Error Analysis of NOMA-Based User Cooperation with SWIPT

(Invited Paper)

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***Abstract*—The present contribution analyzes the performance of non-orthogonal multiple access (NOMA)-based user cooperation with simultaneous wireless information and power transfer (SWIPT). In particular, we consider a two-user NOMA-based cooperative SWIPT scenario, in which the near user acts as a SWIPT-enabled relay that assists the farthest user. In this context, we derive analytic expressions for the pairwise error probability (PEP) of both users assuming the both amplify-and-forward (AF) and decode-and-forward (DF) relay protocols. The derived expressions are expressed in closed-form and have a tractable algebraic representation which renders them convenient to handle both analytically and numerically. In addition to this, we derive a simple asymptotic closed-form expression for the PEP in the high signal-to-noise ratio (SNR) regime which provide useful insights on the impact of the involved parameters on the overall system performance. Capitalizing on this, we subsequently quantify the maximum achievable diversity order of both users. It is shown that numerical and simulation results corroborate the derived analytic expressions. Furthermore, the offered results provide interesting insights into the error rate performance of each user, which are expected to be useful in future designs and deployments of NOMA based SWIPT systems.**

I. Introduction

The explosive growth of connected devices poses challenging requirements for the fifth generation (5G) wireless networks and beyond. These requirements include, among others, high spectral efficiency, low latency and massive connectivity. In this context, non-orthogonal multiple access (NOMA) was recently proposed as a promising solution that is capable of addressing some of these challenges. As a result, NOMA is envisioned to be one of the key enabling technologies for future generations of mobile communications, enabling increased throughput and connectivity of devices [1].

It is recalled that NOMA can be realized according to different strategies. A popular method is by allocating different power levels to different users according to their encountered channel conditions [2]. This power allocation scheme assists users that experience unfavored fading conditions such as severe multiath fading and shadowing. In addition, it enables the efficient implementation of successive interference cancellation (SIC), which aims at removing multi-user interference in multi-user detection systems. In this case, SIC is performed by the users with the best channel conditions and it is carried out in descending order of the encountered channel [3].

Recently, RF energy harvesting and simultaneous wireless information and power transfer (SWIPT) have been proposed as promising solutions that are capable of providing perpetual energy replenishment for low power energy-constrained devices [4]. In addition, SWIPT has been shown to provide gains in terms of power and spectral efficiency by enabling simultaneous information processing and wireless power transfer [5]. These advantageous characteristics are of paramount important in various emerging wireless technologies and particularly in applications relating to the Internet of Things (IoT), which are largely characterized by high quality of service requirements and stringent energy efficiency levels.

In the same context, user cooperation was proposed as an effective paradigm that exhibits several key advantages as compared to point-to-point systems. Such advantages are typically concerned with lower power consumption, increased throughput, better coverage, and improved diversity performance [6]. Assuming constant energy supplies at user terminals, user cooperation was thoroughly investigated in the literature (see e.g., [7] and the references therein). Motivated by this, the present contribution is concerned with the attempt to effectively integrate NOMA, user cooperation and SWIPT in terms of error rate performance, which in turn is sought to improve spectral efficiency, energy efficiency, and reliability.

A. Related Work

Recent contributions have addressed aspects of NOMA based cooperative communication systems. Specifically, Yang et al. investigated the outage probability performance of a cooperative NOMA configuration with relay selection [8]. Likewise, the corresponding error rate performance was investigated for a point-to-point NOMA system over Nakagami-m fading channels in [9]. Recently, there have been some results on cooperative NOMA with SWIPT [7], [10], while user cooperation in a conventional wireless powered communication network was investigated in [7]. In this contribution, the authors analyzed the weighted sum-rate of a two user scenario by jointly optimizing the time and power allocation of both wireless energy transfer and wireless information transmission. In [10], the authors investigated the sum rate performance of a cooperative-NOMA scheme with full-duplex relaying and energy harvesting. In addition, the impact of self-interference was further analyzed.

However, although there have been considerable research efforts on the performance of cooperative NOMA, only a few contributions have been reported on the error rate analysis of NOMA-based systems [9], which do not include investigations in the context of user-cooperation with energy harvesting. Motivated by this, the present contribution provides interesting insights of theoretical and technical usefulness on the performance of NOMA-based user cooperation with SWIPT.

B. Contributions

In this paper, we analyze the error performance for NOMA-based user cooperation with SWIPT over Rayleigh fading channels. Specifically, the main contributions of this paper are summarized as follows:

- We derive closed-form expressions for the pairwise error probability (PEP) of the near and farthest users over Rayleigh fading channels. The derived expressions of the farthest user assume the AF and DF relaying protocols.
- To quantify the diversity order of both users, accurate asymptotic closed-form PEP expressions are deduced.
- Simulation results are provided to verify the derived expressions in terms of both error performance and maximum diversity order.

To the best of the authors' knowledge, the offered results have not been previously reported in the open literature.

II. System Model

A downlink NOMA system is considered consisting of one base station (BS) and two users, each of which is equipped with single antenna, as depicted in Fig. 1. Without loss of generality, the BS selects two users with distinctive channel gains to perform NOMA. The near user acts as relay with RF energy harvesting capability to forward the information from the BS to the far user. In more details, the near user receiver harvests energy from the received RF signal and utilizes the harvested energy to forward the signal to the far user. It is worth mentioning that the direct link between the BS and the far user is ignored. Based on this and in order to realize the implementation of SIC, the far user is allocated a higher power coefficient. Henceforth, the near user is referred to as user 1 and the far user as user 2, as shown in Fig. 1.

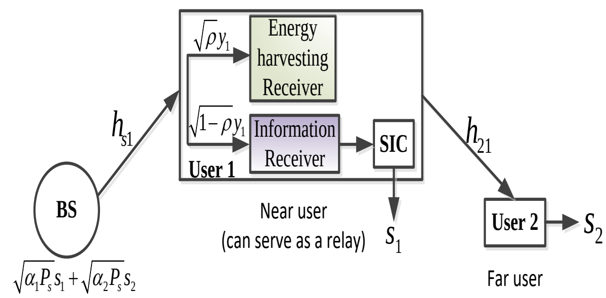

Fig. 1: The illustration of two user NOMA system with SWIPT.

Based on the above, the BS broadcasts the superimposed signal

$$x = \sqrt{\alpha_1 P_s} s_1 + \sqrt{\alpha_2 P_s} s_2, \tag{1}$$

where s_1 and s_2 denote the data symbols of the near and far users, respectively. Moreover, α_1 and α_2 denote the corresponding power allocation coefficients of the near user and far user, respectively, where $\alpha_2 > \alpha_1$ and $\alpha_2 + \alpha_1 = 1$, while P_s represents the total power transmitted by the BS.
For the considered system, the near user not only serves as a relay, but also decodes its own signal. To this end, it receives the signal from the BS, which can be expressed as

$$y_1 = h_{s_1} x + n_1 \tag{2}$$

where $h_{s1} \sim \mathcal{CN}(0, \lambda_1)$ represents the channel between the BS and the near user. Here $\mathcal{CN}(0, \lambda_1)$ denotes a complex Gaussian distribution with zero mean and variance λ_1. Moreover, $n_1 \sim \mathcal{CN}(0, \sigma_n^2)$ indicates the additive Gaussian white noise (AWGN) of the near user.

Recalling that the near user exploits the power splitting (PS) energy harvesting protocol with splitting factor ρ, the received signal y_1 is split into two streams, namely,

$$y_1^E = \sqrt{\rho}(h_{s_1} x + n_1) \tag{3}$$

and

$$y_1^I = \sqrt{1-\rho}(h_{s_1} x + n_1) + n_c. \tag{4}$$

The first stream, y_1^E, is used to harvest energy which is then used to forward the signal to the other user. On the contrary, the second stream, y_1^I, will be forwarded to the information decoder for signal detection. Here, $\rho \in (0, 1)$ is a PS factor and $n_c \sim \mathcal{CN}(0, \sigma_n^2)$ is the RF to baseband conversion noise. Subsequently, the near user forwards its information signal to the far user using the AF or DF relay protocols. For AF relaying, the received signal at the far user can be written as

$$y_2 = G_r h_{21} y_1^I + n_2 \tag{5}$$

which can be equivalently expressed by the following representation

$$y_2 = G_r \sqrt{\rho'} h_{21} h_{s_1} x + G_r \sqrt{\rho'} h_{21} n_1 + G_r h_{21} n_c + n_2, \tag{6}$$

where $\rho' = 1 - \rho$, whereas $h_{21} \sim \mathcal{CN}(0, \lambda_2)$ represents the channel coefficient between the near and far users. Without loss of generality, it is assumed that $\lambda_1 = \lambda_2 = \lambda$. It is also worth noting that h_{s_1} and h_{21} are independent and identically distributed, whereas $n_2 \sim \mathcal{CN}(0, \sigma_n^2)$ is the AWGN at the far user. Considering average power scaling scheme [11], the amplification factor G_r of the near user is represented as follows

$$G_r = \sqrt{\frac{P_1}{\rho' \lambda P_s + (2-\rho)\sigma_n^2}}, \tag{7}$$

where

$$P_1 = \eta \rho \lambda P_s \tag{8}$$

denotes the harvested energy by the near user, whereas $\eta \sim (0, 1]$ represents the corresponding energy conversion efficiency.

III. Pairwise error probability analysis

A. PEP analysis for the near user

Before detecting its own signal, the near user should perform interference cancellation for the far user's signal. Considering the realistic assumption of imperfect SIC, the output signal at the SIC receiver can be then expressed as follows:

$$\tilde{y}_1^I = \sqrt{\rho'}\sqrt{\alpha_1 P_s} h_{s_1} s_1 + \sqrt{\rho'}\sqrt{\alpha_2 P_s} h_{s_1} \Delta_2 + \tilde{n}_1, \tag{9}$$

where

$$\tilde{n}_1 = \sqrt{\rho'} n_1 + n_c \tag{10}$$

and $\Delta_2 = s_2 - \hat{s}_2$. Hence, the conditional PEP of the near user can be expressed as

$$\Pr(s_1 \to \hat{s}_1 | h_{s_1}) = \Pr\left(\left| \tilde{y}_1^I - G_1 h_{s_1} \hat{s}_1 \right|^2 \leq \left| \tilde{y}_1^I - G_1 h_{s_1} s_1 \right|^2 \right) \tag{11}$$

where

$$G_1 = \sqrt{\alpha_1 P_s \rho'}. \tag{12}$$

Based on this and after some mathematical simplifications (11) can be re-written as

$$\Pr(s_1 \to \hat{s}_1 | h_{s_1}) = \Pr\left(2\mathfrak{Re}\left\{ h_{s_1} \hat{\Delta}_1 \tilde{n}_1^* \right\} \leq - \left| h_{s_1} \right|^2 \delta_1 \right) \tag{13}$$

where $\hat{\Delta}_1 = s_1 - \hat{s}_1, s_1 \neq \hat{s}_1$ and

$$\delta_1 = G_1 |\hat{\Delta}_1|^2 + 2\sqrt{\rho' \alpha_2 P_s}\, \mathfrak{Re}\left\{ \hat{\Delta}_1 \Delta_2^* \right\}. \tag{14}$$

Moreover, recalling that (13) is conditioned on h_{s_1}, the decision variable in it follows the normal distribution, namely

$$2\mathfrak{Re}\left\{ h_{s_1} \hat{\Delta}_1 \tilde{n}_1^* \right\} \sim \mathcal{N}\left(0, 2(2-\rho)\sigma_n^2 |h_{s_1}|^2 |\hat{\Delta}_1|^2 \right). \tag{15}$$

Therefore, the conditional PEP is given by

$$\Pr(s_1 \to \hat{s}_1 | g_s) = \mathrm{Q}\left(\frac{g_s \delta_1}{\varphi_1} \right), \tag{16}$$

where

$$\varphi_1 = \sqrt{2(2-\rho)}\sigma_n |\hat{\Delta}_1| \tag{17}$$

and $g_s = |h_{s_1}|$.

According to order statistics [12], the ordered probability distribution function (PDF) of channel gain g_s of the kth user out of K users is given by

$$f_k(g_s) = A_k f(g_s) [F(g_s)]^{k-1} [1 - F(g_s)]^{K-k}, \tag{18}$$

where $k = 1, \cdots, K$, and

$$A_k = \frac{K!}{(k-1)!(K-k)!}. \tag{19}$$

The channel amplitudes from BS to different users are sorted in ascending order, which means that the aforementioned g_s corresponds to the best channel. In other words, for two users system, the order statistic with $k = 2$ in (18) belongs to the ordered channel of the near user.

Referring to the PDF and CDF of Rayleigh random variables [13], the ordered PDF of the near user can be represented as

$$f_s(g_s) = A_2 \frac{g_s}{\sigma^2} e^{\frac{-g_s^2}{2\sigma^2}} \left(1 - e^{\frac{-g_s^2}{2\sigma^2}} \right). \tag{20}$$

Hence, the unconditional PEP can be obtained by averaging the conditional PEP in (16) over the ordered PDF in (20), i.e.

$$\Pr(s_1 \to \hat{s}_1) = \int_0^\infty f_s(g_s) \mathrm{Q}\left(\frac{g_s \delta_1}{\varphi_1} \right) dg_s. \tag{21}$$

By recalling the standard function identity between $Q-$function and error functions,

$$\mathrm{Q}(x) = \frac{1}{2}\left(1 - \mathrm{erf}\left(\frac{x}{\sqrt{2}} \right) \right) \tag{22}$$

and using [14, Eq. 4.3.4], equation (21) can be solved as

$$\Pr(s_1 \to \hat{s}_1) = \frac{1}{2} - \frac{A_2}{2\sqrt{1 + \frac{\varphi_1^2}{\delta_1^2 \sigma^2}}} + \frac{A_2}{4\sqrt{1 + \frac{2\varphi_1^2}{\delta_1^2 \sigma^2}}}, \tag{23}$$

which has a rather simple algebraic representation.

B. PEP analysis for the far user

1) Amplify and Forward (AF) Protocol: Owing to the adopted power allocation scheme, the far user detects its own signal without performing SIC by treating the other signal as noise. To evaluate the PEP of the far user, we express (6) as

$$y_2 = G_r' \sqrt{\alpha_2 P_s} h_{s_1} h_{21} s_2 + G_r' \sqrt{\alpha_1 P_s} h_{s_1} h_{21} s_1 + \tilde{n}_2, \tag{24}$$

where

$$G_r' = G_r \sqrt{\rho'} \tag{25}$$

and

$$\tilde{n}_2 = G'_r h_{21} n_1 + G_r h_{21} n_c + n_2. \tag{26}$$

Hence, the conditional PEP of the far user can be written as

$$\begin{aligned} &\Pr(s_2 \rightarrow \hat{s}_2 | h_{s_1}, h_{21}) \\ &\quad = \Pr\left(|y_2 - G_2 h_{s_1} h_{21} \hat{s}_2|^2 \leq |y_2 - G_2 h_{s_1} h_{21} s_2|^2 \right), \end{aligned} \tag{27}$$

where

$$G_2 = G'_r \sqrt{\alpha_2 P_s}. \tag{28}$$

Alternatively, we can express the conditional PEP of the far user as

$$\Pr(s_2 \rightarrow \hat{s}_2 | g_s, g_w) = \mathrm{Q}\left(\frac{g_s g_w \delta_2}{\varphi_2} \right), \tag{29}$$

where $g_w = |h_{21}|$, whereas

$$\delta_2 = G_2 |\hat{\Delta}_2|^2 + 2G'_r \sqrt{\alpha_1 P_s} \mathfrak{Re}\left\{ \hat{\Delta}_2 s_1^* \right\} \tag{30}$$

and

$$\varphi_2 = \sqrt{2} \sigma_n |\hat{\Delta}_2| \sqrt{1 + G_c^2 g_w^2}, \tag{31}$$

where

$$G_c = \sqrt{G_r^2 + {G'_r}^2}. \tag{32}$$

By averaging over the respective channel power gains, we can rewrite the PEP of the far user based on (29), namely

$$\Pr(s_2 \rightarrow \hat{s}_2) = \int_0^\infty \int_0^\infty f_s(g_s) Q\left(\frac{g_s g_w \delta_2}{\varphi_2} \right) \mathrm{d}g_s f(g_w) dg_w. \tag{33}$$

Having formulated the PEP of the considered setup, the double integral in (33) can be evaluated using two steps. For the first step, one obtains

$$\mathrm{I}_1 = \frac{1}{2} - \frac{1}{2} \int_0^\infty f_s(g_s) \mathrm{erf}\left(\frac{g_s g_w \delta_2}{\sqrt{2} \varphi_2} \right) dg_s \tag{34}$$

which upon using [14, Eq. 4.3.4] it can be expressed as

$$\mathrm{I}_1 = \frac{1}{2} - \frac{A_2}{2\sqrt{1 + \frac{\varphi_2^2}{g_w^2 \delta_2^2 \sigma^2}}} + \frac{A_2}{4\sqrt{1 + \frac{2\varphi_2^2}{g_w^2 \delta_2^2 \sigma^2}}}. \tag{35}$$

In the second step, the PEP derivation of the far user is

$$\Pr(s_2 \rightarrow \hat{s}_2) = \mathrm{I}_2 = \frac{1}{2} - \frac{A_2}{2} \mathrm{I}_{21} + \frac{A_2}{4} \mathrm{I}_{22}, \tag{36}$$

where

$$\mathrm{I}_{21} = \int_0^\infty \frac{1}{\sqrt{1 + \frac{\varphi_2^2}{g_w^2 \delta_2^2 \sigma^2}}} f(g_w) dg_w \tag{37}$$

and

$$\mathrm{I}_{22} = \int_0^\infty \frac{1}{\sqrt{1 + \frac{2\varphi_2^2}{g_w^2 \delta_2^2 \sigma^2}}} f(g_w) dg_w \tag{38}$$

with

$$f(g_w) = \frac{g_w}{\sigma^2} e^{\frac{-g_w^2}{2\sigma^2}}. \tag{39}$$

To this effect and after some algebraic manipulations along with using [15, Eqs. 3.366.2 & 3.364.3], the first integral in (37) can be evaluated as

$$\mathrm{I}_{21} = \int_0^\infty \frac{g_w^2}{\sigma^2 u_a \sqrt{g_w^2 + (1 - \frac{1}{u_a^2}) \frac{1}{G_c^2}}} e^{\frac{-g_w^2}{2\sigma^2}} dg_w \tag{40}$$

which can be expressed in closed-form as follows:

$$\mathrm{I}_{21} = \frac{1}{u_a} \xi_a e^{\xi_a} (K_1(\xi_a) - K_0(\xi_a)), \tag{41}$$

where

$$u_a = \sqrt{1 + \frac{2\sigma_n^2 |\hat{\Delta}_2|^2 G_c^2}{\delta_2^2 \sigma^2}} \tag{42}$$

and

$$\xi_a = \frac{1}{4\sigma^2 G_c^2} \left(1 - \frac{1}{u_a^2} \right), \tag{43}$$

with $K_v(\cdot)$ signifying the modified Bessel functions of the second kind with order v. Similarly, the second integral in (38) can be represented as

$$\mathrm{I}_{22} = \frac{1}{u_b} \xi_b e^{\xi_b} (K_1(\xi_b) - K_0(\xi_b)), \tag{44}$$

where

$$u_b = \sqrt{1 + \frac{4\sigma_n^2 |\hat{\Delta}_2|^2 G_c^2}{\delta_2^2 \sigma^2}} \tag{45}$$

and

$$\xi_b = \frac{1}{4\sigma^2 G_c^2} \left(1 - \frac{1}{u_b^2} \right). \tag{46}$$

Therefore, the exact closed-form PEP expression of the far user can be expressed as follows:

$$\begin{aligned} \Pr(s_2 \rightarrow \hat{s}_2) = &\frac{1}{2} - \frac{A_2}{2u_a} \xi_a e^{\xi_a} (K_1(\xi_a) - K_0(\xi_a)) \\ &+ \frac{A_2}{4u_b} \xi_b e^{\xi_b} (K_1(\xi_b) - K_0(\xi_b)) \end{aligned} \tag{47}$$

which is again given in closed-form expression in terms of the modified Bessel function of the second kind.

2) Decode and Forward (DF) Protocol: Recalling that the near user initially detects the far user's signal in order to perform SIC, the near user forwards the decoded signal of the far user only, instead of the superimposed symbol. Assuming perfect decoding, then the received signal at the far user can be expressed as

$$y_2 = \sqrt{P_1} h_{21} s_2 + n_w \tag{48}$$

which can be re-written as

$$y_2 = \sqrt{\eta \rho P_s} \, |h_{s_1}| \, h_{21} s_2 + n_w. \tag{49}$$

To this effect and following similar steps as in Subsection III-B1, the conditional PEP of the far user for DF protocol is represented as

$$\Pr(s_2 \rightarrow \hat{s}_2 | g_w) = \int_0^\infty f_s(g_s) \mathrm{Q}\left(\frac{g_s g_w \delta_{2d}}{\varphi_{2d}} \right) dg_s \tag{50}$$

which can be expressed in closed-form as

$$\begin{aligned}\Pr(s_2 \to \hat{s}_2|g_w) = \frac{1}{2} - \frac{A_2}{2}\frac{g_w\delta_{2d}\sigma}{\sqrt{g_w^2\delta_{2d}^2\sigma^2+\varphi_{2d}^2}} \\ + \frac{A_2}{4}\frac{g_w\delta_{2d}\sigma}{\sqrt{g_w^2\delta_{2d}^2\sigma^2+2\varphi_{2d}^2}},\end{aligned} \tag{51}$$

where

$$\delta_{2d} = \sqrt{\eta\rho P_s}|\hat{\Delta}_2|. \tag{52}$$

and $\varphi_{2d} = \sqrt{2}\sigma_n$.

Finally, the closed-form PEP expression can be obtained by averaging $\Pr(s_2 \to \hat{s}_2|g_w)$ over the PDF of g_w, namely

$$\Pr(s_2 \to \hat{s}_2) = \int_0^\infty \Pr(s_2 \to \hat{s}_2|g_w)f(g_w)dg_w \tag{53}$$

which yields

$$\begin{aligned}\Pr(s_2 \to \hat{s}_2) = \frac{1}{2} - \frac{A_2}{2}\tau_a e^{\tau_a}(K_1(\tau_a)-K_0(\tau_a)) \\ + \frac{A_2}{4}\tau_b e^{\tau_b}(K_1(\tau_b)-K_0(\tau_b))\end{aligned} \tag{54}$$

where

$$\tau_a = \frac{\varphi_{2d}^2}{4\delta_{2d}^2\sigma^4} \tag{55}$$

and

$$\tau_b = \frac{\varphi_{2d}^2}{2\delta_{2d}^2\sigma^4}. \tag{56}$$

IV. Asymptotic Pairwise Error probability

Diversity order is commonly used to quantify the asymptotic performance of wireless systems, which is defined as the slope of the PEP in log-scale in the high SNR regime, namely

$$d = \lim_{\bar{\gamma}\to\infty} -\frac{\log \Pr(s\to\hat{s})}{\log\bar{\gamma}}, \tag{57}$$

where $\bar{\gamma} = 1/\sigma_n^2$ indicates the average SNR. In this section, we investigate the achievable diversity order of the near and far users. To this end, we first derive the asymptotic PEP. Therefore, considering Chernoff bound, the conditional asymptotic PEP of the near user is

$$\Pr(s_1 \to \hat{s}_1 \mid \gamma_1) \le \exp\left(-\frac{\gamma_1\delta_1^2}{4(2-\rho)|\hat{\Delta}_1|^2}\right), \tag{58}$$

where $\gamma_1 = g_s^2/\sigma_n^2 = \bar{\gamma}\,\Omega_s$ is the instantaneous SNR. Moreover, $\Omega_s = |h_{s_1}|^2$ follows the exponential distribution with the PDF and CDF represented as

$$f(\Omega_s) = \exp(-\Omega_s) \tag{59}$$

and

$$F(\Omega_s) = 1-\exp(-\Omega_s) \tag{60}$$

respectively. To this effect, the ordered PDF of Ω_s can be represented by referring to (18), as

$$f_s(\Omega_s) = A_2\left[\exp\left(-\frac{\gamma_1}{\bar{\gamma}}\right) - \exp\left(-\frac{2\gamma_1}{\bar{\gamma}}\right)\right]. \tag{61}$$

To this effect, the asymptotic PEP of the near user can be written as

$$\Pr(s_1 \to \hat{s}_1) \le \int_0^\infty f_s(\Omega_s)\exp\left(-\frac{\Omega_s\bar{\gamma}\delta_1^2}{4(2-\rho)|\hat{\Delta}_1|^2}\right)d\Omega_s \tag{62}$$

which after the necessary change of variables it can be expressed as

$$\Pr(s_1 \to \hat{s}_1) = \int_0^\infty \frac{1}{\bar{\gamma}}f_s\left(\frac{\gamma_1}{\bar{\gamma}}\right)\exp(-\gamma_1 a_1)d\gamma_1, \tag{63}$$

where

$$a_1 = \frac{\delta_1^2}{4(2-\rho)|\hat{\Delta}_1|^2}. \tag{64}$$

Based on this, the involved integral in (63) can be readily expressed in closed-form, yielding

$$\Pr(s_1 \to \hat{s}_1) \le \frac{A_2}{(1+a_1\bar{\gamma})(2+a_1\bar{\gamma})}, \tag{65}$$

which as $\bar{\gamma} \to \infty$, the asymptotic PEP in (65) converges satisfactorily, namely

$$\lim_{\bar{\gamma}\to\infty}\log\left(\frac{A_2}{(1+a_1\bar{\gamma})(2+a_1\bar{\gamma})}\right) \approx \log\bar{\gamma}^{-2}. \tag{66}$$

Based on this, it follows that the diversity order can be approximated as

$$d \approx -\frac{\log\bar{\gamma}^{-2}}{\log\bar{\gamma}} = 2. \tag{67}$$

Following the same approach and considering the DF protocol, the conditional asymptotic PEP of the far user is expressed as follows:

$$\Pr(s_2 \to \hat{s}_2 \mid \gamma_2) \le \exp\left(-\frac{\gamma_2\delta_{2d}^2}{4|\hat{\Delta}_2|^2}\right), \tag{68}$$

where

$$\gamma_2 = \frac{\Omega_s\Omega_w}{\sigma_n^2} = \bar{\gamma}\Omega_s\Omega_w \tag{69}$$

is the instantaneous SNR and $\Omega_w = |h_{21}|^2$, with PDF $f(\Omega_w) = \exp(-\Omega_w)$. Given Ω_w, the ordered PDF of $\Omega_s = \frac{\gamma_2}{\Omega_w\bar{\gamma}}$ is evaluated as

$$\begin{aligned}f_s(\Omega_s) &= A_2 f(\Omega_s)\mathrm{F}(\Omega_s) && (70)\\ &= A_2\left[\exp(-\Omega_s) - \exp(-2\Omega_s)\right]. && (71)\end{aligned}$$

Consequently, the conditional CDF of γ_2 can be written as

$$\mathrm{F}(\gamma_2|\Omega_w) = \int_0^{\frac{\gamma_2}{\Omega_w\bar{\gamma}}} f_s(\Omega_s)d\Omega_s \tag{72}$$

which is expressed in closed-form as follows:

$$\mathrm{F}(\gamma_2|\Omega_w) = A_2\left[\frac{1}{2} - \exp\left(-\frac{\gamma_2}{\Omega_w\bar{\gamma}}\right) + \frac{1}{2}\exp\left(-\frac{2\gamma_2}{\Omega_w\bar{\gamma}}\right)\right]. \tag{73}$$

Likewise, the unconditional CDF of γ_2 is given by

$$\mathrm{F}(\gamma_2) = \int_0^\infty \mathrm{F}(\gamma_2|\Omega_w)f(\Omega_w)d\Omega_w \tag{74}$$

which can be expressed in closed-form as

$$\mathrm{F}(\gamma_2) = \frac{A_2}{2} - 2A_2\sqrt{\frac{\gamma_2}{\bar{\gamma}}}K_1\left(2\sqrt{\frac{\gamma_2}{\bar{\gamma}}}\right) + A_2\sqrt{\frac{2\gamma_2}{\bar{\gamma}}}K_1\left(2\sqrt{\frac{2\gamma_2}{\bar{\gamma}}}\right). \tag{75}$$

To obtain the PDF, we determine the first derivative of (75) with respect to γ_2, yielding

$$f(\gamma_2) = \frac{2A_2}{\bar{\gamma}}\left[K_0\left(2\sqrt{\frac{\gamma_2}{\bar{\gamma}}}\right) - K_0\left(2\sqrt{\frac{2\gamma_2}{\bar{\gamma}}}\right)\right]. \tag{76}$$

Therefore, the unconditional asymptotic PEP of the far user is expressed as

$$\Pr(s_2 \to \hat{s}_2) \leq \int_0^\infty f(\gamma_2)\exp\left(-a_2\gamma_2\right)d\gamma_2, \tag{77}$$

where

$$a_2 = \frac{\delta_{2d}^2}{4|\hat{\Delta}_2|^2}, \tag{78}$$

which, using [15, Eq. 6.614.4], can be further simplified as

$$\Pr(s_2 \to \hat{s}_2) \leq \frac{A_2}{\sqrt{\bar{\gamma}a_2}}\exp\left(\frac{1}{2\bar{\gamma}a_2}\right)W_{-\frac{1}{2},0}\left(\frac{1}{\bar{\gamma}a_2}\right) - \frac{A_2}{\sqrt{2\bar{\gamma}a_2}}\exp\left(\frac{1}{\bar{\gamma}a_2}\right)W_{-\frac{1}{2},0}\left(\frac{2}{\bar{\gamma}a_2}\right), \tag{79}$$

where $W_{a,b}(.)$ is the Whittaker function. The achievable diversity order of the far user is evaluated numerically in Section V. It is worth highlighting here that in the case of the DF protocol, the achievable diversity order of the far user is unity.

V. Numerical Results

In this section, both Monte Carlo simulation and numerical results are provided to validate the analysis and investigate the error performance of NOMA-based user cooperation with SWIPT. To this end, we consider a two user scenario with binary phase shift keying (BPSK) modulation. The power allocation coefficients of the two NOMA users are selected to be $\alpha_1 = 0.28$ and $\alpha_2 = 0.72$, the energy conversion efficiency is $\eta = 0.8$, and the average $\mathrm{SNR} = P_s/\sigma_n^2$ and $P_s = 1$. It is also noted that the simulation results are shown with lines, whereas markers are used to illustrate the respective numerical results, which are based on (23) for near user and on (47) for the far user. It is observed that all results demonstrate a perfect match between the simulated and the derived analytic results.

Fig. 2 demonstrates the union bound performance of the near and far users under different power splitting factors, i.e, $\rho = 0.3$ and $\rho = 0.6$. For the near user, increasing ρ from 0.3 to 0.6 implies the power reduction of information receiver. Hence, the union bound with $\rho = 0.3$ is superior to $\rho = 0.6$, for the near user. On the contrary, a noticeable improvement in the far user performance is observed as ρ increases.

It is illustrated in Fig. 3 that at SNR = 25dB and assuming the DF protocol, the performance of the far user is affected not only by the harvested energy but also by the power

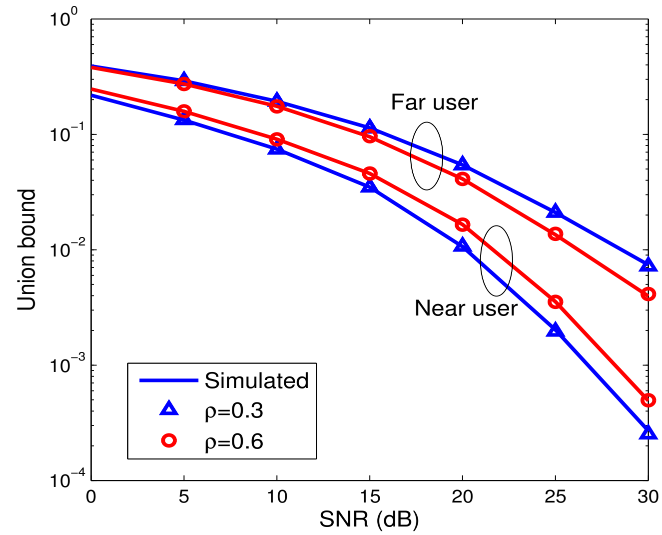

Fig. 2: Union bound vs. SNR for the near and far users with different ρ.

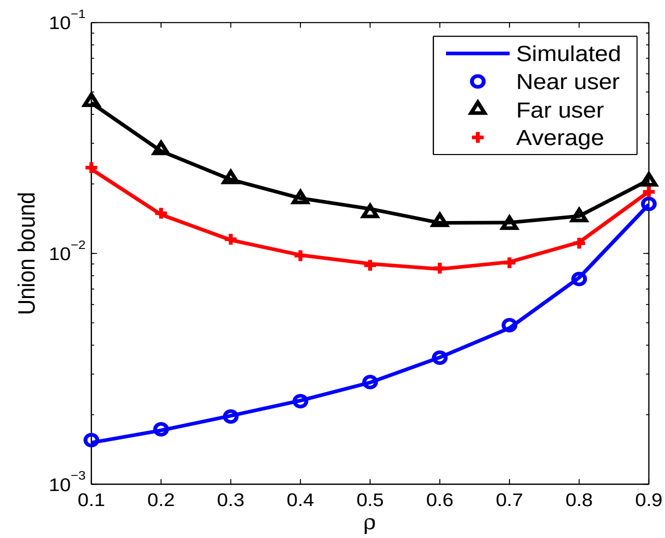

Fig. 3: Average and individual Union bounds vs. power splitting ratio (ρ) for two users.

of (4). It is clear from Fig. 3 that the choice of ρ is an essential requirement for better performance. In particular, for high ρ values, both users experience degradation in the error rate performance, which is primarily due to the low power allocated for the near user's decoding receiver. On the other hand, small ρ values lead to an enhancement in the near user performance only. Clearly, ρ = 0.6 yields the best average rate performance while guaranteeing users' fairness.

For the far user, we compare its error performance with AF and DF schemes given two ρ values as depicted in Fig. 4, where analytical curves are generated by (47) and (54). Under the assumption of perfect decoding, DF achieves

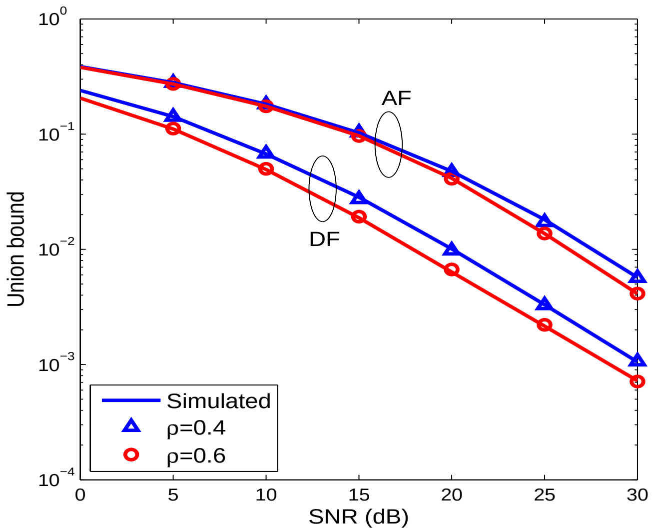

Fig. 4: Union bound of the far user with AF and DF relaying protocols, for different ρ values.

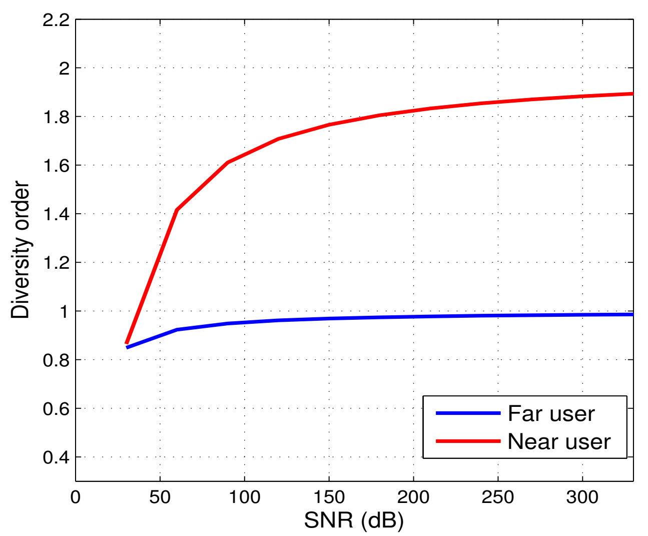

Fig. 5: Achievable diversity order for the near and far users.

superior performance over AF. However, the slopes of two curves remain approximately identical, which implies that the diversity order is preserved.

For the near and far users, their numerical diversity order curves are plotted according to (65) and (79), respectively. Finally, as SNR approaches infinity, the diversity order of the near user approaches two, while the asymptotic diversity order of the far user is unity.

VI. Conclusion

We investigated the performance of NOMA-based user-assisted transmission with SWIPT. For the underlying scenario, closed-form expressions of the PEPs, assuming the AF and DF protocols, were derived. Our asymptotic analysis revealed that the maximum achievable diversity orders are one and two for the far and near users, respectively. Simulation and numerical results demonstrated that the optimization of the power splitting factor is crucial for the effective realization and robust operation of self-sustainable NOMA-based cooperative systems.

VII. Acknowledgment

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