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Estimating maximum shear modulus (G_0) using adaptive neuro-fuzzy inference system (ANFIS)

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ABSTRACT

Realistic estimation of soil behavior is dependent on considering very small and small strain domains. Lengthy formulas proposed in the literature have limited predictive power for estimation of maximum shear modulus, G_0. The aim of this study is to overcome this drawback. Theoretical aspects of fuzzy sets and adaptive neuro-fuzzy inference system (Anfis) are presented. Then, Anfis is implemented within a logical platform that adapts itself with available data to estimate and describe G_0.

1. Introduction

As shown in equation (1) only two parameters of shear wave velocity V_s and total mass density ρ are needed for measuring the maximum shear modulus, G_0 which is a key property of the soil at very small strain region. However, shear modulus G starts decreasing significantly right after passing the very small strain threshold (around 0.001%). In addition, shear modulus depends on many factors, but it is mainly affected by stress, stress history and density [1,2]. The in-situ measured value might thus not be the correct one for an engineering project. While laboratory determination is easily influenced by sample disturbances and need high accuracy of measurements, it is fair to say that determining shear modulus at the small strain region is not straightforward.

$$G_0 = V_s^2 \rho \tag{1}$$

One review covering 372 experiments from 140 different soils with a high amount of scattering in terms of state properties pointed out confining stress p' and initial void ratio e_0 as the main parameters affecting G_0 [3]. This finding is exploited to estimate G_0 in this study.

2. Database

Dataset used in this study contains 62 experimental results [4–12]. This data has a good divergence in terms of confining stress p', and void ratio e_0. To better understanding the wide domain of soils studied in this research, confining stress versus void ratio graph is presented in Fig. 1. Thorough details and evaluation of this valuable dataset are presented in Refs. [13,14].

The performance of two equations in the literature (equations (2) and (3)) was checked for the considered database [4–12]. The suitability for selection of considered relations was that adequate information was available on considered data to enable evaluation to be conducted. Estimation accuracy of these equations is shown in Fig. 2. As shown in this figure, the formula presented by Ref. [3] gives better approximation compared to the other formula based on calculated error criteria. The proposed solution for better approximation of G_0 is explained in the following sections [3,15].

$$\frac{G_0}{\sigma_{ref}} = \frac{630}{\left(1 + 1.2\left(\frac{PI}{LL}\right)^{0.5}\right)}\left(\frac{p'}{\sigma_{ref}}\right)^{0.5}\left(\frac{1}{e_0}\right)^{1.3} \tag{2}$$

$$G_0 = \frac{358 - 3.8PI}{0.4 + 0.7\, e_0} p' \tag{3}$$

3. Fuzzy sets

Fuzzy inference system (FIS) represents a versatile platform to describe complex phenomena within a simple mathematical framework. As once stated by the Father of fuzzy logic, Professor Zadeh: “In almost every case, you can build the same product without fuzzy logic, but fuzzy is faster and cheaper” [16]. There are quite many references related to theoretical aspects of fuzzy logic following the well-known paper in which Professor Zadeh introduced a unique way of

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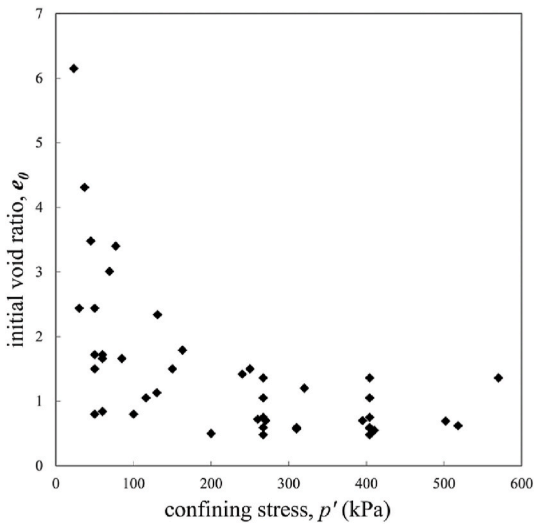


Fig. 1. Confining stress versus initial void ratio.

characterizing non probabilistic uncertainties in 1965 [17]. Among few available publications about application of fuzzy logic in soil mechanics, there is a very good state of the art paper that discusses suitability of this out of the box idea for different types of geotechnical problems [18]. Besides, fuzzy systems have shown the benefit of simplicity, decreasing the number of time-consuming tests, and getting rid of lengthy formulations in approximation of nonlinear stress-strain behavior [19]. In mathematical language classical and fuzzy sets are presented via equations (4) and (5), respectively. Where X is the universe of discourse, x is an element in the X and, $\mu_F(x)$ is the membership function of x in F.

$$C = \{x | x \in X\} \tag{4}$$

$$F = \{(x,\ \mu_F(x)) | x \in X,\ \mu_F(x) \in [0, 1]\} \tag{5}$$

In general, FIS consists of 5 steps. In the following, brief description of each step is given:

1. Fuzzification: Input variables are crisp values, but FIS converts these crisp values into one or more subgroups (i.e. An input can belong to various groups but with different membership degrees) via definition of membership functions which take the values from zero to one.
2. Inference mechanism and rule base: Next step is setting If-Then rules between input and output variables and applying fuzzy operators (AND or OR operators) on fuzzy inputs to come up with a result for each rule. In general, AND operators are minimum and product while OR operators are maximum and probabilistic methods.
3. Implication: Implication operators are the same as AND operators (i. e., minimum which truncates or product which scales output fuzzy sets) acting on consequent of each rule.
4. Aggregation: In this step, fuzzy outcomes of each rule are combined by simple mathematical methods like summation of the rule outputs.
5. Defuzzification: In the last step, aggregation results are "defuzzified" through different methods like centroid, bisector, average, etc. Therefore, result will be a crisp number again.

Similar to input, output can also be defined as membership functions which is called Mamdani type FIS [20] or it can be an equation instead and in this case is referred to as Sugeno type FIS [21]. Schematic of Mamdani type and Sugeno type FISs are shown in Fig. 3a and Fig. 3b, respectively. Where p_i, q_i, and r_i are called consequent parameters and for a zero-order Sugeno type FIS, outputs are constant values (i.e., $p_i = q_i = 0$).

4. Adaptive neuro-fuzzy inference system (Anfis)

There are critics regarding FIS application since it has the spirit of "rule of thumb" used by humans rather than a precise engineering tool, and its results are highly dependent on expert knowledge and preference who sets the membership functions and fuzzy rules. To overcome such drawbacks, authors decided to use a developed version of FIS called adaptive neuro-fuzzy inference system (Anfis) which was introduced in 1993 in this research [22]. Anfis is a fuzzy inference system with parameters adapted via neural networks which minimizes expert's interpretation using a hybrid learning procedure [22]. The decision-making process in Anfis (i.e., tuning membership function parameters and setting the rules) is handled by neural networks instead of a human expert.

Among quite many intelligent models available in the literature Anfis has a unique characteristic which is discussed briefly in the following.

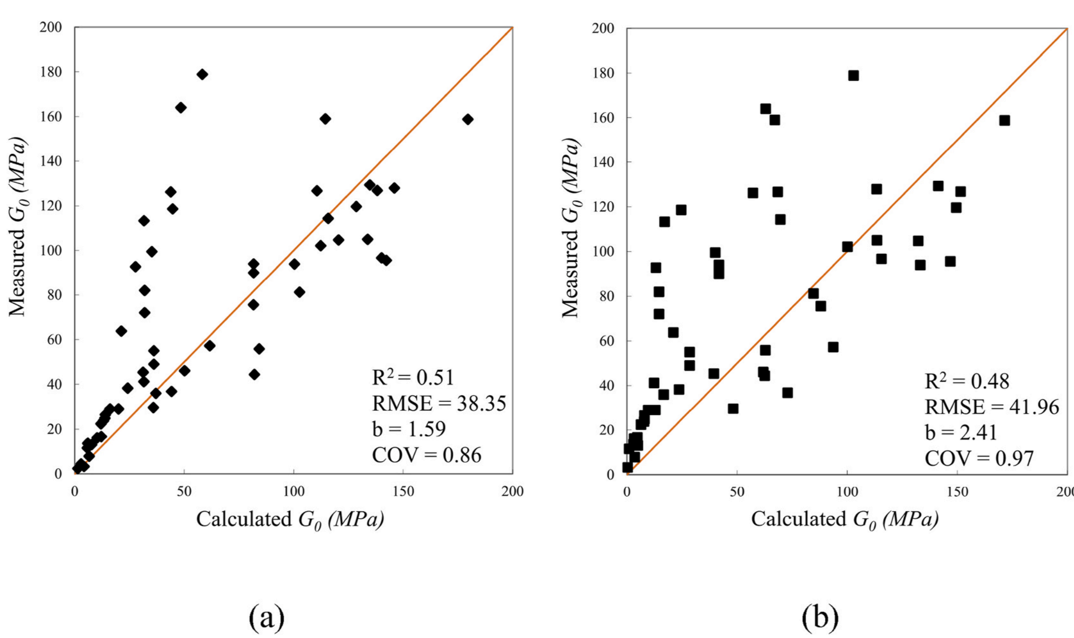
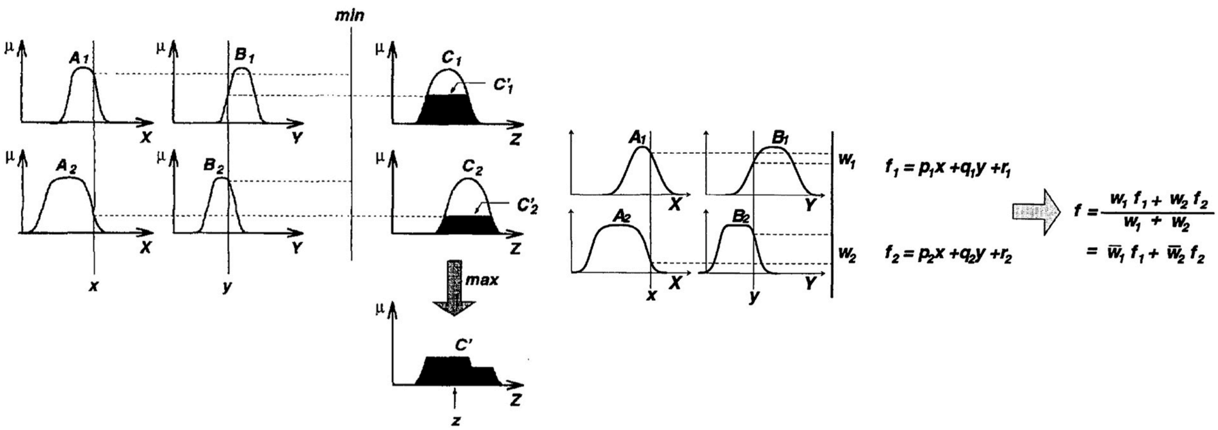


Fig. 2. Performance of two formulas proposed in literature for estimating G_0: a) [3] and b) [15]. In addition to measured versus calculated data, a 1:1 line is given.



(a) (b)

Fig. 3. Mamdani (a) and Sugeno (b) type FISs schematics [22,23].

Both FIS and neural networks have their advantages and disadvantages. Since the FIS concept originated from human decision-making it has an understandable and clear procedure but may not have sufficient precision. On the other hand, neural network provides definite solutions but lacks proper clarification of decision making. As a result, combination of these two methods created a complementary concept (i.e., Anfis) that benefits from both methods' bright sides which are transparency and accuracy. These capabilities of Anfis makes it a potentially useful tool for geotechnical problems where accuracy of estimation and transparency of input-output relations are a challenge.

The architecture of a two inputs-one output Anfis structure equivalent to the Sugeno type FIS presented in Fig. 3b is illustrated in Fig. 4. As shown in this figure, Anfis consists of five layers (here the output of node i in layer *1* is denoted as O_i^l):

1. Layer 1 (Fuzzification layer): This layer takes crisp input values and assigns membership functions to them. Here Gaussian membership function is used where z and m are tuning parameters.

$$O_i^1 = g(x; z, m) = \mu_{Ai}(x) = e^{\frac{-(x-m)^2}{2z^2}} \quad (6)$$

2. Layer 2 (Rule layer): In this layer firing strength of each rule is generated by product operation (Π).

$$O_i^2 = w_i = \mu_{Ai}(x).\mu_{Bi}(y), \; i = 1, 2 \quad (7)$$

3. Layer 3 (Normalization layer): In this step, firing strength of each rule is normalized by the total firing strength.

$$O_i^3 = \overline{w}_i = \frac{w_i}{\Sigma_{i=1}^{n} w_i} \quad (8)$$

4. Layer 4 (Defuzzification layer): This layer receives normalized values and consequent parameters (p_i, q_i, and r_i) as input. After combination of these parameters the defuzzified values are returned.

$$O_i^4 = \overline{w}_i f_i = \overline{w}_i (p_i x + q_i y + r_i) \quad (9)$$

5. Layer 5 (Output layer): The resulting output is the weighted average of each rules' output.

$$O_i^5 = \frac{\Sigma_{i=1}^{n} \overline{w}_i f_i}{\Sigma_{i=1}^{n} \overline{w}_i} \quad (10)$$

5. Estimation result

In the following, G_0 is approximated utilizing two physically meaningful variables mentioned earlier as G_0 governing parameters (i.e., e_0 and p'). Anfis results for 52 training data are shown in Fig. 5. As illustrated in this figure, due to very high scatter of the data, parameter coupling effects are too complicated. Nevertheless, Anfis was capable of estimating G_0 by only four simple fuzzy rules. Moreover, one example of Anfis computation is presented in Fig. 5a where **If** $e_0 = 1.72$ **AND** $p' = 50$

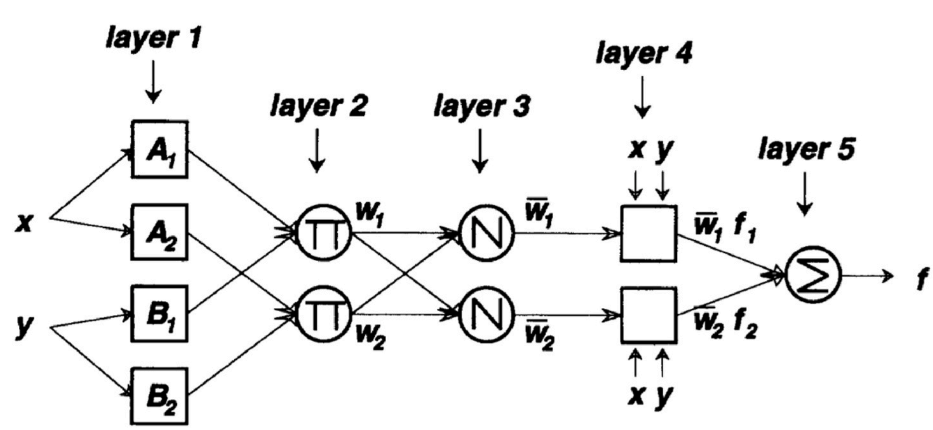


Fig. 4. A typical Anfis structure [22].

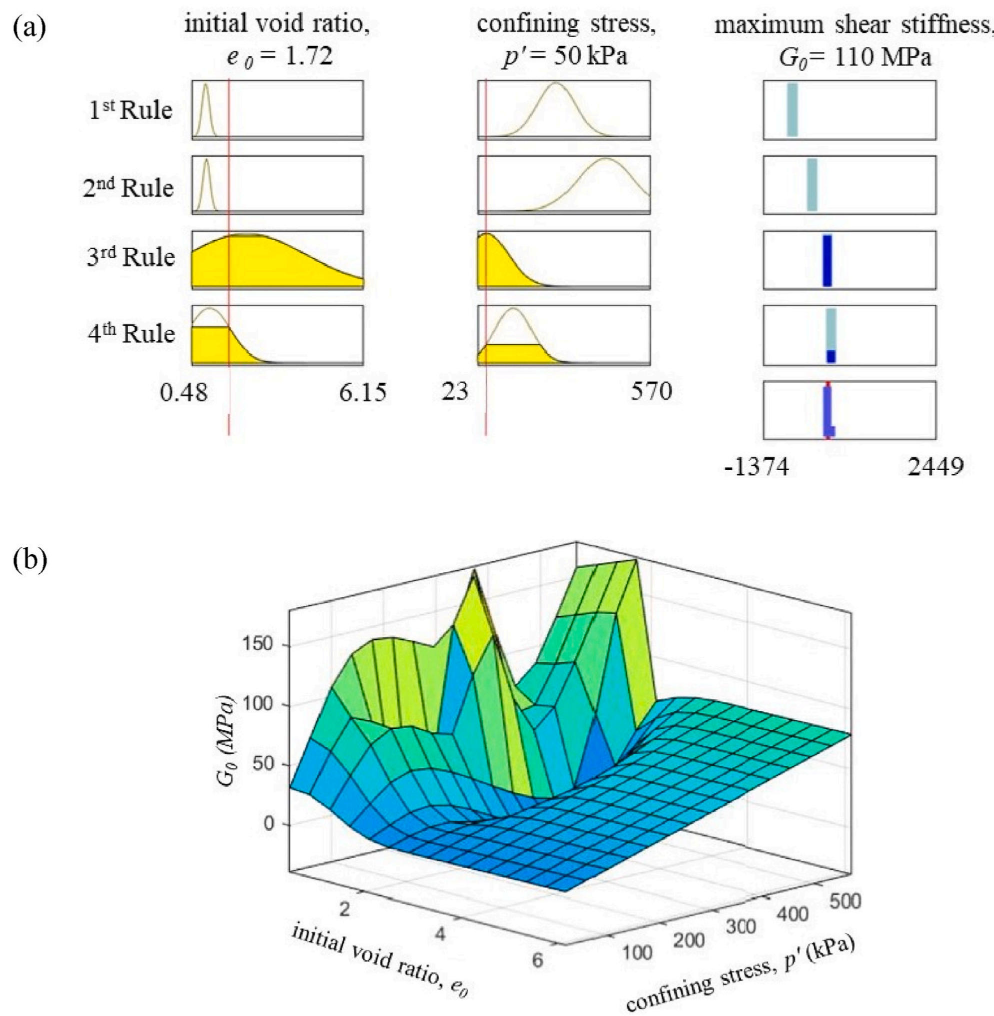


Fig. 5. G_0 computation block (a), and its resulting input-output relation (b).

kPa, **Then** $G_0 = 110$ MPa. While only the last two rules are activated. It means that for this particular case (i.e., $e_0 = 1.72$ and $p' = 50$ kPa) data does not belong to some fuzzy clusters and as the result their membership degree for those clusters is zero. However, as shown in the 3rd and 4th rule rows in Fig. 5a, e_0 and p' belong to two fuzzy clusters each with different membership degrees. Fig. 5a clarifies the input-output relation for each possible case and Fig. 5b presents the general 3D coupling schematic of considered parameters. Note that as mentioned earlier, in addition to estimation, Anfis describes continuous coupling effects based on lumped data and forecasts missing datapoints via its fuzzy clustering characteristic which is a significant benefit compared to many other intelligent models.

Anfis accuracy for estimating G_0 is presented graphically in Fig. 6. In addition, error criteria for test data are also given in this figure. Anfis approximates G_0 with $R^2 = 0.89$ for test data which is a considerable enhancement compared to the lengthy equations' performance shown in Fig. 2. Higher estimation accuracy is reachable by adding more complexity to the system, but this amount of error is acceptable since G_0 is usually highly affected (up to 20%) due to soil disturbance [3].

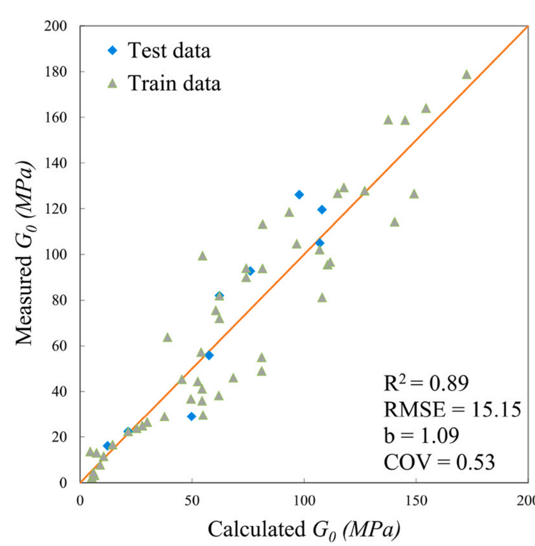


Fig. 6. Performance of Anfis for estimating G_0. In addition to measured versus calculated data, a 1:1 line is given.

6. Conclusion

Fuzzy sets and Anfis concepts were presented and further implemented for approximating G_0. In short, instead of introducing a lengthy formula, a logical framework was utilized that adapts itself with considered data and forecasts missing data points. G_0 was estimated from few easily obtainable parameters with clear physical meaning (i.e., e_0 and p') utilizing a novel notion called Anfis. Investigating the application of Anfis in other geotechnical problems is recommended for future studies. In addition, interested readers are welcome to contact the authors for further details regarding the implementation of Anfis for research purposes.

Author statement

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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