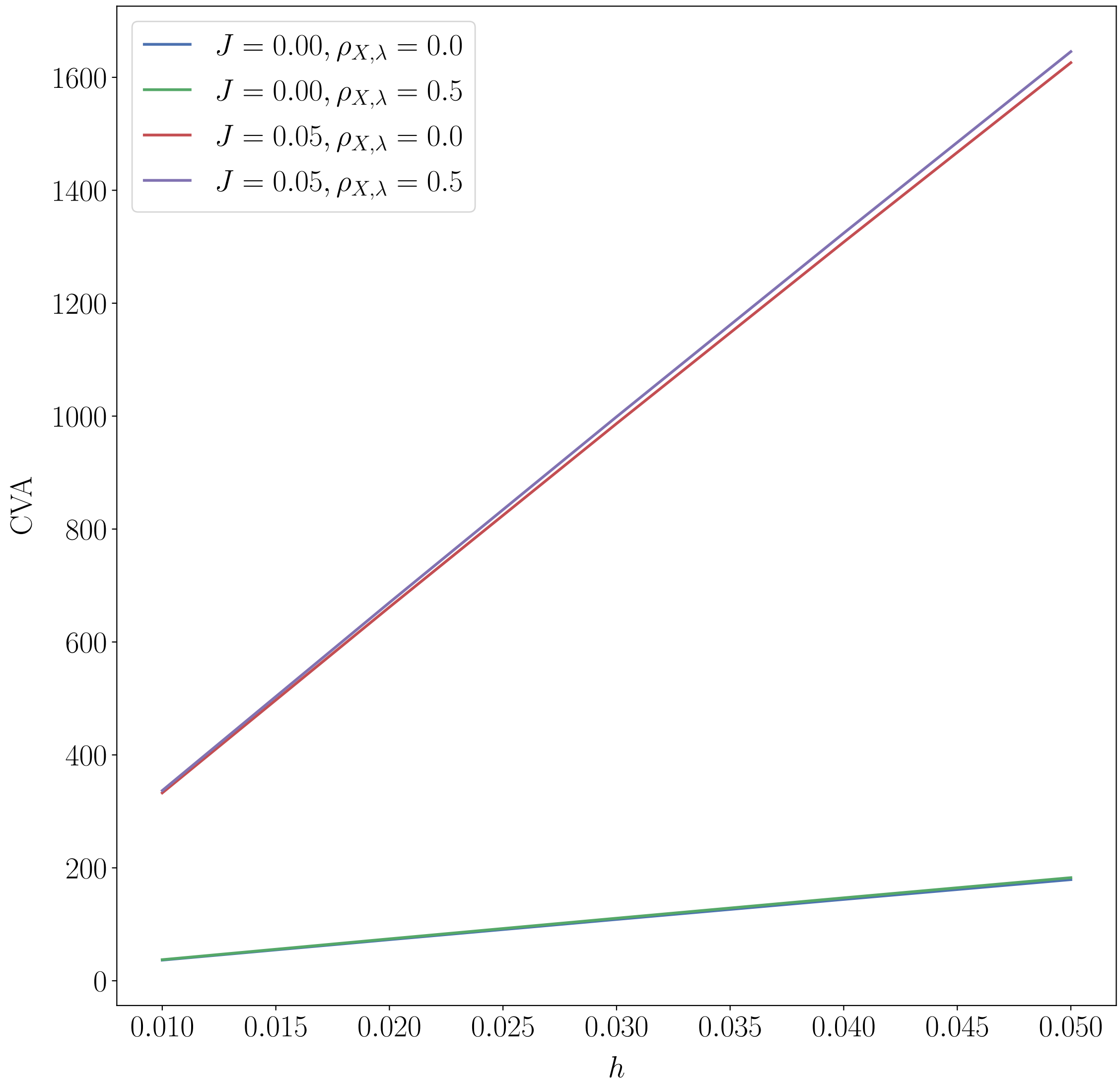

Figure 6.9. *CVA constant hazard rate sensitivity of different modelling choices*

and jump-at-default based CVA WWR models react on the parameter changes. However, the conclusions of CVA methods for collateralized contract remain same as made in the previous section: The constant correlation method doesn't yield notable difference when compared to the independent CVA method, and the constant correlation method's differences with the jump-at-default method remain large with all tested parameter values. The constant correlation is the only tested parameter which can decrease the difference. The constant hazard rate sensitivity for jump-at-default method is higher due to reason discussed above. Nevertheless, the difference remains notable even with a very low constant hazard rate of 1%.

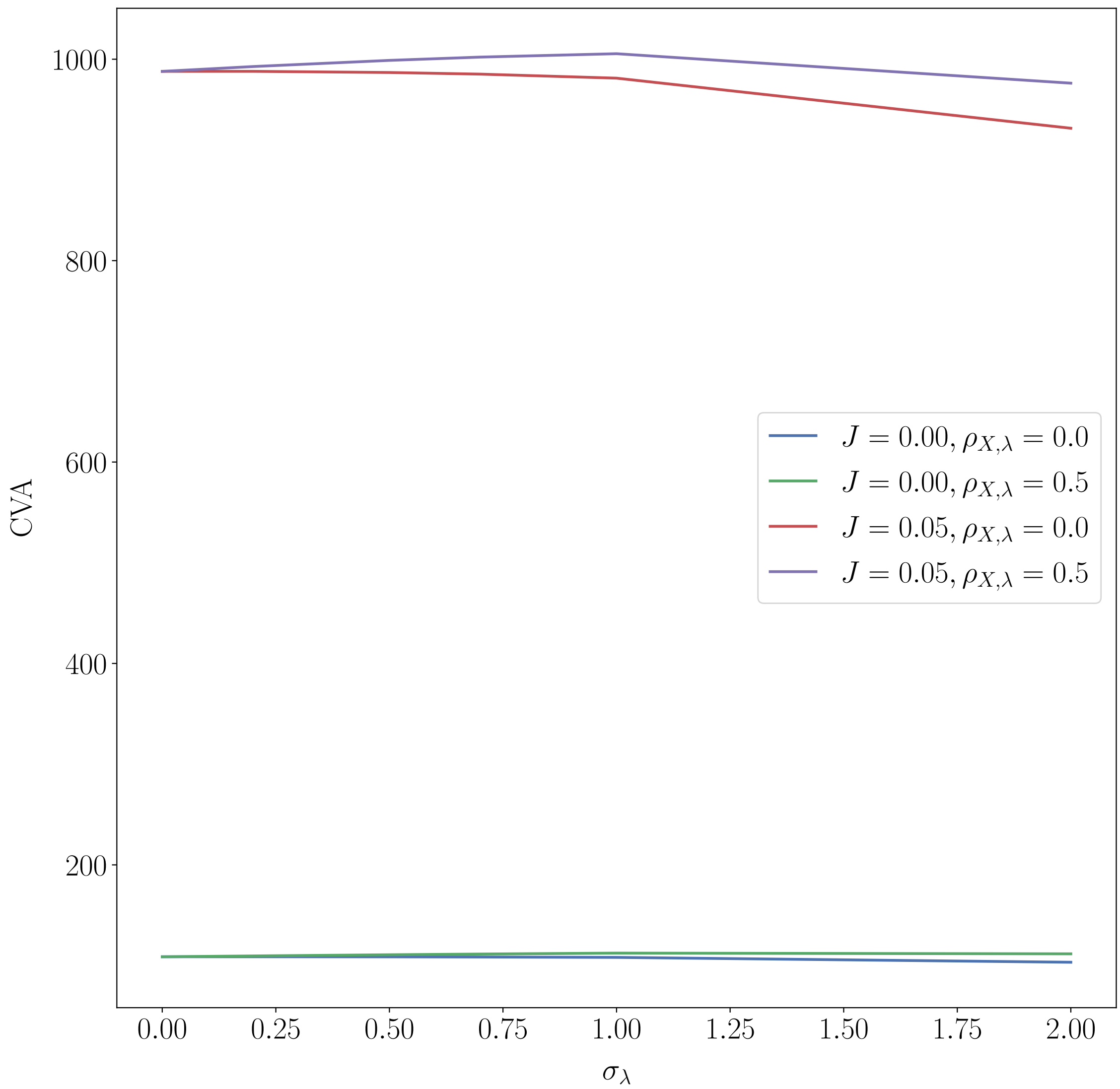

Figure 6.10. *CVA default intensity volatility sensitivity of different modelling choices*

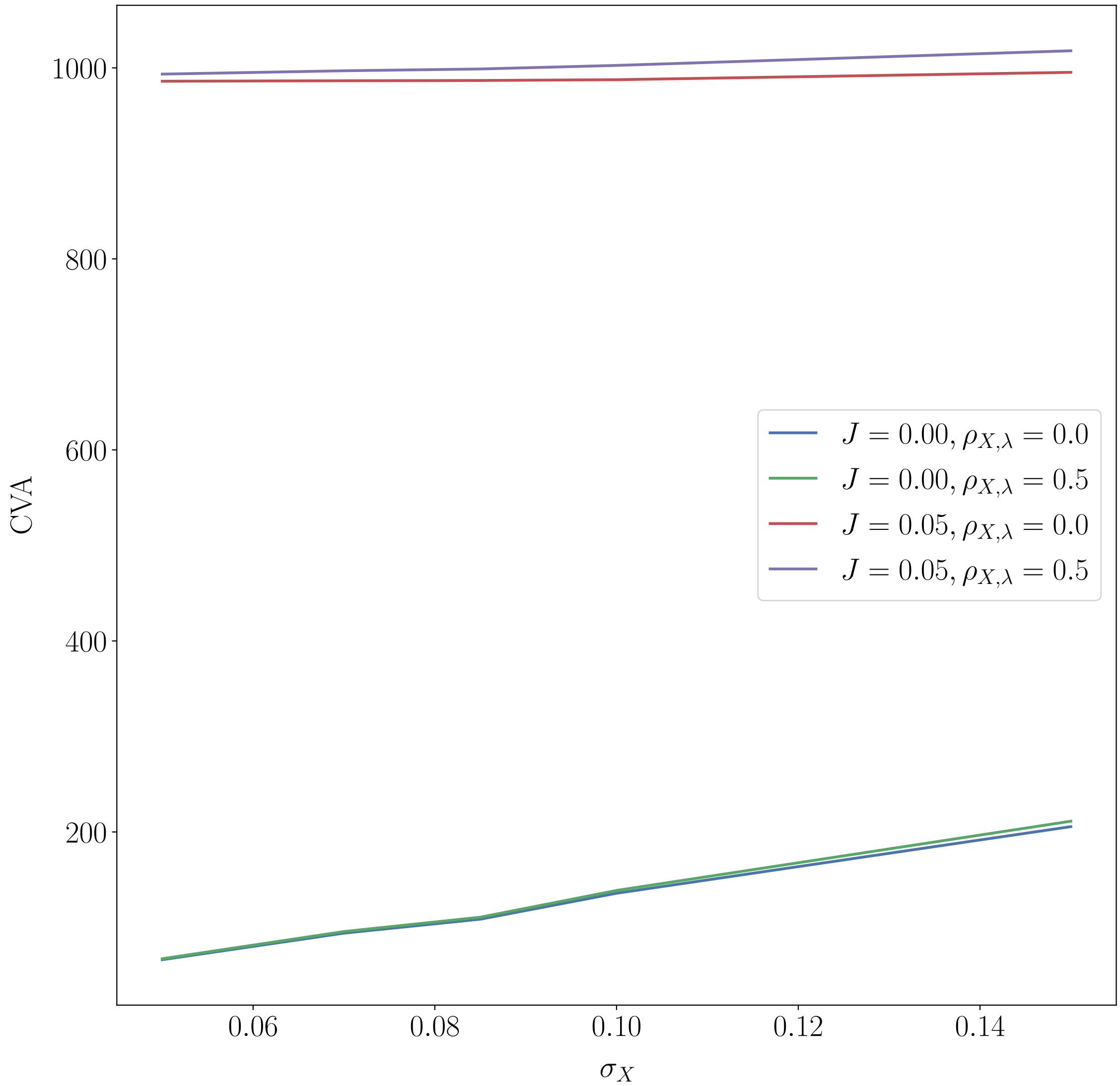

Figure 6.11. *CVA FX volatility sensitivity of different modelling choices*

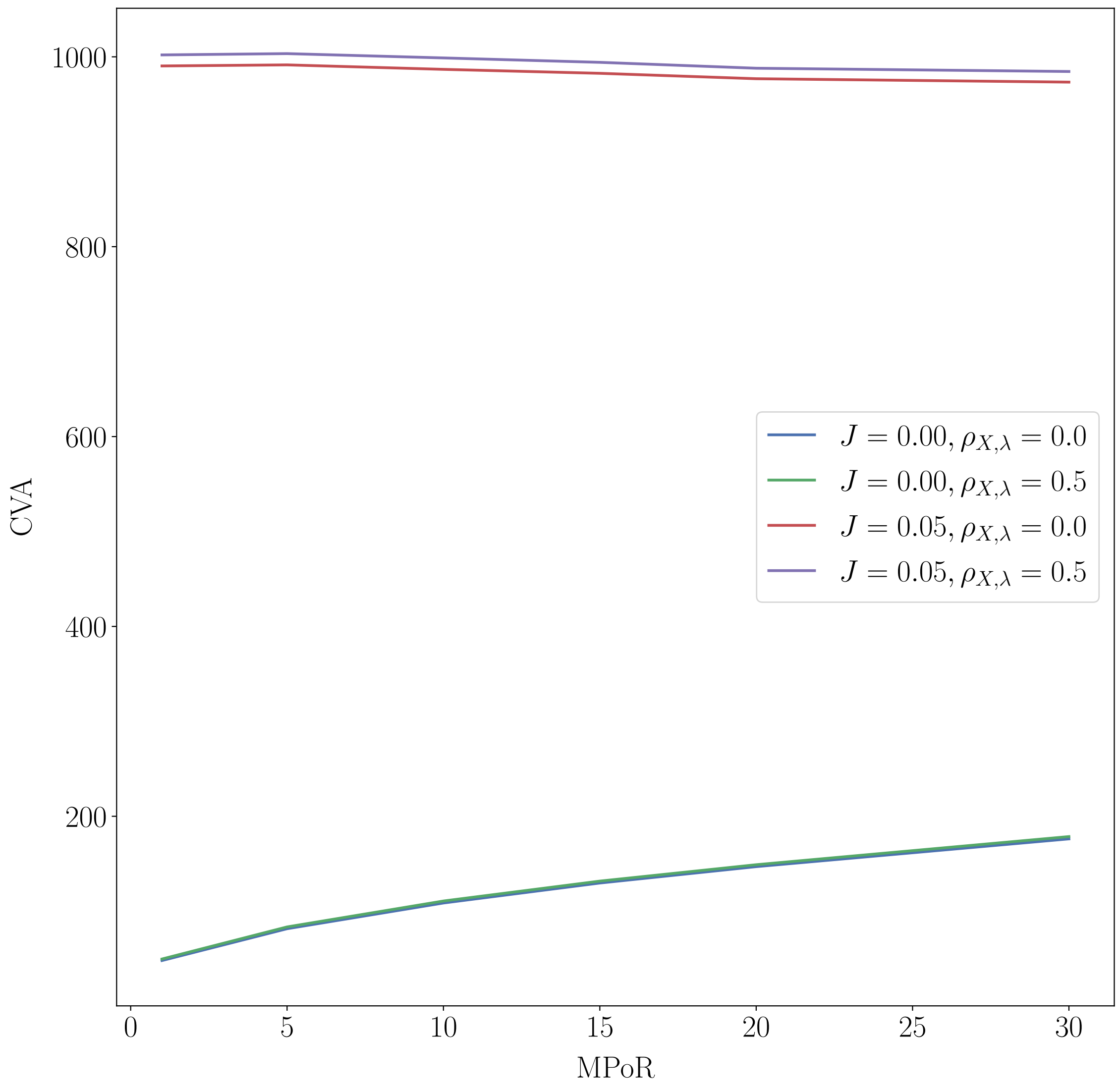

Figure 6.12. *CVA MPoR sensitivity of different modelling choices*

7. CONCLUSION

In this thesis the focus was on CVA WWR effect measurement of a cross-currency basis swap contract, where the counterparty is a significant financial institution in same currency area. The first research question was: *How CVA WWR should be modelled in OTC derivatives contracts made with eurozone G-SIB, when the underlying risk factor is USD/EUR FX rate and both parties post collateral?* Based on portfolio assumptions and requirements imposed by theoretical and practical considerations, the identified candidates were the *jump-at-default* and *linear correlation* approaches. Both models can be calibrated to be arbitrage-free, and they are general enough to be implemented and used in complex portfolios to measure WWR.

The second research question was: *How significant is the WWR effect in the example contract based on the selected model(s) compared to a CVA model without WWR and how it is affected by modelling assumptions?* The CVA calculation results show similar patterns observed in the previous literature: Assumed linear correlation does not have significant effect on independent CVA, when collateralized portfolio is considered. On the other hand, jump-at-default, even with small relative jump size can dramatically affect CVA of collateralized transaction.

From the results of empirical section, it seems evident that events potentially unfolding at the time of a counterparty default should not be ignored when measuring counterparty credit risk. The issue is highlighted with directional portfolios and instruments with high market risk factor sensitivity, like cross-currency swaps used to hedge FX risk. As Pykhtin and Sokol (2013) conclude their study of systemic WWR risk:

“ Counterparty credit exposure is meaningful only when measured conditional on default, and any dependence between exposure and counterparty default time results in divergence between conditional and unconditional exposures. ”

Based on simulation results of this thesis and earlier literature, one can confidently say that the *divergence* can be material, even if the underlying market risk factor’s shock is modest.

Conversely, as empirical WWR literature suggest, simple linearly correlated WWR model is not sufficient in capturing a measurable WWR effect on collateralized portfolio. Low market risk factor volatility and high credit quality of the counterparty combined with collateral posting is effectively mitigating the correlation effect of market risk factor and default intensity. Linear correlation based WWR model can, in the given conditions, severely underestimate the WWR effect on CVA.

7.1 Implications and recommendations

From risk management perspective, the practical implications of this thesis are, that measuring WWR or at least monitoring it in FX derivatives contracts made with large domestic counterparties is necessary. Ignoring the potential dependence structure or modelling it with the linear correlation method can pose significant model risk: Even if the potential jump-at-default effect is modest, the relative impact on CVA is high. Making risk management decisions or pricing based on models without the jump component includes a high change of errors, because in case of market-credit risk combinations like the one discussed in this thesis, the exposure at default can with one counterparty be significantly higher than with another one.

The jump-at-default WWR studied in this thesis can theoretically be avoided by not making OTC FX derivative transactions with counterparties, whose defaults might devalue the currency held long position in the contract. However, more sensible approach is to model the jump-at-default effect, do the pricing adjustment as usual and then draw conclusions about counterparty level allocations. In this analysis CVA with WWR is a useful tool because, by definition, it quantifies the price of counterparty credit risk, which can then be used to control trading decisions.

The effect of CVA WWR could also be hedged, if undesired market risk factor and counterparty combinations are unavoidable. One strategy is to avoid directional characteristics in portfolios with jump-at-default risky counterparties, if it is possible. The effect of jump-at-default WWR can then be expected to be lower (Chung and Gregory 2019). Another approach for risk mitigation could be macro level hedging of whole CVA, including WWR. Van der Zwaard et al. (2021) consider hedging CVA market risk, when the underlying follows jump-diffusion process. However, they assume no WWR and focus on a portfolio with European options on a stock. For an institution using derivatives to hedge foreign currency funding, adjusting portfolio structure might not be an option because for them derivative portfolios are by nature directional.

For directional portfolios the effect of jump-at-default might be best mitigated by more comprehensive collateralization: Initial margin combined with VM might be effective tool in reducing gap risk related to events unfolding in MPoR (Gregory 2015, section 6.2.4). In fact, valuation adjustments CVA and funding valuation adjustment combined can be seen as the cost of imperfect collateralization (Zwaard, Grzelak, and Oosterlee 2022). Since IM is intended to reduce risk of adverse market factor movements in MPoR (Basel Committee on Banking Supervision 2011b), it seems to be the regulatory answer to default-conditional exposure changes, like jump-at-default effect. However, it is another question how high IMs counterparties are willing to suffer. Especially, if we consider the case of directional portfolio, which is theoretically avoidable and rising from the bank's own choices, the counterparty is unlikely willing to compensate with higher IM for this undiversified gap risk.

Another view to market-credit dependency which was only briefly addressed in this thesis is that instead of WWR benefiting RWR can be obtained, if counterparty's default devalues the currency being hedged, relative to the currency held the long position. The conclusion is confirmed by empirical results presented in appendix B. According to Ruiz (2015, p. 169) a trading desk of an institution should be encouraged generating RWR similarly as generating WWR should be discouraged.

7.2 Limitations and future research

In this work the market-credit dependency is investigated only on one transaction type: one year cross-currency basis swap. This is the main limitation of the work, since CVA numbers are normally reported on a netting set level where multiple transactions with same counterparty are netted, and the collateral is posted depending on netted value. With directional portfolios having FX trades one can expect similar results, as Chung and Gregory (2019) show, but with exotic derivatives, different risk factors and long maturities results may differ. A collateralized portfolio's WWR sensitivity with MPoR and multiple transaction types is left for future research.

While this thesis extends previous work of jump-at-default WWR studies by discussing jump-at-default and constant correlation WWR effect at the same time with collateralization and some modelling details, like MPoR and realistic term structures of interest rates, it is still simplified version and could be extended to be more realistic. Firstly, jump and correlation parameters could be modelled with deterministic or stochastic models. For example, stochastic correlation has been shown to produce more adverse WWR effects than constant correlation due to better captured tail dependence of market risk factors and default intensity (Kumar, Markus, and Hari 2021). In jump-at-default component the size could be made stochastic, or the rapid market movement modelled with continuous change instead of instant jump at the credit event: The credit event could be partly expected by the market, in which case the collateral can offset some part of the rapid movement. The partly expected credit event can be modelled for example with a collateral delivery ratio (Pykhtin and Sokol 2013). Another jump related extension would be multiple allowed jumps in spot FX rate across whole portfolio of derivatives contracts. A systemic crisis in financial markets could cluster credit events and thus increase CVA across the whole portfolio of contracts with different counterparties.

Secondly, the FX and default intensity models could be improved with stochastic modelling of other parameters. For example, stochastic interest rate modelling can be included, which would be more important for interest rate sensitive products. With volatility sensitive products, like constant maturity swaps (Veilex 2010), stochastic volatilities could also be necessary. Thirdly, in this thesis WWR effects were discussed only in the case of unilateral credit valuation adjustment and other valuation adjustments were ignored. To extend the discussion to other valuation adjustments the first step would be to model also own default risk and potential FX jump. In addition to bilateral CVA, other valuation adjustment could be studied. The field of valuation adjustments is broad, and while the topic has been discussed extensively after the financial crisis, the best practices are still to be formed.

Finally, causality effects and systemic counterparties are a fruitful topic for future studies, since they show potential adverse effects in FX sensitive portfolios, as shown by results of this thesis. While heuristic calibration of jump-at-default effect with quanto CDS spreads is possible as discussed in chapter 5, liquidity and existence of CDS instruments limit the set of counterparties for which the calibration is possible. Thus, the jump-at-default literature could benefit of peer-group analysis methods and study of economic drivers of jumps. Naturally, quanto instrument valuation and liquidity improvements would also increase the usability of the risk-neutral jump calibration.

For risk and portfolio management the assumed jumps can have interesting implications as discussed in the previous section. While some recommendations were given, further studies of macro-level management of jump-at-default risk are necessary. One point that may affect the results of analysis made in this thesis and in portfolio level are the initial margin requirements set for OTC derivatives (Basel Committee on Banking Supervision 2011b), which can theoretically offset large market movements during MPoR. Large IM combined with frequent VM could potentially mitigate the jump risk completely and hence make the constant correlation more important driver of WWR in collateralized portfolio. This topic, along with other collateral agreement details and strategic portfolio management are left for future studies.

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APPENDIX A: DATA

Table A.1. *Data description*

Data type	Source	Data points	Observation period
EUR/USD FX rate	Euro FX time series (European Central Bank 2023)	Daily	1.4.2020–31.3.2023
Implied vol of EUR/USD FX option	Bloomberg	1Y, at-the-money	31.3.2023
€STR yield curve	Bootstrapped from swap data of Bloomberg	OIS observed rate 1W, 2W and 1M,2M,...,12M swap rates	31.3.2023
SOFR yield curve	Bootstrapped from swap data of Bloomberg	1W, 2W, 3W and 1M, 2M,..., 12M swap rates	31.3.2023
EUR vs. USD cross-currency basis curve	Observed cross-currency basis quotes from Bloomberg	3M, 6M, 9M, 12M	31.3.2023
USD CSA discounting curve	Bootstrapped and constructed synthetically from €STR, SOFR and EUR vs. USD cross-currency basis data of Bloomberg	Described above	31.3.2023
3M Euribor projection curve	Constructed from Bootstrapped future quotes data and €STR swap data of Bloomberg	Observed rate of the day 1M, 2M,...,12M observed futures rates	31.3.2023

Table A.2. *USD/EUR FX rate historical log-return summary statistics. The distribution has leptokurtic form, since the kurtosis is higher than the Standard normal distribution's* 3.0*, which makes the tails of the distribution thicker. Thus, it is more likely to observe "outliers" like the minimum value of* -3.5% *log-return. The distribution is not stable over time as can be seen from statistics of different time periods.*

Period	Observations	Min value	Max value	Mean	Volatility	Skewness	Kurtosis	Annualized volatility
1.4.2020–31.3.2023	772	-0.0349	0.0183	7.25×10^{-6}	5.10×10^{-3}	-0.216	3.70	0.0967
1.4.2020–31.3.2021	255	-0.0130	0.0112	-2.73×10^{-4}	4.38×10^{-3}	-0.0823	0.129	0.0832
1.4.2021–31.3.2022	259	-0.0156	0.0161	2.11×10^{-4}	3.83×10^{-3}	0.201	3.10	0.073
1.4.2022–31.3.2023	258	-0.0130	0.0183	7.97×10^{-5}	6.65×10^{-3}	-0.319	2.78	0.126

APPENDIX B: RWR RESULTS

Table B.1. *Collateralized CVA with RWR effect relative to the independent collateralized CVA with different FX jump and FX rate - default intensity correlation combinations. The measured independent collateralized CVA is* 108.68*€ obtained with* $J = 0$ *and* $\rho_{X,\lambda} = 0$*. The results show how jump-at-default RWR can decrease relative CVA significantly in collateralized portfolio.*

$J \backslash \rho_{X,\lambda}$	0.0	-0.1	-0.2	-0.5	-0.7	-1.0
0.000	1.0000	0.9959	0.9917	0.9784	0.9693	0.9528
-0.005	0.6124	0.6097	0.6069	0.5980	0.5917	0.5805
-0.010	0.3452	0.3435	0.3418	0.3362	0.3321	0.3252
-0.030	0.0129	0.0128	0.0127	0.0125	0.0124	0.0119
-0.050	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-0.100	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

Table B.2. *Uncollateralized CVA with RWR effect relative to the independent uncollateralized CVA with different FX jump and FX rate - default intensity correlation combinations. The measured independent uncollateralized CVA is* 510.60 *€ obtained with* $J = 0$ *and* $\rho_{X,\lambda} = 0$*.*

$J \backslash \rho_{X,\lambda}$	0.0	-0.1	-0.2	-0.5	-0.7	-1.0
0.000	1.0000	0.9574	0.9158	0.7971	0.7237	0.6143
-0.005	0.8961	0.8561	0.8171	0.7065	0.6381	0.5370
-0.010	0.8001	0.7626	0.7261	0.6237	0.5603	0.4674
-0.030	0.4912	0.4639	0.4378	0.3659	0.3211	0.2576
-0.050	0.2869	0.2687	0.2514	0.2037	0.1742	0.1337
-0.100	0.0608	0.0554	0.0503	0.0368	0.0295	0.0199

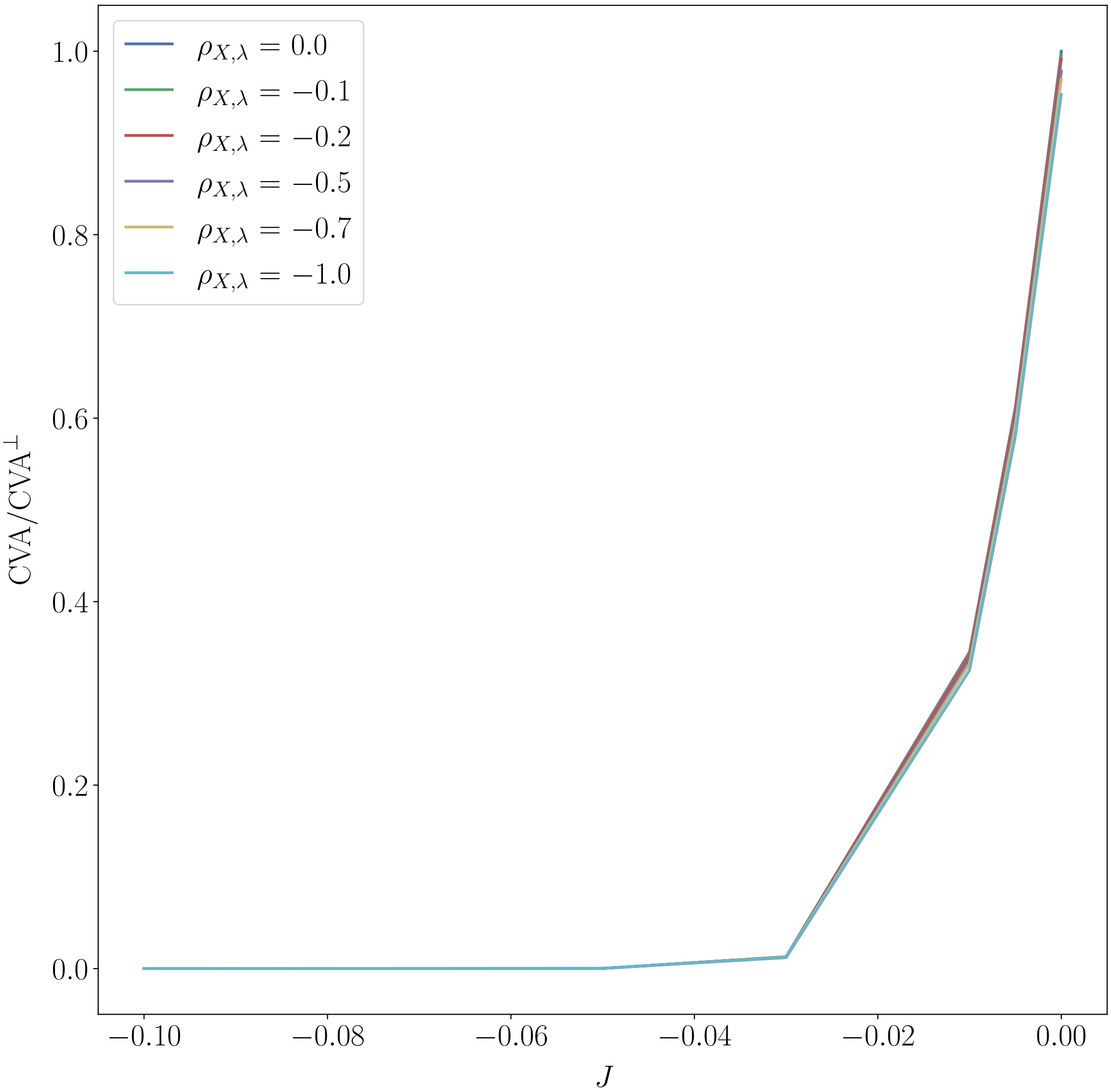

Figure B.1. *Collateralized cross-currency basis swap CVA RWR effect relative to independent CVA with different levels of FX rate and default intensity correlation*

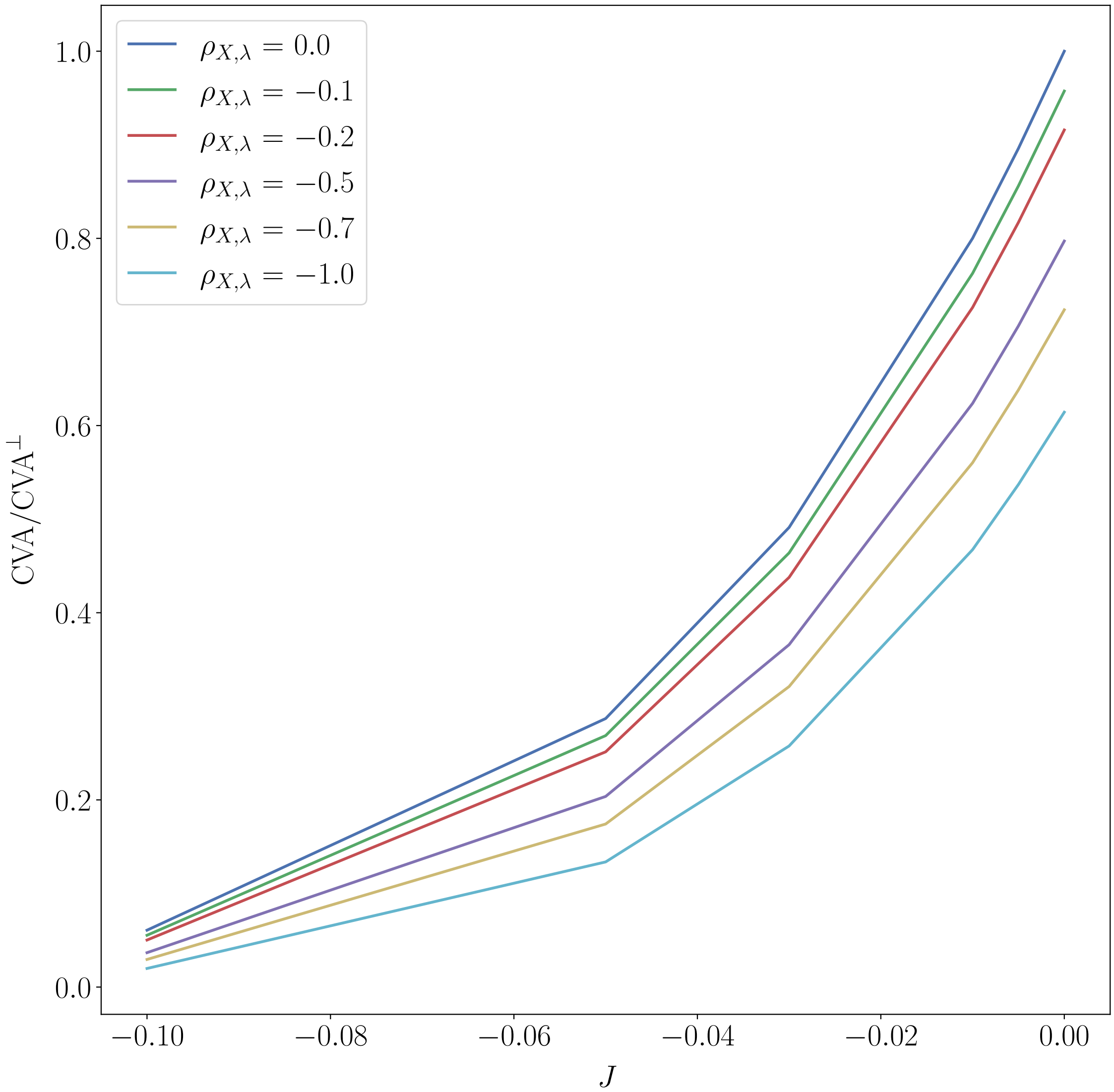

Figure B.2. *Uncollateralized cross-currency basis swap CVA RWR effect relative to independent CVA with different levels of FX rate and default intensity correlation*

APPENDIX C: 10-YEAR CROSS-CURRENCY BASIS SWAP RESULTS

Table C.1. *10-year collateralized cross-currency basis swap's CVA relative to the independent collateralized CVA with different FX jump and FX rate - default intensity correlation combinations. The measured independent collateralized CVA is* 697.31€ *obtained with* $J = 0$ *and* $\rho_{X,\lambda} = 0$*. The results are similar as with the 1Y CCBS, with correlation becoming slightly more important contributor of WWR.*

$J \backslash \rho_{X,\lambda}$	0.0	0.1	0.2	0.5	0.7	1.0
0.000	1.00	1.02	1.03	1.08	1.11	1.15
0.005	1.51	1.53	1.56	1.62	1.67	1.73
0.010	2.14	2.17	2.20	2.29	2.36	2.44
0.030	5.35	5.43	5.50	5.73	5.88	6.10
0.050	8.88	9.00	9.12	9.49	9.74	10.09
0.100	17.61	17.84	18.08	18.79	19.27	19.96

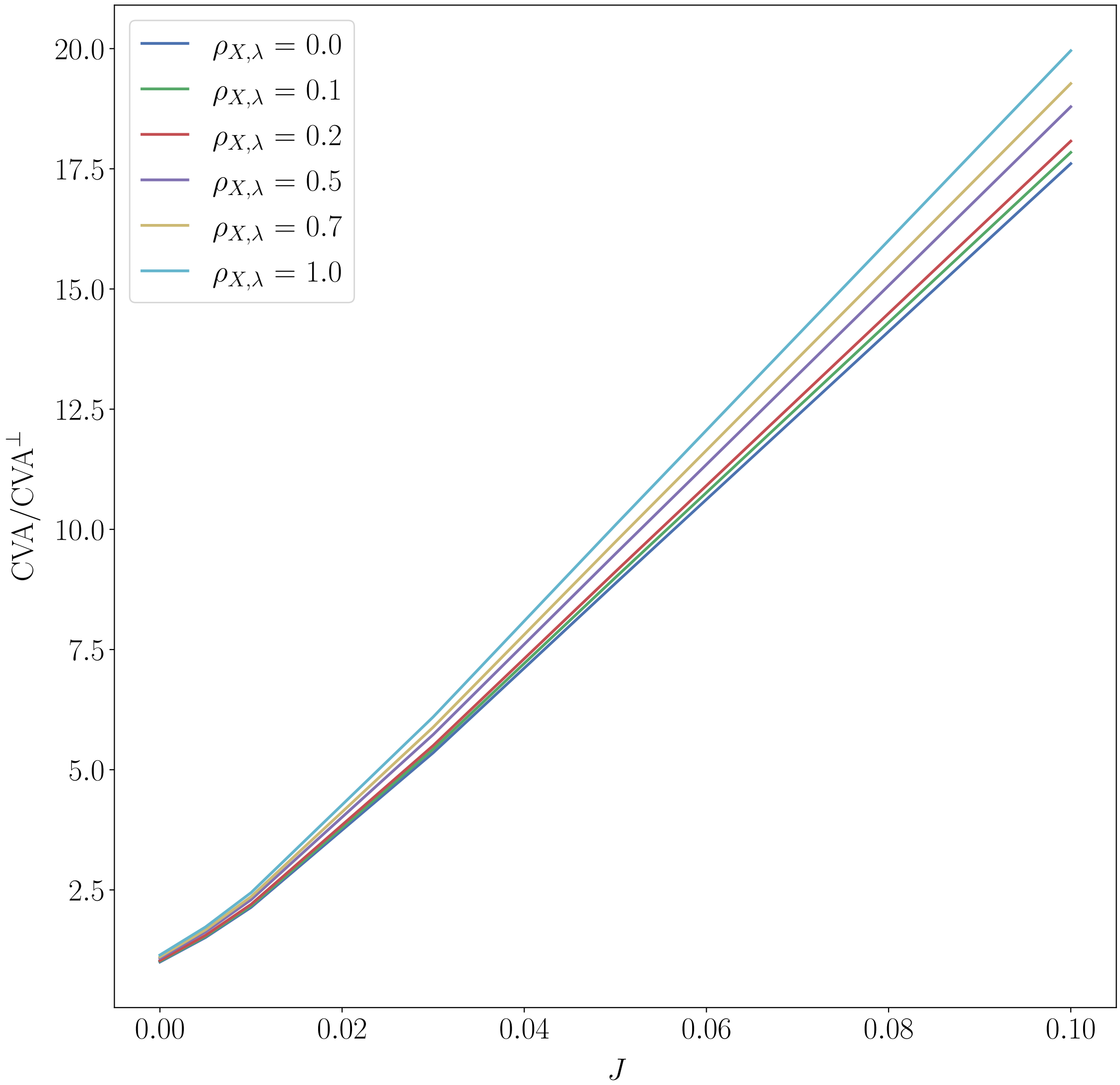

Figure C.1. *Collateralized 10-year cross-currency basis swap CVA WWR effect relative to independent CVA with different levels of FX rate and default intensity correlation*