The Double Shadowed κ-μ Fading Model

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***Abstract*—In this paper, we introduce a new fading model which is capable of characterizing both the shadowing of the dominant component and composite shadowing which may exist in wireless channels. More precisely, this new model assumes a κ-μ envelope where the dominant component is fluctuated by a Nakagami-m random variable (RV) which is preceded (or succeeded) by a secondary round of shadowing brought about by an inverse Nakagami-m RV. We conveniently refer to this as the double shadowed κ-μ fading model. In this context, novel closed-form and analytical expressions are developed for a range of channel related statistics, such as the probability density function, cumulative distribution function, and moments. All of the derived expressions have been validated through Monte-Carlo simulations and reduction to a number of well-known special cases. It is worth highlighting that the proposed fading model offers remarkable flexibility as it includes the κ-μ, η-μ, Rician shadowed, double shadowed Rician, κ-μ shadowed, κ-μ/inverse gamma and η-μ/inverse gamma distributions as special cases.**

I. Introduction

A great number of statistical distributions have been proposed to characterize fading in wireless channels [1]. Shadowing introduced by topographical elements and objects obstructing the propagation path is commonly modeled using the lognormal distribution. On the other hand, multipath fading is described by several other distributions, such as the Rayleigh, Rice, Nakagami-m, Hoyt, Weibull, and more recently κ-μ and η-μ [2] distributions. However, although the fading models mentioned above have been used to characterize propagation channels, unfortunately they are unable to account for any fluctuations of the line-of-sight (LOS) component or scattered signal contributions brought about by composite fading effects. Hence, several composite fading models have been proposed which aim to address these shortcomings. The shadowing in these composite fading models is considered to be either LOS, when the dominant signal component of the envelope is shadowed, or multiplicative when the total power of the dominant (if present) and scattered signal components are shadowed. Some multiplicative fading models include the Nakagami-m/gamma [3], κ-μ/gamma [4], η-μ/gamma [5], κ-μ/inverse gamma and η-μ/ inverse gamma models [6], and some examples of LOS shadow fading models include the shadowed Rician distribution [7], κ-μ/lognormal [8], and κ-μ shadowed fading model [9].

In the present contribution, we focus our attention on the κ-μ shadowed fading model, which assumes that the encountered multipath fading is due to fluctuations brought about by a κ-μ RV, and the shadowing is shaped by a Nakagami-m RV. Notably, this model includes the κ-μ, η-μ and Rician shadowed distributions [7] as special cases. Furthermore, it has been shown to provide excellent agreement with field measurements obtained for land-mobile satellite channels [7], underwater acoustic communications [10] and body-centric fading channels [11]. Motivated by this, we introduce the double shadowed κ-μ fading model which assumes that the dominant component of a κ-μ signal undergoes variations that can be modelled by the Nakagami-m distribution. It also considers that the root mean square (rms) power of the dominant component and scattered waves undergo a secondary round of shadowing characterized by an inverse Nakagami-m RV. Following from this, we derive novel closed-form and analytical expressions for many of its fundamental statistics of interest, namely the probability density function (PDF), cumulative distribution function (CDF), and moments. These results are then used to obtain the amount of fading (AF) and the corresponding outage probability (OP). It is also shown that the novel formulations presented in this paper unify a number of popular fading scenarios such as the the κ-μ, η-μ, Rician shadowed, double shadowed Rician [12], κ-μ shadowed, κ-μ/inverse gamma and η-μ/inverse gamma models.

The remainder of this paper is organized as follows: Section II describes the physical model, whilst Section III derives its statistical characteristics. In Section IV, we present some important performance measures, whilst Section V discusses the special cases of the double shadowed κ-μ fading model alongside some numerical results. Lastly, Section VII concludes the paper with some closing remarks.

II. The Physical Model

The double shadowed κ-μ fading model provides a generalization to the κ-μ shadowed fading model [9]. Similar to the κ-μ shadowed fading model, the received signal in a double shadowed κ-μ fading model is composed of clusters of multipath waves propagating in non-homogeneous environments. Within each multipath cluster, the scattered waves have similar delay times and the delay spreads of different clusters are relatively large. The power of the scattered waves in each cluster is assumed to be identical whilst the power of the dominant component is assumed to be arbitrary. Additionally, the dominant component of each cluster can randomly fluctuate because of shadowing [9]. Unlike the κ-μ shadowed model, the double shadowed κ-μ model assumes that the rms power of the dominant and the scattered waves i.e., all of the multipath components, may also be subject to random variations induced by shadowing. In other words, it can be interpreted as a κ-μ fading channel that is subject to LOS shadowing followed by a secondary composite shadowing or vice versa. Physically, this situation may arise when the signal power delivered through the optical path between the transmitter and receiver is shadowed by objects moving within its locality, whilst further shadowing of the received power (combined multipath and dominant paths) may also occur due to obstacles moving in the vicinity of the transmitter and/or receiver.

The signal envelope, R, of a double shadowed κ-μ fading channel can be expressed in terms of its in-phase and quadrature phase components as follows:

$$R^2 = A^2 \sum_{i=1}^{\mu} \left(X_i + \xi p_i\right)^2 + \left(Y_i + \xi q_i\right)^2 \tag{1}$$

where μ is the number of multipath clusters, X_i and Y_i are mutually independent Gaussian random processes with mean $\mathbb{E}\left[X_i\right] = \mathbb{E}\left[Y_i\right] = 0$ and variance $\mathbb{E}\left[{X_i}^2\right] = \mathbb{E}\left[{Y_i}^2\right] = \sigma^2$, where $\mathbb{E}[\cdot]$ denotes the expectation operator. Here p_i and q_i are the mean values of the in-phase and the quadrature phase components of the multipath cluster i, and ξ is a Nakagami-m RV with shape parameter m_d where $\mathbb{E}\left[\xi^2\right] = 1$, and A denotes an inverse Nakagami-m RV with shape parameter m_s where $\mathbb{E}\left[A^2\right] = 1$. The PDF of A is given by

$$f_A\left(\alpha\right) = \frac{2(m_s-1)^{m_s}}{\Gamma\left(m_s\right)\alpha^{2m_s+1}} \mathrm{e}^{-\frac{m_s-1}{\alpha^2}} \tag{2}$$

where $\Gamma(\cdot)$ represents the Gamma function [13, 8.310.1].

III. Statistical Characteristics

The distribution of the received signal envelope, R, in a double shadowed κ-μ fading channel can be obtained by averaging the following conditional probability over the statistics of the shadowing process, namely

$$f_R\left(r\right) = \int_0^{\infty} f_{R|A}\left(r|\alpha\right) f_A\left(\alpha\right) d\alpha \tag{3}$$

where $f_{R|A}\left(r|\alpha\right)$, follows a κ-μ shadowed distribution and

$$f_{R|A}\left(r|\alpha\right) = \frac{2\mu^{\mu}\left(1+\kappa\right)^{\mu} r^{2\mu-1} m_d^{m_d}}{\Gamma\left(\mu\right)\left(m_d+\mu\kappa\right)^{m_d}\alpha^{2\mu}\hat{r}^{2\mu}} \mathrm{e}^{-\mu(1+\kappa)\frac{r^2}{\alpha^2\hat{r}^2}} {}_1F_1\left(m_d;\mu;\frac{\mu^2\kappa\left(1+\kappa\right)r^2}{\alpha^2\hat{r}^2\left(m_d+\mu\kappa\right)}\right). \tag{4}$$

Here, κ is the ratio of the total power of the dominant components (d^2) to that of the scattered waves ($2\mu\sigma^2$), $\mu > 0$ is related to the number of clusters, $\hat{r} = \sqrt{\mathbb{E}[R^2]}$ represents the rms power of R, the mean signal power is given by $\mathbb{E}[R^2] = 2\mu\sigma^2 + d^2$, and ${}_1F_1\left(\cdot;\cdot;\cdot\right)$ denotes the confluent hypergeometric function [13, Eq. 9.210.1]. The PDF of R for the double shadowed κ-μ fading model can now be obtained in closed-form via Theorem 1 below.

Theorem 1. *For κ, μ, m_d, r, $\hat{r} \in \mathbb{R}^+$ and $m_s > 1$, the PDF of the double shadowed κ-μ fading model can be expressed as*

$$f_R(r) = \frac{2(m_s-1)^{m_s} m_d^{m_d} \mathcal{K}^{\mu} r^{2\mu-1}\hat{r}^{2m_s}}{\left(m_d+\mu\kappa\right)^{m_d} B(m_s,\mu)\left(\mathcal{K}r^2+(m_s-1)\hat{r}^2\right)^{m_s+\mu}} \times {}_2F_1\left(m_d, m_s+\mu;\mu;\frac{\mathcal{K}_1\mu\kappa r^2}{(\mathcal{K}r^2+(m_s-1)\hat{r}^2)}\right) \tag{5}$$

where $\mathcal{K} = \mu\left(1+\kappa\right)$, $\mathcal{K}_1 = \frac{\mathcal{K}}{(m_d+\mu\kappa)}$, $B(\cdot,\cdot)$ represents the Beta function [13, Eq. 8.384] and ${}_2F_1\left(\cdot,\cdot;\cdot;\cdot\right)$ denotes the Gauss hypergeometric function [13, Eq. 9.100].

Proof: See Appendix A. ■

Now letting γ represent the instantaneous signal-to-noise-ratio (SNR) of a double shadowed κ-μ fading channel, the corresponding PDF, $f_\gamma\left(\gamma\right)$, can be obtained from the envelope PDF given in (5) via a transformation of variables $\left(r = \sqrt{\gamma\ \hat{r}^2/\bar{\gamma}}\right)$ as follows.

Corollary 1. *For κ, μ, m_d, γ, $\bar{\gamma} \in \mathbb{R}^+$ and $m_s > 1$, the PDF of γ for the double shadowed κ-μ fading model can be written as*

$$f_\gamma\left(\gamma\right) = \frac{(m_s-1)^{m_s} m_d^{m_d} \mathcal{K}^{\mu}\gamma^{\mu-1}\bar{\gamma}^{m_s}}{\left(m_d+\mu\kappa\right)^{m_d} B\left(m_s,\mu\right)\left(\mathcal{K}\gamma+(m_s-1)\bar{\gamma}\right)^{m_s+\mu}} \times {}_2F_1\left(m_d, m_s+\mu;\mu;\frac{\mathcal{K}_1\mu\kappa\gamma}{(\mathcal{K}\gamma+(m_s-1)\bar{\gamma})}\right) \tag{6}$$

where $\bar{\gamma} = \mathbb{E}[\gamma]$ denotes the corresponding average SNR.

Having obtained a closed form expression for the PDF of γ for the double shadowed κ-μ fading channel, a closed-form expression for its CDF, $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, can be expressed via Lemma 1 as follows.

Lemma 1. *For κ, μ, m_d, γ, $\bar{\gamma} \in \mathbb{R}^+$, and $m_s > 1$, the CDF of γ for the double shadowed κ-μ fading model can be written as*

$$F_\gamma(\gamma) = \left(\frac{m_d}{m_d+\kappa\mu}\right)^{m_d}\left(\frac{\mathcal{K}\gamma}{\bar{\gamma}(m_s-1)}\right)^{\mu}\sum_{i=0}^{\infty}\left(\frac{\mathcal{K}_1\mu\kappa\gamma}{\bar{\gamma}(m_s-1)}\right)^{i} \times \frac{(m_d)_i(i+\mu)_{m_s}}{i!\Gamma(m_s)(i+\mu)}{}_2F_1(i+\mu, i+\mu+m_s; i+\mu+1; \mathcal{T}) \quad (7)$$

where $(x)_{n'} = \frac{\Gamma(x+n')}{\Gamma(x)}$ denotes the Pochhammer symbol [13] and $\mathcal{T} = \frac{-\mathcal{K}\gamma}{\bar{\gamma}(m_s-1)}$. For the case when $\bar{\gamma}(m_s-1)(m_d+\kappa\mu) > \kappa\mu^2(1+\kappa)\gamma$, (7) can be expressed in closed-form as follows:

$$F_\gamma(\gamma) = \left(\frac{m_d}{m_d+\kappa\mu}\right)^{m_d}\left(\frac{\mathcal{K}\gamma}{\bar{\gamma}(m_s-1)}\right)^{\mu}\frac{\Gamma(m_s+\mu)}{\Gamma(m_s)\Gamma(\mu+1)} \times F^{2,1,0}_{1,1,0}\left(\begin{matrix} m_s+\mu, \mu; & m_d; & -; \\ 1+\mu; & \mu; & -; \end{matrix} \frac{\mathcal{K}_1\mu\kappa\gamma}{\bar{\gamma}(m_s-1)}, -\frac{\mathcal{K}\gamma}{\bar{\gamma}(m_s-1)}\right). \quad (8)$$

where $F^{2,1,0}_{1,1,0}\left(\begin{matrix} \cdot,\cdot; & \cdot; & \cdot; \\ \cdot; & \cdot; & \cdot; \end{matrix} \cdot,\cdot\right)$ denotes the Kampé de Fériet function [14].

Proof: See Appendix A. ■

Based on the derived PDF representation, we can readily derive a closed-form expression for the corresponding moments.

Lemma 2. *For κ, μ, m_d, γ, $\bar{\gamma} \in \mathbb{R}^+$, and $m_s > n$ the n-th order moment of the double shadowed κ-μ fading model can be evaluated such that $\mathbb{E}[\gamma^n] \triangleq \int_0^\infty \gamma^n f_\gamma(\gamma)\,d\gamma$, as*[1]

$$\mathbb{E}[\gamma^n] = \left(\frac{m_d}{m_d+\kappa\mu}\right)^{m_d}\frac{B(m_s-n, n+\mu)}{B(m_s,\mu)} \times \left(\frac{(m_s-1)\bar{\gamma}}{\mu(1+\kappa)}\right)^{n}{}_2F_1\left(m_d, n+\mu; \mu; \frac{\kappa\mu}{m_d+\kappa\mu}\right). \quad (9)$$

Proof: See Appendix B. ■

IV. Performance Analysis

Capitalizing on the derivation of the key statistical metrics in Section III, we derive simple expressions for the amount of fading and outage probability in double shadowed κ-μ fading channels.

[1]By recalling the definition of the incomplete beta function, the following term can also be expressed as $\frac{B(m_s-n,n+\mu)}{B(m_s,\mu)} = \frac{\Gamma(m_s-n)\Gamma(\mu+n)}{\Gamma(m_s)\Gamma(\mu)}$

A. Amount of Fading

The amount of fading is often used to quantify the severity of fading experienced during transmission over fading channels. It is defined in [15, Eq. 1.27] as

$$\mathrm{AF} \triangleq \frac{\mathbb{V}[\gamma]}{\mathbb{E}[\gamma]^2} = \frac{\mathbb{E}[\gamma^2] - \mathbb{E}[\gamma]^2}{\mathbb{E}[\gamma]^2} = \frac{\mathbb{E}[\gamma^2]}{\mathbb{E}[\gamma]^2} - 1 \quad (10)$$

where $\mathbb{V}(\cdot)$ denotes the variance operator. A closed-form expression for the AF can be obtained via Corollary 2.

Corollary 2. *For κ, μ, $m_d \in \mathbb{R}^+$, and $m_s > 2$ the* AF *of the double shadowed κ-μ fading model can be expressed as*

$$\mathrm{AF} = \frac{(m_s-1)}{(m_s-2)(1+\kappa)^2}\left[\frac{\kappa^2+m_d(1+\kappa)^2}{m_d} + \frac{(1+2\kappa)}{\mu}\right] - 1. \quad (11)$$

Proof: See Appendix B. ■

B. Outage Probability

The outage probability of a communication system is defined as the probability that the instantaneous SNR drops below a given threshold, γ_{th}

$$P_{\mathrm{OP}}(\gamma_{\mathrm{th}}) \triangleq P[0 \le \gamma \le \gamma_{th}] = F_\gamma(\gamma_{\mathrm{th}}). \quad (12)$$

Therefore, the OP of the double shadowed κ-μ fading model is readily obtained by replacing γ in (7) or (8) with γ_{th}.

V. Special Cases and Numerical Results

The PDF given in (5) represents an extremely versatile fading model as it inherits all of the generalities of the κ-μ shadowed, κ-μ/inverse gamma and η-μ/inverse gamma fading models. For example, letting $m_s \to \infty$ in (5), the PDF of the κ-μ shadowed model is obtained, and letting $\hat{r}^2 = \frac{\hat{r}^2 m_s}{(m_s-1)}$, $m_d \to \infty$ the PDF of the κ-μ/inverse gamma fading model is obtained. Of course, allowing $m_s \to \infty$ and $m_d \to \infty$ yields the PDF of the κ-μ fading model. These special case results are illustrated in Fig. 1 and are in exact agreement with the Monte-Carlo simulations. The PDF of the η-μ/inverse gamma fading model can also be obtained from the double shadowed κ-μ fading model by letting $\hat{r}^2 = \frac{\hat{r}^2 m_s}{(m_s-1)}$, $m_d \to \mu$, $\kappa = \frac{(1-\eta)}{2\eta}$, and $\mu = 2\mu$. Letting $m_s \to \infty$, $m_d \to \mu$, $\kappa = \frac{(1-\eta)}{2\eta}$, and $\mu = 2\mu$ the PDF of the η-μ fading model is obtained. In a similar manner, the PDFs of the double shadowed Rician, Rician shadowed, and Rician fading models can be obtained from (5) by first setting $\mu = 1$, followed by appropriate substitutions for the m_d and m_s parameters. Fig. 1 shows the shape of the PDF for these special cases which are indicated in red. Table I summarizes the special cases of the double shadowed κ-μ fading model.

To provide some insights into the effect of shadowing upon the dominant and scattered multipath signal in

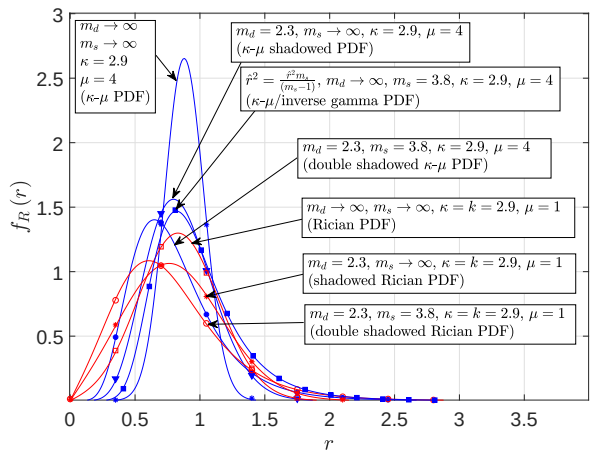

Fig. 1. The PDF of the double shadowed κ-μ fading model (blue circle markers) reduced to some of its special cases: κ-μ (blue astrix markers), κ-μ shadowed (blue triangle markers) κ-μ/inverse gamma (blue square markers), Rician (red triangle markers), Rician shadowed (red astrix markers), double shadowed Rician (red circle markers). Here, $\hat{r} = 0.8$, lines represent analytical results, and markers represent simulation results.

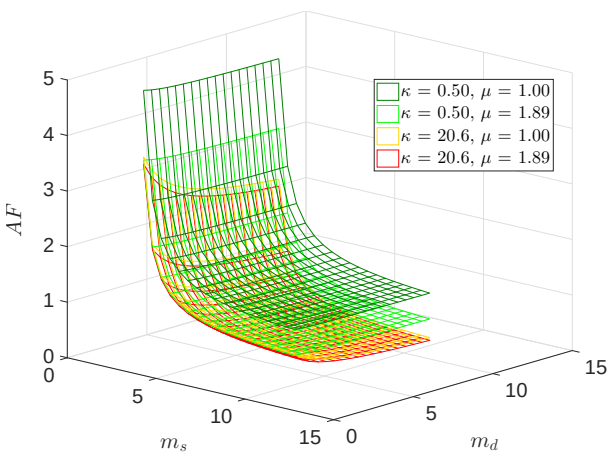

Fig. 2. The AF in double shadowed κ-μ fading channels for a range of m_s and m_d when $\{\kappa, \mu\} = \{0.5, 1.89\}$ and $\{20.6, 1.89\}$. The AF of the double shadowed Rician fading model ($\mu = 1$), is shown as a special case and is indicated by dark green ($\kappa = K = 0.5$) and yellow ($\kappa = K = 20.6$) lines.

double shadowed κ-μ fading channels, Fig. 2 shows the calculated AF for different values of m_d and m_s. It is observed that the greatest AF occurs for severe multiplicative shadowing (m_s), when compared to the shadowing of the LOS component (m_d). For instance, the AF observed when $\{m_s, m_d, \kappa, \mu\} = \{2.5, 3.0, 20.6, 1.89\}$ is 3.05 which is greater than the AF observed when $\{m_s, m_d, \kappa, \mu\} = \{3.0, 2.5, 20.6, 1.89\}$, which is 1.8. Fig. 2 also illustrates the case when the AF observed for the double shadowed κ-μ model coincides with the AF of the double shadowed Rician model [12]. For example, the AF observed for the double shadowed Rician model when $\{m_s, m_d, \kappa, \mu\} = \{2.5, 3.0, 20.6, 1.0\}$ is greater than that observed for the double shadowed κ-μ fading model when $\{m_s, m_d, \kappa, \mu\} = \{2.5, 3.0, 20.6, 1.89\}$.

Fig. 3 shows the outage probability versus $\bar{\gamma}$ for different multipath and shadowing conditions. As expected, we observe that the outage probability increases for

TABLE I
SPECIAL CASES OF THE DOUBLE SHADOWED κ-μ FADING MODEL

Fading models	Double shadowed κ-μ parameters
κ-μ shadowed [9]	$\underline{m_s} \to \infty, \underline{m_d} = m_d$ $\underline{\kappa} = \kappa, \underline{\mu} = \mu$
κ-μ/inverse gamma [6]	$\underline{\hat{r}^2} = \frac{\hat{r}^2 m_s}{(m_s-1)}, \underline{m_s} = m_s$ $\underline{m_d} \to \infty, \underline{\kappa} = \kappa, \underline{\mu} = \mu$
η-μ/inverse gamma [6]	$\underline{\hat{r}^2} = \frac{\hat{r}^2 m_s}{(m_s-1)}, \underline{m_s} \to m_s$ $\underline{m_d} \to \mu, \underline{\kappa} = \frac{(1-\eta)}{2\eta}, \underline{\mu} = 2\mu$
κ-μ	$\underline{m_s} \to \infty, \underline{m_d} \to \infty$ $\underline{\kappa} = \kappa, \underline{\mu} = \mu$
η-μ	$\underline{m_s} \to \infty, \underline{m_d} \to \mu$ $\underline{\kappa} = \frac{(1-\eta)}{2\eta}, \underline{\mu} = 2\mu$
double shadowed Rice [12]	$\underline{m_s} = m_s, \underline{m_d} = m_d$ $\underline{\kappa} = K, \underline{\mu} = 1$
Rician shadowed [7]	$\underline{m_s} \to \infty, \underline{m_d} = m_d$ $\underline{\kappa} = K, \underline{\mu} = 1$
Rician	$\underline{m_s} \to \infty, \underline{m_d} \to \infty$ $\underline{\kappa} = K, \underline{\mu} = 1$
Nakagami-q (Hoyt) [16]	$\underline{m_s} \to \infty, \underline{m_d} = 0.5$ $\underline{\kappa} = \frac{(1-q^2)}{2q^2}, \underline{\mu} = 1$
Nakagami-m	$\underline{m_s} \to \infty, \underline{m_d} \to \infty$ $\underline{\kappa} \to 0, \underline{\mu} = m$
Rayleigh	$\underline{m_s} \to \infty, \underline{m_d} \to \infty$ $\underline{\kappa} \to 0, \underline{\mu} = 1$
One-sided Gaussian	$\underline{m_s} \to \infty, \underline{m_d} \to \infty$ $\underline{\kappa} \to 0, \underline{\mu} = 0.5$

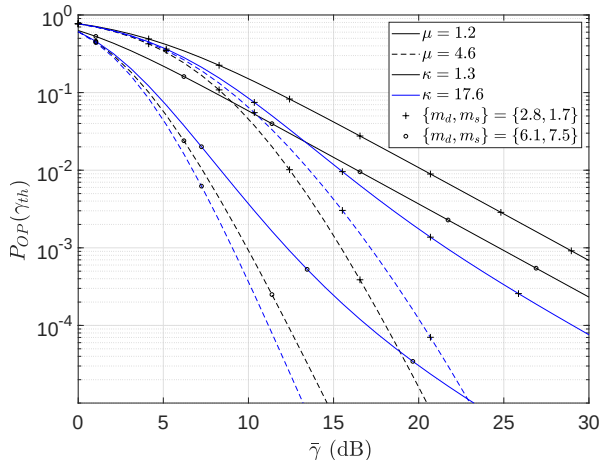

Fig. 3. Outage probability versus $\bar{\gamma}$ for different values of κ, μ, m_d and m_s. Here γ_{th} = 0 dB.

lower values of κ, μ, m_d and m_s parameters. Moreover, the rate at which the outage probability decreases is faster as these parameters grow large.

VI. Conclusion

This paper proposed the double shadowed κ-μ fading model which arises when a κ-μ fading channel undergoes LOS shadowing and then experiences additional multiplicative shadowing or vice versa. The proposed model has a clear physical interpretation, and unifies the shadowing of the dominant component with multiplicative shadowing. Its PDF, CDF, and moments were obtained and performance metrics such as the AF and outage probability were also discussed. It is worth remarking that the double shadowed κ-μ fading model is a very general statistical model that unifies many of the well-known fading models for LOS and NLOS conditions.

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Appendix A
Proof of (5), (7) and (8)

A closed form expression for the PDF of the double shadowed κ-μ fading model can be obtained by substituting (2) and (4) in (3) as follows:

$$f_R(r) = \frac{4 m_d^{m_d} (m_s - 1)^{m_s} \mu^{\mu} (1+\kappa)^{\mu} r^{2\mu-1}}{\Gamma(\mu)\Gamma(m_s)(m_d + \mu\kappa)^{m_d} \hat{r}^{2\mu}} \times \int_0^{\infty} \alpha^{-2m_s-2\mu-1} \mathrm{e}^{-\left(\mu(1+\kappa)\frac{r^2}{\hat{r}^2} + (m_s-1)\right)\frac{1}{\alpha^2}} \times {}_1F_1\left(m_d; \mu; \frac{\mu^2\kappa(1+\kappa) r^2}{\alpha^2 \hat{r}^2 (m_d + \mu\kappa)}\right) d\alpha. \quad (13)$$

The above integral is identical to [13, Eq. 7.621.4]. Now performing the necessary transformation of variables followed by some simple mathematical manipulations, we obtain (5).

Replacing the Gauss hypergeometric function with its series representation [17, Eq. 07.23.02.0001.01] i.e, ${}_2F_1(a;b;c;z) = \sum_{i=0}^{\infty} \frac{(a)_i (b)_i z^i}{(c)_i i!}$ in (6), followed by substituting the resultant expression in $F_\gamma(\gamma) = \int_0^{\gamma} f_\gamma(t)\, dt$ and then solving the integral using [13, Eq. 3.194.5], we obtain the CDF of the double shadowed κ-μ fading model shown in (7).

Furthermore, expanding the Gauss hypergeometric function in (7) with its series representation, and using the definition of the Kampé de Fériet function [14], the CDF of the instantaneous SNR of the double κ-μ fading model can be obtained in closed-form as given in (8). This completes the proof.

Appendix B
Proof of (9) and (11)

A closed form expression for the n-th order moment of the double shadowed κ-μ fading model is obtained by substituting the series representation of the Gauss hypergeometric function [17, 07.23.02.0001.01] in (6), then using the resultant expression in $\mathbb{E}[\gamma^n] \triangleq \int_0^{\infty} \gamma^n f_\gamma(\gamma)\, d\gamma$, and finally solving the integral using [13, eq. 3.194.3] as follows

$$\mathbb{E}[\gamma^n] = \sum_{i=0}^{\infty} \frac{m_d^{m_d} (\bar{\gamma}(m_s-1))^n (\kappa\mu)^i (m_d)_i}{((1+\kappa)\mu)^n (m_d + \kappa\mu)^{i+m_d}\, i!} \times \frac{B(m_s - n, i+n+\mu)}{B(m_s, i+\mu)}. \quad (14)$$

It is worth remarking that by recalling the definition of the incomplete beta function, the following term can also be expressed as $\frac{B(m_s-n, n+\mu)}{B(m_s,\mu)} = \frac{\Gamma(m_s-n)\Gamma(\mu+n)}{\Gamma(m_s)\Gamma(\mu)}$. Now using the identity $\sum_{i=0}^{\infty} \frac{(a)_i (b)_i z^i}{(c)_i i!} = {}_2F_1(a;b;c;z)$ [17, 07.23.02.0001.01] in (14), followed by some basic simplifications, we obtain the result shown in (9).

Substituting n = 1 and then 2 into (9), we obtain the first and second moments of the double shadowed κ-μ fading model. Utilizing these in the AF formulation (see Corollary 1), followed by using [17, Eq. 07.23.02.0001.01] and finally simplifying the resultant expression, we obtain the AF of the double shadowed κ-μ fading model shown in (11). This completes the proof.

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