Effective Rate over $\mathcal{F}$ Composite Fading Channels

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***Abstract*—The $\mathcal{F}$ composite fading model was recently proposed as an accurate and tractable statistical model for the characterization of the composite fading conditions encountered in realistic wireless communication scenarios. In the present contribution we capitalize on the distinct properties of this composite model to evaluate the achievable effective rate over $\mathcal{F}$ composite fading channels. To this end, we derive an exact closed-form expression for the effective rate, which is subsequently used as a benchmark for the derivation of tight upper and lower bounds, as well as of an accurate approximation. The derived analytic expressions are provided in closed-form and benefit from being tractable both analytically and numerically. This enables the development of meaningful insights on the effect of fading conditions and/or latency on the overall system performance. Also, it allows the accurate quantification of the signal to noise ratio required in target quality of service requirements under different composite fading conditions.**

I. Introduction

It is well-known that wireless transmission is subject to multipath fading which is mainly caused by the constructive and destructive interference between two or more versions of the transmitted signal. Since multipath fading is typically detrimental to the performance of wireless communications systems, it is important to characterize and model multipath fading channels accurately in order to understand and improve their behavior. In this context, numerous fading models such as Rayleigh, Rice and Nakagami-m have been utilized in an attempt to characterize multipath fading, depending on the nature of the radio propagation environment [1]–[3].

Based on the above, extensive analyses on the performance of various wireless communication systems have been reported in [4]–[13] and the references therein. Specifically, the authors in [4]–[6] introduced the concepts of capacity analysis under different adaptation policies and carried out an extensive analysis over Rayleigh and Nakagami-m fading channels. Likewise, the ergodic capacity over correlated Rician fading channels and under generalized fading conditions was investigated in [7] and [8], respectively. In the same context, comprehensive capacity analyses over independent and correlated generalized fading channels were performed in [9]–[11] for different diversity receiver configurations. Also, a lower bound for the ergodic capacity of distributed multiple input multiple output (MIMO) systems was derived in [12], while the effective throughput over generalized multipath fading in multiple input single output (MISO) systems was analyzed in [13].

In many practical wireless scenarios, the transmitted signal may not only undergo multipath fading, but also simultaneous shadowing. Shadowing can be typically modeled with the aid of lognormal, gamma, inverse Gaussian and inverse gamma distributions [14]–[19]. Following from this, the simultaneous occurrence of multipath fading and shadowing can be taken into account using any one of the composite fading models, introduced in the open technical literature [20]–[25]. Capitalizing on this, the performance of digital communications systems over composite fading channels has been analyzed in [26]–[39]. The majority of these contributions are concerned with analyses relating to outage probability and error analyses in conventional and diversity based communication scenarios. A corresponding analysis of the channel capacity has only been partially addressed. Many of the existing studies are either limited to an ergodic capacity analysis for the case of independent and correlated fading channels in conventional, relay and multi-antenna communication scenarios or to the effective capacity and channel capacity under different adaptation policies for the case of conventional communication scenarios. In addition, these analyses have been comprehensively addressed only for the case of gamma distributed shadowing and partially for composite models based on lognormal or IG shadowing effects.

Motivated by this, the authors in [40] recently proposed the use of the Fisher-Snedocor $\mathcal{F}$ distribution to describe the composite fading conditions encountered during realistic wireless transmission. This composite model is based on the key assumption that the root mean square (rms) power of a Nakagami-m signal is subject to variation induced by an inverse Nakagami-m random variable (RV). It was shown in [40] that this assumption renders the $\mathcal{F}$ fading model capable of providing a better fit to measurement data than the widely used generalized-K fading model. Additionally, the algebraic representation of the $\mathcal{F}$ composite fading distribution is fairly tractable and simpler than that of the generalized-K distribution, which until now has largely been considered the most analytically tractable composite fading model.

As a result, this model is characterized by its distinct combination of accurate modeling capability and algebraic tractability. In the present contribution, we first derive additional analytic expressions for the key statistical metrics of the $\mathcal{F}$ composite fading model. These formulations are generic and thus, well suited to information-theoretic analyses. Capitalizing on them, we derive a novel exact analytic expression for the effective capacity (C_{eff}) over $\mathcal{F}$ composite fading channels. Based on this, we derive tight upper and lower bounds as well as an accurate approximate expression that provide meaningful insights on the impact of the involved parameters on the overall system performance and limitations. This is useful in numerous emerging wireless applications, such as body area networks and vehicular communications, which are largely characterized by stringent quality of service and low latency requirements.

II. The $\mathcal{F}$ Composite Fading Model

Similar to the physical signal model proposed for the Nakagami-m fading channel [41], the received signal in an $\mathcal{F}$ composite fading channel is composed of separable clusters of multipath in which the scattered waves have similar delay times, with the delay spreads of different clusters being relatively large. However, in contrast to the Nakagami-m signal, in an $\mathcal{F}$ composite fading channel, the rms power of the received signal is subject to random variation induced by shadowing. Based on this, the received signal envelope, R, can be expressed as

$$R = \sqrt{\sum_{i=1}^{m} \alpha^2 I_i^2 + \alpha^2 Q_i^2} \tag{1}$$

where m represents the number of clusters of multipath, I_i and Q_i are independent Gaussian RVs which denote the in-phase and quadrature phase components of the multipath cluster i, where $\mathbb{E}[I_i] = \mathbb{E}[Q_i] = 0$ and $\mathbb{E}[I_i^2] = \mathbb{E}[Q_i^2] = \sigma^2$, with $\mathbb{E}[\cdot]$ denoting statistical expectation. In (1), α is a normalized inverse Nakagami-m RV where m_s is the shape parameter and $\mathbb{E}[\alpha^2] = 1$, such that

$$f_\alpha(\alpha) = \frac{2(m_s-1)^{m_s}}{\Gamma(m_s)\ \alpha^{2m_s+1}} \exp\left(-\frac{m_s-1}{\alpha^2}\right) \tag{2}$$

where $\Gamma(\cdot)$ represents the gamma function [42, eq. (8.310.1)].

Following the approach in [40], we can obtain the corresponding PDF[1] of the received signal envelope, R, in an $\mathcal{F}$ composite fading channel, namely

$$f_R(r) = \frac{2\, m^m (m_s-1)^{m_s}\, \Omega^{m_s} r^{2m-1}}{B(m, m_s)\left[mr^2 + (m_s-1)\ \Omega\right]^{m+m_s}} \tag{3}$$

which is valid for $m_s > 1$, while $B(\cdot,\cdot)$ denotes the beta function [42, eq. (8.384.1)]. The form of the PDF in (3) is functionally equivalent to the $\mathcal{F}$ distribution[2]. In terms of its physical interpretation, m denotes the fading severity whereas m_s controls the amount of shadowing of the rms signal power. Moreover, $\Omega = \mathbb{E}[r^2]$ represents the mean power. As $m_s \to 0$, the scattered signal component undergoes heavy shadowing. In contrast, as $m_s \to \infty$, there exists no shadowing in the wireless channel and therefore it corresponds to a standard Nakagami-m fading channel. Furthermore, as $m \to \infty$ and $m_s \to \infty$, the $\mathcal{F}$ composite fading model becomes increasingly deterministic, i.e., it becomes equivalent to an additive white Gaussian noise (AWGN) channel.

Based on (3), the PDF of the instantaneous SNR, γ, in an $\mathcal{F}$ composite fading channel can be straightforwardly deduced by using the variable transformation $\gamma = \overline{\gamma} r^2/\Omega$, such that

$$f_\gamma(\gamma) = \frac{m^m (m_s-1)^{m_s}\, \overline{\gamma}^{m_s} \gamma^{m-1}}{B(m, m_s)\left[m\gamma + (m_s-1)\,\overline{\gamma}\right]^{m+m_s}} \tag{4}$$

where $\overline{\gamma} = \mathbb{E}[\gamma]$ denotes the corresponding average SNR. To this effect, the redefined moments, $\mathbb{E}[\gamma^n] \triangleq \int_0^\infty \gamma^n f_\gamma(\gamma) \mathrm{d}\gamma$ and the moment-generating function (MGF), $M_\gamma(s) \triangleq \int_0^\infty \exp(-s\gamma) f_\gamma(\gamma) \mathrm{d}\gamma$, [43], are expressed as

$$\mathbb{E}[\gamma^n] = \frac{(m_s-1)^n \overline{\gamma}^n \Gamma(m+n)\Gamma(m_s-n)}{m^n \Gamma(m)\Gamma(m_s)} \tag{5}$$

[1] It is worth highlighting that in the present paper, we have modified slightly the underlying inverse Nakagami-m PDF from that used in [40] and subsequently the PDF for the $\mathcal{F}$ composite fading model. While the PDF in [40] is completely valid for physical channel characterization, it has some limitations in its admissible parameter range when used in analyses relating to digital communications. The redefined PDF in (3), on the other hand, is well consolidated and hence, more useful in practice.

[2] Letting $r^2 = x$, $m = d_1/2$, $m_s = d_2/2$, $\Omega = d_2/(d_2-2)$ and performing the required transformation yields the $\mathcal{F}$ distribution, $f_X(x)$, with parameters d_1 and d_2.

and

$$M_\gamma(-s) = {}_1F_1\left(m; 1-m_s; \frac{s\overline{\gamma}(m_s-1)}{m}\right) + \frac{\Gamma(-m_s)s^{m_s}\overline{\gamma}^{m_s}(m_s-1)^{m_s}}{B(m,m_s)m^{m_s}} \times {}_1F_1\left(m+m_s; 1+m_s; \frac{s\overline{\gamma}(m_s-1)}{m}\right) \tag{6}$$

respectively, with ${}_1F_1(\cdot,\cdot,\cdot)$ denoting the Kummer confluent hypergeometric function [42, eq. (9.210.1)]. Similarly, with the aid of [42, eq. (3.194.1)] the envelope cumulative distribution function (CDF) is expressed as

$$F_R(r) = \frac{m^{m-1}r^{2m}}{B(m,m_s)(m_s-1)^m\Omega^m} \times {}_2F_1\left(m, m+m_s, m+1; -\frac{mr^2}{(m_s-1)\Omega}\right) \tag{7}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function [42, eq. (9.111)], whereas its respective SNR CDF is readily given by

$$F_\gamma(\gamma) = \frac{m^{m-1}\gamma^m}{B(m,m_s)(m_s-1)^m\overline{\gamma}^m} \times {}_2F_1\left(m, m+m_s, m+1; -\frac{m\gamma}{(m_s-1)\overline{\gamma}}\right). \tag{8}$$

It is noted that the above CDF expressions are valid for arbitrary values of the fading parameters m and m_s.

In the sequel, we use the new formulation in the comprehensive analysis of the effective rate under $\mathcal{F}$ composite fading conditions.

III. Effective Rate Analysis

Channel capacity is a core performance metric in conventional and emerging communication systems, and its limits are largely affected by the incurred fading conditions during wireless transmission. Ergodic capacity is the most widely used capacity measure and is concerned with CSI knowledge only at the receiver and a fixed transmit power. However, the effective capacity is also a particularly useful information theoretic measure as it accounts for the achievable capacity subject to the incurred latency relating to the corresponding buffer occupancy. In what follows, we derive novel exact and approximate analytic expressions along with tight bounds for the effective rate over $\mathcal{F}$ composite fading conditions.

A. An Exact Closed-form Expression

It is recalled that the effective rate accounts for the channel capacity as a function of the asymptotic decay rate of the corresponding buffer occupancy. This is an insightful measure, particularly in emerging technologies where latency is a critical quality of service criterion. Next, we derive an exact closed-form expression for the effective rate over $\mathcal{F}$ composite fading channels.

Theorem 1. *For $m, \gamma, \overline{\gamma}, \theta, B, T \in \mathbb{R}^+$ and $m_s > 1$, the effective capacity $C_{\text{eff}} = E_c(\theta)$ under $\mathcal{F}$ composite fading conditions can be expressed as*

$$C_{\text{eff}} = \frac{m_s}{A}\log_2\left(\frac{m}{(m_s-1)\,\overline{\gamma}}\right) + \frac{1}{A}\log_2\left(\frac{(m+m_s)_A}{(m_s)_A}\right) - \frac{\log_2\left({}_2F_1\left(A+m_s, m+m_s; A+m+m_s; \mathcal{D}_1\right)\right)}{A} \tag{9}$$

where $\mathcal{D}_1 = (m-(m_s-1)\,\overline{\gamma})/m$ whereas $(x)_n \triangleq \Gamma(x+n)/\Gamma(x)$ is the Pochhammer symbol [42] and $A = BT\theta/\ln(2)$ is a metric of delay constraint. Also, B and T denote the system bandwidth and the block/frame length, respectively, whereas θ is the quality of service (QoS) exponent in terms of the asymptotic decay rate of the buffer occupancy.

Proof. Given the instantaneous service rate of a system as $R = TB\log_2(1+\gamma)$, the corresponding effective rate can be expressed as $E_c(\theta) = -A^{-1}\log_2\left(\mathbb{E}\left[e^{-\theta R}\right]\right)$, which can be re-written as [47], [48]

$$C_{\text{eff}} = -\frac{1}{A}\log_2\left(\int_0^\infty e^{-\theta TB\log_2(1+\gamma)}f_\gamma(\gamma)\mathrm{d}\gamma\right) \tag{10}$$

where $f_\gamma(\gamma)$ accounts for the corresponding fading statistics. Therefore, for the case of $\mathcal{F}$ composite fading channels, we substitute the redefined PDF in (4) into (10), which after some algebraic manipulations yields

$$C_{\text{eff}} = \frac{1}{A}\log_2\left(\frac{B\left(m,m_s\right)}{m^m(m_s-1)^{m_s}\,\overline{\gamma}^{m_s}}\right) - \frac{1}{A}\log_2\left(\int_0^\infty \frac{\gamma^{m-1}\mathrm{d}\gamma}{(1+\gamma)^A\left[m\gamma+(m_s-1)\,\overline{\gamma}\right]^{m+m_s}}\right). \tag{11}$$

The integral in (11) can be expressed in closed-form with the aid of [42, eq. (3.259.3)]. Hence, by performing the necessary change of variables and after some algebraic manipulations one obtains the following closed-form expression

$$C_{\text{eff}} = -\frac{1}{A}\log_2\left\{\frac{B(m,A+m_s)}{B(m,m_s)}\left(\frac{(m_s-1)\overline{\gamma}}{m}\right)^{m_s} \times {}_2F_1\left(A+m_s, m+m_s; A+m+m_s; \mathcal{D}_1\right)\right\}. \tag{12}$$

To this effect and by also applying the properties and identities of the logarithm, gamma and beta functions, (12) reduces to the compact form of (9), which completes the proof. □

It is noted that a similar expression for the effective capacity was derived in [39]. However, this expression is limited due to the constrained consideration of the SNR PDF of the $\mathcal{F}$ fading model in [40]. As a result, the derived result in Theorem 2 is more valid and suitable since it is based on the well consolidated SNR PDF in (4). In addition, this expression can be used as a benchmark for the derivation of simple tight bounds and an accurate approximation which provide useful insights on the impact of the involved parameters on the system performance.

Proposition 1. *For $m, \gamma, \overline{\gamma}, \theta, B, T \in \mathbb{R}^{+}$, $m_s > 1$ and assuming $m_s + m >> A$ and $\overline{\gamma} > 5$dB, the effective rate under $\mathcal{F}$ composite fading conditions can be bounded by the following inequalities[3]:*

$$C_{\text{eff}}^{\text{UB}} < \frac{\log_2\left((m_s+A)_m\right) - \log_2\left((m_s)_m\right)}{A} + \log_2\left(\overline{\gamma}\right) + \log_2\left(m_s - 1\right) - \log_2\left(m\right) \tag{13}$$

and

$$C_{\text{eff}}^{\text{LB}} > \log_2\left(\overline{\gamma}\right) + \log_2\left(\frac{m_s-1}{m}\right) - \frac{\log_2\left((m_s)_m\right)}{A} \tag{14}$$

which constitute tight upper and lower bounds, respectively.

Proof. It is evident that $A + m + m_s \approx m + m_s$ when $m + m_s >> A$. As a result, equation (9) can be tightly upper bounded as follows:

$$C_{\text{eff}}^{\text{UB}} < -\frac{1}{A}\log_2\left\{\frac{(m_s)_A}{(m_s+m)_A}\left(\frac{(m_s-1)\overline{\gamma}}{m}\right)^{m_s} \times {}_2F_1\left(A+m_s, m+m_s; m+m_s; \mathcal{D}_1\right)\right\}. \tag{15}$$

Importantly, given that

$${}_2F_1\left(A+m_s, m+m_s; m+m_s; \mathcal{D}_1\right) = {}_1F_0\left(A+m_s; ; \mathcal{D}_1\right) \tag{16}$$

and by recalling that ${}_1F_0(n; ; 1+x) \triangleq (-1)^n / x^n$, $n \in \mathbb{R}$, equation (15) can reduce to

$$C_{\text{eff}}^{\text{UB}} < -\frac{1}{A}\log_2\left(\frac{(m_s)_A(m_s-1)^{m_s}\overline{\gamma}^{m_s}}{(m_s+m)_A m^{m_s}}\left(\frac{m}{(m_s-1)\overline{\gamma}}\right)^{A+m_s}\right). \tag{17}$$

To this effect and after some algebraic manipulations, the closed-form upper bound in (13) is deduced.

Based on (13) and recalling that $A + m + m_s \approx m + m_s$ when $m + m_s >> A$, the left hand side term on the fraction of (13) can be eliminated. This readily yields (14), which is a tight lower bound to the exact expression in (9) for the given conditions and thus, it completes the proof. □

It is noted here that (13) and (14) are particularly insightful and they can be also expressed in terms of the involved average SNR, namely

$$\overline{\gamma}_{\text{eff}} \simeq \frac{m 2^{C_{\text{eff}}^{\text{UB}}}}{m_s - 1}\left(\frac{(m_s)_A}{(m_s+m)_A}\right)^{\frac{1}{A}} \simeq \frac{m 2^{C_{\text{eff}}^{\text{LB}}}}{m_s - 1}\left((m_s)_A\right)^{\frac{1}{A}} \tag{18}$$

which is rather accurate when $m + m_s >> A$. Importantly, this allows the determination of $\overline{\gamma}$ for different values of m, m_s and A along with specific values of C_{eff}. This is useful in determining the required SNR for specific fading conditions and target quality of service requirements, particularly in emerging wireless communication systems.

In the same context as the derived bounds in Proposition 1, an accurate approximate expression to (9) can be additionally derived.

[3]It is noted that $(m_s+m)_A/(m_s)_A = (m_s+A)_m/(m_s)_m = \Gamma(m_s+m+A)\Gamma(m_s)/(\Gamma(m_s+m)\Gamma(m_s+A))$.

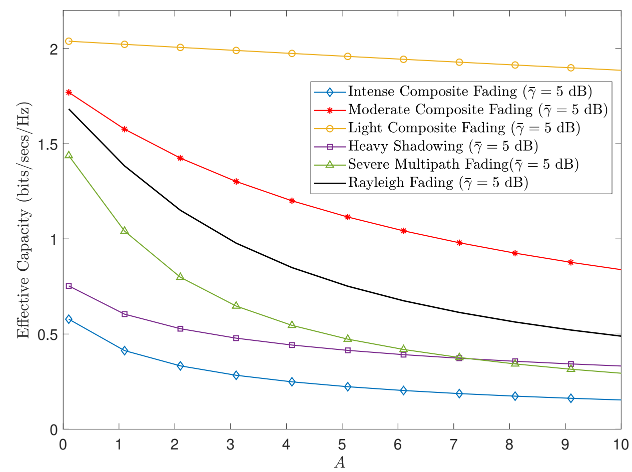

Fig. 1: Effective capacity versus A under $\mathcal{F}$ fading channels for different values of the m and m_s parameters when $\overline{\gamma} = 5$ dB.

Proposition 2. *For $m, \gamma, \overline{\gamma}, B \in \mathbb{R}^{+}$, $m_s > 1$ and $\overline{\gamma} >> 0$, the effective capacity under $\mathcal{F}$ composite fading conditions can be accurately approximated as follows:*

$$C_{\text{eff}}^{\text{appr.}} \simeq -\frac{\log_2\left({}_2F_1(A, m_s; A+m+m_s; 1-\overline{\gamma})\right)}{A}. \tag{19}$$

Proof. In the high SNR regime, i.e. $\overline{\gamma} >> 0$, it readily follows that $\overline{\gamma} >> m$, $\overline{\gamma} >> m_s$ and $\overline{\gamma} >> A$. To this effect and by expanding the logarithmic terms in (9), one obtains

$$\frac{(m_s)_A}{(m+m_s)_A}\left(\frac{(m_s-1)\overline{\gamma}}{m}\right)^{m_s} \simeq \overline{\gamma}^{m_s}. \tag{20}$$

Based on this and after some algebraic manipulations, equation (19) is deduced, which completes the proof. □

It is evident that (19) can be also expressed in terms of the average SNR, namely

$$\overline{\gamma}_{\text{eff}} \simeq 1 - {}_2F_1^{-1}\left(A, m_s; A+m+m_s; 2^{-AC_{\text{eff}}^{\text{appr.}}}\right) \tag{21}$$

where ${}_2F_1^{-1}(\cdot, \cdot; \cdot; \cdot)$ denotes the inverse Gauss hypergeometric function.

It is noted here that the tightness of the derived bounds and the accuracy of the proposed approximation are rather high. This is evident by the fact that the insightful equalities $\lceil C_{\text{eff}}^{\text{LB}} \rceil = \lfloor C_{\text{eff}} \rfloor$ and $\lceil C_{\text{eff}} \rceil = \lfloor C_{\text{eff}}^{\text{UB}} \rfloor$ hold for the two derived bounds and $\lceil C_{\text{eff}}^{\text{appr.}} \rceil = C_{\text{eff}}$ or $\lfloor C_{\text{eff}}^{\text{appr.}} \rfloor = C_{\text{eff}}$ hold for the proposed approximation for the vast majority of possible combinations, where $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ denote the ceiling and floor functions, respectively [42].

To the best of the authors knowledge, the provided analytic expressions have not been previously reported in the open technical literature. Also, the offered analytic expression provide meaningful insights on the impact of the involved parameters on the system performance, which are useful in the design and efficient operation of future wireless systems with stringent quality of service and low latency requirements.

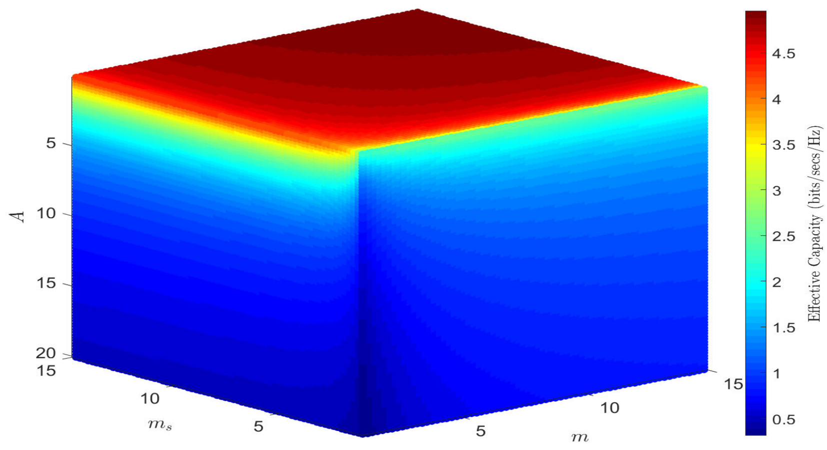

Fig. 2: Effective capacity in an $\mathcal{F}$ fading channel as a function of the m, m_s and a parameters for $\overline{\gamma} = 15$ dB.

IV. Numerical Results

In this section, we utilize the analytic results obtained in the previous section to quantify the effective rate for various communication scenarios under realistic multipath fading and shadowing conditions.

It is recalled that the tightness of the upper and lower bounds and the accuracy of the simple approximation are high. Also, the identities $\lceil C_{\rm eff}^{\rm LB} \rceil = \lfloor C_{\rm eff} \rfloor$ as well as $\lceil C_{\rm eff} \rceil = \lfloor C_{\rm eff}^{\rm UB} \rfloor$ and $\lceil C_{\rm eff}^{\rm appr.} \rceil = C_{\rm eff}$ or $\lfloor C_{\rm eff}^{\rm appr.} \rfloor = C_{\rm eff}$ hold for the vast majority of possible combinations, which verifies the usefulness of these simplified expressions.

Regarding the exact results, Fig. 1 demonstrates how the C_E per unit bandwidth varies as a function of the delay constraint over $\mathcal{F}$ composite fading channels. Five different combinations of the m and m_s parameters were considered for a case of low average SNR, i.e. $\overline{\gamma} = 5$ dB, which makes the impact of the incurred delay more critical. It is evident that the spectral efficiency is affected considerably by the value of A across all types of fading conditions, with the impact on intense fading conditions being the most detrimental. In the same context, the effects of the multipath fading and shadowing are shown in Fig. 2, where the performance of the C_E is illustrated along with different values of A and $\overline{\gamma} = 15$ dB. In all cases, we consider broad ranges of the involved parameters, namely $1 < m \leq 15$, $1 < m_s \leq 15$ and $0 \leq A \leq 20$ in order to consider all types of fading severity and incurred delays, as these are encountered in realistic communication scenarios. As expected, the spectral efficiency increases as the m and m_s parameters are greater ($m, m_s \to 15$) and A is smaller ($A \to 0$), i.e., light composite fading conditions with no delay constraint. Conversely, the performance of the C_E is rather poor for the case of intense composite fading conditions with excessive delay constraint, i.e., $m, m_s \to 1$ and $A \to 20$. In general, it is shown that even if one of the parameters is unfavorable i.e. excessive delay constraint or severe multipath fading or shadowing, the corresponding achievable C_E will lie at moderate levels, regardless of how favorable the values of the other parameters are. This verifies the need for accurate channel modeling and latency estimation and reduction in the deployment of efficient wireless technologies.

V. Conclusion

In this paper, we presented a comprehensive effective rate analysis over $\mathcal{F}$ composite fading channels. In this context, we derived novel exact analytic expressions along with simple upper and lower bounds and an accurate approximation. When comparing these expressions with those for the generalized-K fading channels given in [26], the $\mathcal{F}$ fading model exhibits lower complexity and provides more insights on the impact of the involved parameters on the overall system performance. Based on this, it was shown that the spectral efficiency changes considerably even at slight variations of the average SNR, buffer occupancy and the severity of the multipath fading and shadowing conditions. The impact of different types of $\mathcal{F}$ composite fading was also investigated through comparisons with the respective capacity for the case of a Rayleigh fading channel. This has highlighted that different types of composite fading can have a profound effect which is beyond the range of the fading conditions experienced in a conventional Rayleigh fading environment. Finally, the new results and insights provided here will be useful in the design and deployment of future communications systems. For example when assessing technologies such as channel selection and spectrum aggregation for use in heterogeneous networks, telemedicine and vehicular communications, to name but a few.

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