

© 2017 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

# Coupling dynamic electromagnetic finite element models to circuit simulators by using model order reduction

Antero Marjamäki and Paavo Rasilio

Tampere University of Technology, Finland

**Abstract**— Two model reduction methods, proper orthogonal decomposition (POD) and discrete empirical interpolation method (DEIM) are applied to a finite-element model of a one-phase transformer. The resulting reduced order model is strongly coupled with a system simulator and the performance, feasibility and accuracy of the resulting system is studied. The results show that the workload of evaluating the finite element model drops drastically as the amount of degrees of freedom reduces by 99 % and the amount of involved elements reduces by 65 %. The accuracy of the results is feasible. The time to calculate a single timestep reduces on average 75 %. The full simulation time drops on average by 45 %. In this case the performance of the naive coupling is bad because it introduces two nested nonlinear solvers. This reduces the total speed up of the simulation.

## 1. INTRODUCTION

In the process of product development one often needs to create highly detailed models of electromagnetic devices to simulate the internal phenomena of the device. Many devices also work as a part of some larger system and system level simulation must be done to verify the external behaviour of the device. This is usually obtained by generating some lumped parameter circuit model of the device [1] by either coupling the field system directly into the circuit equations to achieve strong coupling [2] or then solving the two systems in parallel and exchanging data between them when needed [3].

Model reduction methods [4, 5, 6] offer a tool to extract the behaviour of the detailed system by creating a reduced order model which approximates the behaviour of the device accurately enough but is orders of magnitude lighter to evaluate. This enables us to couple the detailed model of the device directly into a system simulator to help investigate the behaviour of the device.

In this paper a new approach to couple electromagnetic field equations to circuit analysis is taken. Two model reduction methods, proper orthogonal decomposition (POD) and discrete empirical interpolation method (DEIM) are used to create a reduced order model. The created model is then strongly coupled to a system simulator to investigate the performance, feasibility and accuracy of the technique. The system investigated in this paper is a one-phase EI transformer which has a laminated core. Nonlinear behaviour and eddy currents are taken into account and the finite element (FE) model is formulated so that hysteresis can be taken into account later on.

## 2. FORMULATION OF THE FINITE ELEMENT SYSTEM

In the formulation of the electromagnetic field equations we account for nonlinear material and eddy currents. A low-frequency approximation for eddy currents in a laminated core which is described in [7] is used. The magnetic constitutive relation is assumed to be arbitrary. The technique of deriving an FE system with these assumptions is described in [8]. Many magnetic materials are hysteretic which is why we formulate the equations so that hysteresis can be accounted for in following studies. In this study we focus on the reduction techniques and the coupling between FE system and the circuit simulator.

We are using a two-dimensional magnetic vector potential formulation where the vector potential is  $\mathbf{A}(\mathbf{x}) = (0, 0, A_z(\mathbf{x}))$  and the magnetic flux density is given as  $\mathbf{B}(\mathbf{x}) = \text{curl}(\mathbf{A}(\mathbf{x}))$  which results in the following partial differential equation (PDE)

$$\text{curl} \left( \sigma \frac{L^2}{12} \frac{d}{dt} \text{curl}(\mathbf{A}(\mathbf{x})) + \mathbf{H}(\mathbf{A}(\mathbf{x}), \mathbf{x}) \right) = \mathbf{J}_s(\mathbf{x}, t), \quad (1)$$

where  $\sigma$  is the conductivity of the core material,  $L$  is the thickness of a single plate in the core,  $\mathbf{H}$  is the magnetic field strength and  $\mathbf{J}_s$  is the source current density in the coils. We are using a

Dirichlet boundary condition where  $\mathbf{A}(\mathbf{x}) = \mathbf{0}$  over the whole boundary  $\partial\Omega$ . In the rest of the paper we mostly omit the notation of the spatial dependency of the fields and functions for simplicity, i.e., we denote  $\mathbf{A}(\mathbf{x}) = \mathbf{A}$ .

In (1) the first term with the partial derivative with respect to time is the contribution of the eddy currents in the laminated core. The magnetic field strength  $\mathbf{H}$  is a function of the vector potential. In this form we don't assume anything about the  $\mathbf{B}$ - $\mathbf{H}$  relation.

The FE discretization for (1) results in a nonlinear ordinary differential equation (ODE) system. To derive the ODE system we follow the method of weighted residuals and in the finite-element derivations we use Petrov-Galerkin formulation. The ODE system can be given in matrix notation as

$$\mathbb{T} \frac{d}{dt} \mathbf{a} + \mathbf{f}(\mathbf{a}) = \mathbf{F}(t) , \quad (2)$$

where  $\mathbb{T}$  is referred to as a *damping matrix*,  $\mathbf{f}$  is referred to as the *nonlinear term* and  $\mathbf{F}$  is the *source term*. They can be calculated elementwise as

$$\mathbb{T}_{ij} = \int_{\Omega} \text{curl}(\mathbf{N}_i) \cdot \sigma \frac{L^2}{12} \text{curl}(\mathbf{N}_j) d\Omega \quad (3)$$

$$\mathbf{f}_i(\mathbf{a}) = \int_{\Omega} \text{curl}(\mathbf{N}_i) \cdot \mathbf{H}(\mathbf{a}) d\Omega \quad (4)$$

$$\mathbf{F}_i(t) = \int_{\Omega} \text{curl}(\mathbf{N}_i) \cdot \mathbf{J}_s(t) d\Omega , \quad (5)$$

where  $\mathbf{N}_i$  are the finite element shape functions. The equation system (2) is solved using a Backward-Euler time integration scheme and Newton-Raphson (NR) method. For the NR method we need to derive the residual term and the Jacobian. After time discretization the residual term is

$$\mathbf{R}(\mathbf{a}_t) = \mathbb{T} \frac{\mathbf{a}_t - \mathbf{a}_{t-1}}{\Delta t} + \mathbf{f}(\mathbf{a}_t) - \mathbf{F}_t , \quad (6)$$

where  $\Delta t$  is the length of the time step. The Jacobian is calculated based on the residual as

$$\mathbf{J}(\mathbf{a}) = \frac{d}{d\mathbf{a}} \mathbf{R}(\mathbf{a}) = \frac{1}{\Delta t} \mathbb{T} + \mathbf{J}_f(\mathbf{a}_t) , \quad (7)$$

where the Jacobian of the nonlinear term  $\mathbf{J}_f$  is calculated componentwise as

$$(\mathbf{J}_f(\mathbf{a}))_{ij} = \int_{\Omega} \text{curl}(\mathbf{N}_i) \cdot \frac{\partial \mathbf{B}}{\partial \mathbf{H}}(\mathbf{a}) \text{curl}(\mathbf{N}_j) d\Omega . \quad (8)$$

The term  $\frac{\partial \mathbf{B}}{\partial \mathbf{H}}$  is called *differential reluctivity tensor* and it can be computed based on the constitutive model. It is represented here by a  $2 \times 2$  matrix valued function of the magnetic vector potential.

### 3. APPLYING THE REDUCTION TECHNIQUES

Proper orthogonal decomposition (POD) and discrete empirical interpolation method (DEIM) are applied to the equation system derived in the previous section. POD [5, 9] is originally based on principal component analysis. The idea behind POD is to capture snapshots out of the state of the system, arrange them into a snapshot matrix and use singular value decomposition (SVD) to capture the largest eigenvalues and corresponding eigenvectors of the matrix. The eigenvectors are used to form a projection mapping which reduces the state vector to some subspace whose dimension is much smaller than the dimension of the full state space.

For time dependent systems as (2) the snapshots are taken at different time instances from the state vector  $\mathbf{a}$ . We will denote the snapshot matrix with  $\mathbf{S} = [\mathbf{a}_t^\mu]$  and its column vectors are the state vector values at different time instances  $t$  and different *parameter configurations*  $\mu$ .

The model being reduced needs to be parametrized in those functioning domains where we want to have a good approximation of the behaviour of the system. For example in our case of a dynamic model of a transformer the parameters are chosen so that primary and secondary current frequency, amplitudes and phases are embedded as changeable parameters. When the reduced order model is

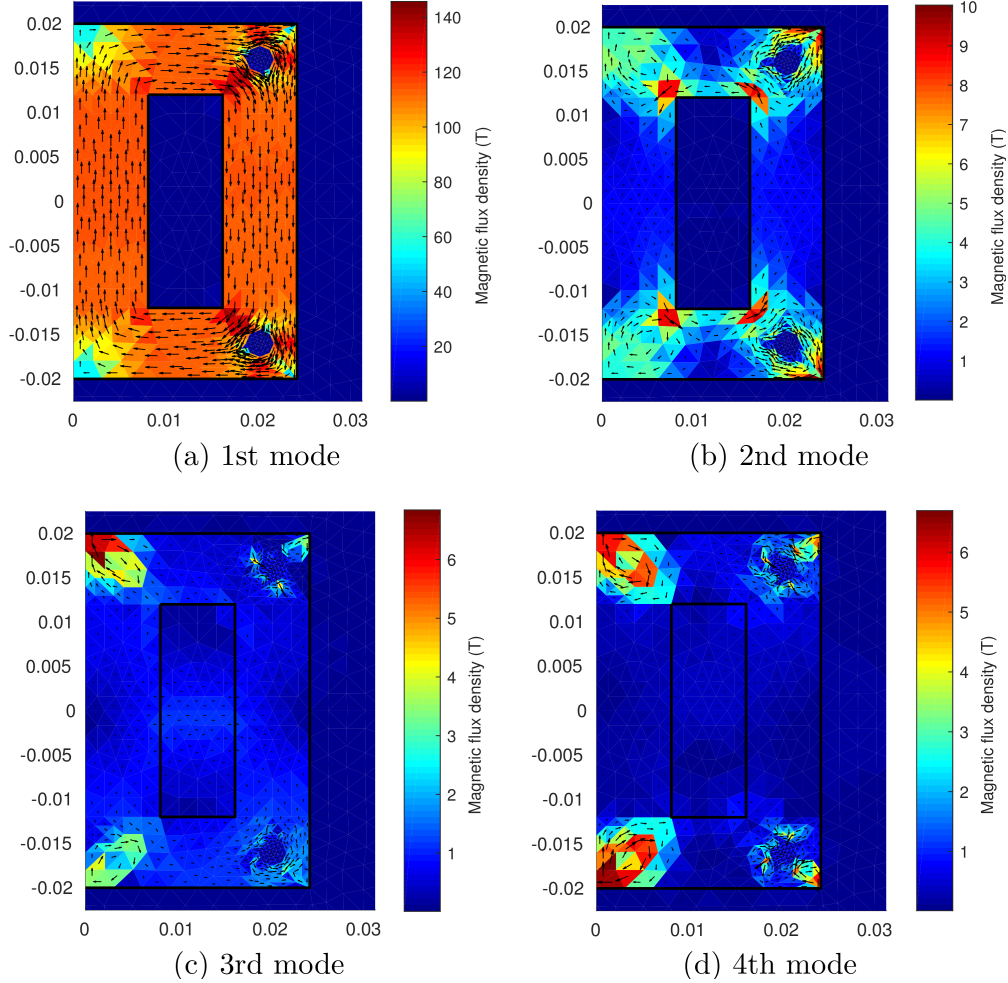


Figure 1: The flux densities  $\mathbf{B}$  calculated from the four POD modes for vector potential  $\mathbf{A}$  of the system. All solutions of the reduced model are linear combinations of these modes.

made it will then give approximately same results as the full model with the same parameters and inputs.

SVD decomposes the snapshot matrix  $\mathbf{S}$  into a product of three matrices

$$\mathbf{S} = \mathbf{V}^T \mathbf{\Sigma} \mathbf{W}, \quad (9)$$

where  $\mathbf{V}$  contains the left singular vectors,  $\mathbf{W}$  contains the right singular vectors and  $\mathbf{\Sigma}$  is a diagonal matrix which contains the singular values of  $\mathbf{S}$  in its diagonal ordered from the largest to the smallest.

The projection operator  $\Psi : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a  $m \times n$  matrix where  $n$  is the original dimension and  $m$  the reduced dimension. It is formed from the rows of the product  $\mathbf{V}^T \mathbf{\Sigma}$ . To form a sufficiently accurate reduced model we only need to pick the singular vectors that correspond to the most dominant singular values. The tolerance limit affects the dimension  $m$  of the reduced space.

The cut-off criterion suggested in [10] is used to determine the required amount of singular values. First we pick an error tolerance parameter  $\epsilon$  which will be the upper bound for the  $L_2$  error norm of the POD solution. Then we can solve the index  $l$  of the smallest singular value from

$$\sum_{i=1+l}^k \sigma_i^2 < \epsilon, \quad (10)$$

where  $k$  is the total amount of singular values. Each column of  $\Psi$  can be thought of as a POD mode and the solution of the reduced order model will be a linear combination of all POD modes which are taken into account. Figure 1 shows all four POD modes of the reduced system produced for the EI transformer used as an example.

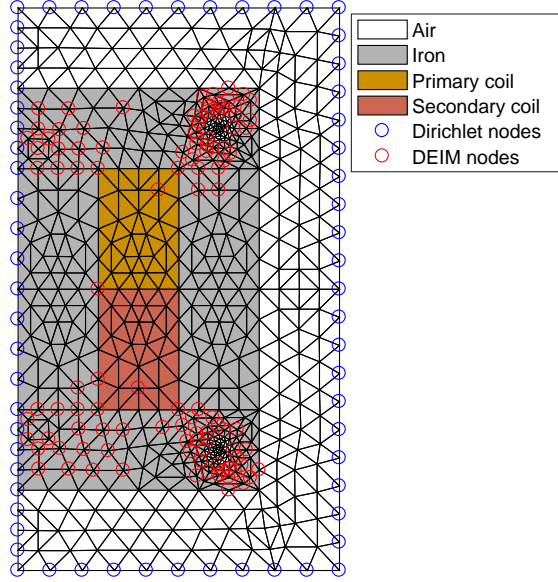


Figure 2: The FE mesh and the significant nodes picked by DEIM algorithm. As can be seen the significant nodes are focused inside the nonlinear core in corners and near the attachment holes of the plate.

DEIM [6] focuses to reduce the workload needed to evaluate vector valued functions. It uses a POD basis to pick the most significant elements of the vector and creates interpolation coefficients to interpolate the less significant ones. As a result only a small number of actual elements of the vector function needs to be calculated and the rest can be evaluated using a matrix product.

A POD basis for the nonlinear term in equation (2) is produced similarly as for the state vector. After this the DEIM algorithm is used to pick the most significant indices. These indices correspond to nodes of the finite element mesh. The DEIM nodes of the example transformer are presented in Figure 2. The results of POD-DEIM procedure are the POD projection matrix  $\mathbf{U}$  and selector matrix  $\mathbf{P}$ . With these mappings the nonlinear term  $\mathbf{f}$  can be approximated as

$$\mathbf{f}(\mathbf{a}) \approx \mathbf{U}(\mathbf{P}^T \mathbf{U})^{-1} \mathbf{P}^T \mathbf{f}(\mathbf{a}) . \quad (11)$$

The key thing to note here is that multiplying with  $\mathbf{P}$  just picks the most significant indices of  $\mathbf{f}$  and the product  $\mathbf{P}^T \mathbf{f}(\mathbf{a})$  can be evaluated by only evaluating a subset of the elements of  $\mathbf{f}$ . Term  $\mathbf{U}(\mathbf{P}^T \mathbf{U})^{-1}$  can be precomputed only once and reused.

When both methods are applied to (2) we end up with a reduced system. First we replace the state with  $\mathbf{a} = \Psi \tilde{\mathbf{a}}$  and then multiply the equation from left with  $\Psi^T$ . Furthermore we insert the DEIM approximation of  $\mathbf{f}$  and obtain

$$\Psi^T \mathbf{T} \Psi \frac{\partial}{\partial t} \tilde{\mathbf{a}} + \Psi^T \mathbf{U} (\mathbf{P}^T \mathbf{U})^{-1} \mathbf{P}^T \mathbf{f}(\Psi \tilde{\mathbf{a}}) = \Psi^T \mathbf{F} . \quad (12)$$

In (12) the dimension of the state space and the workload to evaluate  $\mathbf{f}$  has reduced greatly. We only need to calculate those components of  $\mathbf{f}$  which correspond to DEIM nodes. Furthermore in NR method the dimension of the residual term and Jacobian matrix have reduced resulting in a significant drop in amount of work when solving the next iterate. The reduced versions of the residual in (6) is

$$\tilde{\mathbf{R}}(\tilde{\mathbf{a}}_t) = \Psi^T \mathbf{T} \Psi \frac{\tilde{\mathbf{a}}_t - \tilde{\mathbf{a}}_{t-1}}{\Delta t} + \Psi^T \mathbf{U} (\mathbf{P}^T \mathbf{U})^{-1} \mathbf{P}^T \mathbf{f}(\Psi \tilde{\mathbf{a}}_t) - \Psi^T \mathbf{F}_t , \quad (13)$$

and the reduced Jacobian is

$$\tilde{\mathbf{J}}(\tilde{\mathbf{a}}) = \Psi^T \mathbf{U} (\mathbf{P}^T \mathbf{U})^{-1} \mathbf{P}^T \mathbf{J}_f(\Psi \tilde{\mathbf{a}}) \Psi . \quad (14)$$

In (14) it is noteworthy that evaluating the matrix product  $\mathbf{P}^T \mathbf{J}_f(\Psi \tilde{\mathbf{a}})$  corresponds to picking only those rows of the Jacobian which correspond to DEIM nodes. The dropped rows don't have to be computed.

When the reduced nonlinear term and the Jacobian are evaluated the full state vector is not required. We only need to project the nodal values of all nodes which belong to elements which contain at least one DEIM node. It is only required to pick the necessary components from  $\Psi\tilde{\mathbf{a}}$ . This can be achieved by selecting only those rows from  $\Psi$  in the multiplication which correspond to DEIM nodes and the nodes which are involved in elements containing DEIM nodes.

#### 4. CASE STUDY: ONE-PHASE TRANSFORMER

The theory presented in this paper was applied to a single-phase EI transformer. The transformer was modelled so that eddy currents and nonlinear core effects were taken into account. A nonlinear FE solver was developed using MathWorks Matlab software. The model was reduced and coupled with a simple circuit presented in Figure 3 modelled with MathWorks Simscape circuit simulator.

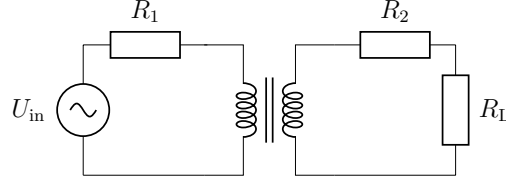


Figure 3: Simple circuit used to benchmark the coupling between FE solver and the simulator. The transformer was replaced with the reduced order model derived from FE analysis.  $R_1$  and  $R_2$  are the resistances of the primary and secondary coil respectively.  $U_{in}$  is the feeding voltage and  $R_L$  the load resistance.

The time-discretized equation system for the voltage controlled full order model is

$$\begin{cases} R_1 i_1 + \frac{1}{\Delta t} C_1 (\mathbf{a} - \mathbf{a}_p) - U_{in}(t) = 0 & (15) \\ \frac{1}{\Delta t} C_2 (\mathbf{a} - \mathbf{a}_p) + (R_2 + R_L) i_2 = 0 & (16) \\ \frac{1}{\Delta t} T (\mathbf{a} - \mathbf{a}_p) + \mathbf{f}(\mathbf{a}) - D_1 i_1 - D_2 i_2 = 0, & (17) \end{cases}$$

where  $U_{in}$  is the input voltage,  $i_1$  and  $i_2$  are the currents of the coils,  $R_1$ ,  $R_2$  are the coil resistances,  $R_L$  is the load resistance,  $\mathbf{a}_p$  is the solution of the previous timestep,  $C_1$ ,  $C_2$  are matrices which are used to calculate the integral of the flux through the coils in the core and  $D_1$ ,  $D_2$  are matrices which divide the currents of the coils evenly to the coil area of the domain.

During the benchmark calculations the developed FE solver class is used to calculate the induced voltages from the input currents. Therefore the system of equations which needs to be solved is

$$\begin{cases} R_1 i_1 + u_1(i_1, i_2) - U_{in}(t) = 0 & (18) \\ u_2(i_1, i_2) + (R_2 + R_L) i_2 = 0, & (19) \end{cases}$$

where  $u_1$  and  $u_2$  are the induced voltages over the primary coil and secondary coil respectively given by the solver class. This equation system was solved with the full model at each timestep using Matlab's `fsolve` function. This made the model of the transformer work like a black box and made it possible to easily switch between the reduced order model and the full order model to compare results. Later the results were compared to the actual Simscape model coupled with the reduced FE model.

The Simscape model of the benchmark circuit is presented in Figure 4. The transformer was formulated as a subsystem presented in Figure 5 with primary coil terminals and secondary coil terminals. The subsystem contains the connections between the physical signals of Simscape and Simulink models. The actual transformer model connected to the Simulink environment was implemented using a Matlab System block which lets the user to implement a customized class with inputs and outputs.

The drawback of this approach is that we end up with bad performance as Simscape uses a nonlinear solver to solve the circuit equations and the solver uses another nonlinear solver to solve the field equations in each timestep. Although the reduced order model is lightweight to solve having two nested nonlinear solvers will impact performance negatively.

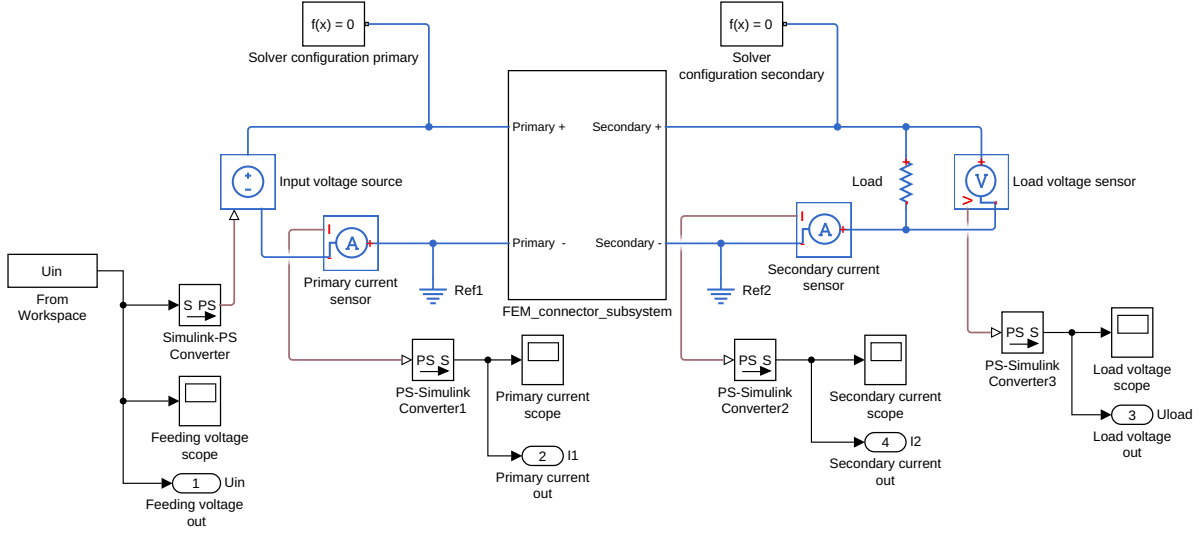


Figure 4: The main circuit implemented with Simscape. The primary side circuit with voltage source on the left and the secondary side circuit with load resistance on the right. The subsystem in the middle is presented in Figure 5.

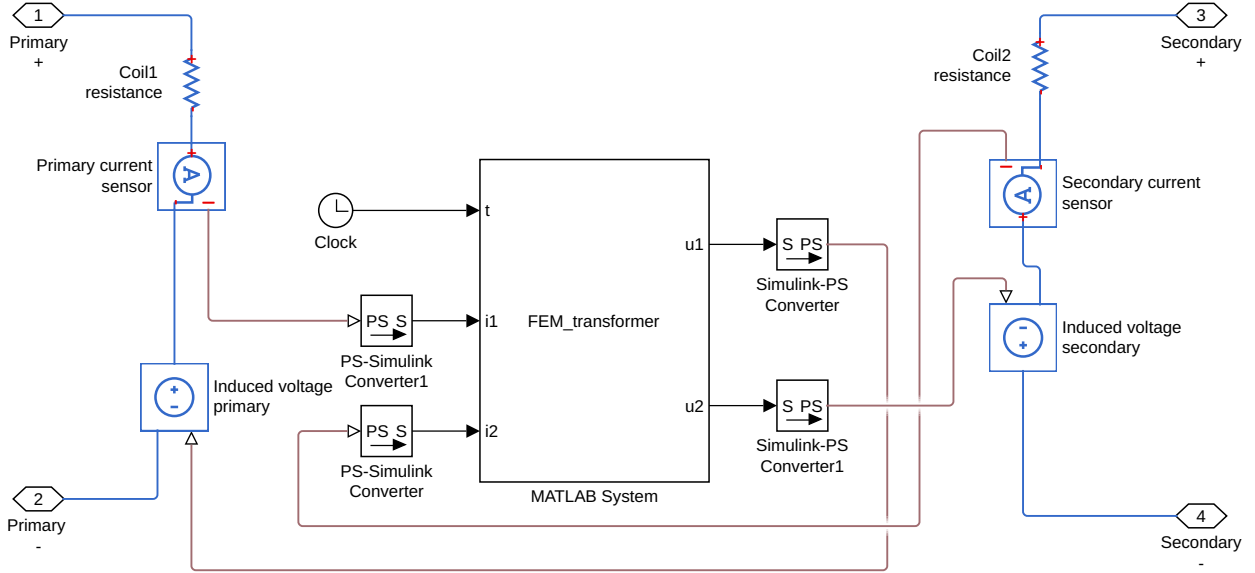


Figure 5: The subsystem contains a Matlab system block which calculates the reduced FE solution for each timestep.

## 5. NUMERICAL RESULTS AND DISCUSSION

The generated reduced order model was benchmarked using a simple circuit into which the transformer was inserted. The circuit consists of a feeding voltage source and a load resistor. Numerous simulations were made with different kinds of feeding voltages and load resistances. The speed of the calculation was benchmarked at the timestep level and at the system level over multiple periods of the feeding voltage.

The reduction in state space dimension was over 99% from 593 degrees of freedom to 4 degrees of freedom. The amount of indices to calculate from the nonlinear term  $\mathbf{f}$  reduced from 593 to 109 which is a reduction of 82%. Furthermore to evaluate these indices it was required to incorporate only 389 elements out of original 1120 elements.

Some convergence issues were detected when different reduced order model instances were tested. It was observed that if the number of DEIM nodes is too small the NR method used to solve the equations ceases to converge. Decreasing the time step size fixed this problem in some cases. The

issue seemed also to be related to the parameter configurations of the captured snapshots. Different model instances with the same reduced dimension and DEIM node amount behaved differently. The convergence, accuracy and optimality of the reduced order models which was generated needs further investigation.

The behaviour of the transformer was most nonlinear when the transformer was idling with a low frequency feeding voltage. Increasing both the frequency or decreasing the load resistance causes the flux density in the core to decrease. The largest errors in the results of the reduced model are present in the nonlinear region. Figure 6 shows the waveforms calculated with both full and reduced models when the input voltage is sinusoidal and pushes the core into nonlinear region and the transformer is idling. The amplitude of the feeding voltage was  $\sqrt{2} \cdot 24$  V with the frequency of 50 Hz. A small error can be seen in the primary current  $I_1$  in Figure 6. The curves of the load voltage are basically overlapping.

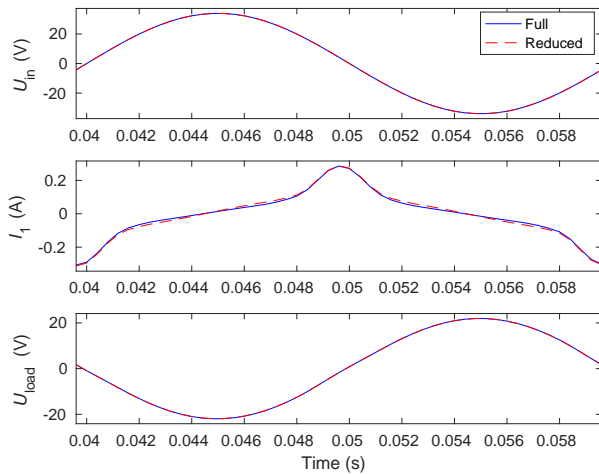


Figure 6: The final period of the input voltage, primary current and output voltage when the transformer is idling.

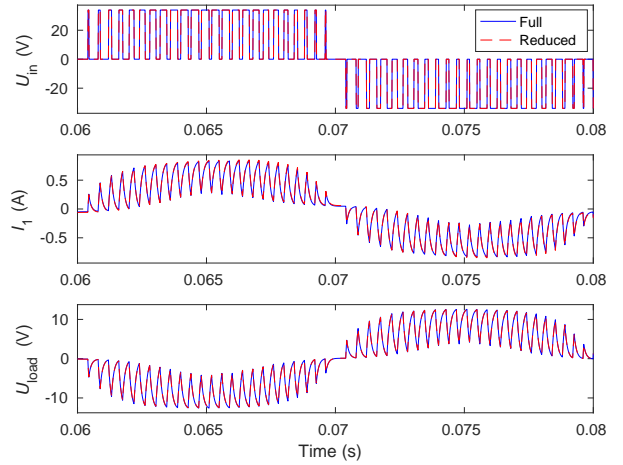


Figure 7: The final period of the input voltage, primary current and output voltage. Transformer was fed with a PWM signal.

To demonstrate the limits of the generated reduced model it was used to simulate the output voltage of a PWM modulated feeding voltage in Figure 7. The feeding voltage amplitude is  $\sqrt{2} \cdot 24$  V and frequency is 50 Hz. Carrier wave frequency was 2.5 kHz. The value of the load resistance was  $10 \Omega$ . The reduced order model was generated using sinusoidal currents as inputs to primary and secondary coils. Still the model can deliver results with feasible accuracy even with extreme voltage waveforms. The curves in both plots  $I_1$  and  $U_{load}$  in Figure 7 are overlapping.

The calculation times were compared with a moderate laptop computer running Matlab. The side effects, for example memory allocations, were not factored out from these times as they are a part of the real world computation anyway. It is clear from the reduction rates of the degrees of freedom and involved elements that the workload of the calculation has reduced greatly. The only difference in a reduced order model run in addition to a full model run was to pick the correct elements corresponding the DEIM nodes from necessary terms by using Matlab logical and linear indexing.

The mean value of the time it took to compute a single timestep using the full model was 12.6 ms. The time for the reduced order model was 3.1 ms so the single step time reduced 75 %. A full simulation run of three periods with 50 timesteps per period took in average 186 seconds from the full model and 102 seconds from the reduced order model. The full simulation time reduced 44 %.

The speed up rates for a single timestep are promising but for both full and reduced simulation run the overhead of the current Simulink coupling causes bad performance. The nested nonlinear solvers cause unnecessary many calls to the FE solver. We believe that the speed of computations can be improved by optimizing the coupling between the systems. The difference will also become more significant if the full order model contains more degrees of freedom.



## 6. CONCLUSION

The model reduction methods POD and DEIM seem to be well applicable to nonlinear dynamic electromagnetic problems. Furthermore they can be used to strongly couple an FE model into a circuit simulator. The performance for this particular setup was poor. The overall speed up compared to full model case was only 45% but the accuracy of the reduced order model seems good and the reduction rates were good, 99% for the state and 82% for the nonlinear term. Furthermore a reduced order system like this could be plugged in any circuit simulator capable of nonlinear circuit analysis which allows user to derive a custom component with relatively low effort. Therefore we can conclude that model reduction is a promising technique to be applied to couple FE models with circuit models.

## ACKNOWLEDGMENT

The foundation of Emil Aaltonen is acknowledged for financial support.

## REFERENCES

1. Lange, E. and Henrotte, F. and Hameyer, K., "An efficient field-circuit coupling based on a temporary linearization of fe electrical machine models," *IEEE Transactions on Magnetics*, vol. 45, No. 3, 1258–1261, March 2009.
2. Melgoza, E. and Venegas, V. and Escarela-Perez, R. and Guardado, J. L. and Hernandez, M., "Strong coupling of circuit and field solvers for simulation of rotating electrical machines," *The XIX International Conference on Electrical Machines - ICEM 2010*, 1–6, September 2010.
3. Gausling K. and Bartel A., "Coupling Interfaces and Their Impact in Field/Circuit Co-Simulation," *IEEE Transactions on Magnetics*, vol. 52, No. 3, 1–4, March 2016.
4. Benner, P. and Gugercin, S. and Willcox, K., "A survey of projection-based model reduction methods for parametric dynamical systems," *SIAM Review*, vol. 57, No. 4, 483–531, November 2015.
5. Henneron, T. and S. Clénet, "Model-order reduction of multiple-input nonlinear systems based on POD and DEI methods," *IEEE Transactions on Magnetics*, Vol. 51, No. 3, 2015.
6. Chaturantabut, S. and D. C. Sorensen, "Nonlinear Model Reduction via Discrete Empirical Interpolation," *SIAM Journal on Scientific Computing*, Vol. 32, No. 5, 2737–2764, 2010.
7. Knight, A. M. and Salmon, J. C. and Ewanchuk, J., "Integration of a First Order Eddy Current Approximation With 2D FEA for Prediction of PWM Harmonic Losses in Electrical Machines," *IEEE Transactions on Magnetics*, vol. 49, No. 5, 1957–1960, May 2013.
8. Gyselinck, J. and Dular, P. and Geuzaine, C. and Legros, W. "Harmonic-balance finite-element modeling of electromagnetic devices: a novel approach," *IEEE Transactions on Magnetics*, Vol. 38, No. 2, 521–524, Mar 2002.
9. Paquay, Y., O. Bruls, and C. Geuzaine, "Nonlinear Interpolation on Manifold of Reduced-Order Models in Magnetodynamic Problems," *IEEE Transactions on Magnetics*, Vol. 52, No. 103 18–21, 2016.
10. Farzamfar, M. and Belahcen, A. and Rasilo, P. and Clnet, S. and Pierquin, A., "Model Order Reduction of Electrical Machines With Multiple Inputs," *IEEE Transactions on Industry Applications*, Vol. 53, No. 4, 3355–3360, July-Aug. 2017.