Efficient Inverse Covariance Matrix Estimation for Low-Complexity Closed-Loop DPD Systems

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***Abstract*—This paper studies closed-loop digital predistortion systems, with special focus on linearization of mmW active antenna arrays. Considering the beam-dependent nonlinear distortion and very high DPD processing rates, a modified self-orthogonalized (SO) learning solution is proposed, which is capable of reducing the computational complexity compared to other similar solutions, while at the same time obtaining a comparable linearization performance. The modified SO consists of a novel method for efficiently calculating the inverse of the input data covariance matrix. Thorough RF measurement results at 28 GHz band featuring a state-of-the-art 64 element active array and channel bandwidths up to 800 MHz, are reported. A complexity analysis is also carried out which, together with the obtained results, allow to asses the performance-complexity trade-offs. Altogether, the results show that the proposed methods can facilitate efficient mmW active antenna array linearization.**

***Index Terms*—Array transmitters, mmW frequencies, nonlinear distortion, digital predistortion, self-orthogonalization, covariance matrix, real-time complexity, EVM, TRP ACLR.**

I. Introduction

Modern communication systems utilize spectrally efficient waveforms which typically infer high peak-to-average power ratio (PAPR) values, complicating the power-efficient operation of power amplifiers (PAs). In order to ensure a high efficiency in the transmitter system while still maintaining low levels of distortion, digital predistortion (DPD) can be applied. Various DPD techniques have been studied in the literature, with good examples shown in [1], [2], and references therein.

One particular modern DPD use case is the linearization of mmW antenna arrays [3], [4]. Linearizing such frequency range 2 (FR-2) systems is generally challenging, as the effective nonlinear distortion has been reported to be dependent on the array beam direction [4], [5]. This issue calls for fast DPD tracking to estimate the DPD as the beam is steered. Also, the 3GPP 5G NR Release 15 [6] already considers very wide channel bandwidths (BWs), which lead to high DPD processing rates. These issues call for low-complexity DPD solutions, which is the main focus of this paper.

Good overviews of 5G mmW array transmitters can be found in [3], [4], while [7]–[10] focus more deeply on DPD and linearization methods. In [7], authors proposed an efficient memory polynomial (MP) DPD with decorrelation based learning. However, an additional basis function (BF) orthogonalization was applied, increasing the processing complexity of the model. In [8], authors proposed a closed-loop 1-bit observation system in combination with a sign-based GN algorithm. In [9], a closed-loop MP model was presented, where the DPD model was estimated with damped Gauss-Newton (GN), in combination with a sign regressor algorithm

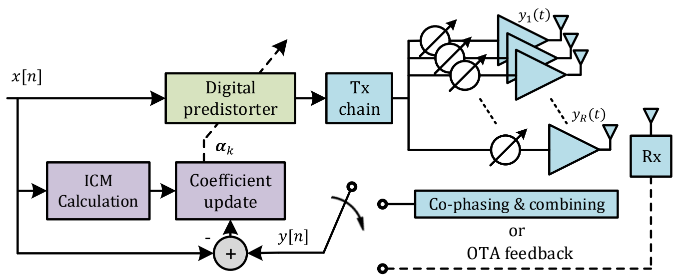


Fig. 1. Closed-loop DPD system for mmW active array linearization. The main novelty in this paper is in the DPD learning system.

(SRA). The SRA, however, can lead to rank-deficiencies in the BF matrix, while extra Walsh-Hadamard matrix transformations are also needed. Finally, [10] presented an MP model where the polynomials were replaced with look-up tables (LUTs), reducing the overall complexity. However, the model performance was also degraded with respect to canonical MP model, due to the quantization effects of the limited-size LUTs.

In this article, in the context of active array transmitters shown in Fig. 1, a closed-loop MP DPD system in combination with a novel reduced-complexity self-orthogonalized (SO) learning rule is adopted. Specifically, an efficient approach to estimate the inverse covariance matrix (ICM) is presented, which constitutes the most complex term in the SO rule. This is particularly so in dynamic scheduling based systems where the waveform is also dynamic, in terms of e.g. modulation order and allocated BW. The modified learning rule achieves great complexity reductions in the learning path, while at the same time obtaining a similar linearization performance when compared to the original SO or reference GN methods. The adopted closed-loop DPD system together with the inherent low complexity of the novel SO learning algorithm, allow for continuous learning and fast real-time adaptation. Finally, to assess the performance of the proposed techniques, RF measurements at 28 GHz mmW band are carried out, utilizing a state-of-the-art 64 element active antenna array and channel BWs up to 800 MHz. The obtained results, along with a complexity analysis, show that efficient linearization can be obtained through the proposed methods, meeting in all cases the 3GPP NR Release 15 [6] requirements at FR-2.

II. DPD System and Proposed Method

The proposed method builds on the classical MP DPD model [1], whose input vector $\psi[n]$ at time instant n, with polynomial order P and memory depth M, is shown in (1). It

$$\psi[n] = \left(x[n] \ \cdots \ x[n-M+1] \ x[n]|x[n]|^2 \ \cdots x[n-M+1]|x[n-M+1]|^2 \cdots \ x[n]|x[n]|^{P-1} \cdots \ x[n-M+1]|x[n-M+1]|^{P-1}\right)^T \quad (1)$$

is combined with the closed-loop SO learning rule to estimate the DPD coefficients, $\boldsymbol{\alpha}_k$, with low complexity. The SO reads

$$\boldsymbol{\alpha}_{k+1} = \boldsymbol{\alpha}_k + \mu_{\text{s}}(\mathbf{R}^*)^{-1}\boldsymbol{\Omega}_k^H \mathbf{e}_k, \quad (2)$$

where k indicates the iteration index, μ_{s} is the learning rate, $\mathbf{e}_k$ is the closed-loop error signal, $\boldsymbol{\Omega}_k$ is the BF matrix stacking $\psi[n]$ for different time instants, and $\mathbf{R} = \text{E}[\psi[n]\psi^H[n]]$ is the ensemble covariance matrix (CM). Using the classical sample estimation method, the CM can in practice be estimated as

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N} \psi[n]\psi^H[n], \quad (3)$$

where N is the total number of input vectors over which the estimate is calculated. The proposed method, instead of directly calculating (3), utilizes the Bussgang's theorem to estimate the CM. Thus, a Gaussian input signal is assumed in the following derivations, while the practical experiments are carried out with true OFDM-based waveforms.

The first step is to express the CM exclusively as a function of the autocorrelation function of the input signal, denoted herein as $x[n]$. The autocorrelation function is defined as

$$R_{\text{X}}(\tau) = \text{E}\{x[n]x^*[n-\tau]\}, \quad (4)$$

where τ is a sample delay. The second-order terms in $\mathbf{R}$, on its first line, are obtained directly from (4), as $\mathbf{R}(1,1){=}R_{\text{X}}(\tau{=}0)$, $\mathbf{R}(1,2){=}R_{\text{X}}(\tau{=}1)$, and, in general, $\mathbf{R}(1,M){=}R_{\text{X}}(\tau{=}M)$. Higher order terms in the CM (e.g., $\text{E}\{x[n]|x[n-\tau]|^p\}$, $p = 2, 4, \ldots$), stemming from the nonlinear BFs, can be expressed through the cross-correlation function, as

$$R_{\text{XY}}(\tau) = \text{E}\{x[n]y^*[n-\tau]\}, \quad (5)$$

where $y[n] = f(x[n])$ is a nonlinear function of $x[n]$. To this end, the Bussgang's theorem states that the cross-correlation of a Gaussian signal passing through a nonlinear operator can be expressed as the product between its autocorrelation and a scaling constant. Formally, this is expressed as

$$R_{\text{XY}}(\tau) = \xi R_{\text{X}}(\tau), \quad (6)$$

where the Bussgang's coefficient, ξ, can be obtained assuming complex-circular Gaussian distribution as

$$\xi = \frac{1}{\pi\sigma_x^4}\int_{-\infty}^{\infty} x^*[n]f(x[n])e^{-\frac{|x[n]|^2}{\sigma_x^2}}dx. \quad (7)$$

In (10), σ_x^2 denotes the variance of $x[n]$, which is typically smaller than 1 to obtain a small condition number in the CM. Using (6), the higher-order CM terms can be expressed as

$$\text{E}\{x[n]x^*[n]|x[n]|^2\} = \xi_1 R_{\text{X}}(\tau=0), \quad (8)$$

$$\text{E}\{x[n]x^*[n-1]|x[n-1]|^2\} = \xi_1 R_{\text{X}}(\tau=1), \quad (9)$$

$$\text{E}\{x[n]x^*[n]|x[n]|^4\} = \xi_2 R_{\text{X}}(\tau=0), \quad (10)$$

$$\text{E}\{x[n]x^*[n-1]|x[n-1]|^4\} = \xi_2 R_{\text{X}}(\tau=1), \quad (11)$$

$$\vdots$$

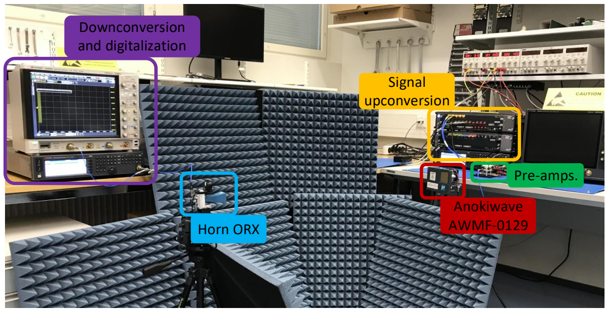


Fig. 2. The 5G mmW/FR-2 OTA measurement setup utilized to carry out the experimental measurements at 28 GHz.

where ξ_{x} is the corresponding Bussgang's coefficient. By having all terms in the CM expressed as a function of $R_{\text{X}}(\tau)$ and the Bussgang's coefficients, the complete CM reads, for a generic polynomial order P and memory depth M, as

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_0 & \mathbf{R}_1 & \mathbf{R}_2 & \cdots & \mathbf{R}_{\lfloor \frac{P}{2} \rfloor} \\ \mathbf{R}_1 & \mathbf{R}_2 & \mathbf{R}_3 & \cdots & \mathbf{R}_{\lfloor \frac{P}{2} \rfloor+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{\lfloor \frac{P}{2} \rfloor} & \mathbf{R}_{\lfloor \frac{P}{2} \rfloor+1} & \mathbf{R}_{\lfloor \frac{P}{2} \rfloor+2} & \cdots & \mathbf{R}_{2\lfloor \frac{P}{2} \rfloor} \end{bmatrix}, \quad (12)$$

where each subindex indicates the corresponding Bussgang's coefficient. Additionally, the sub-matrix $\mathbf{R}_k \in \mathbb{C}^{M\times M}$ is an Hermitian Toeplitz matrix, defined by the vector $\mathbf{v}_k = \xi_k[R_{\text{X}}(\tau=0) \ R_{\text{X}}(\tau=1) \ \cdots \ R_{\text{X}}(\tau=M)]^T$, where $\xi_0 = 1$. Note that only the submatrices appearing in the first row and last column of (12) need to be calculated to build the whole CM for a given signal. The expression in (12) can be alternatively expressed as a Kronecker product, as

$$\mathbf{R} = \Xi \otimes \mathbf{R}_0, \quad (13)$$

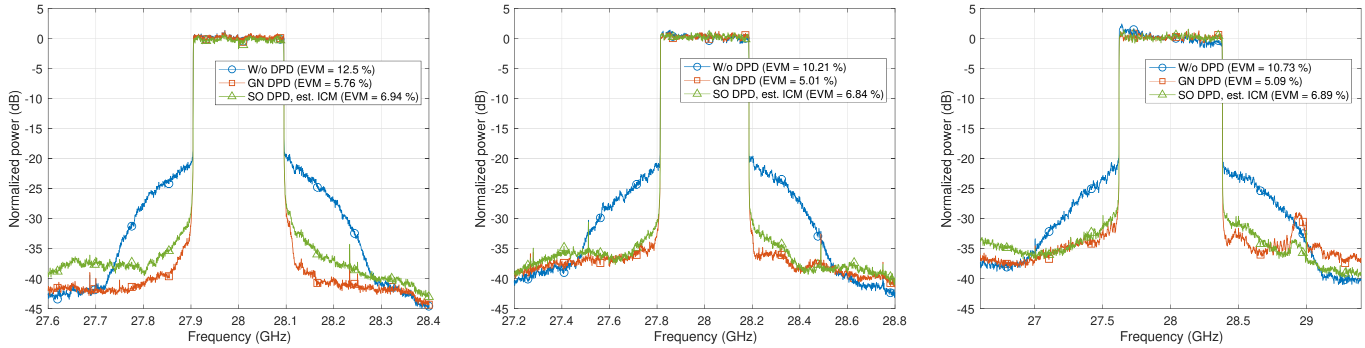
where Ξ contains the set of Bussgang's coefficients. Then, the ICM is directly obtained as

$$\mathbf{R}^{-1} = \Xi^{-1} \otimes \mathbf{R}_0^{-1}. \quad (14)$$

Thus, to calculate the ICM, only the autocorrelation function of $x[n]$ and the Bussgang coefficients are needed, both of which can be calculated in advance. The Kronecker formulation further reduces the required computational complexity.

III. RF Measurement Results

1) Measurement Setup: All the experiments are carried out with the state-of-the-art setup depicted in Fig 2. It consists of a Keysight M8195A to provide the I/Q data samples, additional equipment to upconvert the signal to 28 GHz, a pre-amplification stage, and finally the Anokiwave AWMF-0129 active antenna array. The signal is measured over-the-air (OTA), at an effective isotropic radiated power (EIRP) of +40.5 dBm, while then downconverted, digitized, and taken to a host PC to execute the algorithms. In all cases, the closed-loop MP DPD is configured with $P = 9$, $M = 4$, and $K = 20$



(a) (b) (c)

Fig. 3. OTA linearization performance at EIRP $\approx$ +40.5 dBm with the proposed SO learning rule (SO DPD, est. ICM), in (a) 200 MHz, (b) 400 MHz, and (c) 800 MHz channel bandwidth cases. The GN method is also implemented, measured and shown for reference.

TABLE I
LINEARIZATION PERFORMANCE AND AVERAGE LEARNING COMPLEXITY PER SAMPLE IN THE 800 MHZ CHANNEL BW CASE.

	EVM (%)	TRP ACLR (dBc)	Learning path complexity (real mul. per lin. sample)
Classical SO DPD	6.02	31.2	186.9
Proposed SO DPD	6.89	30.4	81.42
Reference GN DPD	5.09	32.3	1,681

ksamples per iteration, with a total of 20 DPD iterations. The results also include and show the GN learning solution [8], [9] as a reference, which reads

$$\boldsymbol{\alpha}_{k+1} = \boldsymbol{\alpha}_k + \mu_{\mathrm{g}} \left(\boldsymbol{\Omega}_k^H \boldsymbol{\Omega}_k\right)^{-1} \boldsymbol{\Omega}_k^H \mathbf{e}_k, \tag{15}$$

where μ_{g} is the learning rate. The performance of this model is generally better compared to SO, since the term $\left(\boldsymbol{\Omega}_k^H \boldsymbol{\Omega}_k\right)^{-1}$ is calculated in all DPD iterations, however, it also clearly infers larger complexity.

The adopted signals are 5G NR Rel-15 compliant OFDM waveforms, with subcarrier spacing of 120 kHz, covering BWs of 200 and 400 MHz at FR-2. An experiment with a further extended BW of 800 MHz is also included, with the aim of pushing the performance boundaries of the DPD system.

2) Measurement Results: The measured PSDs are depicted in Fig. 3. In all BW cases, the proposed SO learning solution provides a very favorable linearization performance, very close to the GN solution. The error vector magnitude (EVM), adjacent channel leakage ratio (ACLR), and complexity numbers are shown in Table I, for the widest signal BW of 800 MHz, where the ACLR is measured using the total radiated power (TRP) approach. The 5G NR EVM limit of 8%, and the TRP ACLR limit of 28 dBc [6] are fulfilled in all cases, regardless of the 56% complexity reduction with respect to classical SO and 95% complexity reduction with respect to GN. These performance and complexity results demonstrate that the proposed solution is an intriguing approach for the efficient linearization of mmW active antenna arrays.

IV. CONCLUSIONS

In this paper, an efficient method to estimate the inverse covariance matrix of the DPD basis function samples was proposed. The proposed method was shown to provide large complexity reductions in the context of self-orthogonalized (SO) DPD learning applications. The complexity-performance trade-offs of the proposed overall DPD system were demonstrated through extensive RF measurement results at 28 GHz, featuring a state-of-the-art Anokiwave AWMF-0129 active array, and very wide channel BWs up to 800 MHz. The obtained performance results, together with the complexity analysis, indicate that efficient mmW active antenna array linearization can be achieved through the proposed technique.

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