

Chapter 1

Introduction



1.1 Mathematics Education in EU for STEM Disciplines

Seppo Pohjolainen (✉)

Tampere University of Technology (TUT), Laboratory of Mathematics, Tampere, Finland

e-mail: seppo.pohjolainen@tut.fi

Good competency in mathematics is important in science, technology and economy; mathematics can be considered as the language of nature and technology and also is an important methodology in economics and social sciences. A study by Hanushek and Wößman [21] shows that the quality of education has a strong positive influence on economic growth. In their research, students' skills were measured using 13 international tests, which included mathematics, science, and reading. An OECD report on mathematics in industry [42] states that the remarkable development of the natural sciences and of engineering since the Renaissance is a consequence of the fact that all nature's known laws can be expressed as mathematical equations. The Financial Times outlined their news on February 13th, 2006, as "Mathematics offers business a formula for success".

Despite the fact that the value of mathematics in society and economics is understood, in recent decades students' mathematics skills have deteriorated in western countries. The report "Mathematics for the European Engineer" [50] by the European Society for Engineering Education SEFI¹ states that this phenomenon prevails in Europe. According to the SEFI report, universities in the western world

¹SEFI (<http://www.sefi.be>).

have observed a decline in mathematical proficiency among new university students and have taken action to remedy the situation. The most common measures are: reducing syllabus content; replacing some of the harder material with more revisions of lower level work; developing additional units of study; establishing mathematics support centres. But sometimes one does nothing.

The decline in mathematical competency may have serious consequences as Henderson and Broadbridge [22] point out. Their message is that industry can only be internationally competitive through mathematical know-how. The number of students majoring in mathematics e.g., in Australia, has decreased, while the number of positions requiring mathematical skills has increased.

The union of Academic Engineers and Architects in Finland² (TEK) published a report in 2009 with the recommendation “Knowledge of mathematics and natural sciences must be emphasized more strongly as part of common cultivation and their appreciation should be improved in the society”. The report also points out that good command of mathematics and natural sciences is one of the strongest features in engineering studies.

As mathematical proficiency is a prerequisite for studying technical sciences, weak mathematical skills slow down studies. For instance, in Germany the drop-out rate of students sometimes goes up to 35% and one of the primary reasons is the lack of mathematical skills. This caused the industrial Arbeitgeberverband Gesamtmetall to raise an alarm. Drop-out rates in engineering studies are high Europe-wide.³

For example, less than 60% of B.Sc. students starting their studies in Finland at Tampere University of Technology (TUT) in 2005 had completed all mandatory first year mathematics courses in four and a half years. Students who had progressed fastest in their studies had typically completed first year mathematics courses according to the recommended schedule. Students who faced problems in studying mathematics more often progressed slowly with their studies in general.

The problems universities are facing with their enrolling students’ mathematical proficiency are partly due to school mathematics. The level of school mathematics is being assessed internationally by PISA (The Programme for International Student Assessment), and TIMSS (Trends in International Mathematics and Science Study). PISA is an internationally standardised assessment for 15-year-olds in schools testing literacy in reading, mathematics and science. TIMSS collects educational achievement data at the 4th and 8th grades to provide information on quantity, quality, and content of instruction. The test results from 2012 [44] and 2011 [37] confirm that East-Asian countries are on the top but they are criticised for teacher centred education, large amounts of homework, rote learning etc. EU-countries are doing relatively well, but lagging behind the East-Asian nations, and developing countries can be seen at the bottom.

²<http://www.tek.fi/>.

³http://ec.europa.eu/information_society/apps/projects/factsheet/index.cfm?project_ref=ECP-2008-EDU-428046.

Learning outcomes in mathematics are not dependent solely on good teaching, sufficient resources or other external considerations with bearing on learning. Factors with bearing on what the student does include attitudes: orientations, intentions and motivations. In order to achieve learning objectives, activity on the part of the learner is required. As student's attitudes and motivational factors are individual, good teaching should take into account student's different learning styles [25].

The recent report 'Mathematics in Europe: Common Challenges and National Policies' by EURYDICE [17] points out that many European countries are confronted with declining numbers of students of mathematics, science and technology, and they face a poor gender balance in these disciplines. The report gives recommendations on how to increase motivation to learn mathematics and encourage the take-up of mathematics-related careers. The report also suggests that the mathematics curriculum should be broadened from contents to competences. Student motivation should be increased by demonstrating and finding evidence how mathematics is used in industry and society, in students' everyday life, and in their future career. New teaching approaches, such as problem-based learning and inquiry-based methods, should be taken into use. Addressing low achievement is important to decrease the drop-out figures, and gender issues should be considered to make mathematics more tempting to female students. Education and professional development of mathematics teachers also plays a key role in this reform.

The European Society for Engineering Education (SEFI), mentioned above, is an international non-profit organisation established in 1973 in Belgium and founded by 21 European Universities. It is an association directly linking the institutions of higher engineering education as an international forum for discussing problems and identifying solutions relating to engineering education. Today, SEFI is the largest network of institutions of higher engineering education, individuals, associations and companies in Europe. Its mission is to contribute to the development and improvement of engineering education in Europe and to the enhancement of the image of both engineering education and engineering professionals in society.

SEFI has set up several working groups on developing engineering education. Among them is the Working Group on Mathematics and Engineering Education, established in 1982. The major outcome of the group resulted in a "Core Curriculum in Mathematics for the European Engineer", first published 1992 and then revised in 2002 as "Mathematics for the European Engineer" [50], and updated 2013 as "A Framework for Mathematics Curricula in Engineering Education" [49].

These documents clearly reflect the European understanding of what the mathematics is that engineers need, and how it should be learned and taught. The 1992 version of the Core Curriculum answers mainly the question: what should be the contents of mathematics courses for engineers? It presented a list of mathematical topics, which are itemised under the headings of Analysis and Calculus, Linear Algebra, Discrete Mathematics and Probability and Statistics.

The SEFI 2002 document identifies four content levels defined as Cores 0, 1, 2, 3. The entry level Core 0 and Core level 1 comprise the knowledge and skills which are necessary and essential for most engineering areas and they should be

mandatory for all engineering education, whereas from the other two different parts (Cores 2 and 3) contents will be chosen for the various engineering disciplines. The document also specifies learning outcomes for all the topics and contains additional comments on teaching mathematics.

The most recent report: “A Framework for Mathematics Curricula in Engineering Education”, SEFI 2013 [49], proposes a pedagogical reform for engineering mathematics to put more emphasis on what students should know instead of what they have been taught. The learning goals are described as competencies rather than learning contents. Contents should be embedded in a broader view of mathematical competencies that the mathematical education of engineers strives to achieve. Following the Danish KOM project [39], SEFI recommends that the general mathematical competence for engineers is “the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role”. The general mathematical competence can be divided into eight sub-competencies which are: thinking mathematically, reasoning mathematically, posing and solving mathematical problems, modelling mathematically, representing mathematical entities, handling mathematical symbols and formalism, communicating in, with, and about mathematics, and making use of aids and tools.

Following the SEFI 2013 document we briefly introduce the eight subcompetencies:

Thinking Mathematically This competency comprises the knowledge of the kind of questions that are dealt with in mathematics and the types of answers mathematics can and cannot provide, and the ability to pose such questions. It includes the recognition of mathematical concepts and an understanding of their scope and limitations as well as extending the scope by abstraction and generalisation of results. This also includes an understanding of the certainty mathematical considerations can provide.

Reasoning Mathematically This competency includes the ability to understand mathematical argumentation (chain of logical arguments), in particular to understand the idea of mathematical proof and to understand its central ideas. It also contains the knowledge and ability to distinguish between different kinds of mathematical statements (definition, if-then-statement, iff-statement etc.). On the other hand it includes the construction of logical arguments and transforming heuristic reasoning into unambiguous proofs (reasoning logically).

Posing and Solving Mathematical Problems This competency comprises on the one hand the ability to identify and specify mathematical problems (pure or applied, open-ended or closed) and the ability to solve mathematical problems with adequate algorithms. What really constitutes a problem is not well defined and it depends on personal capabilities.

Modelling Mathematically This competency has two components: the ability to analyse and work with existing models and to perform mathematical modelling (set up a mathematical model and transform the questions of interest into mathematical questions, answer the questions mathematically, interpret the results in reality and

investigate the validity of the model, and monitor and control the whole modelling process).

Representing Mathematical Entities This competency includes the ability to understand and use mathematical representations (symbolic, numeric, graphical and visual, verbal, material objects etc.) and to know their relations, advantages and limitations. It also includes the ability to choose and switch between representations based on this knowledge.

Handling Mathematical Symbols and Formalism This competency includes the ability to understand symbolic and formal mathematical language and its relation to natural language as well as the translation between both. It also includes the rules of formal mathematical systems and the ability to use and manipulate symbolic statements and expressions according to the rules.

Communicating in, with, and about Mathematics This competency includes the ability to understand mathematical statements (oral, written or other) made by others and the ability to express oneself mathematically in different ways.

Making Use of Aids and Tools This competency includes knowledge about the aids and tools that are available as well as their potential and limitations. Additionally, it includes the ability to use them thoughtfully and efficiently.

In order to specify the desired cognitive skills for the topical items and the sub-competences, the three levels described in the OECD PISA document [43] may be used. The levels are: the **reproduction level**, where students are able to perform the activities trained before in the same contexts; the **connections level**, where students combine pieces of their knowledge and/or apply it to slightly different situations; and the **reflection level**, where students use their knowledge to tackle problems different from those dealt with earlier and/or do this in new contexts, so as to have to reflect on what to use and how to use their knowledge in different contexts.

While the necessity of a pedagogical reform is well understood, there still are not enough good pedagogical models scalable to universities' resources available. Modern information and communication technology (ICT) provides a variety of tools that can be used to support students' comprehension and pedagogical reform. Teachers may run their courses using learning platforms like Moodle. In these environments they may distribute course material, support communication, collaboration, and peer learning and organise face-to-face meetings with videoconferencing tools. Students can get feedback on their mathematical skills' from their teacher, peers, and also by using carefully chosen computer generated exercises, which are automatically checked by computer algebra systems (Math-Bridge, System for Teaching and Assessment using a Computer algebra Kernel⁴ (STACK)). There

⁴<http://www.stack.bham.ac.uk/>.

exist mathematical programs like MATLAB⁵ and Mathematica,⁶ which support mathematical modelling of real world problems.

Math-Bridge is one of the learning platforms available to study mathematics. It offers online courses of mathematics including learning material in seven languages: German, English, Finnish, French, Dutch, Spanish and Hungarian. The learning material can be used in two different ways: in self-directed learning of individuals and as a ‘bridging course’ that can be found at most European universities (Math-Bridge Education Solution⁷).

Internet contains a variety of open source mathematical tool programs (R, Octave, Scilab), computational engines (Wolfram alpha), various visualisations, and apps that can be used alongside the studies. Links to external resources can be easily used to show real world applications. Some universities have started to provide Massive Open Online Courses (MOOC’s) available world-wide for their on- and off-campus students.

Information and communication technology can be used to support the learning process in many ways, but great technology cannot replace poor teaching or lack of resources. The use of technology does not itself guarantee better learning results; instead it can even weaken the student performance. This obvious fact has been known for a long time. Reusser [46] among many other researchers, stated that the design of a computer-based instructional system should be based on content-specific research of learning and comprehension and a pedagogical model of the learner and the learning process. In designing computer-based teaching and learning environments real didactic tasks should be considered. One should thoroughly consider what to teach and how to teach. Jonassen [26] has presented qualities of “meaningful learning” that the design and use of any learning environment should meet. The list has been complemented by Ruokamo and Pohjolainen [48].

In a recent report by OECD [41], it was discovered that in those countries where it is more common for students to use the Internet at school for schoolwork, students’ average performance in reading declined, based on PISA data. The impact of ICT on student performance in the classroom seems to be mixed, at best. In fact, PISA results show no appreciable improvements in student achievement in reading, mathematics or science in the countries that had invested heavily in ICT for education. One interpretation of these findings is that it takes time and effort to learn how to use technology in education while staying firmly focussed on student learning.

As mathematics is a universal language, the problems in teaching and learning are globally rather similar. The importance of mathematics is internationally well understood and deterioration in the students’ skills is recognised. Pedagogical reforms are the way EU is going and pedagogically justified use of information technology and tools will play an important role here.

⁵<http://mathworks.com>.

⁶<http://www.wolfram.com/mathematica>.

⁷<http://www.math-bridge.org>.

1.2 TEMPUS Projects MetaMath and MathGeAr

Sergey Sosnovsky

Utrecht University, Utrecht, The Netherlands

e-mail: s.a.sosnovsky@uu.nl

1.2.1 Introduction

The world-wide system of STEM⁸ (science, technology, engineering and math) education faces a range of fundamental challenges. Addressing these challenges in a timely and efficient manner is paramount for any national economy to stay competitive in the long range. It is worth noticing that most of these problems are truly global; they are actual not only for the developing countries, but also for countries characterised by stable engineering sectors and successful educational systems (such as, for example, Germany and USA). Experts identify three main clusters of factors characterising the change in the requirements towards the global system of STEM education [40]:

- Responding to the changes in global context.
- Improving perception of engineering subjects.
- Retention of engineering students.

1.2.1.1 Responding to the Changes in Global Context

The speed of renewal of engineering and technical knowledge and competencies is ever growing. Most engineering sectors observe acceleration of the cycle of innovation, i.e. the time between the birth of a technology and its industrial mainstreaming. Technical skills evolve rapidly, new competencies emerge, old competencies dissolve. In 1920, the average half-life of knowledge in engineering was 35 years. In 1960, it was reduced to 10 years. In 1991, it was estimated that an engineering skill is half-obsolete in 5 years. Nowadays, “IT professional would have to spend roughly 10 h a week studying new knowledge to stay current (or upskilling, in the current lingo)” [13]. This essentially means that modern engineering programmes have to teach students how to obtain and operate skills that are not yet defined and how to work at jobs that do not yet exist. Continuous education and retraining is common already and will only widen in the future. This increase in intensity of education and diversity of educational contexts renders the

⁸A similar term has been suggested in the German literature as well—MINT (“Mathematik, Informatik, Naturwissenschaft un Technik”).

traditional system of STEM education inadequate. New forms of education powered by new educational technologies are required.

At the same time, the engineering and technical problems themselves are changing. Technology has become an integral part of most sectors of a modern economy, technical systems have become more complex and interconnected, even our daily life activities have been more and more penetrated by technology. Finding solutions for this kind of problems and management of this kind of systems requires new approaches that take into account not only their technical side, but also their relations to the social, ecological, economical and other aspects of the problem. Effective teaching of future engineers to deal with these interdisciplinary problems by applying more comprehensive methods should require significant redesign of STEM courses and curriculae. One more group of factors influencing STEM education come from the ever growing globalisation of economic and technological relations between countries and companies. Markets and manufacturers have internationalised and the relations between them have become more dynamic. Solutions for typical engineering problems have become a service, basic engineering skills and competencies have transformed into a product that has a market value and can be offered by engineers of other nations. Countries that have not invested in own STEM education and do not possess strong enough engineering workforce will be forced to pay other countries for engineering problems by outsourcing.

1.2.1.2 Improving Perception of Engineering Subjects

While the demand for engineer professionals is increasing world-wide, the number of engineering graduates is not growing, and in some countries has even dropped over the last several years. Potential students often consider engineering and technical professions less interesting. Those driven by financial stimuli do not consider engineering as an attractive, money-making career and choose business-oriented majors. Young people motivated rather by a social mission and a public value of their future profession also seldom believe that STEM answers their life goals and remain in such fields as medicine and humanities. Such beliefs are clearly misconceptions as engineering professions are both well paid and societally important. Yet, if the appeal of STEM careers is often not apparent to potential students it is the responsibility of STEM programme administrations and teachers to properly advertise their fields. In addition to that, many students simply consider STEM education too complex and formal, and, at the end, boring. Changing such an image of STEM education is an important practical task that every national system of engineering education must address.

1.2.1.3 Retention of Engineering Students

One of the biggest problems for engineering and science education is the high dropout rates (especially among freshmen and sophomores). For example, in American

universities, up to 40% of engineering students change their major to a non-technical one or simply drop out from college [40]. The situation in Europe is similar. For instance, in Germany over the last 15 years, the number of students who do not finish their university programmes has grown by about 10% for most engineering specialisations. Now, depending on the program, this number fluctuates between 25 and 30% nation-wide. For degrees that include an intensive mathematics component, the rate goes up to 40% of all enrolling students [23]. Similar trends can be observed in other developed EU nations (e.g., The Netherlands, Spain, United Kingdom). One of the reasons for this is the fact the traditional structure of engineering education does not enable students to develop their engineer identity for several academic semesters. All engineering programmes start with a large number of introductory “101” courses teaching formal math and science concepts. Only on the second or third year, the actual “engineering” part of the engineering educational programs starts. For a large percentage of students this comes too late. How can engineering students be sooner exposed to the competencies, requirements, and problems and use cases of engineering is a big methodological problem. Should the structure of the engineering curriculum be modified to gradually introduce engineering subjects in parallel with the introductory science courses, or should the structure of the “101” courses be changed to become more attractive to students and include more engineering “flavour”, or, maybe, can new educational technologies make these subjects more engaging and help relieve the student retention problem?

1.2.1.4 National Problems of Engineering Education

The three countries addressed in this book (Russia, Georgia and Armenia) have inherited the strong system of school and university STEM education developed in Soviet Union. It was developed to support industrial economy and has used many unique methodological innovations [29]. Yet, after the collapse of Soviet Union, the educational systems of these countries went through a significant transformation. This process has been characterised by several trends, including attempts to resolve the disproportion of the old Soviet education systems that emphasised formal and technical subjects while overlooking the humanities; closing gaps in largely fragmented inherited national educational systems (mostly, the case of Georgia and Armenia); introduction and implementation of elements of the Bologna process and ECTS. Despite the economic and political turmoil of the 1990s, significant progress has been achieved. Yet, there still exist a number of problems impeding further development of these countries’ systems of education. This book takes a closer look at the problems pertaining to the mathematics component of STEM education.

1.2.1.5 Role of Mathematics in STEM Education

All the problems mentioned above are especially important when it comes to the mathematics component of STEM education. Math is a key subject for all

technical engineering and science programs without exception. In many respects mathematics serves as a lingua franca for other more specialised STEM subjects. The level of math competencies of a student is critical for successful engineering college education, especially in the beginning, when possible learning problems are amplified and math represents a large share of his/her studies. Differences between the requirements of school and university math education can be rather large, especially since different schools can have very different standards of math training. In addition, students themselves often underestimate the volume of math knowledge required to succeed in an engineering university program. In Georgia and Armenia, these problems are especially actual. There is a massive gap between the level of technical competencies of GE/AM school graduates and the requirements they face once enrolling in universities. This gap has emerged as a result of asynchronous reforms of secondary and tertiary education in these countries. According to the statistics of the National Assessment and Examinations Centre of Georgia, about a third of the university entrants fail the national exam. In Armenia, the data of the Ministry of Education and Science shows that the average score of school graduates in math reaches only about 50%. In Russia, the situation is not that drastic. Yet these problems also exist due to unique national circumstances. After the introduction of the unified state exam (USE) in the beginning of the 2000s, Russian universities have abolished the common habit of year-long preparatory math courses that many potential students took. This resulted in a considerable decrease in math competences of freshmen in provincial universities (more prestigious universities of Moscow and St. Petersburg are less affected, as they can select stronger students based on the result of their USE). Finally, from the organisational perspective, a deficit of STEM students coupled with increased market demands for engineering specialists makes many universities loosen enrollment standards, especially with regards to mathematics. Georgia provides a particular example of this situation. Several years ago, when Georgian national tests did not stipulate mandatory subjects, it was a common practice for students with weak school math grades, to not take a math test, yet get accepted to engineering programs. Such a practice not only reduces the overall level of students but also adds an extra load on university teachers. At the end, this will unavoidably result in a decrease in the quality of engineering programme graduates [1].

1.2.2 Projects MathGeAr and MetaMath

The complete titles of the projects are:

- **MetaMath: Modern Educational Technologies for Math Curricula in Engineering Education of Russia;**
- **MathGeAr: Modernisation of Mathematics curricula for Engineering and Natural Sciences studies in South Caucasian Universities by introducing modern educational technologies.**

They both have been supported under the 6th call of the Tempus-IV Program financed by the Education, Audiovisual and Culture Executive Agency (EACEA) of EU. The project have been executed in parallel from 01/12/2013 until 28/02/2017.

1.2.2.1 Objectives

MetaMath and MathGeAr projects aimed to address a wide spectrum of the listed problem of math education in engineering programs of Russian, Georgian and Armenian universities. To solve these problems, the projects rely on a comprehensive approach including studying international best practices, analytical review and modernisation of existing pedagogical approaches and math courses. The objectives of the projects include:

- Comparative analysis of the math components of engineering curricula in Russia, Georgia, Armenia and EU and detection of several areas for conducting reforms;
- Modernisation of several math courses within the selected set of programs with a special focus on introduction of technology-enhanced learning (TEL) approaches.
- Localisation of European TEL instrument for partner universities, including digital content localisation with a focus on the introduction of the intelligent tutoring platform for mathematical courses Math-Bridge [51].
- Building up technical capacity and TEL competencies within partner universities to enable the application of localised educational technologies in real courses.
- Pilot evaluation of the modernised courses with real students validating the potential impact of the conducted reform on the quality of engineering education.
- Disseminate results of the projects.

1.2.2.2 Consortia

Projects consortia consisted of organisation from EU and partner countries (Russia for MetaMath and Georgia/Armenia for MathGeAr). The set of EU partners was the same for the two projects and included:

- Universität des Saarlandes—USAAR (Saarbrücken, Germany),
- Université Claude Bernard Lyon I—UCBL (Lyon, France),
- Tampere University of Technology—TUT (Tampere, Finland),
- Deutsches Forschungszentrum für Künstliche Intelligenz—DFKI (Saarbrücken, Germany),
- Technische Universität Chemnitz—TUC (Chemnitz Germany).

Additionally, the MetaMath consortium contained six more partners form Russia:

- the Association for Engineering Education of Russia—AEER (Tomsk, Russia),
- Saint Petersburg Electrotechnical University—LETI (St. Petersburg, Russia),

- Lobachevsky State University of Nizhni Novgorod—NNSU (Nizhni Novgorod, Russia),
- Tver State University—TSU (Tver, Russia),
- Kazan National Research Technical University named after A.N. Tupolev—KNRTU (Kazan, Russia),
- Ogarev Mordovia State University—OMSU (Saransk, Russia).

The MathGeAr consortium, instead of Russian participants, contained five organisation from Georgia and four from Armenia:

- Georgian Technical University—GTU (Tbilisi, Georgia),
- University of Georgia—UG (Tbilisi, Georgia),
- Akaki Tsereteli State University—ATSU (Kutaisi, Georgia),
- Batumi Shota Rustaveli State University—BSU (Batumi, Georgia),
- Georgian Research and Educational Networking Association—GRENA (Tbilisi, Georgia),
- National Centre for Educational Quality Enhancement—NCEQE (Tbilisi, Georgia),
- National Polytechnic University of Armenia—NPUA (Yerevan, Armenia),
- Armenian State Pedagogical University after Khachatur Abovian—ASPU (Yerevan, Armenia),
- Armenian National Centre For Professional Education and Quality Assurance—ANQA (Yerevan, Armenia),
- Institute for Informatics and Automation Problems of the National Academy of Sciences of the Republic of Armenia—IIAP (Yerevan, Armenia).

Partner organisations had different roles in the projects based on their main competencies. Figures 1.1 and 1.2 graphically represent the structure of the two consortia including the main role/expertise for each organisation. This book is a collective effort of all partners from the two consortia.

1.2.2.3 Execution

The projects have been conducted in three main phases. The results of the first phase are essentially the subject of this book. It included the following tasks:

- Development of the methodology for comparative analysis of math courses.
- Pairwise comparative analysis of math courses between EU and partner universities.
- Development of recommendations for the consequent reform of structural, pedagogical/technological/administrative aspects of the target courses.
- Identification of areas where TEL would bring about the most impact, and selection of TEL instruments to lead to this impact.

The second phase built up on the results of the previous one. It included the following activities:

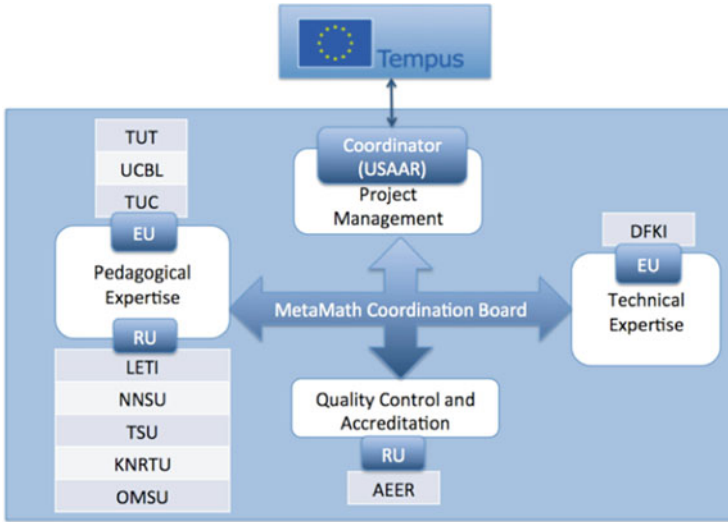


Fig. 1.1 Structure of the MetaMath project consortium

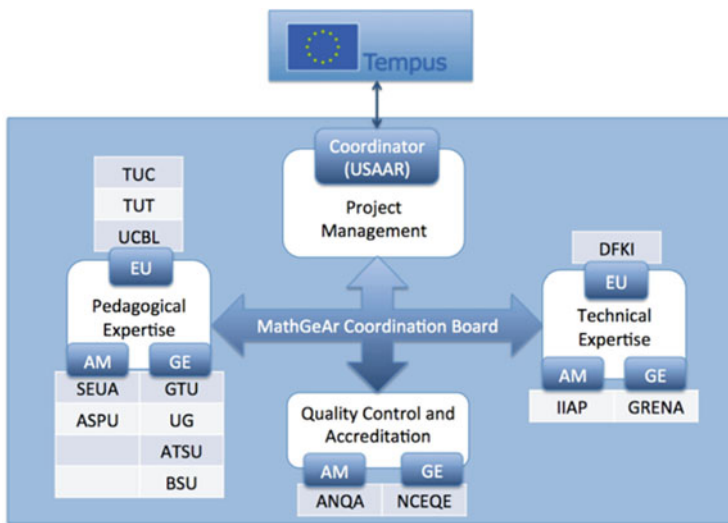


Fig. 1.2 Structure of the MathGear project consortium

- Modification of a set of math courses taught to students of engineering programs in partner universities.
- Localisation of the Math-Bridge platform and its content into Russian, Georgian and Armenian.
- Training of teaching and technical personnel of partner universities to use Math-Bridge and other TEL tools for math education.

Finally, the third phase contained:

- Implementation of the (parts of) modified courses into the Math-Bridge platform.
- Planning and conduction of a large-scale pedagogical experiment across three countries and eleven universities examining the effect of the modernised courses on different learning parameters of engineering students.
- Analysis and dissemination of the project results.

1.2.3 Learning Platform Math-Bridge

Both projects plans have been especially focussed on applying TEL approaches. This decision has been motivated by recent advancements in developing intelligent and adaptive systems for educational support, such as Math-Bridge, especially for STEM subjects. For example, the use of computers to improve students' performance and motivation has been recognised in the final report of the Mathematical Advisory Panel in the USA: "Research on instructional software has generally shown positive effects on students achievement in mathematics as compared with instruction that does not incorporate such technologies. These studies show that technology-based drill and practice and tutorials can improve student performance in specific areas of mathematics" [18]. As the main TEL solution, the projects rely on Math-Bridge, which is an online platform for teaching and learning courses in mathematics. It has been developed as a technology-based educational solution to the problems of bridging courses taught in European universities. As its predecessor—the intelligent tutoring system ActiveMath [36]—Math-Bridge, has a number of unique features. It provides access to the largest in the world collection of multilingual, semantically annotated learning objects (LOs) for remedial mathematics. It models students' knowledge and applies several adaptation techniques to support more effective learning, including personalised course generation, intelligent problem solving support and adaptive link annotation. It facilitates direct access to LOs by means of semantic search. It provides rich functionality for teachers allowing them to manage students, groups and courses, trace students' progress with the reporting tool, create new LOs and assemble new curricula. Math-Bridge offers a complete solution for organizing TEL of mathematics on individual, course and/or university level.

1.2.3.1 Math-Bridge Content

The Math-Bridge content base consists of several collections of learning material covering the topics of secondary and high school mathematics as well as several university-level subjects. They were originally developed for teaching bridging courses by mathematics educators from several European universities. Compared to the majority of adaptive e-learning applications, Math-Bridge supports a mul-

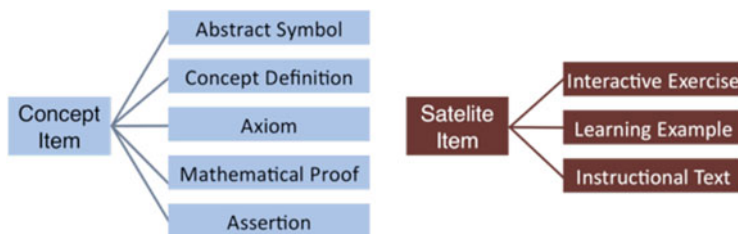


Fig. 1.3 Hierarchy of LO types in Math-Bridge

titude of LO types. The OMDoc language [30] used for representing content in Math-Bridge defines a hierarchy of LOs to describe the variety of mathematical knowledge. On the top level, LOs are divided into concept objects and satellite objects. Satellite objects are the main learning activities; they structure the learning content, which students practice with: exercises, examples, and instructional texts. Concept objects have a dualistic nature: they can be physically presented to a student, and she/he can browse them and read them; at the same time, they are used as elements of domain semantics, and, as such, employed for representing knowledge behind satellite objects and modelling students' progress. Figure 1.3 provides further details of the types of LOs supported in Math-Bridge.

1.2.3.2 Learning Support in Math-Bridge

The Math-Bridge platform provides students with multilingual, semantic and adaptive access to mathematical content. Its interface consists of three panels (Fig. 1.4). The left panel is used for navigation through learning material using the topic-based structure of the course. The central panel presents the math content associated with the currently selected (sub)topic. The right panel provides access to the details of the particular LO that a student is working with, as well as some additional features, such as semantic search and social feedback toolbox.

Math-Bridge logs every student interaction with learning content (e.g., loading a page or answering an exercise). The results of interactions with exercises (correct/incorrect/partially correct) are used by the student-modelling component of Math-Bridge to produce a meaningful estimation of the student's progress. For every math concept the model computes the probabilities that the student has mastered it. Every exercise in Math-Bridge is linked with one or several concepts (symbols, theorems, definitions etc.) and the competencies that the exercise is training for these concepts. A correct answer to the exercise is interpreted by the system as evidence that the student advances towards mastery and will result in the increase of corresponding probabilities. Math-Bridge implement three technologies for intelligent learning support:

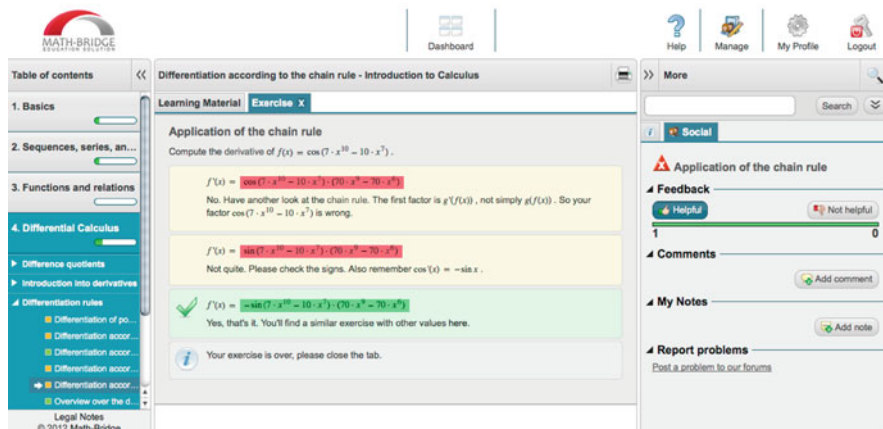


Fig. 1.4 Math-Bridge student interface

- **Personalised courses.** The course generator component of Math-Bridge can automatically assemble a course optimised for individual students' needs and adapted to their knowledge and competencies.
- **Adaptive Navigation Support.** Math-Bridge courses can consist of thousands of LOs. The system helps students find the right page to read and/or the right exercise to attempt by implementing a popular adaptive navigation technique—adaptive annotation [9]. The annotation icons show the student how much progress she/he has achieved for the corresponding part of learning material.
- **Interactive Exercises and Problem Solving Support.** The exercise subsystem of Math-Bridge can serve multi-step exercises with various types of interactive elements and rich diagnostic capabilities. At each step, Math-Bridge exercises can provide students with instructional feedback, ranging from mere flagging the (in)correctness of given answers to presenting adaptive hints and explanations.

1.2.4 Conclusion

Now that both MetaMath and MathGeAr have finished, it is important to underline that their success was in many respects dependent on the results of the comparative analyses conducted during their first phase. Although the overall project approach was defined in advance, individual activities have been shaped by the findings of projects partners contrasting various aspects of math education in Russia/Georgia/Armenia and the EU. The rest of this book presents these findings in detail.

1.3 Perceptions of Mathematics

Christian Mercat and Mohamed El-Demerdash and Jana Trgalova
IREM Lyon, Université Claude Bernard Lyon 1 (UCBL), Villeurbanne, France
e-mail: christian.mercat@math.univ-lyon1.fr; jana.trgalova@univ-lyon1.fr

Pedro Lealdino Filho
Université Claude Bernard Lyon 1 (UCBL), Villeurbanne, France
e-mail: pedro.lealdino@etu.univ-lyon1.fr

1.3.1 Introduction

The global methodology of this comparative study project is based on the analysis of the proposed curriculum and of the actual way this curriculum is implemented in the classroom, in order to identify venues for improvements and modernisations, implement them and study their effect.

In the literature, the philosophical features of the scientific spirit are evident in the sciences which need more objectiveness, chiefly mathematics [11]. From the ingenuous perception of a phenomenon, a pre-scientific spirit needs to overcome a set of epistemological obstacles to reach a scientific stage. We consider this scientific stage an important factor in order to learn, acquire and improve mathematical competencies. The definition of mathematical competence on this project follows the one used in the Danish KOM project and adapted in the SEFI Framework. It is defined as “the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role”. The attitude towards mathematics is a long standing strand of research and uses reliable measuring tools such as the seminal ATMI [52]. But our research identified dimensions which we find specific for engineering, especially through its relationship with reality.

Mathematics is considered as the foundation discipline for the entire spectrum of Science, Technology, Engineering, and Mathematics (STEM) curricula. Its weight in the curriculum is therefore high [1]. In Armenia, Georgia and Russia, all university students pursuing this kind of curriculum are obliged to take a three semester standard course in higher mathematics. Special studies in Europe suggest that a competencies gap in mathematics is the most typical reason for STEM students to drop out of study [4, 5, 10, 24, 31].

Several research studies show that students’ perceptions of mathematics and of mathematics teaching have an impact on their academic performance of mathematics [12, 38], and a positive attitude and perceptions toward the subject will encourage an individual to learn the subject matter better. In a broader sense, perceptions towards mathematics courses are also important to take into account in order to grasp the *cultural differences* between all the different institutions, from the point of view of the students of the course, of their teachers and of the engineers themselves.

The fact that culture does influence these beliefs, while seemingly obvious, is not widely studied. Therefore this study fills a gap in the literature. Without assuming cultural determination, we do show significant differences between institutions. We present in this section a study that investigates these issues, the methodology of data acquisition, the main themes that the study investigates, and the main results.

In order to evaluate students' perceptions of mathematics we elaborated an online survey spread out over all participant countries (Armenia, France, Finland, Georgia and Russia). The survey was elaborated to investigate the three main dimensions of the mathematical courses:

- The usefulness of mathematics.
- Mathematical courses in engineering courses—Contents and Methods.
- Perception of Mathematics.

The survey was spread with a web tool and translated into each partner language to ensure that the meaning of the questions was adequately taken into account. A total of 35 questions were answered by 1548 students from all participant countries.

After collecting the data from the online survey we used the statistical package R to analyse the data and draw conclusions.

We performed a Principal Component Analysis to verify whether there were some patterns in the students' responses. Using the graphical representation of data, we can propose the hypothesis that the methodology of teaching mathematics of each country shapes the average students' perception towards mathematics. Thus, the first conclusion of this analysis is that in the European universities the mathematics are taught as tools to solve problems, that is, mathematics by practicing, while in the non-European universities, the mathematics are taught focussing on proofs and theorems, that is, mathematics by thinking.

1.3.2 Theoretical Background and Research Question

Furinghetti and Pehkonen [19] claim that students' beliefs and attitudes as regards mathematics have a strong impact on their learning outcomes. Mathematics-related perceptions are referred to as a belief system in the literature [15, 19]. Furinghetti and Pehkonen point out that there is a diversity of views and approaches to the study of beliefs in the field of mathematics education, and they conclude that the definition of the concept of belief itself remains vague. Some researchers acknowledge that beliefs contain some affective elements [35], while others situate beliefs rather on the cognitive side [53]. Furinghetti and Pehkonen (ibid.) bring to the fore a variety of concepts used by researchers to address issues related to beliefs, such as conceptions, feelings, representations or knowledge. Some authors connect these concepts, for example beliefs and conceptions, as Lloyd and Wilson [34]: "We use the word conceptions to refer to a person's general mental structures that encompass knowledge, beliefs, understandings, preferences, and views". Others distinguish clearly between the two concepts, as Ponte [45]: "They [beliefs] state that something

is either true or false, thus having a propositional nature. Conceptions are cognitive constructs that may be viewed as the underlying organizing frame of concepts. They are essentially metaphorical.” Furinghetti and Pehkonen (ibid.) attempt to relate beliefs or conceptions and knowledge by introducing two aspects of knowledge: “objective (official) knowledge that is accepted by a community and subjective (personal) knowledge that is not necessarily subject to an outsider’s evaluation” (p. 43).

The purpose of this study is not to contribute to the theoretical discussion on these concepts, but rather to study how engineering students view mathematics and its teaching in their schools. We therefore adopt the word ‘perceptions’ to address these students’ views and opinions.

Breiteig, Grevholm and Kislenko [8] claim that “there are four sets of beliefs about mathematics:

- beliefs about the nature of mathematics,
- beliefs about teaching and learning of mathematics,
- beliefs about the self in context of mathematics teaching and learning,
- beliefs about the nature of knowledge and the process of knowing.”

Our interest has thus been oriented toward such perceptions of mathematics found with students in engineering courses. These students, engaged in the sciences, have nevertheless different positions, whether philosophical, practical or epistemological, towards mathematics.

The study thus investigates the following question:

“How far do the students’ perceptions of mathematics in engineering courses regarding the usefulness of mathematics in real life, the teaching of mathematics (contents and methods) and the nature of mathematics knowledge differ in terms of university, country (France, Finland, Russia, Georgia, Armenia), region (Caucasian, European, Russian) and gender (female, male)?”

1.3.3 Method and Procedures

Drawing on prior studies [16, 20, 38] related to students’ mathematics perceptions, and in particular the four sets of beliefs about mathematics suggested by Breiteig et al. [8], we have designed a questionnaire to gather and assess students’ perceptions of mathematics and their mathematics courses, and to get concrete indicators of their beliefs about the following:

1. Usefulness of mathematics.
2. Teaching of mathematics in engineering schools, its contents and methods.
3. Nature of mathematical knowledge.

Given the target audience, namely students in engineering courses, we decided not to address beliefs about ‘the self in context of mathematics teaching and

Table 1.1 Questionnaire dimensions and numbers of corresponding items

Questionnaire dimension	Number of questions
1. Usefulness of mathematics	8
2. Teaching of mathematics in engineering schools, contents and methods	15
3. Nature of mathematics knowledge	12
Total	35

Table 1.2 Items related to the dimension “usefulness of mathematics”

Item n	Usefulness of mathematics
1	Mathematics has an interest, especially for solving concrete problems.
2	Mathematics is used mostly in technical domains.
3	Mathematics is useless in everyday life.
4	Only applied mathematics is interesting.
5	Mathematics serves no purpose in human sciences.
6	Natural phenomena are too complex to be apprehended by mathematics.
7	Mathematics can be applied to man crafted objects and much less to objects found in nature.
8	Learning mathematics in early classes serves mostly the purpose to help children get around in life.

learning’ because this perception is not our focus here: mathematics cannot be avoided and has to be confronted with.

Based on the three above-mentioned dimensions of the questionnaire, we developed 35 questions that cover these dimensions as shown in Table 1.1.

The first dimension of the questionnaire (Table 1.2) explores the students’ beliefs about the usefulness of mathematics. Chaudhary [14] points out that mathematics is useful in everyday life: ‘since the very first day at the starting of the universe and existence of human beings, mathematics is a part of their lives’ (p. 75) and in some professions, such as architects who ‘should know how to compute loads for finding suitable materials in design’, advocates who ‘argue cases using logical lines of reasons; such skill is developed by high level mathematics courses’, biologists who ‘use statistics to count animals’, or computer programmers who develop software ‘by creating complicated sets of instructions with the use of mathematical logic skills’ (p. 76). In line with such a view of the utility of mathematics, we proposed eight items addressing the utility of mathematics in everyday life (items 1, 3, 8), as well as in technical (item 2), natural (items 6, 7) and human (item 5) sciences. Item 4 questions the perception of the usefulness of mathematics in relation with the distinction between pure and applied mathematics.

A higher education evaluation conducted in France in 2002 [6] focussed, among others, on the mathematics teaching in engineering schools. The study concludes that mathematics takes a reasonable place among the subject matters taught: the amount of mathematics courses is 16% of all courses in the first year of study, in the second year 10% and in the third one only 6%. Another result of this study

Table 1.3 Items related to the dimension “teaching mathematics in engineering schools”

Item n	Teaching mathematics in engineering schools
9	In engineering school, mathematics are mostly pure and abstract.
10	We need more applied mathematics in engineer training.
11	In engineering school, theory are taught without taking into account their applications.
12	There is almost no connection between math teaching and the engineer job reality.
13	Math teaching does not try to establish links with other sciences.
14	Mathematics weigh too much in engineer training.
15	Mathematics cannot be avoided in engineer training.
16	A teacher’s only purpose is to bring knowledge to students.
17	The structure of math courses does not allow learning autonomy.
18	With new means available to students, learning is no longer required; one just has to quickly find solutions to problems that are encountered.
19	Courses have not changed in the last decades when the world is evolving greatly and fast.
20	The mathematics courses are extremely theoretic.
21	The mathematics courses are not theoretical enough.
22	The mathematics courses are extremely practical.
23	The mathematics courses are not practical enough.

pointed out that engineering students are mostly taught basic mathematics and do not encounter enough applications. Based on these results, we wished to gather students’ perceptions about the teaching of mathematics they are given in their engineering schools. Thus, in the second dimension of the questionnaire (Table 1.3), we decided to address the students’ perceptions of the place mathematics takes in their engineering education (item 14 and 15), of the balance between theoretical and practical aspects of mathematics (items 9, 10, 11, 20, 21, 22, and 23) and of the links established between mathematics and other subject matters (item 13). In addition, we wanted to know whether the students feel that the mathematics teaching they receive prepares them for the workplace (item 12). Finally, a set of items addressed the students’ perceptions of the methods of teaching of mathematics (items 16, 17, 18, and 19).

The third dimension attempted to unveil the students’ implicit epistemology of mathematics, i.e. their perceptions of “what is the activity of mathematicians, in what sense it is a theoretical activity, what are its objects, what are its methods, and how this all integrates with a global vision of science including the natural sciences” [7]. The items related to this dimension (Table 1.4) addressed the relations of mathematics with reality (items 27 and 30), with the truth (items 28, 29 and 32), with other sciences (item 25), with creativity (items 24 and 26), and with models (item 31). Moreover, they questioned the students’ perceptions of the nature of mathematical objects (item 35) and the accessibility of mathematical knowledge (items 33 and 34).

Table 1.4 Items related to the dimension “perception of mathematics”

Item n	Perception of mathematics
24	In mathematics, there is nothing left to discover.
25	Mathematics are only a tool for science.
26	There is no room for creativity and imagination in mathematics.
27	Mathematics raised from concrete needs.
28	There is no room for uncertainty in mathematics.
29	Only math can approach truth.
30	Mathematics is only an abstraction, it does not deal with reality.
31	A mathematical model is necessarily limited.
32	A mathematical theory cannot be refuted.
33	Mathematics cannot be the subject of a conversation (contrarily to literature or philosophy).
34	Mathematics is better left to experts and initiated people.
35	Mathematics is a human construction.

Table 1.5 Participants of the study

Country	Number of students	Number of students with completed responses
Armenia	24	12
Finland	189	112
France	430	245
Georgia	285	179
Russia	612	410
Total	1548	958

We calculated the reliability coefficient (Cronbach’s alpha) by administering an online version of the questionnaire to a sample of 1548 students from all participant countries (see the Sample section). Students’ responses were analysed to calculate the scores of each student. The reliability coefficient (Cronbach’s alpha) of 0.79 is high enough to consider our questionnaire as a whole construct a reliable measuring tool.

The experimental validity of the questionnaire as an estimation of the tool validity is also calculated by taking the square root of the test reliability coefficient [3]. Its value of 0.89 shows that the questionnaire has a high experimental validity.

An operational definition of students’ perception of their engineering courses is therefore defined for us as a random variable taking vector values represented by the Likert score of the students on the 35 items of the prepared questionnaire on a 1–6 Likert-type scale.

The population on which we base this study are students from partner universities in two Tempus projects, MetaMath in Russia and MathGeAr in Georgia and Armenia, and French and Finnish students on the European side. Within this population a sample of 1548 students filled in the survey with 958 complete responses (incomplete surveys discarded)—see Table 1.5.

1.3.4 Data Analysis

After collecting the data from the online survey we used the statistical package R to analyse the data and draw preliminary conclusions. We performed a Principal Component Analysis (PCA) [27, 47, 54] to investigate patterns in the students’ responses. Although the students’ responses are not strictly speaking continuous but are a Likert scale between 1 and 6, Multiple Factor Analysis, where different Likert values are not numerically linked but used as simply ordered categories, did not yield finer results. PCA uses a vector space transform to reduce the dimensionality of large data sets giving some interpretation to variability. The original data set, which involves many variables, can often be interpreted by projecting it on a few variables (the principal components).

We used PCA to reveal patterns in students’ responses. Using the two first principal components, explaining almost a quarter of the variability, we identify the main common trends and the main differences. Principal components are described in Table 1.6 and Fig. 1.5. In particular, the main result is that we can verify the hypothesis that the methodology of teaching mathematics of each partner, and in particular each country, shapes the average students’ perception of mathematics.

During students’ interviews and study visits in the project, we could point out the main trends in the way mathematics is taught in the partner institutions. Thus it appeared that mathematics in Europe is taught as a sophisticated tool addressing

Table 1.6 Importance of the components

	PC1	PC2	PC3	PC4
Standard deviation	2.3179	1.69805	1.39846	1.2321
Proportion of variance	0.1535	0.0823	0.05588	0.0433
Cumulative proportion	0.1535	0.2358	0.29177	0.3351

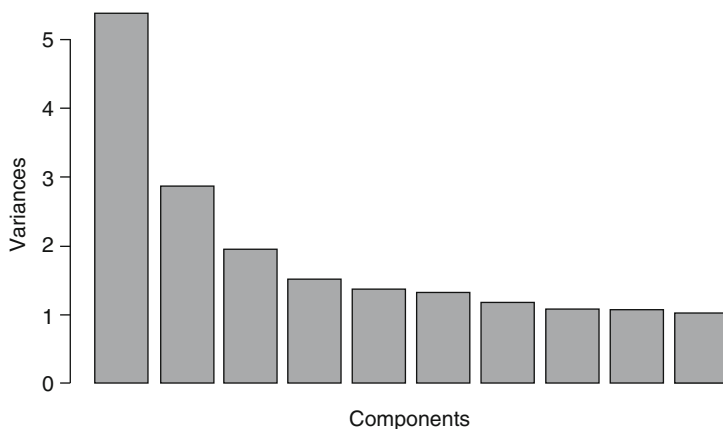


Fig. 1.5 Variance of the first 10 principal components

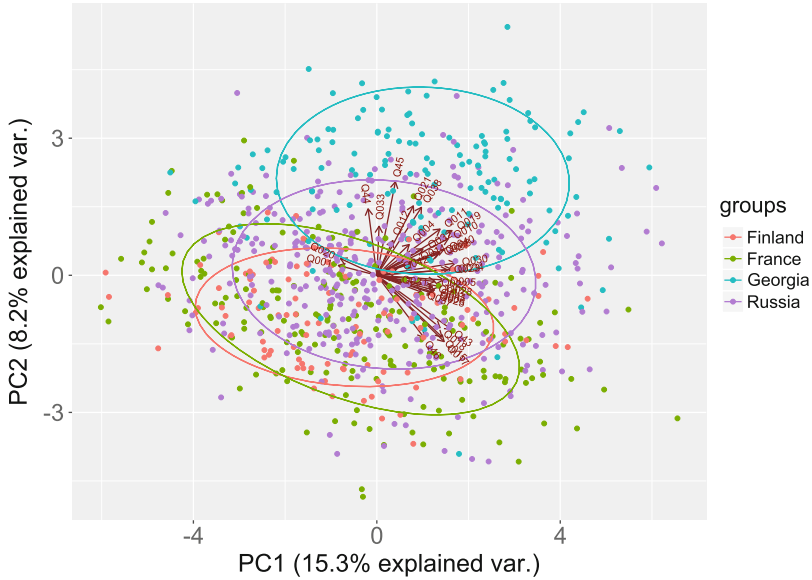


Fig. 1.6 PCA grouped by countries

real engineer's issues; it stands out with respect to a more theoretical approach in the East. This fact does show in the data.

The analysis shows that all engineering students responding to the questionnaire (15.2% of the variability) feel that mathematics teaching is too theoretical, is not practical enough and does not have enough connection with other sciences and the reality of an engineer's job. Therefore, modernised curricula for engineers should address these issues. On the other hand, we identify that Finnish and French students (Fig. 1.6) share most of their perceptions, while the Caucasian students notably differ from them, the Russian students lying in between with a broader variability even given their size. The semantic analysis of the second principal component (8.6% of variability) reveals that in the European universities, mathematics is taught as a tool to solve problems, that is to say, by practicing of applying mathematics to problems, while in the Caucasian universities, mathematics is taught focussing on theorems and proofs, that is to say, mathematics is an abstract subject matter. The Caucasian students tend to perceive of mathematics as consisting of knowledge rather than competencies, mainly of theoretical interest, with a discrepancy between early practical mathematics and theoretical engineering mathematics (Fig. 1.7).

The European students feel that advanced mathematics is useful, that the role of a teacher is more to help students to apply mathematics than to only transmit knowledge. The Russian students fall in between the two groups and are more diverse in their opinions [32, 33].

Apart from the country and the institution, which do explain a lot of the variability, we looked for characters separating students into groups in a statistically

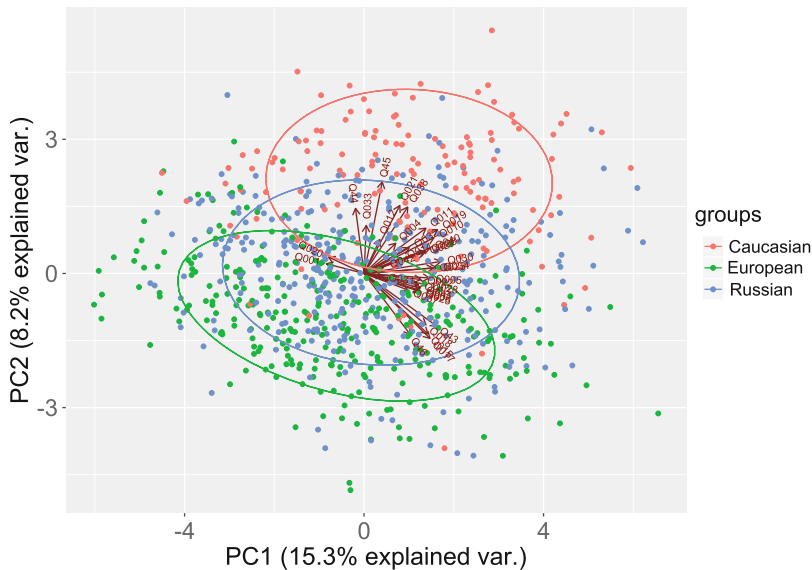


Fig. 1.7 PCA grouped by regions

significant way. In engineering courses, gender is a major differentiating trait [2, 28]. And, to our surprise, the partner’s institution explains much better the differences between students than gender: male and female students have very similar responses, only 6 out of 35 questions are statistically distinguishable (p -value < 0.05) and we have no clear-cut semantic explanation of the slight differences: male students tend to disagree a little bit more strongly to the proposal that mathematics can be applied more easily to man crafted objects than to objects found in nature, while female students tend to find slightly more that mathematics courses are practical enough. But the differences are much higher between partner institutions than between genders: there are statistically greater differences between the answers of a student in St Petersburg Electrotechnical University (LETI) and another in Ogarev Mordovia State University (OMSU), both in Russia (much lower p -values, with 16 out of 35 being less than 0.05) than between a male and a female student in each university (Figs. 1.8 and 1.9). And the differences are even higher between institutions belonging to different countries. We have to look at the seventh principal component, which is almost meaningless, in order to get a dimension whose interpretation of the variability relates to gender. The same relative irrelevance with respect to age appears: students’ perceptions depend on the year of study, but to an extent much lower than the dependency on the institution. We find these results remarkable.

The main finding of this analysis is that there are indeed great differences between students’ responses in partners’ higher education institutions, with homogeneous European universities tending to see engineering mathematics as a profes-

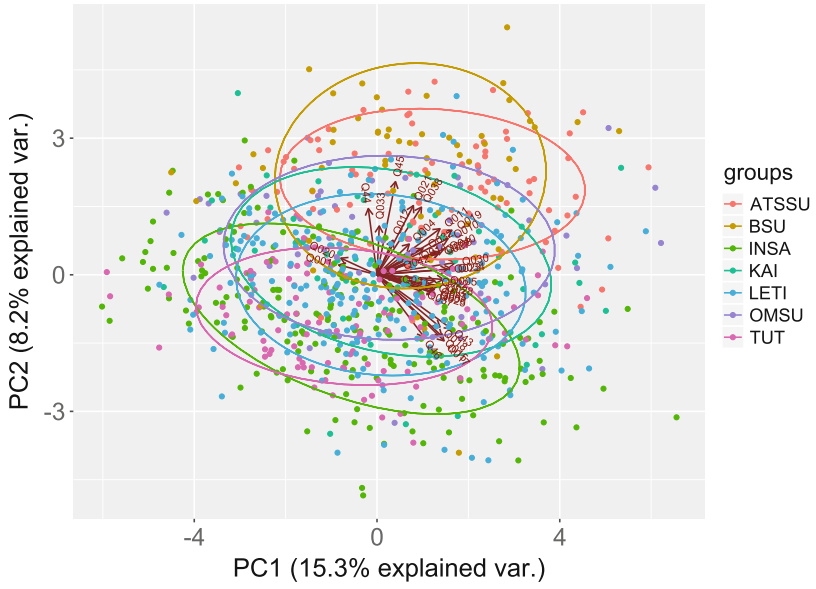


Fig. 1.8 PCA grouped by institutions



Fig. 1.9 PCA grouped by gender

sional tool on the one side; homogeneous Caucasian universities on the other, where advanced mathematics are felt as dealing with abstraction, and Russian universities in between.

1.3.5 Conclusions

In this study, we observed that European countries on the one hand and South Caucasian countries on the other are quite aligned. However, Russian students' perception is more spread out and in between those of the European and South Caucasian students. The country factor has a large influence but within these differences, institutions can be more finely differentiated and this difference is higher than most other criteria, like gender: a student can be linked to his/her university in a more confident way than to his/her gender or his/her year of study. Comparison with other institutions would be interesting.

The main implication for the MetaMath and MathGeAr projects from this study is that if the European way is to be promoted, the project should put forward the applications of advanced mathematics and focus on competencies rather than on transmission of knowledge.

This questionnaire has some limitations. For instance, its item-internal consistency reliability was not high enough regarding the three dimensions of the questionnaire which we identified a priori. The item-internal consistency reliabilities measured by Cronbach's alpha are 0.52, 0.65, and 0.62, which tells us that reality is more complex than our question choices based on epistemology. It evokes the need to further study to qualify the questionnaire with a bigger homogeneous sample and/or redesign of the current questionnaire by adding more items related to these dimensions or qualify these dimensions better.

Because perceiving mathematics in a positive way would influence students' motivation and performance, it is desirable to change the mathematics contents and the way we teach it in order to address the negative aspects of the perceptions identified here, for instance teaching mathematics as a powerful modelling tool not abstractly, but in actual students' projects.

We might as well try to directly modify students' perceptions by better informing them about some aspects of mathematics; its usefulness in engineer's profession for example. Therefore, we need to know which type of mathematics in-service engineers do use in a conscious way, and what their perceptions are of the mathematics they received during their education.

The current study suggests further investigation avenues: the first one is to study deeper the influence of engineering students' perceptions on mathematics performance for each partner institution. The second one is the elaboration of questionnaires targeting engineers in order to study the perceptions and actual usage of mathematics by professionals. Because the link between students and engineers goes through teachers, we need to study as well the perceptions of teachers themselves. We have already adapted this questionnaire in order to address these two

targets and it will be the subject of subsequent articles. This study is only the first real size pilot of a series of further studies to come.

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