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On the regularisation property of the Kachanov-Rabotnov continuum damage model - a finite element study

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Summary. In this paper the regularising properties of the Kachanov-Rabotnov type continuum damage constitutive model are studied using a simple one-dimensional dynamic problem. It is found that the width of the localisation zone is independent of the mesh size if the loading rate is below a certain threshold value.

Key words: continuum damage mechanics, Kachanov-Rabotnov model, strain softening, localisation, dynamic problem, finite element analysis

Introduction

A well known model to describe continuous degradation of a material is the Kachanov-Rabotnov model, which was first introduced in 1958 [3, 7]. Since then, continuum damage mechanics has developed into an important and active field of continuum mechanics, see e.g. [2, 4, 5, 6, 8].

As it is shown in [1], two different formulations either based on stiffness or flexibility approaches can be considered to be identical when certain relations are fullfilled between the model parameters. The stiffness formulation of an elastic-damaging material model can be described by the equations

$$\boldsymbol{\sigma} = (1-D)\boldsymbol{C}_{\mathrm{e}} : \boldsymbol{\varepsilon}, \tag{1}$$

$$\dot{D} = \frac{1}{t_{\mathrm{d}}(1-D)^p}\left(\frac{Y}{Y_{\mathrm{r}}}\right)^r, \tag{2}$$

where $\boldsymbol{C}_{\mathrm{e}}$ is the elasticity tensor, $\boldsymbol{\sigma}, \boldsymbol{\varepsilon}$ stress and strain tensors and : denotes the double dot-product. For the flexibility formulation the corresponding constitutive equations are

$$\boldsymbol{\varepsilon} = (1+\alpha)\boldsymbol{C}_{\mathrm{e}}^{-1} : \boldsymbol{\sigma}, \tag{3}$$

$$\dot{\alpha} = \frac{(1+\alpha)^m}{t_{\mathrm{d}}^{\mathrm{c}}}\left(\frac{Z}{Z_{\mathrm{r}}}\right)^n. \tag{4}$$

Expressions for the thermodynamic forces Y and Z can be written as

$$Y = \frac{1}{2}\boldsymbol{\varepsilon} : \boldsymbol{C}_{\mathrm{e}} : \boldsymbol{\varepsilon} = \frac{1}{2(1-D)^2}\boldsymbol{\sigma} : \boldsymbol{C}_{\mathrm{e}}^{-1} : \boldsymbol{\sigma}, \tag{5}$$

$$Z = \frac{1}{2}\boldsymbol{\sigma} : \boldsymbol{C}_{\mathrm{e}}^{-1} : \boldsymbol{\sigma} = \frac{1}{2(1+\alpha)^2}\boldsymbol{\varepsilon} : \boldsymbol{C} :_{\mathrm{e}} \boldsymbol{\varepsilon}. \tag{6}$$

A reasonable choice for the reference values Y_r and Z_r is

$$Y_\mathrm{r} = Z_\mathrm{r} = \frac{\sigma_\mathrm{r}^2}{2E} = \frac{1}{2}E\varepsilon_\mathrm{r}^2, \tag{7}$$

where $\sigma_\mathrm{r} = E\varepsilon_\mathrm{r}$ is a reference stress, and ε_r is a corresponding reference strain.

Physically the damage variable D can be interpreted as a ratio of the differential damaged area element to the original area element. However, interpretation of the damage parameter α is not so straightforward.

It is shown in [1] that the two models yield identical stress-strain responses if

$$n = r, \quad m = p + 2n + 2 \quad \text{and} \quad t_\mathrm{d}^\mathrm{c} = t_\mathrm{d}. \tag{8}$$

However, it should be noticed that the damage variables are not identical: $D \neq \alpha$.

Strain and damage localisation

As in viscoplasticity, the damage evolution equations (2) and (4) contain a material parameter which has the unit of time, i.e. t_d or t_d^c. It is therefore assumed that the apparent viscosity could regularise the governing equilibrium equations or the equations of motion in dynamic analysis.

For strain-softening in inviscid solids localisation takes place in a plane of zero thickness. Viscosity added to either plasticity or damage models may bring in the desired non-zero material length-scale. To investigate the regularising behaviour of the Kachanov-Rabotnov continuum damage model, a one-dimensional finite element analysis is carried out. A semi-infinite bar subjected to a linearly increasing displacement boundary condition $u(0,t) = \eta\epsilon_\mathrm{r}Lt/t_\mathrm{d}$ has been analysed with different uniform mesh sizes. The computational domain is chosen to be large enough that reflections from the other boundary do not occur. The length L is chosen as $L = c_\mathrm{e}t_\mathrm{d}$, where c_e is the elastic wave speed $c_\mathrm{e} = \sqrt{E/\rho}$. A standard central difference scheme is used to integrate the equations of motion with a constant time-step equal to the critical time step of the elastic bar.

From the numerical computations presented in [1], it can be concluded that the width of the localisation zone l_loc is constant if

$$(\dot{\varepsilon}t_\mathrm{d})^{2r} = \text{constant}, \tag{9}$$

however, the situation is more subtle and here a more detailed investigation of the regularising properties of the Kachanov-Rabotnov model is carried out.

A localisation study is performed with varying prescribed rate η and the damage localisation width is defined as the measure of the domain where

$$D^* \leq x \leq 1, \tag{10}$$

where D^* is the damage value at fracture stress for quasi-static constant strain-rate loading

$$D^* = 1 - \left(\frac{2r-1}{2r+p+2}\right)^{1/(p+1)}, \tag{11}$$

and it is independent of the aplied strain-rate.

The width of the localisation zone is shown as a function of the loading rate η in figure 1 for three different mesh sizes $h = L/100, L/1000$ and $L/10000$ for the cases $r = 2, p = 1$, and $r = 4, p = 1$. As it can be seen, the width of the damage localisation zone is mesh-size independent if the loading rate satisfies $\eta < 0.75$.

In figure 2 the damage profiles for the case $p = 1, r = 2, \eta = 0.5$ at times $t_\mathrm{d}, 2t_\mathrm{d}, 3t_\mathrm{d}$ and at the fracture $t = 3.76t_\mathrm{d}$ are shown for a mesh with $h = L/1000$.

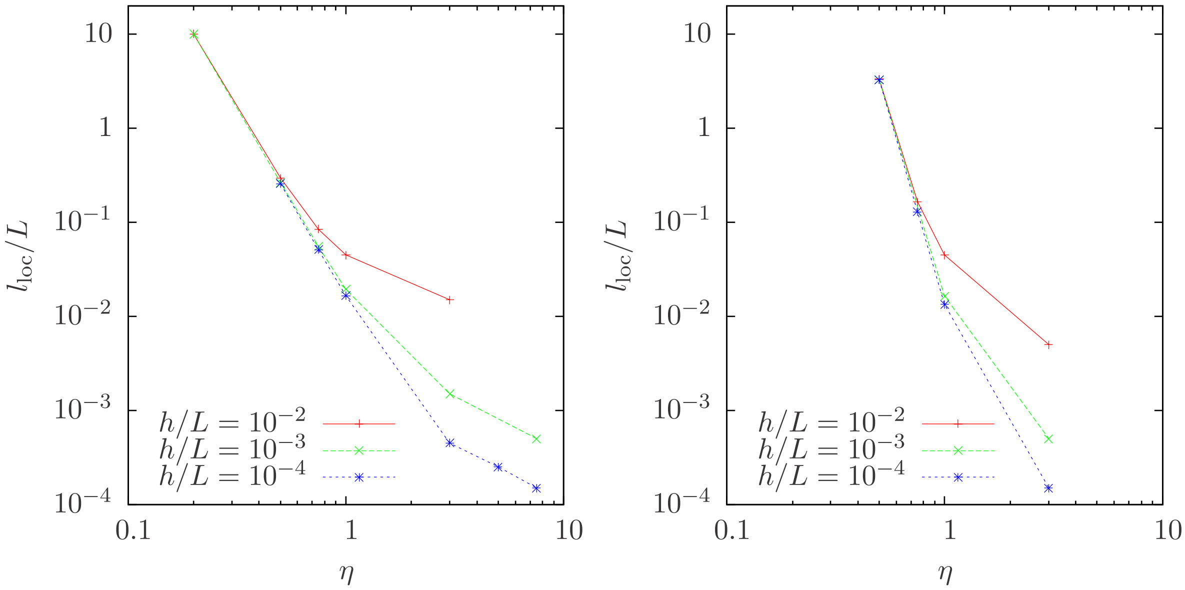

Figure 1. Damage localisation width as a function of the prescribed loading rate, $r = 2$ (lhs), $r = 4$ (rhs). In both cases $p = 1$.

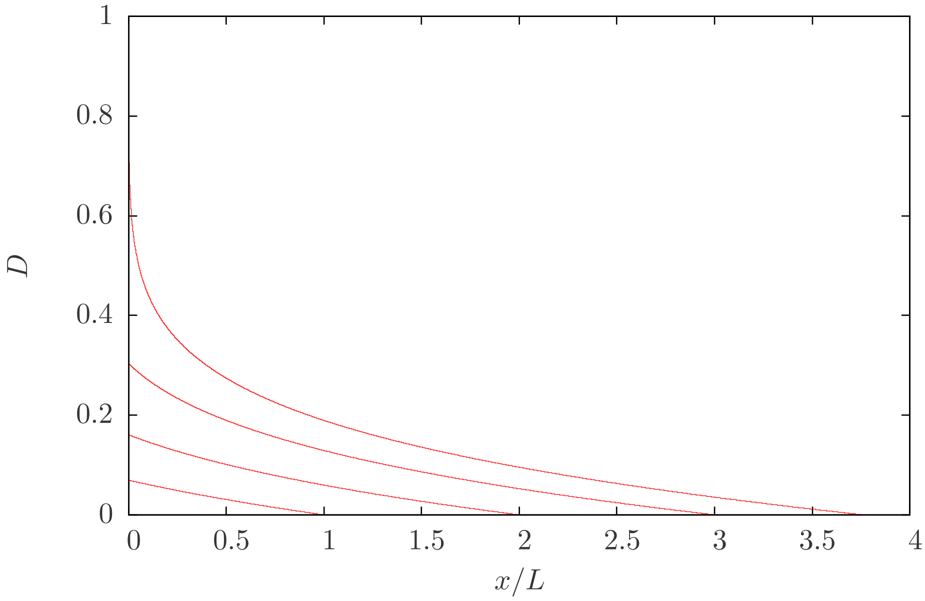

Figure 2. Damage profiles for the case $r = 2, p = 1$ and $\eta = 0.5$ at times $t_{\mathrm{d}}, 2t_{\mathrm{d}}, 3t_{\mathrm{d}}$ and at the fracture $t = 3.76t_{\mathrm{d}}$. Mesh size is $h = L/1000$.

Concluding remarks

A preliminary finite element study on the lcalisation behaviour of the Kachanov-Rabotnov type continuum damage model is performed. It is found that the width of the damage localisation zone is independent of the mesh size if the loading rate is below a certain treshold depending on the material parameters of the model.

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