# Using Games to Understand and Create Randomness 

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#### Abstract

Massive growth of data and communication encryption has created growing need for non-predictable, random data, needed for encryption keys creation. Need for randomness grows (nearly) linearly with growth of encryption, but randomness is very important ingredient also e.g. in quickly growing industry of game programming. Computers are deterministic devices and their basic operations $\wedge$ (and), $\vee$ (or), $\longrightarrow$ (not), which are used to compose all their operations cannot create random results, computer procedures can generate only pseudo-random (looking random) data. For true randomness is needed some outside information - time and placement of user's keystrokes, fluctuations of current, interrupt requests in computer processor etc. But even those sources can often not comply with requests from our increasingly randomness-hunger environment of ciphered communications and data.

Growing need for randomness has created a market of randomness sources; new sources are proposed constantly. These sources differ in their properties (ease of access, size of required software etc.) and most importantly - ease of estimating their quality.

However, there is an easily available good source for comparing quality of randomness and also creating new randomness computer games. The growing affectionateness of users to play digital games makes this activity very attractive for comparing quality of randomness sources and using as a source of new randomness. In the following are analyzed possibilities for investigating and extracting randomness from digital gameplay and demonstrated some experiments with simple stateless games which allow to compare existing sources of (pseudo) randomness and generate new randomness.


Categories and Subject Descriptors: • Information systems~Browsers • Computing methodologies~Computer graphics •Computing methodologies~Graphics file formats

General Terms: Entropy, Randomness, Learning, Games, Algorithms
Additional Key Words and Phrases: learning, JavaScript, gameplay

## 1 INTRODUCTION

All businesses and private persons are increasingly reliant on many formats of digital data, which is essential in our personal lives, economic prosperity and security. But criminals have also understood, that digital data has become the most valuable resource. Data breaches are increasing by more than 20 percent in a year [BSA. Encryption 2018] and they have become the most worrying feature of Internet [Fortinet 2018].

The main method to protect our data and communications is encryption. Use of encryption for data and communications protection is growing rapidly. Cisco, the largest networking company in the world shows that encrypted traffic has increased by more than 90 percent year over year [Cisco 2018]. Currently already at least of half of websites are already encrypting traffic and world's leading research and advisory company Gartner predicts that by 2019 already 80 percent of web traffic will be encrypted [Gartner 2018].

Every encrypted bit in a message is somehow transformed by (at least one) bit of encryption code. For different messages different encryption bits should be used - if some encryption scheme is repeated often it becomes easier to break. Thus the need for randomness grows (nearly) linearly with the amount of encrypted traffic and data.

New computing environments - the coming era of IoT (Internet of Things), virtual/cloud servers) etc. all increase need for randomness, thus there are already proposals for special services even from governments [Chen2018] to serve entropy, i.e. random data [NIST 2016 ]. In order to deliver provided entropy to users is proposed a special new protocol, 'Entropy as a Service Protocol' [EaaSP 2018 ]. But for delivery entropy should be encrypted - this is a new source needing 'fresh' entropy, thus it is not clear, whether this service will reduce the need for entropy or contrary, increase it. Everything is much simpler if the entropy is generated and collected there where it is needed.

In the following is proposed a method for creating entropy/randomness from computer's gameplay and comparing quality of randomness (unpredictability) of already established sources. The inputs for the generated sequence are either human's gameplay or an already established source of randomness,
the output - computer-generated sequence of moves or the sequence of game states, i.e. pairs (arrays in multiplayer mode) of inputs from all players. The main idea is that using the maximally similar (to the current one) situation from the previous gameplay allows computer to improve its play, thus produce better responses (win) and also produce better randomness against which the established sources are compared. Adding the (unpredictable) component of human gameplay can improve randomness of the output. As the established sources of randomness are used the functions random() and window.crypto.getRandomValues () from JavaScript compilers of Firefox or Google Chrome browsers and a downloaded table of random integers from RANDOM.ORG.

In the next section is given overview of currently used methods to create randomness in computers and problems with establishing, whether a sequence of numbers is random (unpredictable). After that is in section 3 described a simple game, which requires for winning producing uniformly distributed random binary sequences. In section 4 is presented a principle of learning from the maximally similar situation in gameplay history and in section 5 described acting on this principle computer algorithm, which makes computer player superior against for (most of) human players. The algorithm can be used also against established sources of randomness, i.e. to evaluate their quality and its output (or the sequence of inputs from all players) can also be used as a new source of randomness - in the section 6 are described results from experiments and tests. In the section 7 is presented a generalization of the previous results employing binary sequences to $m$-ary sequences, $m>2$.

## 2 RANDOMNESS AND INFORMATION

It is impossible to generate random values using computers basic operations - binary operations conjunction $\wedge$ (and), disjunction $\vee$ (or), and unary negation $\neg$ (not) - all combinations of these connectives return single determined value (if not, then the computer is severely broken). All computers are finite deterministic devices and after a time they all go into loop, will repeat already generated values, which definitely are not random. John von Neumann commented on this: "Anyone who attempts to generate random numbers by deterministic means is, of course, living in a state of sin." All algorithms used on programming language's translators for generating random values are actually pseudorandom number generators (PRNG) - big loops, which after their period start repeating output values. One of the best known is the formula (a linear congruential generator) used in popular standard C-language library glibc, which produces in a pseudorandom order a cycle with length 2^31:

$$
X_{i+1}=\left(1103515245^{\star} x_{i}+12345\right) \bmod ^{\wedge} 31
$$

The cycle length of the Mersenne Twister, the default PRNG in many computing environments is even bigger - $2^{19937}-1$. But the size of these loops does not guarantee that generated numbers are unpredictable - the more computers use the same PRNG, the less unpredictable it comes. And sometimes randomness is just faked by a finite list of integers, e.g. in the very popular game Doom randomness was introduced by a list of 256 integers [Doom 1997].

A seemingly good source of randomness is time and user inputs. In first computers, which did not have time functions, games got randomness e.g. counting game frames which were already played before user pressed "Start" or performed some other actions. But this kind of sources are very limited, besides, players tend to perform their actions in similar temp, thus after some plays 'random' becomes 'non-random'. Use in computers of the 'real' time has also several problems. Many computing processes are started by timer, i.e. in pre-defined time, thus all derivatives of 'time' also become predictable. Predictable are also many other seemingly random values extracted from computer software and/or hardware. The 'father of all browsers' Netscape used the time of day, the process and the parent process's ID-s for seeding its PRNG, but the resulting values were shown to be relatively predictable thus the resulting encryption become insecure.

Many programming environments even do not show the most precise value of time available or present it in different formats. In most browsers the JavaScript function

Date.now();
produces integer value, but in Firefox the result is rounded to even value. Another time function

## performance.now();

produces in Firefox even integer, but in Chrome and IE 11 - a real number with 12 decimals.
We use and check randomness with finite devices and when their memory (number of states) grows, some previously hidden regularities appear and 'random' becomes 'non-random'. Many formulae for producing seemingly random values are at their introduction considered 'good enough', but later found to be not 'good enough' as e.g. the RC4 (Rivest Cipher 4) which is/was used in several commonly used encryption protocols and standards, e.g. in the TLS (Transport Layer Security), but was some years ago prohibited; widely known was periodicity in the random function of Microsoft PHP translator.

Randomness is an evasive concept to define. The widely accepted definition is the KolmogorovChaitin definition [Kolmogorov 1965]:
a sequence is random if it can't be expressed by any algorithm or device which can be described using less symbols than what are in the sequence.

This definition and consequent definitions [Martin-Löf 1966], [Schnorr 1971] are theoretical, using infinite sets of concepts ('any algorithm'), thus useless for evaluating quality of a source of randomness. Available randomness tests [NIST 2016], [Marsaglia 2002] require installation of specific software and higher than average computer skills.

## 3 GAMES

New, 'fresh' randomness is also very important for game developers - a rapidly growing area of software development. Humans are not very good sources of randomness, but employing different aspects of human gameplay, e.g. human errors in gameplay allows to create quite good, unpredictable randomness [Alimomeni 2014]. In the following are analyzed possibilities for comparing quality of various sources of randomness and demonstrated experiments with simple static games which allow to generate good randomness.

For creating randomness and investigating randomness created during play could be used static games, where the game data structure does not change (not like in chess) and the game simply checks correspondence of player's inputs to rules, defining the result; the rules do not change during the play. Such a game is just a finite table for determining wins, loses and draws.

There are numerous examples of such games. A simple example is the game 'even-odd' (or 'Matching pennies' [Matching pennies]): two players (call them 'even' and 'odd') simultaneously select one of two options ' 0 ' or ' 1 ' (integers); 'even' wins, if the sum of their selections is even, 'odd' - if it is odd.

Table. 1. The decision table for the game 'even-odd'.

| 'odd' 'even' | '0' | '1' |
| :---: | :---: | :---: |
| '0' | win = 'even' | win = 'odd' |
| '1' | win = 'odd' | win = 'even' |

Game decision tables have often various symmetries, e.g. the above table allows reflection - change of rows to columns and vice versa; this helps considering game properties.

Players select their moves with certain probabilities, collection of these probabilities is called player's strategy. Suppose the strategy of the player 'even' for selecting ' 0 ' or ' 1 ' is

$$
p_{e}=\left[p_{e 0}, 1-p_{e 0}\right]
$$

Here $P_{e 0}$ is the probability that player 'even' selects ' 0 ' and $1-p_{e 0}=p_{e 1}$ - probability, that he selects ' 1 '. The strategy for player 'odd' is

$$
p_{0}=\left[p_{00}, 1-p_{00}\right]
$$

Player 'even' wins, if both players select the same values and loses, if the select different values, thus he wants to maximize probability of winning, i.e. the quantity

$$
P_{e W}=p_{e 0} p_{o 0}+\left(1-p_{e 0}\right)\left(1-p_{o 0}\right)=1-p_{e 0}-p_{o 0}+2 p_{e 0} p_{o 0}
$$

The probability of winning for player 'odd' is

$$
P_{o W}=p_{e 0}\left(1-p_{o 0}\right)+\left(1-p_{e 0}\right) p_{o 0}=p_{e 0}+p_{o 0}-2 p_{e 0} p_{o 0}
$$

The probability of outcome for any of players is
$W=P_{e W}-P_{o W}$
The expression for probability of win for player 'even' is a function of its probability $P_{e 0}$ to select ' 0 '. A function is growing if its derivative is positive and in the maximum - equals to zero, thus for player 'even' the best strategy would be:

$$
\frac{d P_{e W}}{d p_{e 0}}=p_{o 0}-\left(1-p_{o 0}\right)-\left(1-p_{o 0}\right)+p_{o 0}=0
$$

From here it follows
$p_{00}=1 / 2$
Thus $p_{01}=1-p_{o 0}=1 / 2$ and from the symmetry of the game also $p_{e 0}=p_{e 1}=1 / 2$.
This is the 'Nash Equilibrium' [Nash 1950] - an optimal strategy if other players do not change their strategy. Nash proved existence of equilibrium points, but did not show, how to find them. It turned out to be quite a complex problem [Babichenko Rubinstein 2016], but for this very simple game it was easy to calculate the optimum.

When playing, players generally do not take derivatives to calculate their optimum strategy - they come to the optimum with step-by-step modifications of their strategies. But it is very easy to 'slip down' from the equilibrium point, since this is a saddle point of the function $W$ :


Fig. 1. The 3D plot of the function W and the equilibrium point - change of strategy for any player will change probability of winning for both players.

## 4 LEARNING

Real (human) players usually quickly understand, that they should follow what the opponent does - they should learn from the sequence of already made moves.

A player with very limited memory considers only the last move and adheres the simplest learning strategy (learning with deph 1):
if I won - repeat the last move; if I lost-change the move.
Consider this game as a finite automaton, where states are pairs $[0,0],[0,1],[1,0],[1,1]$ of players moves (first the move of player 'even', then - player 'odd), transforming the current state to the state designated with this move and producing outputs 'o','e' which indicate, who won (player 'even' or player 'odd'), the output is shown after the state. It is easy to see, that whatever was the first move
(where they could yet not follow the above rule), the game converges to four-step cycle. For instance, suppose the first move was $[0,0]$ and both players follow the above rule:

$$
[0,0] e \xrightarrow{0,1}[0,1] O \xrightarrow{1,1}[1,1] e \xrightarrow{1,0}[1,0] \circ \xrightarrow{0,0}[0,0] e \rightarrow \ldots
$$

It is easy to check, that whatever was the first move, players always create this four-state deterministic automaton. Very boring, this is not a game ...

Thus a wise player tries to confuce the opponent, make random moves and also learn from previous game situations. Every 'novel' move, i.e. a move that has not been used before in current situation adds to the above automaton a new transition and the automaton becomes non-deterministic. When game advances, players could add frequences to transitions in order to approximate probabilities. But this strategy requires lot of memory. If the game has $n$ states and $/$ user inputs, then players should follow and memorize frequencies of all possible $2 \times n \times I$ transitions ( $2-$ both directions) - this task becomes quickly very difficult.

## 5 ALGORITHM

It is a much easier to learn from 'looking further back', to use longest repeating sequence to predict the opponent's move. To see what opponent has done earlier, i.e. learn with depth > 1:

- look back and when you see a situation maximally similar to the current one make the move that then (in the previous similar situation) would make you winning.

More precisely: suppose the sequence of moves from the first state $S_{0}$ to the the current state $S_{C}$ of the game is

$$
S_{0}, S_{1}, \ldots, S_{n}, \ldots, S_{k}, S_{c}, S_{c+1}, \ldots, S_{n}, \ldots, S_{k}, S_{c}
$$

From the current state $S_{c}$ search backwards for the longest subsequence which already has occurred before the current state $S_{c}$; suppose it is $S_{n}, \ldots, S_{k}, S_{c}$ :
$S_{0}, S_{1}, \ldots S_{n}, \ldots, S_{k}, S_{c}, S_{c+1}, \ldots, S_{n}, \ldots, S_{k}, S_{c}$
For instance, for the list
$1,7,2,1,2,3,4,2,1,2,4,3,1,2,3,5,7,5,9,8,9,1,2,3,4,7,4,3,1,2,3,5$
the longest suffix which the algorithm would return is
$4,3,1,2,3,5$
The longer the subsequence is, the more probable is that the opponent will repeat its last move in this situation, i.e. the move that created the state $S_{C+1}$. Now select your move correspondingly, i.e. if the state $S_{C+1}$ was for you winning, repeat your move; if it was not, change it.

## 6

CREATING RANDOMNESS
In tests with students (and authors of this paper) this simple strategy for computer has turned out to be very good - if the length of the game is $>50$, in most cases human players are already behind (you can test yourself at http://staff.ttu.ee/~jaak/games/). We can not remeber long sequences of moves and we are not sufficiently random to beat computer, especially if the memory requirements (length of the game) grows; it seems that here also works the famous human short-term memory principle
$7 \pm 2$ [Miller 1956]. The best strategy for human players is to get access to some established source of random binary numbers - use Javascript's function Math.floor ( $2 *$ Math. random () or window.crypto.getRandomValues(), download/lookup a table of random binary numbers from https://www.random.org/ etc and enter your moves from this source.

This mode of playing expels humans. This is 'the table of random numbers vs computer' and this is a rather good test of quality of randomness presented in this table, i.e. evaluation of the quality of the source of randomness.

All sequenced produced by deterministic finite automata are ultimately periodic, i.e. after some number of steps they become periodic. The reason is easy to understand - any finite deterministic device/program 'runs into cycle' in sufficiently long time, i.e. starts to produce periodically repeating sequences of outputs.

The descibed above algorithm for searching longest previously already occurred suffix in the list of moves actually tests, wheather the game automaton is already in cycle, i.e. repeating moves. If it is, it breaks this cycle with probability growing with the length of the cycle - in every move probability, that one of player's (table of random numbers or computer) does not repeat the previously used values (i.e. the probability that the cycle will be broken) is $\approx 0.5$. Lack of long cycles is a good evidence for randomness of the sequence.

The described above computer algorithm was tested against three established sources of randomness: Javascript function random () (traditional source of randomness in Javascript, works in all major browsers), window. crypto.getRandomValues () - the new, 'stronger' Javascript function, and downloading (at the beginning of test) a list of random numbers from RANDOM. ORG (https://www.random.org/), where randomness is based on atmospheric noise.

In series of 10 tests in Firefox, every test with 10000 moves (JavaScript random () against computer) the number $n$ of occurrences of cycles of length $\lambda$ is presented the table below. Tests for other opponents: window. crypto.getRandomValues () and RANDOM.ORG produced similar results.

Table. 2. Occurrence of cycles and their length in series of 10000 tests:

| $\lambda$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $n$ | 982 | 932 | 756 | 416 | 153 | 46 | 11 | 5 | 3 | 1 | 0 |

The computer played quite well against all three opponents. Below are results of 10 cumulative tests 'player1 = window. crypto.getRandomValues (), player2 = computer', each for 10000 moves executed in Google Chrome. While in the first 5 series computer had difficulties, but after that learned constanly and at the end clearly outperformed the opponent.

Table. 3. Cumulative results of $3 x 10$ tests à 10000 moves, player $1=$ window.crypto.getRandomValues (), player2 = computer.

| Better player1 |  |  |  | Better player2 |  |  |  |  | Draw |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Firefox |  |  | Chrome |  |  | Firefox |  |  | Chrome |  |  | Firefox |  |  |  | Chrome |  |  |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 2 | 2 | 2 | 3 | 2 | 1 | 2 | 2 | 2 | 1 | 2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 3 | 2 | 2 | 3 | 2 | 1 | 2 | 3 | 3 | 2 | 3 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 4 | 3 | 3 | 3 | 2 | 1 | 2 | 3 | 3 | 3 | 4 | 5 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 5 | 4 | 4 | 3 | 3 | 1 | 2 | 3 | 3 | 4 | 4 | 6 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 6 | 5 | 5 | 3 | 3 | 1 | 2 | 3 | 3 | 5 | 5 | 7 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 6 | 5 | 5 | 3 | 3 | 1 | 3 | 4 | 4 | 6 | 6 | 8 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 7 | 6 | 6 | 3 | 4 | 2 | 3 | 4 | 4 | 7 | 6 | 8 | 0 | 0 | 0 | 0 | 0 | 0 |  |

This and results of other similar tests (computer against Javascript's random () or table of random values from RANDOM.ORG) show, that the described above algorithm quite well handles these conventional sources of randomness, i.e. its own randomness is on the same leval. From the above table it is seen, that there are some differences in browsers - in Chrome computer outperformed the conventional sources of randomness, but in Firefox the result was opposite - implementation of the function window. crypto.getRandomValues() differs in these browsers.

Creating randomness requires memory. In the above experiments the longest subsequence was searched from the whole list of played states, i.e. the algorithm could have as much memory as it needed. If the available memory is restricted, the results are slightly worse, but the difference is marginal. Below are results of tests, if access to memory was restricted - computer could use only the last half of the list of moves (he 'forgets' the earlier moves).
Table. 4. Cumulative results of $3 \times 10$ tests à 10000 moves, player1 $=$ window.crypto.getRandomValues 0 , player2 $=$ computer; computer could use only the last half of the list of made moves.

| Better player1 |  |  |  | Better player2 |  |  |  |  | Chrome |  |  |  |  | Firefox |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Firefox |  |  | Chrome |  |  | Firefox |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 2 | 1 | 2 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 2 | 2 | 2 | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 2 | 3 | 3 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 3 | 3 | 4 | 2 | 3 | 2 | 2 | 2 | 1 | 3 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 3 | 4 | 4 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 4 | 4 | 4 | 3 | 4 | 4 | 3 | 3 | 3 | 4 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 4 | 5 | 4 | 4 | 5 | 4 | 4 | 3 | 4 | 4 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 5 | 6 | 5 | 5 | 5 | 5 | 4 | 3 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 5 | 6 | 6 | 6 | 5 | 6 | 5 | 4 | 4 | 4 | 5 | 0 | 0 | 0 | 0 | 0 | 0 |

## 7 M-ARY SEQUENCES

The game 'even-odd' works with binary sequences, but it is easy to design similar static games for investigating randomness of $m$-ary sequences, i.e. sequences of of $m$-ary residues $0,1, \ldots, m-1, m>2$. Well-known example of such game is the game rock-paper-scissors, $m=3$, where players inputs are compared in one-directional order.


Fig. 2. Decision schema for the rock-paper-scissors game $(m=3)$ and for the general case $(m=7)$.
In m-ary game players select one of integers $0,1, \ldots, \mathrm{~m}-1, \mathrm{~m}>2$; winner is the player, who can 'shoot' the other, if ranges of their wepons are less than $\mathrm{m} / 2$ (the shooting is possible only in one direction), i.e. on the above schema for $\mathrm{m}=7$ winner is player p 1 (variations of this game are also available in http://staff.ttu.ee/~jaak/games). The same way as above it is easy to show that the optimal strategy (the Nash equilibrium) for this kind of games is also uniform randomness, thus the above algorithm wins against human players, who often use persistent cyclic motions, predicted by the evolutionary game theory because of humans bounded rationality [Wang,Hu,Zhou 2014].

## 8 CONCLUSIONS

Games are a convenient and easy to use environment for comparing quality of sources of randomness. They can also be used for creation of new randomness and entropy; this would please many humans in our over-organized non-random environment, where entropy/randomness is becoming an endangered species.

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