Extending a Digital Fraction Game Piece by Piece with Physical Manipulatives

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Abstract. This paper reports results from an ongoing project that aims to develop a digital game for introducing fractions to young children. In the current study, third-graders played the Number Trace Fractions prototype in which they estimated fraction locations and compared fraction magnitudes on a number line. The intervention consisted of five 30 minutes playing sessions. Conceptual fraction knowledge was assessed with a paper based pre- and posttest. Additionally, after the intervention students' fraction comparison strategies were explored with game-based comparison tasks including self-explanation prompts. The results support previous findings indicating that game-based interventions emphasizing fraction magnitudes improve students' performance in conceptual fraction tasks. Nevertheless, the results revealed that in spite of clear improvement many students tended to use false fraction magnitude comparison strategies after the intervention. It seems that the game mechanics and the feedback that the game provided did not support conceptual change processes of students with low prior knowledge well enough and common fraction misconceptions still existed. Based on these findings we further developed the game and extended it with physical manipulatives. The aim of this extension is to help students to overcome misconceptions about fraction magnitude by physically interacting with manipulatives.

Keywords: Game-Based Learning, Fraction, Number Line, Conceptual Change, Manipulatives, Serious games, Mathematics

1 Introduction

In recent years, mathematics has been the most researched discipline in the field of game-based learning in primary education [1]. Moreover, a recent meta-analysis [2] indicated that using game-based learning in mathematics is, in general, significantly more effective than other instructional methods. Game-based math trainings founded on number line based mechanics have been particularly successful in improving rational number magnitude understanding [e.g. 3,4,5].

While these studies paint an overall positive picture on game-based interventions to also foster fraction magnitude understanding, students generally struggle with fractions. Importantly, fraction magnitude understanding was found to be a relevant predictor for later knowledge of rational numbers, such as the density of rational numbers [6] and arithmetic operations with rational numbers [7]. Children's problems in understanding fraction magnitude [for a review, see 8] often originates from the tendency to treat denominators and numerators of a fraction as two separate whole numbers instead of considering their relation to each other [e.g., 9] - often referred to as whole number bias [10] or natural number bias [11]. Consequently, children often infer that the value or numerical magnitude, respectively, of a fraction increases when either the numerator or the denominator increases. For instance, 2/3 (0.667) is larger than 3/8 (0.375) although its numerator 2 is smaller than 3 and the denominator 3 is smaller than 8. According to conceptual change theories, these misconceptions originates from the fact that children form an initial conception of numbers as counting units before they encounter fractions, and later they draw incorrectly on this initial understanding to make sense of fractions and rational numbers [9,12]. Therefore, conceptual change seems to be required in the acquisition of the concept of a fraction, because radical changes in the pre-existing concept of number magnitude is needed [12].

1.1 Conceptual Change

Conceptual change refers to situations in which a new information to be learned conflicts with learner's prior knowledge and thus changes in the prior knowledge are needed. Dole and Sinatra [13] have argued that radical changes in student's thinking are usually difficult to attain. The creation of cognitive conflicts has been a dominating instructional strategy in supporting conceptual change [14]. Cognitive conflict is a term used to describe a situation in which a new information makes a learner dissatisfied with his or her existing conception of a phenomenon. The perception of this contradiction between prior knowledge and new information may lead to radical changes in learner's thinking. In order to explain why cognitive conflict does not always support conceptual change, Merenluoto and Lehtinen [15] proposed a theoretical model of the dynamics of motivational, cognitive, and metacognitive processes in conceptual change. The model distinguishes three possible learning paths: the experience of conflict, the illusion of understanding, and having no relevant perception.

In the experience of conflict path, the conflict may lead to radical conceptual change. The experience of conflict reduces learner's certainty about the phenomenon and thus the learner is ready to change his or her knowledge beliefs. If learner's tolerance of ambiguity is high, the learner may feel that the conflict is solvable. However, if the tolerance of ambiguity is low, sensitivity to perceive novel features of the tasks may decrease or the situation leads to a loss of trust, resulting in avoidance behavior.

In the illusion of understanding path, the learner does not notice the conflict because of overconfidence. The learner recognizes some familiar elements in the new phenomenon, but his prior knowledge is not adequate for paying attention to the novel aspects. Familiar elements of the phenomenon arouse an illusion of understanding, which leads to an enrichment of existing naïve models or the construction of synthetic models.

In the no relevant perception path, the learner misses the conflict because of his or her broad cognitive distance to the phenomenon to be learned. Cognitive overload usually confuses the learner and may lead to avoidance behavior or routine activity that does not involve processing of the aspects of the new phenomenon. These learners can be supported by providing them with information that is needed to understand the phenomenon and consequently increase the probability to perceive a cognitive conflict. One aim of this paper is to consider how physical manipulatives could be integrated to a digital game to trigger reflective processes leading to conceptual change. Reflection refers to an activity in which a learner recaptures his/her experience, think about it, mull it over, and evaluate it. The outcome of reflection in game-based learning may be personal synthesis of knowledge, validation of a hypothesis laid during the formation of a playing strategy or a new strategy to be tested.

1.2 Present Study

The present pilot study builds on previous research that has proposed new game mechanics for learning fractions [16]. We report results from an ongoing project that aims to develop a digital game for introducing fractions to young children. We present the results of a pilot study in which we studied the effectiveness of the Number Trace: Fractions (NT Fractions) game among students who are new to fractions. Moreover, with self-explanation prompts we explored students' fraction comparison strategies to identify possible misconceptions. Based on the results we present an extension to the game in which physical manipulatives are integrated to the game. Physical manipulatives refer to concrete objects (e.g. base-ten blocks, colored counters, patterning materials, and different sized elements) that aim to help learners to better understand abstract mathematical concepts or properties by allowing them to manipulate the objects. The aim of the physical manipulatives is, in case of existing misconceptions of fraction magnitude, to create more explicit cognitive conflicts in students and help them to solve these conflicts by interacting with manipulatives.

2 The NT Fractions Prototype

The aim of the NT Fractions game is to introduce fractions to young children with limited previous knowledge of fractions. The core gameplay is based on number line estimation and magnitude comparison mechanics. In the game, players control a dog character and have to trace bones a cat has hidden in a forest (see Fig. 1). The locations of the bones are shown as symbolic or non-symbolic fractions (target number; left book in Fig. 1) reflecting locations on a number line ranging from zero to one. Players are supposed to direct the dog to the correct location on the number line and dig out the hidden bones by pressing the answer button (bone button in Fig. 1). The estimation accuracy determines whether players find a bone and get points or loses health.

Accuracy is visualized also with the size of the found bone and the correct location of the bone is shown on the number line. The number line may also contain hidden obstacles that players have to avoid. The locations of the obstacles are indicated with symbolic or non-symbolic fractions (obstacle number; right book in Fig. 1). Some estimation tasks have monsters wandering around that players have to avoid or destroy. Most of the monsters have mathematical meaning. For example, monsters can move by jumping unit fractions and that way provide hints for the player. The aim of the monsters is to motivate players to observe the game world more mathematically.

The magnitude comparison task is implemented according to a recently proposed mechanic that integrates a comparison task into a number line estimation task to draw users attention to the spatial locations of fractions in a comparison task [17]. This combined task starts with a dialog in which a mole character asks players whether removing the obstacle is required. In case the obstacle is located between the dog and the bone – i.e. the obstacle number 1/6 is smaller than the target number 4/6 (see Fig. 1) - removing the obstacle is necessary. In contrast, removing obstacle is not necessary when obstacle is located beyond the bone. In practice, players have to compare explicitly the two values and answer either “Yes” or “No”. Players get immediate feedback about correctness of their answer. Feedback to a correct answer: a) If the obstacle is on dog’s way to the bone, it is removed and the location of the removed obstacle is shown on the number line. b) If the obstacle is not on dog’s way to the bone, the location of the obstacle is shown on the number line. Feedback of an incorrect answer: Players lose health, the obstacle is removed, and the location of the inactive obstacle is shown on the number line. After the comparison dialog the task continues as a basic estimation task. However, players can utilize the location of an obstacle that is marked on the number line in estimating the location of the bone. Ninaus et al. [16] have argued that the strength of this novel number line based comparison task is the feedback, which is visually linked to the number line. In other words, the feedback allows players to see the relation of the compared magnitudes on the number line.

Fig. 1. An example of the task that consists estimation and comparison task mechanic; left book = Target number; right book = obstacle number.

When players progress in the game they earn special skills that are designed to support the development of conceptual fraction knowledge and to help them to perform well in the game. Players can activate these skills using in-game currency (diamonds). In each level, players can use only a limited number of diamonds. The skills influence either the mathematical or general gameplay challenges. Mathematical skills can be used to reveal the approximate location of the bone, divide the number line into convenient sections, transform a fraction number into a pie chart, reduce the fraction to lowest terms (e.g. 4/8 -> 1/2), see player's latest answer on the same task, and limit dog's movements only to unit fraction jumps (1/b of a fraction a/b). Gameplay skills make the tasks easier by removing obstacles, monsters or time pressure. Watch a video for more complete description of the features at https://youtu.be/xrL2kOR4-yU

3 Method

3.1 Participants

Eleven Finnish third graders with no previous knowledge of fractions participated in the study (N = 11; 6 females; mean age = 9.18 years; SD = 0.40 years).

3.2 Measures

Pre- and Posttest

We created two versions of a paper-based conceptual fraction test. Only fractions between 0 and 1 were used. The tests included eight number line estimation items (e.g. Indicate the position of 1/5 on the number line below ranging from 0 to 1), six magnitude comparison items (e.g. "Circle the larger fraction. When the fractions are equal circle both.", e.g., 3/9 vs. 2/3), and three magnitude ordering items (e.g. "Put the numbers in order from smallest to largest": 1/2; 5/8; 2/6). Each item was scored as correct or incorrect. Estimations that were performed at least with 90% accuracy were considered as correct. The test versions were balanced according to difficulty (e.g. distance between compared magnitudes, whole number consistency).

Fraction Comparison with Self-Explanations

The magnitude comparison task of the Semideus game [4] was extended with self-explanation prompts and used to explore students' fraction comparison strategies. In the comparison task, students had to arrange stones with fractions depicted on them in ascending order (left to right). In the case of equivalent fractions, students were instructed to pile up the stones. The items in the fraction comparison task were designed in a way that identification of students strategies that are incorrectly based on whole number properties is possible. Students played through one level that included four training items and eight test items with self-explanation prompts. After each comparison the game prompted a recording dialog and the students were asked to explain their solution. The spoken explanations were converted to text by Speech API of IOS and the texts were recorded to a server. In order to make the explanation of comparisons easier, the colors of the fractions to be compared were different. With self-explanations we aimed to explore what kind of misconceptions students have about fraction magnitudes and what kind of strategies they use to compare magnitudes.

3.3 Procedure

First, two lessons about fractions were given to the students by their own teacher. Second, the students completed the paper based pretest. Third, students played the Number Trace game for five times 30 minutes during a two-week period. During the playing phase students did not get any other teaching about fractions. Fourth, students completed the posttest. Finally, students completed the Semideus based fraction comparison level along with the self-explanations.

4 Results of the Pilot Study

4.1 Learning Gains

During the intervention students played on average *(SD)* 276 (47) estimation tasks and 62 (16) comparison tasks. On average *(SD)* students used special skills 109 (19) times. From these skills 26% were math based skills and the rest were gameplay skills. The fact that gameplay skills were cheaper and more often available than math based skills might partly explain the difference. Fig. 2 shows students' mean performance in the pre- and posttest according to task types. Students' performance seem to have improved from pre- to posttest in each task type. The overall mean *(SD)* accuracy also increased from 42.25% (20.81) to 64.17%. The student who used math based skills most, also improved most from pretest (24%) to posttest (82%). Due to the small sample we decided to not follow up our descriptive analysis with inferential statistical tests.

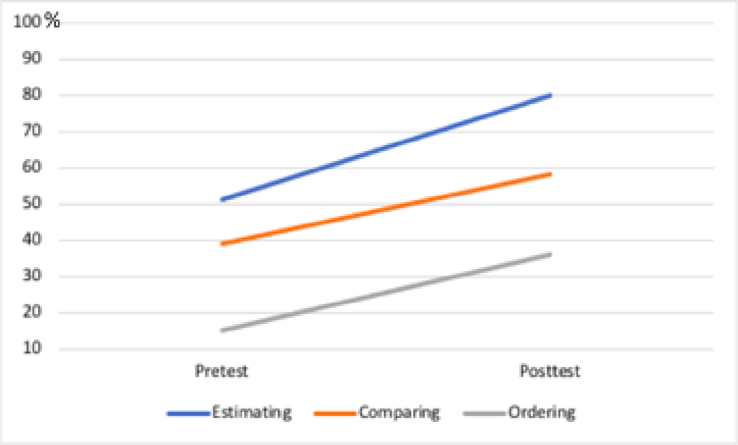

Fig. 2. Mean percentage scores in the pre- and posttest according to task types

4.2 Results from the Self-Explanation Phase

Self-Explanations

Students' explanations of their fraction comparison solutions showed that students relied on several faulty strategies or misconceptions, respectively. One of the most common comparison strategies was the missing pieces strategy, while 6 out of 11 students tended to use it. For example, one student explained that 1/9 is smaller than 1/6 based on the following reasoning. "There needs to be eight so that it makes a whole and the other needs only five to make a whole.". In this case, the missing pieces strategy worked, but in some cases it did not work. For example, one student failed to compare 3/9 and 1/3 while he or she reasoned that "It requires two to make three and other requires more." When solving the same tasks another student explained that "The red one is bigger [3/9] while it has bigger numbers than the blue one [1/3]". This strategy originates from clear whole number bias as the student do not understand fraction as a ratio between numerator and denominator, but considers numbers componentially as whole numbers. Popularity of the missing pieces strategy may have resulted from the fraction introduction that the students got in the beginning in which only part-whole interpretation was emphasized.

Two out of 11 students used a strategy that was based on pieces, but in an advanced way as they did not calculate the missing pieces, but based on pieces they compared fraction magnitudes to common fractions that they understood. For example, "If one is added to red [2/6] it makes only half, but if one is added to blue [⅚] it makes a whole." In fact, based on our observations, it seems that half was well understood among students and some students could utilize the half when making sense of compared magnitudes. For example, One student explained that "Two parts of three is much over half way and four parts of seven is maybe only little more than half."

We identified some occasions in which students noticed that they had used their strategy only superficially without thinking enough and when they explained their solution they realized that the strategy that they used did not eventually work. That is, self-explanation might have worked as a cognitive conflict and the explanation led students to think critically about their conceptions. However, self-explanation based conflicts did not lead to conceptual change in our study, because the game did not provide feedback that would have helped students to understand the fraction concept. For instance, one student who realized that the missing pieces strategy does not always work, did not experience this conflict as solvable, leading to confusion and consequently continued playing by guessing the right order.

5 Extending the NT Fractions Game with Manipulatives

The current results indicate that when young students reason about fraction magnitudes they may rely on faulty strategies as they do not understand the concept of fractions. For example, some of the students used component-based strategies and compared nominators and denominators separately. Moreover, the feedback of our game seems not be enough to cause cognitive conflicts and consequently facilitate conceptual change in students with very low prior knowledge in fractions. One problem of fraction comparison tasks is that some false comparison strategies such as missing pieces strategy may work with some comparison items leading to strengthening the naive conception of fractions and illusion of understanding. Consequently, for example an overconfident player who has an illusion of understanding the subject, may ignore the feedback (miss the conflict) that the game provides, because the same strategy has worked earlier in a different context and thus the player tends to rely on his/her prior experiences. Therefore, to support conceptual change we need to create stronger cognitive conflicts, provide enough information about fractions for students to facilitate perception of the conflicts, and motivate students to solve the conflicts. One way to reach this is to supplement gameplay with other instructional methods. In fact, a recent meta-analyses about game-based learning [2] supports this idea as the results indicated that, players learned more, relative to those taught with conventional instructional methods, when the game was supplemented with other instructional methods. Because a meta-analysis about math manipulatives indicated that manipulatives can be effective particularly in learning fractions [17], we extended the game with physical manipulatives. We modified the game in a way that fraction manipulatives can be used along the game to provide more direct and explicit support for fraction magnitude reasoning. In practice, color coding is used in the panels showing the locations of the bones and obstacles (Fig. 3) and through this color coding the game elements can be linked to fraction manipulatives that use the same color coding.

Fig. 3. Student comparing fractions with manipulatives along with the game providing more explicit and physically graspable support for fraction magnitude reasoning.

Each unit fraction (e.g. 1/2, 1/3,…) has its own color. In our example (Fig. 3), 1/4 is coded with blue color and consequently 2/4, 3/4 and 4/4 are blue as well. Fig. 3 also shows how a student has used paper-based physical manipulatives to decide whether he should remove the obstacle (4/5) or not. Using the physical manipulatives the player can reason that 4/5 (four pink units) is larger than 3/4 (three blue units) and that it is not necessary to remove the obstacle with the mole. Let's assume that the player has earlier used the missing pieces strategy – the most common strategy in the current study. According to missing-pieces reasoning the player assumes that the fractions are equal and he or she should remove the obstacle. However, now when the player uses manipulatives along with the game, he or she notices that the fractions are not equal and it is not necessary to remove the obstacle. At the same time the manipulatives provide visual information that players can utilize to solve the conflict which might help players to understand why the missing pieces strategy cannot be used and how units can be used to reason about magnitudes. Therefore, future versions of the NT Fractions game can be accompanied with a set of physical manipulatives that may support players who need more concrete experiences to make sense of fraction magnitudes. The manipulatives are only addons and the use of manipulatives are not required to play the game. Furthermore, we will also implement virtual manipulatives into the game and study the usefulness and differences of these different manipulative approaches.

6 Conclusion and Future Work

The current study explored the usefulness of the NT Fractions game prototype for introducing fractions to students. In general, results of the pilot study are promising as students' performance in fraction tasks improved. It seems that our new task, combining mechanics of number line estimation and number magnitude comparison tasks, can be used in number line based games. However, we noticed that many students used false fraction magnitude comparison strategies after the intervention. It seems that the used game mechanics and the feedback that the game provided did not support conceptual change processes well enough and some misconceptions remained. Thus, based on the results we further developed the game and extended it with physical manipulatives. The aim of this extension is to trigger students to reflect on their conception of fraction and help them to overcome possible misconceptions of fraction magnitude.

The results also indicated that players utilized game character's special skills quite a lot, but, unfortunately, the use of the gameplay skills were more popular than math based skills designed to support the development of conceptual fraction knowledge. Thus, we will redesign the gameplay skills in a way that they are better integrated to the learning content. Furthermore, we are currently implementing adaptive features to the game. In fact, the game already detects students' misconceptions and the next version of the game will automatically activate math based skills that should support conceptual change processes.

Future studies need to study (i) learning effectiveness of the game with and without manipulatives, (ii) how students use in-game special skills and how the use of the skills influence learning outcomes, (iii) what game elements or interaction patterns lead to conceptual change and in what kind of context, and (iv) compare the effectiveness of in-game digital manipulatives and physical manipulatives used along a game.

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