Coupled 2-D FEM and 1-D Micromagnetic Model for Transverse Anisotropy Tape-Wound Magnetic Cores

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We present a new approach for modeling high-frequency losses in transverse anisotropy magnetic tapes by solving a 1-D eddy-current problem coupled to a micromagnetic constitutive law. Contrary to earlier models, the model is derived using a magnetic flux density conforming formulation. The model allows coupling the tape-level magnetization process to a 2-D finite element model for analyzing larger cores by homogenizing the tape layers. The developed model predicts the high-frequency losses in good agreement with previously presented measured results and models, demonstrating potential for increased accuracy in the calculation of losses in tape-wound cores.

***Index Terms*— Eddy currents, finite element method, homogenization, magnetic losses, micromagnetics, tape-wound magnetic cores.**

I. Introduction

TRANSVERSE field annealed (TFA) amorphous or nanocrystalline tapes are attractive materials for manufacturing high-permeability and low-loss magnetic cores e.g., in power electronics applications [1]. Owing to their transverse 180° domain wall (DW) structure, their longitudinal magnetization processes above the MHz-range are dominated by domain rotations, making losses related to DW movement negligible [2]. In addition, their μm-range thicknesses efficiently suppress classical eddy currents. Simplified illustrations of the domain structure and a wound core are given in Fig. 1.

Power losses in TFA tapes have been analyzed by coupling a magnetic-field conforming 1-D formulation for the eddy currents in the tape to a 1-D micromagnetic constitutive law derived from the Landau–Lifshitz–Gilbert (LLG) equation [3], [4]. Such an approach has been shown to provide a reasonable estimate of the losses of thin toroidal test samples at frequencies ranging from 0.1-10 MHz up to 1 GHz [3]. However, the 1-D approaches do not allow accounting for inhomogeneous flux density distributions present in larger magnetic cores in practical applications.

In this article, we derive a new magnetic flux density conforming formulation for the coupled 1-D eddy-current-LLG problem and present an approach for coupling this to a 2-D finite element (FE) model through a homogenization approach. This allows accounting for the tape-level losses in larger cores. The developed models will be referred to as the 1-D micromagnetic model and the coupled 2-D/1-D model, respectively. The coupled 2-D/1-D model is validated against measurements and modeling results presented in [3].

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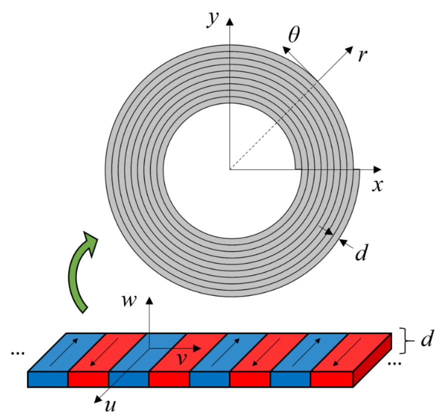

Fig. 1. Domain structure in a TFA tape with thickness d, illustration of a core wound from the tape, and definition of relevant coordinate systems.

II. Model

A. 1-D Micromagnetic Model

Following the notation in Fig. 1, the 1-D diffusion equation describing the dynamics of the magnetic field in the tape thickness $w \in [(-d/2),\ (d/2)]$ is given by

$$\frac{\partial^2 h(w,t)}{\partial w^2} = \sigma \frac{\partial b(w,t)}{\partial t} \tag{1}$$

where h and b are the v-components of the magnetic field strength and flux density, respectively, and σ is the electrical conductivity of the tape. Following [5] and [6], the diffusion problem (1) is solved by expressing $b(w,t)$ using a set of cosinusoidal basis functions $\alpha_j(w) = \cos(2\pi j w/d)$ and their time-dependent coefficients $b_j(t)$, $j = 0, \ldots, n_\mathrm{b} - 1$, which can be solved from the system

$$\boldsymbol{h}_\mathrm{s}(t) = \frac{1}{d}\int_{-d/2}^{d/2} h_\mathrm{Fe}(b(w,t))\boldsymbol{\alpha}(w)dw + \frac{\sigma d^2}{12}\boldsymbol{C}\frac{d}{dt}\boldsymbol{b}(t) \tag{2}$$

where $h_{\mathrm{Fe}}(b)$ represents the constitutive law and $\boldsymbol{C}$ is a constant matrix. The vectors $\boldsymbol{h}_{\mathrm{s}}(t) = [h_{\mathrm{s}}(t) \;\; 0 \ldots]^{\top}$ $\boldsymbol{\alpha}(w) = [\alpha_0(w) \;\; \ldots \;\; \alpha_{n_{\mathrm{b}}-1}(w)]^{\top}$ and $\boldsymbol{b}(t) = [b_0(t) \;\; \ldots \;\; b_{n_{\mathrm{b}}-1}(t)]^{\top}$ are also introduced. From (2), the magnetic flux density distribution in the tape can be solved either for given surface field strength $h_{\mathrm{s}}(t)$ or average flux density $b_0(t)$.

The micromagnetic model describing the magnetization process in the tape thickness is included in the constitutive law by writing

$$h_{\mathrm{Fe}}(w,t) = \nu_0 b(w,t) - M_{\mathrm{v}}(w,t) \tag{3}$$

where M_{v} is the v-component of the magnetization vector $\boldsymbol{M}$ and $\nu_0 = 1/\mu_0$ is the reluctivity of free space. By discretizing the LLG equation with the Forward-Euler time-stepping scheme with time-step length Δt, we can write M_{v} at time-step k as

$$\begin{aligned} M_{\mathrm{v}}^{k}(w) = M_{\mathrm{v}}^{k-1}(w) - \frac{\Delta t|\gamma_{\mathrm{G}}|\mu_0}{1+\alpha_{\mathrm{G}}^2}\Bigg\{\Bigg[\boldsymbol{M}^{k-1}(w)\times\Bigg(\boldsymbol{H}_{\mathrm{ex,an,ms}}^{k-1}(w) \\ +\frac{\alpha_{\mathrm{G}}}{M_{\mathrm{s}}}\boldsymbol{M}^{k-1}(w)\times\boldsymbol{H}_{\mathrm{ex,an,ms}}^{k-1}(w)\Bigg)\Bigg]\cdot\boldsymbol{u}_{\mathrm{v}} \\ +\frac{\alpha_{\mathrm{G}}}{M_{\mathrm{s}}}\Big(\big(M_{\mathrm{u}}^{k-1}(w)\big)^2+\big(M_{\mathrm{w}}^{k-1}(w)\big)^2\Big)h_{\mathrm{Fe}}^{k}(w)\Bigg\} \end{aligned} \tag{4}$$

where $|\gamma_{\mathrm{G}}|$ is the electron gyromagnetic ratio and α_{G} is the phenomenological damping parameter of the LLG equation, M_{s} is the saturation magnetization, and $\boldsymbol{H}_{\mathrm{ex,an,ms}}$ contains the exchange, anisotropy, and magnetostatic contributions to the effective field vector as described in [3]. The effective field term of the LLG equation is defined to be the sum of $\boldsymbol{H}_{\mathrm{ex,an,ms}}$ and $h_{\mathrm{Fe}}\boldsymbol{u}_{\mathrm{v}}$. The unit vector in the v-direction is denoted with $\boldsymbol{u}_{\mathrm{v}}$. It is noted that the magnetostatic field is calculated for a straight piece of tape. Uniaxial anisotropy in the u-direction is assumed in the calculation of the anisotropy field contribution in $\boldsymbol{H}_{\mathrm{ex,an,ms}}$.

Equation (4) can be shortly expressed as $M_{\mathrm{v}}^{k}(w) = a^{k-1}(w) + c^{k-1}(w)h_{\mathrm{Fe}}^{k}(w)$, where $a^{k-1}(w)$ and $c^{k-1}(w)$ contain terms from the previous time-steps. Combining this result with (3) and applying the Backward-Euler method to discretize the time derivative of the magnetic flux density coefficients in (2) finally allows writing (2) as

$$\begin{aligned} \boldsymbol{h}_{\mathrm{s}}^{k} = \underbrace{\left[\frac{1}{d}\int_{-d/2}^{d/2}\frac{\nu_0}{1+c^{k-1}(w)}\boldsymbol{\alpha}(w)\boldsymbol{\alpha}(w)^{\top}dw\right]}_{\boldsymbol{V}^{k-1}}\boldsymbol{b}^{k} \\ -\underbrace{\frac{1}{d}\int_{-d/2}^{d/2}\frac{a^{k-1}(w)}{1+c^{k-1}(w)}\boldsymbol{\alpha}(w)dw}_{\boldsymbol{F}^{k-1}}+\frac{\sigma d^2}{12}\boldsymbol{C}\frac{\boldsymbol{b}^{k}-\boldsymbol{b}^{k-1}}{\Delta t} \end{aligned} \tag{5}$$

where an auxiliary constant matrix $\boldsymbol{V}^{k-1}$ and an auxiliary constant vector $\boldsymbol{F}^{k-1}$ have been defined. Using (5), the coupled 1-D eddy-current-LLG problem can be solved for an arbitrary average flux density waveform $b_0(t)$. Afterward, the energy-loss density in the tape can be obtained by integrating the rate-of-change of the field energy density over one fundamental period T in the steady state [7]

$$w_{\mathrm{loss}} = \int_T h_{\mathrm{s}}(t)\frac{db_0(t)}{dt}dt. \tag{6}$$

The losses calculated with (5) and (6) have been observed to be in good agreement with the previously presented 1-D models [3], [4]. During the time-stepping, the conservation of the norm of the magnetization $\|\boldsymbol{M}\|$ is ensured by adopting a norm-conserving formalism based on the Cayley transform according to [8]. This method is used when simultaneously time-stepping the LLG equation to obtain all the components of $\boldsymbol{M}$.

B. Coupled 2-D/1-D Model

Following the idea of [6], the 1-D micromagnetic model (5) can be coupled to 2-D FE analysis of a wound core (in the xy-plane according to Fig. 1), where the tape layers are accounted for by using a homogenization approach. In 2-D, we assume that due to surface roughness, the core has a macroscopic reluctivity of ν_0 in the radial direction, while the magnetization in the tangential direction obeys (5). The spiral structure of the wound core is not taken into account, and instead the tape layers are assumed to be circular.

For each 1-D flux density coefficient b_j^k we define a corresponding 2-D quantity $\boldsymbol{B}_j^k(x,y)$ which is expressed using an out-of-plane magnetic vector potential $A_j^k(x,y)\boldsymbol{u}_{\mathrm{z}}$ as $\boldsymbol{B}_j^k(x,y) = B_{j,\mathrm{x}}^k(x,y)\boldsymbol{u}_{\mathrm{x}} + B_{j,\mathrm{y}}^k(x,y)\boldsymbol{u}_{\mathrm{y}} = \nabla\times(A_j^k(x,y)\boldsymbol{u}_{\mathrm{z}})$. Similarly, each row i of $\boldsymbol{h}_{\mathrm{s}}^k$ in (5) has a 2-D counterpart $\boldsymbol{H}_{\mathrm{s},i}^k(x,y)$ which obeys the Ampere's law $\nabla\times\boldsymbol{H}_{\mathrm{s},i}^k(x,y) = 0$. The coordinate transformation between xy- and $r\theta$-coordinates is expressed using a rotation matrix $\boldsymbol{R}^{\top}$. The weak form of the Ampere's law for $\boldsymbol{H}_{\mathrm{s},i}^k(x,y)$ is discretized using the standard 2-D Galerkin FE method and can be written as

$$\begin{aligned} \int_{\Omega}\boldsymbol{D}_{\mathrm{xy}}^{\top}\boldsymbol{R}\Bigg(\sum_{j=0}^{n_{\mathrm{b}}-1}\begin{bmatrix}\nu_0\delta_{ij} & 0 \\ 0 & V_{ij}+\frac{\sigma d^2}{12}\frac{C_{ij}}{\Delta t}\end{bmatrix}\boldsymbol{R}^{\top}\boldsymbol{D}_{\mathrm{xy}}\boldsymbol{a}_j^k \\ -\begin{bmatrix}0 \\ F_i\end{bmatrix}\Bigg)d\Omega - \boldsymbol{f}_i = 0 \end{aligned} \tag{7}$$

where Ω denotes the core region. In (7), $\boldsymbol{D}_{\mathrm{xy}}$ is a matrix containing the curls of the FE shape functions (in the xy-coordinates), $\boldsymbol{a}_j^k$ is a vector containing the nodal values of vector potential A_j^k, δ_{ij} is the Kronecker delta function, V_{ij}, C_{ij}, and F_i are elements of matrices $\boldsymbol{V}^{k-1}$, $\boldsymbol{C}$ and vector $\boldsymbol{F}^{k-1}$ used in (5), and $\boldsymbol{f}_i$ contains terms from the previous time-step.

The system of equations obtained by writing out all i rows of (7) is observed to be linear. When all the nodal values of the vector potentials $\boldsymbol{a}_j$ are gathered to one vector $\boldsymbol{a}$, the system of equations can be expressed as $\boldsymbol{S}\boldsymbol{a} = \boldsymbol{g}$. The linearity of the system results from the explicit Forward-Euler discretization of the LLG equation.

III. Results and Discussion

Coarse 2-D FE meshes containing $\sim$30 first-order elements in the core region are created for 5° sectors of toroidal

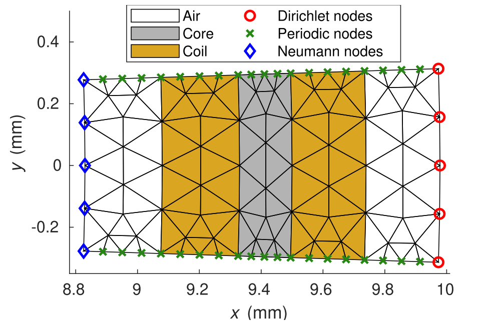

Fig. 2. Used mesh when a core with 17 μm thick tape is modeled. The core region has the dimensions of a core consisting of ten layers of tape.

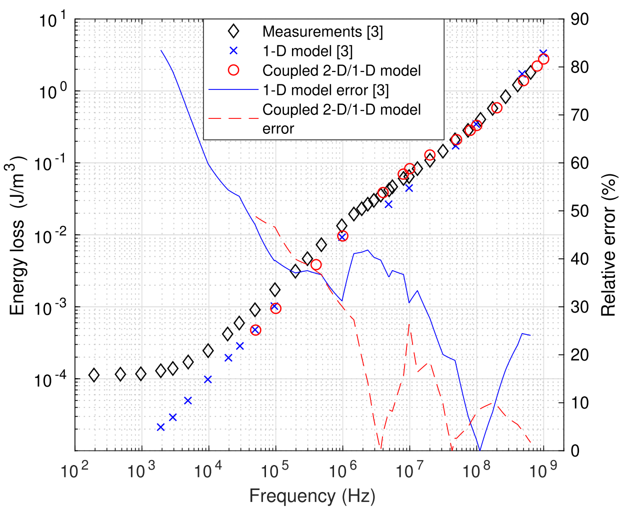

Fig. 3. Comparison of losses w_{loss} simulated with the coupled 2-D/1-D model against measurements and modeling results extracted from [3] for a core with 17 μm thick $Co_{71}Fe_4B_{15}Si_{10}$. Additionally, the relative errors of both models with respect to the measurements are plotted.

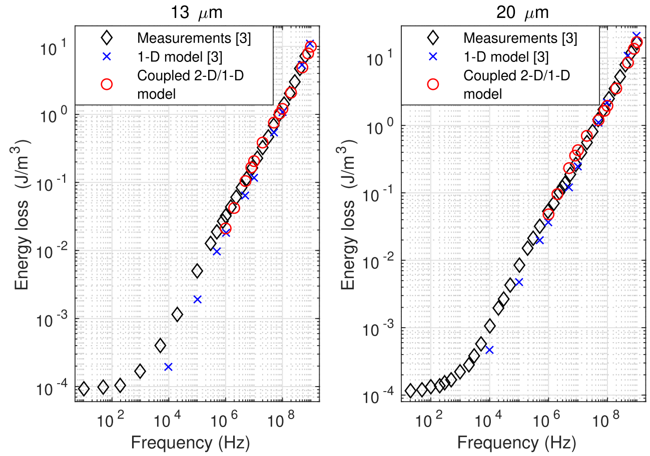

Fig. 4. Comparison of high-frequency losses w_{loss} simulated with the coupled 2-D/1-D model against measurements and modeling results extracted from [3] for $Co_{67}Fe_4B_{14.5}Si_{14.5}$ cores. The tape thickness in the examined core is 13 μm (left) and 20 μm (right).

inductors made of 13, 17, and 20 μm thick TFA tapes analyzed in [3]. The material and micromagnetic parameters for the tapes are extracted from [3]. The cores are surrounded by a stranded winding region, which is supplied by 50 kHz – 1 GHz sinusoidal ac voltage such that the desired magnetic flux density amplitude is obtained in the core. Additionally, a small air region surrounding the conductor is included in the modeling domain. Fig. 2 shows an example mesh when a core made of 17 μm thick tape is modeled.

The air and stranded conductor domains are modeled with standard 2-D FE practices whereas (7) describes the core region. Periodic boundary conditions are set along the radial boundaries of the modeling domain and a homogeneous Dirichlet condition on the outer radius. Since the flux density inside the center region of the toroid is negligible, only few layers of air elements are modeled inside the inner coil and a homogeneous Neumann condition is set on the inner radius.

Fig. 3 shows that the losses w_{loss} in a core wound from 17 μm thick $Co_{71}Fe_4B_{15}Si_{10}$ obtained with the coupled 2-D/1-D model and (6) correspond well to both the measurements and modeling results extracted from [3]. The core was simulated with an average magnetic flux density of 5 mT and damping parameter value of $\alpha_G = 0.04$. The relative errors plotted in Fig. 3 suggest the coupled 2-D/1-D model to be more accurate at high frequencies. Both the models lose accuracy at frequencies below 1 MHz as the assumption of negligible DW effects does not hold anymore.

To further validate the coupled 2-D/1-D model, the losses were calculated also for cores with different tape thicknesses and magnetic flux density amplitudes. Fig. 4 demonstrates the high-frequency losses for cores wound from 13 and 20 μm thick $Co_{67}Fe_4B_{14.5}Si_{14.5}$ with the magnetic flux density amplitude of 10 mT. Again, good agreement between the models and measurements is observed. Further analysis, however, suggests that the losses in the 20 μm thick tape are slightly less accurately predicted compared to the 13 and 17 μm thick tapes The difference can be due to uncertainty in the optimal choice of some model parameters. For the loss calculation in Fig. 4, the damping parameter value of $\alpha_G = 0.10$ was used. This contradicts earlier works suggesting increasing α_G as the tape thickness increases [9]. With the coupled 2-D/1-D model, it was observed that increasing α_G was necessary to better predict the losses with increased magnetic flux density amplitude. Rigorous methods to choose the exactly correct values do not seem to exist. Values of α_G between 0.04 and 0.22 are used in [3], [4], and [9]

The unconventional time-discretization used in the coupled 2-D/1-D model motivates to examine more closely the performance of the model with respect to the chosen time-step length. The chosen time-stepping scheme follows commonly used practices of choosing an implicit scheme for the eddy current problem [6] and an explicit scheme for the LLG equation [10], and thus the resulting time-discretization is a mix of implicit and explicit methods. Due to the lack of an analytical solution, the dependence on Δt is studied based on the dependency of the losses on the used time-step $w_{\text{loss}}(\Delta t)$. Fig. 5 shows the losses calculated with the coupled 2-D/1-D model with different time-steps for different excitation frequencies. The calculation was done for the $Co_{71}Fe_4B_{15}Si_{10}$ core with 17 μm thick tape layers under the same conditions as in Fig. 3. The results show that a

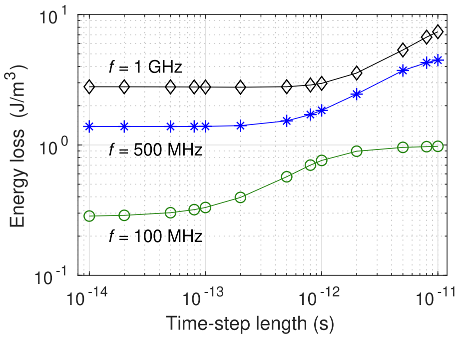

Fig. 5. Dependency of the calculated losses on the time-step length $w_{\mathrm{loss}}(\Delta t)$. The losses were calculated with the coupled 2-D/1-D model for a $Co_{71}Fe_4B_{15}Si_{10}$ core with tape thickness of 17 μm. The calculation was carried out under excitation frequencies of 1 GHz, 500 MHz, and 100 MHz.

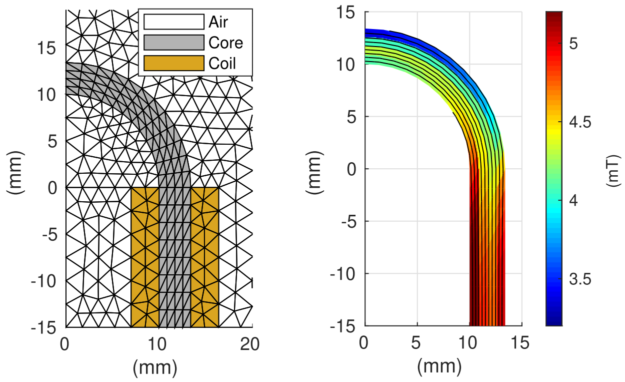

Fig. 6. Simulated quarter of the racetrack-shaped core consisting of 200 layers of 17 μm thick $Co_{71}Fe_4B_{15}Si_{10}$. Used mesh (left). Distribution of $||\boldsymbol{B}_0||$ in the core at the maximum of excitation (right).

small time-step Δt is required to obtain accurate results. A small time-step of $\sim$1 ps is required to accurately model the dynamics of the LLG equation [8], but the observed required time-step lengths are rather small even for micromagnetic simulations. The losses in Figs. 3 and 4 were calculated with a time-step of $\Delta t = 0.1$ ps which is justified according to the analysis of Fig. 5. However, it is noted that the required Δt seems to decrease as the frequency decreases. Larger time-steps could possibly be used if higher-order time-stepping schemes would be utilized. The current implementation is, however, sufficient to provide a good engineering estimate for the losses. The computational burden of the coupled 2-D/1-D model is high with low frequencies due to the large number of time-steps.

As the last example, a racetrack-shaped tape-wound core is simulated. This demonstrates the capability of the coupled 2-D/1-D model to take into account inhomogeneous flux density distributions. The simulation is carried out for a core consisting of 17 μm thick $Co_{71}Fe_4B_{15}Si_{10}$ with 200 layers of tape. The simulation parameters are set to match those used in Fig. 3, but now the frequency was fixed to 500 MHz and a flux density of 5 mT is imposed on the core at the middle of the coil. Fig. 6 shows the used mesh and distribution of $\|\boldsymbol{B}_0\|$. The calculated losses are 1.19 J/m^3 whereas for the toroidal core they were 1.39 J/m^3. The lower losses are due to the lower average $\|\boldsymbol{B}_0\|$ in the racetrack-shaped core.

IV. Conclusion

A new model was developed to calculate the high-frequency losses in tape-wound cores constructed from TFA tape materials. The losses calculated with the coupled 2-D/1-D model were shown to be in good agreement with a reference model and measurements from the literature under high frequencies of excitation. Potential for more accurate high-frequency loss prediction was also demonstrated.

To confirm the more accurate loss prediction of the coupled 2-D/1-D model, the effect of the model parameters to the predicted losses should be further investigated e.g., with sensitivity analysis. More advanced time-stepping schemes could be used to reduce the computational burden of the model.

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