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Nonlinear PDE control of flexible robotic arms for state tracking and link-deflection mitigation

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Abstract: This work presents a novel nonlinear control scheme for simultaneous state tracking and mitigation of undesired deflection effects in flexible robotic manipulators. The presented method directly incorporates the partial differential equations (PDEs) used for describing dynamics of the mechanism into the corresponding control calculations and assumes no form of reduction in PDEs (which in this work have been derived according to extended Hamilton principle for a rigid-flexible manipular when considering the flexible arm as an Euler-Bernoulli beam). Hence, the presented methodology should be considered as considerably more feasible for a wide range of applications in comparison with the conventional strategies which use assumed modes to analyze link flexibility dynamics or vibration effects. Furthermore, the proposed controller ensures state tracking and link-deflection boundedness only using standard control inputs to the mechanism without incorporating additional boundary inputs (which essentially represents satisfaction of more control objectives than the limited number of inputs would allow in conventional controllers), which would render it a viable choice for robotic applications where additional inputs cannot be easily exerted to end effector or cases where this strategy would require significant modifications in existing devices. Numerical simulations indicate the effectiveness of the presented control scheme.

Keywords: PDE Control; Robotic Manipulator; Nonlinear Control; Nonlinear Continuous- Dynamics System; Rigid-Flexible Mechanism.

1. INTRODUCTION

Control of flexible robotic manipulators inherently involves many different issues pertaining to the fields of multi degree-of-freedom (DOF) robotics, nonlinear control theory, and continuous-system dynamics. To achieve objectives such as development of fully automated industrial sites or space robotics, control schemes assigned to robotic manipulators should consider all the aforementioned effects. In such cases, conventional control algorithms often cannot ensure functionality of the closed-loop system and therefore are not able to achieve desired characteristics such as precise tracking control and elimination or mitigation of undesired effects such as displacement of a beam due to link flexibility.

Over the years, researchers have applied increasingly more sophisticated control theories to ensure that appropriate performance quality of a flexible system is maintained throughout a given operation despite the added complexity. Many iterations of such researches focus mostly on mitigation of undesired vibrational effects [1] but in recent years increasing attention has been devoted to investigations pertaining to qualities such as closed-loop stability, reference tracking and optimality. Furthermore, the great majority of the studies in this field have often considered the flexible system dynamics to be estimable by sets of ordinary differential equations (ODE) [2]. However, this can be at best considered an approximation as flexible systems feature distributed states and should be considered as infinite-dimensional for precise modeling and control tasks. Modelling procedure for various classes of infinite-dimensional flexible system is well-documented in the existing literature [3–5]. The obtained PDE-based sets of equations are difficult to work with for control purposes which is why their approximations as ODEs using methods such as finite-element method or assumed-mode methods are often used in their place. Naturally, such schemes do not feature all characteristics of distributed systems as they cannot describe the dynamical effects existing due to link flexibility and hence cannot reliably maintain high-quality control performance in physical applications. For example, while desired joint angles may be maintained using conventional scheme, the precision of end-point trajectory would suffer significantly due to flexibility-based displacement effects.

To address the issues regarding lack of precision and feasibility of conventional schemes for control of flexible manipulators, some researchers have adapted alternate control strategies where distributed state calculations are directly incorporated in control calculations instead. The work of Zhang et al. [6] presenting a PID-based PDE control scheme for flexible systems can be considered as an early milestone in this field. Later modifications such as the use of boundary control strategies (which are based on exertion of control action onto boundary conditions of the mechanism) allow consideration of additional effects such as mitigation of end-point deflection [7,8]. Some researchers such have also proposed the use of additional modifications including incorporation of state or disturbance observers for estimating unavailable states or external disturbances [9,10]. It should be noted that even when using PDE-based control strategies, various simplifications are often considered when assigning a control scheme for flexible systems. The existing literature is often based on derivation of flexible link dynamics based on estimation of deflections as perfect arcs [11,12]. This leads to obtaining a set of linear PDEs and boundary conditions which are relatively easy to work with but cannot be considered as extremely accurate. Assigning an appropriate control scheme using a fully nonlinear set of equations would be considerably more difficult. Furthermore, the studies addressing the topic of simultaneous stabilization and vibration mitigation normally adapt boundary control strategies which would require exertion of additional inputs at boundary points. This approach may not be feasible in applications where modifications to existing hardware would not be possible or efficient. Hence, alternate schemes capable of satisfying the discussed objectives using standard control inputs must be considered.

Noting the aforementioned issues, this study presents a novel nonlinear PDE control scheme capable of ensuring reference tracking for system states alongside with mitigation and boundedness of link flexibility effects for a robotic manipulator considering link flexibility effects. The proposed controller solely works with fully nonlinear set of PDEs and no reduction in control calculations or derivation of governing dynamical equations have been employed (other than the ones directly corresponding to Euler-Bernoulli beam theory). The presented controller also ensures maintenance of bounded displacement effects over the length of the flexible beam, which essentially means satisfaction of more control objectives than the number of used inputs would allow when using standard schemes. Therefore, the proposed scheme should be considered as more feasible for many existing applications. Numerical simulations demonstrate the effectiveness of proposed strategies.

2. PROBLEM STATEMENT

To address the problem of control of robotic manipulators with flexible arms, the commonly considered case of coupled rigid-flexible manipulator [7,13] depicted in Fig. 1 is investigate in this study. The mechanism constitutes of one rigid link and one flexible link, which will be considered as an infinite dimensional system due to distributed displacement states $w(\xi,t)$ where ξ indicates the position of an element of flexible beam alongside its longitudal axis. t is the system time. l_i, m_i, A_i, ρ_i, E_i and I_i respectively indicate the length, surface area, mass density, elastic modulus and surface moment of the inertia of the beam i. θ_i expresses the angle of the beams. F_1 and F_2 are system inputs respectively located at the distances l_{01} and l_{02} from the corresponding joints alongside the beams' longitudal axis and exerted at a perpendicular angle to the beam surface. The lumped mass M_0 is installed at the end of the flexible beam and acts as the payload of mechanism.

The extended Hamilton principle [3] is used in this study to obtain the mathematical model for the described dynamical system. To this end, the position vector $\bar{\boldsymbol{r}}_\xi(t)$ for an infinitesimal element of flexible beam located at the distance ξ from the connecting joint and the position vector for $\bar{\boldsymbol{r}}_{\boldsymbol{M}}(t)$ for the payload are expressed as in Eqs. (1-3).

$$\bar{\boldsymbol{r}}_\xi(t) = \begin{bmatrix} r_{x,\xi}(t) \\ r_{y,\xi}(t) \end{bmatrix} \tag{1}$$

$$r_{x,\xi}(t) = l_1 \cos\theta_1(t) + \xi\cos\theta_2(t) - w(\xi,t)\sin\theta_2(t) \tag{2}$$

$$r_{y,\xi}(t) = l_1 \sin\theta_1(t) + \xi\sin\theta_2(t) + w(\xi,t)\cos\theta_2(t) \tag{3}$$

$$\bar{\boldsymbol{r}}_M(t) = \bar{\boldsymbol{r}}_{l_2}(t)$$

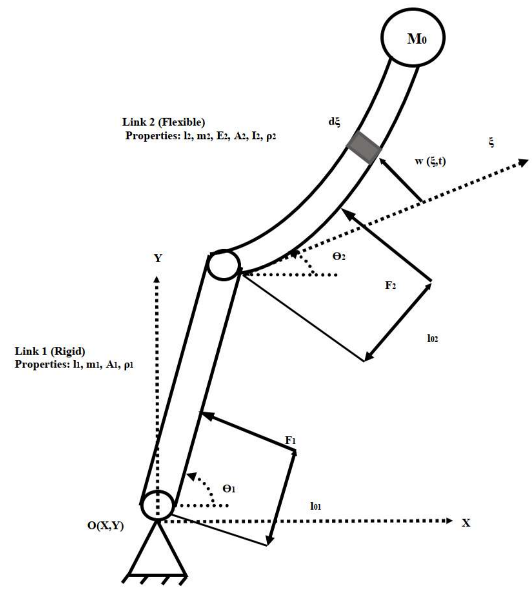


Fig. 1. Coupled rigid-flexible mechanism

According to the Euler-Bernoulli beam theory, the kinetic energy $T(t)$, potential energy $V(t)$ and generalized force corresponding to the entire mechanism with position vector $\bar{\boldsymbol{r}}_\xi(t)$ for the elements of flexible beam are described as follows.

$$T(t) = \frac{1}{2} I_{m_1}\dot{\theta}_1^2(t) + \frac{1}{2}\rho_2 A_2 \int_0^{l_2} \dot{\bar{\boldsymbol{r}}}_\xi^T(t)\dot{\bar{\boldsymbol{r}}}_\xi(t)d\xi + M_0\dot{\bar{\boldsymbol{r}}}_M^T(t)\dot{\bar{\boldsymbol{r}}}_M(t) \tag{4}$$

$$V(t) = \frac{1}{2} m_1 g l_1 \sin\theta_1(t) + \frac{1}{2} m_2 g l_2 \sin\theta_2(t) + m_2 g l_1 \sin\theta_1(t) + M_0 g l_1 \sin\theta_1(t) + M_0 g l_2 \sin\theta_2(t) + M_0 g w(\xi,t) cos\,\theta_2(t) + \frac{1}{2} E_2 I_2 \left[\frac{\partial^2 w}{\partial\xi^2}(\xi,t)\right]^2 d\xi \tag{5}$$

$$Q_{\theta_j} = \sum_{i=1}^{2} F_i \frac{\partial \bar{r}_{l_0 i}}{\partial\theta_j}, j = 1,2 \tag{6}$$

$[\dot{\cdot}]$ expresses derivation with respect to time and $I_{m_i} = \frac{1}{3} m_i l_i^2$ is the mass moment of inertia of the beams without considering link flexibility effects (which are described by the other terms in Eqs. (4-6)).

Substituting the aforementioned terms in extended Hamilton principle $\delta(T - V + Q) = 0$, after some lengthy but straightforward calculations, the mathematical model for system dynamics featuring governing equations and boundary conditions is obtained as follows. For brevity, we will use the expressions $w(\xi) = w(\xi, t)$, $\theta_i = \theta_i(t)$ and $(-)_\xi = \frac{d(-)}{d\xi}$.

$$\left(I_{m1} + \frac{1}{2}m_2 l_1^2 + M_0 l_1^2\right)\ddot{\theta}_1 + \left[\frac{1}{2}m_2 l_1 l_2 \cos(\theta_1 - \theta_2) + M_0 l_1 l_2 \cos(\theta_1 - \theta_2) + M_0 l_1 w(l_2)\sin(\theta_1 - \theta_2)\right]\ddot{\theta}_2 + \frac{1}{2}m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - \frac{1}{2}(m_1 + m_2) g l_1 \cos\theta_1 - M_0 g l_1 \cos(\theta_1) + M_0 l_1 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + 2M_0 l_1 \dot{w}(l_2)\dot{\theta}_2 \sin(\theta_1 - \theta_2) - M_0 l_1 \dot{\theta}_2^2 w(l_2)\cos(\theta_1 - \theta_2) - \rho_2 A_2 \int_0^{l_2}\left[l_1 \ddot{\theta}_2 \sin(\theta_1 - \theta_2)\, w(\xi) - l_1 \dot{\theta}_2^2 \cos(\theta_1 - \theta_2)\, w(\xi) - l_1 \ddot{w}(\xi)\cos(\theta_1 - \theta_2)\right] d\xi + M_0 l_1 \cos(\theta_1 - \theta_2)\,\ddot{w}(l_2) = F_1 l_{01} + F_2 l_1 \tag{7}$$

$$\left[\frac{1}{2}m_2 l_1 l_2 \cos(\theta_1 - \theta_2) + M_0 l_1 l_2 \cos(\theta_1 - \theta_2) + M l_1 w(l_2)\sin(\theta_1 - \theta_2)\right]\ddot{\theta}_1 + [I_{m2} + M_0 l_2^2 + M_0^2 w(l_2)]\ddot{\theta}_2 - \left(\frac{1}{2}m_2 + M_0\right) l_1 l_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + M_0 l_2 \ddot{w}(l_2) + 2M_0 w(l_2)\dot{w}(l_2)\dot{\theta}_2 + M_0 l_1 \dot{\theta}_1^2 w \cos(\theta_1 - \theta_2) - M_0 g l_2 \cos(\theta_2) - \rho_2 A_2 \int_0^{l_2}\left[-w^2(\xi)\ddot{\theta}_2 + l_1 \ddot{\theta}_1 \sin(\theta_1 - \theta_2)\, w(\xi) + l_1 \dot{\theta}_1^2 \cos(\theta_1 - \theta_2) w(\xi) - \xi \ddot{w}(\xi) + 2 l_1 \dot{\theta}_1 \sin(\theta_1 - \theta_2)\,\dot{\omega}(\xi)\right] d\xi = F_2 l_{02} \tag{8}$$

$$\rho_2 A_2 \ddot{w}(\xi) - \rho_2 A_2 \dot{\theta}_2^2 w(\xi) + \rho_2 A_2 \xi \ddot{\theta}_2 + \rho_2 A_2 l_1 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - \rho_2 A_2 l_1 \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2)\sin(\theta_1 - \theta_2) + E_2 I_2 w_{\xi\xi\xi\xi}(\xi) = 0 \tag{9}$$

$$\ddot{w}(l_2) - w(l_2)\dot{\theta}_2^2 + l_2 \ddot{\theta}_2 + l_1 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - l_1 \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2)\sin(\theta_1 - \theta_2) + g w(l_2)\sin\theta_2 - g\cos(\theta_2) + E_2 I_2 w_{\xi\xi\xi} = 0 \tag{10}$$

$$w(o) = w_{\xi\xi}(0) = 0 \tag{11}$$

$$w_{\xi\xi}(l_2) = 0 \tag{12}$$

Remark 1 (Control Objective). Control objectives are considered asymptotic stability of closed-loop system and tracking of position reference signals $\theta_{1r}(t)$ and $\theta_{2r}(t)$ with respect to system states $\theta_1(t)$ and $\theta_2(t)$. Furthermore, the objective of mitigating link deflection $w(\xi), \xi \in [0, l_2]$ and ensuring its boundedness is considered. Furthermore, the set of PDEs Eqs. (7-12) should be directly incorporated in control calculations without any reduction for increased feasibility.

In Section 3, a nonlinear PDE-based control scheme will be presented satisfying the described control objective for dynamical system described by Eqs. (7-12).

3. CONTROLLER DESIGN

This section will detail the procedure of designing the nonlinear PDE-based controller for flexible manipulators (NPCFM). To present control calculations in a concise and orderly format, Eqs. (7,8) will be expressed as state-space equations.

$$\boldsymbol{M}_{rr}(t)\ddot{\bar{\boldsymbol{q}}}_r(t) + \bar{\boldsymbol{h}}_r(t) = \boldsymbol{B_{rr}}(t)\bar{\boldsymbol{u}}(t) \tag{13}$$

$$\bar{\boldsymbol{q}}_r(t) = [\theta_1, \quad \theta_2]^T \tag{14}$$

$$\bar{\boldsymbol{u}}(t) = [F_1(t), \quad F_2(t)]^T \tag{15}$$

$$\boldsymbol{M}_{rr}(t) = \begin{bmatrix} M_{rr_{11}}(t), & M_{rr_{12}}(t) \\ * & M_{rr_{22}}(t) \end{bmatrix} \tag{16}$$

$$M_{rr_{11}} = I_{m1} + \frac{1}{2}m_2 l_1^2 + M_0 l_1^2 - \rho_2 A_2 \int_0^{l_2} l_1 \sin(\theta_1 - \theta_2)\, w(\xi) d\xi \tag{17}$$

$$M_{rr_{12}} = \frac{1}{2}m_2 l_1 l_2 \cos(\theta_1 - \theta_2) + M_0 l_1 l_2 \cos(\theta_1 - \theta_2) + M_0 l_1 w(l_2)\sin(\theta_1 - \theta_2) + \rho_2 A_2 \int_0^{l_2} w^2(\xi)\, d\xi \tag{18}$$

$$M_{rr_{22}} = I_{m2} + M_0 l_2^2 + M_0^2 w(l_2) \tag{19}$$

$$\bar{\boldsymbol{h}}_r(t) = [h_1(t), \quad h_2(t)] \tag{20}$$

$$h_1(t) = \frac{1}{2}m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - \frac{1}{2}(m_1 + m_2) g l_1 \cos\theta_1 - M_0 g l_1 \cos(\theta_1) + M_0 l_1 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + 2M_0 l_1 \dot{w}(l_2)\dot{\theta}_2 \sin(\theta_1 - \theta_2) - M_0 l_1 \dot{\theta}_2^2 w(l_2)\cos(\theta_1 - \theta_2) - \rho_2 A_2 \int_0^{l_2}\left[-l_1 \dot{\theta}_2^2 \cos(\theta_1 - \theta_2)\, w(\xi) - l_1 \ddot{w}(\xi)\cos(\theta_1 - \theta_2)\right] d\xi + M_0 l_1 \cos(\theta_1 - \theta_2)\,\ddot{w}(l_2) \tag{21}$$

$$h_2(t) = -\left(\frac{1}{2}m_2 + M_0\right) l_1 l_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + M_0 l_2 \ddot{w}(l_2) + 2M_0 w(l_2)\dot{w}(l_2)\dot{\theta}_2 + M_0 l_1 \dot{\theta}_1^2 w \cos(\theta_1 - \theta_2) - M_0 g l_2 \cos(\theta_2) - \rho_2 A_2 \int_0^{l_2}\left[l_1 \dot{\theta}_1^2 \cos(\theta_1 - \theta_2) w(\xi) - \xi \ddot{w}(\xi) + 2 l_1 \dot{\theta}_1 \sin(\theta_1 - \theta_2)\,\dot{\omega}(\xi)\right] d\xi \tag{22}$$

$$\boldsymbol{B_{rr}} = \begin{bmatrix} l_{01} & l_1 \\ 0 & l_{02} \end{bmatrix} \tag{23}$$

Theorem 1 will detail the procedure of assigning an appropriate controller capable of ensuring satisfaction of control objectives i.e., tracking of assigned position references and mitigation of undesired link flexibility effects. To address this, a candidate Lyapunov function (CLF) will be assigned featuring measures of tracking errors and link flexibility effects. Through mathematical manipulations of CLF and without introducing additional boundary inputs, it will be ensured that the controllable system states will converge to desired references. Simultaneously, boundedness of distributed system states and mitigation of undesired flexibility effects will be maintained.

Theorem 1. The control input Eq. (24) will ensure the satisfaction of control objectives of Remark 1 for the flexible dynamical system described by Eqs. (7-12).

$$\bar{\boldsymbol{u}}(t) = \boldsymbol{B}_{rr}^{-1}\left[\boldsymbol{M}_{rr}(t)\ddot{\bar{\boldsymbol{q}}}_{rf}(t) + \bar{\boldsymbol{h}}_r(t)\right], \tag{24}$$

where $\ddot{\bar{\boldsymbol{q}}}_{rf}(t)$ is the solution for a control inequality that will be defined in Eq. (40).

Proof. The CLF $V(t)$ is defined in Eq. (25) as follows.

$$V(t) = V_1(t) + c_f V_2(t) \tag{25}$$

$$V_1(t) = \frac{1}{2}\bar{\boldsymbol{e}}^T(t)\bar{\boldsymbol{e}}(t) \tag{26}$$

$$V_2(t) = \frac{1}{2}\int_0^{l_2}\left[\dot{w}^2(\xi) + \frac{E_2 I_2}{\rho_2 A_2} w_{\xi\xi}^2(\xi)\right] d\xi \tag{27}$$

$\bar{\boldsymbol{e}}(t)$ is the error vector defined as:

$$\bar{\boldsymbol{e}}(t) = [e_1(t), \quad e_2(t)]^T, \tag{28}$$

$$e_i(t) = \theta_i(t) - \theta_{ri}(t) + \lambda\left[\dot{\theta}_i(t) - \dot{\theta}_{ri}(t)\right]. \tag{29}$$

$\lambda \in (0,1)$ and $c_f > 0$ are control parameters.

To calculate derivative of CLF, $\dot{\bar{\boldsymbol{e}}}(t)$ is expressed as:

$$\dot{\bar{\boldsymbol{e}}}(t) = \boldsymbol{M}_{rr}^{-1}(t)\boldsymbol{B}_{rr}(t)\bar{\boldsymbol{u}}(t) - \lambda \boldsymbol{M}_{rr}^{-1}(t)\bar{\boldsymbol{h}}_r(t) + \bar{\boldsymbol{\delta}}(t), \tag{30}$$

$$\bar{\boldsymbol{\delta}}(t) = \begin{bmatrix} \dot{\theta}_1(t) - \dot{\theta}_{1r}(t) - \lambda\ddot{\theta}_{1r}(t) \\ \dot{\theta}_2(t) - \dot{\theta}_{2r}(t) - \lambda\ddot{\theta}_{2r}(t) \end{bmatrix}. \tag{31}$$

It follows that:

$$\dot{V}_1(t) = \bar{\boldsymbol{e}}^T(t)\dot{\bar{\boldsymbol{e}}}(t) = \bar{\boldsymbol{e}}^T(t)\boldsymbol{M}_{rr}^{-1}(t)\boldsymbol{B}_{rr}(t)\bar{\boldsymbol{u}}(t) - \bar{\boldsymbol{e}}^T(t)\boldsymbol{M}_{rr}^{-1}\bar{\boldsymbol{h}}_r(t) + \bar{\boldsymbol{e}}^T(t)\bar{\boldsymbol{\delta}}(t), \tag{32}$$

$$\dot{V}_2(t) = \int_0^{l_2}\left[\dot{w}(\xi)\ddot{w}(\xi) + \frac{E_2I_2}{\rho_2A_2}w_{\xi\xi}(\xi)\dot{w}_{\xi\xi}(\xi)\right]d\xi. \tag{33}$$

To simply $\dot{V}_2(t)$, based on Eq. (9), the acceleration value $\ddot{w}(\xi)$ is expressed as Eq. (34) instead.

$$\ddot{w}(\xi) = \dot{\theta}_2^2 w(\xi) - \frac{E_2I_2}{\rho_2A_2}w_{\xi\xi\xi\xi}(\xi) + f(\bar{\boldsymbol{q}}_r, \dot{\bar{\boldsymbol{q}}}_r, \ddot{\bar{\boldsymbol{q}}}_r) \tag{34}$$

$$f(\bar{\boldsymbol{q}}_r, \dot{\bar{\boldsymbol{q}}}_r, \ddot{\bar{\boldsymbol{q}}}_r) = -\xi\ddot{\theta}_2 - l_1\ddot{\theta}_1\cos(\theta_1-\theta_2) + l_1\dot{\theta}_1(\dot{\theta}_1-\dot{\theta}_2)\sin(\theta_1-\theta_2) \tag{35}$$

Similarly, using consecutive integration by parts for $\int_0^{l_2}[w_{\xi\xi}(\xi,t)\dot{w}_{\xi\xi}(\xi,t)]\,d\xi$ and substituting boundary conditions Eqs. (11,12), it is obtained that:

$$\dot{V}_2(t) = \int_0^{l_2}\left\{\left[\dot{\theta}_2^2 w(\xi) - \frac{E_2I_2}{\rho_2A_2}w_{\xi\xi\xi\xi}(\xi) + f(\bar{\boldsymbol{q}}_r, \dot{\bar{\boldsymbol{q}}}_r, \ddot{\bar{\boldsymbol{q}}}_r)\right]\dot{w}(\xi)\right\}d\xi + \frac{E_2I_2}{\rho_2A_2}\int_0^{l_2}w_{\xi\xi\xi\xi}(\xi)\dot{w}(\xi)\,d\xi + \frac{E_2I_2}{\rho_2A_2}\left(\dot{w}_\xi\,\dot{w}_{\xi\xi} - \dot{w}w_{\xi\xi\xi}\right)_0^{l_2} = \int_0^{l_2}\{[\dot{\theta}_2^2 w(\xi) + l_1\dot{\theta}_1(\dot{\theta}_1 - \dot{\theta}_2)\sin(\theta_1-\theta_2)]\dot{w}(\xi)\}\,d\xi - \int_0^{l_2} l_1\cos(\theta_1-\theta_2)\,\dot{w}(\xi)d\xi\ddot{\theta}_1 - \int_0^{l_2}\xi\dot{w}\,(\xi)d\xi\ddot{\theta}_2 - \frac{E_2I_2}{\rho_2A_2}\dot{w}(l_2)w_{\xi\xi\xi}(l_2). \tag{36}$$

Then, from Eq. (32) and Eq. (36), $\dot{V}(t)$ is expressed as:

$$\dot{V}(t) = \bar{\boldsymbol{\gamma}}^T\ddot{\bar{\boldsymbol{q}}}_r + \bar{\boldsymbol{e}}^T(t)\bar{\boldsymbol{\delta}}(t) - c_f\frac{E_2I_2}{\rho_2A_2}\dot{w}(l_2)w_{\xi\xi\xi}(l_2) + c_f\int_0^{l_2}\{[\dot{\theta}_2^2 w(\xi) + l_1\dot{\theta}_1(\dot{\theta}_1-\dot{\theta}_2)\sin(\theta_1-\theta_2)]\dot{w}(\xi)\}, \tag{37}$$

$$\bar{\boldsymbol{\gamma}} = \begin{bmatrix} \lambda e_1(t) - \int_0^{l_2} l_1\cos(\theta_1-\theta_2)\,\dot{w}(\xi)d\xi \\ \lambda e_2(t) - \int_0^{l_2}\xi\dot{w}\,(\xi)d\xi \end{bmatrix}. \tag{38}$$

Now, based on Eq. (37), $\dot{V}(t) < 0$ is solvable for $\ddot{\bar{\boldsymbol{q}}}_r$ and therefore for $\bar{\boldsymbol{u}}$. as $\boldsymbol{M}_{rr}\ddot{\bar{\boldsymbol{q}}}_r + \bar{\boldsymbol{h}}_r = \boldsymbol{B}_{rr}\bar{\boldsymbol{u}}$ according to state space representations above. The solution for $\ddot{\bar{\boldsymbol{q}}}_r$ is expressed as $\ddot{\bar{\boldsymbol{q}}}_{rf}$. To obtain the solution, inequality $\dot{V}(t) < 0$ is transformed using slack variable α as follows.

$$\bar{\boldsymbol{\gamma}}^T\ddot{\bar{\boldsymbol{q}}}_r = \left[\lambda e_1(t) - \int_0^{l_2} l_1\cos(\theta_1-\theta_2)\,\dot{w}(\xi)d\xi\right]\ddot{\theta}_{1f} + \left[\lambda e_2(t) - \int_0^{l_2}\xi\dot{w}\,(\xi)d\xi\right]\ddot{\theta}_{2f} = c_f\frac{E_2I_2}{\rho_2A_2}\dot{w}(l_2)w_{\xi\xi\xi}(l_2) - c_f\int_0^{l_2}\{[\dot{\theta}_2^2 w(\xi) + l_1\dot{\theta}_1(\dot{\theta}_1-\dot{\theta}_2)\sin(\theta_1-\theta_2)]\dot{w}(\xi)\}\,d\xi - \bar{\boldsymbol{e}}^T(t)\bar{\boldsymbol{\delta}}(t) - \alpha^2. \tag{39}$$

Then, the solution $\ddot{\bar{\boldsymbol{q}}}_{rf}$ is calculated as Eqs. (40,41).

$$\ddot{\bar{\boldsymbol{q}}}_{rf} = \left[\ddot{\theta}_{1f}, \quad \ddot{\theta}_{2f}\right]^T \tag{40}$$

$$\ddot{\theta}_{1f} = \frac{\beta(t)}{\lambda e_1(t) - \int_0^{l_2} l_1\cos(\theta_1-\theta_2)\dot{w}(\xi)d\xi}\left[-c_f\frac{E_2I_2}{\rho_2A_2}f_b(t) - c_f\int_0^{l_2}\{[\dot{\theta}_2^2 w(\xi) + l_1\dot{\theta}_1(\dot{\theta}_1-\dot{\theta}_2)\sin(\theta_1-\theta_2)]\dot{w}(\xi)\}\,d\xi - e_1(t)\delta_1(t) - \alpha^2\right] \tag{41}$$

$$\ddot{\theta}_{2f} = \frac{1-\beta}{\lambda e_2(t) - \int_0^{l_2}\xi\dot{w}(\xi)d\xi}\left[-c_f\frac{E_2I_2}{\rho_2A_2}f_b(t) - c_f\int_0^{l_2}\{[\dot{\theta}_2^2 w(\xi) + l_1\dot{\theta}_1(\dot{\theta}_1-\dot{\theta}_2)\sin(\theta_1-\theta_2)]\dot{w}(\xi)\}\,d\xi - e_2(t)\delta_2(t) - \alpha^2\right] \tag{42}$$

$\beta(t) \in [0,1]$ is a control parameter. From Eq. (13) and Eq. (40), it follows that control input $\bar{\boldsymbol{u}}(t)$ satisfying control objectives expressed in Remark 1 is calculated as Eq. (24), which completes the proof.

Remark 2. The significance of deflection mitigation or state tracking can be adjusted in CLF $V(t)$ through the control parameter c_f. Higher magnitudes of c_f would correspond to better vibration mitigation, whereas smaller values would prioritize state tracking.

Remark 3. A potential limitation of the proposed controller is the requirement for availability of data corresponding to distributed states as they may be difficult to measure. This issue could be addressed by assigning observation or estimation schemes or assigning a set of corresponding sensors to the flexible beam.

4. NUMERICAL RESULTS

This section will present the results of numerical simulations for the control scheme developed in Sections 2 and 3. To this end, numeric values presented in Table 1 are considered for investigated mechanical system.

Table 1. Properties of mechanical system and control parameters

Parameter	Value	Parameter	Value
$\boldsymbol{\rho_i A_i}$	$1.872[kg/m]$	l_{0i}	$l_i/3$
$\boldsymbol{E_i I_i}$	$5.76e7[N.m^2]$	g	$9.8\ [m/s^2]$
Sample time	0.001 [s]	β	0.0
$\boldsymbol{\alpha(t)}$	$10.0e_i(t)$	C_f	1.0
$\boldsymbol{M_0}$	$1.0[kg]$	λ	0.8
$\boldsymbol{l_1}$	$1.0[m]$	l_2	$2.0[m]$

Remark 3. While no reduction has been considered in design of control scheme, the nonlinear set of PDEs (7-12) have no analytical solution. Therefore, for numerical simulations and verification of controller efficiency, the Galerkin method [3] is employed in this study. To this end, the distributed estimation of deflection Eq. (43) is considered in numerical simulations.

$$w(\xi,t) = \sum_{i=1}^{n_m}\eta_i(t)\phi_i(\xi), \tag{43}$$

where $\eta_i(t)$ are unknown transformed states that are determined during numerical calculations and $\phi_i(\xi)$ describe the mode shapes of flexible beam assumed to be in the form of Eq. (47).

$$\phi_i(\xi) = c_{1i}(\cos\beta_i\xi + \cosh\beta_i\xi) + c_{2i}(\cos\beta_i\xi - \cosh\beta_i\xi) + c_{3i}(\sin\beta_i\xi + \sinh\beta_i\xi) + c_{4i}(\sin\beta_i\xi - \sinh\beta_i\xi) \tag{44}$$

β_i, c_{1i}, c_{2i}, c_{3i} and c_{4i} are determined by substituting Eq. (43) in boundary conditions Eqs. (10-12).

$$c_1 = c_2 = 0 \tag{45}$$

$$\begin{bmatrix} M_{11}, & M_{12} \\ M_{21}, & M_{22} \end{bmatrix} \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \hbar_2 \end{bmatrix} \tag{46}$$

$$M_{11} = -\sin \beta_i \xi + \sinh \beta_i \xi \tag{47}$$

$$M_{12} = -\sin \beta_i \xi - \sinh \beta_i \xi \tag{48}$$

$$M_{21} = E_2 I_2 \beta^3 (-\cos \beta_i \xi + \cosh \beta_i \xi) + M_0 [-\dot{\theta}_2^2 + g] \tag{49}$$

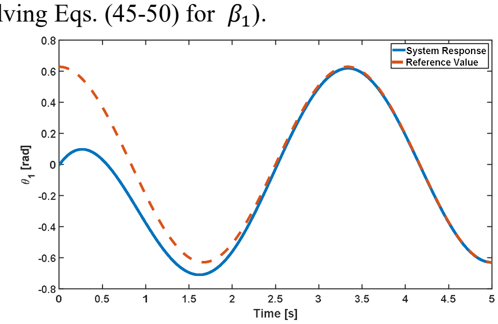
$$M_{22} = E_2 I_2 \beta^3 (\sin \beta_i \xi - \sinh \beta_i \xi) + M_0 [-\dot{\theta}_2^2 + g] \tag{50}$$

$$\hbar_2 = l_2 \ddot{\theta}_2 + l_1 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - l_1 \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2) - g \cos(\theta_2) \tag{51}$$

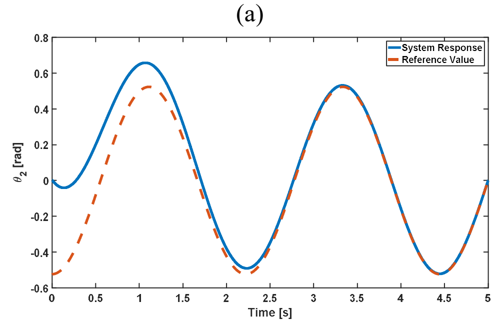
Remark 4. To obtain a numerically solvable set of ODEs, Eq. (43) is substituted in governing equations Eqs. (7-9). After substituting Eq. (43) in Eq. (9), the resulting equation is multiplied by $\phi_j(\xi)$ for $j = 1, \dots, n_m$ and integrated over $[0, l_2]$ [13]. This would result in n_m equations corresponding to η_1, η_2, … and η_{n_m}.

As it was expressed, the procedure of calculation of control inputs do not require any internal variable used in Galerkin method and are only based on system states and inputs. In other words, no reduction or assumed mode is employed within the control scheme itself.

The performance of the closed-loop system for the described configuration is described in Figs. 3-5 . In this study, tracking response of the system for assigned references $\theta_{ir}(t) = a_{ir} \cos \omega_{ir} t$ $[m]$ with $a_{1r} = \pi/5$, $a_{2r} = -\pi/6$, $\omega_{1r} = 0.6\,\pi$ and $\omega_{2r} = 0.9\,\pi$ is investigated which activates the first vibrational mode of the considered mechanism (which can be calculated by solving Eqs. (45-50) for β_1).

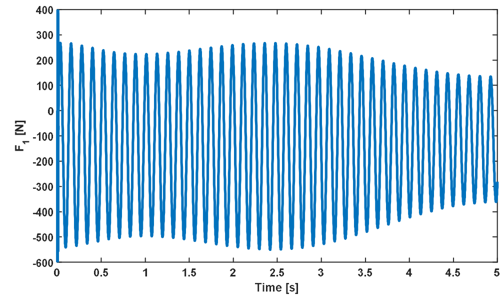


(a)

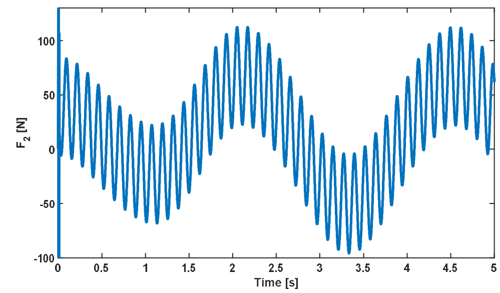


(b)

Fig 2. State tracking performance (a) θ_1 (b) θ_2



(a)



(b)

Fig 3. Control input (a) F_1 (b) F_2

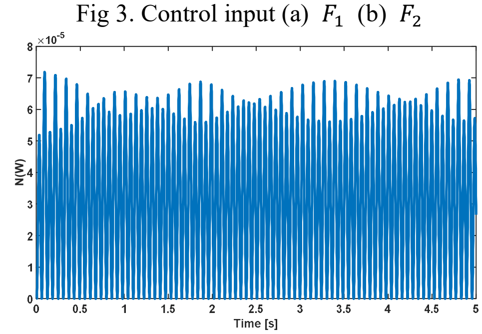


Fig 4. Analysis of boundedness of distributed deflection for flexible beam

As it is shown by Fig. 2, the control system successfully tracks the assigned reference for joint angles signals in finite time. Fig. 3 depicts control inputs F_1 and F_2 . In Fig. 4, the magnitude of distributed displacement over the simulation time is investigated using the displacement measure $N(w) = \int_0^{l_2} w^2(\xi)\, d\xi$ which retain bounded values over the length of flexible beam, as proven by direct incorporation of PDE calculations in the proposed control scheme.

One of the main features of the presented controller in comparison with existing schemes is that it does not require incorporation of any other additional input to the boundary conditions, which renders this method as readily applicable to many applications. In this scheme, the effects of nonlinear link-flexibility dynamics and interactions between different bodies are directly incorporated into inputs exerted at conventional locations which is different from previous studies including Control of Two-Link Manipulator (CTLM) [6] and Boundary Control for Flexible Manipulator (BCFM) [14]. Inputs ensure satisfaction of control objectives without the need to assign control gains corresponding to velocity and position tracking error, which in our tests had significant impact on controllability of the overall system using previous methods. Results of performance comparisons with existing methods are described in Fig. 5. Previous methods cannot be considered as feasible for the considered system because they require exertion of additional inputs or were derived for simpler systems.

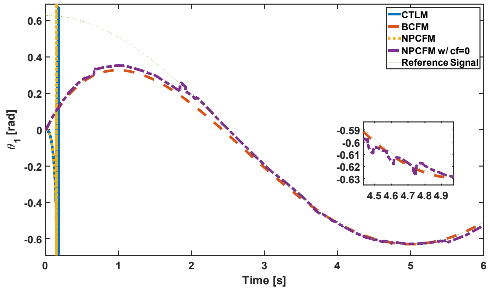


Fig 5. Convergence analysis for NPCFM compared to CTLM, BCFM and NPCFM without link flexibility effects

5. CONCLUSIONS

In this paper, a novel nonlinear PDE-based method was proposed for tracking control of multi-DOF manipulators. The presented control scheme does not include any form of reduction or assumption for transforming PDEs to solvable ODEs. As a result, it should be considered as considerably more feasible for various applications where accurate estimation of PDEs would not be possible or in cases where that would be too cost intensive. Furthermore, the investigated dynamical model uses a fully nonlinear set of equations for describing dynamics of flexible mechanism rather than linear models which are based on estimation of arc-length of flexible beam. This scheme does not require exertion of additional boundary inputs for satisfying control objectives including state tracking and maintenance of bounded vibration effects. Hence, based on combination of realistic modeling and PDE-based control calculations, the presented method can be considered as a feasible algorithm for general classes of manipulators with flexible arms. Future research opportunities include design of distributed state observers for practical applications as well as construction of convenient controllers for high-DOF flexible robotics.

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