Proceedings of the 32nd Nordic Seminar on Computational Mechanics
A.H. Niemi and H. Koivurova (Eds.)

Numerical modelling of concrete fracture: a mesoscopic approach based on embedded discontinuity FEM

Timo Saksala

Tampere University, Civil Engineering, Finland, timo.saksala@tuni.fi

Summary. This article presents a numerical study on concrete failure under uniaxial compression. For this end, a fracture model based on the embedded discontinuity finite elements is employed. Concrete mesostructure, i.e. the aggregates and the cement matrix, is explicitly described by convex polygons. Numerical simulations (in 2D) demonstrate that the present approach can predict the salient features of concrete fracture modes even when the crack initiation is based solely on the first principal stress criterion.

Key words: mesoscopic model, concrete fracture, finite elements, embedded discontinuity

Introduction

Numerical modelling of concrete fracture processes is an important and challenging task in Civil Engineering. The classical approach based on homogenisation of the aggregates-in-cement matrix mesostructure is still the foremost method in analyses of concrete structures (especially large structures such as dams). However, the mesoscopic approach describing the aggregate-mortar structure explicitly is indispensable in the analyses of the effect of aggregate shape, size and mechanical properties on the overall behaviour of a specific concrete. Moreover, as aggregates introduce various fracture toughening mechanisms, such as crack stopping, redirection and branching [1], their explicit description is crucial.

This paper presents a numerical modelling approach where the concrete mesostructure is explicitly described and the concrete fracture is modelled based on the embedded discontinuity finite elements [2]. The original model presented by Saksala [3] is modified here by allowing cracks to initiate only in mode I, i.e. by the first principal stress (Rankine) criterion.

Concrete fracture model

Concrete fracture is described by the embedded discontinuity finite elements. In the present context this means that an ordinary constant strain triangle (CST) in enriched with special functions to model discontinuities [2, 3]. For a CST element with a strong discontinuity (crack) illustrated in Figure 1a, the displacement and strain fields are

$$\mathbf{u} = N_i \mathbf{u}_i^e + (H_{\Gamma_d} - \varphi)\boldsymbol{\alpha}_d, \quad \boldsymbol{\varepsilon} = \nabla N_i \otimes \mathbf{u}_i^e - \nabla\varphi \otimes \boldsymbol{\alpha}_d$$

$$\nabla\varphi = \arg\left(\max_{k=1,2} \frac{\left|\Sigma_{i=1}^{k} \nabla N_i \cdot \mathbf{n}\right|}{\left\|\Sigma_{i=1}^{k} \nabla N_i\right\|} \right) \tag{1}$$

where $\boldsymbol{\alpha}_d$ is the displacement jump vector, N_i and $\mathbf{u}_i^e$ are the standard interpolation function and displacement vector at node *i* (with summation on repeated *i*), and H_{Γ_d} is the Heaviside function at discontinuity Γ_d with normal **n**. Finally, φ is a function that restricts the effect of the displacement jump within the corresponding finite element so that the essential boundary conditions remain unaffected. This function is chosen by criterion (1).

A bi-surface, plasticity inspired model for solving the displacement jump vector and the traction updates, as well as to control the softening behavior at the discontinuity. The components of this model are

$$\phi_{\mathrm{t}}(\mathbf{t}_{\Gamma_d}, \kappa, \dot{\kappa}) = \mathbf{n} \cdot \mathbf{t}_{\Gamma_d} - (\sigma_{\mathrm{t}} + q(\kappa, \dot{\kappa})), \quad \phi_{\mathrm{s}}(\mathbf{t}_{\Gamma_d}, \kappa, \dot{\kappa}) = |\,\mathbf{m} \cdot \mathbf{t}_{\Gamma_d}\,| - (\sigma_{\mathrm{s}} + \tfrac{\sigma_{\mathrm{s}}}{\sigma_{\mathrm{t}}} q(\kappa, \dot{\kappa}))$$

$$\dot{\boldsymbol{\alpha}}_d = \dot{\boldsymbol{\alpha}}_{\mathrm{I}} + \dot{\boldsymbol{\alpha}}_{\mathrm{II}} = \dot{\lambda}_{\mathrm{t}} \frac{\partial \phi_{\mathrm{t}}}{\partial \mathbf{t}_{\Gamma_d}} + \dot{\lambda}_{\mathrm{s}} \frac{\partial \phi_{\mathrm{s}}}{\partial \mathbf{t}_{\Gamma_d}}, \quad \dot{\mathbf{t}}_{\Gamma_d} = -\mathbf{E} : (\nabla\varphi \otimes \dot{\boldsymbol{\alpha}}_d) \cdot \mathbf{n}, \quad \dot{\kappa} = -\dot{\lambda}_{\mathrm{t}} \frac{\partial \phi_{\mathrm{t}}}{\partial q} - \dot{\lambda}_{\mathrm{s}} \frac{\partial \phi_{\mathrm{s}}}{\partial q} \tag{2}$$

$$q = h\kappa + s\dot{\kappa}, \quad h = -g\sigma_{\mathrm{t}} \exp(-g\kappa), \quad \dot{\lambda}_i \geq 0, \quad \phi_i \leq 0, \quad \dot{\lambda}_i \phi_i = 0, \quad i = \mathrm{t,s}$$

where ϕ_{t} and ϕ_{s} are the tension (mode I) and shear (mode II) loading functions, respectively, $\kappa, \dot{\kappa}$ are the internal variable and its rate, **m** is the crack tangent vector, σ_{t} and σ_{s} are elastic limits in tension and shear, respectively, $\boldsymbol{\alpha}_{\mathrm{I}}, \boldsymbol{\alpha}_{\mathrm{II}}$ are the mode I and II crack opening vectors, **E** is the elasticity tensor, $\dot{\lambda}_{\mathrm{t}}, \dot{\lambda}_{\mathrm{s}}$ are the mode I and II opening increments, respectively, *h* is the softening modulus, and *s* is the constant viscosity modulus. The softening slope parameter *g* is defined by the mode I fracture energy G_{Ic} by $g = \sigma_t/G_{\mathrm{Ic}}$. It should be noted that that last equations in (2) are the consistency conditions. Viscosity is thus included in the spirit of the viscoplastic consistency approach.

A fracture occurs, i.e. a discontinuity is embedded in an element, upon violation of the Rankine criterion. However, once a crack is introduced, it can fail in both shear and tensile mode, as governed by model (2). The material point level behaviour is linear elastic up to fracture. However, as heterogeneity is naturally included in the model by the hard aggregates and the weak microcracks in the cement matrix, global nonlinear pre-peak behaviour can be captured, as will be shown in the numerical simulations.

The global equations of motion are solved with an explicit time integrator.

Concrete mesostructure

Concrete is described as a bi-phasic material with explicit aggregates in a mortar matrix. The aggregates are taken to be convex polygons. The resulting mortar-aggregate structure is meshed with the ordinary CST elements. Concrete contains inherent microcrack populations induced by hydration processes, for example. These cracks have, expectedly, a non-negligible influence on the concrete failure processes.

In the numerical concrete mesostructure in Figure 1b, the aggregates are represented by 50 polygons of different shapes and sizes. The blue lines (598 in total) represent the inherent microcrack population. The interface transition zone (ITZ) is here accounted for by lowering the strength to one half of the intact value of the elements surrounding the aggregates.

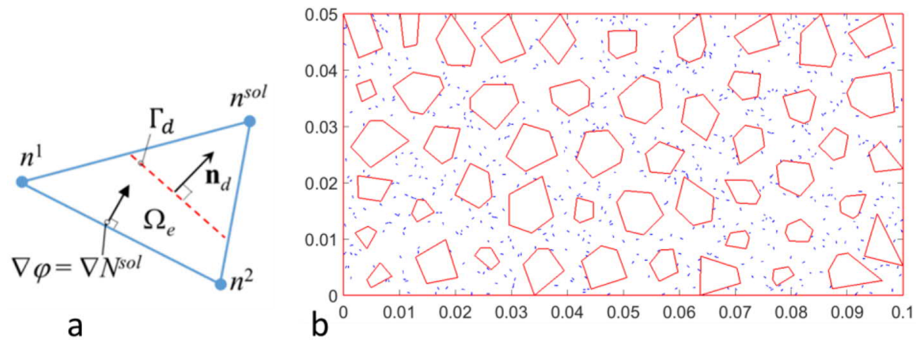

Figure 1. CST element with a discontinuity (a), and numerical concrete mesostructure (b).

The mortar is taken as the Portland cement while the aggregates are granitic rock Some of the key material properties are for mortar: E = 27.5 GPa; ν = 0.2; ρ = 2400 kg/m^3; σ_t = 3.5 MPa; σ_s = 7 MPa; G_{Ic} = 0.02 N/m and for aggregates: E = 60 GPa; ν = 0.17; ρ = 2400 kg/m^3; σ_t = 10 MPa; σ_s = 25 MPa; G_{Ic} = 0.04 N/m.

Numerical example

Uniaxial compression test is simulated as a numerical example. In addition to the original Rankine cracking criterion, where a crack is parallel to the first principal direction, a crack reorientation is tested: from among the element edge normals, the one that is most parallel to the first principal direction is selected. This scheme should result in a different kind of failure mode as, unlike in the Rankine criterion, the crack normal is not orthogonal to the loading direction. The simulation results are shown in Figure 2.

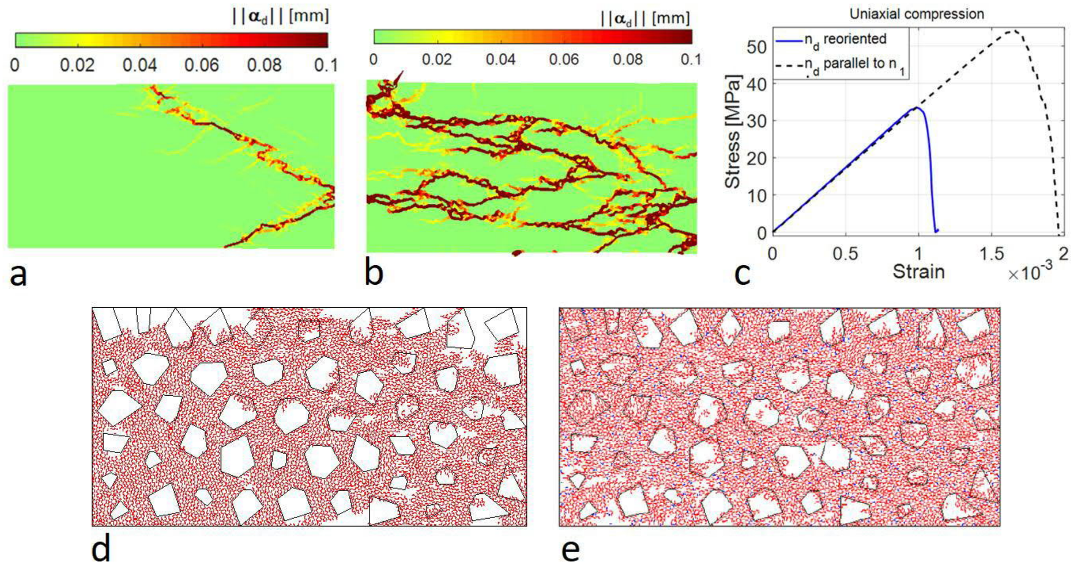

Figure 2. Uniaxial compression test: failure mode for crack reorientation (initial cracks and ITZ not included) (a) and for original orientation (initial cracks and ITZ included) (b), stress-strain responses (c), and crack distributions for crack reorientation (d) and for original orientation (e).

The failure modes (plotted as the magnitude of crack opening vectorss) shown in Figure 2a and b indeed attest different failure modes: the reorientation scheme results in a typical shear failure mode observed in the experiments [4] while the original Rankine criterion results in a multiple axial splitting mode, which is also observed in the experiments. The corresponding compressive strengths, as readable in Figure 2c, are 33 MPa and 54 MPa – typical values for normal and intermediate high strength concretes. Finally, Figure 2d and c show the final crack distributions. Cracks are almost everywhere in the numerical samples but the deformation (crack opening) localizes only in part of them to form macrocracks. Cracking occurs also in aggregates.

Conclusion

The concrete fracture modelling approach presented here show some predictive capabilities. Namely, by giving the mortar cement and the aggregates realistic material properties, low and high strength concrete behavior, including the peak stresses and the failure modes, in uniaxial compression can be replicated. Moreover, the simulation results with the first principal stress criterion corroborate the hypothesis that brittle materials failure in uniaxial compression is a violent multiple axial splitting mode.

Acknowledgements

This research was funded by Academy of Finland under grant number 298345.

References

[1] E.N. Landis, J.E. Bolander JE. Explicit representation of physical processes in concrete fracture. *Journal of Physics D: Applied Physics* 2009; 42:214002.

[2] A.E. Huespe, J. Oliver J. Crack Models with Embedded Discontinuities. In: G. Hofstetter, G. Meschke G (eds) Numerical Modeling of Concrete Cracking. pp 161-219, CISM, Udine 2011.

[3] T. Saksala. Numerical modelling of concrete fracture processes under dynamic loading: meso-mechanical approach based on embedded discontinuity finite elements. *Engineering Fracture Mechanics* 2018; 201: 282-297.

[4] J.G.M van Mier. *Fracture processes of concrete: assessment of material parameters for fracture models*, Boca Raton, CRC Press, 1996.